Semileptonic Form Factors for $B_s \rightarrow K \ell \nu$ decays

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31$^{\text{st}}$ July 2023
Lattice 2023

Phys. Rev. D 107, 114512 (2023)
arXiv:2303.11280
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Motivation

- $b$-physics continues to play an important role in the search for new physics at the precision frontier
- Large $m_b$ allows us to probe high energy scales
- Physical applications include
  - Shape of QCD form factors
  - CKM matrix elements
  - Lepton flavour universality tests

Goal

- $V_{ub}$ enters the $B_s \to K\ell\nu$ differential decay rate:

$$\frac{d\Gamma(B_s \to K\ell\nu)}{dq^2} \bigg|_{\text{Experiment}} = \left| V_{ub} \right|^2 \times \left( \kappa_1 |f_+(q^2)|^2 + \kappa_2 |f_0(q^2)|^2 \right)$$

- $q^\mu = p_B^\mu - p_K^\mu$
- $\kappa$ — Known factors

- Form factors require non-perturbative computation:

$$\langle K(\vec{p}_K) | \mathcal{V}^\mu | B_s(\vec{p}_{B_s}) \rangle = 2f_+(q^2) \left( p_B^\mu - \frac{p_{B_s} \cdot q}{q^2} q^\mu \right) + f_0(q^2) \left( \frac{M_B^2 - M_K^2}{q^2} q^\mu \right)$$

$$\mathcal{V}^\mu = \bar{u} \gamma^\mu b$$
Relativistic Heavy Quark (RHQ) action for $b$ quarks


- Builds on original Fermilab action [El-Khadra et al. PRD 55 (1997) 3933]
- Anisotropic clover action
- Uses 3 parameters ($m_0a,c_p,\zeta$) that can be non-pertubatively tuned to remove $O((m_0a)^n)$, $O((p a)(m_0a)^n)$ errors [Aoki et al. PRD 86 (2012) 116003]

Shamir Domain-Wall Fermions (DWF) for $l,s$


Relate continuum and lattice currents via renormalisation constant [El-Khadra et al. PRD 64 (2001) 014502]

$$\langle K|\nu_\mu|B_s\rangle = Z_{bl}^{\nu}\langle K|V_\mu|B_s\rangle; \quad Z_{bl}^{\nu} = \rho_{bl}^{\nu} \sqrt{Z_{bb}^{\nu}Z_{ll}^{\nu}}$$

- $O(a)$-improved $V_\mu$ at one-loop
**Ensembles**

<table>
<thead>
<tr>
<th></th>
<th>$L^3 \times T / a^4$</th>
<th>$a^{-1} / \text{GeV}$</th>
<th>$m_\pi / \text{MeV}$</th>
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<tbody>
<tr>
<td>C1</td>
<td>$24^3 \times 64$</td>
<td>1.78</td>
<td>340</td>
</tr>
<tr>
<td>C2</td>
<td>$24^3 \times 64$</td>
<td>1.78</td>
<td>430</td>
</tr>
<tr>
<td>M1</td>
<td>$32^3 \times 64$</td>
<td>2.38</td>
<td>300</td>
</tr>
<tr>
<td>M2</td>
<td>$32^3 \times 64$</td>
<td>2.38</td>
<td>360</td>
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<tr>
<td>M3</td>
<td>$32^3 \times 64$</td>
<td>2.38</td>
<td>410</td>
</tr>
<tr>
<td>F1S</td>
<td>$48^3 \times 96$</td>
<td>2.79</td>
<td>270</td>
</tr>
<tr>
<td>C0*</td>
<td>$48^3 \times 96$</td>
<td>1.73</td>
<td>139</td>
</tr>
</tbody>
</table>

- 2+1f ensembles: degenerate light quarks
- Domain-wall fermions and Iwasaki gauge action
- **F1S ensemble**: new for this analysis
  - *C0 currently under analysis.
- Addition of C0 disentangles chiral and continuum effects
Form Factor Fits

- For lattice data in the $B_s$-meson rest frame, easier to decompose matrix elements as
  \[ f_{\parallel} = \frac{\langle K|V^0|B_s \rangle}{\sqrt{2M_{B_s}}} , \quad f_{\perp} p^i = \frac{\langle K|V^i|B_s \rangle}{\sqrt{2M_{B_s}}} \]
- Neatly separates into spatial and temporal components
- Linear combination recovers $f_0$ and $f_+$:
  \[ f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}} \left[ f_{\parallel}(q^2) + (M_{B_s} - E_K)f_{\perp}(q^2) \right] \]
  \[ f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 + E_K^2} \left[ (M_{B_s} - E_K)f_{\parallel}(q^2) + (E_K^2 - M_K^2)f_{\perp}(q^2) \right] \]
Simultaneously fit two-point functions and three-point function ratios on each ensemble over all momenta

\[
\frac{C_3^\mu(t)}{\sqrt{C_2^K(t)C_2^{Bs}(t_{\text{sink}} - t)}} = \langle K|V^{\mu}|B_s \rangle \sqrt{\frac{4E_K E_{Bs}}{e^{-tE_K} e^{-(t_{\text{sink}} - t)E_{Bs}}}}
\]

+ excited state contrib.

Use lattice dispersion relation to constrain kaon energies in three-point fits
Form factor fits on F1S ensemble
Chiral-Continuum Fits

- Extrapolate to physical kaon mass and zero lattice spacing simultaneously
- Use NLO hard-pion (kaon) SU(2) HMχPT [PRD 67 (2003) 054010]

\[ f(M_{\pi}^{\text{sim}}, E_K, a, L) = \frac{\Lambda}{E_K + \Delta_{\text{pole}}} \times \left( c^{(0)} \times (1 + \text{chiral log}) + c^{(1)} \frac{\Delta M_{\pi}^2}{\Lambda^2} + c^{(2)} \frac{E_K}{\Lambda} + c^{(3)} \left( \frac{E_K}{\Lambda} \right)^2 + c^{(4)} \left( a\Lambda \right)^2 \right) \]

\[ \Delta M_{\pi}^2 = (M_{\pi}^{\text{sim}})^2 - (M_{\pi}^{\text{phys}})^2 \]

\[ \Delta_{\text{pole, 0}} = M_{B^*(0^+)} - M_{B_s} \approx 263 \text{ MeV} \]

\[ \Delta_{\text{pole, +}} = M_{B^*(1^-)} - M_{B_s} \approx -42.1 \text{ MeV} \]
Extrapolation in terms of $f_+, f_0$ and $f_\perp, f_\parallel$

- Choice of chiral-continuum extrapolation strategy:
  - Take continuum limit of $f_+, f_0$ constructed from $f_\perp, f_\parallel$
  - Reconstruct $f_+, f_0$ from continuum-limit $f_\perp, f_\parallel$

- Latter strategy assumes $f_\perp, f_\parallel$ continuum limit is described well by $f_+, f_0$ pole energies

- **Significant difference** between the two strategies for $f_0$

- **Kinematic constraint at** $q^2 = 0$ **couples** $f_+$ **and** $f_0$ - influences $q^2$ extrapolation of both form factors!
Chiral-Continuum Fits

- $E_K^2$ term unresolved for $f_+$ and dropped
- Continuum form factor given by $f(M_{\pi}^p, E_K, a = 0, L \to \infty)$
- Variations on the continuum fit ansatz to assess systematic errors
Bayesian-inferential $z$-expansion

- Extrapolate over full $q^2$ range using a $z$-expansion
- Fit to synthetic data at reference $q^2$ points
- Limited data points restricts available number of terms in frequentist $z$-expansion
- Adopt a Bayesian strategy that can easily explore truncation errors for the BGL parameterisation


- **Details in Andreas Jüttner’s talk in this session at 14:30!**

![Graphs showing $f_X(q^2)$ and $f_0(q^2)$](image-url)
 observables - $|V_{ub}|$

- **Combine experimental inputs:**
  - $R_{BF} = \frac{\mathcal{B}(B_s^0 \rightarrow K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \rightarrow D^- \mu^+ \nu_\mu)}$ [LHCb PRL 126 (2021) 081804]
  - $\mathcal{B}(B_s^0 \rightarrow D^{-} \mu^+ \nu_\mu)$ [LHCb PRD 101 (2020) 072004]
  - $B_s^0$ lifetime $\tau_{B_s^0}$ [PDG PTEP 2022 (2022) 083C01] [HFLAV arXiv:2206.07501]

- **Lattice contribution:** Reduced decay rate $\Gamma_0 = \Gamma / |V_{ub}|^2$

$$|V_{ub}| = \sqrt{\frac{R_{BF} \mathcal{B}(B_s^0 \rightarrow D^- \mu^+ \nu_\mu)}{\tau_{B_s^0} \Gamma_0(B_s \rightarrow K\ell\nu)}}$$

- $|V_{ub}|_{RBC-UKQCD 2023}$ exclusive, $B_s \rightarrow K\ell\nu = 3.78(61) \times 10^{-3}$ [PRD 107 (2023) 114512]
- $|V_{ub}|_{FLAG 2021}$ exclusive, $B \rightarrow \pi \ell\nu = 3.74(17) \times 10^{-3}$ [FLAG EPJC 82 (2022) 869]
- $|V_{ub}|_{PDG 2022}$ inclusive, $B$ decays $= 4.13(26) \times 10^{-3}$ [PDG PTEP 2022 (2022) 083C01]

- Consistent with both exclusive and inclusive averages
Observables - LFU-testing ratios

- Standard R-ratio takes the form

\[ R_{B_s \to K} = \frac{\int_{m^2_\tau}^{q^2_{\text{max}}} dq^2 \frac{d\Gamma(B_s \to K\tau\nu_\tau)}{dq^2}}{\int_{m^2_\ell}^{q^2_{\text{max}}} dq^2 \frac{d\Gamma(B_s \to K\ell\nu_\ell)}{dq^2}} \]

- This ratio is insensitive to the region \( m^2_\ell < q^2 < m^2_\tau \)
- We can form a ratio with an equally-weighted parts by
  - Reweighting the integrand, \([\text{Isidori and Sumensari EPJC 80 (2020) 1078}]\)
  - Unifying the integration ranges \([\text{Freytsis et al. PRD 92 (2015) 054018}]\) \([\text{Bernlochner and Ligeti PRD 95 (2017) 014022}]\)
- We obtain the alternative R-ratio

\[ R_{B_s \to K}^{\text{imp}} = \frac{\int_{m^2_\tau}^{q^2_{\text{max}}} dq^2 \frac{d\Gamma(B_s \to K\tau\nu_\tau)}{dq^2}}{\int_{m^2_\ell}^{q^2_{\text{max}}} dq^2 \frac{\omega_{\tau}(q^2)}{\omega_{\ell}(q^2)} \frac{d\Gamma(B_s \to K\ell\nu_\ell)}{dq^2}} \]

\[ R_{B_s \to K} = 0.77(16) [21\%] \quad R_{B_s \to K}^{\text{imp}} = 1.72(11) [6.4\%] \]
Next Steps
Next steps - C0 Ensemble

- Physical-mass light, strange, and bottom quarks
- $L^3 \times T = 48^3 \times 96; \ a^{-1} = 1.73\GeV$
- Point sources replaced with $Z_2$ sources for increased precision
  
  [Dong and Liu PLB 328 (1994) 130-136]

- Accelerated light + strange quark solves by exploiting AMA-corrected zMöbius DWF action + deflation
  
  [Blum et al. PRD 88 (2013) 094503]  [McGlynn PoS(LATTICE 2015)]

- Data generated with Grid and Hadrons
  
  [https://github.com/paboyle/Grid]  [https://github.com/aportelli/Hadrons]

- Analysis in progress!
Next steps - Reduced-parameter fits

- We might gain better control over fits by removing parameters not directly related to our physics goals.
- The **ground-state amplitudes** cancel from the ratio in use,

\[
\frac{C_3(t, t')}{\sqrt{C_{2,A}(t)C_{2,B}(t')}} = f_{00} \sqrt{\frac{e^{-E_A^{(0)} t} e^{-E_B^{(0)} t'}}{4E_A^{(0)} E_B^{(0)}}}
\]

- ...but are still present in two-point correlators, and so cannot be removed from a combined fit.

\[
C_2(t) = \frac{Z^{(0)}}{2E^{(0)}} \left( e^{-E^{(0)} t} + e^{-E^{(0)} (T-t)} \right)
\]

- However, they **do** cancel in the effective mass.

\[
\frac{C_2(t + 1) + C_2(t - 1)}{2C_2(t)} = \cosh(E^{(0)})
\]

- This can be extended to first-excited states.
Next steps - Reduced-parameter fits

Three-point ratio (ground state only):

\[
\frac{C_3(t, t')}{\sqrt{C_{2,A}(t)C_{2,B}(t')}} = f_{00}\sqrt{\frac{e^{-E_A(0)t}e^{-E_B(0)t'}}{4E_A^{(0)}E_B^{(0)}}}
\]

Two-point effective mass (ground state only):

\[
\frac{C_2(t + 1) + C_2(t - 1)}{2C_2(t)} = \cosh(E^{(0)})
\]
Next steps - Reduced-parameter fits

Three-point ratio (ground + 1st excited states):

\[
\frac{C_3(t, t')}{\sqrt{C_{2,A}(t)C_{2,B}(t')}} = f_{00} \sqrt{\frac{e^{-E_A^{(0)} t} e^{-E_B^{(0)} t'}}{4E_A^{(0)} E_B^{(0)}}} \\
\times \left( 1 + \frac{f_{10}}{f_{00}} g_A(t) + \frac{f_{01}}{f_{00}} g_B(t') + \frac{f_{11}}{f_{00}} g_A(t) g_B(t') \right)
\]

Two-point effective mass (ground + 1st excited state):

\[
\frac{C_{2,X}(t+1) + C_{2,X}(t-1)}{2C_{2,X}(t)} = \cosh(E_X^{(0)}) \left( 1 + \alpha_X g_X^{ATW}(t) \frac{\cosh(E_X^{(1)})}{\cosh(E_X^{(0)})} \right) \\
\]

\[
\alpha_X = \frac{Z_X^{(1)}}{Z_X^{(0)}}, \quad g_X(t) = \alpha_X \frac{E_X^{(0)}}{E_X^{(1)}} e^{-E_X^{(0)} t} + \alpha_X \frac{E_X^{(1)}}{E_X^{(0)}} e^{-E_X^{(1)} (T-t)}
\]

\[
g_X^{ATW}(t) = \alpha_X \frac{E_X^{(0)}}{E_X^{(1)}} e^{-E_X^{(0)} t} + \alpha_X \frac{E_X^{(1)}}{E_X^{(0)}} e^{-E_X^{(1)} (T-t)}
\]
RBC-UKQCD 2023 result for $B_s \rightarrow K\ell\nu$ now published

[PRD 107 (2023) 114512] [arXiv:2303.11280]

- Directly extrapolating in $f_+, f_0$ vs. $f_\perp, f_\parallel$ can produce incompatible results for $f_0$ at low $q^2$
- **Assessable truncation errors** in determination of $z$-expansion coefficients *via* Bayesian fit
  → Andreas Jüttner, **14:30 today**
- $|V_{ub}| = 3.78(61) \times 10^{-3}$
- **Alternative LFU-testing ratios** can yield significantly more precise theory predictions
- Update for $B \rightarrow \pi\ell\nu$ and physical-point data in progress

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Backup Slides
Extrapolation in terms of $f_+, f_0$ and $f_\perp, f_\parallel$

- Literature results consistent for $f_+$, in tension for $f_0$
- $>3\sigma$ shift in RBC-UKQCD result between choice to extrapolate in $f_+, f_0$ or $f_\perp, f_\parallel$
- Results for the two choices using RBC-UKQCD data are 100% correlated and include all systematics
Extrapolation in terms of $f_+, f_0$ and $f_\perp, f_\parallel$

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Chiral-Continuum Fits

- All five terms resolved for $f_0$

![Graph showing the Chiral-Continuum Fits for $f_0$ as a function of $(E_K/M_{B_s})^2$]

- $(E_K/M_{B_s})^2$
- $f_{B_s \to K}$
- Central, M1, M2, C1, C2, M3, F1S

---

(a) omit FV
(b) omit $(a\Lambda)^2$
(c) omit $\Delta M^2_{\pi}$
(d) omit chiral log
(e) omit $(a\Lambda)^2$ and chiral log
(f) exclude $n^2 = 0$
(g) exclude $n^2 = 4$
(h) constant dispersion relation $(a\Lambda)^2$
(i) varying $f_{\pi}$
(j) $\Delta_s$ by $\pm 100$ MeV

---

![Graph showing the error in $f_{B_s \to K}$ as a function of $q^2$]

- $q^2$ [GeV$^2$]
- $f_{B_s \to K}$
- Error $2 \%$
- Statistics, fit systematics, discretization (heavy), renormalization, isospin breaking, discretization (light), RHQ inputs