Semileptonic Form Factors for $B_s \to K\ell\nu$ decays

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Lattice 2023

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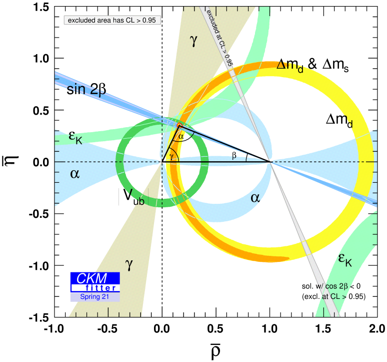
Motivation

- b-physics continues to play an important role in the search for new physics at the precision frontier
- Large m_b allows us to probe high energy scales
- Physical applications include
  - Shape of QCD form factors
  - CKM matrix elements
  - Lepton flavour universality tests



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

Goal

- V_{ub} enters the $B_s \to K\ell\nu$ differential decay rate:

$$\underbrace{\frac{d\Gamma(B_s \to K\ell\nu)}{dq^2}}_{\text{Experiment}} = \underbrace{|V_{ub}|^2}_{CKM} \times \left(\kappa_1 \underbrace{|f_+(q^2)|^2}_{non-pert.} + \kappa_2 \underbrace{|f_0(q^2)|^2}_{non-pert.}\right)$$

$$q^\mu = p^\mu_{B_s} - p^\mu_K$$
κ — Known factors

- Form factors require non-perturbative computation:

$$\langle K(\vec{p}_K)|\mathcal{V}^\mu|B_s(\vec{p}_{B_s})\rangle = 2f_+(q^2)\left(p^\mu_{B_s} - \frac{p_{B_s}\cdot q}{q^2}q^\mu\right) + f_0(q^2)\left(\frac{M^2_{B_s} - M^2_K}{q^2}q^\mu\right)$$

$$\mathcal{V}^\mu = \bar{u}\gamma^\mu b$$

Quark Actions

- Relativistic Heavy Quark (**RHQ**) action for b quarks

  [Christ et al. PRD 76 (2007) 074505] [Lin and Christ PRD 76 (2007) 074506]

  - Builds on original Fermilab action [El-Khadra et al. PRD 55 (1997) 3933]
  - Anisotropic clover action
  - Uses 3 parameters (m_0a,c_p,ζ) that can be non-pertubatively tuned to remove $\mathcal{O}((m_0a)^n)$, $\mathcal{O}((\vec{p}a)(m_0a)^n)$ errors [Aoki et al. PRD 86 (2012) 116003]

- Shamir Domain-Wall Fermions (**DWF**) for l, s

  [Shamir Nucl. Phys. B406 (1993) 90] [Furman and Shamir Nucl. Phys. B439 (1995) 54]

- Relate continuum and lattice currents *via* renormalisation constant [El-Khadra et al. PRD 64 (2001) 014502]

$$\langle K|\mathcal{V}^\mu|B_s\rangle = Z_V^{bl}\langle K|V^\mu|B_s\rangle; \quad Z_V^{bl} = \rho_V^{bl}\sqrt{Z_V^{bb}Z_V^{ll}}$$

- $\mathcal{O}(a)$-improved V^μ at one-loop

Ensembles

	$L^3 \times T\ /\ a^4$	$a^{-1}\ /\ \mathrm{GeV}$	$m_\pi\ /\ \mathrm{MeV}$
C1	$24^3 \times 64$	1.78	340
C2	$24^3 \times 64$	1.78	430
M1	$32^3 \times 64$	2.38	300
M2	$32^3 \times 64$	2.38	360
M3	$32^3 \times 64$	2.38	410
F1S	$48^3 \times 96$	2.79	270
C0*	$48^3 \times 96$	1.73	139

- 2+1f ensembles: degenerate light quarks
- Domain-wall fermions and Iwasaki gauge action
- **F1S** ensemble: new for this analysis

  *C0 currently under analysis.
- Addition of C0 disentangles chiral and continuum effects

Form Factor Fits

- For lattice data in the B_s-meson rest frame, easier to decompose matrix elements as

$$f_{\parallel} = \frac{\langle K|V^0|B_s\rangle}{\sqrt{2M_{B_s}}}, \qquad f_{\perp} p^i = \frac{\langle K|V^i|B_s\rangle}{\sqrt{2M_{B_s}}}$$

- Neatly separates into spatial and temporal components
- Linear combination recovers f_0 and f_+:

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}}\left[f_{\parallel}(q^2) + (M_{B_s} - E_K)f_{\perp}(q^2)\right]$$

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 + E_K^2}\left[(M_{B_s} - E_K)f_{\parallel}(q^2) + (E_K^2 - M_K^2)f_{\perp}(q^2)\right]$$

Form Factor Fits

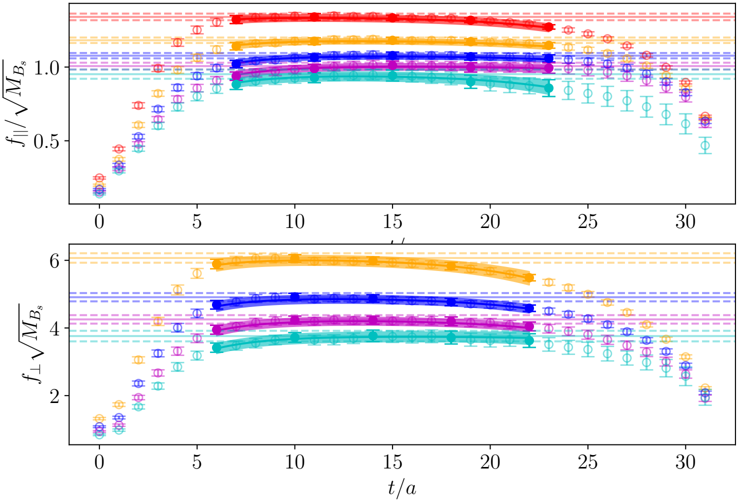
- Simultaneously fit two-point functions and three-point function ratios on each ensemble over all momenta

$$\frac{C_3^{\mu}(t)}{\sqrt{C_2^K(t)C_2^{B_s}(t_{\rm sink}-t)}} = \langle K|V^{\mu}|B_s\rangle \sqrt{\frac{4E_K E_{B_s}}{e^{-tE_K}e^{-(t_{\rm sink}-t)E_{B_s}}}}$$

$$+ \text{ excited state contrib.}$$

- Use lattice dispersion relation to constrain kaon energies in three-point fits

Form Factor Fits



Form factor fits on F1S ensemble

Chiral-Continuum Fits

- Extrapolate to physical kaon mass and zero lattice spacing simultaneously
- Use NLO hard-pion (kaon) SU(2) HMχPT [PRD 67 (2003) 054010]

$$f(M_\pi^{\text{sim}}, E_K, a, L) = \frac{\Lambda}{E_K + \Delta_{\text{pole}}}$$

$$\times\left(\left(c^{(0)} \times (1 + \text{chiral log})\right) + c^{(1)} \frac{\Delta M_\pi^2}{\Lambda^2} + c^{(2)} \frac{E_K}{\Lambda} + c^{(3)} \left(\frac{E_K}{\Lambda}\right)^2 + c^{(4)} \left(a\Lambda\right)^2 \right)$$

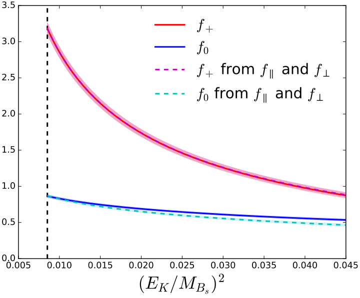
$$\Delta M_\pi^2 = \left(M_\pi^{\text{sim}}\right)^2 - \left(M_\pi^{\text{phys}}\right)^2$$

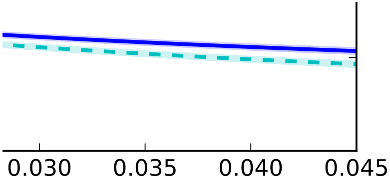
$$\Delta_{\text{pole, }0} = M_{B^*(0^+)} - M_{B_s} \approx 263 \text{ MeV}$$

$$\Delta_{\text{pole, }+} = M_{B^*(1^-)} - M_{B_s} \approx -42.1 \text{ MeV}$$

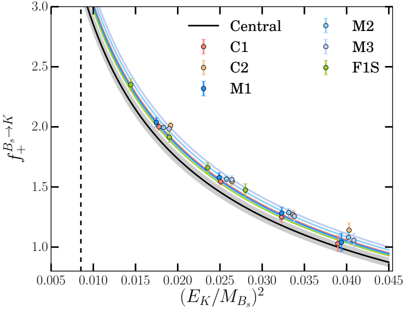
Extrapolation in terms of f_+, f_0 and $f_\perp$, $f_\parallel$

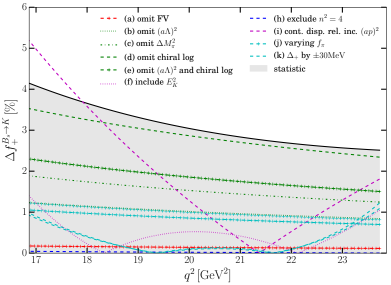
- Choice of chiral-continuum extrapolation strategy:
  - Take continuum limit of f_+, f_0 constructed from $f_\perp$, $f_\parallel$
  - Reconstruct f_+, f_0 from continuum-limit $f_\perp$, $f_\parallel$
- Latter strategy assumes $f_\perp$, $f_\parallel$ continuum limit is described well by f_+, f_0 pole energies
- **Significant difference** between the two strategies for f_0
- **Kinematic constraint at $q^2 = 0$ couples f_+ and f_0 - influences q^2 extrapolation of both form factors!**

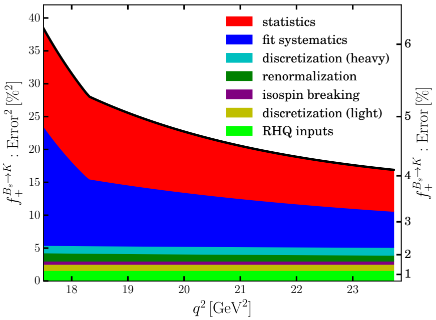




Chiral-Continuum Fits







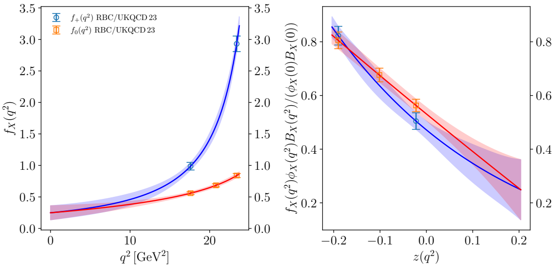
- E_K^2 term unresolved for f_+ and dropped
- Continuum form factor given by $f(M_\pi^{\mathrm{p}}, E_K, a = 0, L \to \infty)$
- Variations on the continuum fit ansatz to assess systematic errors

Bayesian-inferential z-expansion

- Extrapolate over full q^2 range using a z-expansion
- Fit to synthetic data at reference q^2 points
- Limited data points restricts available number of terms in frequentist z-expansion
- Adopt a Bayesian strategy that can easily explore truncation errors for the BGL parameterisation

  [Flynn, Jüttner, and Tsang: arXiv:2303.11285]

- **Details in Andreas Jüttner's talk in this session at 14:30!**



Observables - $|V_{ub}|$

- Combine experimental inputs:
  - $R_{BF} = \frac{\mathcal{B}(B_s^0 \to K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)}$ [LHCb PRL 126 (2021) 081804]
  - $\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)$ [LHCb PRD 101 (2020) 072004]
  - B_s^0 lifetime $\tau_{B_s^0}$ [PDG PTEP 2022 (2022) 083C01] [HFLAV arXiv:2206.07501]
- Lattice contribution: Reduced decay rate $\Gamma_0 = \Gamma / |V_{ub}|^2$

$$|V_{ub}| = \sqrt{\frac{R_{BF}\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)}{\tau_{B_s^0}\Gamma_0(B_s \to K\ell\nu)}}$$

- $|V_{ub}|^{\text{RBC-UKQCD 2023}}_{\text{exclusive, } B_s \to K\ell\nu} = 3.78(61) \times 10^{-3}$ [PRD 107 (2023) 114512]
- $|V_{ub}|^{\text{FLAG 2021}}_{\text{exclusive, } B \to \pi\ell\nu} = 3.74(17) \times 10^{-3}$ [FLAG EPJC 82 (2022) 869]
- $|V_{ub}|^{\text{PDG 2022}}_{\text{inclusive, B decays}} = 4.13(26) \times 10^{-3}$ [PDG PTEP 2022 (2022) 083C01]
- Consistent with both exclusive and inclusive averages

Observables - LFU-testing ratios

- Standard R-ratio takes the form

$$R_{B_s\to K} = \frac{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \frac{d\Gamma(B_s\to K\tau\nu_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{\max}} dq^2 \frac{d\Gamma(B_s\to K\ell\nu_\ell)}{dq^2}}$$

- This ratio is insensitive to the region $m_\ell^2 < q^2 < m_\tau^2$
- We can form a ratio with an equally-weighted parts by
  - Reweighting the integrand, [Isidori and Sumensari EPJC 80 (2020) 1078]
  - Unifying the integration ranges

    [Freytsis et al. PRD 92 (2015) 054018] [Bernlochner and Ligeti PRD 95 (2017) 014022]
- We obtain the alternative R-ratio

  [Atwood and Soni PRD 45 (1992) 2405] [Flynn et al. PoS(LATTICE2021)306]

$$R^{\text{imp}}_{B_s\to K} = \frac{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \frac{d\Gamma(B_s\to K\tau\nu_\tau)}{dq^2}}{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_s\to K\ell\nu_\ell)}{dq^2}}$$

$R_{B_s\to K} = 0.77(16)$ [21%] $\quad$ $R^{\text{imp}}_{B_s\to K} = 1.72(11)$ [6.4%]

Next Steps

Next steps - C0 Ensemble

- **Physical-mass** light, strange, and bottom quarks
- $L^3 \times T = 48^3 \times 96$; $a^{-1} = 1.73 GeV$
- Point sources replaced with Z_2 sources for increased precision

  [Dong and Liu PLB 328 (1994) 130-136]

- Accelerated light + strange quark solves by exploiting AMA-corrected zMöbius DWF action + deflation

  [Blum et al. PRD 88 (2013) 094503] [McGlynn PoS(LATTICE 2015)]

- Data generated with Grid and Hadrons

  [https://github.com/paboyle/Grid] [https://github.com/aportelli/Hadrons]

- Analysis in progress!

Next steps - Reduced-parameter fits

- We might gain better control over fits by removing parameters not directly related to our physics goals.
- The **ground-state amplitudes** cancel from the ratio in use,

$$\frac{C_3(t,t')}{\sqrt{C_{2,A}(t)C_{2,B}(t')}} = f_{00}\sqrt{\frac{e^{-E_A^{(0)}t}e^{-E_B^{(0)}t'}}{4E_A^{(0)}E_B^{(0)}}}$$

- ...but are still present in two-point correlators, and so cannot be removed from a combined fit.

$$C_2(t) = \frac{Z^{(0)}}{2E^{(0)}}\left(e^{-E^{(0)}t} + e^{-E^{(0)}(T-t)}\right)$$

- However, they **do** cancel in the effective mass.

$$\frac{C_2(t+1)+C_2(t-1)}{2C_2(t)} = \cosh(E^{(0)})$$

- This can be extended to first-excited states.

Next steps - Reduced-parameter fits

Three-point ratio (ground state only):

$$\frac{C_3(t,t')}{\sqrt{C_{2,A}(t)C_{2,B}(t')}} = f_{00}\sqrt{\frac{e^{-E_A^{(0)}t}e^{-E_B^{(0)}t'}}{4E_A^{(0)}E_B^{(0)}}}$$

Two-point effective mass (ground state only):

$$\frac{C_2(t+1)+C_2(t-1)}{2C_2(t)} = \cosh(E^{(0)})$$

Next steps - Reduced-parameter fits

Three-point ratio (ground + 1st excited states):

$$\frac{C_3(t,t')}{\sqrt{C_{2,A}(t)C_{2,B}(t')}} = f_{00}\sqrt{\frac{e^{-E_A^{(0)}t}e^{-E_B^{(0)}t'}}{4E_A^{(0)}E_B^{(0)}}} \times \left(\frac{1+\frac{f_{10}}{f_{00}}g_A(t)+\frac{f_{01}}{f_{00}}g_B(t')+\frac{f_{11}}{f_{00}}g_A(t)g_B(t')}{\sqrt{(1+\alpha_A g_A(t))(1+\alpha_B g_B(t'))}}\right)$$

Two-point effective mass (ground + 1st excited state):

$$\frac{C_{2,X}(t+1)+C_{2,X}(t-1)}{2C_{2,X}(t)} = \cosh(E_X^{(0)})\left(\frac{1+\alpha_X g_X^{\mathrm{ATW}}(t)\frac{\cosh(E_X^{(1)})}{\cosh(E_X^{(0)})}}{1+\alpha_X g_X^{\mathrm{ATW}}(t)}\right)$$

$$\alpha_X = \frac{Z_X^{(1)}}{Z_X^{(0)}},\ g_X(t) = \alpha_X\frac{E_X^{(0)}}{E_X^{(1)}}\frac{e^{-E_X^{(1)}t}}{e^{-E_X^{(0)}t}},\ g_X^{\mathrm{ATW}}(t) = \alpha_X\frac{E_X^{(0)}}{E_X^{(1)}}\frac{e^{-E_X^{(1)}t}+e^{E_X^{(1)}(T-t)}}{e^{-E_X^{(0)}t}+e^{E_X^{(0)}(T-t)}}$$

Summary

- RBC-UKQCD 2023 result for $B_s \to K\ell\nu$ now published

  [PRD 107 (2023) 114512] [arXiv:2303.11280]
- Directly **extrapolating in** f_+**,** f_0 **vs.** $f_\perp$**,** $f_\parallel$ can produce incompatible results for f_0 at low q^2
- **Assessable truncation errors** in determination of z-expansion coefficients *via* Bayesian fit
  $\to$ Andreas Jüttner, **14:30 today**
- $|V_{ub}| = 3.78(61) \times 10^{-3}$
- **Alternative LFU-testing ratios** can yield significantly more precise theory predictions
- Update for $B \to \pi\ell\nu$ and physical-point data in progress

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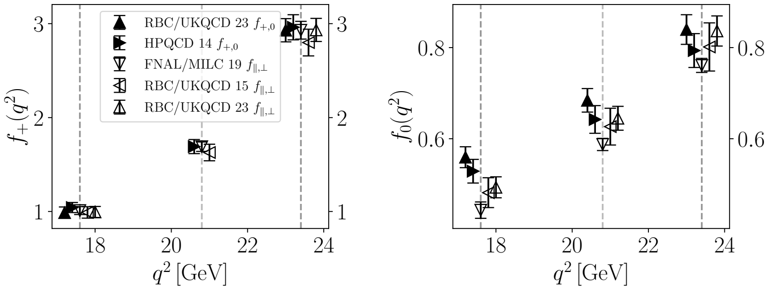
Backup Slides

Extrapolation in terms of f_+, f_0 and $f_\perp$, $f_\parallel$



- Literature results consistent for f_+, in tension for f_0
- $> 3\sigma$ shift in RBC-UKQCD result between choice to extrapolate in f_+, f_0 or $f_\perp$, $f_\parallel$
- Results for the two choices using RBC-UKQCD data are 100% correlated and include all systematics

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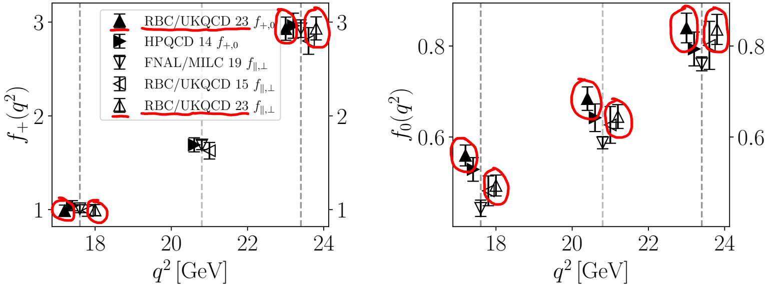

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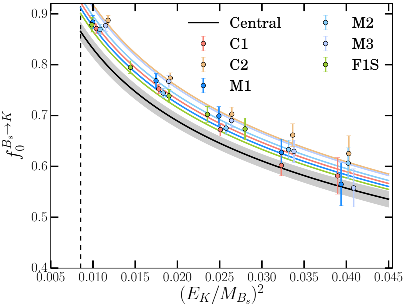

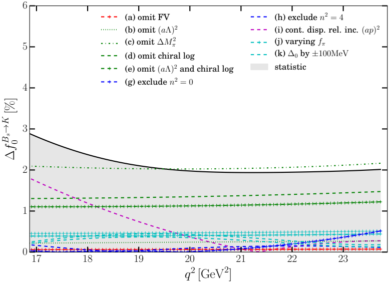

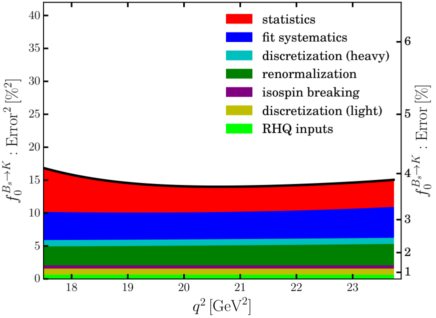

- All five terms resolved for f_0