The 40th International Symposium on Lattice Field Theory

Towards a high-precision description of the ρ and K^* resonances

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Motivations

- Multi-hadron states in Standard Model

$$\left.\begin{array}{l}\text{rare decays, e.g. } B \to K^* l^+ l^- (\to K\pi l^+ l^-) \\ \text{multibody decays, e.g. } B \to K\pi\pi,\ D \to \pi\pi, K\bar{K}\end{array}\right\} \text{insights into New Physics}$$

- Hadronic resonances **non-perturbatively** on the lattice [Briceño, Dudek, Young - RevModPhys.90.025001, 2018]

- Towards *high-precision* on $K^*(892)$ and $\rho(770)$ [Fischer et al, PhysLettB 819.136449, 2021] [Paul et al, Lattice21, arXiv:2112.07385v1] [Rendon et al, PhysRevD 102.114520, 2021]

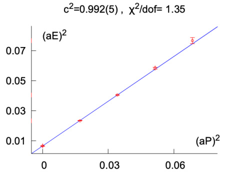
$\to$ *Physical* m_π and $N_f = 2 + 1$

Lattice Details

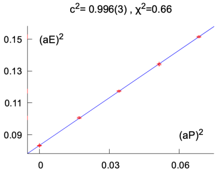
- Möbius domain-wall $N_f = 2 + 1$ fermion action [Blum, Boyle, Christ et al - PRD.93.074505, 2016]
- Measurements on 90 configs, inversions on *all* 96 time slices

volume	$48^3 \times 96$
a	≈ 0.114 fm
L	≈ 5.5 fm
$m_\pi L$	≈ 3.8
m_π m_K	≈ 139 MeV ≈ 499 MeV

ensemble features



(a) Pion



(b) Kaon

Dispersion relation from $\langle(\bar{q}\Gamma q)(\bar{q}\Gamma q)\rangle^\dagger$ (P)

Variational basis through (exact) distillation method [Peardon, Bulava, Foley et al - PRD.80.054506, 2009]

- $N_{vec} = 64$ ($R_{smear} \approx 1fm$) [Lachini et al - PoS LATTICE2022 076]
- Kinematics: $0 \leq \mathbf{p}_1^2, \mathbf{p}_2^2, \mathbf{P}^2 \leq 4\left(\frac{2\pi}{L}\right)^2$ for meson field momenta $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}$

Lattice Code

Open-source and free software

GRID

Data parallel C++ lattice library
[github.com/paboyle/Grid]

Hadrons

Grid-based lattice workflow management system
[Portelli et al, v1.3, 10.5281/ZENODO.6382460, 2022]

Distillation within `Grid` and `Hadrons` [https://aportelli.github.io/Hadrons-doc/#/mdistil]

- Supports any **`Grid`** solver, inversions on CPU and GPU (same code)
- Meson fields on CPU, account for time-sparsity

GEVP

Solve

$$C(t)u_n(t,t_0) = \lambda_n(t,t_0)C(t_0)u_n(t,t_0), \qquad \lambda_n(t,t_0) \xrightarrow{t,t_0 \text{ large}} A_n e^{-tE_n}(1+\mathcal{O}(e^{-t\Delta E_n}))$$

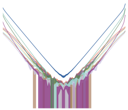
Operator basis

$$\text{bilinear } O_{\bar{q}\gamma_i q}(\mathbf{P}) \begin{cases} q = s,d \\ q = u,d \end{cases} + \quad \text{two-bilinear } \begin{cases} O_{K\pi}^{I=1/2}(\mathbf{p}_1,\mathbf{p}_2) \\ O_{\pi\pi}^{I=1}(\mathbf{p}_1,\mathbf{p}_2) \end{cases}$$

Projected onto (2-particle only)

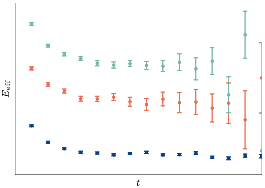
Channel	Irreps $(\mathbf{P}^2, \Lambda)$
$K\pi^{I=1/2}$	$(0,T_{1u})$; $(1,E)$; $(2,B_1)$; $(2,B_2)$; $(3,E)$; $(4,E)$
$\pi\pi^{I=1}$	all above + $(1,A_1)$; $(2,A_1)$; $(3,A_1)$; $(4,A_1)$

Lüscher-type method



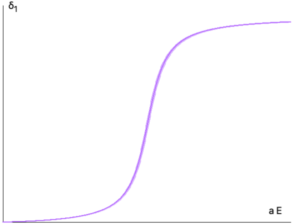
Lattice data

→ GEVP



(1)

⟶ Quantisation Condition

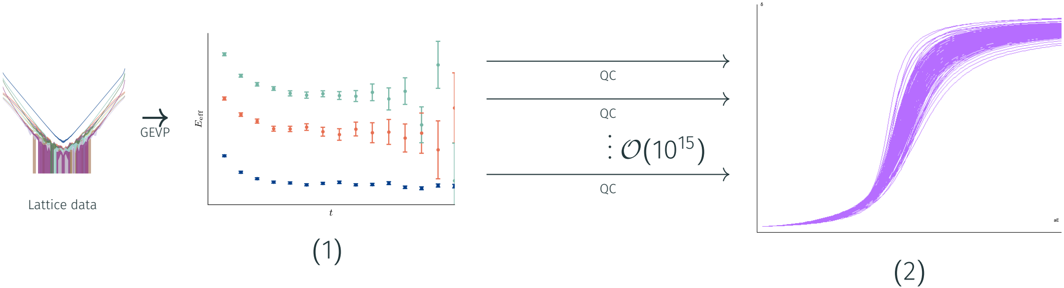


(2)

- Choice of fit ranges from (1) to (2) introduces non-negligible systematics

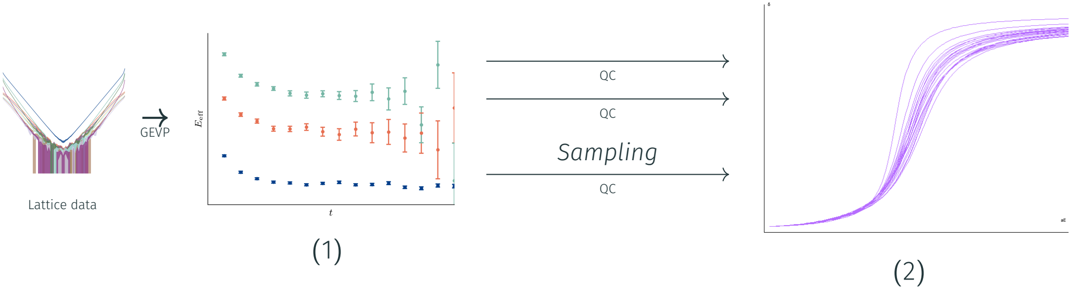
Workflow

Lüscher-type method



- Choice of fit ranges from (1) to (2) introduces non-negligible systematics
- Fit range space has very high multiplicity

Lüscher-type method

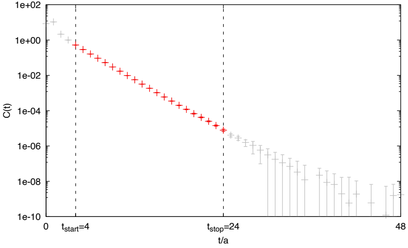


- Choice of fit ranges from (1) to (2) introduces non-negligible systematics
- Fit range space has very high multiplicity

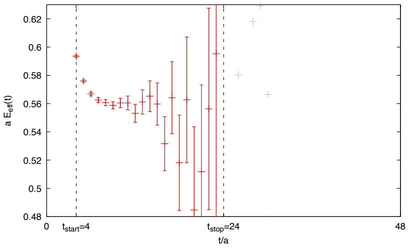
$\rightarrow$ Importance sampling of GEVP eigenvalue fit ranges (+ weighting at the end)

Correlator Fit Scan

Perform every one-state fit to each λ_i in the range $[t_{start}, t_{stop}]$ with $t_f - t_i \geq 3$



Eigenvalue



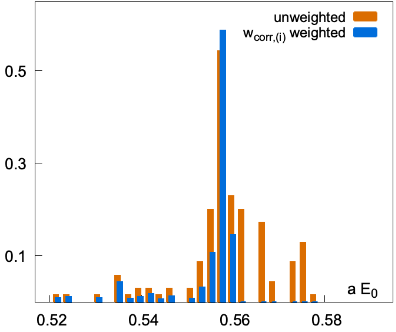
Effective mass

Example of principal correlator(eigenvalue), effective energy and its scan limits

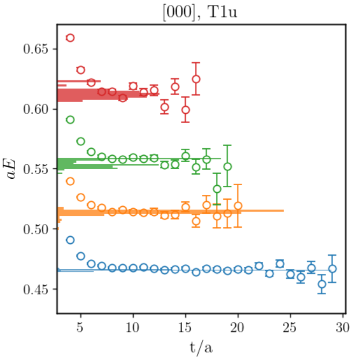
Weight fit ranges [Borsanyi, Fodor, Guenther et al - Nature, 2021] [Jay & Neil - PRD.103.114502, 2021]

$$AIC_{corr,i} = \chi^2_{corr,i} + 2n^{par}_{corr,i} - n^{data}_{corr,i}, \qquad \text{for level } i = (\mathbf{d}^2, \Lambda, n)$$

Sample $n = 1, \ldots, N_{scan}$ fit ranges for each i using $w_{corr,i} = \exp\left(-\frac{1}{2}AIC_{corr,i}\right)$



Example of histograms of an energy central value



Effective mass and w_{corr,i_k} for $i_k = (0, T_{1u}, k)$ levels

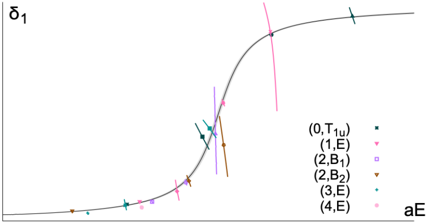
→ Combine the n^{th} sample of all levels to form the n^{th} fit range sample

Spectrum Fit

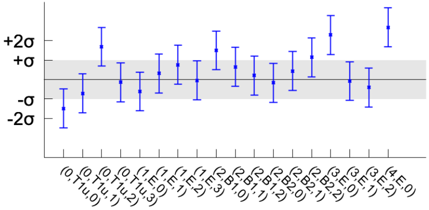
For the n^{th} fit range sample: fit spectrum to model $E_i^{mod}(\boldsymbol{\alpha})$ [Guo, Dudek, Edwards et al - PRD.88.014501, 2013]

$$\chi^2_{spec}(\alpha) \equiv \sum_{ij} \left[E_i^{mod}(\boldsymbol{\alpha}) - E_i^{data}\right] \mathrm{Cov}_{ij}^{-1} \left[E_j^{mod}(\boldsymbol{\alpha}) - E_j^{data}\right]$$

and assign an AIC_{spec}



Breit-Wigner (BW) phase shift $\boldsymbol{\alpha} = (g, M)$



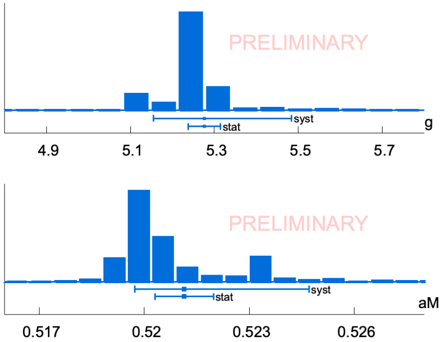
Normalised residuals ($LL^T = \mathrm{Cov}^{-1}$)

Spectrum fit example

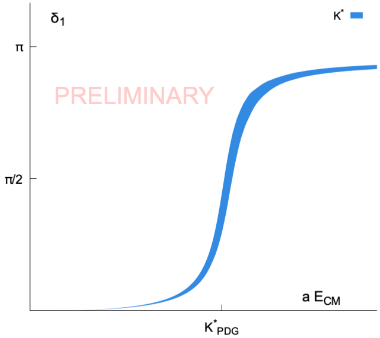
K^* Parameters

Sampling $N_{scan} = 50000$ fit ranges and using only a Breit-Wigner

- statistical: variance of scan means across bootstrap samples
- systematic: $(2\%, 98\%)$-percentile interval of scan mean on central value



K^*: $\sum_i {}^{AIC_{corr,i} + AIC_{spec}}$-weighted central values



Phase-shift quadrature band (lattice units)

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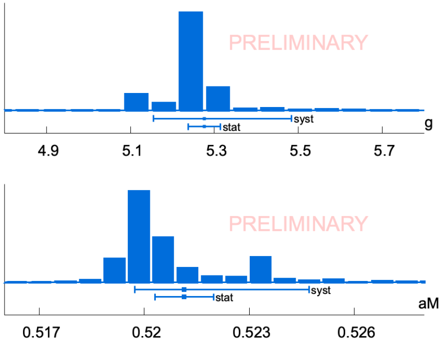

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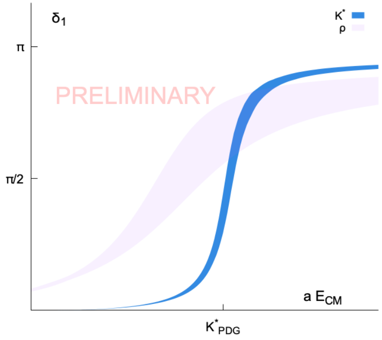


Phase-shift quadrature band (lattice units)

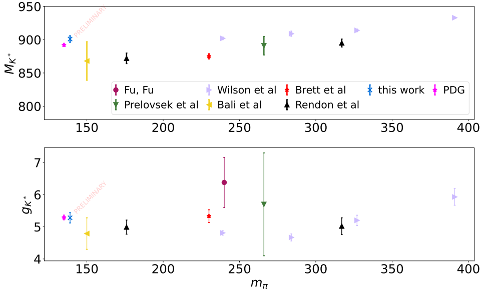
Summary

	BW_{K*}
g	$5.28(4)^{(21)}_{(12)}(\ldots)$
aM	$0.5211(8)^{(36)}_{(14)}(\ldots)$
$a\Gamma$	$0.0262(4)^{(28)}_{(6)}(\ldots)$
M(MeV)	$901(1.4)^{(6.2)}_{(2.6)}(2)(\ldots)$
Γ(MeV)	$45.4(7)^{(48)}_{(11)}(0.9)(\ldots)$

Value (statistical) (fit range systematic) (scale setting) (other)

Source	M_{K*}	$g_{K*K\pi}$	Γ_{K*}
statistical	0.16%	0.7%	1.5%
fit range	(0.69%) (0.27%)	(3.9%) (2.3%)	(11%) (2.5%)

Partial error budget (BW)



PDG values and published lattice QCD results computing the K^* resonance parameters M_{K*}, $g_{K*K\pi}$ [Fu, Fu, 1209.0350] [Prelovsek et al, 1307.0736] [Wilson et al, 1411.2004,1904.03188] [Bali et al, 1512.08678] [Brett et al, 1802.03100] [Rendon et al, 2006.14035.]

- Other non-negligible systematics to be estimated: quark mass mismatch; higher thresholds; discretisation
- Final number for ρ still being worked out with the same analysis method

Outlook

- Model-averaging applied to Lüscher-type scattering calculation
- Light vector P-wave resonance parameters

  First K^* computation at physical pion mass

  On the way: ρ at physical pion mass (first with $N_f = 2 + 1$)
- Soon: model-average different parametrizations
- Outlook

  Scalar resonance κ (and K^* with all irreps)

  Weak decays with 3-point functions (code ready)

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