Pion and kaon generalized form factors from lattice QCD

Joseph Delmar

Temple University

Collaborators (ETMC): C. Alexandrou, S. Bacchio, I. Cloët, M. Constantinou, G. Koutsou

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Outline

- Introduction
- Theoretical setup – Calculation parameters
- Pion generalized form factors A_{20}, B_{20}
- Kaon generalized form factors A_{20}, B_{20}
- SU(3) flavor symmetry breaking
- Summary

Updated results for scalar, vector, tensor form factors @ APS GHP 2023
https://indico.jlab.org/event/667/contributions/12289/

PHYSICAL REVIEW D **103,** 014508 (2021)

Mellin moments $\langle x \rangle$ and $\langle x^2 \rangle$ for the pion and kaon from lattice QCD

Constantia Alexandrou,[1,2] Simone Bacchio,[1,2] Ian Cloët,[3] Martha Constantinou,[4]
Kyriakos Hadjiyiannakou,[1,2] Giannis Koutsou,[2] and Colin Lauer[3,4]

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PHYSICAL REVIEW D **104,** 054504 (2021)

Pion and kaon $\langle x^3 \rangle$ from lattice QCD and PDF reconstruction from Mellin moments

Constantia Alexandrou,[1,2] Simone Bacchio,[2] Ian Cloët,[3] Martha Constantinou,[4]
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PHYSICAL REVIEW D **105,** 054502 (2022)

Scalar, vector, and tensor form factors for the pion and kaon from lattice QCD

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Motivation

- Understanding the structure of pion and kaon is as important for describing QCD dynamics as the proton (pions & kaons arise from dynamical chiral symmetry breaking)
- Important for studying SU(3) flavor symmetry breaking
- Pion and kaon less studied than proton:
  Most experimental, theoretical, and lattice studies on pion structure regard the vector form factor
- Increased interest in comparing the pion and proton structure
  (e.g., simultaneous extraction of PDFs and TMDs)
  [JAM collaboration: P. Barry et al., arXiv:2302.01192]
- Meson structure under experimental study at JLab 12 GeV
  (e.g., Measurement of the Charged Pion Form Factor in JLab Hall C)
- Pion and kaon structure will be studies at the Electron-Ion Collider (EIC)
  [EIC Yellow Report, arXiv:2103.05419; Aguilar et al., EPJA 55, 190 (2019)]

Important to complement experimental effort from first-principle calculations

Theoretical Setup

- Form factors and generalizations obtained from matrix elements of local operators

$$\mathcal{O}^{\{\mu\nu\}} \equiv \bar{\psi}\left[\frac{1}{2}\left(\gamma^{\mu}\overleftrightarrow{D}^{\nu}+\gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)-\frac{1}{4}\left(\sum_{\rho=1}^{4}\delta_{\mu\nu}\gamma^{\rho}\overleftrightarrow{D}^{\rho}\right)\right]\psi \qquad \mathcal{O}^{\{\mu\nu\rho\}} \equiv \bar{\psi}\gamma^{\{\mu}D^{\nu}D^{\rho\}}\psi \qquad \mathcal{O}^{\{\mu\nu\rho\sigma\}} \equiv \bar{\psi}\gamma^{\{\mu}D^{\nu}D^{\rho}D^{\sigma\}}\psi$$

- Matrix elements decompose to form factors and generalized form factors
  [P. Hagler, Phys. Rept. 490, 49 (2010)]

$$\left\langle M(p')\,|\,\mathcal{O}^{\{\mu\nu\}}\,|\,M(p)\right\rangle = C\left[2P^{\{\mu}P^{\nu\}}A_{20}(Q^2)+2\Delta^{\{\mu}\Delta^{\nu\}}B_{20}(Q^2)\right]$$

$$\left\langle M(p')\,|\,\mathcal{O}^{\{\mu\nu\rho\}}\,|\,M(p)\right\rangle = C\left[2iP^{\{\mu}P^{\nu}P^{\rho\}}A_{30}(Q^2)+2i\Delta^{\{\mu}\Delta^{\nu}P^{\rho\}}B_{30}(Q^2)\right]$$

$$\left\langle M(p')\,|\,\mathcal{O}^{\{\mu\nu\rho\sigma\}}\,|\,M(p)\right\rangle = C\left[-2P^{\{\mu}P^{\nu}P^{\rho}P^{\sigma\}}A_{40}(Q^2)-2\Delta^{\{\mu}\Delta^{\nu}P^{\rho}P^{\sigma\}}B_{40}(Q^2)-2\Delta^{\{\mu}\Delta^{\nu}\Delta^{\rho}\Delta^{\sigma\}}C_{40}(Q^2)\right]$$

$$B_{20}=A_{22}$$

$$B_{30}=A_{32}$$

$$B_{40}=A_{42}$$

$$C_{40}=A_{44}$$

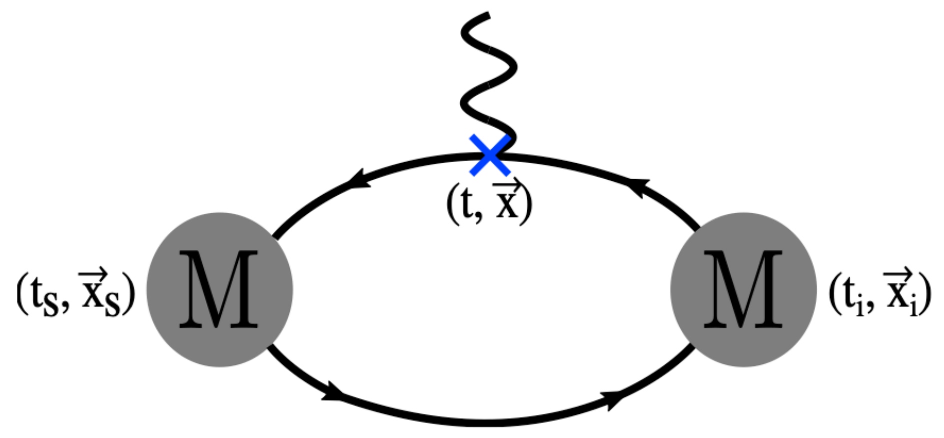
- Form factors are frame independent
- Two methods to extract ground-state contribution
  - fit plateau region to a constant value
  - perform a two-state fit on three-point function
- Non-perturbative renormalization ($\overline{\text{MS}}$ at 2 GeV)

Lattice Details

- Two ensembles of twisted-clover fermions and Iwasaki improved gluons **[C. Alexandrou et al., PRD 104, 074520 (2021)]**

Ensemble	β	a(fm)	$L^3 \times T$	N_f	m_π(MeV)	L(fm)
cA211.30.32	1.726	0.094	$32^3 \times 64$	$2+1+1$	265	3.0
cB211.25.48	1.778	0.079	$48^3 \times 96$	$2+1+1$	250	3.79

- Calculation of connected contributions



- Use of different kinematic frames:
  -- Rest frame: desirable for charges and usual form factors
  -- Boosted frame: necessary for higher Mellin moments of GPDs without power-divergent mixing
- Advantage of boosted frame: access to a denser range of $-t$ $\quad \left(-t = \vec{q} - \left(E(p') - E(p)\right)^2 \right)$
- Boosted frame of choice: fixed final momentum
  (use of sequential method: matrix element for all values of $\vec{q}$ are obtained at once)

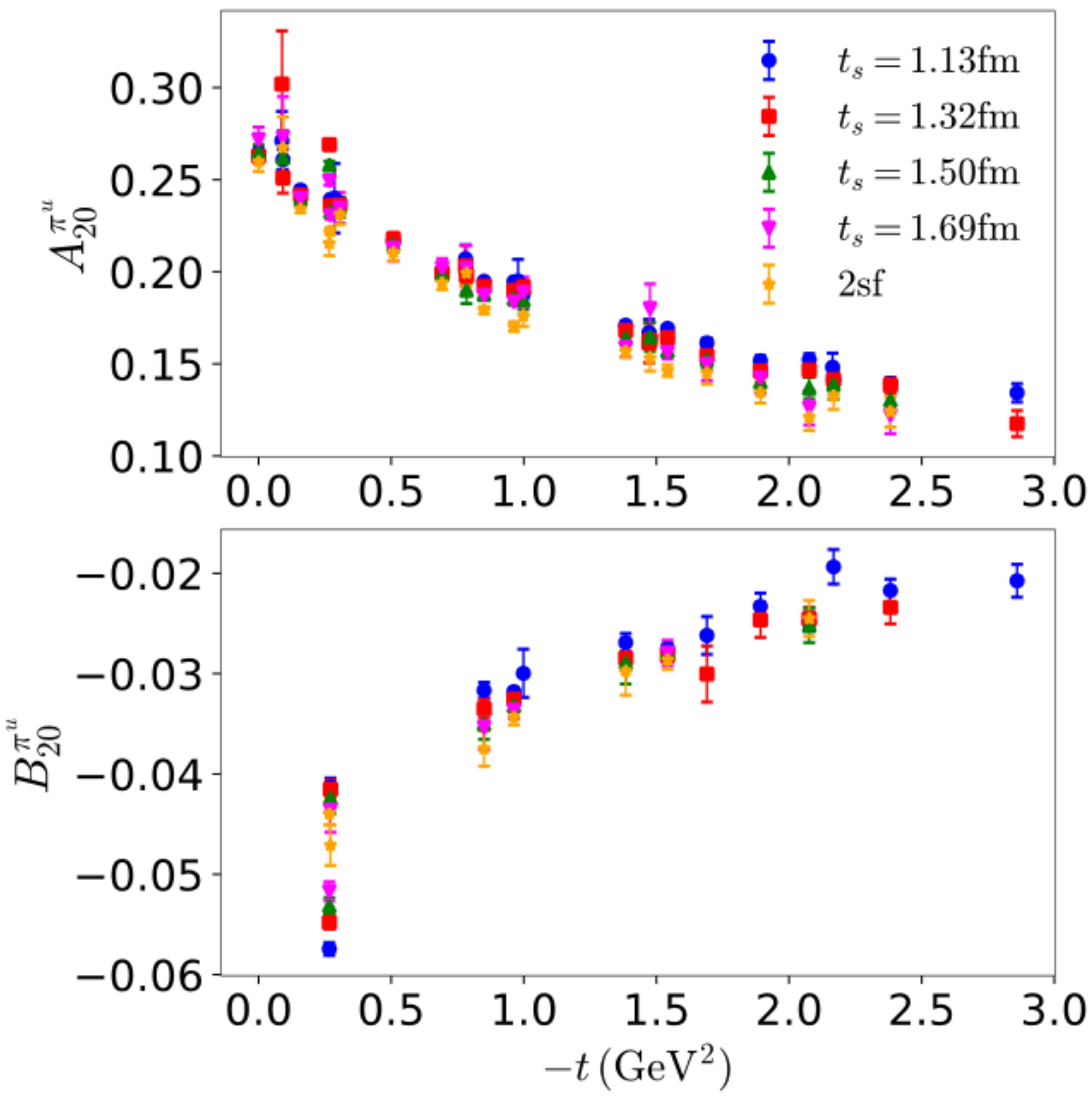
$$\vec{p}\,' = \frac{2\pi}{L}(\pm 1, \pm 1, \pm 1), \quad \vec{p} = \vec{p}\,' - \vec{q} \qquad \vec{q} = \frac{2\pi}{L}(\pm n_x, \pm n_y, \pm n_z) \qquad n_x, n_y, n_z \in [0,8]$$

Statistics

Ensemble	Frame	$\vec{p}\,(2\pi/L)$	t_s/a	t_s (fm)	confs	src pos.	Total
cA211.30.32	R	$(0,0,0)$	12, 14, 16, 18, 20, 24	1.13, 1.32, 1.50, 1.69, 1.88, 2.256	122	16	1,952
cA211.30.32	B	$(\pm1,\pm1,\pm1)$	12, 14, 16, 18	1.13, 1.32, 1.50, 1.69	122	136	132,736
cB211.25.48	B	$(\pm1,\pm1,\pm1)$	14, 16, 18, 20	1.11, 1.26, 1.42, 1.58	76	8	4,864

Pion Generalized Form Factors

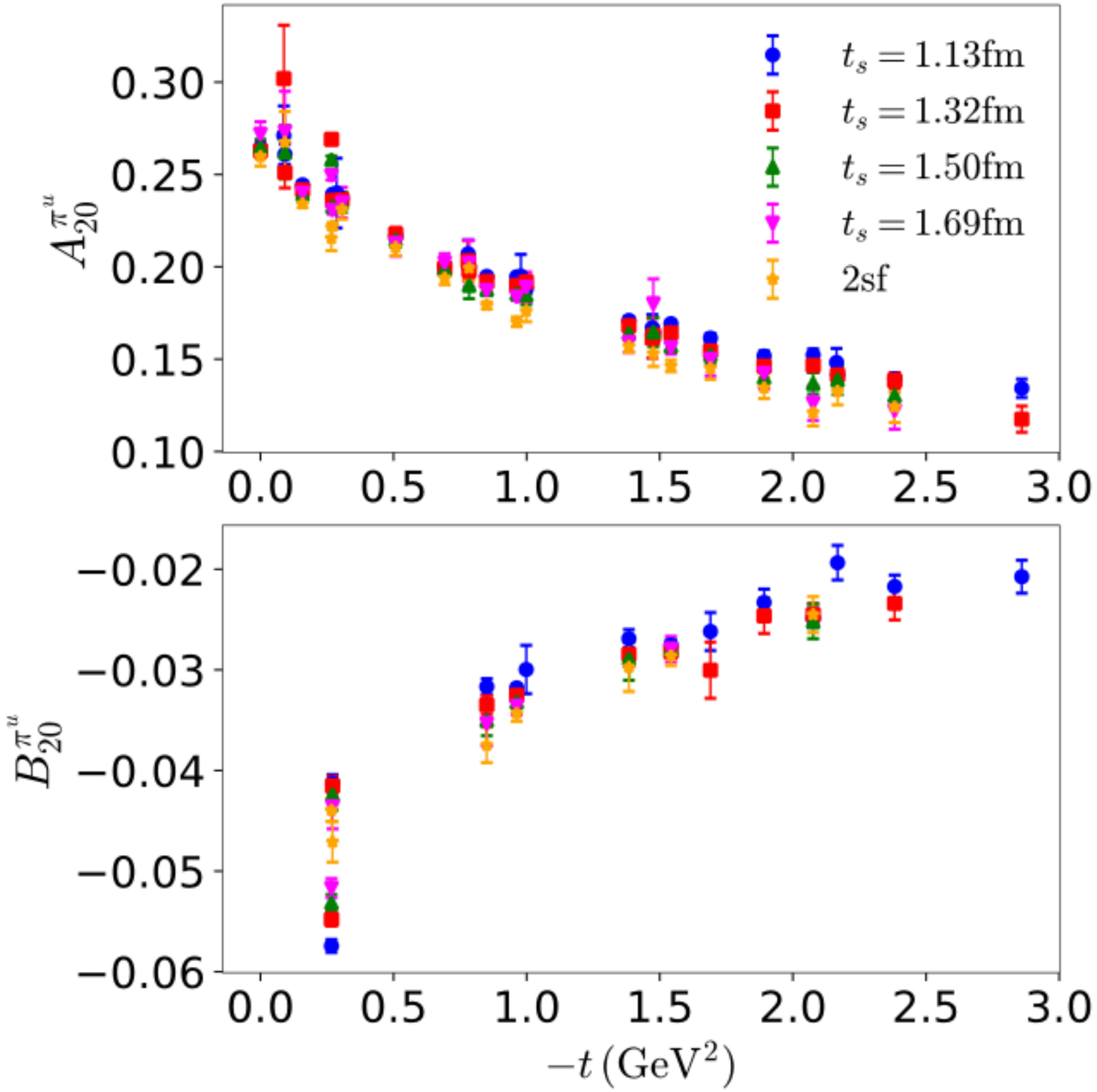
Pion Generalized Form Factors

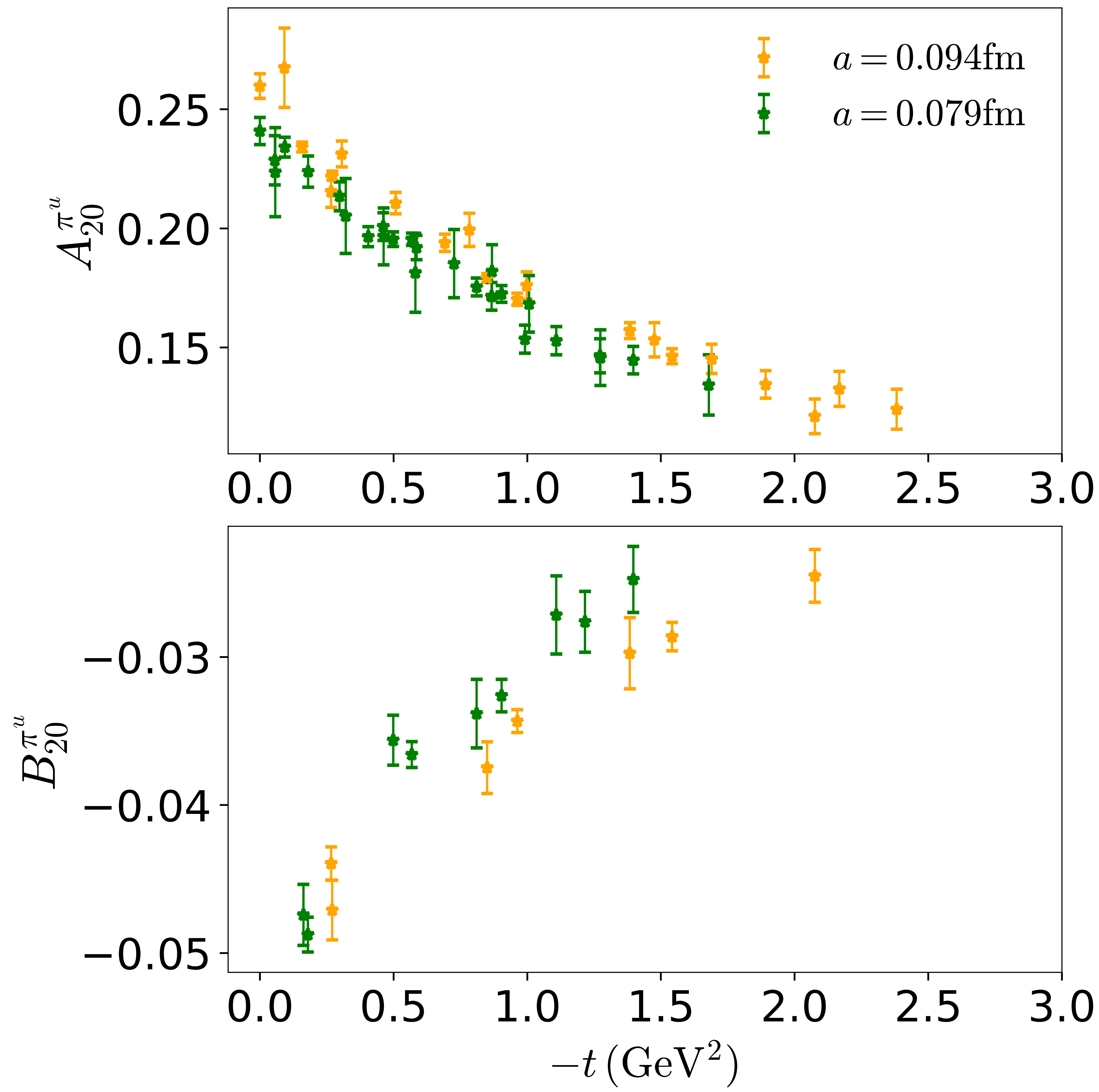


- Increase of statistical error is not linear with $\vec{q}$ $(-t = \vec{q}^{\,2} - (E(p') - E(p))^2)$

→ a careful analysis required to select data points with controlled uncertainties

Pion Generalized Form Factors





Comparison of 2-state fits:

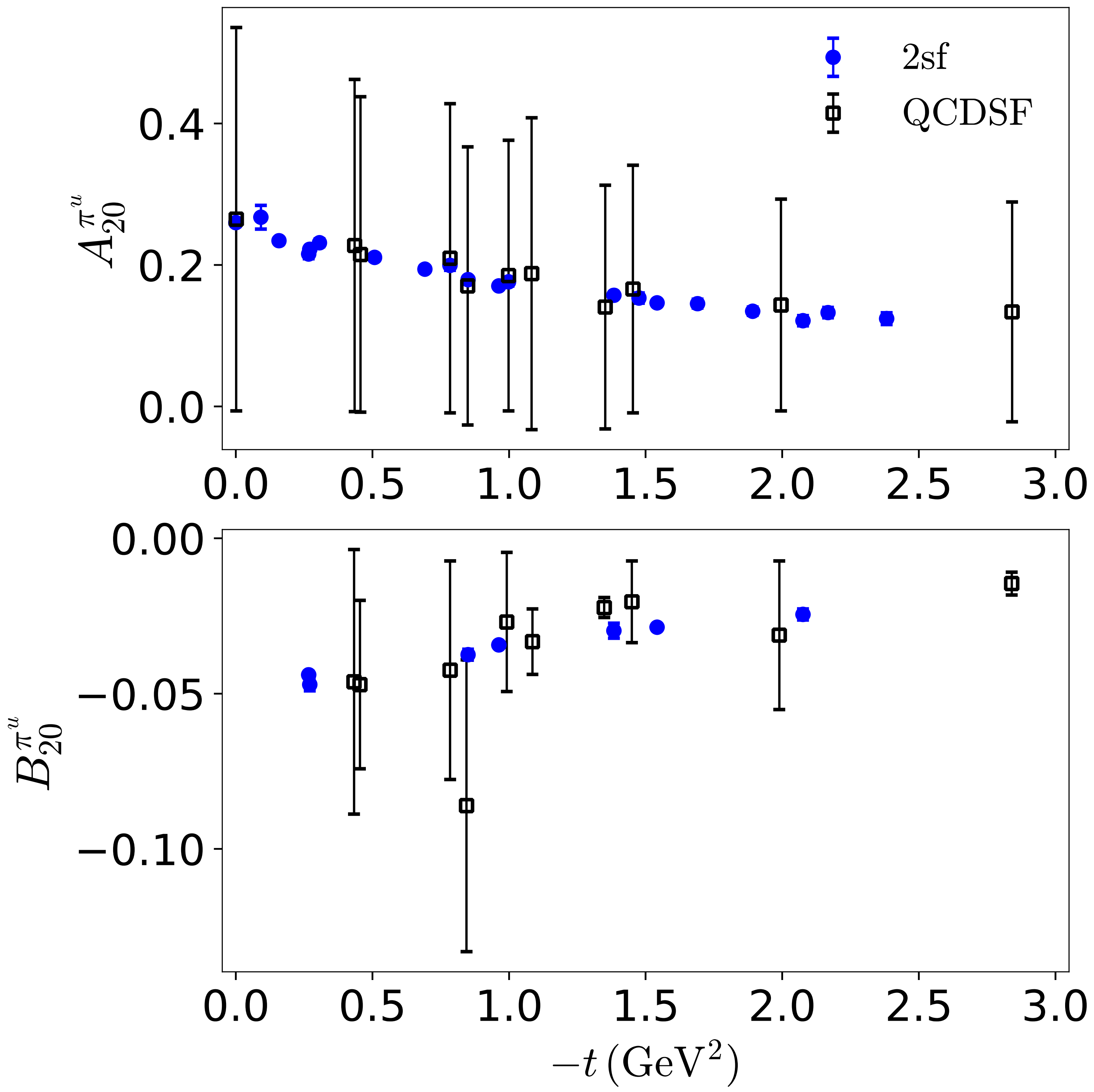
- Systematic effects more prominent in B_{20}
- Discretization effects less significant than scalar, vector, tensor FFs

- Increase of statistical error is not linear with $\vec{q}$ $(-t = \vec{q}^{\,2} - (E(p') - E(p))^2)$

$\rightarrow$ a careful analysis required to select data points with controlled uncertainties

Pion Generalized Form Factors

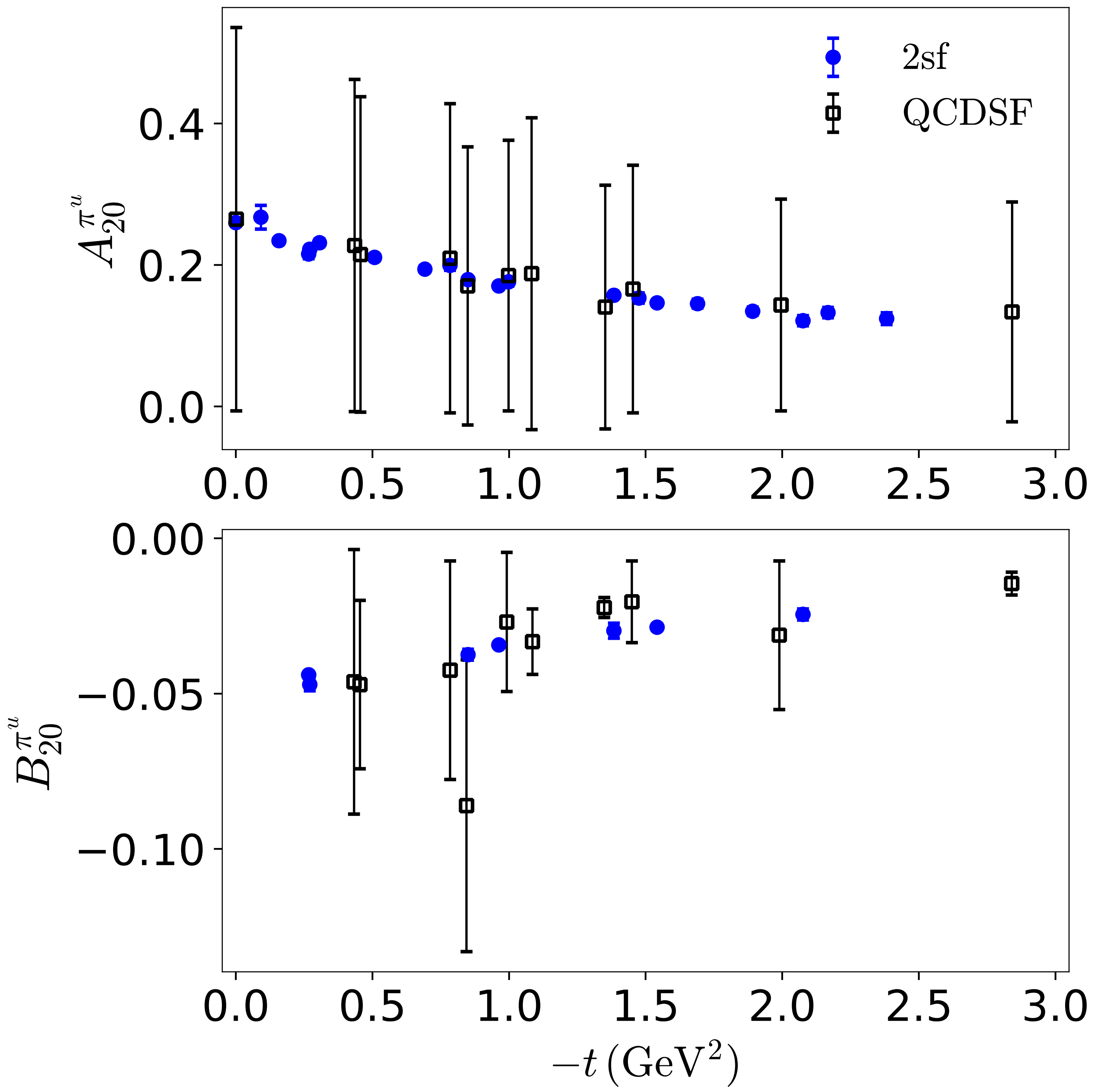
Pion Generalized Form Factors



- Results fully compatible with QCDSF [PoS(LAT2005)360]

Pion Generalized Form Factors

Comparison with other studies



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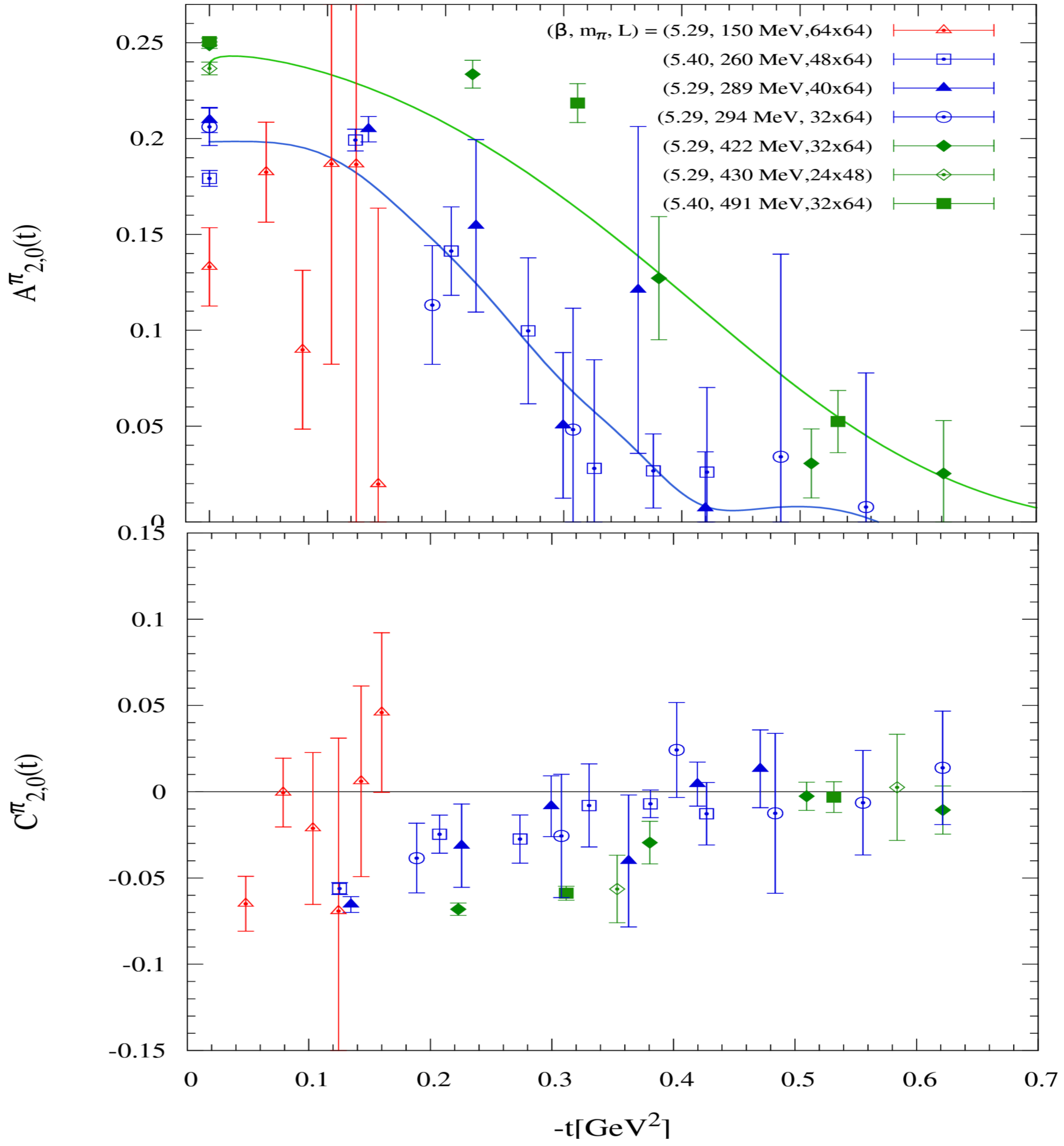

- RQCD demonstrates large systematic uncertainties [PoS LAT2013 (2014) 447]

Pion Generalized Form Factors

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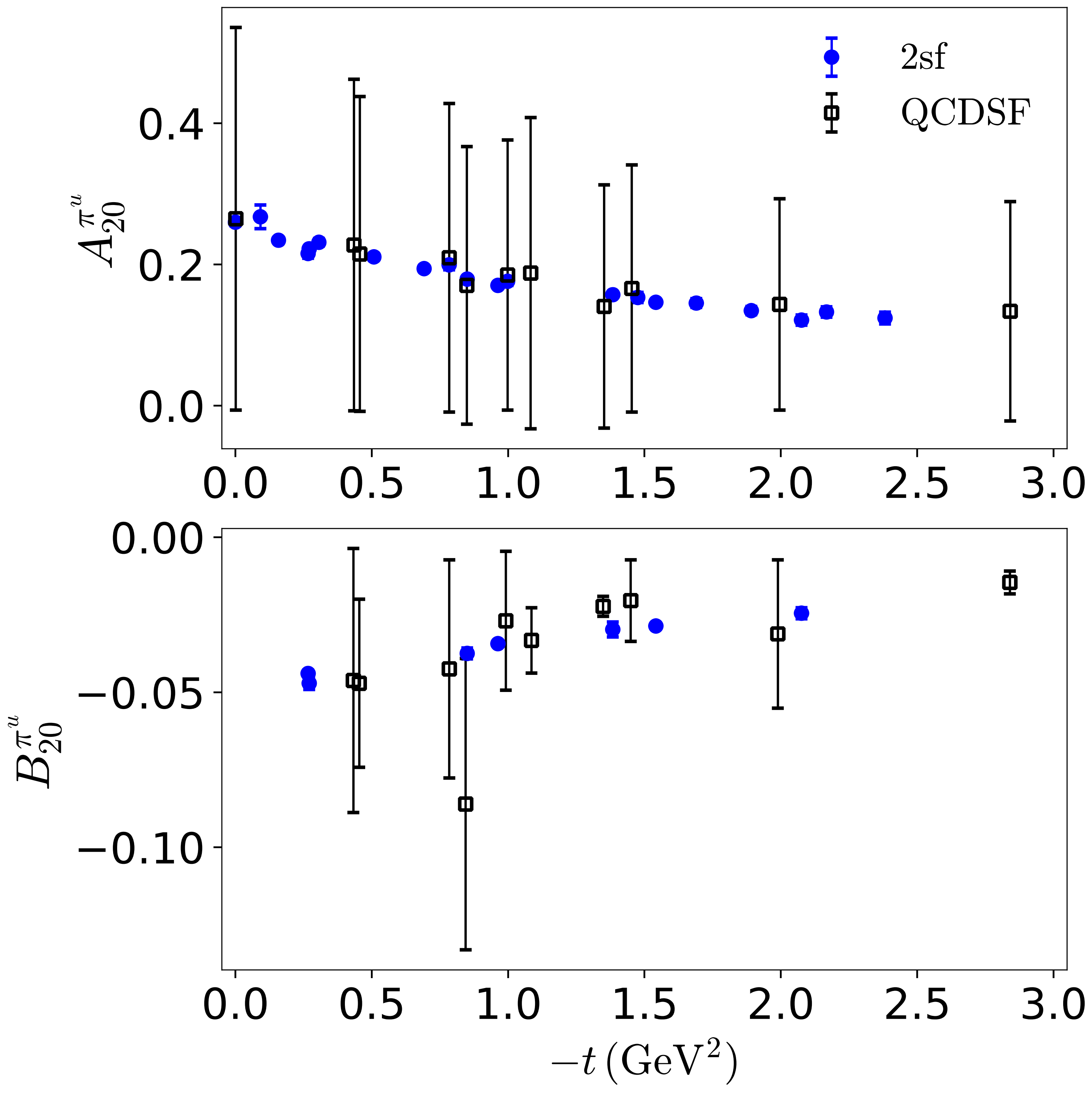
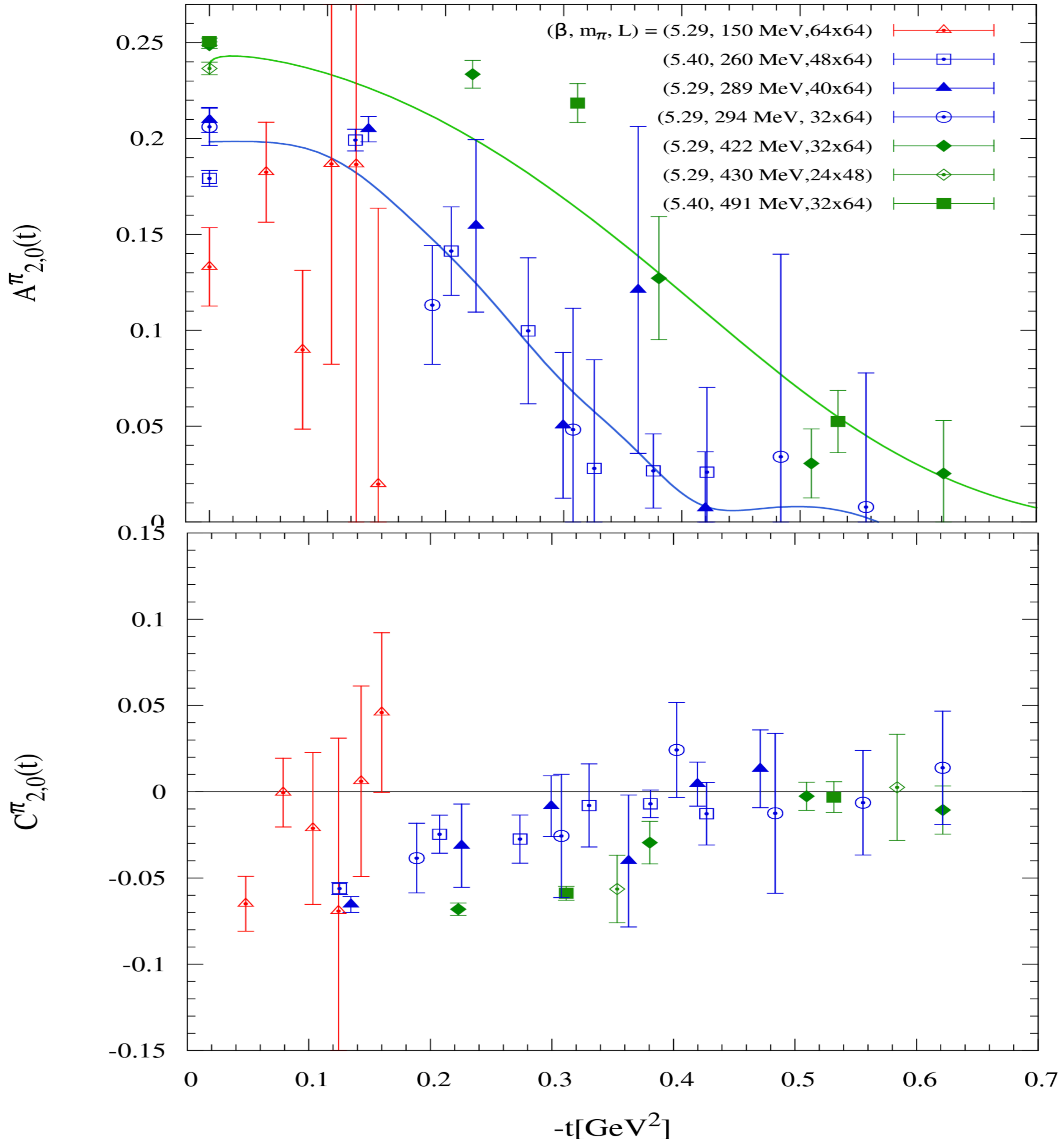
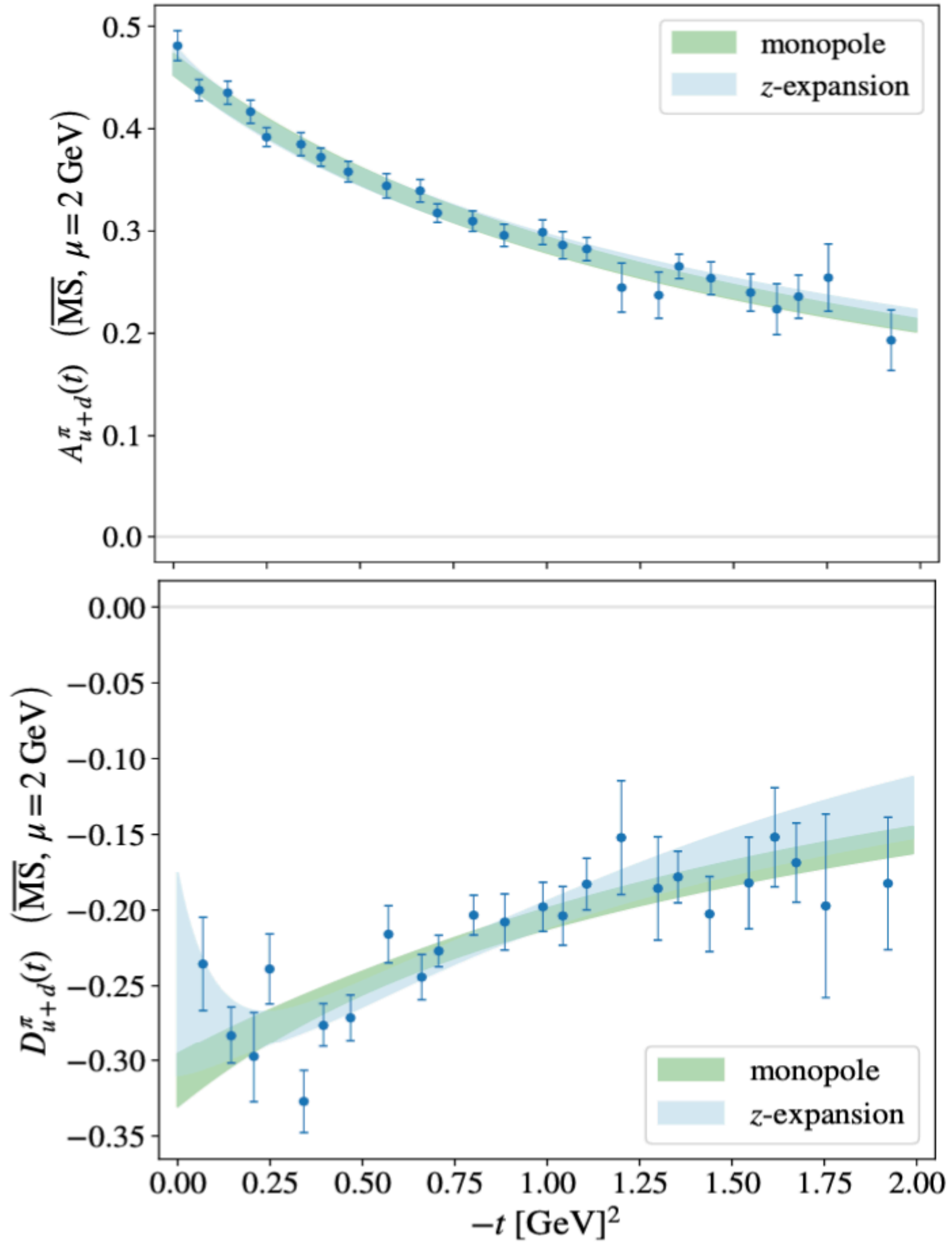


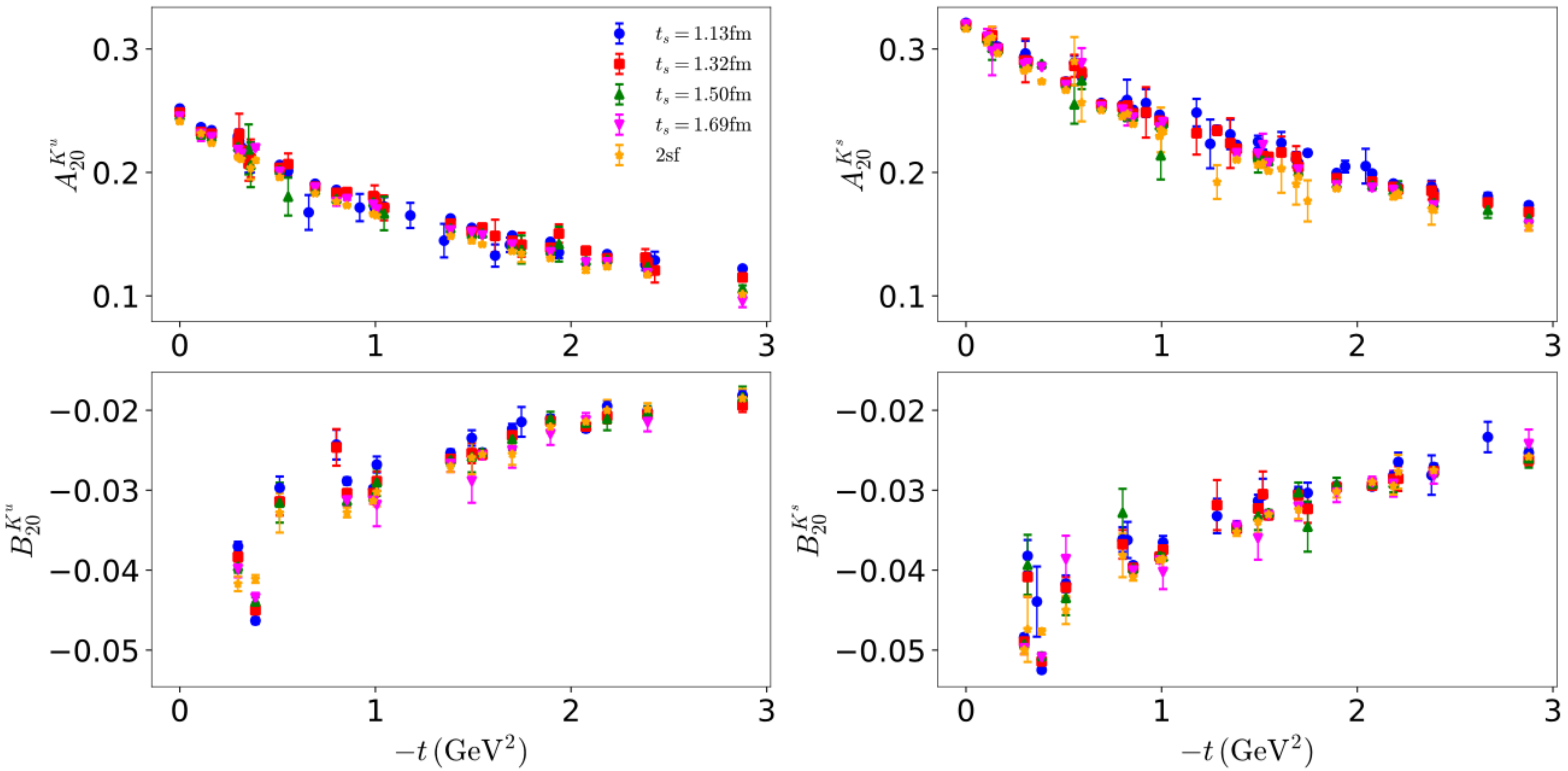




- Results fully compatible with QCDSF [PoS(LAT2005)360]
- RQCD demonstrates large systematic uncertainties [PoS LAT2013 (2014) 447]
- Work by MIT (arXiv:2307.11707) includes disconnected contributions and elimination of mixing with gluon contributions (see talk by D. Pefkou)

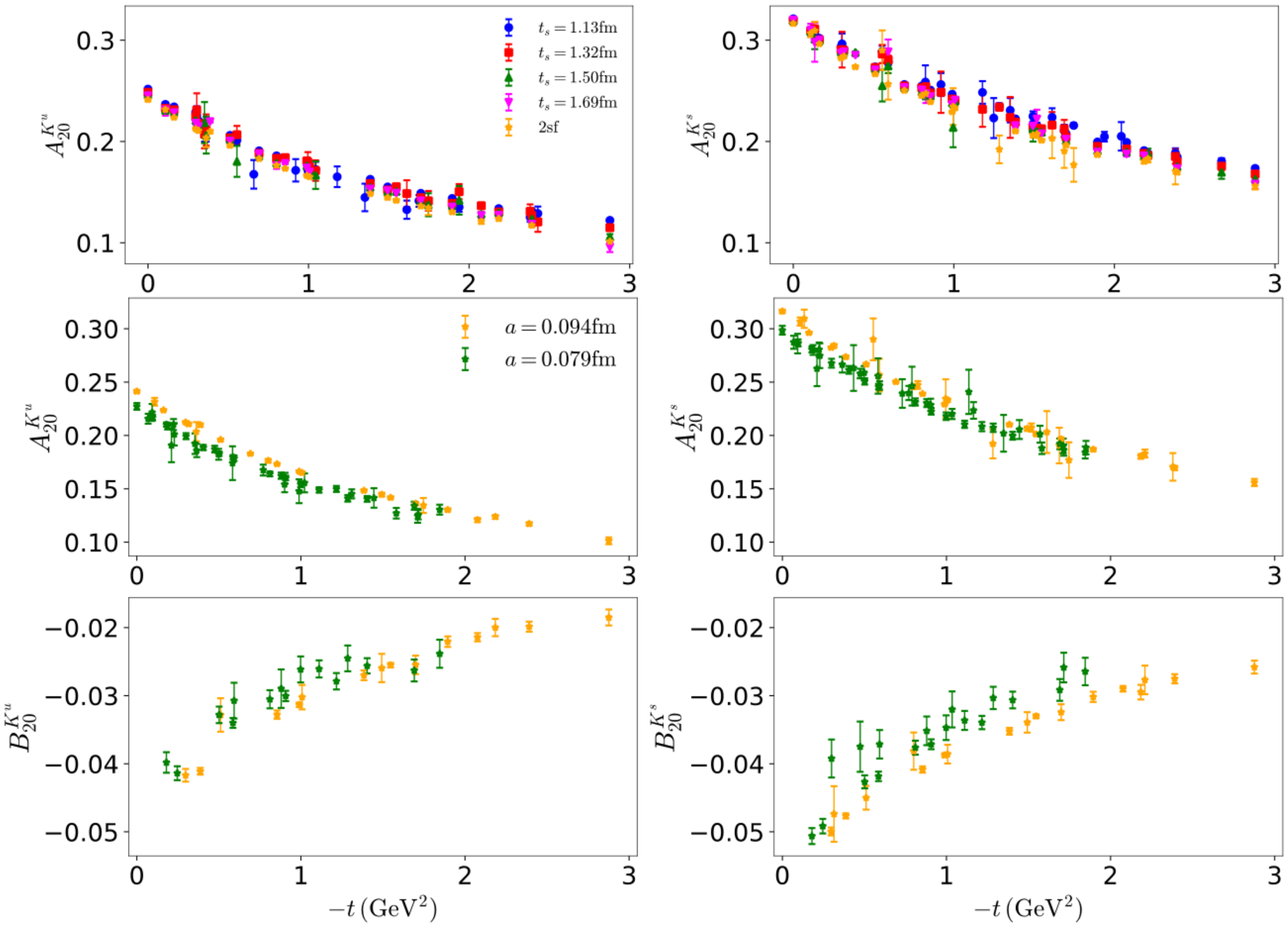
Kaon Generalized Form Factors

Kaon Generalized Form Factors



- Less statistical noise compared to pion
- Dense range of –t values
- Strange contributions dominant in magnitude
- Small excited state effects (typically within errors)

Kaon Generalized Form Factors



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- Dense range of –t values
- Strange contributions dominant in magnitude
- Small excited state effects (typically within errors)

- Accuracy of data reveals systematic effects in both A_{20} and B_{20}

Pion Parameterization

- Generalized form factors provide information on QCD dynamics (eg. SU(3) flavor symmetry effects)

  We parameterize the $-t$ dependence using a monopole (n=1) and an n-pole fit $F_\Gamma(Q^2) = \dfrac{F_\Gamma(0)}{\left(1 + \frac{Q^2}{M_\Gamma^2}\right)^n}$
- Other fit functions to be explored (e.g., z-expansion)

- Examined effect of $-t$ range included in fit (~1 GeV^2, ~3 GeV^2)

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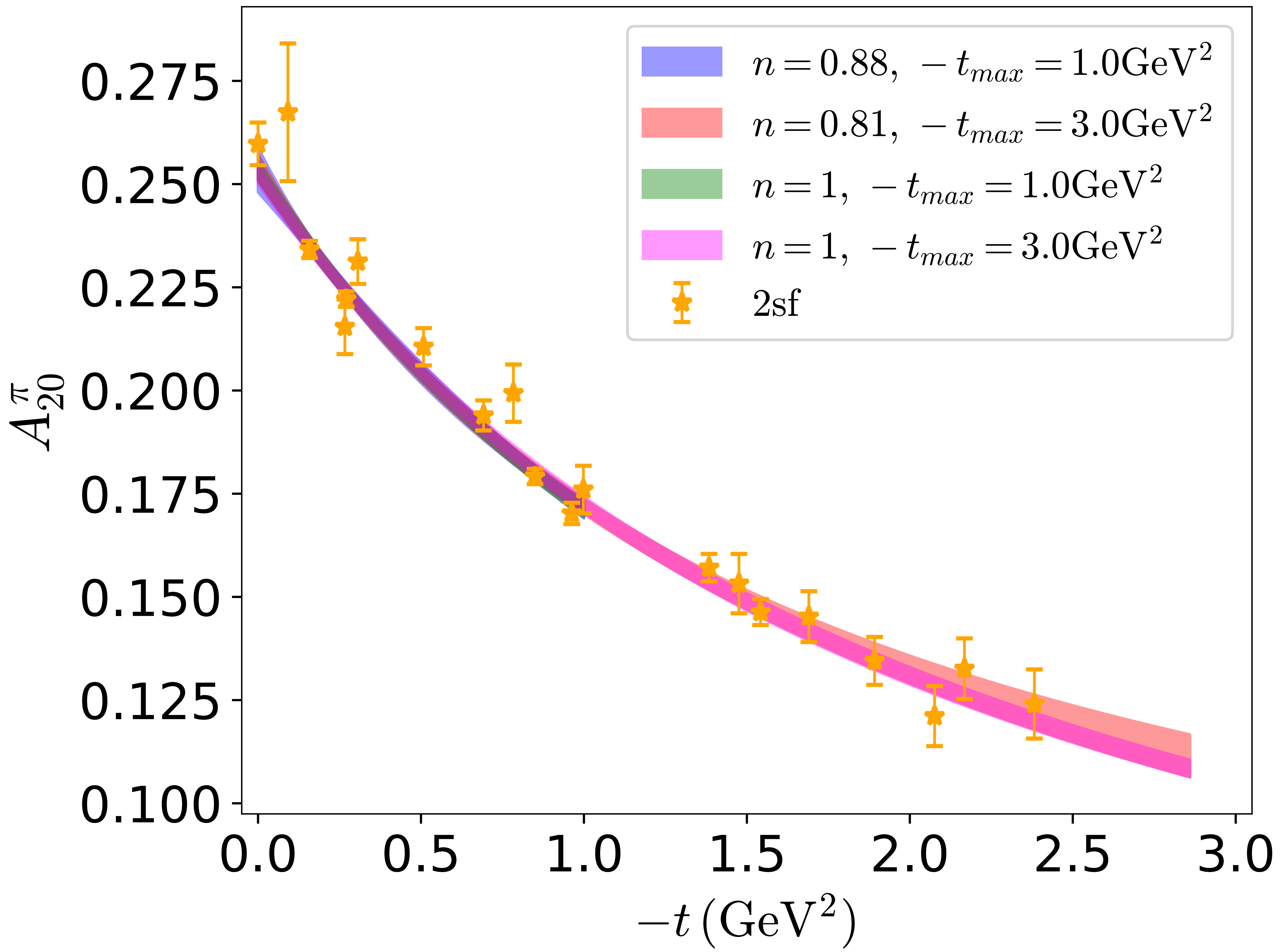

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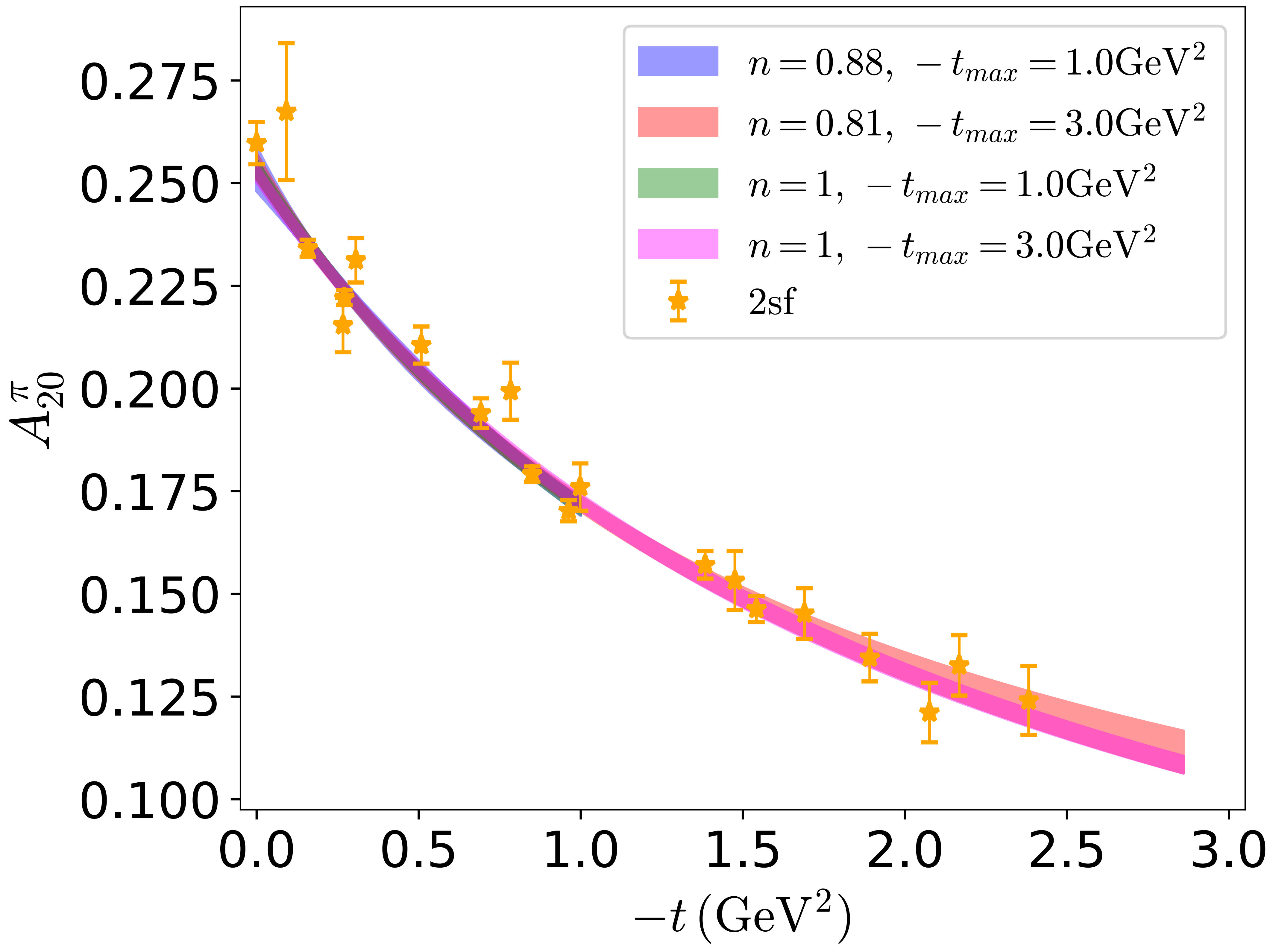

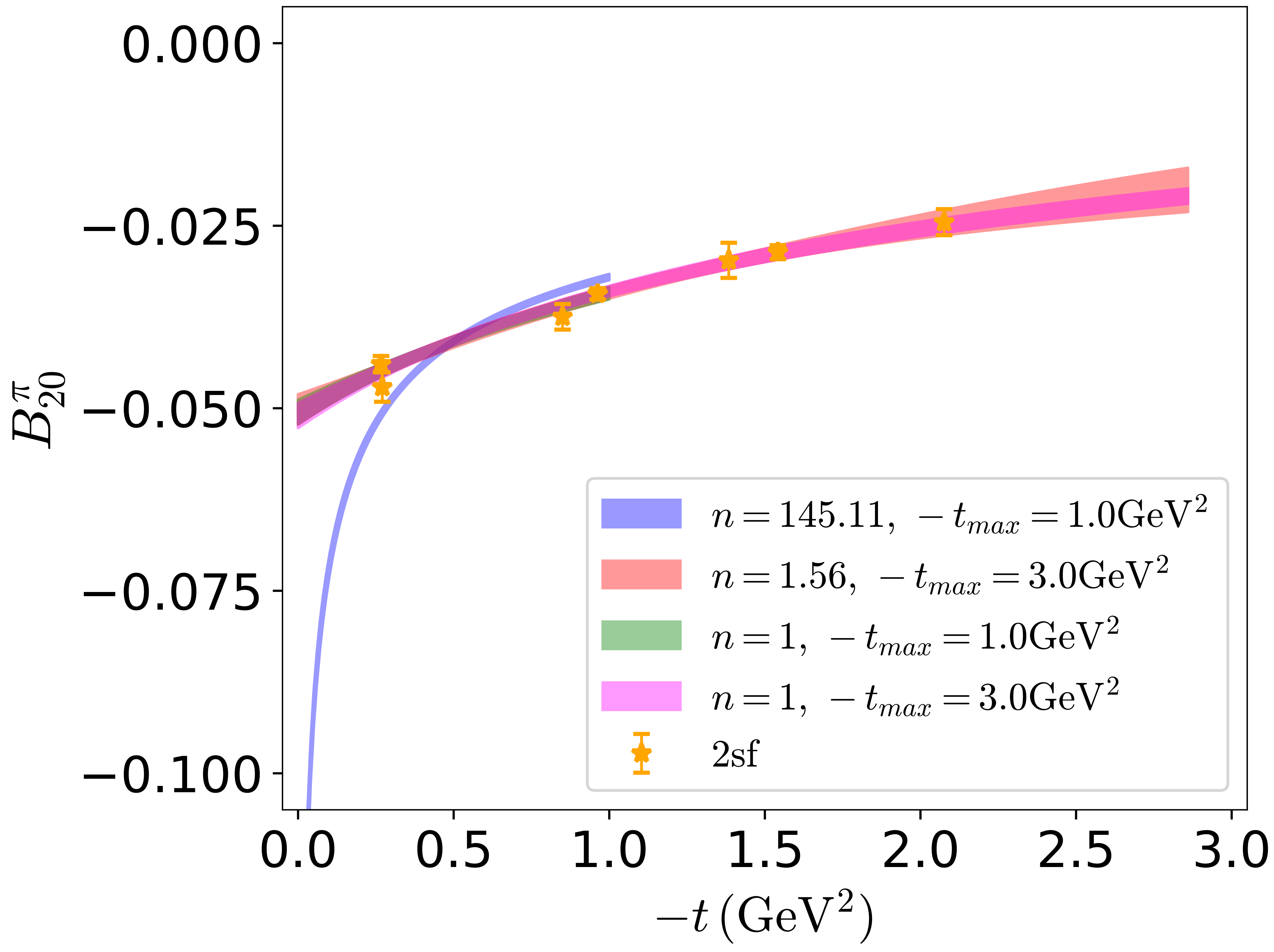

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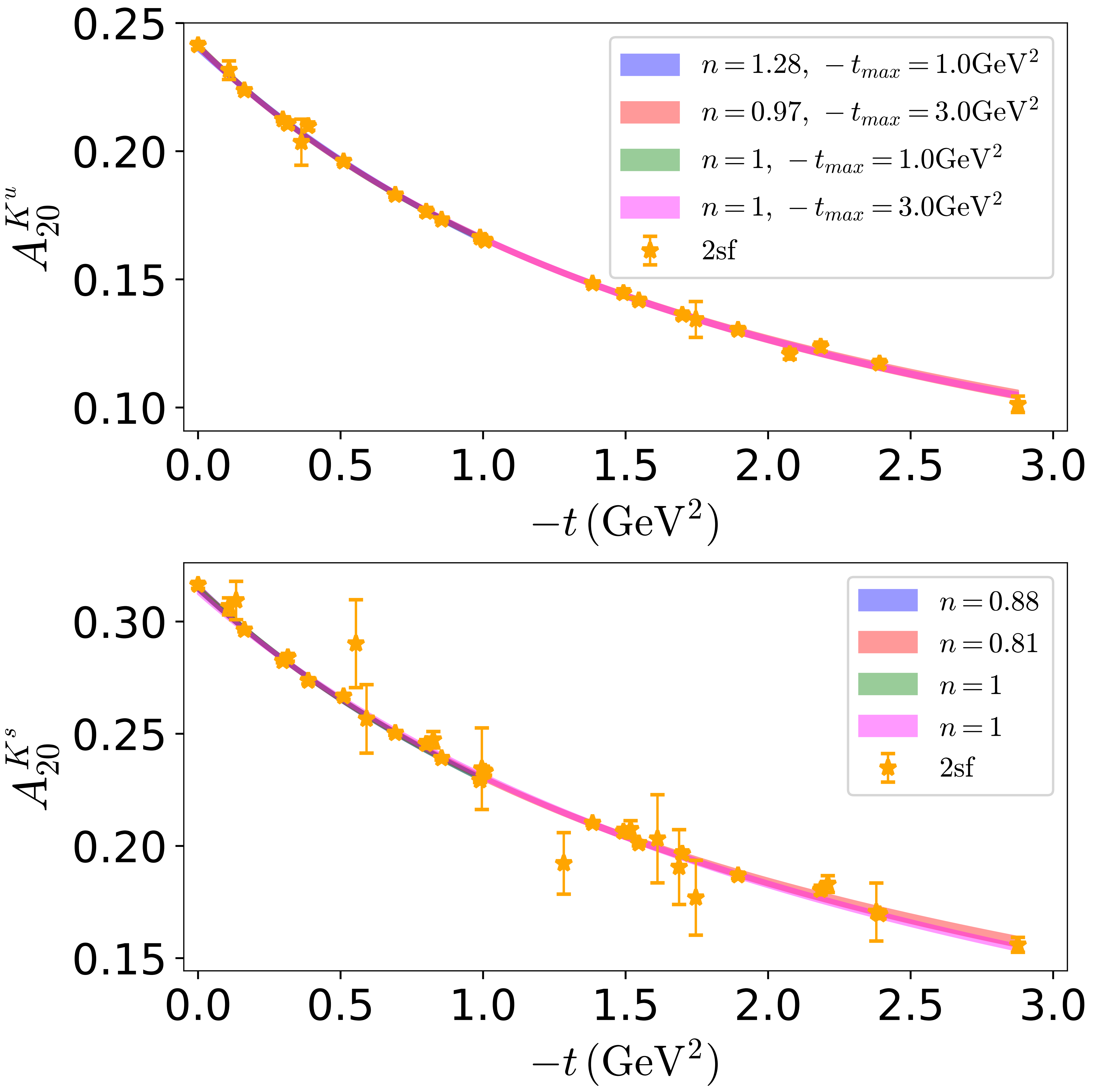

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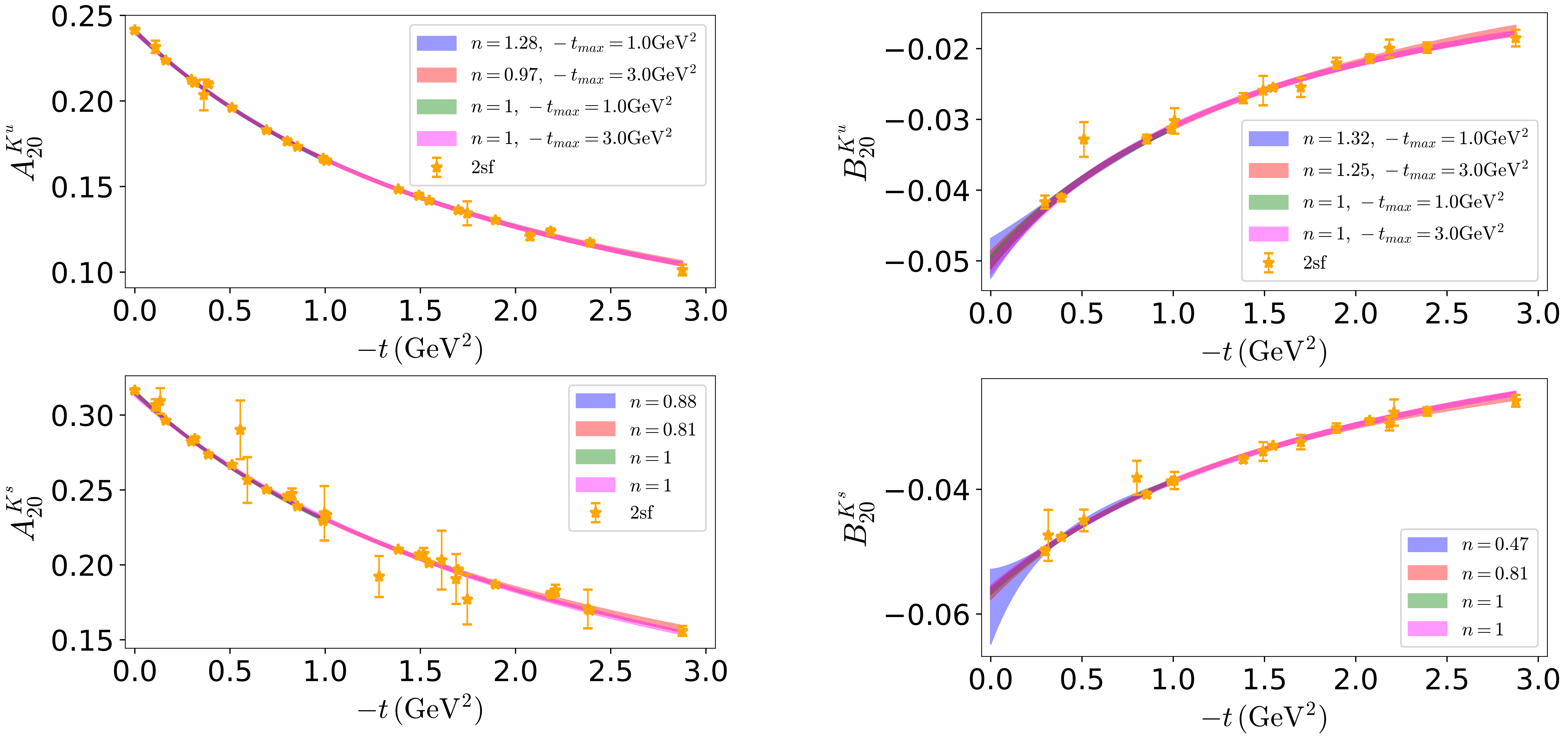

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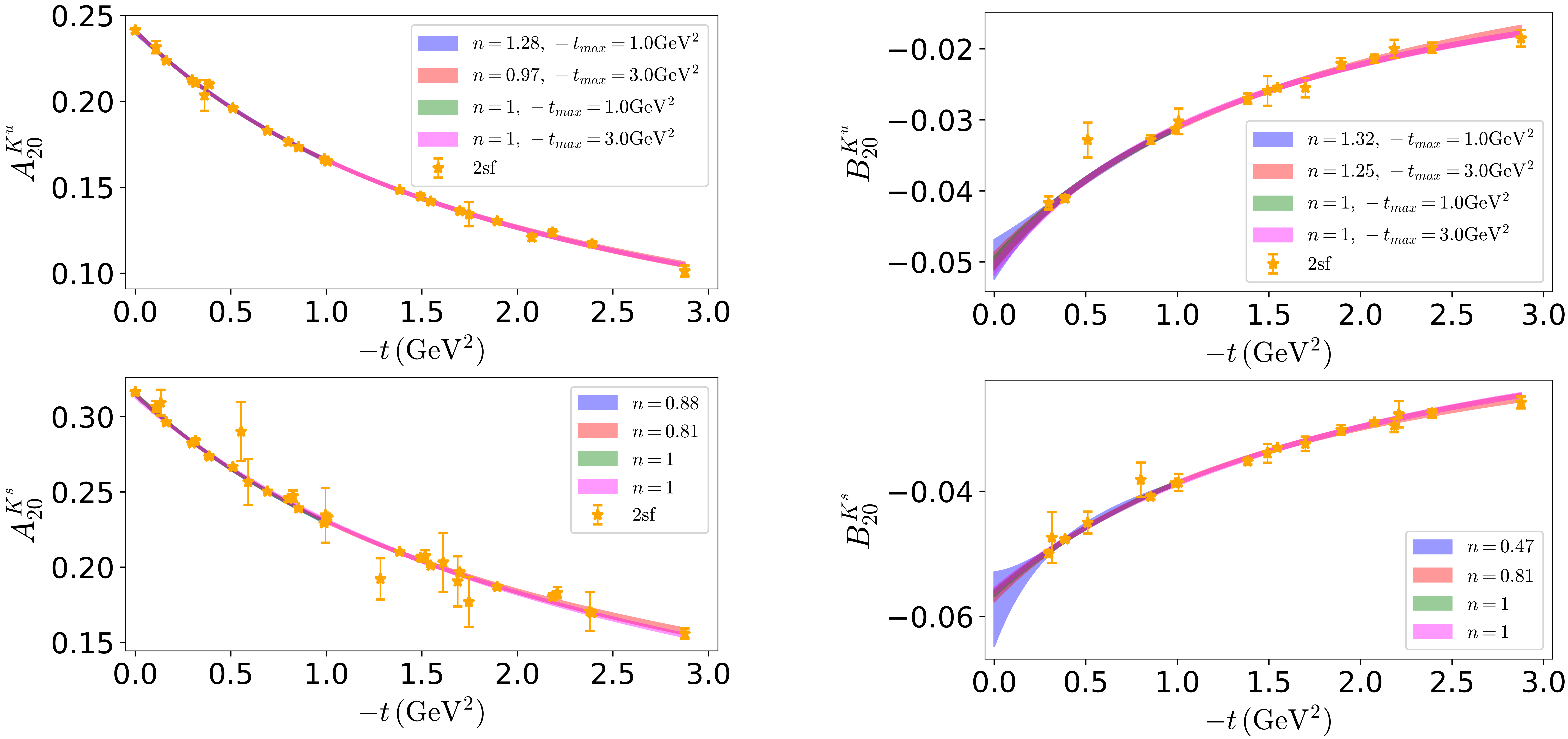

- All parametrizations describe the data very well
- Better stability in the fit for B_{20} in both flavors for the kaon compared to the pion

SU(3) Flavor Symmetry Breaking

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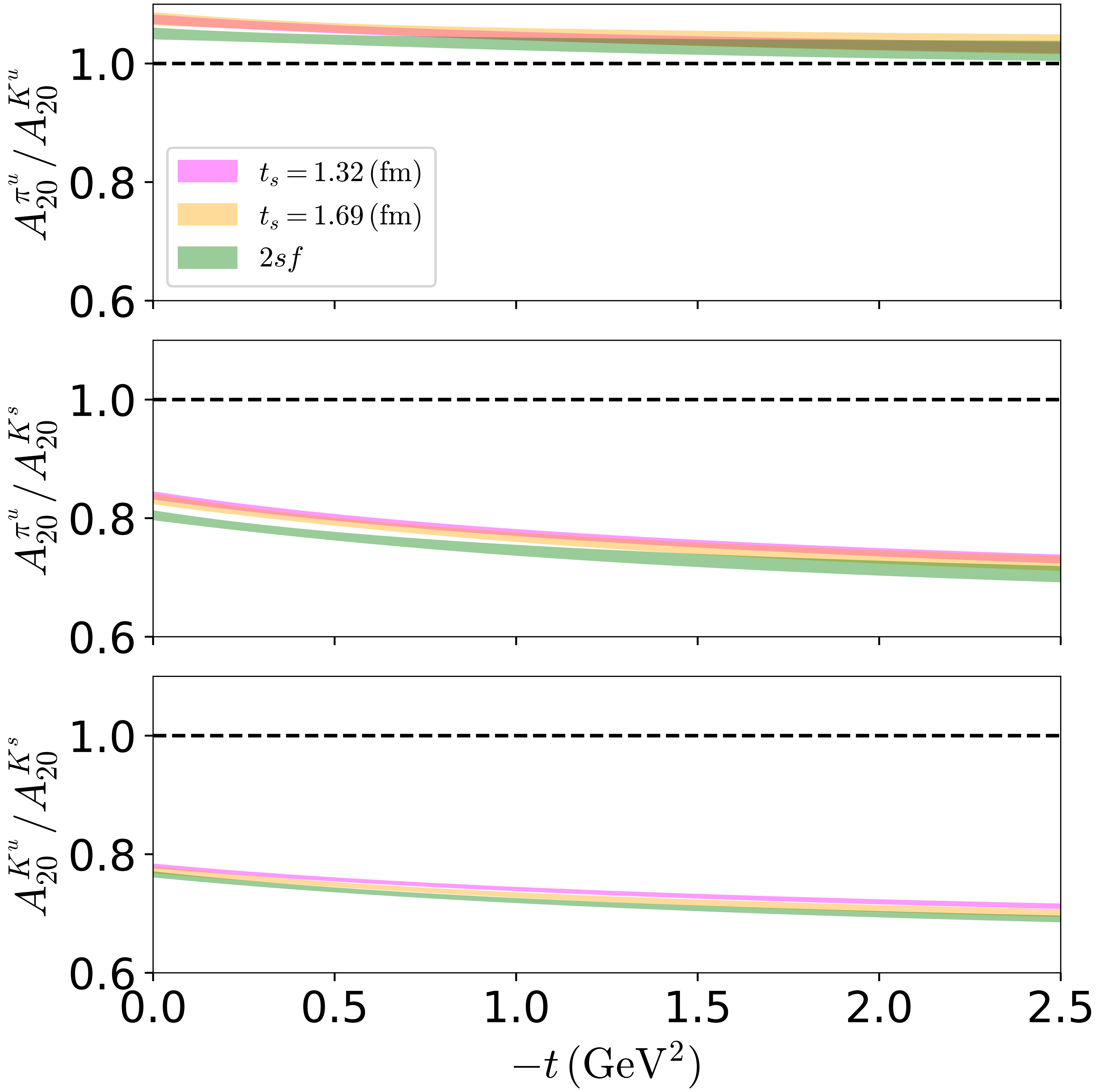

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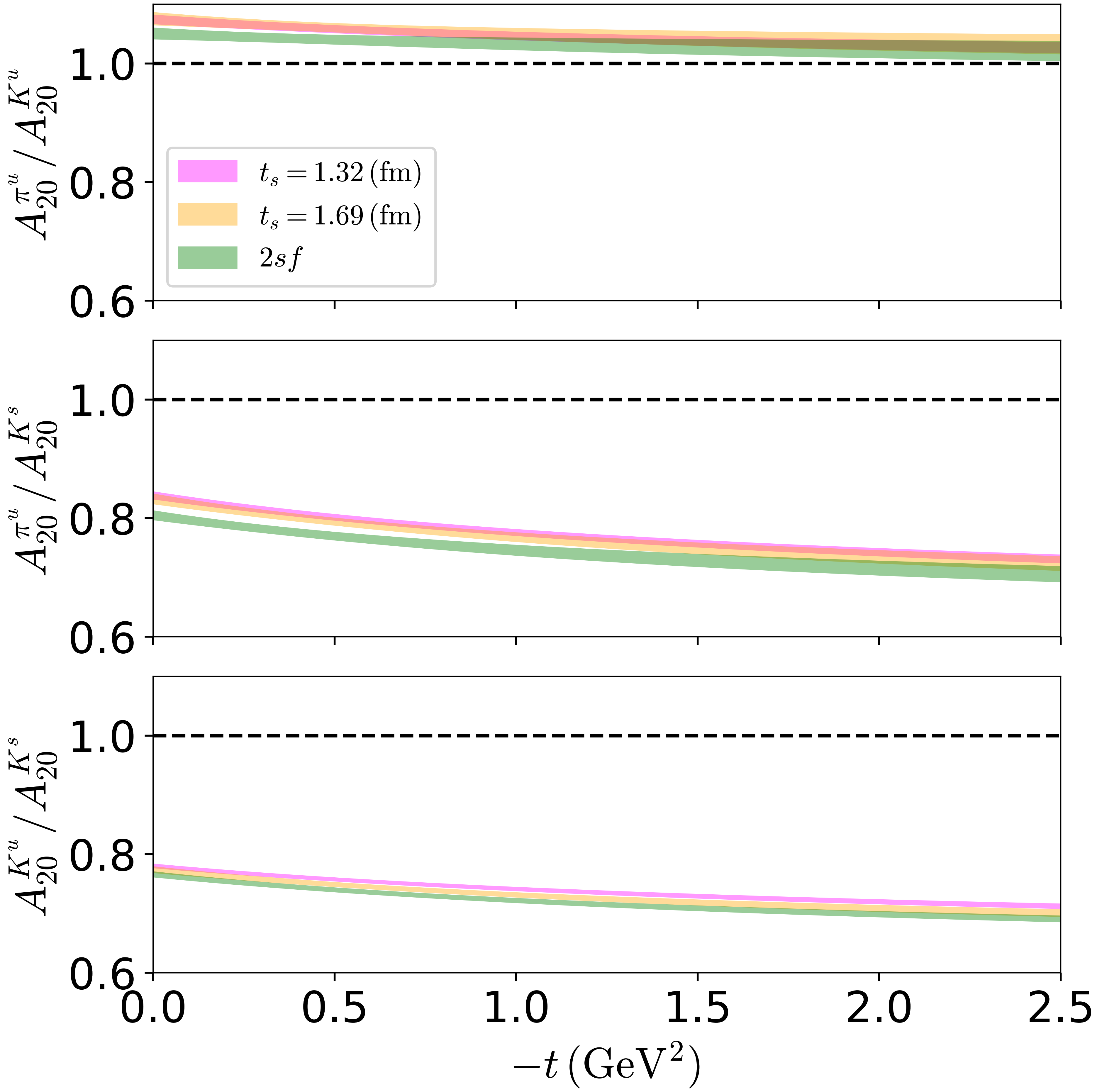
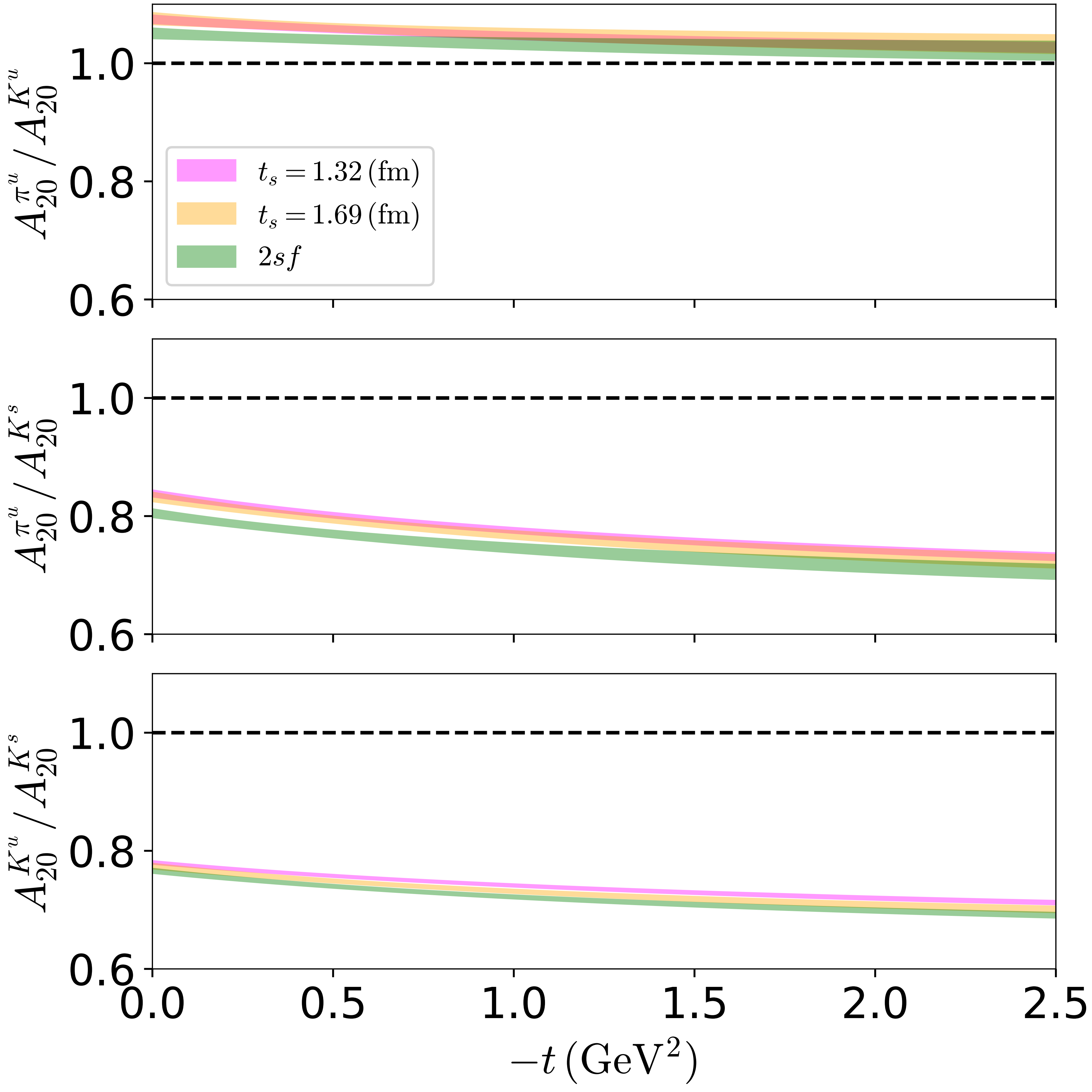


- Plots show ratios of parametrized data (monopole fit)
- Up quark equivalent contribution in both particles (about 5% - 10% higher in pion)
- SU(3) breaking effects between up and strange quarks at 20% for forward limit and up to 30% for $-\mathrm{t} = 2.5\ \mathrm{GeV}^2$

SU(3) Flavor Symmetry Breaking



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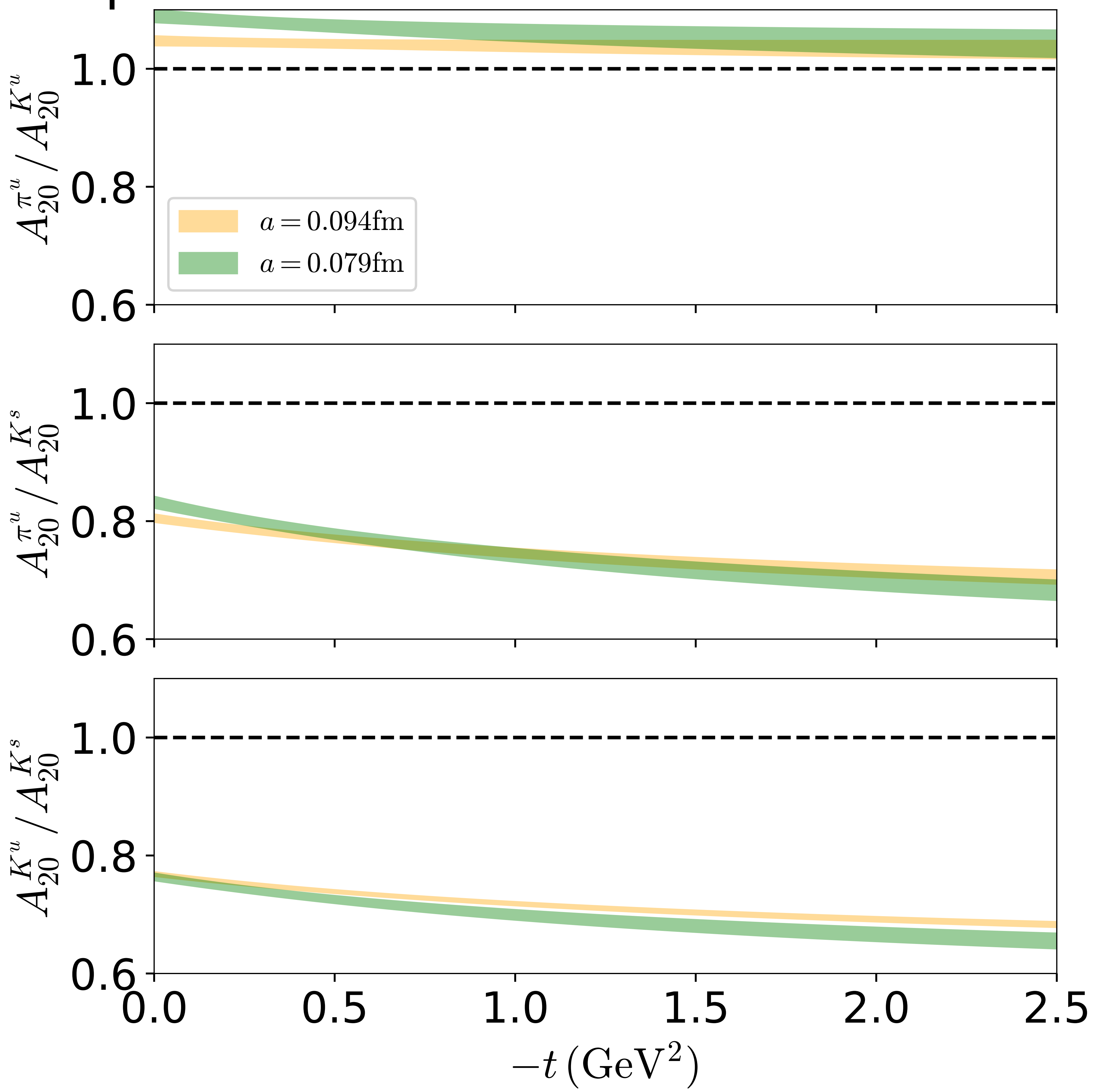

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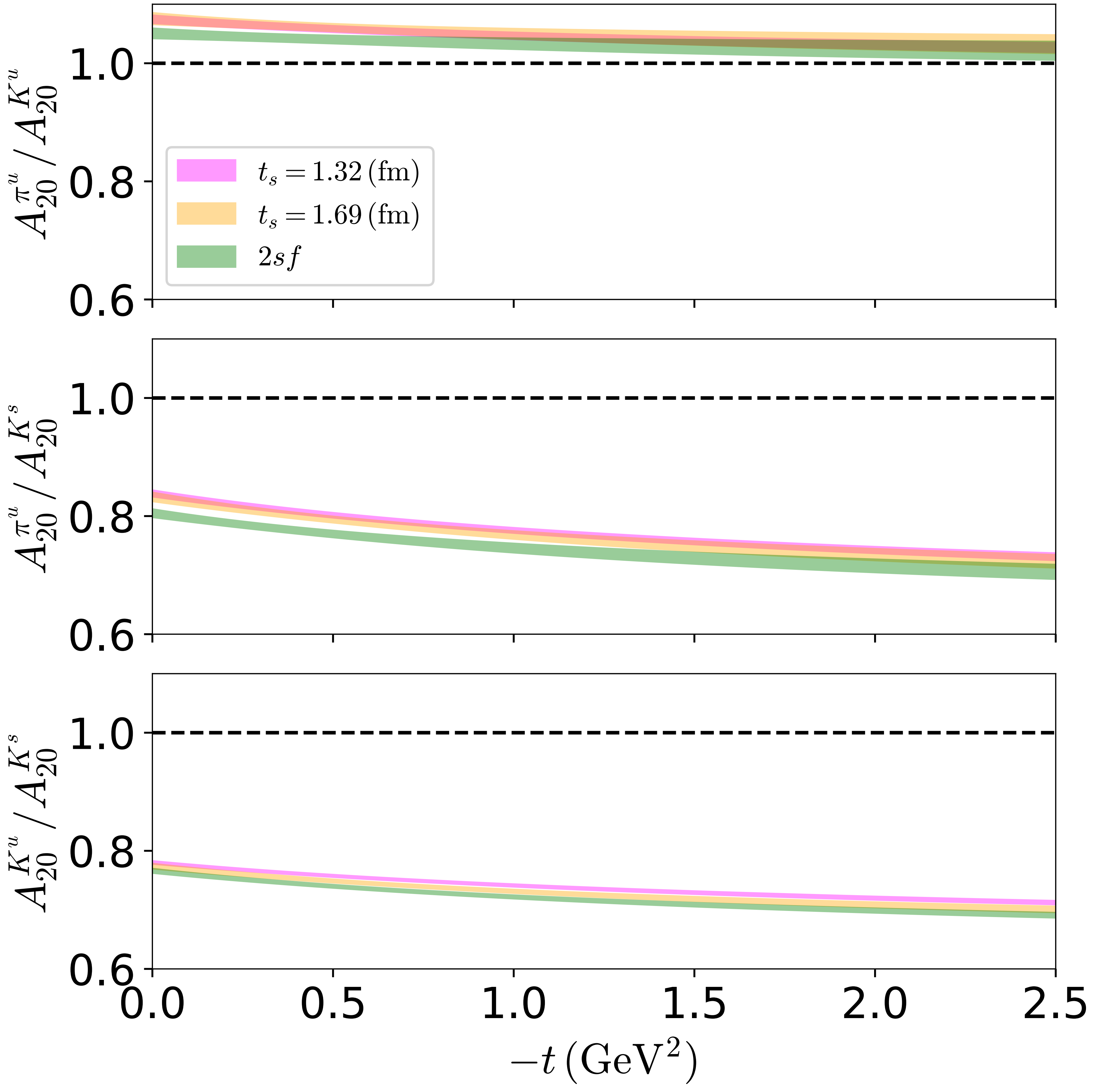

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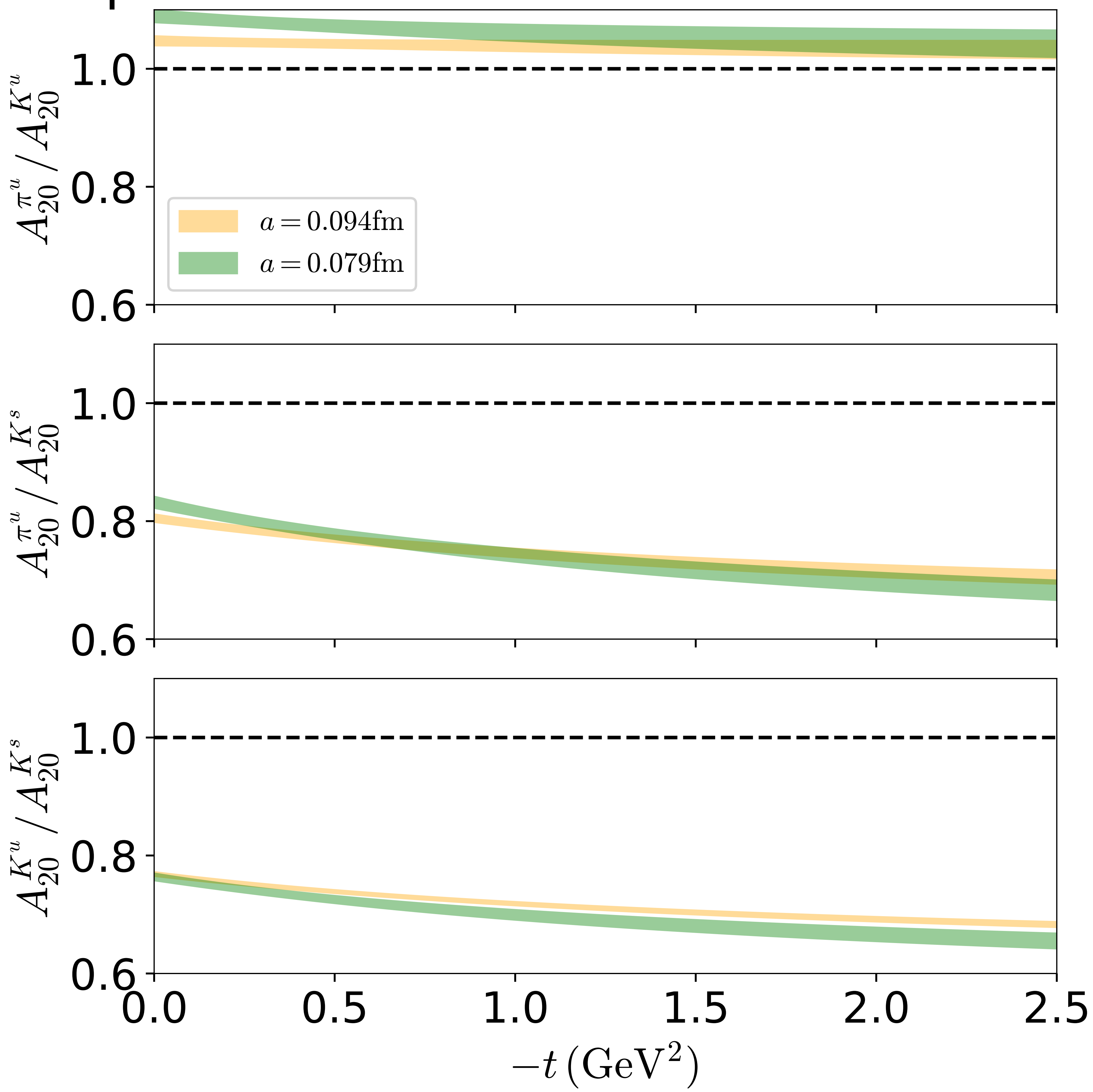
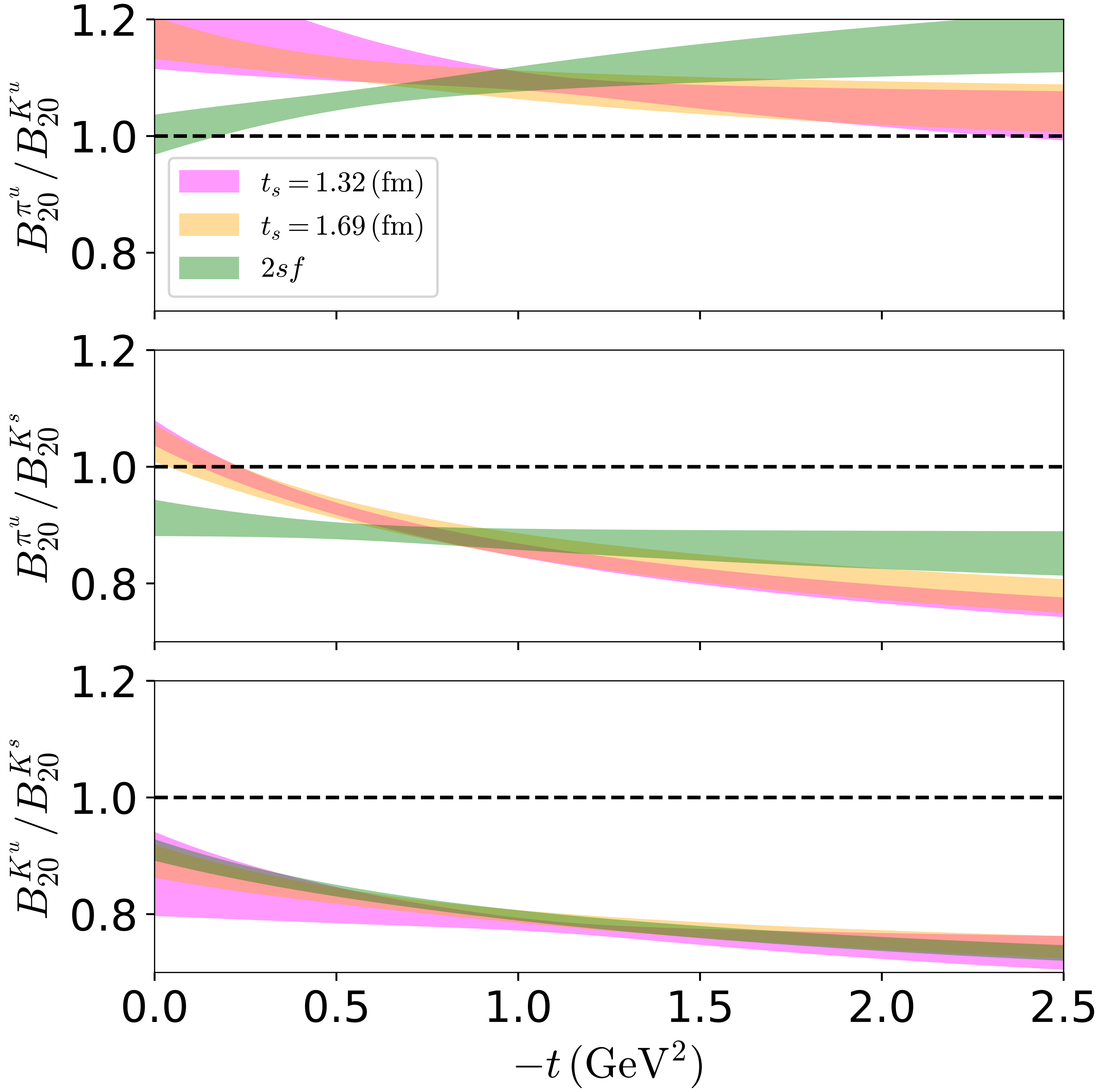


- Similar behavior between ensembles and small differences in certain regions of $-\mathrm{t}$
- $\mathrm{a} = 0.079\ \mathrm{fm}$ has larger errors (lower statistics) but still under control

SU(3) Flavor Symmetry Breaking

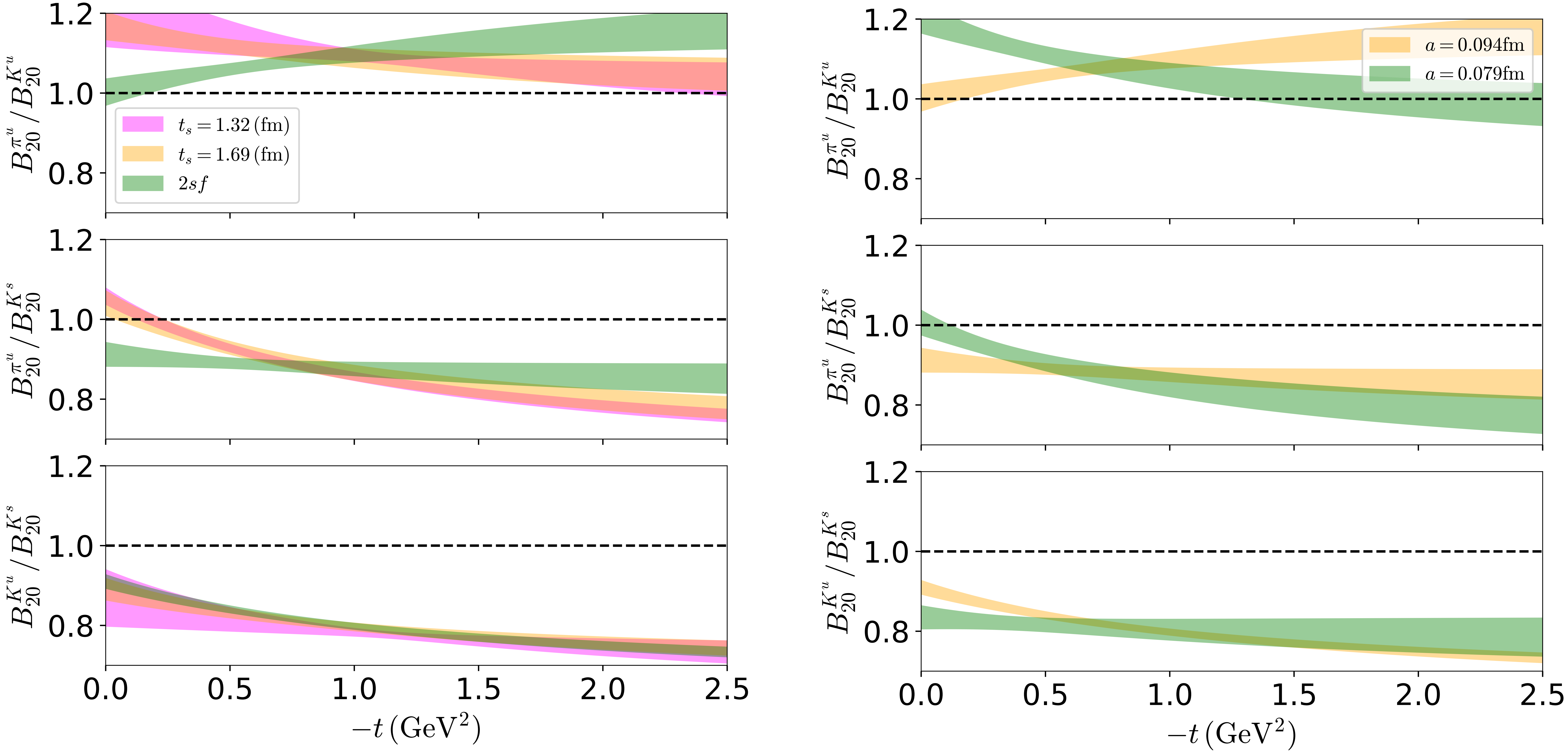


- Ratio affected by small values of B_{20} and large uncertainties (particularly for the comparison of the role of the up-quark in the two particles)

- Excited-states effects more prominent than A_{20}

- 2-state fit: - small SU(3) flavor symmetry breaking between up and strange (~10%)
  - Up quark contribution increases in pion as $-t$ increases (up to ~ 20% effect)

SU(3) Flavor Symmetry Breaking



- Possible discretization and/or volume effects observed

- Ratio affected by small values of B_{20} and large uncertainties (particularly for the comparison of the role of the up-quark in the two particles)

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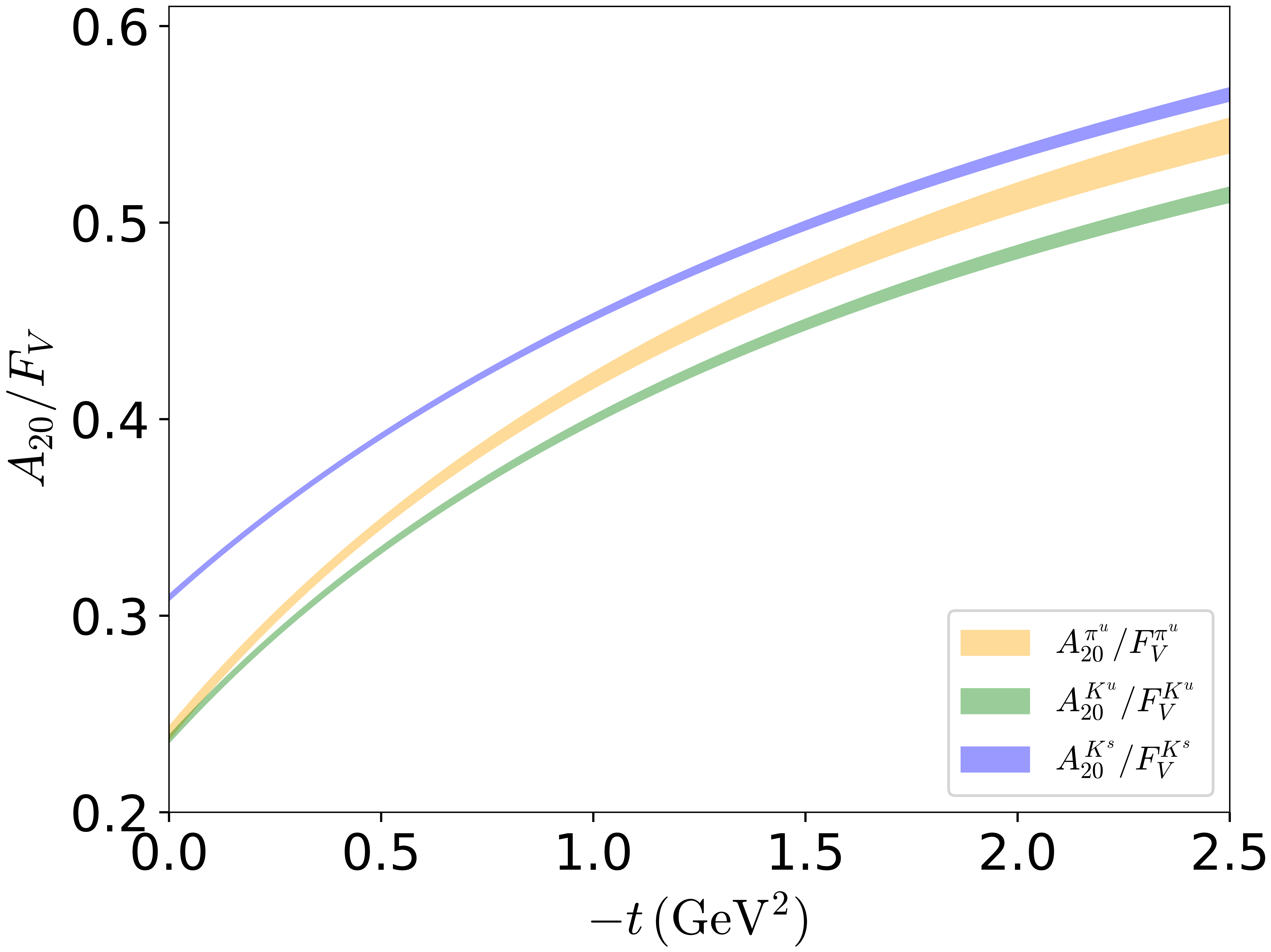
Comparison of different order of Mellin moments



- Higher Mellin moments receive support at higher values of x and are expected to suppress in value compared to lower Mellin moments.

- Ratio at $-t = 0$ equivalent to $A_{20}(0)$
- Slope of of A_{20} different than of F_V (similar behavior for both particles/flavors)
- Values of A_{20} becomes about 50% of F_V at $-\mathrm{t} = 2.5\ \mathrm{GeV}^2$

Mellin Moments of PDFs (Preliminary B-ensemble)

- We calculate the Mellin moments of PDFs up to $\langle x^3 \rangle$ with non-perturbative renormalization using operators that avoid mixing
- Pion Mellin moments $\langle x^n \rangle$:

$$\langle x \rangle_A^{\pi} = 0.273(07) \;\; \langle x^2 \rangle_A^{\pi} = 0.112(06) \;\; \langle x^3 \rangle_A^{\pi} = 0.036(13)$$

$$\langle x \rangle_B^{\pi} = 0.227(14) \;\; \langle x^2 \rangle_B^{\pi} = 0.090(12) \;\; \langle x^3 \rangle_B^{\pi} : \text{too noisy}$$

- Kaon Mellin moments $\langle x^n \rangle$:

$$\langle x \rangle_A^{K^u} = 0.243(5) \;\; \langle x^2 \rangle_A^{K^u} = 0.096(2) \;\; \langle x^3 \rangle_A^{K^u} = 0.042(6)$$

$$\langle x \rangle_B^{K^u} = 0.234(6) \;\; \langle x^2 \rangle_B^{K^u} = 0.089(4) \;\; \langle x^3 \rangle_B^{K^u} = 0.060(11)$$

$$\langle x \rangle_A^{K^s} = 0.307(7) \;\; \langle x^2 \rangle_A^{K^s} = 0.129(4) \;\; \langle x^3 \rangle_A^{K^s} = 0.060(12)$$

$$\langle x \rangle_B^{K^s} = 0.320(4) \;\; \langle x^2 \rangle_B^{K^s} = 0.138(2) \;\; \langle x^3 \rangle_B^{K^s} = 0.082(4)$$

Some differences observed outside statistical errors that lead to indication for systematic uncertainties

Summary

- Calculation of pion and kaon generalized form factors fully accessible from a boosted kinematic frame up to 3 GeV^2
- Lattice data reveal systematic effects attributed to a combination of finite-a and finite volume
- Parametrization of A_{20} and B_{20} favors a monopole fit
- SU(3) flavor symmetry breaking observed in u-quark (both particles) vs s-quark (up to 20-30%)
- SU(3) flavor symmetry breaking effects different between A_{20} and B_{20}
- Ratio of A_{20} and the vector FF shows up to 50% suppression in value

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Future Work

- Kinematic setup designed to access generalized form factors up to 3-derivative operator
- Higher statistics required to go beyond A_{20} and B_{20}
- Extraction of second Mellin moment of the quark probability density in impact parameter space, ρ, with addition of tensor GFFs
- Addition of two more ensembles with finer lattice spacing for continuum limit

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