On the Baryon Octet: Sigma Terms in the continuum limit from $N_f = 2 + 1$ QCD with Wilson fermions

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Why determine the sigma terms?

- quark mass contributions
  → decomposition of the hadron mass
- investigate flavour symmetry breaking in the baryon octet
  ⇒ nucleon N, lambda Λ, sigma Σ and xi Ξ (in our setup $m_u = m_d$)
- WIMP-nucleon scattering cross-sections
  (e.g. XENON1T)
- discrepancies between results for the nucleon pion sigma term from LQCD and phenomenology still to be resolved
- very few determinations for hyperons
How are the sigma terms defined?

\[ \sigma_{qB} = m_q \langle B | \bar{q} 1 q | B \rangle \]

- quark mass \( m_q \) and a scalar current inserted
- \( B \) refers to the ground state of a \textbf{baryon} \( B \) \textbf{at rest}.

We’re interested in:

- \textbf{strange sigma terms} \( \sigma_{sB} \)
- \textbf{pion sigma terms} \( \sigma_{\pi B} = \sigma_{uB} + \sigma_{dB} \)

\textbf{renormalisation} via normalisation factor \( r_m \) (determined by ALPHA, RQCD), the ratio of flavour non-singlet and singlet scalar density renormalisation parameters → accounts for the \textbf{mixing of quark flavours under renormalisation} for Wilson fermions

[ALPHA 2101.10969], [RQCD 2211.03744]
How to access the matrix element

→ spectral decompositions

\[
C_{2pt}(t_f) = \sum_{\vec{x}} \langle \mathcal{O}_{snk}(\vec{x}, t_f) \bar{\mathcal{O}}_{src}(\vec{0}, 0) \rangle = \sum_n |Z_n|^2 e^{-E_n t_f}
\]

where \( Z_n = \langle \Omega | \mathcal{O}_{snk} | n \rangle \) (vacuum state \( \Omega \)) is the overlap of the interpolator \( \mathcal{O}_{snk} \) onto the state \( n \)

\[
C_{3pt}(t_f, t) = \sum_{\vec{x}, \vec{y}} \langle \mathcal{O}_{snk}(\vec{x}, t_f) J(\vec{y}, t) \bar{\mathcal{O}}_{src}(\vec{0}, 0) \rangle - \sum_{\vec{x}, \vec{y}} \langle J(\vec{y}, t) \rangle \langle \mathcal{O}_{snk}(\vec{x}, t_f) \bar{\mathcal{O}}_{src}(\vec{0}, 0) \rangle
\]

\[
= \sum_{n,n'} Z_{n'} Z_n^* \langle n'|J|n \rangle e^{-E_n t} e^{-E_{n'}(t_f-t)}
\]

\( t_f \) is the source-sink separation & \( t \) is the insertion time of the current
Connected and disconnected contributions

\[ \bar{u} \Gamma u \]

\[ \bar{q} \Gamma q \]
How to access the scalar matrix element

**Ratio method** (cannot resolve $c_{11}$ (excited-to-excited) so far)

$$R(t_f, t) = \frac{C_{3pt}(t_f, t)}{C_{2pt}(t_f)} = g_S^q + c_{0\leftrightarrow 1} \left( e^{-\Delta E_1 t} + e^{-\Delta E_1 (t_f-t)} \right) + ...$$

**Summation method** (only have access to a large number of insertion times for $R_{\text{dis}}$)

$$\sum_{t=c}^{t_f-c} R(t_f, t) = g_S^q (t_f - 2c + 1) + \frac{2c_{0\leftrightarrow 1}}{1 - e^{\Delta E_1}} \left( e^{\Delta E_1 (c-t_f)} - e^{\Delta E_1 (1-c)} \right)$$

$$+ c_{11} (t_f - 2c + 1) e^{-\Delta E_1 t_f} + ...$$

**Linear limit**

$$\sum_{t=c}^{t_f-c} R(t, t_f) \to g_S^q (t_f - 2c + 1) + \text{constant}$$

$\Delta E_1 = E_1 - E_0$ energy gap

$c_{0\leftrightarrow 1} : B_0 \leftrightarrow B_1$ matrix elements of transitions 

between the ground and first excited state
Numerical setup - CLS ensembles

Lüscher-Weisz gluon action and the Sheikholeslami-Wohlert fermion action with $N_f = 2 + 1$
Numerical Setup

Quark mass plane

- around 25 out of 32 ensembles (final set)
- do not yet use the more precise sequential data for the nucleon
- for some ensembles not all configurations used yet (especially disconnected)
- six different lattice spacings
- High statistics: error estimation in the analysis via the $\Gamma$-method

[Wolff: arXiv:hep-lat/0306017]
[Ramos: arXiv:1809.01289]

- pyerrors python package

[Joswig et al.: arXiv:2209.14371]
Numerical setup

Connected & Disconnected Three-Point Functions

▶ connected

▶ $m_l = m_s$ ensembles:
  one measurement at $t_f/a = 11$, two at $t_f/a = [14, 16]$ and four at $t_f/a = 19$
▶ otherwise:
  two measurements (forward and backward direction) for each $t_f$ on every configuration
▶ four different source-sink separations typically corresponding to $t_f \approx [0.7 \text{ fm}, 0.9 \text{ fm}, 1 \text{ fm}, 1.2 \text{ fm}]$

▶ disconnected

▶ correlate a quark loop with a baryon two-point function
▶ 20 different spatial source positions on every configuration of the two-point function
  (different for $m_l = m_s$ ensembles e.g. N202: 26, J500: 27)
▶ A reasonable signal is obtained for $t_f$ up to around $1.22 \text{ fm}$. 
Simultaneous fits to connected & disconnected ratios

- $\Xi$ baryon at $a = 0.076$ fm
- $m_\pi = 352$ MeV (S400)
- simultaneous fit:
  - $\chi^2/\chi^2_{\text{exp}} \approx 0.6$
  - $\Delta \approx 720$ MeV
- top:
  - $\bar{u}u$ current (left)
  - $\bar{s}s$ current (right)
- bottom:
  - $\bar{l}l$ current (left)
  - $\bar{s}s$ current (right)
Two-state fits

\[ R(t_f, t) = g_S^q + c_{0\leftrightarrow 1} \left( e^{-\Delta E_1 \cdot t} + e^{-\Delta E_1 \cdot (t_f-t)} \right) + c_{0\leftrightarrow 2} \left( e^{-\Delta E_2 \cdot t} + e^{-\Delta E_2 \cdot (t_f-t)} \right) \]

- set prior for \( \Delta E_1 \) to \( B(1)\pi(-1) \) (lower green bars)
- set prior for \( \Delta E_2 \) to a mean of the results for \( \Delta E_2 \) from simultaneous fits to the four channels \( J \in \{ A, S, T, (V) \} \) setting a prior for \( \Delta E_1 \) to the lowest multi-particle state (orange points)

'Octet baryon isovector charges from \( N_f = 2 + 1 \) LQCD' [RQCD 2305.04717]
Comparison with Octet Baryon Isovector Scalar Charges

- performed one- & two-state fits
- consistency with baryon isovector charges from [RQCD 2305.04717]
- for now: used stochastic data for the nucleon (less precise)
- isovector ground-state matrix element consistent irrespective of a one- or two-state fit being performed
PROCEDURE

Perform extrapolation for different fitting methods to estimate systematics arising from any residual excited state contamination in the final physical sigma terms.
Chiral Perturbation Theory and Cut-off Effects

\[
\begin{align*}
\sigma^B_\pi & \approx (1 + a^2(c + cM^2 + \delta c_B \delta M^2)) \cdot M^2_\pi \left\{ \frac{2}{3} \bar{b} - \delta b_B + \frac{m_0^2}{2(4\pi F_0)^2} \left[ g_{B,\pi} f' \left( \frac{M_\pi}{m_0} \right) + \frac{g_{B,K}}{2M_K} f' \left( \frac{M_K}{m_0} \right) + \frac{g_{B,\eta}}{3M_\eta} f' \left( \frac{M_\eta}{m_0} \right) \right] \right\} \\
\sigma^B_s & \approx (1 + a^2(c + cM^2 + \delta c_B \delta M^2)) \cdot (2M_K^2 - M_\pi^2) \left\{ \frac{2}{3} \bar{b} + 2\delta b_B + \frac{m_0^2}{(4\pi F_0)^2} \left[ g_{B,K} f' \left( \frac{M_K}{m_0} \right) + \frac{2g_{B,\eta}}{3M_\eta} f' \left( \frac{M_\eta}{m_0} \right) \right] \right\}
\end{align*}
\]

Cut-off effects are modelled as \(O(a^2)\) here as

\(O(a)\) improvement term (\(g_S\) neglected in sigma term determination) expected to be small (\(a^{-3} e_S g_0^2\) drops out when performing the vacuum subtraction):

\[
(S_1)^0(x) = S^0(x) + \frac{1}{\sqrt{2N_f}} a^{-3} e_S g_0^2 \mathbb{1} + \frac{1}{\sqrt{2N_f}} ag_s(g_0^2) \widetilde{\text{Tr}}[F_{\mu\nu}(x)F_{\mu\nu}(x)]
\]
Chiral Perturbation Theory and Cut-off Effects

\[ \sigma^B_\pi \approx (1 + a^2 (c + \bar{c} M^2 + \delta c_B \delta M^2)) \]
\[ \cdot \left\{ \frac{2}{3} \frac{\bar{b}}{B} - \delta b_B + \frac{m_0^2}{2 (4\pi F_0)^2} \left[ g_{B,\pi} \frac{f'}{M_\pi} \left( \frac{M_\pi}{m_0} \right) + \frac{g_{B,K}}{2M_K} \frac{f'}{M_K} \left( \frac{M_K}{m_0} \right) + \frac{g_{B,\eta}}{3M_\eta} \frac{f'}{M_\eta} \left( \frac{M_\eta}{m_0} \right) \right] \right\} \]

\[ \sigma^B_s \approx (1 + a^2 (c + \bar{c} M^2 + \delta c_B \delta M^2)) \]
\[ \cdot \left\{ \frac{2}{3} \frac{\bar{b}}{K} + 2 \delta b_B + \frac{m_0^2}{(4\pi F_0)^2} \left[ \frac{g_{B,K}}{2M_K} \frac{f'}{M_K} \left( \frac{M_K}{m_0} \right) + \frac{2 g_{B,\eta}}{3M_\eta} \frac{f'}{M_\eta} \left( \frac{M_\eta}{m_0} \right) \right] \right\} \]

BaryonChPT

- \( F_0 = \) pion decay constant in the chiral limit
- couplings \( g = \) combinations of LO low energy coupling constants \( F \) & \( D: F + D = g_A \)
- \( f' \) derivative of the loop function \( f \) in covariant BChPT in the extended on-mass-shell scheme

Chiral Perturbation Theory and Cut-off Effects

\[
\sigma^B_\pi \approx (1 + a^2 (c + \sqrt{c} M^2 + \delta c_B \delta M^2)) \\
\cdot M^2_\pi \left\{ \frac{2}{3} \bar{b} - \bar{b} B + \frac{m^2_0}{2(4\pi F_0)^2} \left[ \frac{g_{B,\pi}}{M_\pi} f' \left( \frac{M_\pi}{m_0} \right) + \frac{g_{B,K}}{2M_K} f' \left( \frac{M_K}{m_0} \right) + \frac{g_{B,\eta}}{3M_\eta} f' \left( \frac{M_\eta}{m_0} \right) \right] \right\}
\]

\[
\sigma^B_s \approx (1 + a^2 (c + \sqrt{c} M^2 + \delta c_B \delta M^2)) \\
\cdot (2M_K^2 - M^2_\pi) \left\{ \frac{2}{3} \bar{b} + 2\delta \bar{b}_B + \frac{m^2_0}{(4\pi F_0)^2} \left[ \frac{g_{B,K}}{2M_K} f' \left( \frac{M_K}{m_0} \right) + \frac{2g_{B,\eta}}{3M_\eta} f' \left( \frac{M_\eta}{m_0} \right) \right] \right\}
\]

- all dimensionful quantities need to be rescaled
  - lattice spacings are rescaled by \( t^*_0, a = \frac{a}{\sqrt{8t^*_0}} \)
  - parameters 'in the chiral limit' are rescaled by \( t_{0,\text{ch}}, \) e.g. \( F_0 = \sqrt{8t_{0,\text{ch}}} F_0 \)
  - the rest of the parameters are rescaled by \( t_0, \) e.g. \( M_\pi = \sqrt{8t_0} M_\pi \)

[RQCD 2211.03744]
Preliminary Extrapolation - Pion Mass Dependence
based on Two-state fits

$\chi^2/d.o.f. = 0.9$

$\sigma_{\pi} = \text{const.}$, $\text{Tr}M = \text{const.}$, $m_s = m_{\pi}$

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Preliminary Extrapolation - Cut-off Effects
based on Two-state fits

\[ \chi^2 / \text{d.o.f.} = 0.9 \]
Preliminary Physical results for the Sigma Terms based on two-state fit (with priors) vs. one-state fit (rough error estimation!)

<table>
<thead>
<tr>
<th>$\sigma$(MeV)</th>
<th>$N$</th>
<th>$\Lambda$</th>
<th>$\Xi$</th>
<th>$\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>pion-baryon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35(3)</td>
<td>26(2)</td>
<td>20(2)</td>
<td>13(2)</td>
<td></td>
</tr>
<tr>
<td>39(3)</td>
<td>28(2)</td>
<td>23(2)</td>
<td>13(1)</td>
<td></td>
</tr>
<tr>
<td>43.9(4.7)(4.7)</td>
<td>28.2(4.3)(5.4)</td>
<td>25.9(3.8)(6.1)</td>
<td>11.2(4.5)(6.4)</td>
<td></td>
</tr>
<tr>
<td>strange-baryon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50(6)</td>
<td>159(11)</td>
<td>209(17)</td>
<td>299(20)</td>
<td></td>
</tr>
<tr>
<td>53(5)</td>
<td>155(10)</td>
<td>213(14)</td>
<td>291(17)</td>
<td></td>
</tr>
<tr>
<td>16(58)(68)</td>
<td>144(58)(76)</td>
<td>229(65) (70)</td>
<td>311(72)(83)</td>
<td></td>
</tr>
</tbody>
</table>

[RQCD 2211.03744] (indirect determination via Feynman-Hellman theorem)
Preliminary Low Energy Constants

<table>
<thead>
<tr>
<th>LECs</th>
<th>two-state fit</th>
<th>one-state fit</th>
<th>[RQCD 2211.03744]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>0.335(65)</td>
<td>0.259(100)</td>
<td>0.34(4)(5)</td>
</tr>
<tr>
<td>$D$</td>
<td>0.729(51)</td>
<td>0.684(54)</td>
<td>0.57(5)</td>
</tr>
<tr>
<td>$F/D$</td>
<td>0.460(98)</td>
<td>0.379(161)</td>
<td>0.612(14)(12)</td>
</tr>
</tbody>
</table>
Summary and outlook

- variations of summation and ratio methods to cross-check whether we control excited state contributions sufficiently
  1. one- and two-state fits
  2. priors for the first and second excited state
  3. correlated fits
- chiral extrapolation to the physical pion mass accounting for cut-off effects ✓
  - preliminary physical results for the baryon octet sigma terms

to do:
- error estimation: statistical & systematic (model averaging - Akaike information Criterion)
  - extrapolation for different fits (one-/two-state fits ..)
- include finite-volume effects
- use all available configurations/ensembles - final set of 32 ensembles
- use sequential data available for the nucleon (more precise)
- further investigation of the excited states on this larger set of ensembles