On the Baryon Octet: Sigma Terms in the continuum limit from $N_\mathrm{f} = 2 + 1$ QCD with Wilson fermions

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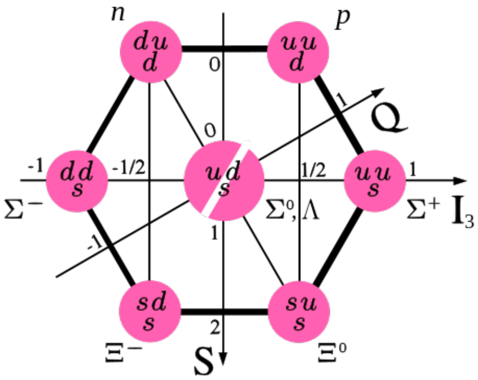
Why determine the sigma terms?

- **quark mass contributions**
  $\rightarrow$ **decomposition of the hadron mass**
- investigate **flavour symmetry breaking** in the **baryon octet**
  $\Rightarrow$ nucleon N, lambda Λ, sigma Σ and xi Ξ (in our setup $m_{\mathrm{u}} = m_{\mathrm{d}}$)
- **WIMP-nucleon scattering cross-sections** (e.g. XENON1T)
- **discrepancies** between results for the **nucleon pion sigma term from LQCD and phenomenology** still to be resolved
- very few determinations for hyperons



How are the sigma terms defined?

$$\sigma_{qB} = m_q \langle B | \bar{q}\, \mathbf{1}\, q | B \rangle$$

- quark mass m_q and a scalar current inserted
- B refers to the ground state of a **baryon B at rest**.

We're interested in:

- **strange sigma terms** $\boldsymbol{\sigma_{sB}}$
- **pion sigma terms** $\boldsymbol{\sigma_{\pi B} = \sigma_{uB} + \sigma_{dB}}$

- **renormalisation** via normalisation factor $\mathbf{r_m}$ (determined by ALPHA , RQCD), the ratio of flavour non-singlet and singlet scalar density renormalisation parameters $\rightarrow$ accounts for the **mixing of quark flavours under renormalisation** for Wilson fermions

[ALPHA 2101.10969], [RQCD 2211.03744]

How to access the matrix element

$\rightarrow$ **spectral decompositions**

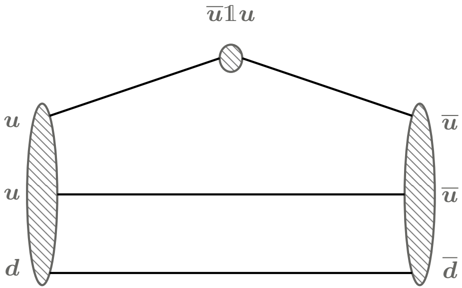
$$C_{2\text{pt}}(t_\text{f}) = \sum_{\vec{x}} \left\langle \mathcal{O}_\text{snk}(\vec{x}, t_\text{f}) \bar{\mathcal{O}}_\text{src}(\vec{0}, 0) \right\rangle = \sum_n |Z_n|^2 e^{-E_n t_\text{f}}$$

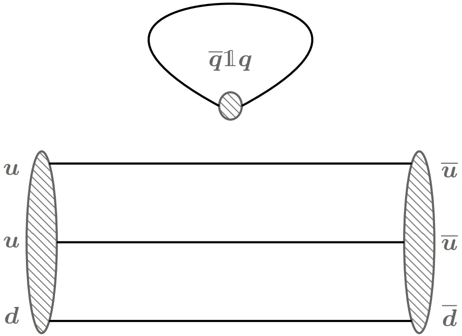
where $Z_n = \langle \Omega | \mathcal{O}_\text{snk} | n \rangle$ (vacuum state Ω) is the overlap of the interpolator $\mathcal{O}_\text{snk}$ onto the state n

$$\begin{aligned} C_{3\text{pt}}(t_\text{f}, t) &= \sum_{\vec{x},\vec{y}} \left\langle \mathcal{O}_\text{snk}(\vec{x}, t_\text{f}) J(\vec{y}, t) \bar{\mathcal{O}}_\text{src}(\vec{0}, 0) \right\rangle - \sum_{\vec{x},\vec{y}} \langle J(\vec{y}, t) \rangle \left\langle \mathcal{O}_\text{snk}(\vec{x}, t_\text{f}) \bar{\mathcal{O}}_\text{src}(\vec{0}, 0) \right\rangle \\ &= \sum_{n,n'} Z_{n'} Z_n^* \langle \mathbf{n'} | \mathbf{J} | \mathbf{n} \rangle e^{-E_n t} e^{-E_{n'}(t_\text{f} - t)} \end{aligned}$$

t_f is the source-sink separation & t is the insertion time of the current

Connected and disconnected contributions





How to access the scalar matrix element

ratio method (cannot resolve $\mathbf{c}_{11}$ (excited-to-excited) so far)

$$R(t_\mathrm{f}, t) = \frac{C_\mathrm{3pt}(t_\mathrm{f}, t)}{C_\mathrm{2pt}(t_\mathrm{f})} = g_S^q + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1 \cdot t} + \mathrm{e}^{-\Delta E_1 \cdot (t_\mathrm{f} - t)}\right) + \dots$$

summation method (only have access to a large number of insertion times for R^dis)

$$\sum_{t=c}^{t_\mathrm{f}-c} R(t_\mathrm{f}, t) = g_S^q (t_\mathrm{f} - 2c + 1) + \frac{2c_{0\leftrightarrow 1}}{1 - \mathrm{e}^{\Delta E_1}}\left(\mathrm{e}^{\Delta E_1 (c - t_\mathrm{f})} - \mathrm{e}^{\Delta E_1 (1-c)}\right)$$
$$+ c_{11}(t_\mathrm{f} - 2c + 1)\mathrm{e}^{-\Delta E_1 t_\mathrm{f}} + \dots$$

linear limit

$$\sum_{t=c}^{t_\mathrm{f}-c} R(t, t_\mathrm{f}) \to g_S^q (t_\mathrm{f} - 2c + 1) + \mathbf{constant}$$

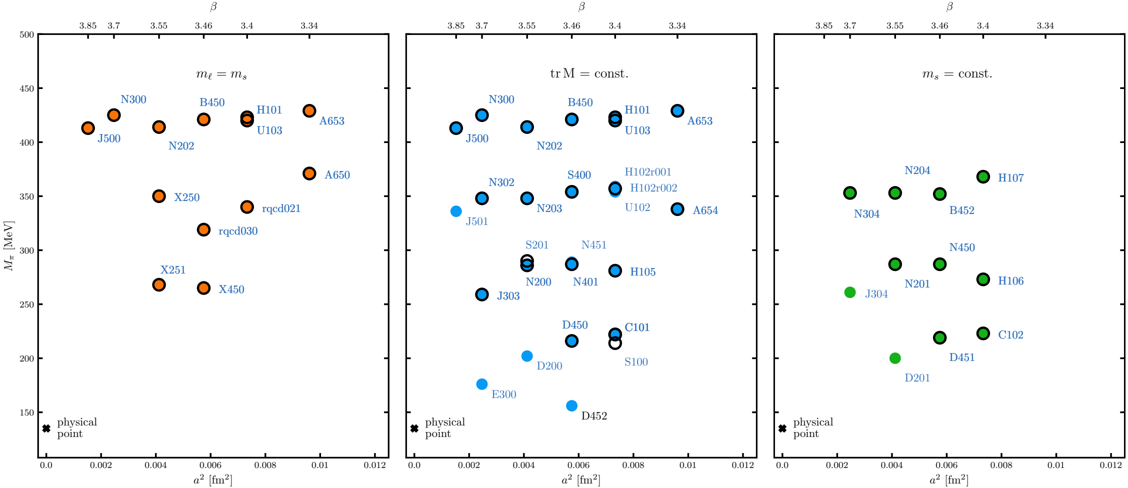
$\Delta E_1 = E_1 - E_0$ energy gap

$c_{0\leftrightarrow 1} : B_0 \leftrightarrow B_1$ matrix elements of transitions

} between the ground and first excited state

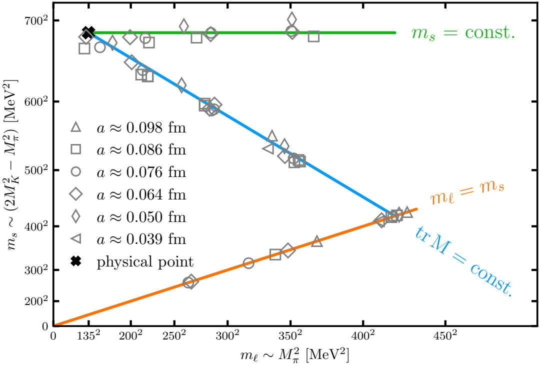
Numerical setup - CLS ensembles

Lüscher-Weisz gluon action and the Sheikholeslami-Wohlert fermion action with $N_\mathrm{f} = 2 + 1$



Numerical Setup

Quark mass plane



- around 25 out of 32 ensembles (final set)
- do not yet use the more precise sequential data for the nucleon
- for some ensembles not all configurations used yet (especially disconnected)
- six different lattice spacings
- High statistics: error estimation in the analysis via the Γ-method

[Wolff: arXiv:hep-lat/0306017]

[Ramos: arXiv:1809.01289]

  - pyerrors python package

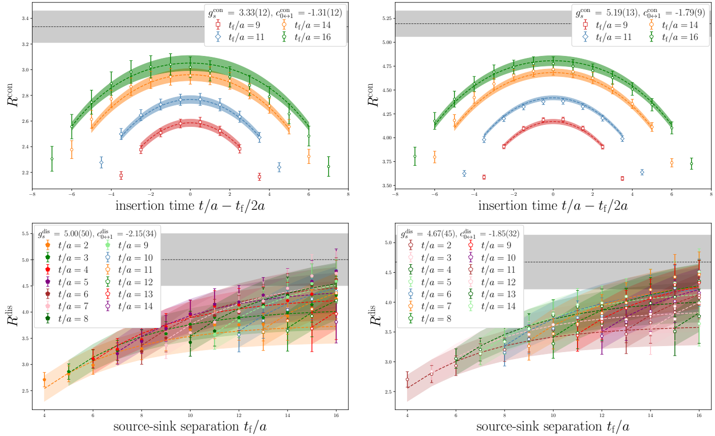
[Joswig et al.: arXiv:2209.14371]

Numerical setup

Connected & Disconnected Three-Point Functions

- **connected**
  - $m_l = m_s$ ensembles:
    one measurement at $t_\mathrm{f}/a = 11$, two at $t_\mathrm{f}/a = [14, 16]$ and four at $t_\mathrm{f}/a = 19$
  - otherwise:
    two measurements (forward and backward direction) for each t_f on every configuration
  - four different source-sink separations typically corresponding to
    $t_\mathrm{f} \approx [0.7\,\mathrm{fm}, 0.9\,\mathrm{fm}, 1\,\mathrm{fm}, 1.2\,\mathrm{fm}]$
- **disconnected**
  - correlate a quark loop with a baryon two-point function
  - 20 different spatial source positions on every configuration of the two-point function (different for $m_l = m_s$ ensembles e.g. N202: 26, J500: 27)
  - A reasonable signal is obtained for t_f up to around $1.22\,\mathrm{fm}$.

Simultaneous fits to connected & disconnected ratios



- Ξ **baryon** at $a = 0.076\,\mathrm{fm}$
- $m_\pi = 352\,\mathrm{MeV}$ (S400)
- **simultaneous fit:**

$\rightarrow \chi^2/\chi^2_{\mathrm{exp}} \approx 0.6$

$\rightarrow \Delta \approx 720\,\mathrm{MeV}$

- **top:**

$\rightarrow \bar{u}u$ current (left)

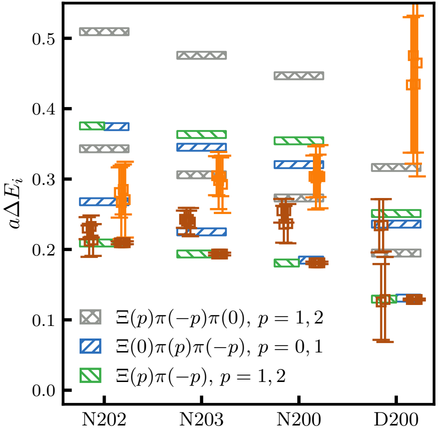
$\rightarrow \bar{s}s$ current (right)

- **bottom:**

$\rightarrow \bar{l}l$ current (left)

$\rightarrow \bar{s}s$ current (right)

Two-state fits

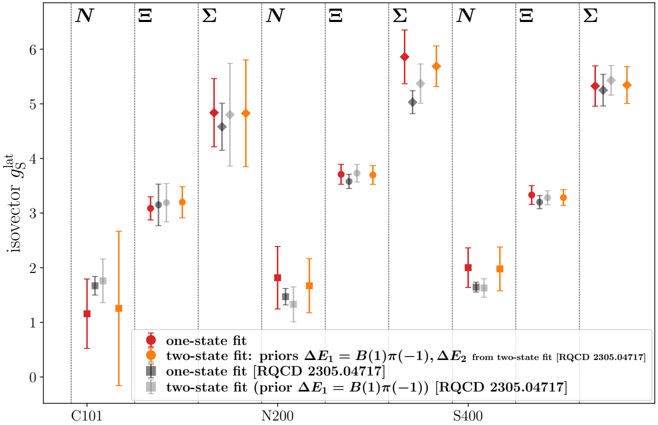


'Octet baryon isovector charges from $N_\mathrm{f} = 2 + 1$ LQCD' [RQCD 2305.04717]

$$R(t_\mathrm{f}, t) = g_S^q + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1 \cdot t} + \mathrm{e}^{-\Delta E_1 \cdot (t_\mathrm{f} - t)}\right) + c_{0\leftrightarrow 2}\left(\mathrm{e}^{-\Delta E_2 \cdot t} + \mathrm{e}^{-\Delta E_2 \cdot (t_\mathrm{f} - t)}\right)$$

- set prior for ΔE_1 to $\boldsymbol{B(1)\pi(-1)}$ (lower green bars)
- set prior for ΔE_2 to **a mean of the results for $\boldsymbol{\Delta E_2}$** from simultaneous fits to the four channels $J \in \{A, S, T, (V)\}$ setting a prior for ΔE_1 to the lowest multi-particle state (orange points)

Comparison with Octet Baryon Isovector Scalar Charges



- performed one- & two-state fits
- consistency with baryon isovector charges from [RQCD 2305.04717]
- for now: used stochastic data for the nucleon (less precise)
- isovector ground-state matrix element consistent irrespective of a one- or two-state fit being performed

PROCEDURE

Perform **extrapolation for different fitting methods**
to **estimate systematics**
arising from any **residual excited state contamination**
in the final **physical sigma terms**.

Chiral Perturbation Theory and Cut-off Effects

$$\sigma_\pi^B \approx \left(1 + \mathbb{a}^2(c + \bar{c}\overline{\mathbb{M}}^2 + \delta c_B \delta\mathbb{M}^2)\right) \cdot \mathbb{M}_\pi^2 \left\{ \frac{2}{3}\bar{\mathbb{b}} - \delta\mathbb{b}_B + \frac{\mathbb{m}_0^2}{2(4\pi\mathbb{F}_0)^2} \left[\frac{g_{B,\pi}}{\mathbb{M}_\pi} f'\left(\frac{\mathbb{M}_\pi}{\mathbb{m}_0}\right) + \frac{g_{B,K}}{2\mathbb{M}_K} f'\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \frac{g_{B,\eta}}{3\mathbb{M}_\eta} f'\left(\frac{\mathbb{M}_\eta}{m_0}\right) \right] \right\}$$

$$\sigma_s^B \approx \left(1 + \mathbb{a}^2(c + \bar{c}\overline{\mathbb{M}}^2 + \delta c_B \delta\mathbb{M}^2)\right) \cdot (2\mathbb{M}_{\mathbb{K}}{}^2 - \mathbb{M}_\pi^2) \left\{ \frac{2}{3}\bar{\mathbb{b}} + 2\delta\mathbb{b}_B + \frac{\mathbb{m}_0^2}{(4\pi\mathbb{F}_0)^2} \left[\frac{g_{B,K}}{2\mathbb{M}_K} f'\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \frac{2g_{B,\eta}}{3\mathbb{M}_\eta} f'\left(\frac{\mathbb{M}_\eta}{\mathbb{m}_0}\right) \right] \right\}$$

Cut-off effects are modelled as $\mathrm{O}(a^2)$ here as

$\mathrm{O}(a)$ improvement term (g_S neglected in sigma term determination) expected to be small ($a^{-3}e_S(g_0^2)$ drops out when performing the vacuum subtraction):

$$(S_\mathrm{I})^0(x) = S^0(x) + \frac{1}{\sqrt{2N_\mathrm{f}}} a^{-3} e_S(g_0^2)\mathbb{1} + \frac{1}{\sqrt{2N_\mathrm{f}}} a g_s(g_0^2)\widetilde{\mathrm{Tr}}[F_{\mu\nu}(x)F_{\mu\nu}(x)]$$

Chiral Perturbation Theory and Cut-off Effects

$$\sigma_\pi^B \approx (1 + \mathbb{a}^2(c + \bar{c}\overline{\mathbb{M}}^2 + \delta c_B \delta\mathbb{M}^2)) \cdot \mathbb{M}_\pi^2 \left\{ \frac{2}{3}\bar{\mathbb{b}} - \delta\mathbb{b}_B + \frac{\mathbb{m}_0^2}{2(4\pi\mathbb{F}_0)^2} \left[\frac{g_{B,\pi}}{\mathbb{M}_\pi} f'\left(\frac{\mathbb{M}_\pi}{\mathbb{m}_0}\right) + \frac{g_{B,K}}{2\mathbb{M}_K} f'\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \frac{g_{B,\eta}}{3\mathbb{M}_\eta} f'\left(\frac{\mathbb{M}_\eta}{m_0}\right) \right] \right\}$$

$$\sigma_s^B \approx (1 + \mathbb{a}^2(c + \bar{c}\overline{\mathbb{M}}^2 + \delta c_B \delta\mathbb{M}^2)) \cdot (2\mathbb{M}_\mathbb{K}{}^2 - \mathbb{M}_\pi^2) \left\{ \frac{2}{3}\bar{\mathbb{b}} + 2\delta\mathbb{b}_B + \frac{\mathbb{m}_0^2}{(4\pi\mathbb{F}_0)^2} \left[\frac{g_{B,K}}{2\mathbb{M}_K} f'\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \frac{2g_{B,\eta}}{3\mathbb{M}_\eta} f'\left(\frac{\mathbb{M}_\eta}{\mathbb{m}_0}\right) \right] \right\}$$

BaryonChPT [Lehnhart et al.: arXiv:hep-ph/0412092]

- F_0 = pion decay constant in the chiral limit
- couplings g = combinations of LO low energy coupling constants F & D: $\boldsymbol{F} + \boldsymbol{D} = \boldsymbol{g}_\mathbf{A}$
- $\boldsymbol{f'}$ derivative of the loop function $\boldsymbol{f}$ in covariant BChPT in the extended on-mass-shell scheme

[Bernard et al.: Nucl. Phys. B 388 (1992) 315; Gasser et al.: Nucl. Phys. B 307 (1988) 779]

Chiral Perturbation Theory and Cut-off Effects

$$
\begin{aligned}
\sigma_\pi^B \approx & (1 + \mathbb{a}^2(c + \bar{c}\overline{\mathbb{M}}^2 + \delta c_B \delta\mathbb{M}^2)) \\
& \cdot \mathbb{M}_\pi^2 \left\{ \frac{2}{3}\bar{\mathbb{b}} - \delta\mathbb{b}_B + \frac{\mathbb{m}_0^2}{2(4\pi\mathbb{F}_0)^2} \left[\frac{g_{B,\pi}}{\mathbb{M}_\pi} f'\left(\frac{\mathbb{M}_\pi}{\mathbb{m}_0}\right) + \frac{g_{B,K}}{2\mathbb{M}_K} f'\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \frac{g_{B,\eta}}{3\mathbb{M}_\eta} f'\left(\frac{\mathbb{M}_\eta}{m_0}\right) \right] \right\}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_s^B \approx & (1 + \mathbb{a}^2(c + \bar{c}\overline{\mathbb{M}}^2 + \delta c_B \delta\mathbb{M}^2)) \\
& \cdot (2\mathbb{M}_{\mathbb{K}}{}^2 - \mathbb{M}_\pi^2) \left\{ \frac{2}{3}\bar{\mathbb{b}} + 2\delta\mathbb{b}_B + \frac{\mathbb{m}_0^2}{(4\pi\mathbb{F}_0)^2} \left[\frac{g_{B,K}}{2\mathbb{M}_K} f'\left(\frac{\mathbb{M}_K}{\mathbb{m}_0}\right) + \frac{2g_{B,\eta}}{3\mathbb{M}_\eta} f'\left(\frac{\mathbb{M}_\eta}{\mathbb{m}_0}\right) \right] \right\}
\end{aligned}
$$

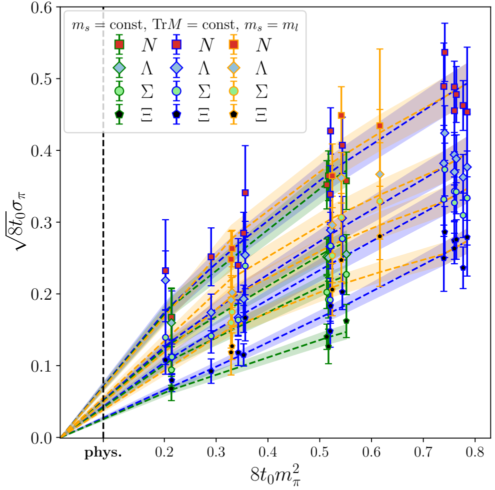
- all dimensionful quantities need to be rescaled
  - lattice spacings are rescaled by t_0^*, $\mathbb{a} = \frac{a}{\sqrt{8t_0^*}}$
  - parameters 'in the chiral limit' are rescaled by $t_{0,\mathrm{ch}}$, e.g. $\mathbb{F}_0 = \sqrt{8t_{0,\mathrm{ch}}}F_0$
  - the rest of the parameters are rescaled by t_0, e.g. $\mathbb{M}_\pi = \sqrt{8t_0}M_\pi$

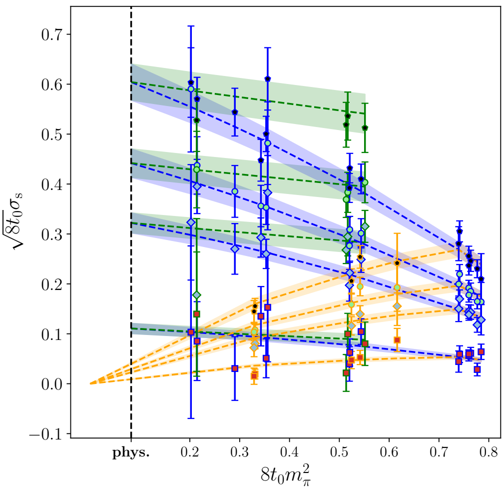
[RQCD 2211.03744]

Preliminary Extrapolation - Pion Mass Dependence

based on Two-state fits

$\chi^2/\mathrm{d.o.f.} = 0.9$

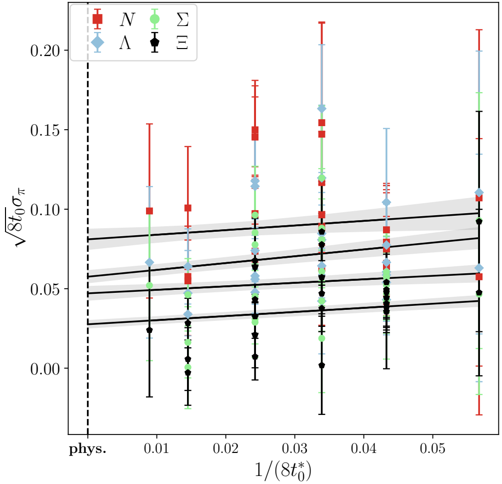


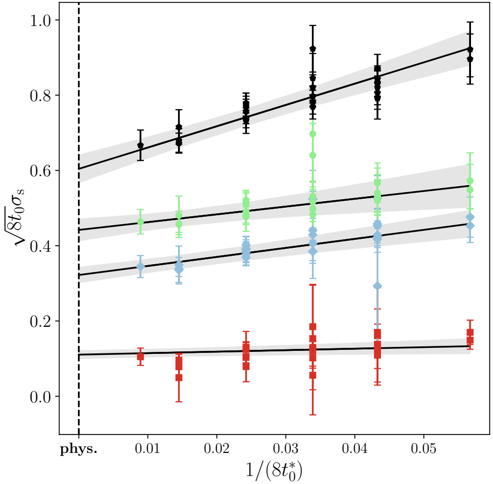


Preliminary Extrapolation - Cut-off Effects

based on Two-state fits

$\chi^2/\text{d.o.f.} = 0.9$





Preliminary Physical results for the Sigma Terms

based on two-state fit (with priors) vs. one-state fit (rough error estimation!)

σ(MeV)	N	Λ	Ξ	Σ
pion-baryon	35*(3)*	26*(2)*	20*(2)*	13*(2)*
	39*(3)*	28*(2)*	23*(2)*	13*(1)*
	43.9(4.7)(4.7)	28.2(4.3)(5.4)	25.9(3.8)(6.1)	11.2(4.5)(6.4)
strange-baryon	50*(6)*	159*(11)*	209*(17)*	299*(20)*
	53*(5)*	155*(10)*	213*(14)*	291*(17)*
	16(58)(68)	144(58)(76)	229(65) (70)	311(72)(83)

[RQCD 2211.03744] (indirect determination via Feynman-Hellman theorem)

Preliminary Low Energy Constants

LECs	two-state fit	one-state fit	[RQCD 2211.03744]
F	$0.335(65)$	$0.259(100)$	$0.34(4)(5)$
D	$0.729(51)$	$0.684(54)$	$0.57(5)$
F/D	$0.460(98)$	$0.379(161)$	$0.612(14)(12)$

Summary and outlook

- variations of summation and ratio methods to cross-check whether we **control excited state contributions** sufficiently
  1. **one- and two-state fits**
  2. **priors for the first and second excited state**
  3. **correlated fits**
- **chiral extrapolation** to the physical pion mass accounting for **cut-off effects** ✓
  - preliminary physical results for the baryon octet sigma terms

to do:

- error estimation: statistical & **systematic** (model averaging - Akaike information Criterion)
  - **extrapolation for different fits** (one-/two-state fits ..)
- include **finite-volume effects**
- use all available configurations/ensembles - **final set of 32 ensembles**
- use sequential data available for the nucleon (more precise)
- further investigation of the excited states on this larger set of ensembles