Exploring lattice supersymmetry with variational quantum deflation

David Schaich (U. Liverpool)

Lattice 2023, August 3

arXiv:2112.07651 arXiv:2301.02230

and more to come with Chris Culver

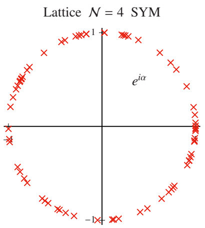
Overview and plan

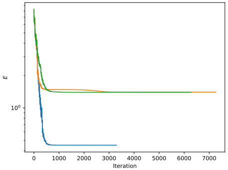
Spontaneous susy breaking in Wess–Zumino model
is compelling target for near-term quantum computing

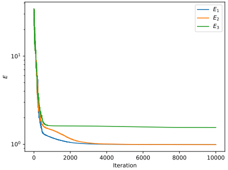
Lattice supersymmetry motivations

Sign problems and spontaneous susy breaking

Wess–Zumino model and variational quantum deflation







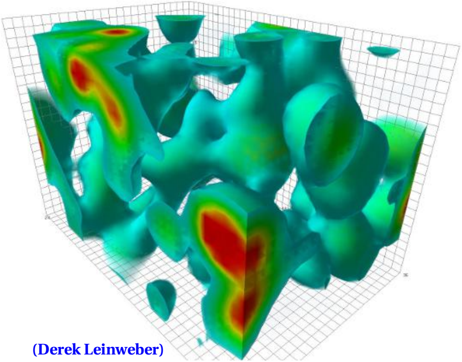
Lattice supersymmetry motivations

arXiv:2208.03580

Lattice field theory promises first-principles predictions
for strongly coupled supersymmetric QFTs

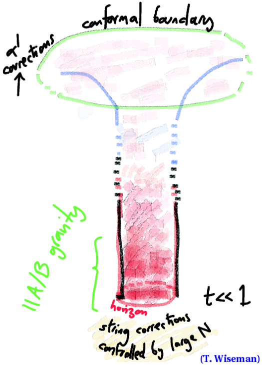
BSM

QFT



(Derek Leinweber)

Holography



(T. Wiseman)

Lattice supersymmetry motivations

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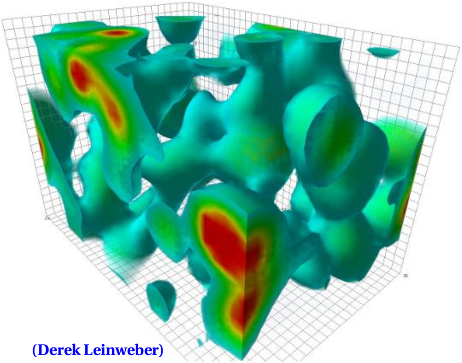
QFT



(Derek Leinweber)

(1+1)d Wess–Zumino model

Simplest QFT
with spontaneous supersymmetry breaking

Lattice supersymmetry sign problems

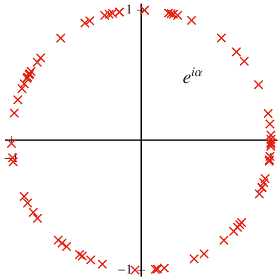
Recall phase reweighting in lagrangian formalism

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]} \longrightarrow \frac{\left\langle \mathcal{O} \; e^{i\alpha} \right\rangle_{\text{pq}}}{\left\langle e^{i\alpha} \right\rangle_{\text{pq}}}$$

Sign problem

$$\left\langle e^{i\alpha} \right\rangle_{\text{pq}} = \frac{\mathcal{Z}}{\mathcal{Z}_{\text{pq}}} \longrightarrow 0 \;\; \text{exponentially quickly}$$

Lattice $\mathcal{N} = 4$ SYM



Lattice supersymmetry sign problems

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$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]} \longrightarrow \frac{\left\langle \mathcal{O} \; e^{i\alpha} \right\rangle_{\text{pq}}}{\left\langle e^{i\alpha} \right\rangle_{\text{pq}}}$$

Spontaneous supersymmetry breaking

Requires vanishing **Witten index** $\mathcal{W} = \text{Tr}\left[(-1)^F e^{-iHt}\right] = \text{Tr}_B\left[e^{-iHt}\right] - \text{Tr}_F\left[e^{-iHt}\right]$

Periodic BCs $\longrightarrow$ $\mathcal{W} = \mathcal{Z} \propto \left\langle e^{i\alpha} \right\rangle_{\text{pq}} = 0$ $\longrightarrow$ maximally bad sign problem

[Avoided with fermion loop formulation by Steinhauer–Wenger, arXiv:1410.6665;
with tensor networks by Kadoh et al., arXiv:1801.04183]

Apply quantum computing

Evade sign problems by changing perspective

Path integral $\longrightarrow$ continuous-time hamiltonian H on spatial lattice

Generic targets:

Find ground state $|\Omega\rangle$ $\longrightarrow$ test spontaneous symmetry breaking

Real-time evolution $|\Psi(t)\rangle = e^{-iHt}\,|\Psi(0)\rangle \sim \left(\exp\left[-iH\delta_T\right]\right)^{N_T}|\Psi(0)\rangle$

(1+1)-dimensional Wess–Zumino model

arXiv:2301.02230

Supersymmetric $H \longleftrightarrow$ matched boson / fermion d.o.f. at each lattice site:

$$H = Q^2 = \sum_n \left[\frac{p_n^2}{2} + \frac{1}{2}\left(\frac{\phi_{n+1} - \phi_{n-1}}{2}\right)^2 + \frac{1}{2}[W(\phi_n)]^2 + W(\phi_n)\frac{\phi_{n+1} - \phi_{n-1}}{2}\right.$$
$$\left. + (-1)^n W'(\phi_n)\left(\chi_n^\dagger \chi_n - \frac{1}{2}\right) + \frac{1}{2}\left(\chi_n^\dagger \chi_{n+1} + \chi_{n+1}^\dagger \chi_n\right)\right]$$

Prepotential $W(\phi)$ ensures supersymmetric interactions

$W = \phi$ [free theory] $\longrightarrow$ $\mathcal{W} \neq 0$ $\longrightarrow$ supersymmetric $|\Omega\rangle$

$W = \phi^2$ $\longrightarrow$ expect dynamical supersymmetry breaking

Wess–Zumino set up for quantum computing

Map bosons and fermions to finite number of qubits

Fermions: Usual Jordan–Wigner transformation $\longrightarrow$ one qubit per site

Bosons: Retain lowest $\Lambda = 2^B$ harmonic oscillator modes

binary encoding $\longrightarrow$ B qubits per site

Defines operators like H in terms of Pauli strings

Explicit susy breaking from different treatment, removed as $\Lambda \to \infty$

Current focus on exploratory development & testing

$\longrightarrow$ Qiskit simulator for rapid turnaround

[github.com/chrisculver/WessZumino]

Variational quantum eigensolver (VQE)

Well-known 'hybrid' quantum–classical algorithm

Quantum circuit implements wave-function ansatz $|\Psi(\theta_i)\rangle$ with tunable params

Loss function measurements $\longrightarrow$ classical optimizer adjusts θ_i to minimize

$\nwarrow$ shallower circuit $\longrightarrow$ less sensitive to noise / errors

Energy as loss function $\longrightarrow$ approximate ground state

Apply to Wess–Zumino model

Spontaneous supersymmetry breaking $\longleftrightarrow$ non-zero ground-state energy

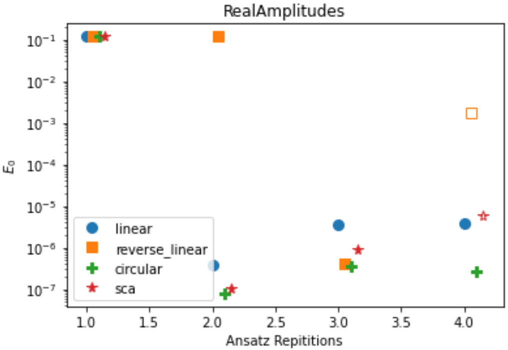
$$E_0 = \langle\Omega\,|H|\,\Omega\rangle = |Q\,|\Omega\rangle|^2$$

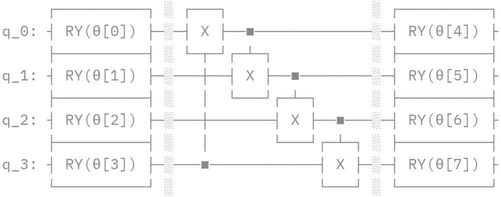
Some Wess–Zumino VQE technical details

Balance expressivity of ansatz vs. number of parameters

Tested various flavors of Qiskit `RealAmplitudes` circuits

alternating CNOTs and parameterized Y rotations



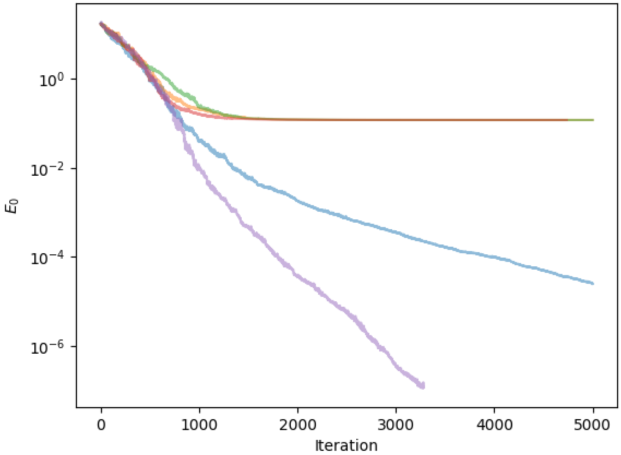


$N_s \times (B + 1) \times (\text{reps} + 1)$ params

COBYLA optimizer

Wess–Zumino VQE

$W = \phi$ with $N_s = 2$ and $\Lambda = 16 \longrightarrow$ 10 qubits, 30 params, ~1000 Pauli strings



Can struggle to find ground state even for free theory

5 VQE runs find $10^{-7} \lesssim E_0 \lesssim 10^{-1}$ [exact $E_1 = E_2 \approx 1.118$]

Challenge:
Distinguishing "small" vs. "zero"

Excited states can help

Supersymmetry $\longrightarrow$ all $E > 0$ eigenstates paired

$$[H, Q] = 0 \longrightarrow H\left(Q\left|\Psi\right\rangle\right) = E_{\Psi}\left(Q\left|\Psi\right\rangle\right)$$

Consider just three smallest energies:

$E_0 < E_1 \approx E_2 \longleftrightarrow$ supersymmetric $\qquad E_0 \approx E_1 < E_2 \longleftrightarrow$ broken

Get k lowest eigenstates from Variational Quantum Deflation (VQD)

[Higgott–Wang–Brierley, arXiv:1805.08138]

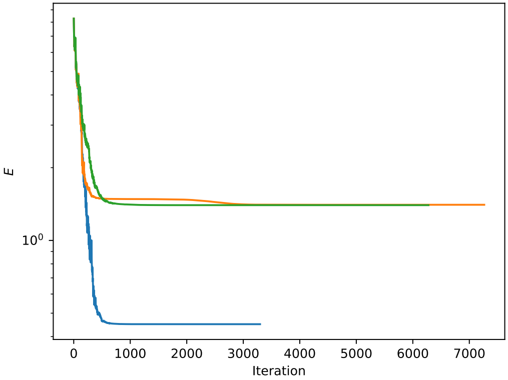
Do k VQE solves, each time deflating previous eigenstates

$$H_k = H + \sum_{i=0}^{k-1} \beta_i \left|\Psi_i\right\rangle \left\langle\Psi_i\right| \qquad \beta_i > E_k - E_i$$

Wess–Zumino VQD

$W = \phi$ with $N_s = 4$ and $\Lambda = 4$

12 qubits, 36 params, $\sim$4000 Paulis

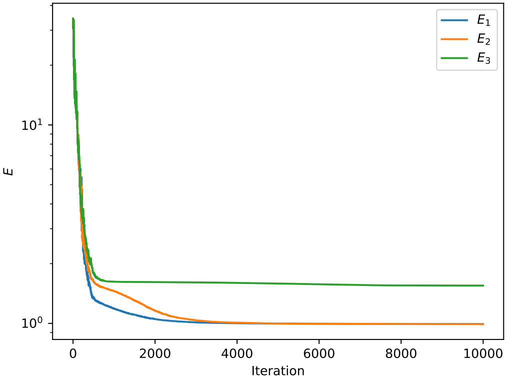


VQD: 0.450, 1.399, 1.405

Exact: -0.001, 0.829, 0.829

$W = \phi^2$ with $N_s = 3$ and $\Lambda = 8$

12 qubits, 36 params, $\sim$6000 Paulis



VQD: 0.991, 0.993, 1.551

Exact: 0.416, 0.416, 1.089

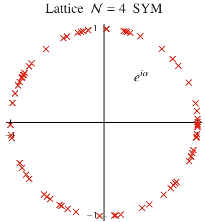
Recap and outlook

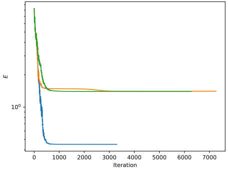
Spontaneous susy breaking in Wess–Zumino model
is compelling target for near-term quantum computing

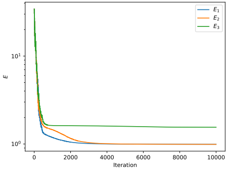
Sign problem motivates quantum computing

Variational quantum deflation distinguishes broken or not

Lots to explore: Optimizations, formulations, real-time evol. . .







Recap and outlook

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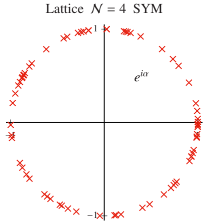

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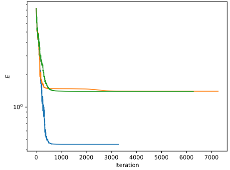
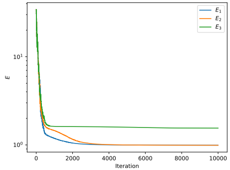
Variational quantum deflation distinguishes broken or not



Lots to explore: Optimizations, formulations, real-time evol. . .

Thanks for your attention! Any questions?



Chris Culver

UK Research and Innovation

Backup: Supersymmetry of Wess–Zumino lattice hamiltonian

Build $H = Q^2$ from lattice supercharge

$$Q = \sum_n \left[p_n \psi_n^+ - \left(\frac{\phi_{n+1} - \phi_{n-1}}{2} + W(\phi_n)\right) \psi_n^-\right]$$

by changing variables

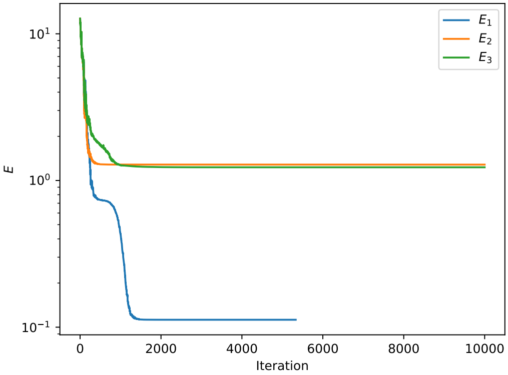
$$\psi_n^\pm = \frac{1 \mp i(-1)^n}{2i^n} \left(\chi_n^\dagger \pm i\chi_n\right)$$

Note exact lattice supersymmetry allowed by continuous time

Backup: Wess–Zumino VQD with Dirichlet BCs

$W = \phi$ with $N_s = 3$ and $\Lambda = 8$

12 qubits, 36 params, $\sim$4250 Paulis

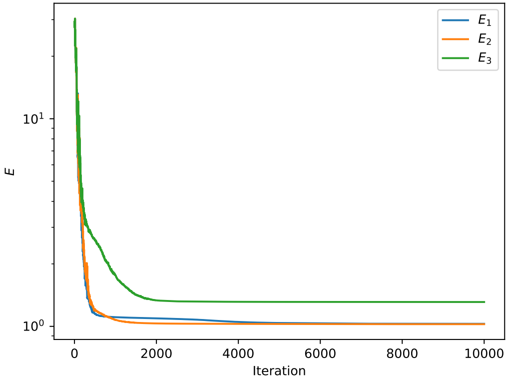


VQD: 10^{-1}, 1.230, 1.283

Exact: 10^{-5}, 1.000, 1.000

$W = \phi^2$ with $N_s = 3$ and $\Lambda = 8$

12 qubits, 36 params, $\sim$3000 Paulis

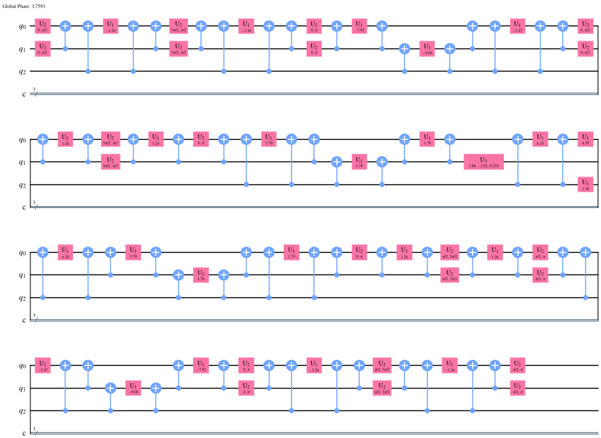


VQD: 1.025, 1.029, 1.310

Exact: 0.493, 0.493, 0.899

Backup: Real-time evolution

For now, need far too many gates for reasonable $\Lambda \gtrsim \mathcal{O}(10)$



$W = g\phi^3 + m\phi$ with $\Lambda = 4$, $N_s = 1$

$\longrightarrow$ supersymmetric quantum mech.

[arXiv:2112.07651]

Smarter Trotterization & transpilation should help

Backup: Supersymmetric quantum mechanics

arXiv:2112.07651

Reduce to single spatial site:

$$H_{\text{SQM}} = \frac{1}{2}\left[\hat{p}^2 + [W(\hat{q})]^2 - W'(\hat{q})\left(\hat{b}^\dagger\hat{b} - \hat{b}\hat{b}^\dagger\right)\right].$$

Spontaneous supersymmetry breaking no longer dynamical

$\longrightarrow$ determined by prepotential

Harmonic oscillator $W_{\text{HO}} = m\hat{q}$ $\longrightarrow$ expect (free) supersymmetric $|\Omega\rangle$

Anharmonic oscillator $W_{\text{AHO}} = m\hat{q} + g\hat{q}^3$ $\longrightarrow$ expect supersymmetric $|\Omega\rangle$

Double-well $W_{\text{DW}} = m\hat{q} + g\left(\hat{q}^2 + \mu^2\right)$ $\longrightarrow$ expect spont. susy breaking

Backup: Supersymmetric quantum mechanics

arXiv:2112.07651

Ground-state energies from VQE

Free theory clearly converges to zero energy

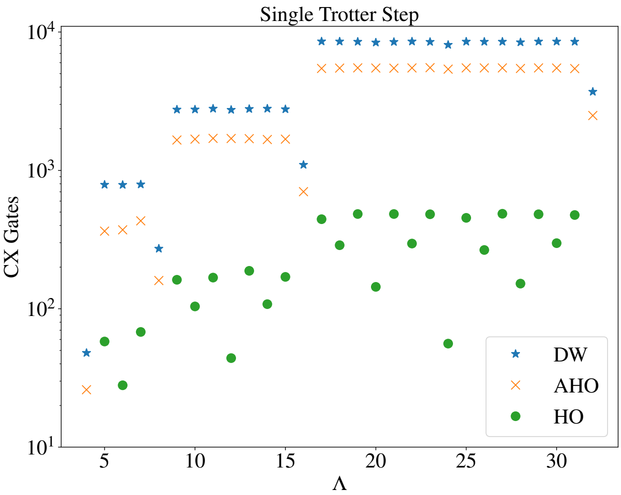
More params $\longrightarrow$ harder to converge, especially for anharmonic oscillator W_{AHO}

Clear non-zero energy (spont. susy breaking) for double-well W_{DW}

Λ	HO	VQE	AHO	VQE	DW	VQE
2	0	5.34e-10	9.38e-01	9.38e-01	1.08e+00	1.08e+00
4	0	1.07e-09	1.27e-01	1.27e-01	9.15e-01	9.15e-01
8	0	4.06e-09	2.93e-02	2.93e-02	8.93e-01	8.93e-01
16	0	1.13e-08	1.83e-03	6.02e-02	8.92e-01	8.94e-01
32	0	4.81e-08	1.83e-05	6.63e-01	8.92e-01	8.95e-01

Backup: Supersymmetric quantum mechanics

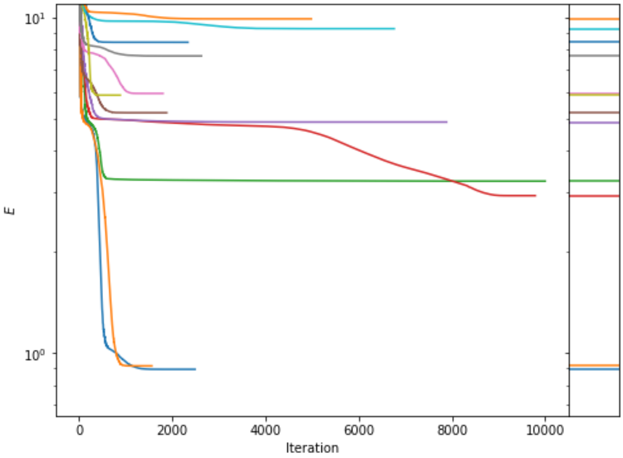
arXiv:2112.07651



Count number of entangling gates
for single Trotter step
(default Qiskit transpilation)

Big improvements when $\Lambda = 2^B$
(note log scale)

Backup: Supersymmetric quantum mechanics VQD



Double-well W_{DW} with $\Lambda = 16$

Decent pairing among lowest 12 Es

VQD: 0.894, 0.915, 3.249, ...
Exact: 0.892, 0.892, 2.734, ...