Trace anomaly form factors of the pion and the nucleon from lattice QCD

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Physics Motivation: Pion Mass Puzzle

- Energy momentum tensor (EMT)

$$T_{\mu\nu} = \frac{1}{4}\overline{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2 \tag{1}$$

- Pion mass can be obtained from the trace of the EMT:

$$m_\pi = \underbrace{\frac{\langle\pi|\int d^3x\left[\frac{\beta(g)}{2g}G^2 + \sum_f \gamma_m(g) m_f \overline{\psi}_f\psi_f\right]|\pi\rangle}{\langle\pi|\pi\rangle}}_{\text{conformal symmetry breaking} \leftrightarrow \text{trace anomaly ME}} + \underbrace{\frac{\langle\pi|\int d^3x \sum_f m_f\overline{\psi}_f\psi_f|\pi\rangle}{\langle\pi|\pi\rangle}}_{f\ \text{term} \sim \frac{1}{2}m_\pi \propto \sqrt{m_q}\,[1]} \tag{2}$$

- 1st order in the chiral symmetry breaking:

$$m_\pi \propto \sqrt{m_q}, \quad \text{for} \quad m_q = m_u = m_d \tag{3}$$

[1] Based on the Gellmann-Oakes-Renner relation and the Feynman-Hellman theorem.

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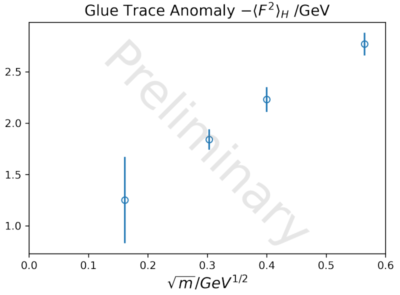
- **Does** the trace anomaly matrix element keep itself proportional to $\sqrt{m_q}$ as $m_q \to 0$?

  F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

$$m_c = \underbrace{\frac{\langle \pi | \int d^3\vec{x} \left(\frac{\beta(g)}{2g} G^2 + \sum_f \gamma_m(g) m_f \overline{\psi}_f \psi_f \right) |\pi\rangle}{\langle \pi|\pi\rangle}}_{\text{conformal symmetry breaking} \leftrightarrow \text{trace anomaly ME}} + \underbrace{\frac{\langle \pi | \int d^3\vec{x} \sum_f m_f \overline{\psi}_f \psi_f |\pi\rangle}{\langle \pi|\pi\rangle}}_{\sigma\ \text{term} \sim \frac{1}{2} m_c \propto \sqrt{m_q}^2} \quad (4)$$



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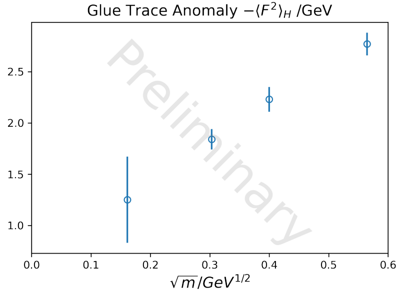

This is a suggestion that the **conformal** (scale) symmetry breaking in the **pion** is linked to the **chiral** symmetry breaking. K.F. Liu arXiv:2302.11600

- **How** does the trace anomaly matrix element keep itself proportional to $\sqrt{m_q}$ as $m_q \to 0$?

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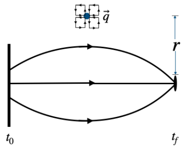
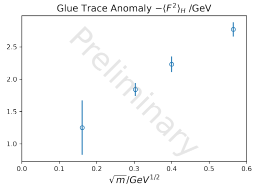
Calculate a denity function

$\rho_H(r)$:



$$\int_0^{\infty} \rho_H(r) r^2 dr \quad \frac{\beta(g)}{2g} G^2 \quad (5)$$



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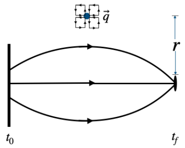

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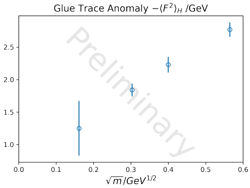
$$\rho_H(r) r^2 dr \quad \frac{\beta(g)}{2g} G^2 \qquad (5)$$

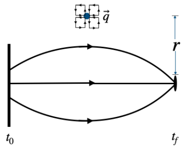


As $m_q \to 0$, the **density function changes sign**.

- **How** does the trace anomaly matrix element keep itself proportional to $^{\text{p}}\overline{m_q}$ as m_q ! 0?

  F. He, P. Sun and Y.B. Yang (*j* QCD) (PRD 2021, 2101.04942)

Calculate a denity function $\rho_H{}^1r^o$:



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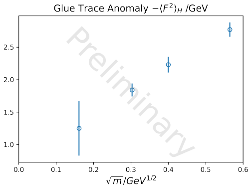
$$\rho_H{}^1r^o r^2 dr \quad \frac{\beta^1 g^o}{2g} G^2 \quad (5)$$



As m_q ! 0, the **density function changes sign**.

- Will the **form factors** changes sign as well? (if they are connected by some sort of Fourier transform)

Definition of the trace anomaly form factors

Normalization convention:

$$1 = \int \frac{d^3p}{(2\pi)^3} |p\rangle \frac{m}{E_p} \langle p| , \quad |p\rangle = \sqrt{\frac{E_p}{m}} a_p^\dagger |0\rangle \tag{6}$$

Define a dimensionless trace anomaly form factor $G_H(Q^2)$, where $Q^2 = -(P' - P)^2$:

for spin-$\frac{1}{2}$ particle like proton:

$$\langle P'|T^\mu_\mu|P\rangle = m_N G_N(Q^2) \bar{u}(P') u(P), \tag{7}$$

for spin-0 particle like pion:

$$\langle P'|T^\mu_\mu|P\rangle = m_\pi G_\pi(Q^2). \tag{8}$$

$G_H(Q^2 = 0)$ is the contribution to the total mass of hadron H.

The trace anomaly terms need renormalization:

$$\mathrm{hcj} \int^{1} d^3 x \text{@W»} \frac{V^1 g^o}{2g} G^2 \text{¸} \tilde{O}_{f}\, W_m{}^1 g^o m_f \bar{k}_f k_f \text{¼gi} \qquad (9)$$

Renormalization method: F. He, P. Sun and Y.B. Yang (j QCD) (PRD 2021, 2101.04942) :

$\frac{V^1 g^o}{2g}$ and $W_m{}^1 g^o$ are independent of the hadron state

Solve the mass sum-rule equations for pseudo-scalar (c) and vector meson (d) at one mass $m_V a = 0{\bullet}3$:

$$M_{PS} \;{}^1\; 1\text{¸} W_m{}^o \mathrm{h}H_m \mathrm{i}_{PS} \; \frac{V^1 g^o}{2g} \mathrm{h}F^2 \mathrm{i}_{PS} = 0\text{–} \qquad (10)$$

$$M_V \;{}^1\; 1\text{¸} W_m{}^o \mathrm{h}H_m \mathrm{i}_V \; \frac{V^1 g^o}{2g} \mathrm{h}F^2 \mathrm{i}_V = 0\text{–} \qquad (11)$$

and obtain the bare W_m and $\frac{V^1 g^o}{2g}$.

Verify the assumption:
sum rule satis ed for other masses

$$R_H = {}^1 1\text{¸} W_m{}^o \mathrm{h}H_m \mathrm{i}_H \text{¸} \frac{V^1 g^o}{2g} \mathrm{h}F^2 \mathrm{i}_H \;{\bullet}\, m_H \qquad (12)$$

- For proton, the f term is small, and glue trace anomaly dominates
- For pion, trace anomaly $\frac{1}{2} m_c$ and W_m is not very large.

! calculate glue trace anomaly form factors

Three-point correlators: Two-point correlators:

Need large statistics for various momentum transfer values:

- Add source and sink momenta using phase factors(no need for extra inversions)

$$C_N^{G}(\vec{p}_f,\vec{p}_i,t)=\frac{1}{n}\sum_{i}^{n} e^{i\vec{p}\cdot\vec{w}_i}\, C_N^{G-W_i}(\vec{p},t),\quad C_N^{G-H}(\vec{p}_f,\vec{p}_i,t) \tag{13}$$

G. Wang, Y.-B. Yang, J. Liang, T. Draper, and K.-F. Liu, Phys. Rev. D 106, 014512.(j QCD)

Calculation of glue trace anomaly FF using two- and three-point correlators with grid source and low-mode substitution (LMS)

Three-point correlators:

Two-point correlators:

Taking ratios of the 3pt and 2pt (using pion as an example):

$$R_c^{S_i S_f}(t-\tau,\vec p_i,\vec p_f) = \frac{C^{S_i S_f}_{c-3pt}(t-\tau,\vec p_i,\vec p_f)}{C^{S_f}_{c-2pt}(t;\vec p_f)} \sqrt{\frac{C^{S_i}_{c-2pt}(t-\tau;\vec p_i)\,C^{S_f}_{c-2pt}(t;\vec p_f)\,C^{S_f}_{c-2pt}(\tau;\vec p_f)}{C^{S_f}_{c-2pt}(t-\tau;\vec p_f)\,C^{S_i}_{c-2pt}(t;\vec p_i)\,C^{S_i}_{c-2pt}(\tau;\vec p_i)}}$$

$$\xrightarrow{t \gg \tau \gg 0} \sqrt{\frac{m_c^2}{E_c(\vec p_i)E_c(\vec p_f)}}\, G_c(Q^2)$$

$$+\, C_1^0 e^{E_i^1 \tau} + C_2^0 e^{E_f^1 (t-\tau)} + C_3^0 e^{E_i^1 \tau\ E_f^1 (t-\tau)}$$

Results using grid-source propagators and LMS preliminary

Overlap fermions on DWF at near-physical pion mass:

Ensemble	$L^3 \times T$	a (fm)	$L(\mathrm{fm})$	m_π (MeV)	N_{conf}	N_{src}
24I	$24^3 \times 64$	0.1105(3)	2.65	340	783	64×2

Three different momentum transfer scenarios, up to $O(100)$ source-sink momentum combinations (with same Q^2 averaged)

source at rest: $|\vec{p}_i| = 0$ with $\vec{q} = \vec{p}_f$

back-to-back: $\vec{p}_f = -\vec{p}_i$ with $\vec{q} = 2\vec{p}_f$

near-back-to-back: $\vec{p}_f < -\vec{p}_i$, $\vec{p}_f$ & $-\vec{p}_i$ $\vec{q} \cdot 2$

small Q^2

large Q^2

other nearby Q^2

Results using grid-source propagators and LMS preliminary

source at rest:
$|\vec{p}| = 0$ with $\vec{q} = \vec{p}$

back-to-back:
$\vec{p} = \vec{p}$ with $\vec{q} = 2\vec{p}$

near-back-to-back:
$\vec{p} < \vec{p}, \vec{p}$ & $\vec{p} \quad \vec{q} \leq 2$

Results for the pion preliminary

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Results for the pion preliminary

Current work: form factors

Previous results: density functions

F. He, P. Sun and Y.B. Yang (j QCD)
(PRD 2021, 2101.04942)

- positive at $Q^2 = 0\,\text{GeV}^2$ (contribution to the pion mass from glue)
- sign change of glue trace anomaly form factors for pion, consistent with the density function.
- form factor calculated up to $Q^2 \quad 4.3\,\text{GeV}^2$

Results for the nucleon preliminary

Current work: form factors

Previous results: density functions

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positive at $Q^2 = 0\,\mathrm{GeV}^2$ (contribution to the proton mass from glue)

NO sign change, monotonically decreasing, consistent with the density function.

pion mass puzzle (motivation):

- trace anomaly matrix element is proportional to $\sqrt{m_q}$ as $m_q \to 0$.
- This is a suggestion that the conformal (scale) symmetry breaking in the pion is linked to the chiral symmetry breaking. K.F. Liu arXiv:2302.11600
- glue trace anomaly density has a sign change to achieve this.

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glue trace anomaly form factors of the EMT (preliminary):

- consistent with hadron mass contribution at $Q^2 = 0\,\mathrm{GeV}^2$.
- UNIQUE: sign change of glue trace anomaly form factor for pion.
- form factor calculated up to $Q^2 \approx 4.3\,\mathrm{GeV}^2$

1. pion mass puzzle (motivation):

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2. glue trace anomaly form factors of the EMT(preliminary):

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    UNIQUE: sign change of glue trace anomaly form factor for pion.
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Outlook

- Extract radius of trace anomaly form factors for π, ρ, and N.
- We expect the calculation on the 48I ensemble will give a prediction of the trace anomaly form factors at physical pion mass.

Thanks for your attention!