Trace anomaly form factors of the pion and the nucleon from lattice QCD

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Physics Motivation: Pion Mass Puzzle

- Energy momentum tensor (EMT)

$$T_{\mu\nu} = \frac{1}{4}\overline{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2 \tag{1}$$

- Pion mass can be obtained from the trace of the EMT:

$$m_\pi = \underbrace{\frac{\langle\pi|\int d^3\vec{x}[\frac{\beta(g)}{2g}G^2 + \sum_f \gamma_m(g) m_f\overline{\psi}_f\psi_f]|\pi\rangle}{\langle\pi|\pi\rangle}}_{\text{conformal symmetry breaking}\leftrightarrow\text{trace anomaly ME}} + \underbrace{\frac{\langle\pi|\int d^3\vec{x}\sum_f m_f\overline{\psi}_f\psi_f|\pi\rangle}{\langle\pi|\pi\rangle}}_{\sigma\, term,\, \frac{1}{2}m_\pi\propto\sqrt{m_q}^{1}} \tag{2}$$

- 1st order in the chiral symmetry breaking:

$$m_\pi \propto \sqrt{m_q}, \quad for \quad m_q = m_u = m_d \tag{3}$$

[1] Based on the Gellmann-Oakes-Renner relation and the Feynman-Hellman theorem.

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- **Does** the trace anomaly matrix element keep itself proportional to $\sqrt{m_q}$ as $m_q \to 0$?
  F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

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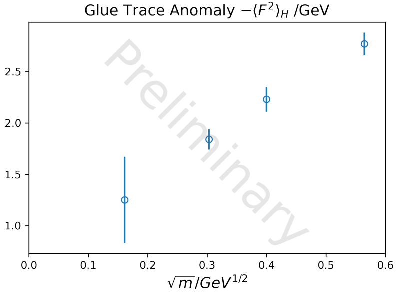

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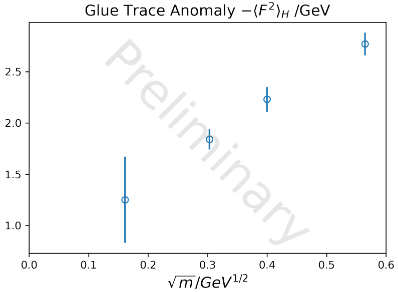

This is a suggestion that the **conformal** (scale) symmetry breaking in the **pion** is linked to the **chiral** symmetry breaking. K.F. Liu arXiv:2302.11600

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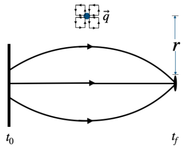
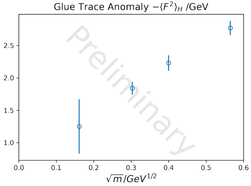
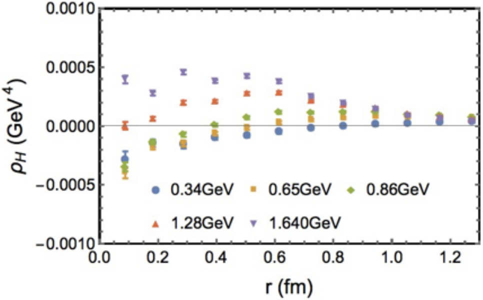
Calculate a denity function $\rho_H(r)$:



$$\int \rho_H(r) r^2 dr \sim \frac{\beta(g)}{2g} G^2 \quad (5)$$





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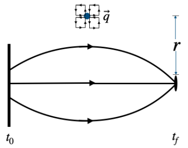

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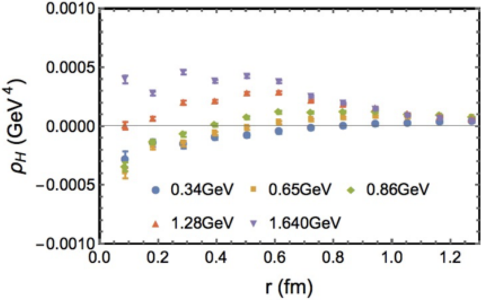

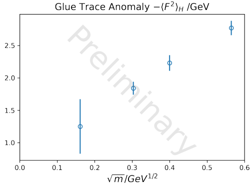

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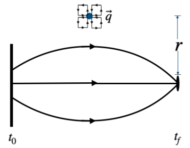

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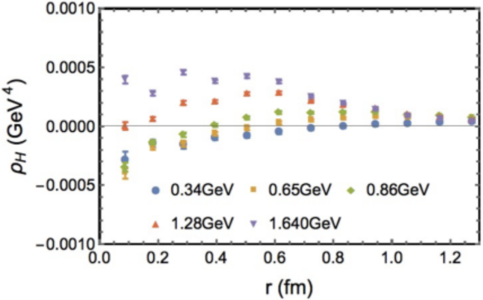
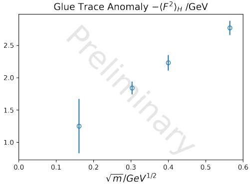




As $m_q \to 0$, the **density function changes sign**.

- Will the **form factors** changes sign as well? (if they are connected by some sort of Fourier transform)

Definition of the trace anomaly form factors

1. Normalization convention:

$$1 = \int \frac{d^3p}{(2\pi)^3} |p\rangle \frac{m}{E_p} \langle p|, \quad |p\rangle = \sqrt{\frac{E_p}{m}} a_p^+ |\Omega\rangle \tag{6}$$

2. Define a dimensionless trace anomaly form factor $G_{\rm H}(Q^2)$, where $Q^2 = -(P' - P)^2$:
   - for spin-$\frac{1}{2}$ particle like proton:

$$\langle P' | T^\mu_\mu | P \rangle = m_{\rm N} G_{\rm N}(Q^2) \bar{u}(P') u(P), \tag{7}$$

   - for spin-0 particle like pion:

$$\langle P' | T^\mu_\mu | P \rangle = m_\pi G_\pi(Q^2). \tag{8}$$

3. $G_H(Q^2 = 0)$ is the contribution to the total mass of hadron H.

Renormalization of the trace anomaly form factors

The trace anomaly terms need renormalization:

$$\langle\pi|\int d^3\vec{x}\gamma[\frac{\beta(g)}{2g}G^2+\sum_f \gamma_m(g)m_f\overline{\psi}_f\psi_f]|\pi\rangle \tag{9}$$

Renormalization method F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942) :

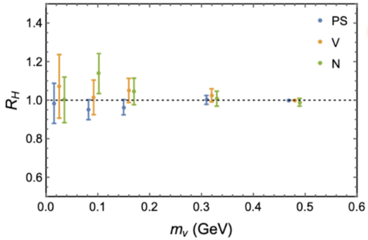
- $\frac{\beta(g)}{2g}$ and $\gamma_m(g)$ are independent of the hadron state
- Solve the mass sum-rule equations for pseudo-scalar(π) and vector meson(ρ) at one mass $m_{\rm v}a = 0.3$:

$$M_{\rm PS}-(1+\gamma_m)\langle H_m\rangle_{\rm PS}-\frac{\beta(g)}{2g}\langle F^2\rangle_{\rm PS}=0, \tag{10}$$

$$M_{\rm V}-(1+\gamma_m)\langle H_m\rangle_{\rm V}-\frac{\beta(g)}{2g}\langle F^2\rangle_{\rm V}=0, \tag{11}$$

and obtain the bare γ_m and $\frac{\beta(g)}{2g}$.

- Verify the assumption: sum rule satisfied for other masses



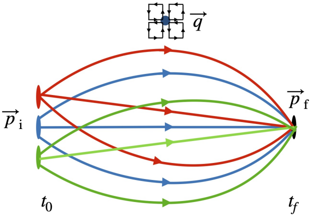
$$R_H=\left[(1+\gamma_m)\langle H_m\rangle_{\rm H}+\frac{\beta(g)}{2g}\langle F^2\rangle_{\rm H}\right]/m_{\rm H} \tag{12}$$

Calculation of glue trace anomaly FF using two- and three-point correlators, with grid source and low-mode substitution (LMS)

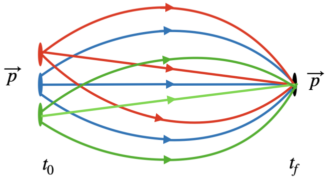
- For proton, the σ term is small, and glue trace anomaly dominates
- For pion, trace anomaly $\sim \frac{1}{2}m_\pi$ and γ_m is not very large.

$\longrightarrow$ calculate glue trace anomaly form factors

Three-point correlators:



Two-point correlators:



Need large statistics for various momentum transfer values:

- Add source and sink momenta using phase factors(no need for extra inversions)

$$C_N^{\mathcal{G}}(\vec{p}_i, \vec{p}_f, t) = \frac{1}{n}\sum_i^n e^{i\vec{p}_i\cdot\vec{w}_i} C_N^{\mathcal{G},\vec{w}_i}(\vec{p}_f, t) + C_N^{\mathcal{G},H}(\vec{p}_i, \vec{p}_f, t) \quad (13)$$

G. Wang, Y.-B. Yang, J. Liang, T. Draper, and K.-F. Liu, Phys. Rev. D 106, 014512 (χQCD)

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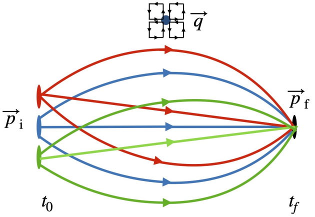

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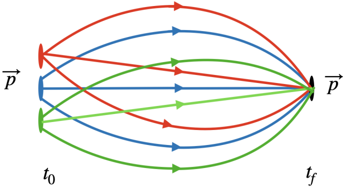


Taking ratios of the 3pt and 2pt (using pion as an example):

$$R_{\pi}^{S_i S_f}(t,\tau;\vec{p}_i,\vec{p}_f) = \frac{C^{S_i S_f}_{\pi,3pt}(t,\tau;\vec{p}_i,\vec{p}_f)}{C^{S_f}_{\pi,2pt}(t;\vec{p}_f)} \sqrt{\frac{C^{S_i}_{\pi,2pt}(t-\tau;\vec{p}_i)\,C^{S_f}_{\pi,2pt}(t;\vec{p}_f)\,C^{S_f}_{\pi,2pt}(\tau;\vec{p}_f)}{C^{S_f}_{\pi,2pt}(t-\tau;\vec{p}_f)\,C^{S_i}_{\pi,2pt}(t;\vec{p}_i)\,C^{S_i}_{\pi,2pt}(\tau;\vec{p}_i)}}$$

$$\xrightarrow{t\gg\tau\gg0} \frac{m_\pi^2}{\sqrt{E_{\pi,\vec{p}_i} E_{\pi,\vec{p}_f}}} G_\pi(Q^2)$$

$$+ C_1' e^{-\Delta E_i^1 \tau} + C_2' e^{-\Delta E_f^1 (t-\tau)} + C_3' e^{-\Delta E_i^1 \tau - \Delta E_f^1 (t-\tau)}$$

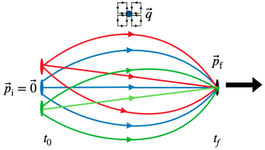
Results using grid-source propagators and LMS preliminary

- Overlap fermions on DWF at near-physical pion mass:

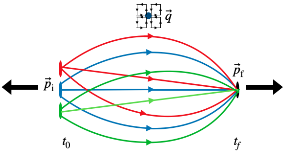
Ensemble	$L^3 \times T$	a (fm)	L(fm)	m_π (MeV)	N_{conf}	N_{src}
24I	$24^3 \times 64$	0.1105(3)	2.65	340	783	64×2

Three different momentum transfer scenarios, up to $\sim \mathcal{O}(100)$ source-sink momentum combinations (with same Q^2 averaged)

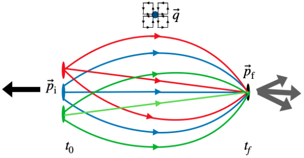
- source at rest: $|\vec{p}_{\mathrm{i}}| = 0$ with $\vec{q} = \vec{p}_{\mathrm{f}}$
- back-to-back: $\vec{p}_{\mathrm{f}} = -\vec{p}_{\mathrm{i}}$ with $\vec{q} = 2\vec{p}_{\mathrm{f}}$
- near-back-to-back: $\vec{p}_{\mathrm{f}} \neq -\vec{p}_{\mathrm{i}}$, $\vec{p}_{\mathrm{f}}$ & $-\vec{p}_{\mathrm{i}} \sim \vec{q}/2$



small Q^2



large Q^2



other nearby Q^2

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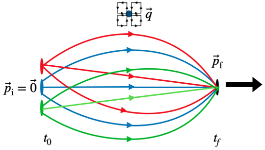

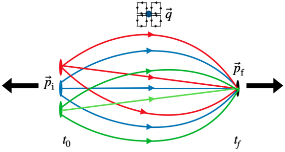

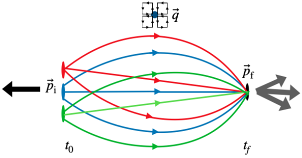

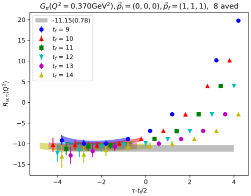

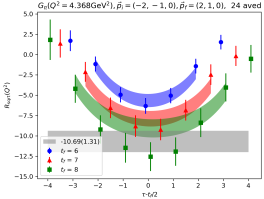

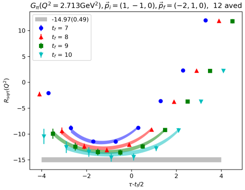

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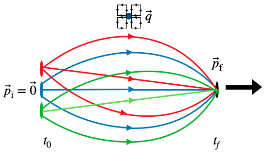
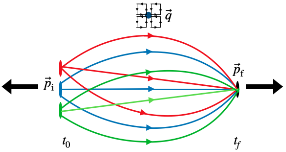
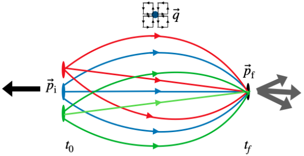
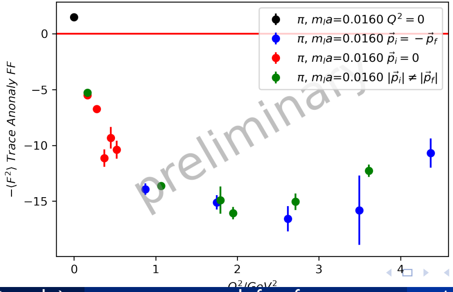


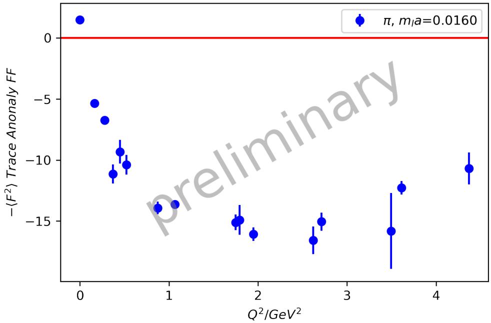






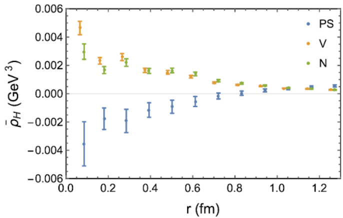
Results for the pion preliminary

Current work: form factors



Previous results: density functions

F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)



- **positive** at $Q^2 = 0\,\mathrm{GeV}^2$(contribution to the pion mass from glue)
- **sign change** of glue trace anomaly form factors for **pion**, consistent with the density function.
- form factor calculated up to $Q^2 \sim 4.3\,\mathrm{GeV}^2$

Results for the nucleon preliminary

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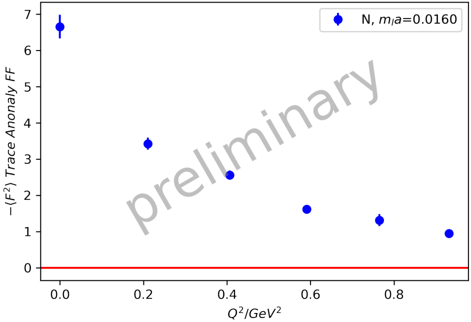

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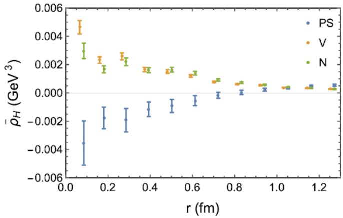

- **positive** at $Q^2 = 0\,\mathrm{GeV}^2$(contribution to the proton mass from glue)
- **NO sign change**, monotonically decreasing, consistent with the density function.

1. pion mass puzzle (motivation):
   - trace anomaly matrix element is proportional to $\sqrt{m_q}$ as $m_q \rightarrow 0$. This is a suggestion that the **conformal** (scale) symmetry breaking in the **pion** is linked to the **chiral** symmetry breaking. K.F. Liu arXiv:2302.11600
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Summary and Outlook

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Outlook

- Extract radius of trace anomaly form factors for π, ρ, and N.
- We expect the calculation on the 48I ensemble will give a prediction of the trace anomaly form factors at physical pion mass.

Thanks for your attention!