Renormalization of Karsten-Wilczek Quarks on a Staggered Background

Daniel A. Godzieba[1]

in collaboration with
S. Borsanyi, Z. Fodor, R. Vig, P. Parotto, C. H. Wong

[1]Department of Physics
The Pennsylvania State University

1 August 2023

Motivation

The implementation of minimally-doubled fermions on the lattice constitutes a multidimensional tuning problem for the selection of appropriate values for bare parameters.

The Karsten-Wilczek Action

The Karsten-Wilczek (KW) implementation of minimally-doubled fermions [L.H. Karsten 1981, F. Wilczek 1987]

$$S_F^{KW} = S_F^N + \sum_x \sum_{j=1}^{3} \bar{\psi}(x) \frac{i\zeta}{2} \gamma^\alpha \left(2\psi(x) - U_j(x)\psi(x+\hat{j}) - U_j^\dagger(x)\psi(x-\hat{j}) \right) \quad (1)$$

explicitly breaks hypercubic symmetry on the lattice and introduces three counterterms:

$$S^{3f} = c \sum_x \bar{\psi}(x) i\gamma^\alpha \psi(x), \quad (2)$$

$$S^{4f} = d \sum_x \bar{\psi}(x) \frac{1}{2} \gamma^\alpha \left(U_\alpha(x)\psi(x+\hat{\alpha}) - U_\alpha^\dagger(x)\psi(x-\hat{\alpha}) \right), \quad (3)$$

$$S^{4g} = d_G \sum_x \sum_{\mu\neq\alpha} \mathrm{Re}\,\mathrm{Tr}(1 - \mathcal{P}_{\mu\alpha}(x)). \quad (4)$$

Numerically, the bare parameters c, d, and d_G can be tuned to their correct values non-perturbatively. (For more details about the KW action, see the previous talk presented by R. Vig.)

The KW c Parameter

Non-physical oscillating contributions to the oscillating meson correlator in the γ_0 channel,

$$C_0(x,y) \sim \left\langle \bar{\psi}(x)\gamma_0\psi(x)\bar{\psi}(y)\gamma_0\psi(y)\right\rangle, \tag{5}$$

occur when the c parameter is detuned. When we average over the symmetric halves of the correlator,

$$C_0'(n) = \frac{1}{2}\left(C_0(N_t/2-n) + C_0(N_t/2+n)\right) \quad \text{for } 0 \le n \le N_t/2, \tag{6}$$

we find the correlator to be well described by the model

$$C_0'(n) \approx A\cosh(mn)\cos(\omega n - \phi) \tag{7}$$

where m is the mass of the γ_0 and $\omega = \omega_c + \pi$ is the oscillation frequency of the correlator. We know empirically that the beat frequency ω_c is zero at the tuned valued of c [J. Weber (2017)].

The Fermionic Anisotropy

The renormalized fermionic anisotropy parameter is given by

$$\xi_f = \frac{a_s}{a_t} = \frac{m_s}{m_t}, \tag{8}$$

where m_s and m_t are the spatial and temporal pion mass parameters respectively. For $\alpha = t$, the bare anisotropy parameter ξ_0 which enters into the momentum space Dirac operator

$$D_{KW} = \frac{i}{a}\left[\sum_{\mu=0}^{3} \gamma_\mu \xi_\mu \sin ak_\mu + \zeta\gamma_0 \sum_{j=1}^{3}(1 - \cos ak_j)\right] \tag{9}$$

also requires tuning to achieve the desired renormalized anisotropy.

A Mixed-Action Study

We present a mixed-action study of a general method of tuning the bare parameters of the KW action.

The previous presentation by R. Vig used measurements from dynamical simulations. This study uses measurements taken with the KW action on 4-stout staggered configurations at the physical point.

Stout configurations were computed with the following parameters:

β	3.6376	3.7089	3.7589	3.8360
$N_s^3 \times N_t$	$40^3 \times 64$	$48^3 \times 64$	$64^3 \times 96$	$64^3 \times 96$

Measurements are taken at a constant pion mass of $\sim$595 MeV.

Fitting Methods

We perform two 1D scans:

1. scanning through values of c at constant ξ_0 for where $\omega_c = 0$,
2. scanning through values of ξ_0 at constant c for where $\xi_f = 1$.

The procedure rests primarily upon tuning the KW c parameter. Since the γ_0 correlator is well-described by the model

$$C_0'(n) \approx A \cosh(mn) \cos(\omega n - \phi) \tag{10}$$

we consider two systematic choices for extracting ω_c at detuned c:

1. a direct 4D fit to the correlator
2. a "mass scan" method and sinusoid fit

Direct Fit Method

We define a modified correlator $C(n) = (-1)^n C_0'(n)$, which oscillates with a frequency of ω_c. The frequency ω_c can be extracted by directly fitting $C(n)$ with the function

$$C(n) = A_0 \cosh(m_0 n) \cos(\omega_0 n - \phi_0), \tag{11}$$

where the amplitude A_0, mass parameter m_0, frequency ω_0, and phase ϕ_0 are the fitting parameters.

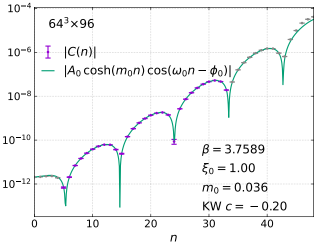
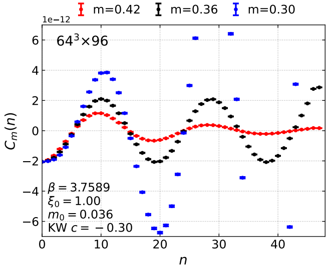


Figure: Example of a direct 4D to a γ_0 correlator at detuned KW c.

Mass Scan Method

We define a second modified correlator

$$C_m(n) = \frac{(-1)^n C_0'(n)}{\cosh(mn)}. \qquad (12)$$



$C_m(n)$ should be a sinusoid with frequency ω_c at the true value of the mass parameter m. We scan through a range of m values, at each value fitting the correlator with the function $C_m(n) = A_0 \cos(\omega_0 n - \phi_0)$. At the value of m where $\chi^2/\mathrm{d.o.f.}$ is minimal, $\omega_0 = \omega_c$.

ω_c-versus-KW c

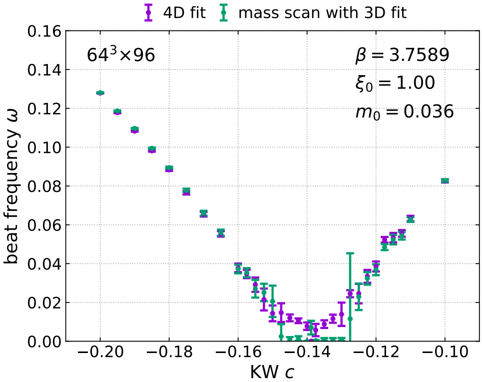
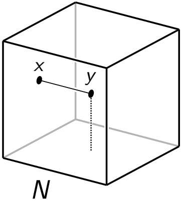
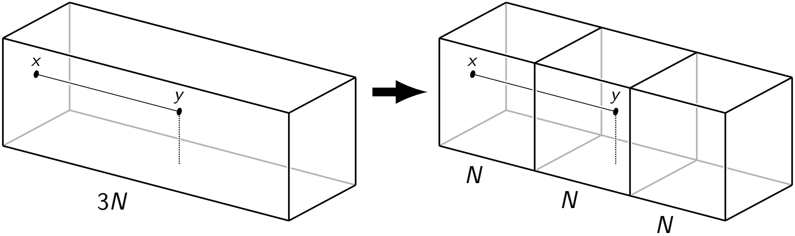


Figure: Beat frequency ω_c of the γ_0 correlator as a function of KW c in the vicinity of tuned c found with both fit methods. The beat vanishes at the tuned c. Both methods become unreliable when the beat wavelength is on the order of the size of the lattice.

Tiling



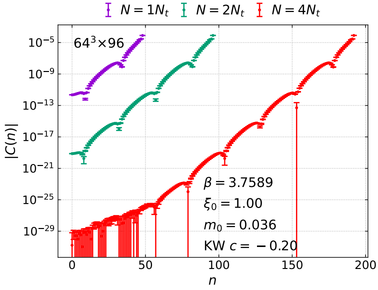
Measurements of spacetime correlators $C(x, y)$ can be taken on stored configurations.



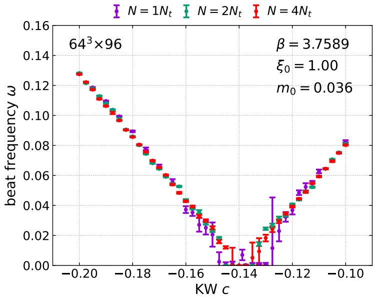
Measurements of correlations at longer distances can be done by tiling stored configurations in direction of the correlation.

Tiling

Tiling increases the number of beat wavelengths in the correlator, leading to a more precise determination of ω_c.

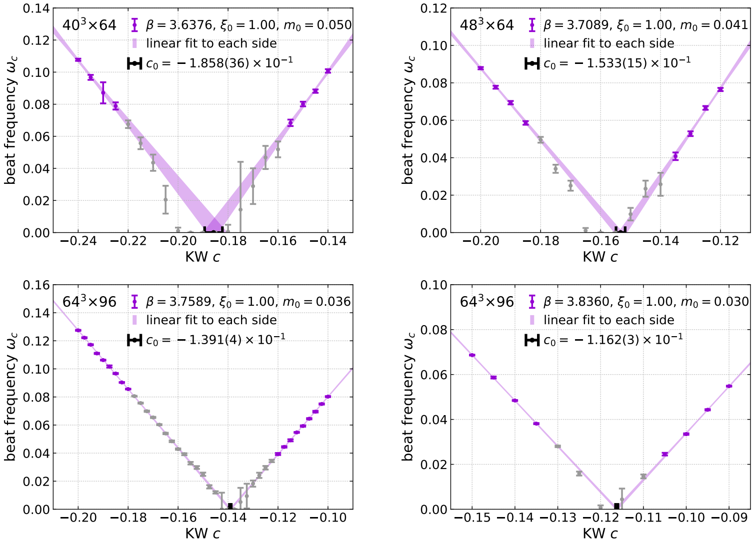


(a) γ_0 correlator at detuned KW c for three different tilings.

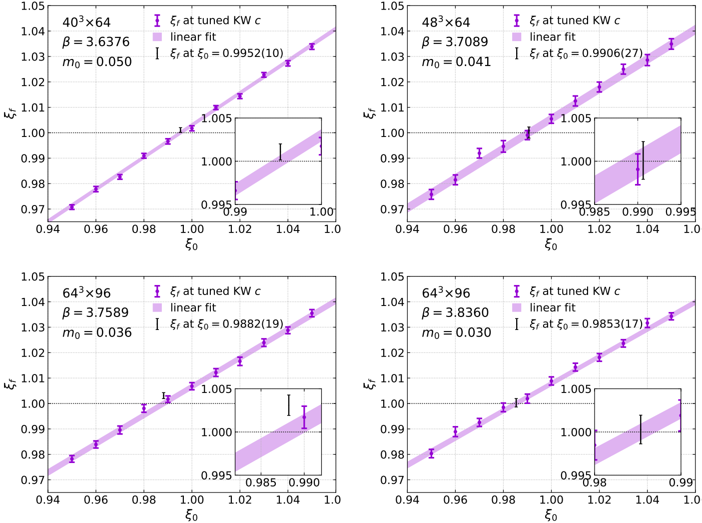


(b) Beat frequency of the γ_0 correlator as a function of KW c for three different tilings using the mass scan method.

Tuning the KW c Parameter



Tuning the Bare Anisotropy ξ_0



Tuning Results

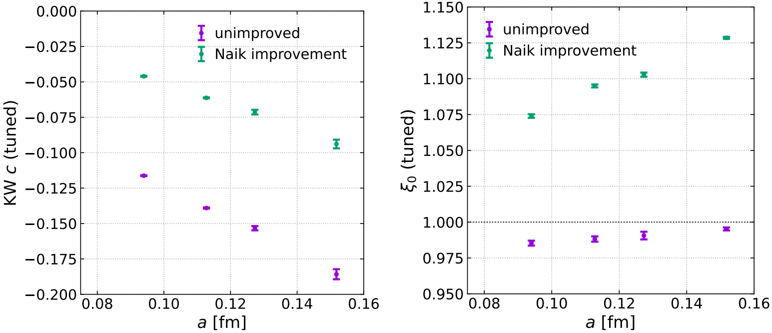


Figure: Tuned KW c and ξ_0 as functions of the temporal lattice spacing. The results of the unimproved and Naik improved KW action are compared. (See previous talk by R. Vig for details about the Naik improvement.)

Difference in the γ_0 and γ_5 Masses at Tuned c and ξ_0

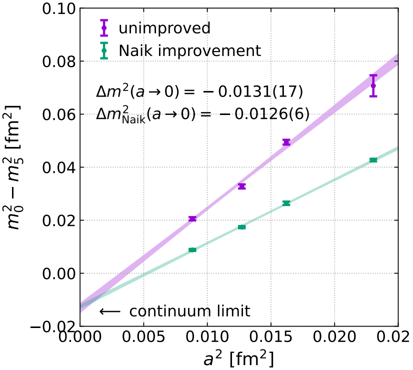


Figure: Initial mixed-action results for the quadratic mass difference between the γ_0 and γ_5 channels as a function of the lattice spacing squared. The results of the unimproved and Naik improved KW action are compared.