How many quantum gates do gauge theories require?

Lattice Conference – Fermilab 2023

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July 31, 2023

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Introduction

Objective

Assess the complexity of simulating gauge theories on near-term quantum computers

Methodology

- Measure the complexity in terms of the CNOT count
- Improve on existing methods to reduce the CNOT count
- Test the method on gauge theories

Articles and Codes Basis for this Talk

- Edison M. Murairi, Michael J. Cervia, Hersh Kumar, Paulo F. Bedaque, and Andrei Alexandru
  "'How many quantum gates do gauge theories require?"
  PRD **106**, 094504 (2022). arXiv:2208.11789.
  Codes https://github.com/emm71201/QC-Hamiltonian-Compilation

- Edison Murairi and Michael J. Cervia
  "Reducing Circuit Depth with Qubitwise Diagonalization."
  arXiv:2306.00170
  Codes https://github.com/emm71201/Reducing-Circuit-Depth-with-Qubitwise-Diagonalization

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Step 0: Trotterization

Consider a Hamiltonian: $H = \sum_j h_j$
The unitary time evolution operator can be approximated as:

$$
\begin{aligned}
U(\delta t) &= e^{-\mathrm{i}\, H\, \delta t} \\
&= e^{-\mathrm{i} \sum_j h_j \delta t} \\
&\approx \prod_j e^{-\mathrm{i}\, h_j \delta t}
\end{aligned}
\tag{1}
$$

Step 1: Expand the Hamiltonian in the Pauli basis

Given a Hamiltonian H, expand H as:

$$H = \sum_i c_i \, P_i \tag{2}$$

where

$$P_i \in \{\mathbb{1}, X, Y, Z\}^{\otimes n} \tag{3}$$

$$c_i = \mathrm{Tr}\left[P_i H\right]/2^n \tag{4}$$

Notation: Drop the $\otimes$ symbol: $X \otimes Z \otimes Y \mapsto X\,Z\,Y$

Step 2: Group commuting Pauli terms

- Two Paulis commute if the number of qubits where the Pauli matrices anti-commute is even.

Figure: Example of two commuting Paulis

Checking commutations of n-qubit Paulis: Complexity: $\mathcal{O}(n)$ operations.

Step 2: Group commuting Pauli terms – Continued

Graph coloring Problem:

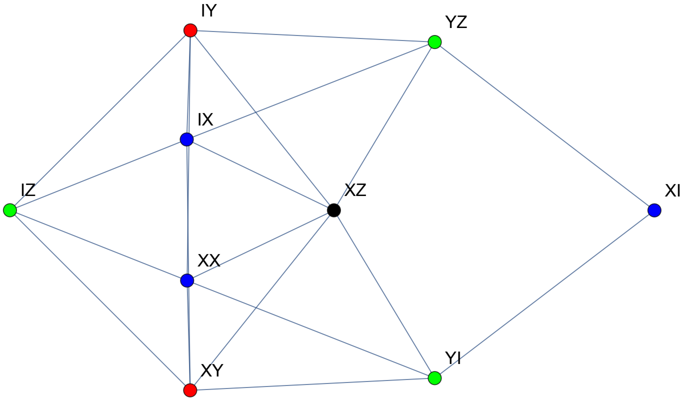


Figure: Grouping commuting Paulis with Graph Coloring.
See Murairi et al., "How many quantum gates do gauge theories require?"

Step 3: Diagonalize each set of commuting Paulis

- Let C be a set of commuting Paulis
- Construct a unitary V simultaneously diagonalizing C
- We guarantee circuit depth of $\boldsymbol{\mathcal{O}\left(n \log r\right)}$ where $\boldsymbol{(r \leq n)}$

Edison M. Murairi and Michael J. Cervia. "Reducing Circuit Depth with Qubitwise Diagonalization". In: (May 2023). arXiv: 2306.00170 [quant-ph]

Code source: https://github.com/emm71201/Reducing-Circuit-Depth-with-Qubitwise-Diagonalization

Step 4: Construct the time evolution circuit of a diagonal Hamiltonian

- A n-qubits diagonal Hamiltonian D has the form:

$$D = \sum_{c_i} P_i \tag{5}$$

where $P_i \in \{I, Z\}^{\otimes n}$

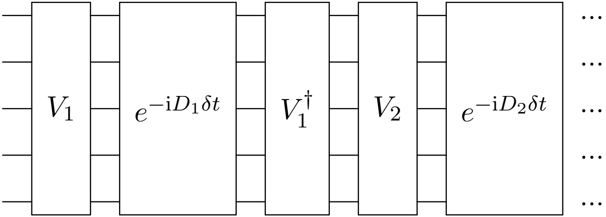
- The quantum circuit of $e^{-\mathrm{i}Dt}$ can be constructed with a tree traversal algorithm.

Edison M. Murairi et al. "How many quantum gates do gauge theories require?" In: *Phys. Rev. D* 106 (9 Nov. 2022), p. 094504. DOI: 10.1103/PhysRevD.106.094504. URL: https://link.aps.org/doi/10.1103/PhysRevD.106.094504

Step 5: Putting everything together

$$H = \sum_i c_i \, P_i = \sum_j h_j \tag{6}$$

where h_j consists only of mutually commuting Paulis. Let V_j be a unitary diagonalizing h_j: $h_j = V_j D_j V_j^\dagger$



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Randomly generated Hamiltonians

- Randomly generate diagonal Hamiltonians with N Paulis.

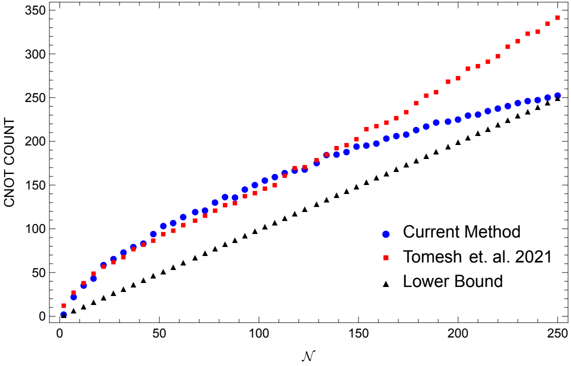

Figure: The average CNOT gate count required to simulate a diagonal Hamiltonian as function of the number of Pauli strings for $n = 8$ qubits. The blue data points are obtained using the compilation method described in step 4.

Lattice Gauge Theories

SU(2) Lattice Gauge Theory	n_{link}	CNOT	$\mathrm{R_z}$
Orland et. al.[1]	2	180	64
D. Horn[2]	3	17,168	16,384

[1] Peter Orland and Daniel Rohrlich. "Lattice gauge magnets: Local isospin from spin". In: *Nuclear Physics B* 338.3 (1990), pp. 647–672. DOI: https://doi.org/10.1016/0550-3213(90)90646-U.

[2] D. Horn. "Finite matrix models with continuous local gauge invariance". In: *Physics Letters B* 100.2 (1981), pp. 149–151. ISSN: 0370-2693. DOI: https://doi.org/10.1016/0370-2693(81)90763-2.

Thank You

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