Taming power divergences with the gradient flow

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Collaborators

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Power divergences

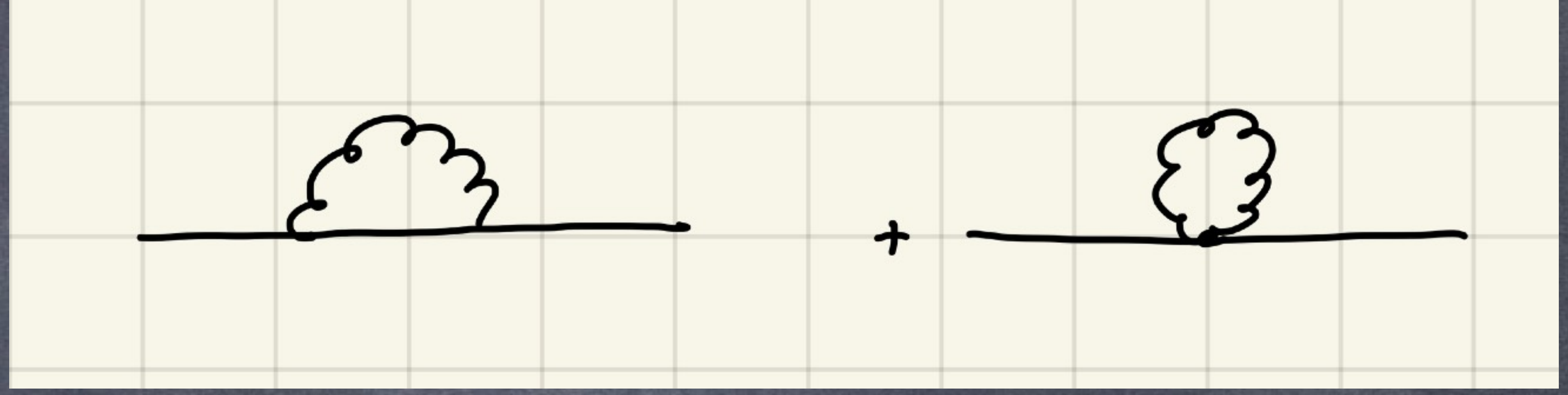
- Renormalization -> mixing lower dimensional operators

Example: critical mass with Wilson-type fermions

$$m_{\rm cr} = \frac{c(g_0)}{a}$$

$$m_0 \overline{\psi}\psi \to Z_m (m_0 - m_{\rm cr}) \overline{\psi}\psi$$

$$m_{\rm PCAC} \propto (m_0 - m_{\rm cr})$$



Example: chiral condensate

$$\left[\overline{\psi}\psi\right]_R = Z_S(a) \left[c_0(a) \frac{1}{a^3} + c_1(a) \frac{m}{a^2} + c_2(a) \frac{m^2}{a} + c_3(a) m^3 + \overline{\psi}\psi \right]$$

Power divergences need to be subtracted non-perturbatively

Maiani, Martinelli, Sachrajda: 1992

Absent for chirally symmetric actions

Gradient flow for fermions

Lüscher: 2013

$$\partial_t \chi(x,t) = \Delta \chi(x,t) \qquad \partial_t \bar{\chi}(x,t) = \bar{\chi}(x,t)\overleftarrow{\Delta}$$

$$\chi(x, t=0) = \psi(x)$$

$$\bar{\chi}(x, t=0) = \bar{\psi}(x)$$

$$x_\mu = (x_0, \mathbf{x}) \qquad t = \mathrm{flow-time} \qquad [t] = -2$$

$$\Delta = D_{\mu,t} D_{\mu,t} \qquad D_{\mu,t} = \partial_\mu + B_{t,\mu}$$

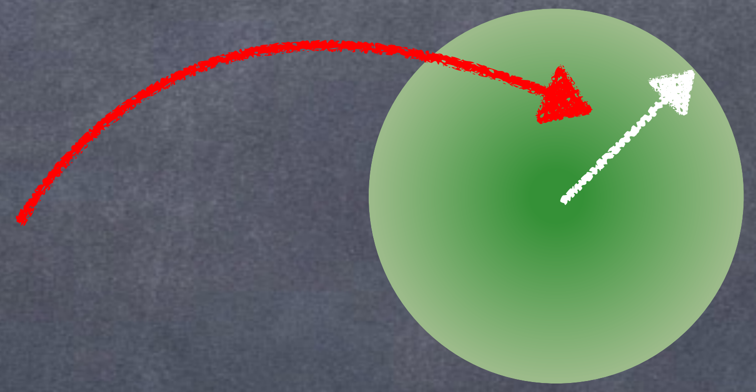
Gradient flow

Lüscher: 2013

$$\chi(x,t) = \int d^4y \; K(x-y,t)\psi(y)$$

$$K(t,x) = \frac{\mathrm{e}^{-\frac{x^2}{4t}}}{(4\pi t)^2}$$

- Smoothing over a range $\sqrt{8t}$



- Gaussian damping at large momenta

$$\chi_R(x,t) = Z_\chi^{1/2}\chi(x,t) \qquad \mathcal{O}(x,t) = \overline{\chi}(x,t)\Gamma(x,t)\chi(x,t) \qquad \mathcal{O}_R = Z_\chi \mathcal{O}$$

$$\Sigma_t = \langle \overline{\chi}(x,t)\chi(x,t)\rangle \qquad \Sigma_{t,R} = Z_\chi \Sigma_t$$

Lüscher: 2010, 2013
Lüscher, Weisz: 2011

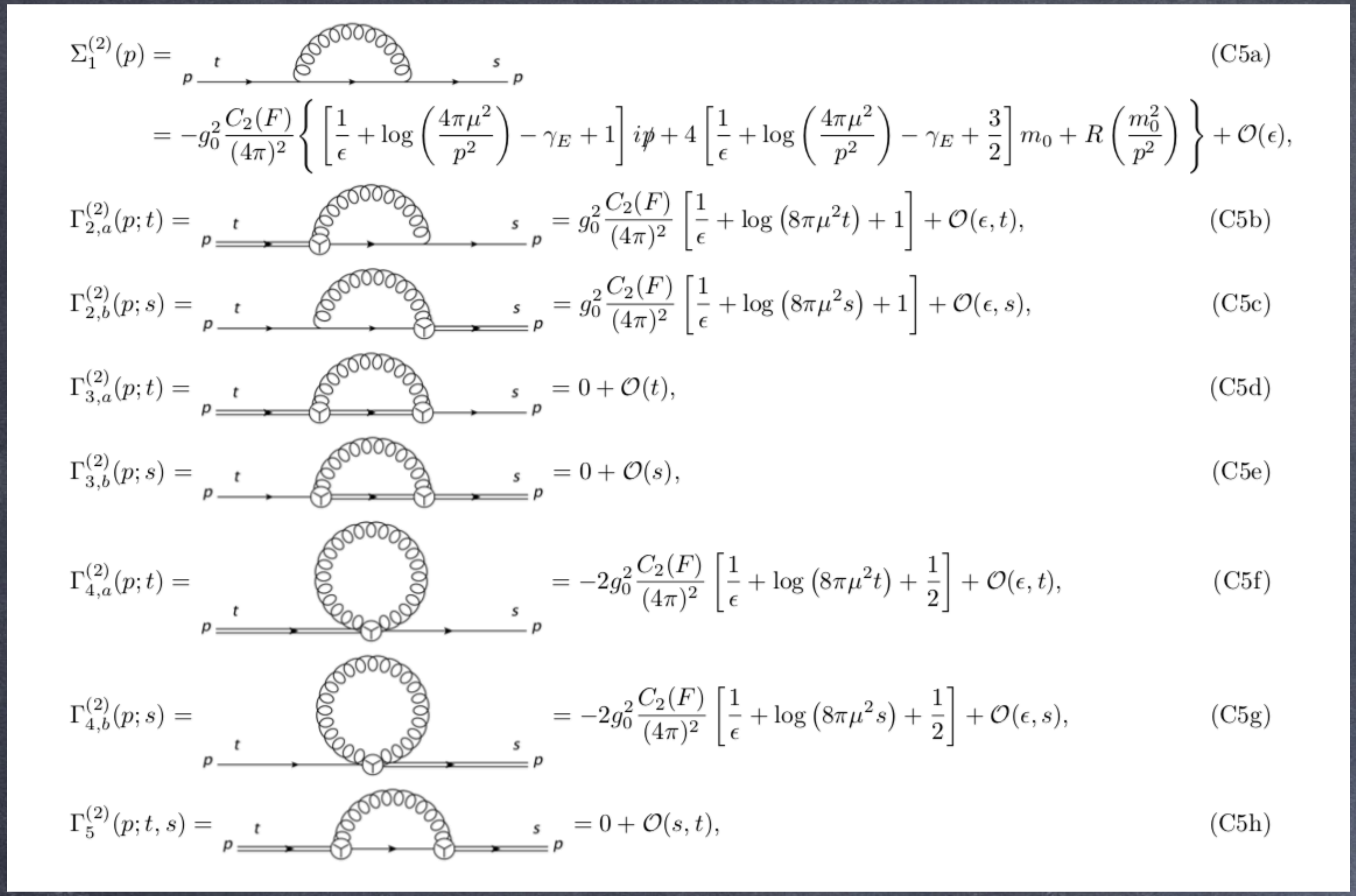
No additive divergences

All fermion operators renormalize multiplicatively with same factor

Flowed fermions renormalization

Makino, Suzuki: 2014

Harlander, Kluth, Lange :2018



$$\Sigma_1^{(2)}(p) = \quad \text{(C5a)}$$

$$= -g_0^2 \frac{C_2(F)}{(4\pi)^2}\left\{ \left[\frac{1}{\epsilon} + \log\left(\frac{4\pi\mu^2}{p^2}\right) - \gamma_E + 1\right] i\not{p} + 4\left[\frac{1}{\epsilon} + \log\left(\frac{4\pi\mu^2}{p^2}\right) - \gamma_E + \frac{3}{2}\right] m_0 + R\left(\frac{m_0^2}{p^2}\right)\right\} + \mathcal{O}(\epsilon),$$

$$\Gamma_{2,a}^{(2)}(p;t) = \; = g_0^2 \frac{C_2(F)}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 t\right) + 1\right] + \mathcal{O}(\epsilon, t), \quad \text{(C5b)}$$

$$\Gamma_{2,b}^{(2)}(p;s) = \; = g_0^2 \frac{C_2(F)}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 s\right) + 1\right] + \mathcal{O}(\epsilon, s), \quad \text{(C5c)}$$

$$\Gamma_{3,a}^{(2)}(p;t) = \; = 0 + \mathcal{O}(t), \quad \text{(C5d)}$$

$$\Gamma_{3,b}^{(2)}(p;s) = \; = 0 + \mathcal{O}(s), \quad \text{(C5e)}$$

$$\Gamma_{4,a}^{(2)}(p;t) = \; = -2g_0^2 \frac{C_2(F)}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 t\right) + \frac{1}{2}\right] + \mathcal{O}(\epsilon, t), \quad \text{(C5f)}$$

$$\Gamma_{4,b}^{(2)}(p;s) = \; = -2g_0^2 \frac{C_2(F)}{(4\pi)^2}\left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 s\right) + \frac{1}{2}\right] + \mathcal{O}(\epsilon, s), \quad \text{(C5g)}$$

$$\Gamma_5^{(2)}(p;t,s) = \; = 0 + \mathcal{O}(s, t), \quad \text{(C5h)}$$

$$Z_\chi^{\mathrm{MS}} = 1 + g^2 \frac{3C_F}{(4\pi)^2}\frac{1}{\epsilon} \qquad C_F = \frac{N^2-1}{2N}$$

Lüscher: 2013

Regularization independent scheme

Artz, Harlander, Lange, Neumann, Prausa: 2019

$$\left\langle \mathring{\bar{\chi}} \overleftrightarrow{\not{D}} \mathring{\chi} \right\rangle = -\frac{2N}{(4\pi)^2 t^2}$$

$$\mathring{\chi}(x,t) = \left[-\frac{2N}{(4\pi)^2 t^2} \frac{1}{\left\langle \bar{\chi} \overleftrightarrow{\not{D}} \chi \right\rangle} \right]^{1/2} \chi(x,t)$$

$$\chi_R^{\overline{MS}} = (8\pi t)^\epsilon \zeta_\chi^{1/2} \mathring{\chi}$$

Finite renormalization

$\overline{MS} \rightarrow$ ringed

Scalar content

A.S., de Vries, Luu: 2014

$$S^{rs}(t) = \overline{\chi}^r(t)\chi^s(t) \qquad P^{rs}(t) = \overline{\chi}^r(t)\gamma_5\chi^s(t)$$

$$S^{rs}(t) = \frac{c_0(t)}{t}M^{rs} + c_1(t)M^{rs}\mathrm{Tr}[M^2] + c_2(t)(M^3)^{rs} + c_3(t)S^{rs} + O(t)$$

$$P^{rs}(t) = c_3(t)P^{rs} + O(t)$$

$$\mathcal{C}^{\rm sub}(t) = \frac{\langle \mathcal{N} S^{rs}(t)\overline{\mathcal{N}}\rangle}{\langle \mathcal{N}\overline{\mathcal{N}}\rangle} - \langle S^{rs}(t)\rangle \qquad \mathcal{C}^{\rm sub}(t) = c_3(t)\mathcal{C}^{\rm sub} + O(t) \qquad c_3(t) = \frac{G_\pi(t)}{G_\pi} + O(t)$$

$$\mathcal{C}^{\rm sub}(t) = \frac{G_\pi}{G_\pi(t)}\left[\frac{\langle \mathcal{N} S^{rs}(t)\overline{\mathcal{N}}\rangle}{\langle \mathcal{N}\overline{\mathcal{N}}\rangle} - \langle S^{rs}(t)\rangle\right]$$

$$g_S^q = Z_P\frac{G_\pi}{G_\pi(t)}\left[\frac{\langle \mathcal{N} S^{qq}(t)\overline{\mathcal{N}}\rangle}{\langle \mathcal{N}\overline{\mathcal{N}}\rangle} - \langle S^{qq}(t)\rangle\right] \qquad \sigma_q = m_q g_S^q$$

Scalar content

$$g_S^q = Z_P \frac{G_\pi}{G_\pi(t)} \left[\frac{\langle \mathcal{N} S^{qq}(t) \overline{\mathcal{N}} \rangle}{\langle \mathcal{N} \overline{\mathcal{N}} \rangle} - \langle S^{qq}(t) \rangle \right]$$

Improved using c_{fl} = 0.5734

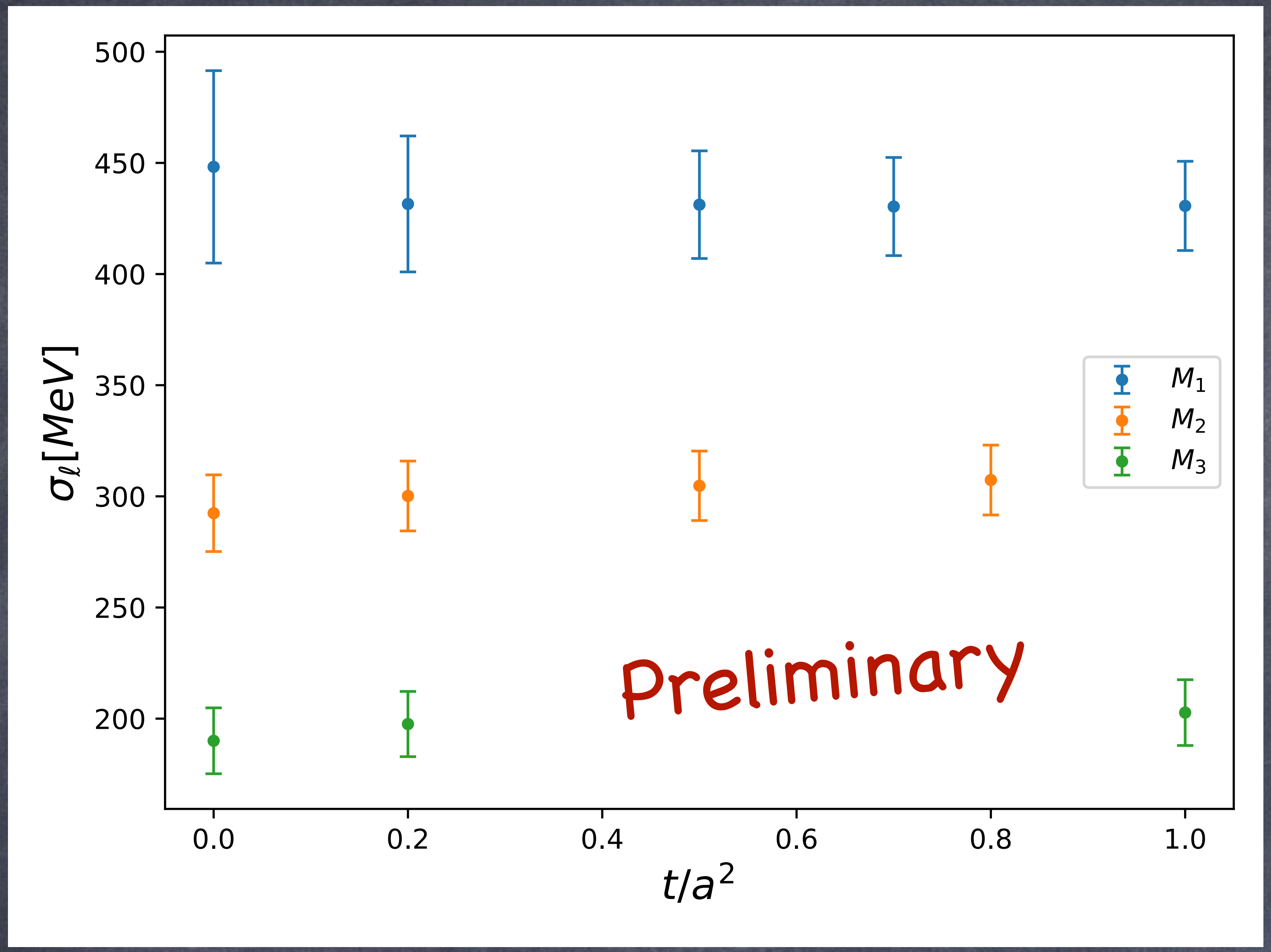
Lüscher: 2013

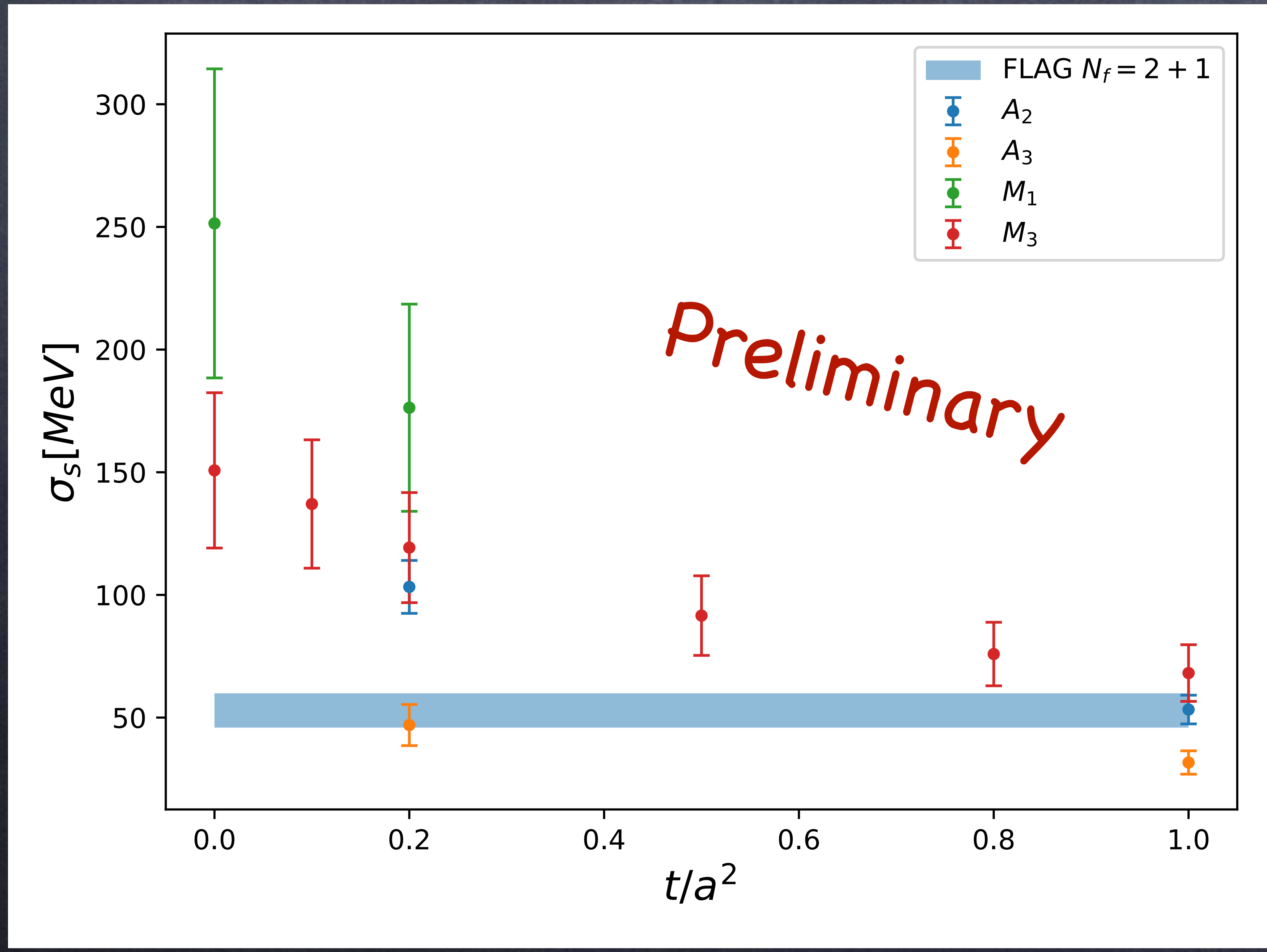
m_π = 500 MeV

a = 0.093fm

Z_P Aoki et al. (PACS-CS): 2010

Scalar content





Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	$\sigma_{\pi N}$ [MeV]	σ_s [MeV]
ETM 19	[150]	2+1+1	A	■	○	★	na/na	○	41.6(3.8)	45.6(6.2)
JLQCD 18	[60]	2+1	A	■	○	○	na/na	○	26(3)(5)(2)	17(18)(9)
χQCD 15A	[56]	2+1	A	○	★	★	na/na	○	45.9(7.4)(2.8)$^\$$	40.2(11.7)(3.5)$^\$$
χQCD 13A	[55]	2+1	A	■	■	○	−/na	○	−	33.3(6.2)$^\$$
JLQCD 12A	[59]	2+1	A	■	○	○	−/na	○	−	$0.009(15)(16)\times m_N{}^\dagger$
Engelhardt 12	[185]	2+1	A	■	○	■	−/na	○	−	$0.046(11)\times m_N{}^\dagger$
ETM 16A	[39]	2	A	■	○	○	na/na	○	$37.2(2.6)(^{4.7}_{2.9})$	$41.1(8.2)(^{7.8}_{5.8})$
RQCD 16	[35]	2	A	○	★	★	na/★	■	35(6)	35(12)
MILC 12C	[190]	2+1+1	A	★	★	★	−/○	○	−	$0.44(8)(5)\times m_s{}^{\P\S}$
MILC 12C	[190]	2+1	A	★	○	★	−/○	○	−	$0.637(55)(74)\times m_s{}^{\P\S}$
MILC 09D	[191]	2+1	A	★	○	★	−/na	○	−	59(6)(8)§

FLAG: 2021

G. Pederiva -> Aug. 4 10:20am
Algorithms and Artificial Intelligence

Renormalization

Bhattacharya, Cirigliano, Gupta, Mereghetti, Yoon: 2015

$$O^{rs}_{CE}(x) = \overline{\psi}^r(x)\gamma_5\sigma_{\mu\nu}G_{\mu\nu}\psi^s(x)$$

$$[O^{rs}_{CE}]_R = Z_{CE}\left[O^{rs}_{CE} - \frac{C}{a^2}P^{rs} + \text{d=4,5 operators}\right]$$

$$P^{rs}(x) = \overline{\psi}^r(x)\gamma_5\psi^s(x)$$

RI-MOM Off-shell

$\frac{1}{a}$ d=4 –> 2 operators + 3 O(m)

$\log a$ d=5 –> 3 operators + (7 + 5) O(m,m²) + 4 "nuisance"

Strategy - Short flow-time expansion

$$[\mathcal{O}_i(t)]_{\mathrm{R}} = \sum_i c_{ij}(t,\mu)\,[\mathcal{O}_i(t=0,\mu)]_{\mathrm{R}} + O(t)$$

LQCD PT - LQCD

$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi} c_{ij}^{(1)}(t,\mu) + O(\alpha_s^2)$$

Lüscher: 2013

A.S., Luu, de Vries: 2014-2015
Dragos, Luu, A.S. de Vries: 2018-2019
Rizik, Monahan, A.S.: 2018-2020
A.S.: 2020
Kim, Luu, Rizik, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer: 2021

- Calculation of matrix elements with flowed fields
  - Multiplicative renormalization (no power divergences and no mixing)
- Calculation of Wilson coefficients
  - Insert OPE in off-shell amputated 1PI Green's functions
- Power divergences subtracted non-perturbatively (LQCD)
- Determination of the physical renormalized matrix element at zero flow-time

Renormalization

$$O^{rr}_{CE}(t) = \overline{\chi}^r(t)\gamma_5\sigma_{\mu\nu}G_{\mu\nu}(t)\chi^r(t) \qquad [O^{rr}_{CE}(t)]_R = Z_{CE}O^{rr}_{CE}(t) \qquad Z_{CE} = Z_\chi \qquad \chi_R(t) = Z_\chi^{1/2}\chi(t)$$

$$\mathring{O}^{rr}_{CE}(t) = \overline{\mathring{\chi}}^r(t)\gamma_5\sigma_{\mu\nu}G_{\mu\nu}(t)\mathring{\chi}^r(t)$$

$$\mathring{O}^{rr}_{CE}(t) = \frac{c_P}{t}P^{rr}_R + \sum_i c_i(t,\mu)\,[O^{rr}_i(\mu)]_R + O(t) \qquad O^{rr}_1 = \overline{\psi}^r\gamma_5\sigma_{\mu\nu}G_{\mu\nu}\psi^r$$

$$\mathring{O}^{rr}_{CE,\mathrm{sub}}(t) = \mathring{O}^{rr}_{CE}(t) - \frac{c_P}{t}P^{rr}_R = \sum_i c_i(t,\mu)\,[O^{rr}_i(\mu)]_R + O(t)$$

c_P depends on the scheme used to renormalize the flowed fields

The dependence on the χ-scheme cancels out if we use the same scheme to calculate $c_i(t,\mu)$

Renormalization

$$\Gamma_{CP}(x_4;t) = a^3 \sum_{\mathbf{x}} \langle \mathcal{O}^{rr}_{CE}(x_4,\mathbf{x};t) P^{rr}(0,\mathbf{0};0)\rangle$$

$$\Gamma_{PP}(x_4) = a^3 \sum_{\mathbf{x}} \langle P^{rr}(x_4,\mathbf{x}) P^{rr}(0,\mathbf{0})\rangle$$

$$[R_P(x_4;t)]_R = t\,\frac{[\Gamma_{CP}(x_4;t)]_R}{[\Gamma_{PP}(x_4)]_R}$$

$$[R_P(x_4;t)]_R = c_P + O(t)$$

$$[\Gamma_{PP}(x_4)]_I = \left(1 + a\; b_P m_{q,rr} + a\; \bar{b}_P \mathrm{Tr} M\right) \Gamma_{PP}(x_4)$$

$$[\Gamma_{CP}(x_4;t)]_I = \left(1 + a\; b_\chi m_{q,rr} + a\; \bar{b}_\chi \mathrm{Tr} M\right) \Gamma_{CP}(x_4;t) + a\; \tilde{c}_P \widetilde{\Gamma}_{CP}(x_4;t)$$

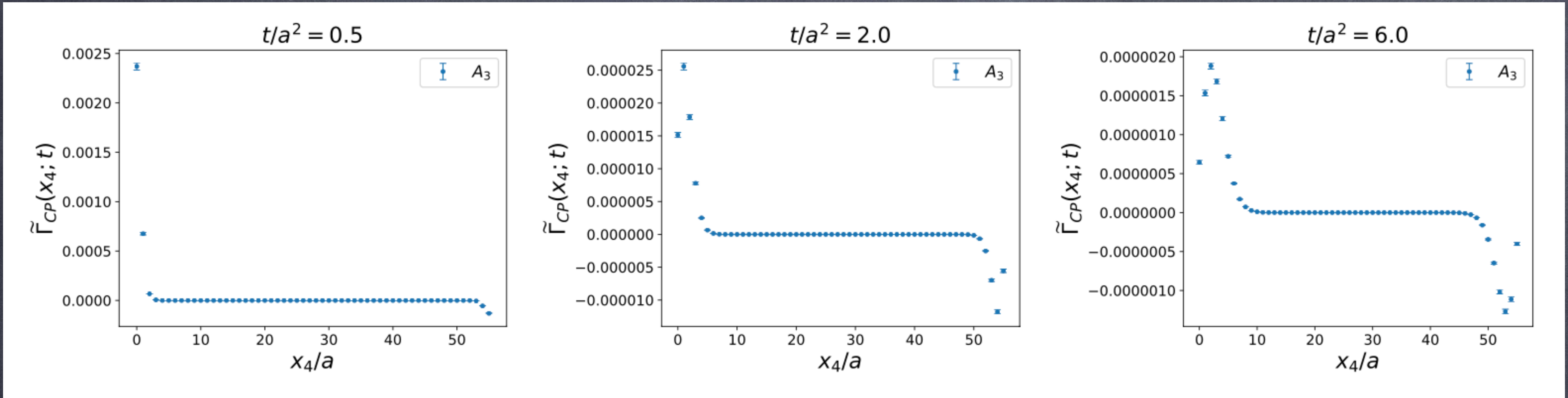
$$b_P^{(0)} = b_\chi^{(0)} = 1$$

$$\bar{b}_X = O(g^4)$$

$$\widetilde{\Gamma}_{CP}(x_4;t) = a^3 \sum_{\mathbf{x}} \left\langle O^{rr}_{CE}(x_4,\mathbf{x};t) \widetilde{P}^{rr}(0,\mathbf{0};0)\right\rangle$$

$$\widetilde{P}^{rs} = \overline{\lambda}^r \gamma_5 \psi^s + \overline{\psi}^r \gamma_5 \lambda^s$$

O(a) improvement

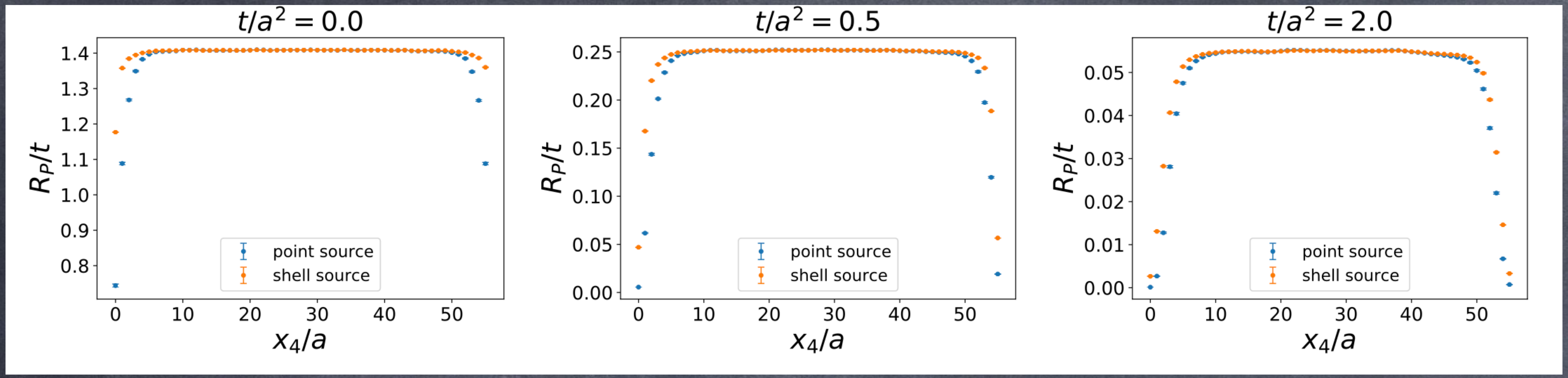


$$[R_P(x_4;t)]_R = t\frac{[\Gamma_{CP}(x_4;t)]_R}{[\Gamma_{PP}(x_4)]_R}$$

residual $O(am\alpha_s)$ cutoff effects

Non-perturbative renormalization (power divergences)

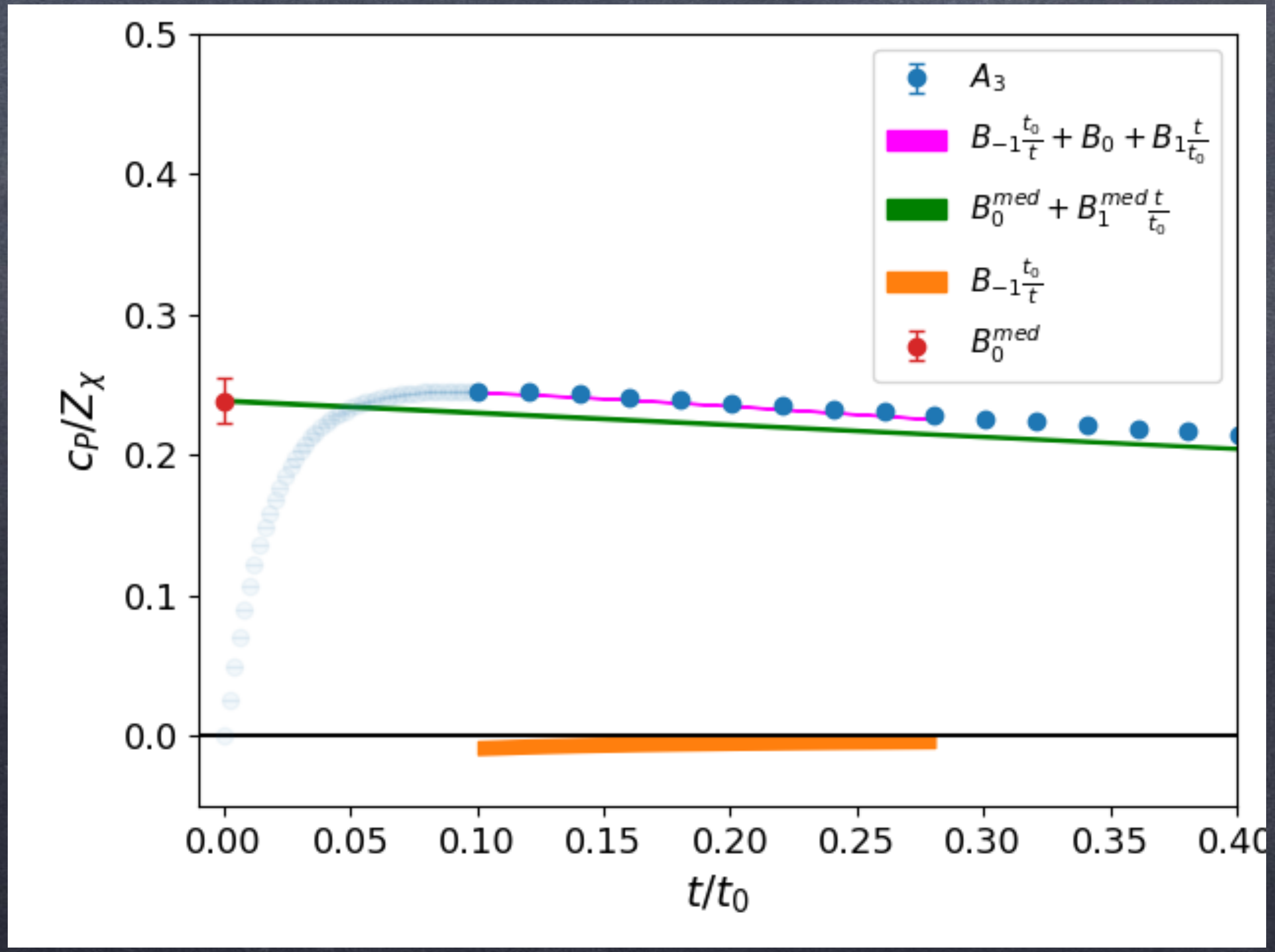
Kim, Luu, Rizik, A.S.:2020

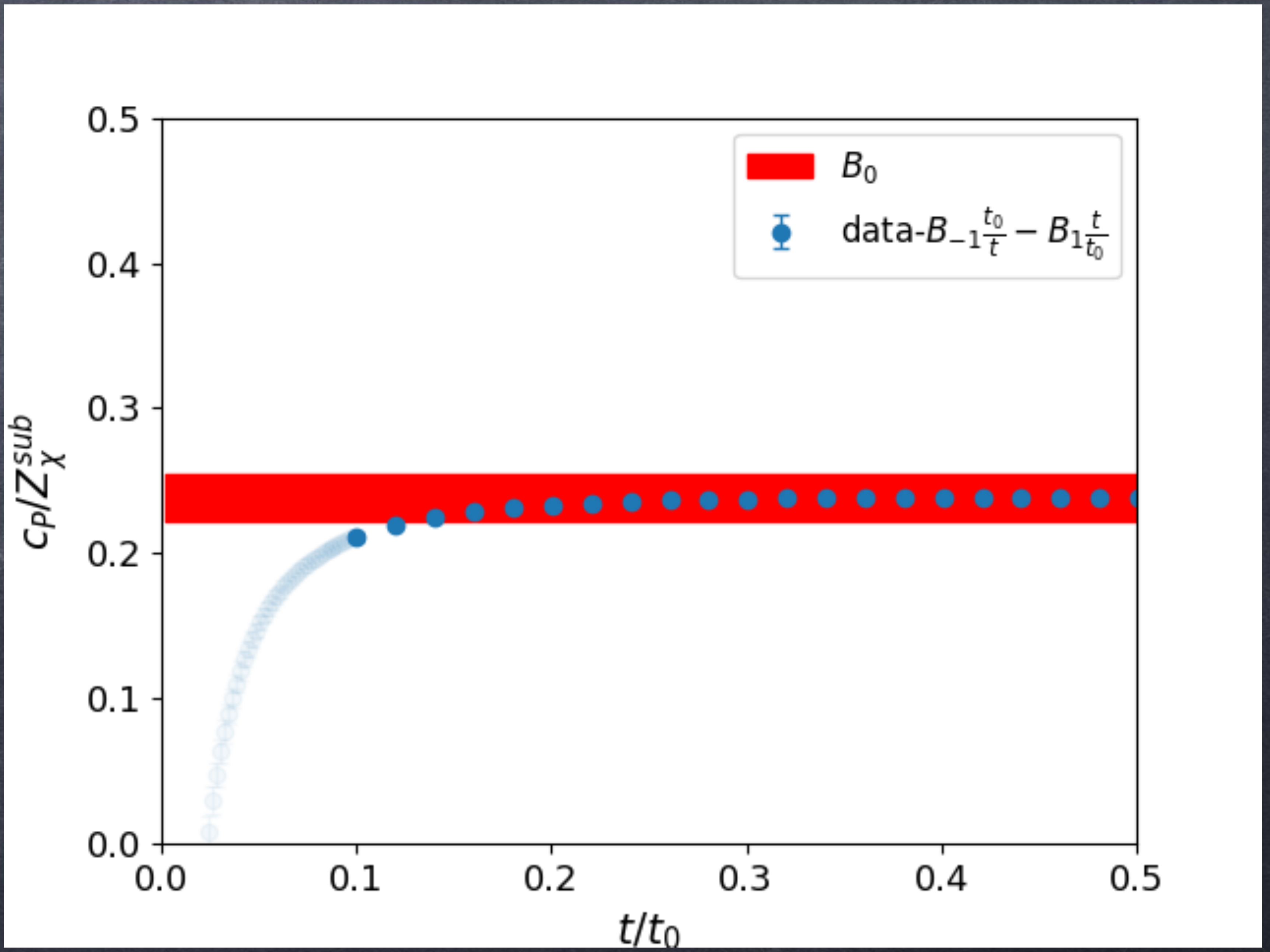


$$\frac{R_P(x_4;t)}{t}$$

$$\frac{1}{Z_\chi} c_P = \lim_{t\to 0} \frac{t}{Z_P} \frac{\langle 0|O_{CE}(t)|PS\rangle}{\langle 0|P(0)|PS\rangle}$$

$$R_{\rm fit}(t) = B_{-1}\frac{t_0}{t} + B_0 + B_1\frac{t}{t_0}$$





Non-perturbative renormalization (power divergences)

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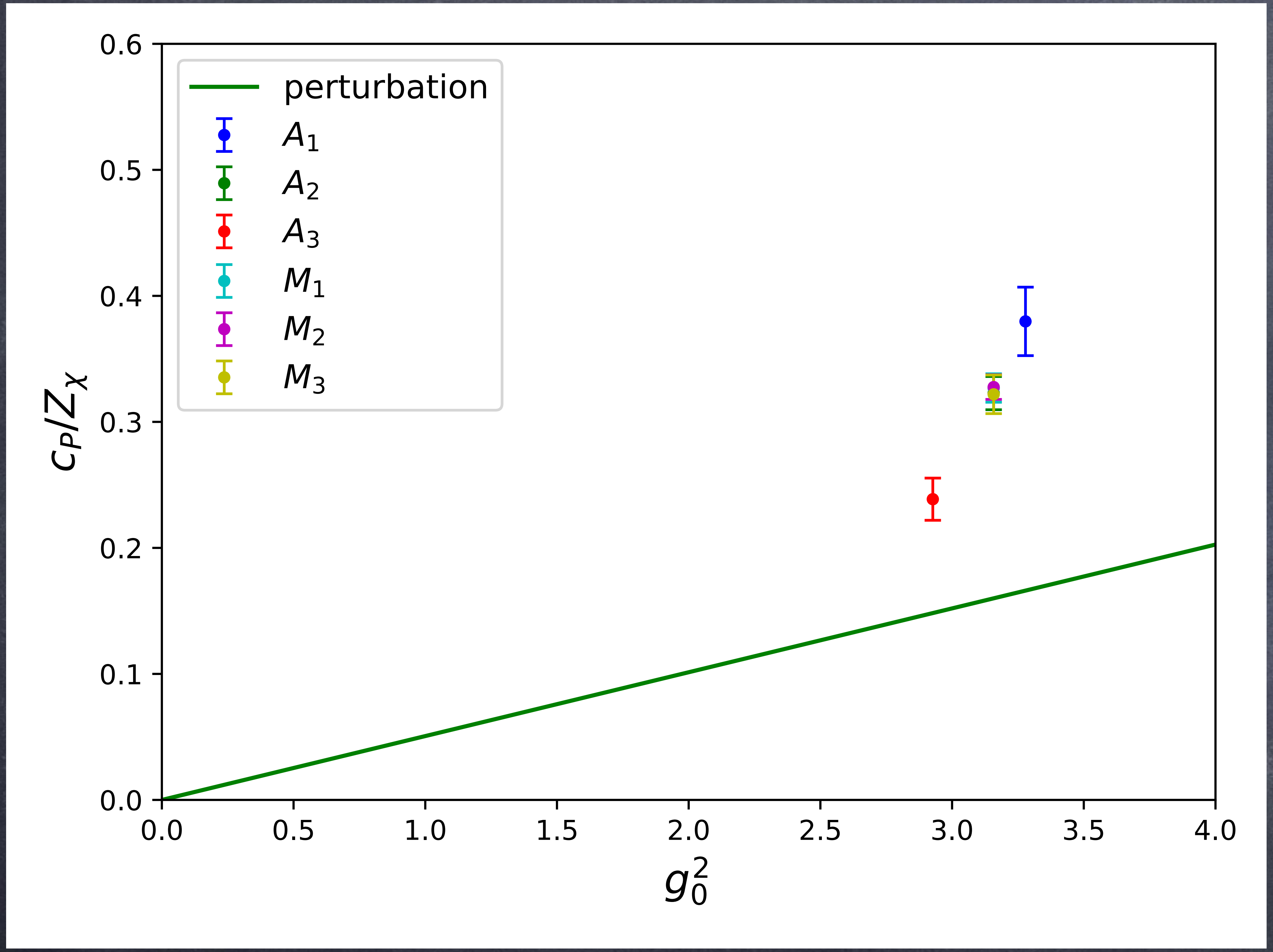
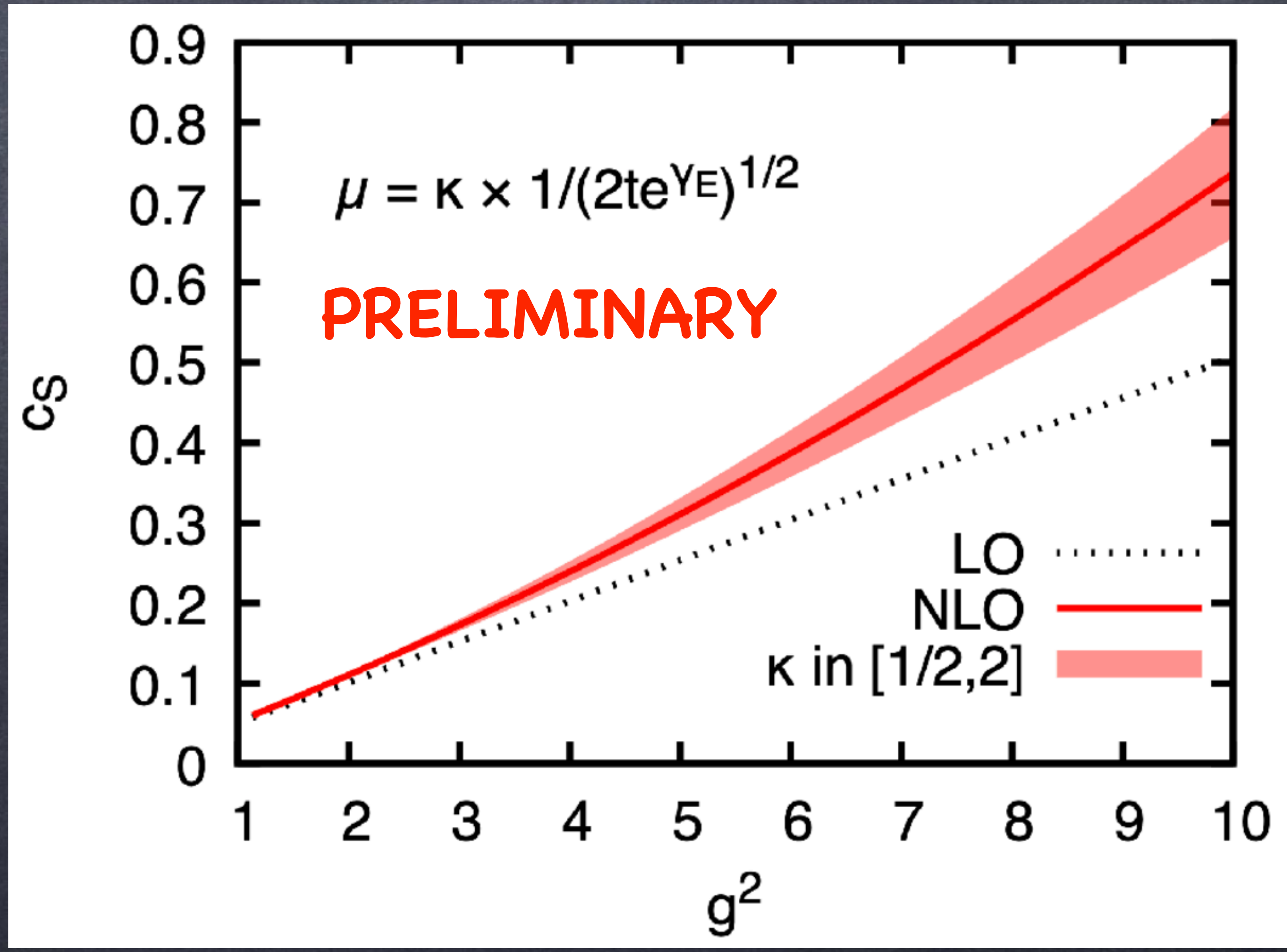


$$c_P(t) = \frac{\bar{g}^2}{2\pi^2} + \left(\frac{\bar{g}^2}{4\pi^2}\right)^2 [x_0 + x_1 \log \mu^2 t]$$

Rizik, Monahan, A.S.: 2020
Borgulat, Harlander, Rizik, A.S.

Warm-up MDM —> 2-loops (226 - 3375 FD)



Ongoing work

- Non-perturbative determination of Z_χ
- Extend the range of flow times —> use of ML algorithms
- qCEDM nucleon matrix element
- Extension to the CP-odd 3-gluon operator
- Perturbation theory is ongoing
- OpenLat: open science initiative. Gauges with SWF open to the whole community

Cuteri, Francis, Fritzsch, Pederiva, Rago, A.S., Walker-Loud, Zafeiropoulos

A. Francis -> Aug. 4 9am
Hadronic and Nuclear Spectrum and Interactions

Ongoing work

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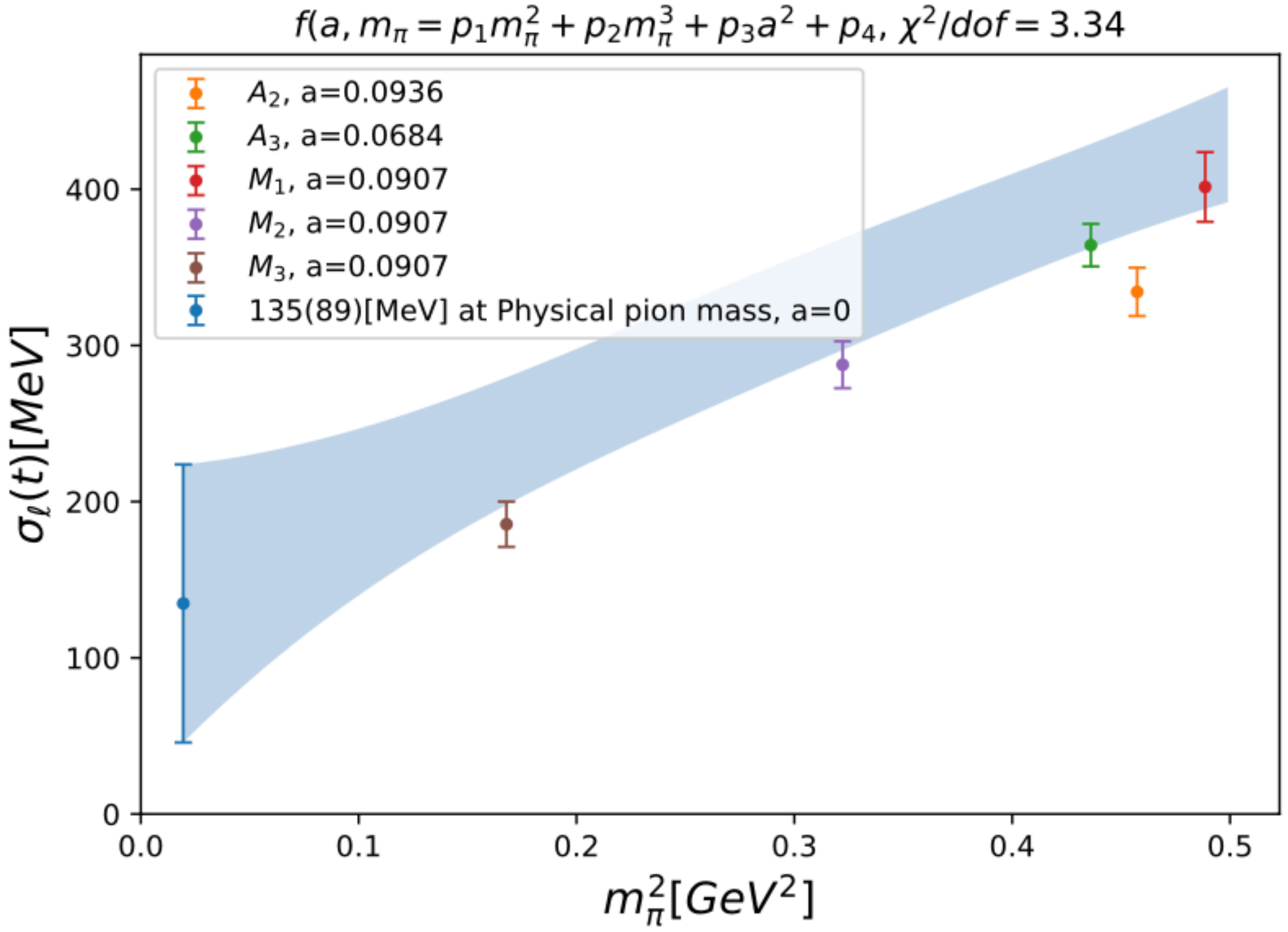
Cuteri, Francis, Fritzsch, Pederiva, Rago, A.S., Walker-Loud, Zafeiropoulos

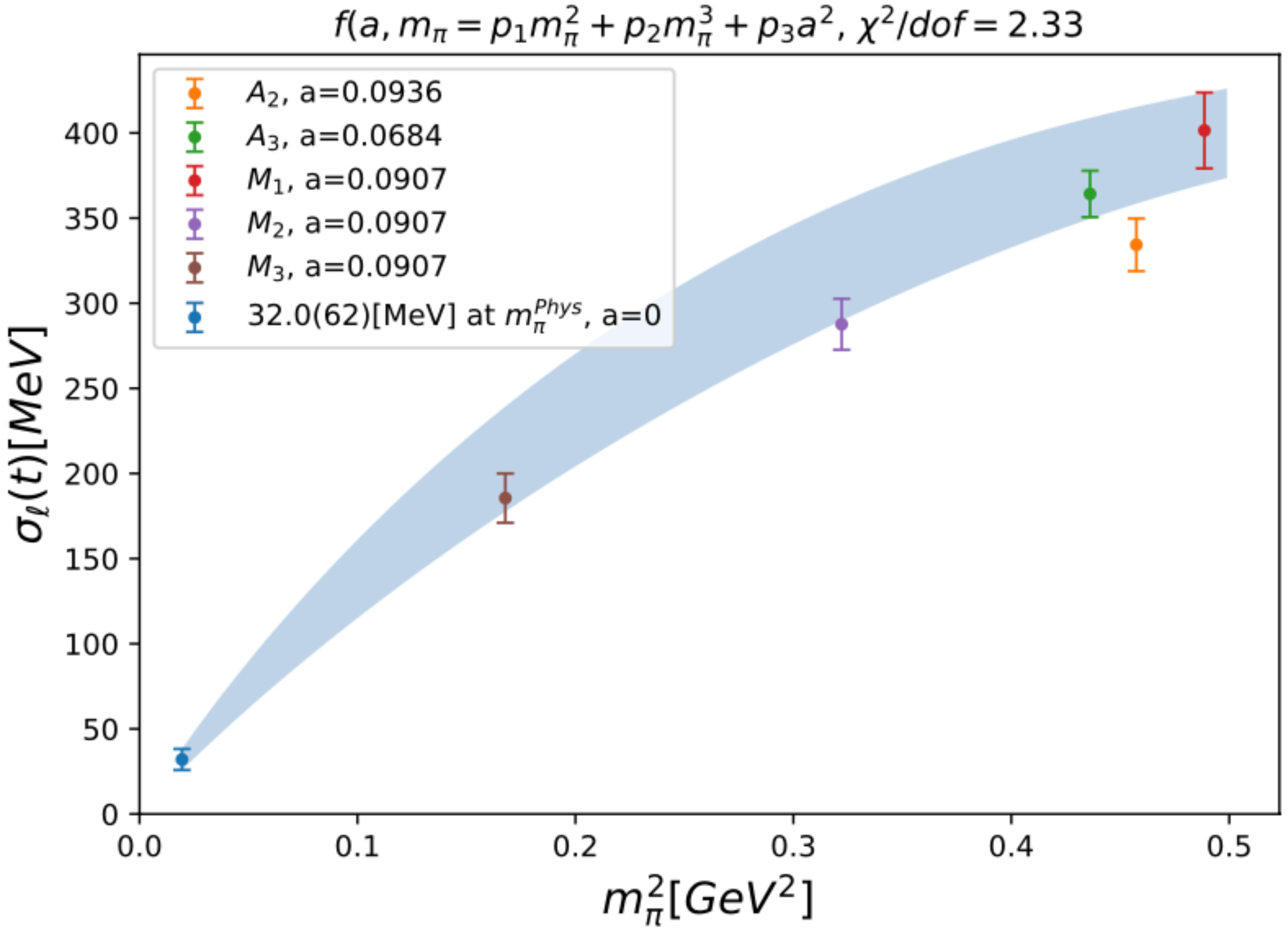
A. Francis -> Aug. 4 9am
Hadronic and Nuclear Spectrum and Interactions

Thank you!

Backup Slides

Chiral extrapolation





Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$\mathcal{O}_{CE}(x) = \bar{\psi}(x)\tilde{\sigma}_{\mu\nu}t^a\psi(x)G^a_{\mu\nu}(x)$$

- Mixes under renormalization with lower dimensional operators
- Taking into account gauge invariance and chiral symmetry

$$\mathcal{O}^R_{CE}(x;t) = c_P(t,\mu)\mathcal{O}^{\rm MS}_P(x;\mu) + c_{m\theta}(t,\mu)\mathcal{O}^{\rm MS}_{m\theta}(x;\mu) + c_E(t,\mu)\mathcal{O}^{\rm MS}_E(x;\mu)$$
$$+ c_{CE}(t,\mu)\mathcal{O}^{\rm MS}_{CE}(x;\mu) + c_{mP^2}(t,\mu)\mathcal{O}^{\rm MS}_P(x;\mu) + O(t)$$

$$\mathcal{O}_P(x) = \bar{\psi}(x)\gamma_5\psi(x) \qquad \mathcal{O}_{m^2P}(x) = m^2\bar{\psi}(x)\gamma_5\psi(x)$$

$$\mathcal{O}_{m\theta}(x) = m\,\mathrm{tr}[G_{\mu\nu}\widetilde{G}_{\mu\nu}] \qquad \mathcal{O}_E(x) = \bar{\psi}(x)\tilde{\sigma}_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

Quark-Chromo EDM: non-perturbative renormalization (power divergences)

Kim, Luu, Rizik, A.S.:2020

- Non-perturbative determination of power divergences
- Continuum limit impossible with other methods. Uncontrolled systematics

$$\Gamma_{CP}(x_4;t) = a^3 \sum_{\mathbf{x}} \left\langle \mathcal{O}_{CE}^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0};0) \right\rangle$$

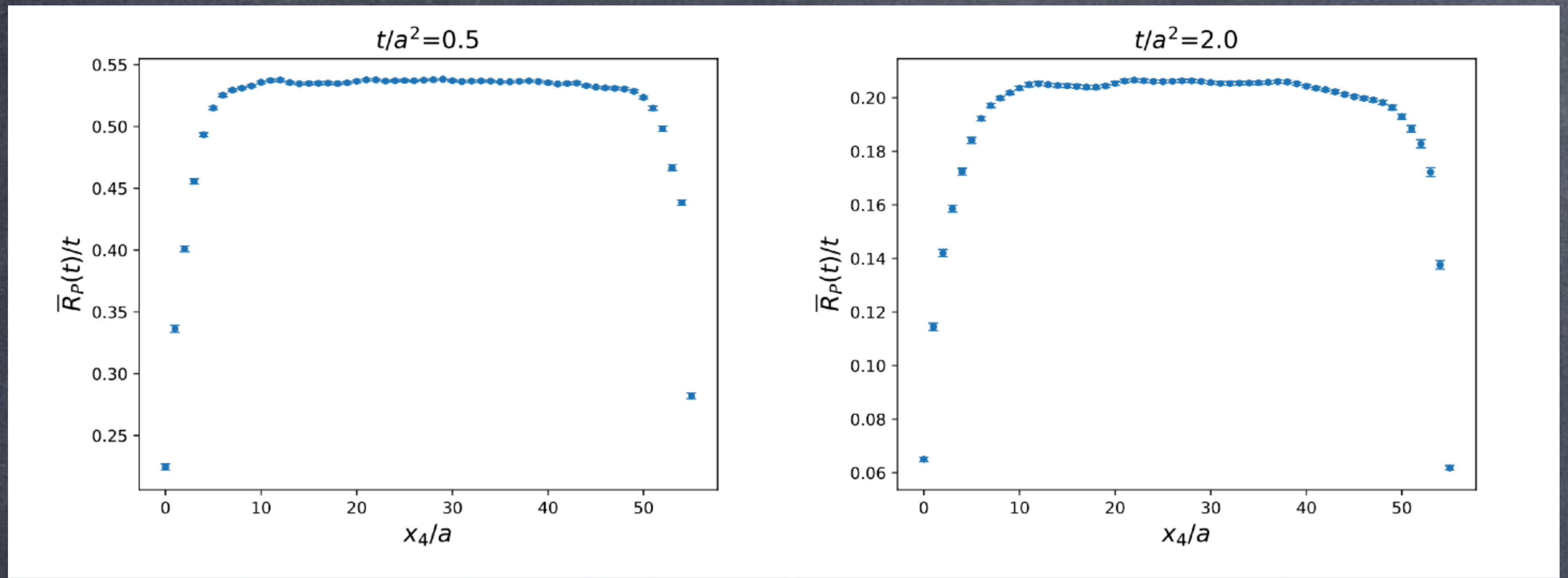
$$\Gamma_{PP}(x_4;t) = a^3 \sum_{\mathbf{x}} \langle P^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0}) \rangle$$

$$\left[\overline{R}_P(x_4;t)\right]_R = t\,\frac{[\Gamma_{CP}(x_4;t)]_R}{[\Gamma_{PP}(x_4;t)]_R}$$

→ Coefficient linear divergence

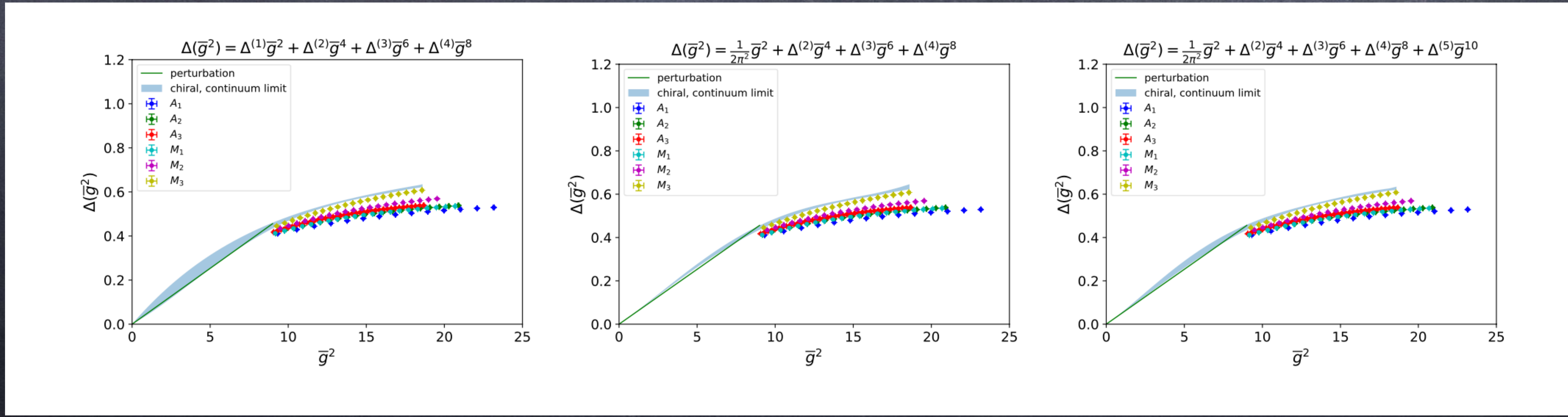
Quark-Chromo EDM: non-perturbative renormalization (power divergences)

Kim, Luu, Rizik, A.S.:2020



$$\frac{\left[\overline{R}_P(x_4;t)\right]_R}{t}$$

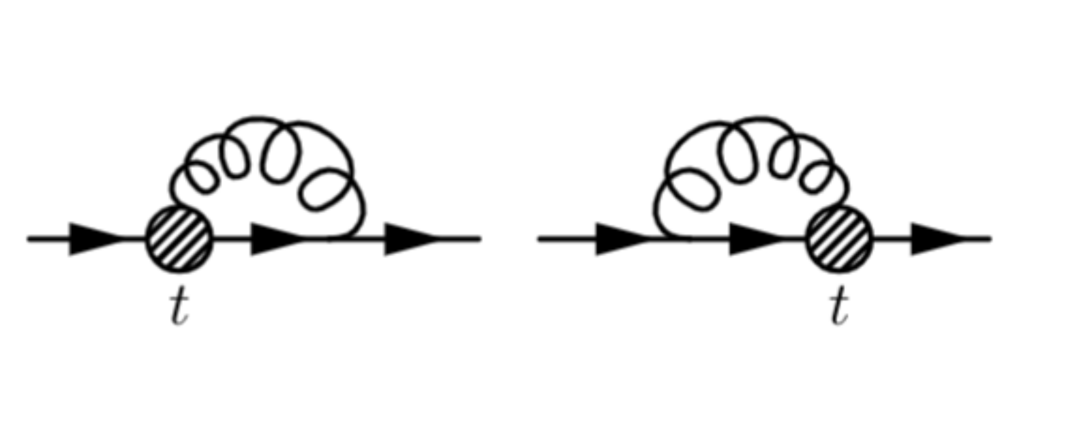
Coefficient linear divergence



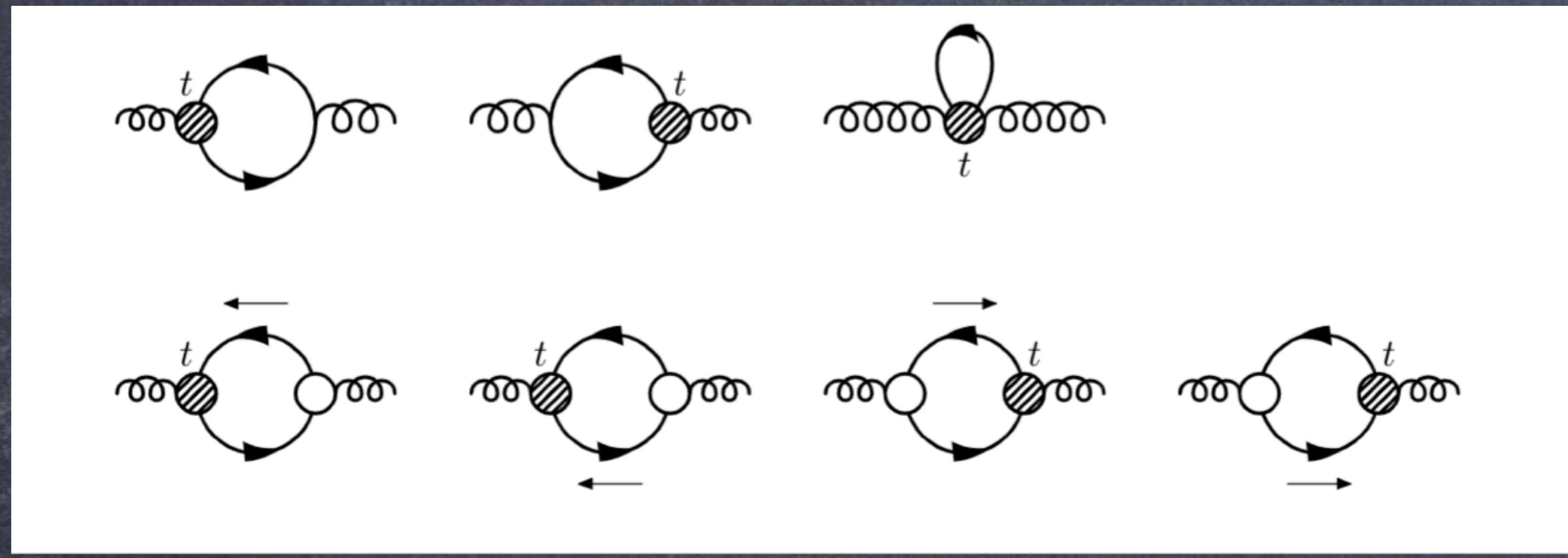
Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

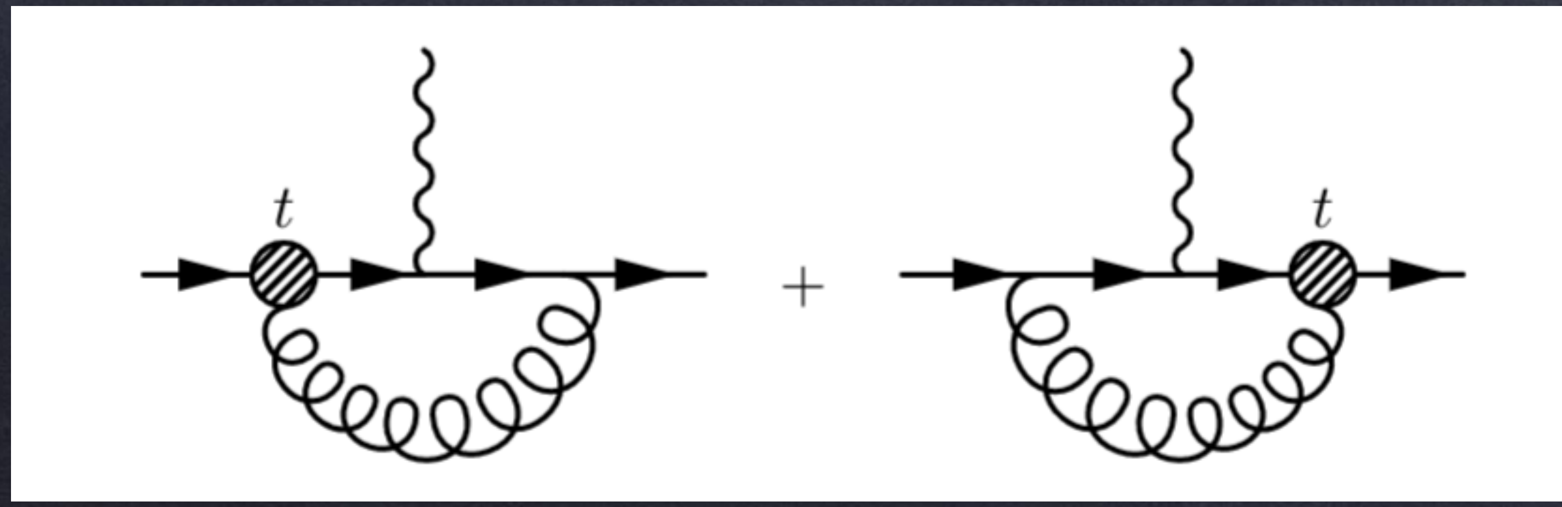
$c_P(t,\mu)$ $c_{m^2P}(t,\mu)$

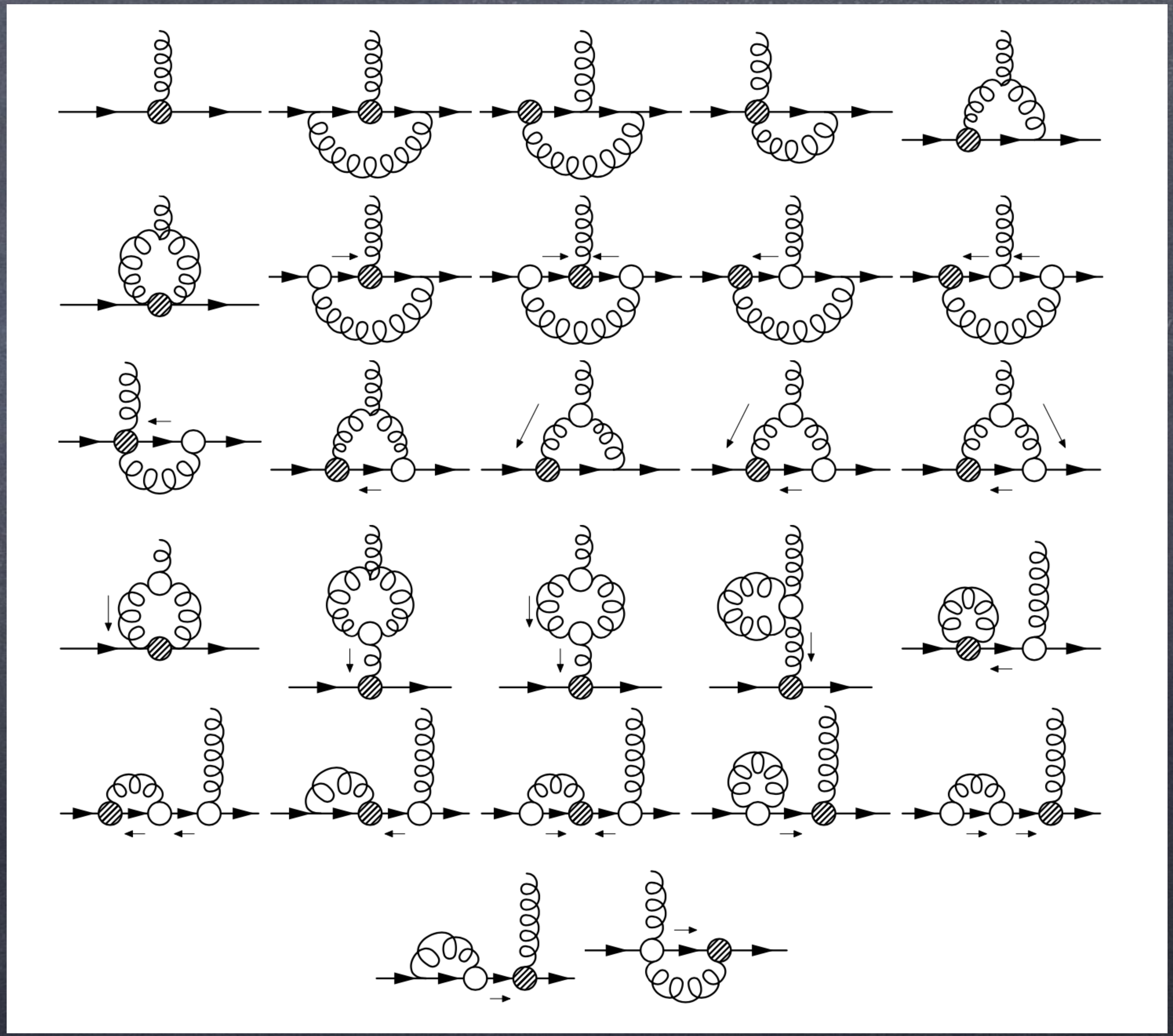


$c_{m\theta}(t,\mu)$



$c_E(t,\mu)$





$c_{CE}(t,\mu)$

Quark-Chromo EDM

Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$\mathcal{O}^R_{CE}(x;t) = \mathring{\bar{\chi}}(x;t)\tilde{\sigma}_{\mu\nu}t^a\mathring{\chi}(x;t)G^a_{\mu\nu}(x;t)$$

$$\begin{aligned}\mathcal{O}^R_{CE}(x;t) &= c_P(t,\mu)\mathcal{O}^{\overline{\rm MS}}_P(x;\mu) + c_{m\theta}(t,\mu)\mathcal{O}^{\overline{\rm MS}}_{m\theta}(x;\mu) + c_E(t,\mu)\mathcal{O}^{\overline{\rm MS}}_E(x;\mu)\\ &\quad + c_{CE}(t,\mu)\mathcal{O}^{\overline{\rm MS}}_{CE}(x;\mu) + c_{mP^2}(t,\mu)\mathcal{O}^{\overline{\rm MS}}_P(x;\mu) + O(t)\end{aligned}$$

$$\begin{aligned}c_{CE}(t,\mu) &= \zeta_\chi^{-1} + \frac{\alpha_s}{4\pi}\left[2(C_F - C_A)\log(8\pi\mu^2 t) - \frac{1}{2}\Big((4+5\delta_{\rm HV})C_A + (3-4\delta_{\rm HV})C_F\Big)\right]\\ &= 1 + \frac{\alpha_s}{4\pi}\Big[(5C_F - 2C_A)\log(8\pi\mu^2 t)\\ &\qquad - \frac{1}{2}\Big((4+5\delta_{\rm HV})C_A + (3-4\delta_{\rm HV})C_F\Big) - \log(432)C_F\Big]\end{aligned}$$

$$c_P(t,\mu) = \frac{\alpha_s C_F}{4\pi}\frac{6i}{t} \qquad c_E(t,\mu) = \frac{\alpha_s C_F}{4\pi}\left(4\log(8\pi\mu^2 t) + 3 + 2\delta_{\rm HV}\right) \qquad c_{m^2P}(t,\mu) = \frac{\alpha_s C_F}{4\pi}i\left(12\log(8\pi\mu^2 t) + \frac{1}{2}(33 - 16\delta_{\rm HV})\right)$$

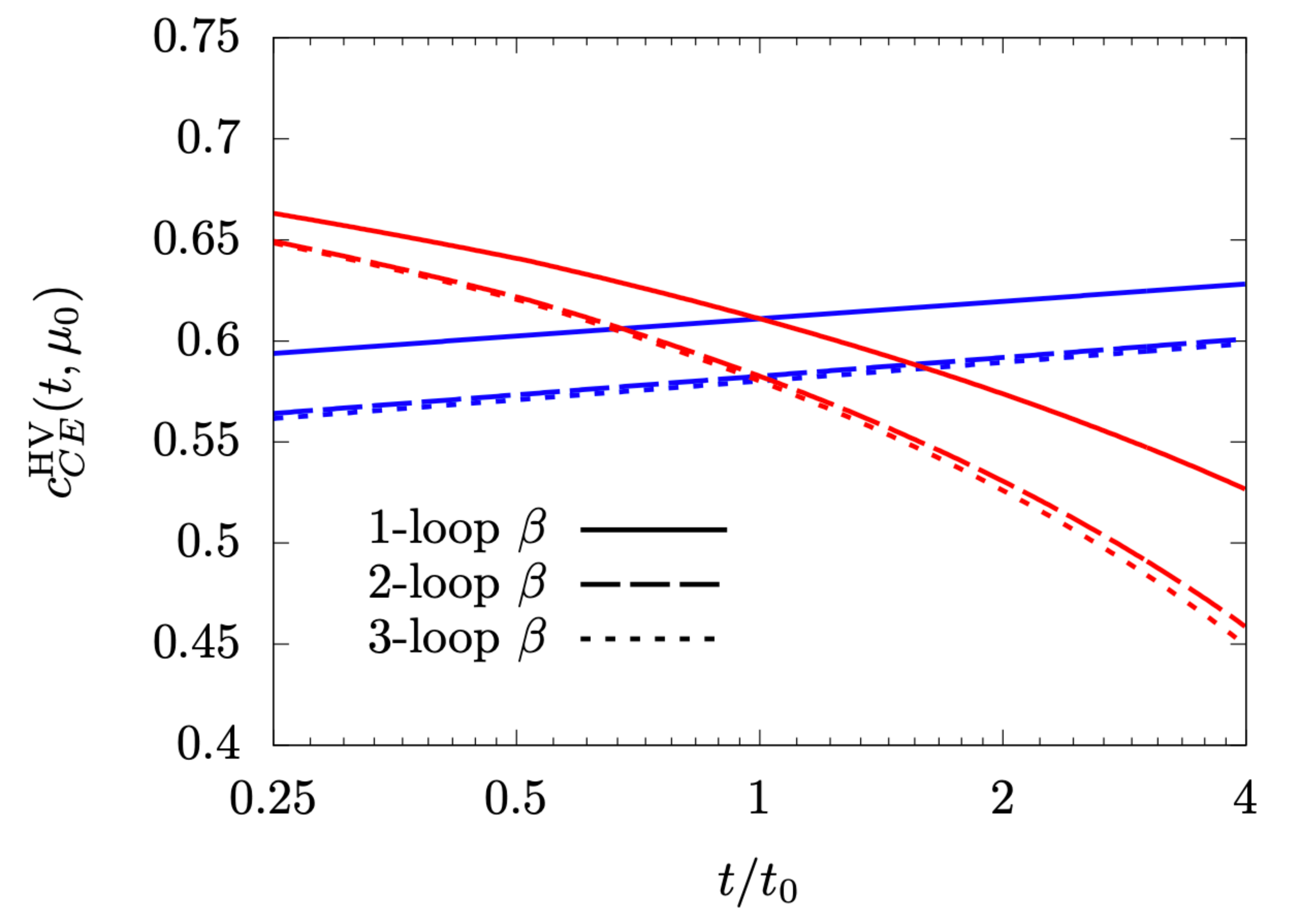
Scale dependence matching coefficients

$\bar{\mu}_0 = 3\ \text{GeV} \to \mu_0 = 1.13\ \text{GeV}$

$$t_0 = \frac{1}{8\pi\mu_0^2}$$

Red - Blue =

$$A_1\alpha_s^2(\mu_0^2)\log^2(8\pi t\mu_0^2) + A_2\alpha_s^2(\mu_0^2)\log(8\pi t\mu_0^2) + O(\alpha_s^3)$$



$t \in [t_0/4, 4t_0]$ 10%-20% uncertainties from PT at 1-loop

4+1 Local field theory

Lüscher
2010-2013

$$S = S_{\rm G} + S_{\rm G,fl} + S_{\rm F,QCD} + S_{\rm F,fl}$$

$$S_{\rm F,fl} = \int_0^\infty dt \int d^4x \left[\overline{\lambda}(t,x)\left(\partial_t - \Delta\right)\chi(t,x) + \overline{\chi}(t,x)\left(\overleftarrow{\partial}_t - \overleftarrow{\Delta}\right)\lambda(t,x)\right]$$

- Wick contractions
- Renormalization. All order proof for gauge sector — Lüscher, Weisz: 2011
- Chiral symmetry and Ward identities — Lüscher: 2013; A.S.:2013
- Wilson twisted mass — A.S.: 2013

4+1 chiral symmetry

A.S. 2013

$$S_{\mathrm{F},fl} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^4x \left[\overline{\lambda}(t,x)\left(\partial_t - \Delta\right)\chi(t,x) + \overline{\chi}(t,x)\left(\overleftarrow{\partial}_t - \overleftarrow{\Delta}\right)\lambda(t,x)\right]$$

$$\begin{cases} \chi(t,x) \to \exp\left\{i\left(\alpha_V^a \frac{T^a}{2} + \alpha_A^a \frac{T^a}{2}\gamma_5\right)\right\}\chi(t,x) \\ \overline{\chi}(t,x) \to \overline{\chi}(t,x)\exp\left\{i\left(-\alpha_V^a \frac{T^a}{2} + \alpha_A^a \frac{T^a}{2}\gamma_5\right)\right\}. \end{cases} \qquad \begin{cases} \lambda(t,x) \to \exp\left\{i\left(\alpha_V^a \frac{T^a}{2} - \alpha_A^a \frac{T^a}{2}\gamma_5\right)\right\}\lambda(t,x) \\ \overline{\lambda}(t,x) \to \overline{\lambda}(t,x)\exp\left\{i\left(-\alpha_V^a \frac{T^a}{2} - \alpha_A^a \frac{T^a}{2}\gamma_5\right)\right\} \end{cases}$$

$$\langle \mathcal{O}_t \delta S\rangle = \langle \delta\mathcal{O}_t\rangle$$

Chiral variation before integrating the Lagrange multipliers

$$\begin{cases} \left\langle \left[\partial_\mu A^a_{R,\mu}(x) - 2mP^a(x) + \widetilde{P}^a_R(0,x)\right]\mathcal{O}_R\left(\{t_0\},x\right)\right\rangle = 0 \quad t=0 \\ \left\langle \left[\partial_s \tilde{P}^a(s,x) + \partial_\mu \mathcal{A}^a_\mu(s,x)\right]\mathcal{O}_R\left(\{t_0\},x\right)\right\rangle = 0 \quad s>0\,, \quad s<\{t_0\} \end{cases}$$

Lüscher: 2013

$$\widetilde{P}^a(t,x) = \overline{\lambda}(t,x)\frac{T^a}{2}\gamma_5\chi(t,x) + \overline{\chi}(t,x)\frac{T^a}{2}\gamma_5\lambda(t,x)$$

Quark-Chromo EDM

$$Z_\chi^{-n/2} \left\langle \left(\psi^{(0)}\right)^{n_\psi} \left(\bar{\psi}^{(0)}\right)^{n_{\bar{\psi}}} \left(G_\mu^{(0)}\right)^{n_G} \mathcal{O}_i^t[\chi^{(0)}, \bar{\chi}^{(0)}, B^{(0)}] \right\rangle^{\text{amp}} =$$

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$= c_{ij}(t) \; (Z_{jk}^{\text{MS}})^{-1} \left\langle \left(\psi^{(0)}\right)^{n_\psi} \left(\bar{\psi}^{(0)}\right)^{n_{\bar{\psi}}} \left(G_\mu^{(0)}\right)^{n_G} \mathcal{O}_k^{(0)}[\psi^{(0)}, \bar{\psi}^{(0)}, G^{(0)}] \right\rangle^{\text{amp}}$$

$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi} c_{ij}^{(1)}(t,\mu) + O(\alpha_s^2)$$

$$\left\langle \mathring{\bar{\chi}}(x;t) \overleftrightarrow{\not{D}} \mathring{\chi}(x;t) \right\rangle = -\frac{2N_c N_f}{(4\pi)^2 t^2}$$

Makino, Suzuki: 2014

$$\chi(x;t) = (8\pi t)^{\varepsilon/2} \zeta_\chi^{1/2} \mathring{\chi}(x;t)$$

$$\zeta_\chi = 1 - \frac{\alpha_s C_F}{4\pi} \left(3 \log\left(8\pi\mu^2 t\right) - \log(432)\right) + O(\alpha_s^2)$$

Harlander, Kluth, Lange :2018

$$\bar{\chi}(x;t) = (8\pi t)^{\varepsilon/2} \zeta_\chi^{1/2} \mathring{\bar{\chi}}(x;t)$$

Artz, Harlander, Lange, Neumann, Prausa: 2019

Perturbation theory with flowed fields

Lüscher, Weisz:2010,2011
Lüscher: 2013

Rizik, Monhahan, A.S.: 2018, 2020

$$B_\mu(x;t) = \int \mathrm{d}^d y \left[K_{\mu\nu}(x-y;t)A_\nu(y) + \int_0^t \mathrm{d}s K_{\mu\nu}(x-y;t-s)R_\nu(y;s) \right],$$

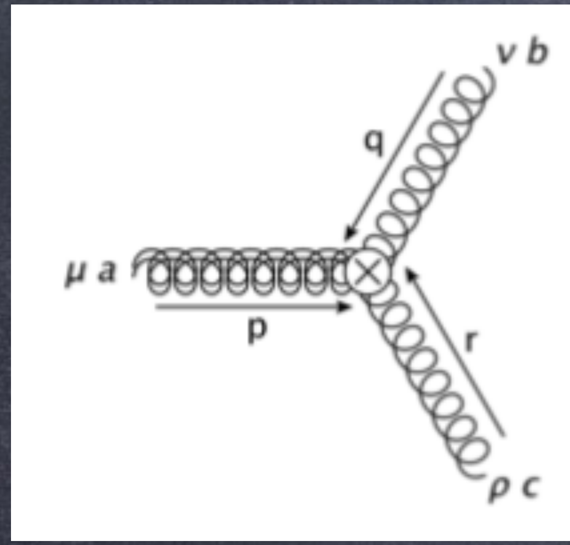
$$\chi(x,t) = \int \mathrm{d}^d y \left[J(x-y;t)\psi(y) + \int_0^t \mathrm{d}s\, J(x-y;t-s)\Delta'\chi(y;s) \right],$$

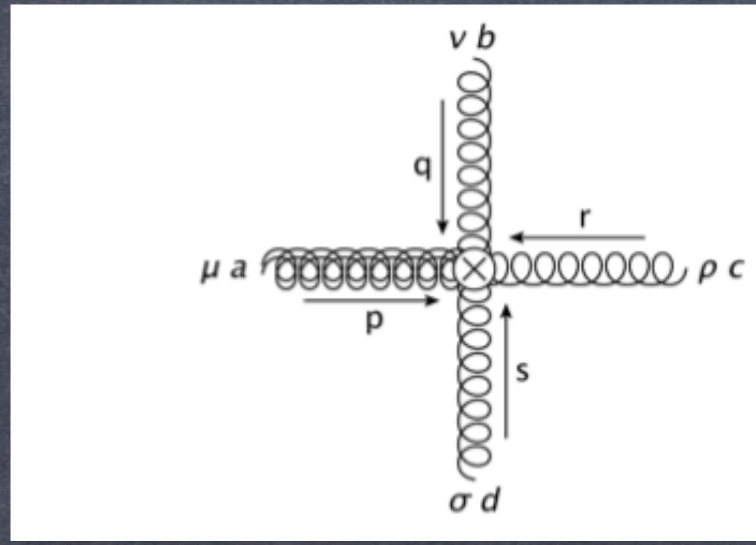
$$\overline{\chi}(x,t) = \int \mathrm{d}^d y \left[\overline{\psi}(y)\bar{J}(x-y;t) + \int_0^t \mathrm{d}s\, \overline{\chi}(y;s)\overleftarrow{\Delta}'\bar{J}(x-y;t-s) \right].$$

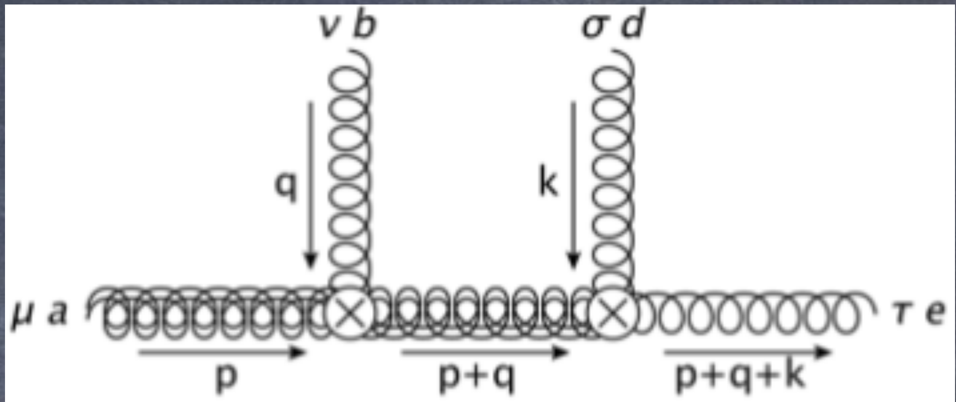
$$\partial_t \chi_t = \Delta \chi_t \qquad \partial_t \overline{\chi_t} = \overline{\chi_t}\overleftarrow{\Delta}$$

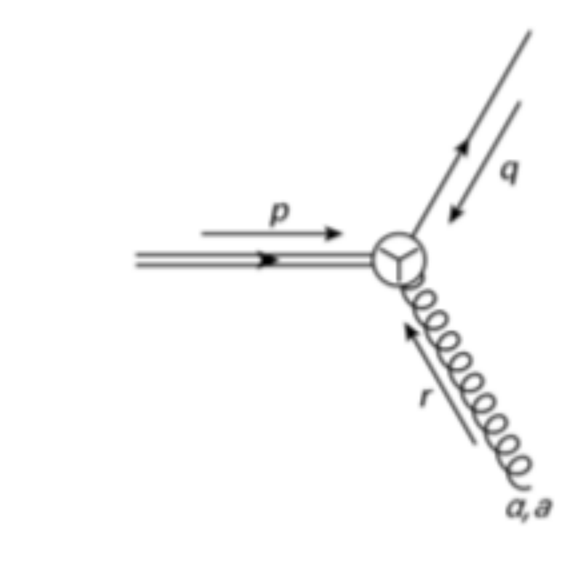
$$\chi_t(x)|_{t=0} = \psi(x)$$

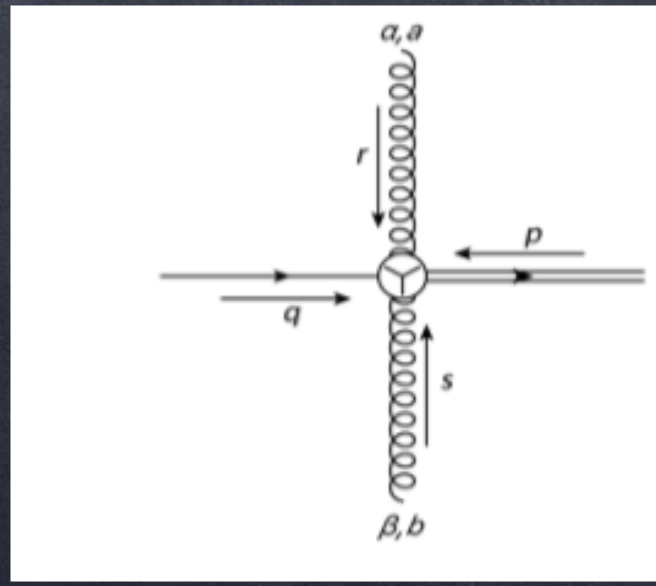
$$\overline{\chi}_t(x)|_{t=0} = \overline{\psi}(x)$$

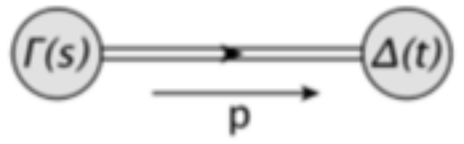




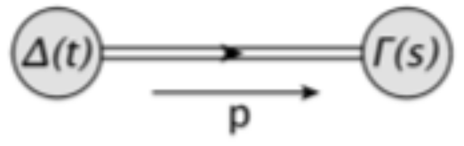








$$= \int_0^\infty ds\ \theta(t-s)\Delta(t)\tilde{J}_{t-s}(p)\Gamma(s)\,,$$

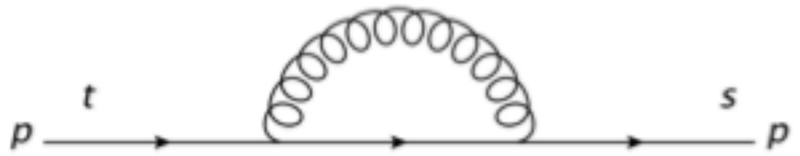


$$= \int_0^\infty ds\ \theta(t-s)\Gamma(s)\tilde{\bar{J}}_{t-s}(p)\Delta(t)\,,$$

Sample calculation: quark propagator

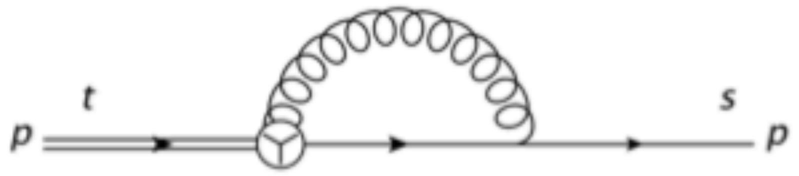
Lüscher: 2013

Rizik, Monhahan, A.S.: 2018, 2020

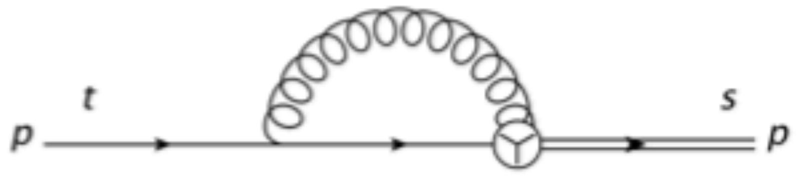
$\Sigma_1^{(2)}(p) =$ 

(C5a)

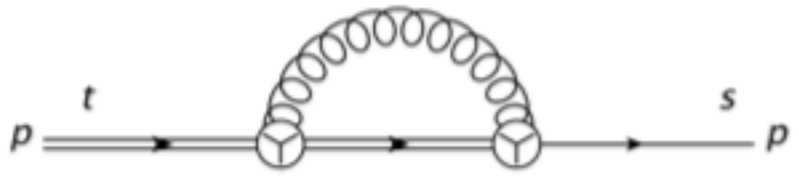
$$= -g_0^2 \frac{C_2(F)}{(4\pi)^2} \left\{ \left[\frac{1}{\epsilon} + \log\left(\frac{4\pi\mu^2}{p^2}\right) - \gamma_E + 1 \right] i\not{p} + 4 \left[\frac{1}{\epsilon} + \log\left(\frac{4\pi\mu^2}{p^2}\right) - \gamma_E + \frac{3}{2} \right] m_0 + R\left(\frac{m_0^2}{p^2}\right) \right\} + \mathcal{O}(\epsilon),$$

$\Gamma_{2,a}^{(2)}(p;t) =$ 

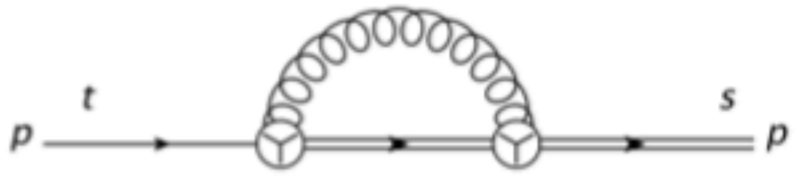
$= g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 t\right) + 1 \right] + \mathcal{O}(\epsilon, t),$ (C5b)

$\Gamma_{2,b}^{(2)}(p;s) =$ 

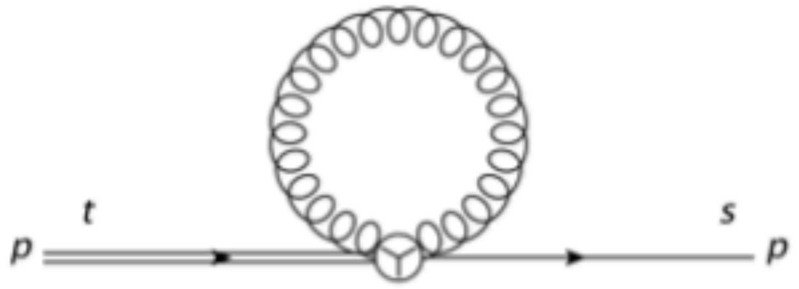
$= g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 s\right) + 1 \right] + \mathcal{O}(\epsilon, s),$ (C5c)

$\Gamma_{3,a}^{(2)}(p;t) =$ 

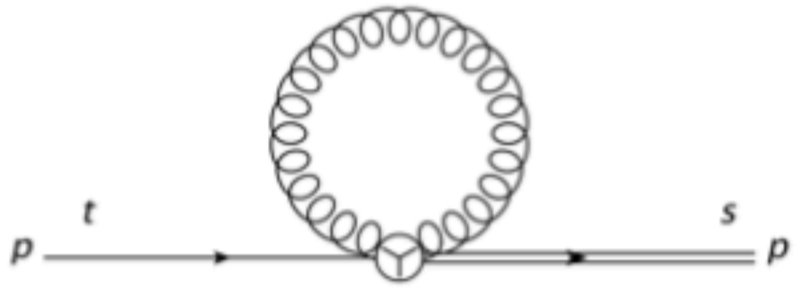
$= 0 + \mathcal{O}(t),$ (C5d)

$\Gamma_{3,b}^{(2)}(p;s) =$ 

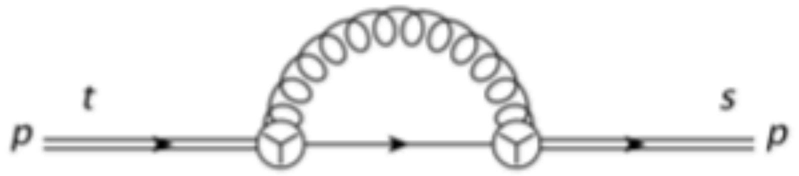
$= 0 + \mathcal{O}(s),$ (C5e)

$\Gamma_{4,a}^{(2)}(p;t) =$ 

$= -2g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 t\right) + \frac{1}{2} \right] + \mathcal{O}(\epsilon, t),$ (C5f)

$\Gamma_{4,b}^{(2)}(p;s) =$ 

$= -2g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 s\right) + \frac{1}{2} \right] + \mathcal{O}(\epsilon, s),$ (C5g)

$\Gamma_5^{(2)}(p;t,s) =$ 

$= 0 + \mathcal{O}(s, t),$ (C5h)

$$Z_\chi = 1 + g_0^2 \frac{C_2(F)}{(4\pi)^2} \left\{ \frac{3}{\epsilon} + \log(4\pi) - \gamma_E + 1 \right\}$$

Numerical details

NP improved Wilson + Iwasaki gauge

a=0.1-0.068 fm
mpi=400-700 MeV

O(L/2a) Stochastic source locations

3 Gaussian smearings

	β	κ_l	κ_s	L/a	T/a	c_{sw}	N_G	N_{corr}
M_1	1.90	0.13700	0.1364	32	64	1.715	322	30094
M_2	1.90	0.13727	0.1364	32	64	1.715	400	20000
M_3	1.90	0.13754	0.1364	32	64	1.715	444	17834
A_1	1.83	0.13825	0.1371	16	32	1.761	800	15220
A_2	1.90	0.13700	0.1364	20	40	1.715	789	15407
A_3	2.05	0.13560	0.1351	28	56	1.628	650	12867

PACS-CS: 2009