Doubly charm tetraquark using meson-meson and diquark-antidiquark interpolators

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LHCb: Double-charm tetraquark $T_{cc}^+ [cc\bar{u}\bar{d}]$

2021: Signal in $D^0D^0\pi^+$ just 0.4 MeV below $D^0D^{*+}$ threshold. \(^1\) \(^2\)

\[ \Gamma = 410 \pm 165 \text{ keV}. \]

From different models expected:

$I(J^P) = 0(1^+)$
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Does this state exist in QCD?

- What is its mass?
- Quantum numbers?

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Available lattice studies on $T^{+}_{cc}$, $J^{P} = 1^{+}$

HALQCD, 2302.04505, $m_{\pi} = 146$ MeV

Padmanath, Prelovsek: 2202.10110, PRL, $m_{\pi} = 280$ MeV

CLQCD 2206.06185, PLB, $m_{\pi} = 348$ MeV

$E_{n}$ obtained with $D(\vec{p}_{1})D^{*}(\vec{p}_{2})$ interpolators. Scattering amplitude extracted via Lüsher eq.

Additional works

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Adittional works $a$ $b$ $c$


Is the $T_{cc}^{+}$ basis with two-mesons sufficient?
Study of $T_{cc}^+$ with $M(\vec{p}_1)M(\vec{p}_2) + [cc][\bar{u}\bar{d}]$

**Hadspec collaboration**: a)

- $m_\pi = 391$ MeV, distillation.
- Resulting spectrum found to be insensitive to addition of the $[cc][\bar{u}\bar{d}]$.
- Only total momentum $\vec{P} = 0$ employed.
- Energy shift of the ground state not resolved.
- Scattering amplitude not extracted.

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Present work

This work is a follow up of the study of $T_{cc}^+$ with $M(\vec{p}_1)M(\vec{p}_2)$ interpolators [M. Padmanath, Prelovsek: 2202.10110, PRL], now adding also $[cc]\bar{3}c[\bar{u}\bar{d}]3c$ interpolators.

Simulation details:

- $N_f = 2 + 1$ CLS ensembles.
- $m_\pi \simeq 280$ MeV
- Spatial lattice extent $N_L = 24, 32$
- $a \simeq 0.086$ fm

We employ two heavy quark masses $m_Q$ for the system $QQ\bar{u}\bar{d}$ with $J^P = 1^+, I = 0$:

$m_Q \simeq m_c : m_D \simeq 1.931$ GeV $m_{D^*} \simeq 2.051$ GeV

$m_Q \simeq m_{\text{"b}} : m_{\text{"B}} \simeq 4.042$ GeV $m_{\text{"B}^*} \simeq 4.075$ GeV

The heavier the quark mass is close the $b$ quark mass.
\( M(\vec{p}_1) M(\vec{p}_2) \) and \([cc]_3 c [\bar{u}d]_3 c \) interpolators within distillation

- Total momenta: \( P = 0 \) (irrep \( T_1^+ \)), \( P = 1 \) (irrep \( A_2 \))
- Color singlet Meson-Meson interpolators \([\bar{u}c]_1 c [\bar{d}c]_1 c \)

\[
O^{D(*)D^*}(\vec{p}_1, \vec{p}_2) = D^{(*)}(\vec{p}_1)D^*(\vec{p}_2) = \sum_{\vec{x}_1} \bar{u}_A^a (\Gamma_1)_{AB} e^{i\vec{p}_1 \vec{x}_1} c_B^a \sum_{\vec{x}_2} d_C^b (\Gamma_2)_{CD} e^{i\vec{p}_2 \vec{x}_2} c_D^b - \{u \leftrightarrow d\}, \quad N^M_{MM} = 60
\]

Several operators
$M(\vec{p}_1)M(\vec{p}_2)$ and $[cc]_{3c}[\bar{u}\bar{d}]_{3c}$ interpolators within distillation

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Several operators

- Diquark-antidiquark interpolators $[cc]_{3c}[\bar{u}\bar{d}]_{3c}$

\[ O^{4q}(\vec{P}) = \sum_{\vec{x}} \epsilon_{abc}c_A^b(\vec{x})(C\gamma_i)_{AB}c_B^c(\vec{x}) \quad \epsilon_{ade}\bar{u}_C^d(\vec{x})(C\gamma_5)_{CD}\bar{d}_D^e(\vec{x})e^{i\vec{P}\vec{x}}, \quad N_{v}^{4q} = 45 \]
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- Distillation method - all quarks fields are smeared $\rightarrow$ spectral decomposition

$$q^b_A(\vec{x}) = \sum_{i=1}^{N_v} v^i_b(\vec{x}) v^{(i)\dagger}_b(\vec{y}) q^b_A(\vec{y}),$$

$N_v$ is the number of eigenvectors
\( M(\vec{p}_1) M(\vec{p}_2) \) and \([cc]_3[\bar{u}\bar{d}]_3 \) interpolators within distillation

Tensors (in distillation space) needed to compute correlators:

- **\( MM \):** single meson kernel

\[
\phi^{ij}(\vec{p}) = \sum_{\vec{x}} \sum_c v_c(i)(\vec{x}) v_c(j)(\vec{x}) e^{i\vec{p}\vec{x}}.
\]
\( M(\vec{p}_1)M(\vec{p}_2) \) and \([cc][\bar{u}\bar{d}]\) interpolators within distillation

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  \]

- **\( 4q \)**: kernel for compact tetraquark \([cc][\bar{u}\bar{d}]\)

  \[
  \phi^{jklm}(\vec{p}) = \sum_{\vec{x}} \sum_{abcde} \epsilon_{abc}\epsilon_{ade}v_c^{(j)}(\vec{x})v_c^{(k)}(\vec{x})v_c^{(l)}(\vec{x})v_c^{(m)}(\vec{x})e^{i\vec{p}\vec{x}}.
  \]

- Costly summation over distillation indices for \( 4q \) \( \implies \) we employ \( N_v^{4q} < N_v^{MM} \).
Effective masses of the diagonal correlators, dependence on $N_v$
Effective energies for $\vec{P} = 0$ (irrep $T_{1+}^+$)

$O^{4q} : [cc]_3[\bar{u}\bar{d}]_3$  
$O^{MM} : D(0)D^*(0)$

$D(1)D^*(-1)|_{l=0}$
$D(1)D^*(-1)|_{l=2}$
$D^*(0)D^*(0)$

$c - quark$

$M(\bar{p}_1)M(\bar{p}_2) + [cc][\bar{u}\bar{d}]$

$M(\bar{p}_1)M(\bar{p}_2)$

$D(1)D^*(1)$
$D(0)*D^*(0)$
$D(0)D^*(0)$

$\Delta E_{\text{eff}}$

$12 14 16$
$t/a$

$M(\bar{p}_1)M(\bar{p}_2) + [cc][\bar{u}\bar{d}]$

$M(\bar{p}_1)M(\bar{p}_2)$

$B(1)B^*(1)$
$B^*(0)B^*(0)$
$B(0)B^*(0)$

$\Delta E_{\text{eff}}$

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$t/a$

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$T_{cc}^+$ with diquark-antidiquark
Effective energies for $\vec{P} = 1$ (irrep $A_2$)

$$O^{4q} : [cc]_3 [\bar{u}\bar{d}]_3$$  \hspace{1cm} $$O^{MM} : D(0)D^*(1)$$

$$D(1)D^*(0)$$  \hspace{1cm} $$D^*(1)D^*(0)$$

$c$ – quark

$$M(\vec{p}_1)M(\vec{p}_2) + [cc][\bar{u}\bar{d}]$$  \hspace{1cm} $$M(\vec{p}_1)M(\vec{p}_2)$$

$$D^*(1)D^*(0)$$

$$D(1)D^*(0)$$  \hspace{1cm} $$D(0)D^*(1)$$

$b$ – quark

$$B^*(1)B^*(0)$$  \hspace{1cm} $$B(1)B^*(0)$$  \hspace{1cm} $$B(0)B^*(1)$$
Conclusions

- Obtaining the scattering amplitude from correlators based on the $M(\vec{p}_1)M(\vec{p}_2) + [cc][\bar{u}\bar{d}]$ interpolators. Studying the dependence of the spectrum as well as the scattering amplitude and pole position on different heavy quark masses $m_Q$.

- In both studied irreps, with $\vec{P} = 0$ and $\vec{P} = 1$, the ground state does not move significantly when diquark-anti-diquark interpolators are included for charm quarks.

- However, for a heavy quark mass close to the bottom sector, there is a significant energy shift of approximately 90 MeV in favor of the basis with diquark-antidiquark, as expected for the b quark sector.

- Computational cost of the diquark-anti-diquark operators increases significantly with $N_v$. Smaller $N_v$ chosen for $O^{4q}$ compared to $O^{MM}$.

- Analyze the contribution of the left hand cuts from the one pion exchange on the pole extraction of the doubly charm tetraquark.
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Thanks!