Generalized Hall current on a finite lattice

Srimoyee Sen and Semeon Valgushev

Iowa State University

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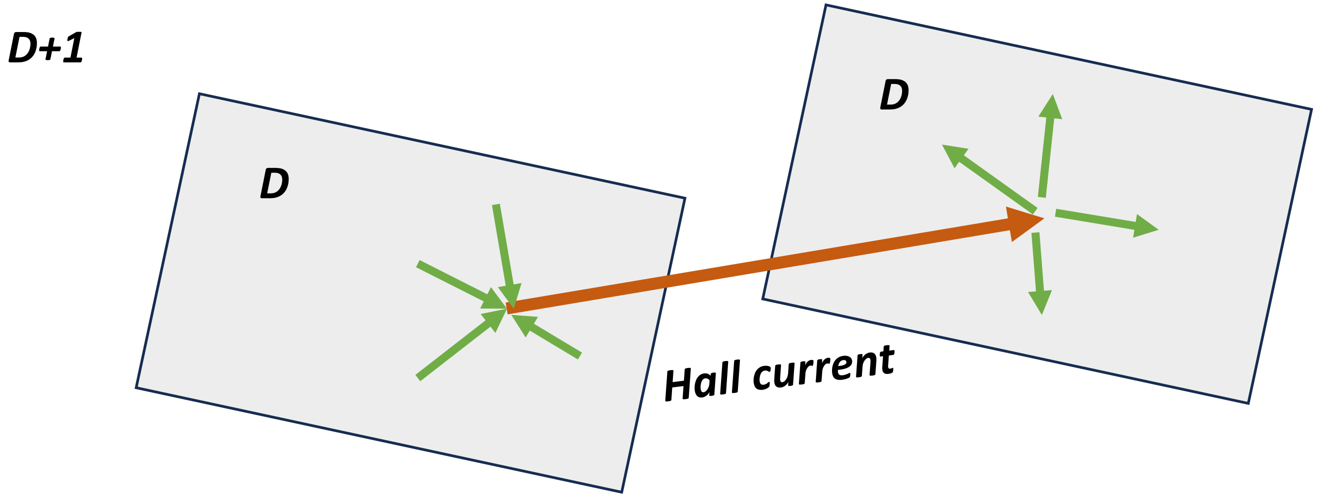
Based on arXiv:2307.04792

Massless modes on defects and Hall current

***Domain wall** fermions in **odd** dimensions:*

massless modes on defects produce an anomaly



Can one say anything about massless modes on defects in even dimensions or for theories without chiral symmetry?

Can one say more about massless modes on defect?

- Yessss
D. Kaplan and S. Sen arXiv:2112.06954

Fermion index:

$$I = \lim_{M\to 0} \mathcal{I}(M)$$

$$\mathcal{I}(M) = \mathrm{Tr}\left(\frac{M^2}{\mathcal{D}^\dagger\mathcal{D}+M^2} - \frac{M^2}{\mathcal{D}\mathcal{D}^\dagger+M^2}\right)$$

$$= \mathrm{Tr}\,\Gamma_\chi \boxed{\frac{M}{K+M}},$$

Looks like a propagator of some new theory:

$$S = \int d^{d+1}x\,\overline{\Psi}(K+M)\Psi$$

$$K = \begin{pmatrix} 0 & -\mathcal{D}^\dagger \\ \mathcal{D} & 0 \end{pmatrix},\ \Gamma_\chi = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ \{K,\Gamma_\chi\} = 0$$

So the index become:

$$\mathcal{I}(M) = -M\int d^{d+1}x\,\langle\overline{\Psi}(x)\Gamma_\chi\Psi(x)\rangle$$

$$= \mathcal{I}(\cancel{\infty})^{\,0} - \frac{1}{2}\int d^{d+1}x\,\boxed{\partial_\mu\langle\overline{\Psi}\Gamma_\mu\Gamma_\chi\Psi\rangle}$$

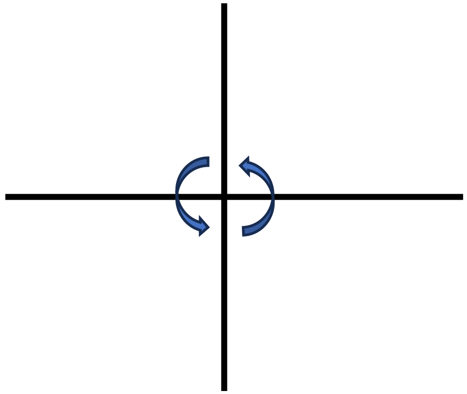
(div of) Generalized Hall Current

Simple example

1+1d fermion and a domain wall with a zero mode:

$$\mathcal{L}_M = \frac{1}{2}\psi^T C\left(i\partial\!\!\!/ - m\right)\psi$$

In Euclidean space:



$$\phi_1 + i\phi_2 = (v+\rho(x))e^{i\theta(x)}$$

Crossed domain wall

$$\mathcal{D} = \left(\partial\!\!\!/ + \phi_1 + \boxed{i\phi_2\gamma_\chi}\right)$$

Diagnostic field to make zeromode normalizable, switch it off in the end

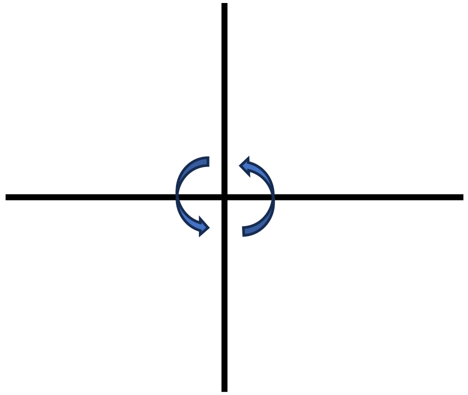
Generalized Hall current and index: $\mathcal{J}_\mu = -\nu_p \frac{\epsilon_{\mu\nu}\partial_\nu\vartheta}{\pi}\,, \qquad \mathcal{I}(0) = \nu_p\nu_\phi$

Manifestly topological

Challenge: how to study effects of interactions?

First step: study realization of Generalized Hall current on the lattice

1+1d fermion and a crossed domain wall:
4 vortices due to periodic BC!



In finite volume zero modes are almost zero…

Wilson-like operator:

$$\mathcal{D}_1 = \sum_\mu \gamma_\mu \nabla_\mu + \frac{R}{2}\nabla_1^2 + i\gamma_\chi \frac{R}{2}\nabla_0^2.$$

Exact zero modes can be analytically found even in finite volume for certain fine tuned parameters

Wilson operator:

$$\mathcal{D}_2 = \sum_\mu \gamma_\mu \nabla_\mu + \frac{R}{2}\sum_\mu \nabla_\mu^2$$

Almost zero modes

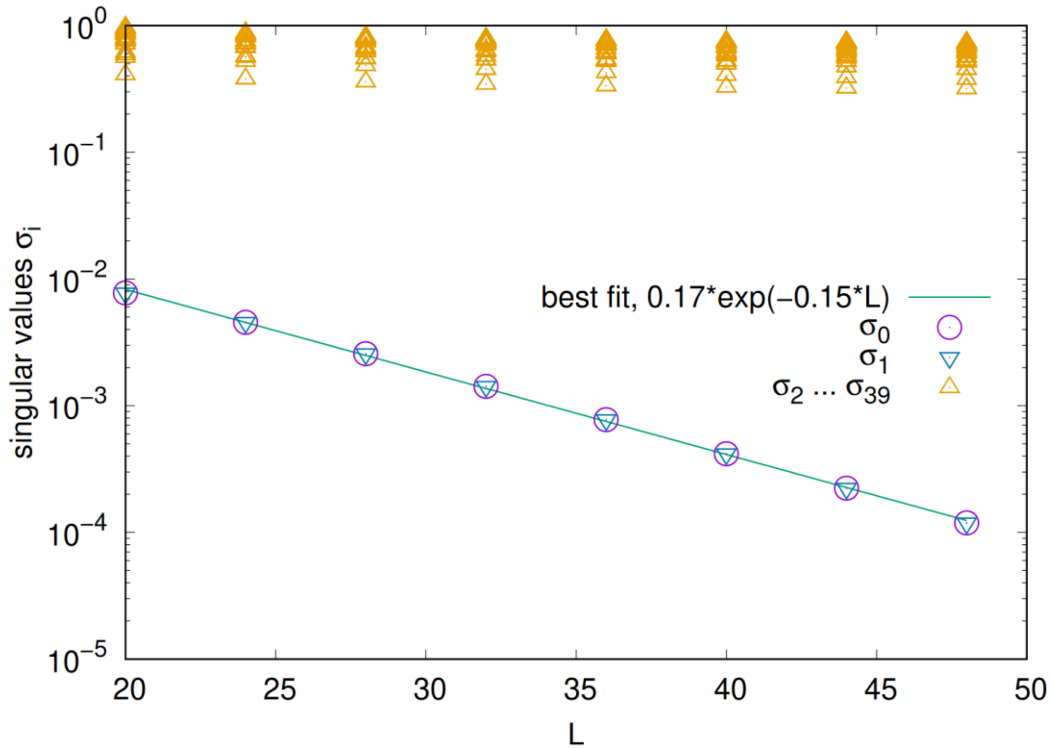
Take infinite volume limit?

Wilson-* operators are not normal. Finite dimensional eigenvectors do not necessarily converge to true zero modes

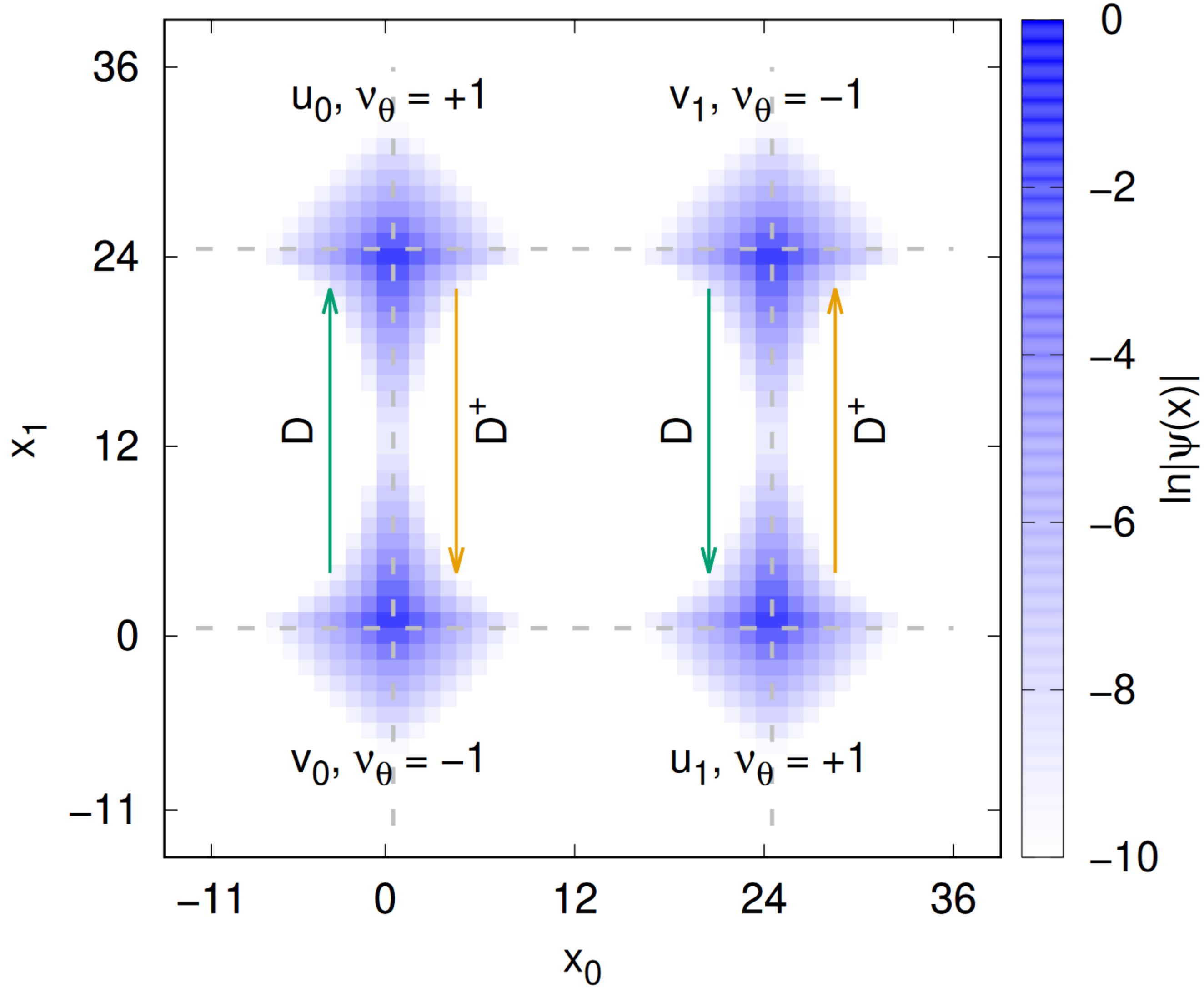
Consider SVD instead:

$$\mathcal{D}\mathcal{D}^\dagger u_i = \sigma_i^2 u_i,$$
$$\mathcal{D}^\dagger \mathcal{D} v_i = \sigma_i^2 v_i$$

Smallest singular value yields minimum of $|\mathcal{D}v'|$ ***and*** $|\mathcal{D}^\dagger u'|$**.**



Almost zero modes



Generalize Hall current on the lattice

We define it analogously to continuum formula:

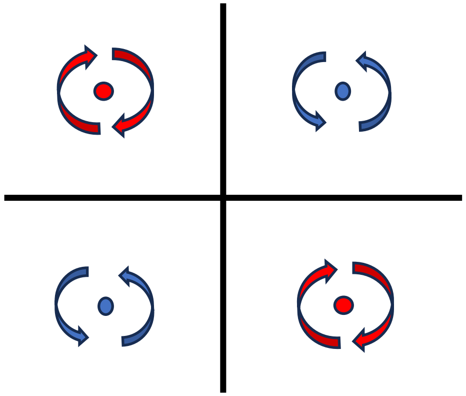
$$J_\mu^H(x) = \bar{\Psi}\tilde{\Gamma}_\mu(x)\Gamma_\chi\Psi \qquad \tilde{\Gamma}_\mu(x) = -i\,\frac{\delta K(A_\mu(x))}{\delta A_\mu(x)}\bigg|_{A_\mu(x)=0}$$

Divergence:

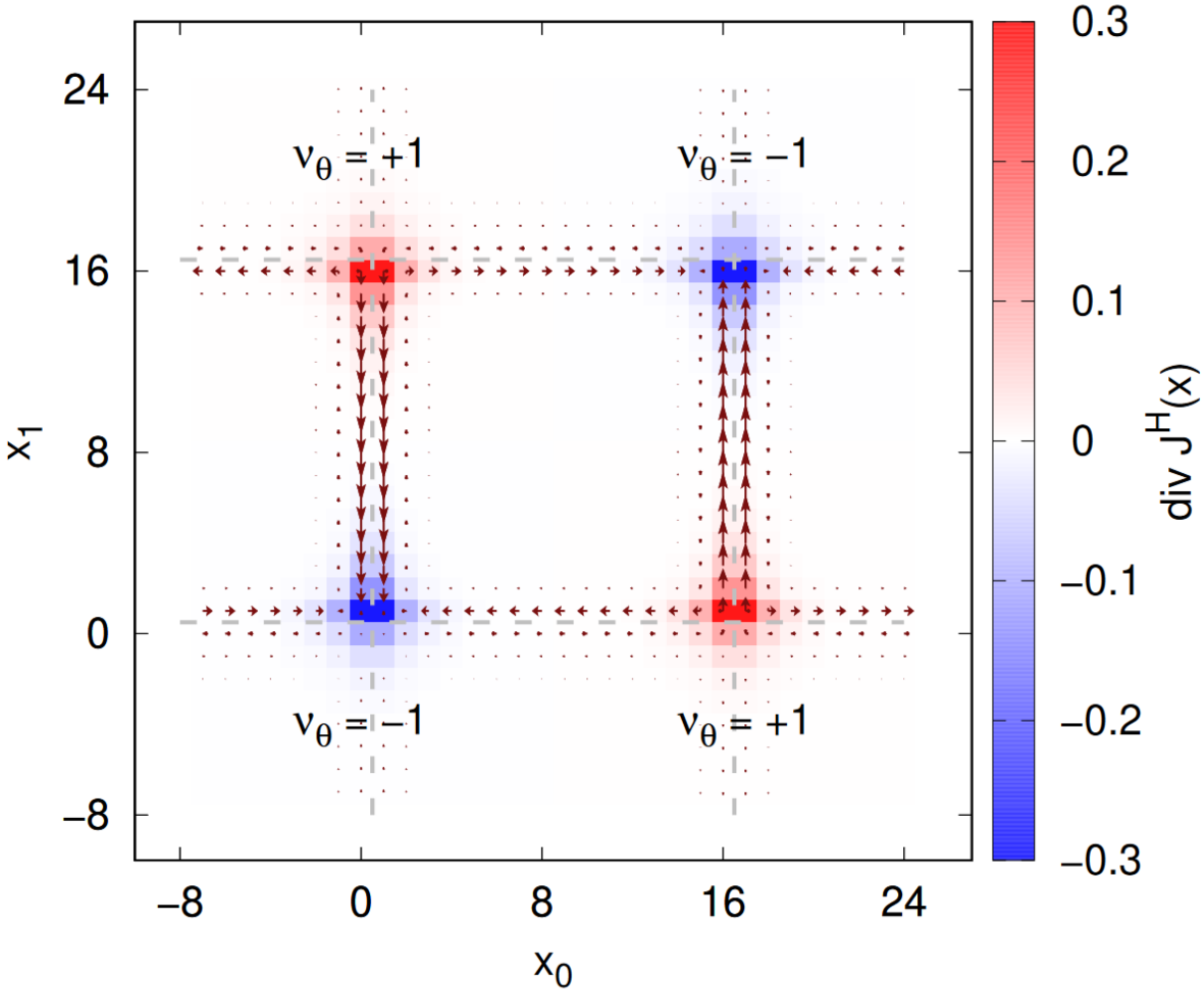
$$\nabla_\mu^B J_\mu^H(x) = \sum_{\mu=0,1}\left(J_\mu^H(x-a_\mu) - J_\mu^H(x)\right)$$

"Index":

$$I_{lat} = -\frac{1}{2}\sum_{x\in S}\nabla_\mu^B J_\mu^H(x)$$

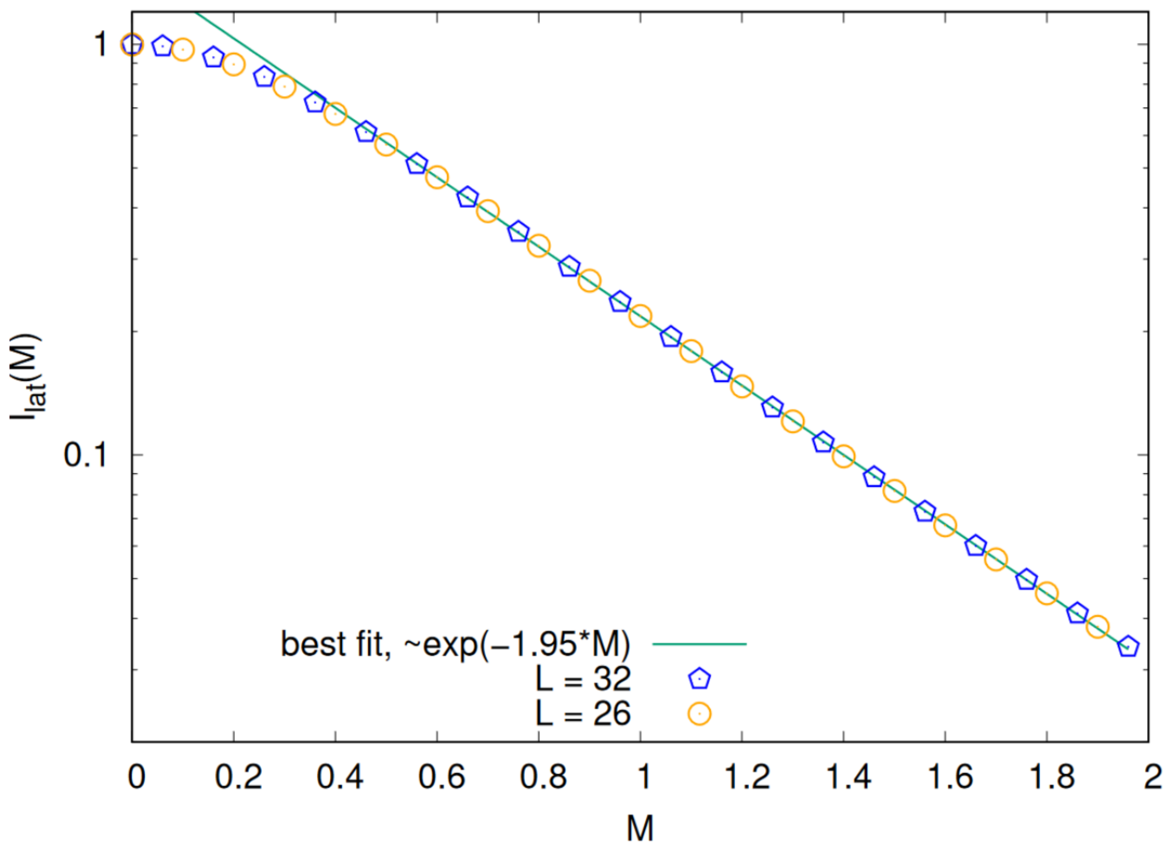


Generalize Hall current on the lattice



Generalize Hall current on the lattice

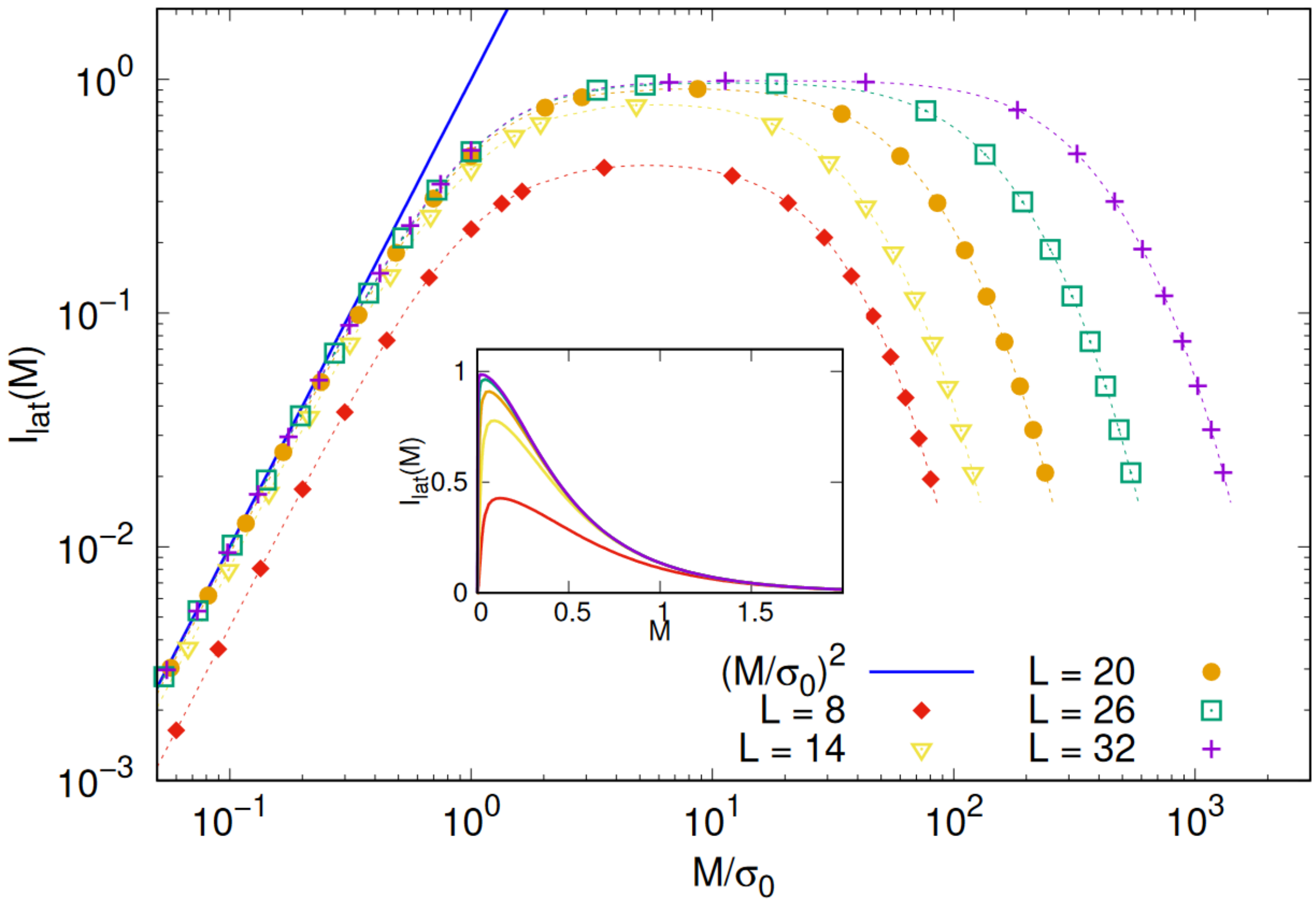
Wilson-like operator + fine tuning (= exact zero mode)



$$\mathcal{I}(M) = \frac{M^2}{M^2 + \mathcal{D}^\dagger \mathcal{D}} - \frac{M^2}{\mathcal{D}\mathcal{D}^\dagger + M^2}$$

Generalize Hall current on the lattice

Wilson operator



Blue line: $$\frac{M^2}{D_2^\dagger D_2 + M^2} = \frac{M^2}{\sigma_0^2} + O\left(\frac{M^4}{\sigma_0^4}\right)$$

Conclusions

1. *We studied realization of Generalized Hall current on the lattice*

2. *We successfully reproduced the infinite volume index*

3. *Intricate finite volume analysis was required due to lattice geometry*

4. *Future work: higher-dimensional theories, interactions and non-perturbative effects*