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FOR 5269



# Optimized Distillation Profiles for Heavy-Light Spectroscopy

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# Distillation Profiles

Distillation operator:

$$S(t) = V(t) \mathbf{J}(\mathbf{t}) V^\dagger(t)$$

$$\underset{\alpha, \beta}{J_{i,j}}(t) = \delta_{ij} \delta_{\alpha\beta} \mathbf{g}(\lambda_i(\mathbf{t}))$$

The perambulator:

$$\tau(t_1, t_2) = V^\dagger(t_1) D^{-1} V(t_2)$$

The elemental:

$$\underset{\alpha, \beta}{\Phi_{i,j}}(t) = V_i^\dagger(t) \Gamma_{\alpha, \beta}(t) \mathbf{g}^*(\lambda_i(\mathbf{t})) \mathbf{g}(\lambda_j(\mathbf{t})) V_j(t)$$

The meson 2pt correlator:

$$-\langle \text{tr} [\Phi_2(t) \tau_{q_a}(t, 0) \bar{\Phi}_1(0) \tau_{q_b}(0, t)] \rangle_{\text{gauge}}$$

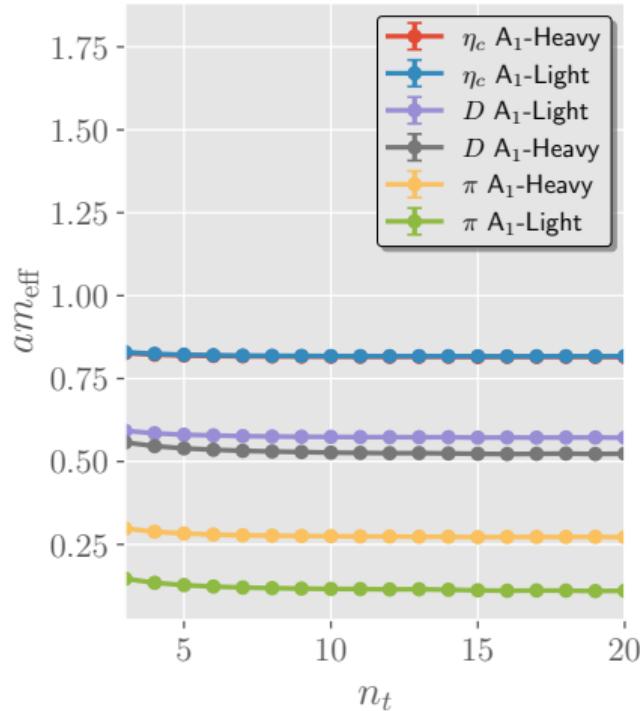
- ▶ Choose basis  $g_n(\lambda) = \exp(-\frac{\lambda^2}{\sigma_n^2})$
- ▶ Optimize  $\Phi$  with GEVP
- ▶ Get profile  $f = \sum_n c_n g_n^* g_n$

[F. Knechtli, T. Korzec, M. Peardon, J. A. Urrea-Niño, Phys. Rev. D106 (2022)]

# Ensembles

Name	$N_f$	$a[\text{fm}]$	$L^3 \times T$	$N_v$	$m_\pi[\text{GeV}]$
A11	3+1	0.054	$32^3 \times 96$	200	$\approx 1/\approx 0.4$
D5	2	0.0653	$24^3 \times 48$	200	0.439

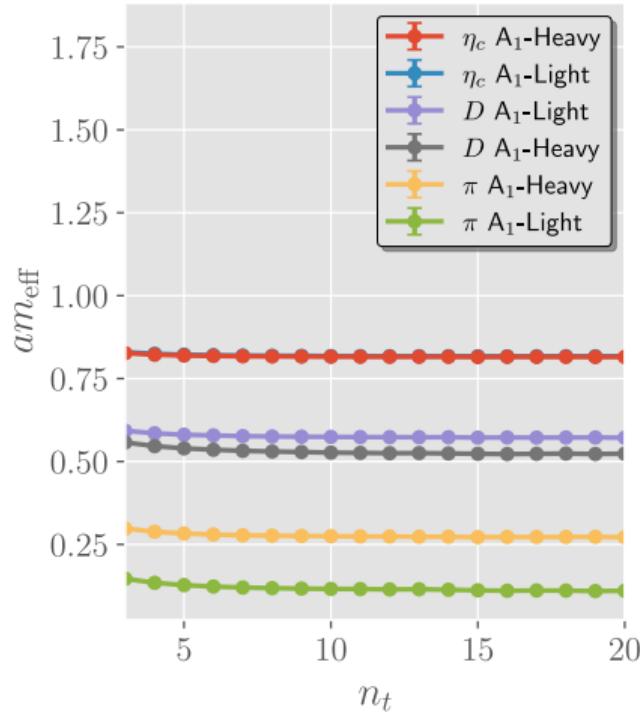
- ▶ A1 at  $SU(3)$  symmetric point
- ▶ We use  $A1_{\text{light}}$
- ▶ Varying  $N_{\text{cfg}}$  used
- ▶ D5: periodic boundary conditions,  $O(a)$  improved Wilson fermions [PoS(Lattice 2011)232]



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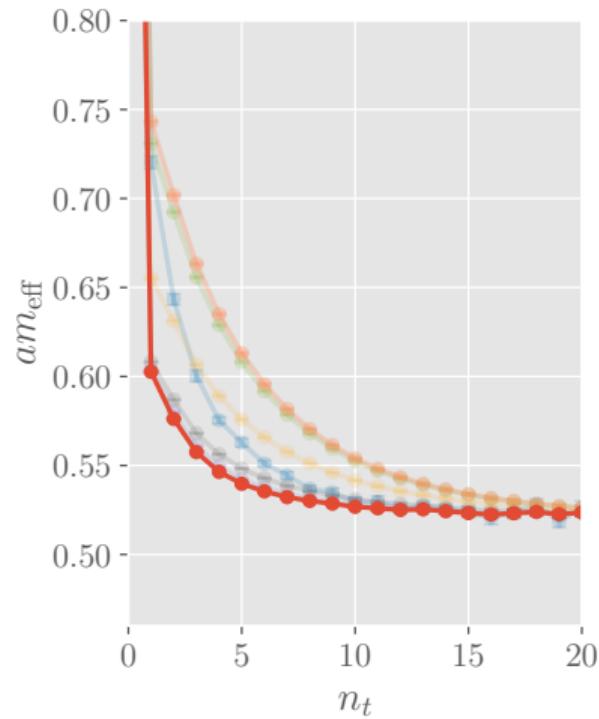
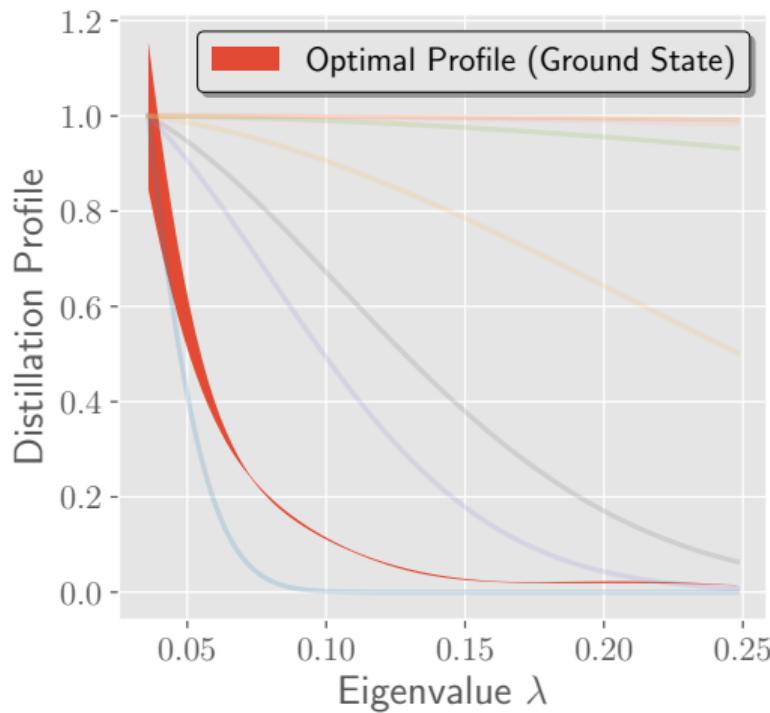
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# Motivation

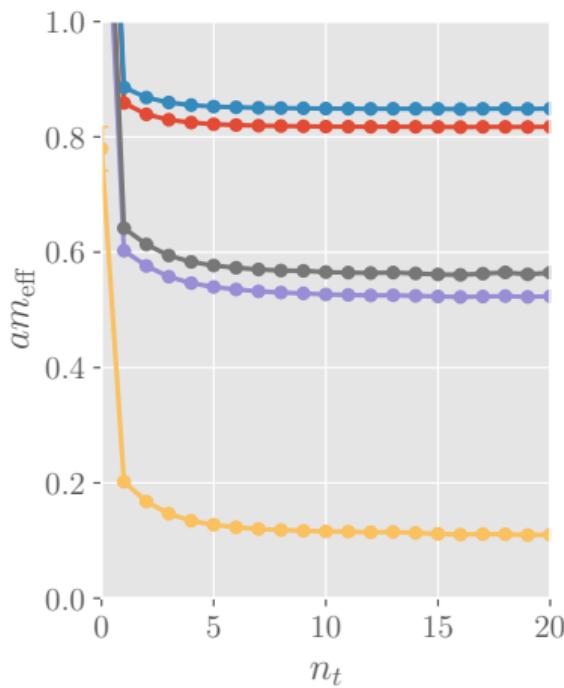
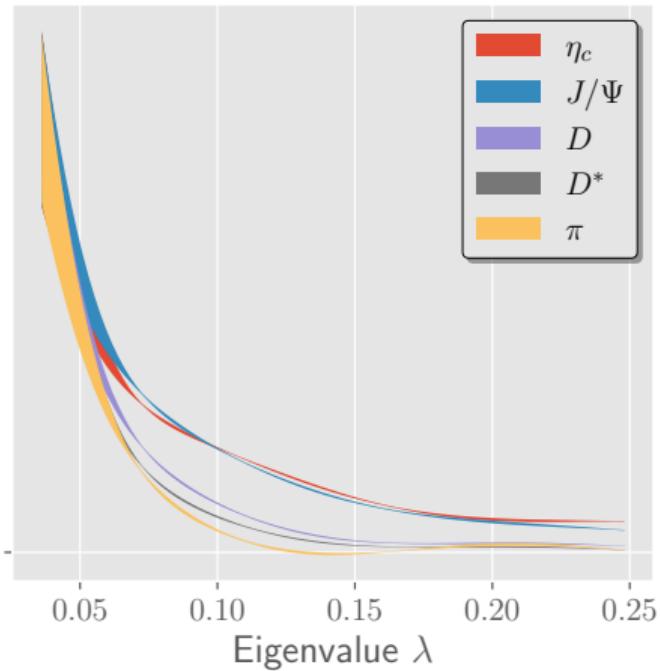
- ▶ Preparation for a future  $D\bar{D}$  scattering analysis
  - ▶ Optimized profiles can be used for different traces/diagrams
- ▶ Distillation profiles improve charmonium excited state spectroscopy
  - ▶ Interest in D5 from [PoS(LATTICE2022)266]

# Demonstration of Heavy-Light Profiles



# Comparing Profiles of Different Particles

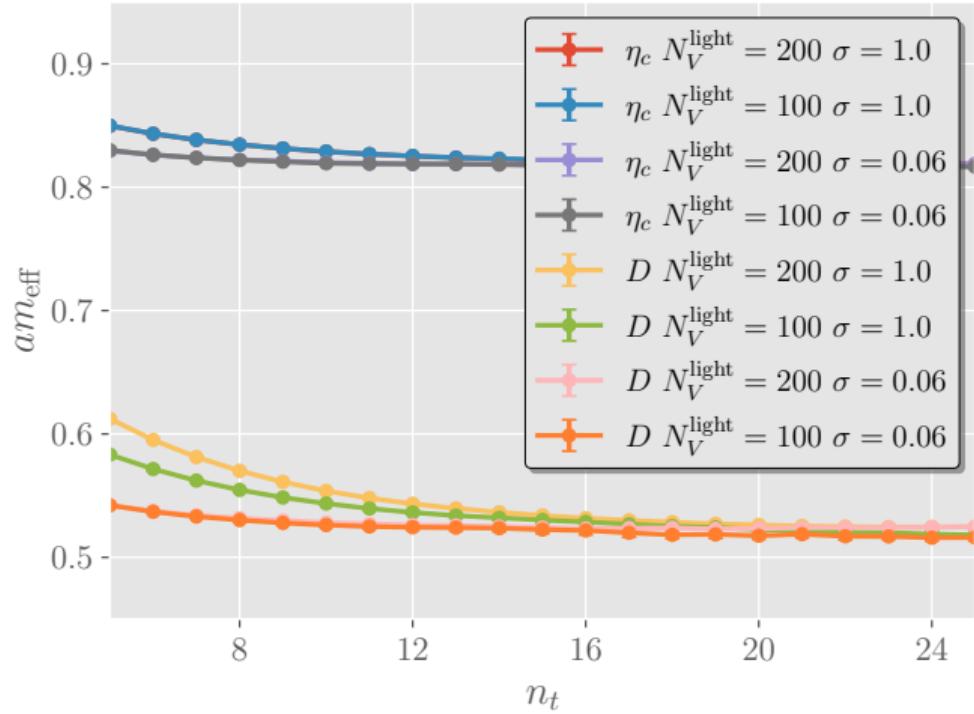
Distillation Profile



- ▶ Charmonium wider than  $D$
- ▶  $D$  wider than  $\pi$
- ▶ **Wider** profile  $\iff$  more **localized** source

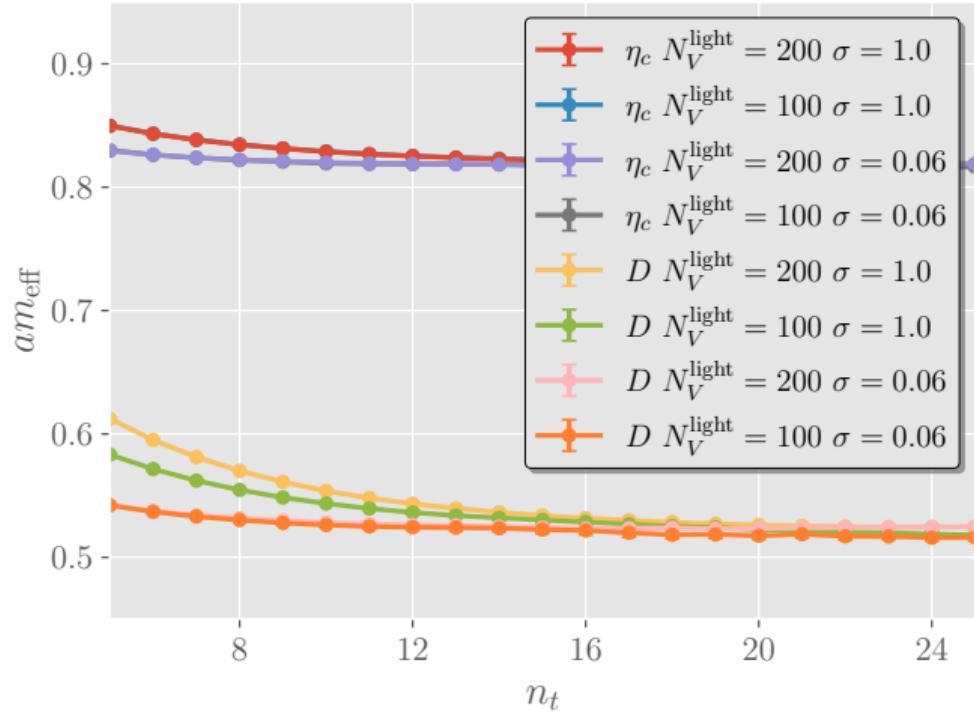
# Restricting $N_V^{\text{Light}}$

- ▶  $4 \times N_v \times T$  inversions
- ▶  $N_v^{\text{Light}}$  and  $N_v^{\text{Charm}}$  can differ  
⇒ rectangular elementals
- ▶ Wider Profiles are more affected
- ▶ Optimal  $D$  profiles are narrow



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# Profiles and Momenta

Elementals with lattice momentum:

$$\Phi_{i,j}(\vec{p}) = \sum_{\vec{x}} V_i^\dagger(\vec{x}) e^{-i\vec{p}\cdot\vec{x}} g^*(\lambda_i) g(\lambda_j) \Gamma V_j(\vec{x})$$

The momentum part can be **precalculated**

$$\Phi_{i,j}(\vec{p}) = \Phi_{i,j}(\vec{0}) \sum_{\vec{x}} V_i^\dagger(\vec{x}) e^{-i\vec{p}\cdot\vec{x}} V_j(\vec{x})$$

This is the **only additional cost**

Alternative:

**Partially twisted periodic boundary conditions**

► Inversion with

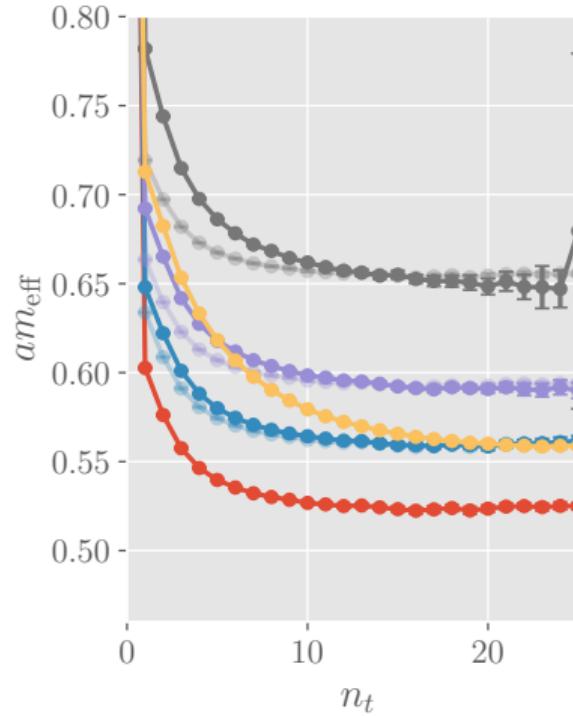
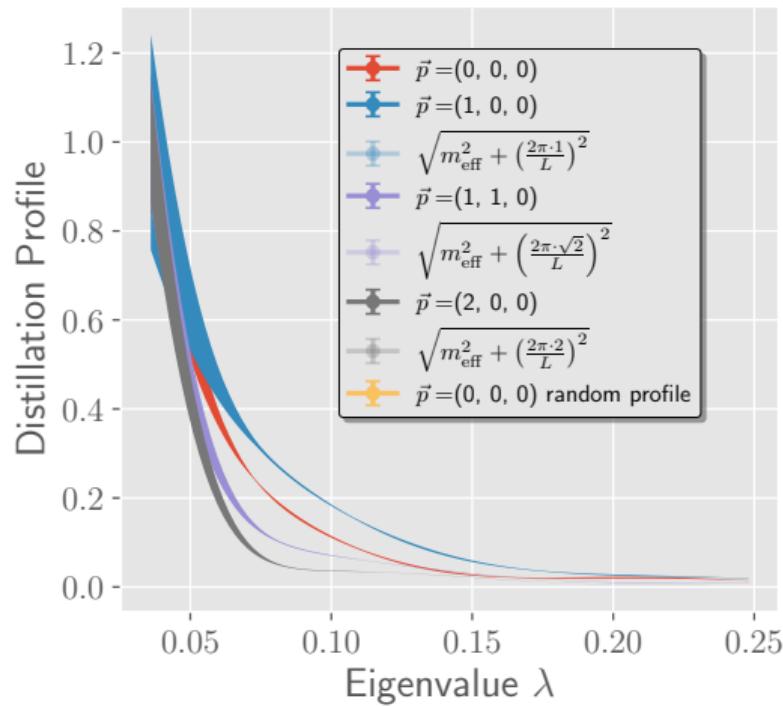
$$\begin{aligned} \psi(x + L) &= e^{i\theta} \psi(x) \\ \implies p &= \frac{2\pi n + \theta}{L} \end{aligned}$$

► Allows **continuos momenta**

► Requires **new inversions**

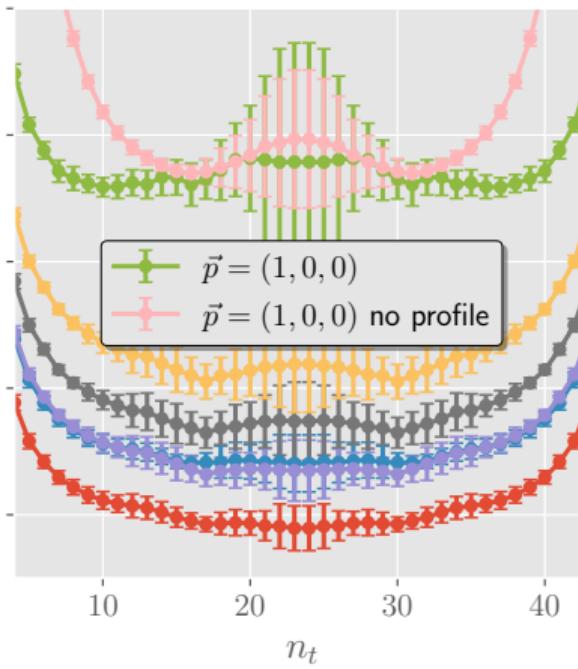
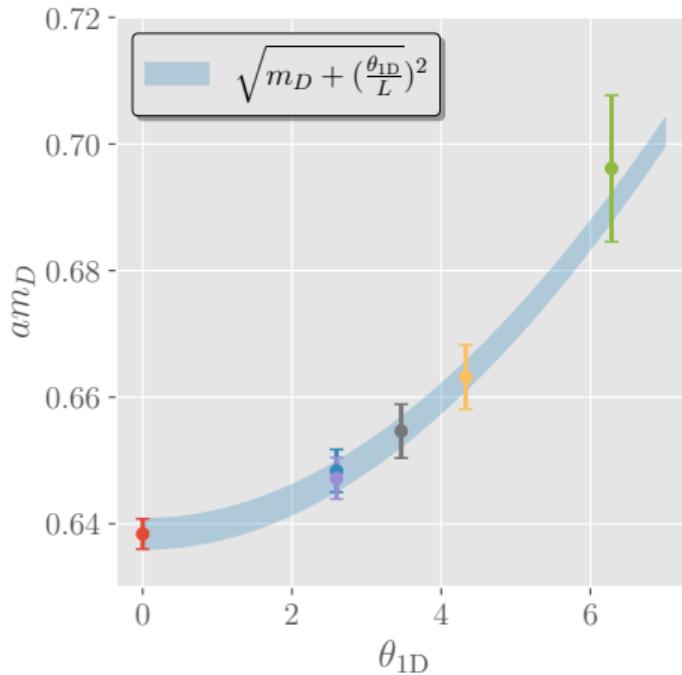
# Profiles and Momenta

A1 ( $N_f = 3 + 1$ )



# Profiles and Momenta

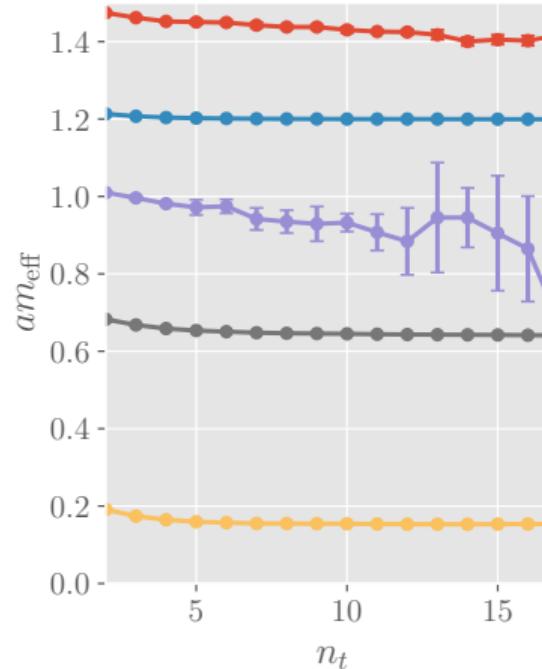
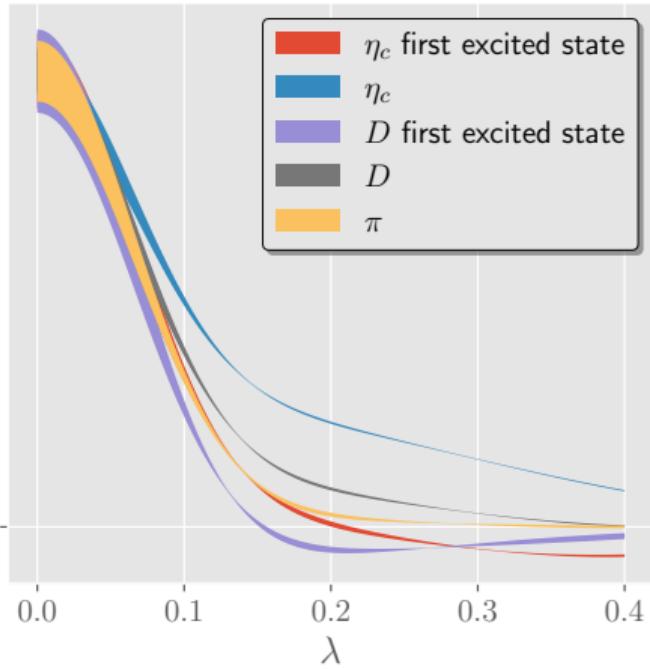
D5 ( $N_f = 2$ )



- ▶ Both methods fulfill **dispersion relation**
- ▶ Profiles are effective at higher momenta

# Profiles on D5

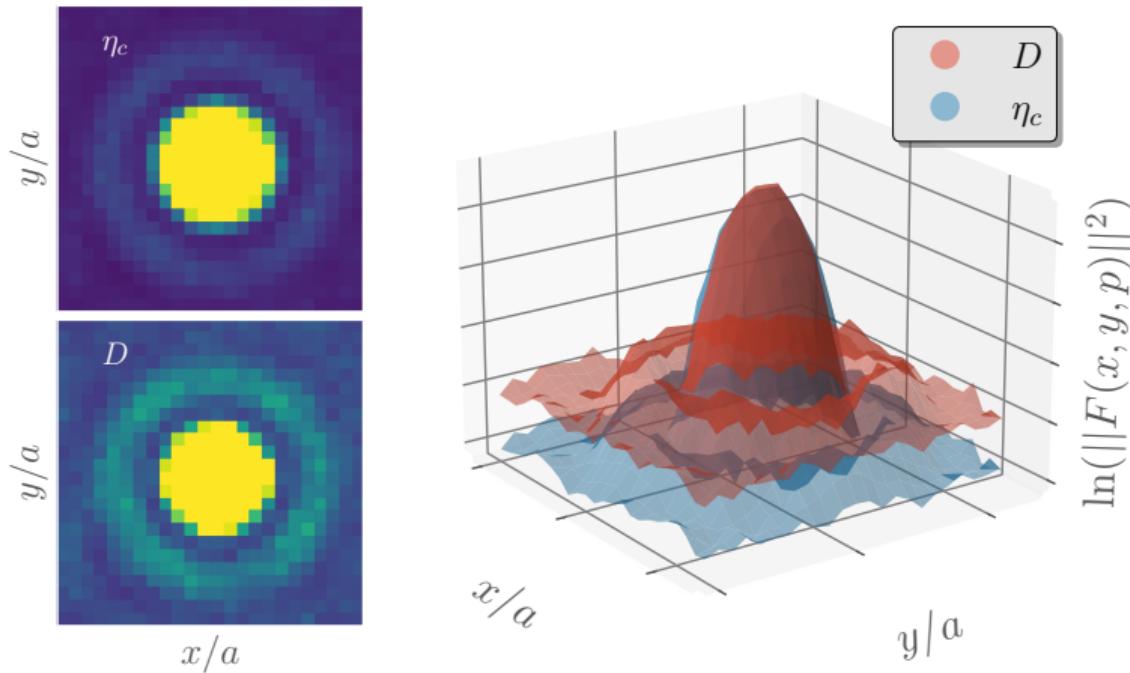
Distillation Profile



- ▶ Profiles resemble those on A1
- ▶ Different  $x$ -axis scaling
- ▶ Different form for excited states

# Profiles on D5

- Reconstructed distillation operator **applied to point source**
- **Spatial slice** and average over  $n_t$  and configurations
- $\text{tr} [\gamma_5 \Gamma]$  and color average for **scalar value**
- $D$  shows non-localized background



# Summary

We have seen that distillation profiles work for:

- ▶ **heavy-light** systems
- ▶ different types of **momenta**

The optimized profiles are:

- ▶ **narrower for  $D$**  compared to charmonium
- ▶ consistent across ensembles

**Thank you  
for your attention!**