A status update of Fermilab/HPQCD/MILC Collaborations muon g-2 project

Lattice 2023, Fermilab.

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On behalf of the Fermilab Lattice, HPQCD and MILC Collaborations.

Outline

- Light quark connected.
  - Low-mode improved data set.
  - Preliminary results for long-distance observables.
- Sub-leading contributions to intermediate window.
  - Strange and Charm
  - Disconnected
- Conclusions/Outlook

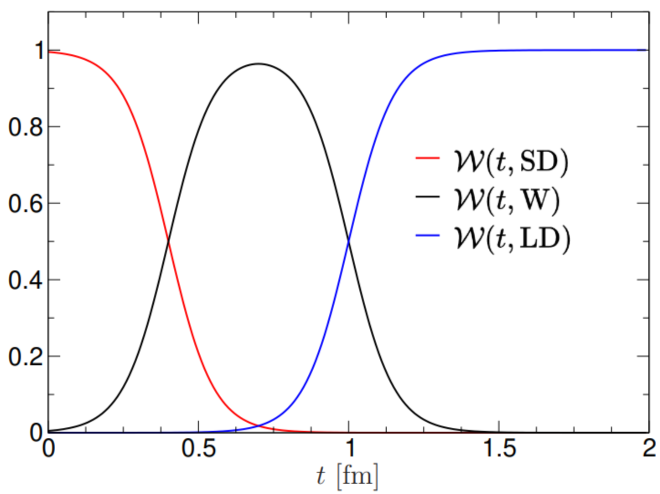
HVP contribution from the Lattice

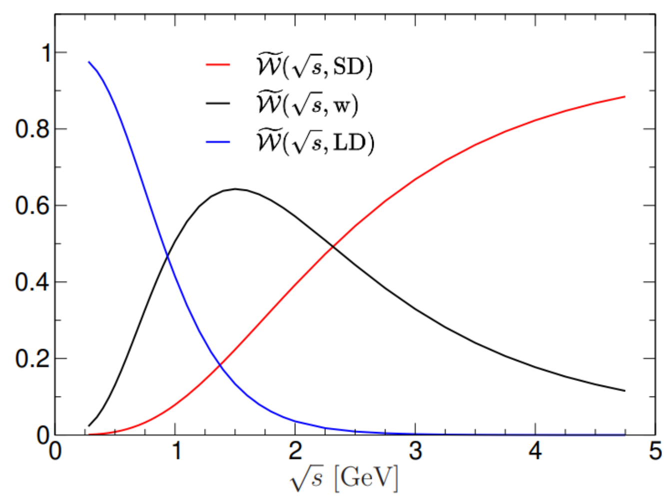
B,M: 1107.4388

$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt \tilde{K}(t) C(t), \quad C(t) = \frac{1}{3}\sum_i^3 \int d^3x \langle J_i(x) J_i(0) \rangle$$

$$J_i(x) = Q_u \bar{u}(x)\gamma_i u(x) + Q_d \bar{d}(x)\gamma_i d(x) + Q_s \bar{s}(x)\gamma_i s(x) + \dots$$

window function: $$\mathcal{W}(t, t_0, t_1, \Delta) = \frac{1}{2}\left[\tanh\left(\frac{t-t_0}{\Delta}\right) - \tanh\left(\frac{t-t_1}{\Delta}\right)\right] + (t \to -t)\,.$$





Window	$[t_0, t_1]$ fm
SD	$[0, 0.4]$
W	$[0.4, 1]$
LD	$[1, \infty]$

$\Delta = 0.15$ fm

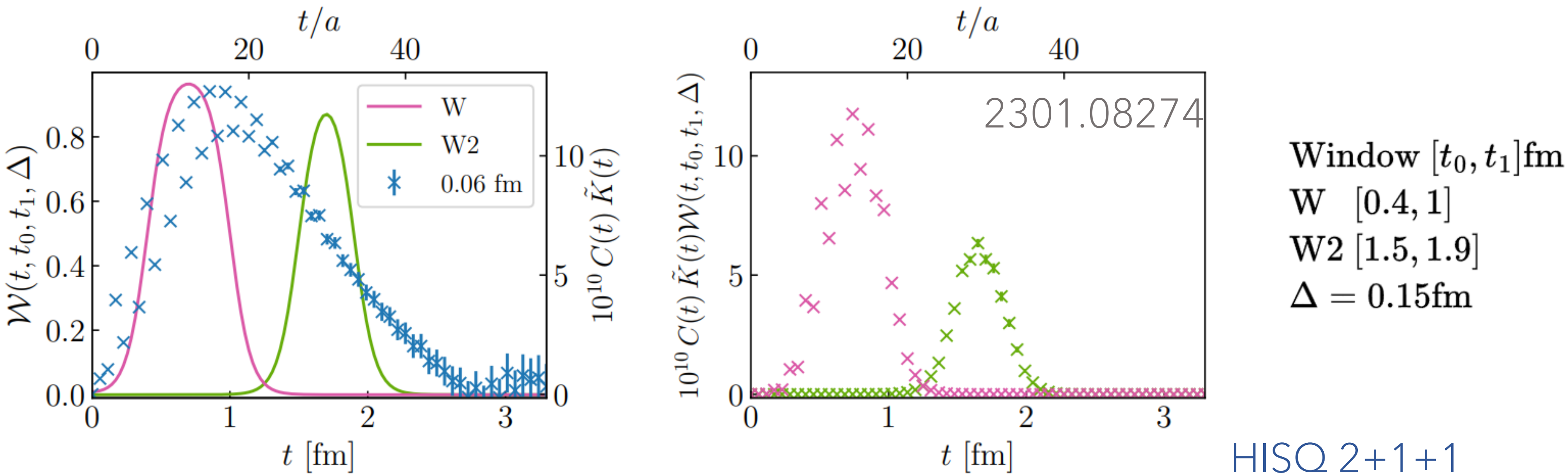
plots taken from 2205.12963

SD+W+LD=Full

Light-quark connected

Intermediate windows

$$a_\mu^{\mathrm{HVP,win}(t_0,t_1,\Delta)} = 4\alpha^2 \int_0^\infty \mathrm{d}t C(t)\tilde{K}(t)\mathcal{W}(t,t_0,t_1,\Delta), \quad C(t) = \frac{1}{3}\sum_i^3 \int d^3x \langle J_i(x)J_i(0)\rangle$$

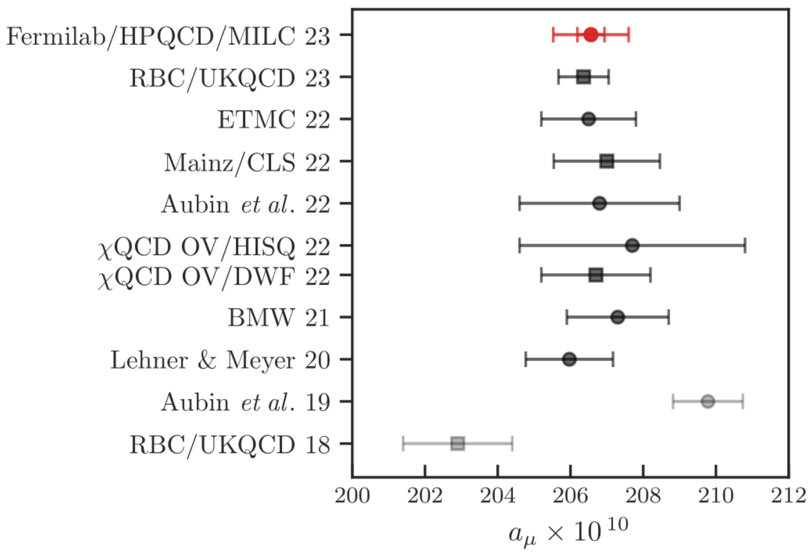


HISQ 2+1+1

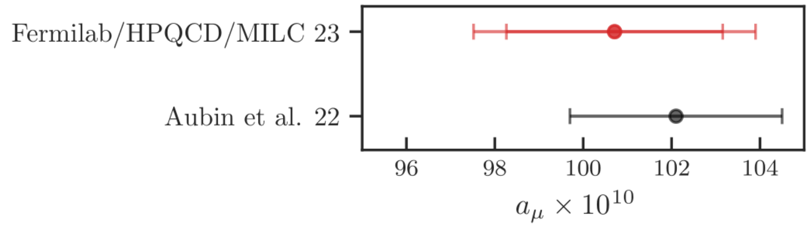
$$a_{\mu,\,\mathrm{Trap.}}^{\mathrm{win}(t_0,t_1,\Delta)} = 4\alpha^2 a \sum_{t=1}^{N_t/2-1} C(t)\tilde{K}(t)\mathcal{W}(t,t_0,t_1,\Delta)$$

- Staggered temporal oscillations found to have a negligible effect.
- Integration scheme errors are similarly negligible.

W [0.4, 1] fm



W2 [1.5, 1.9] fm



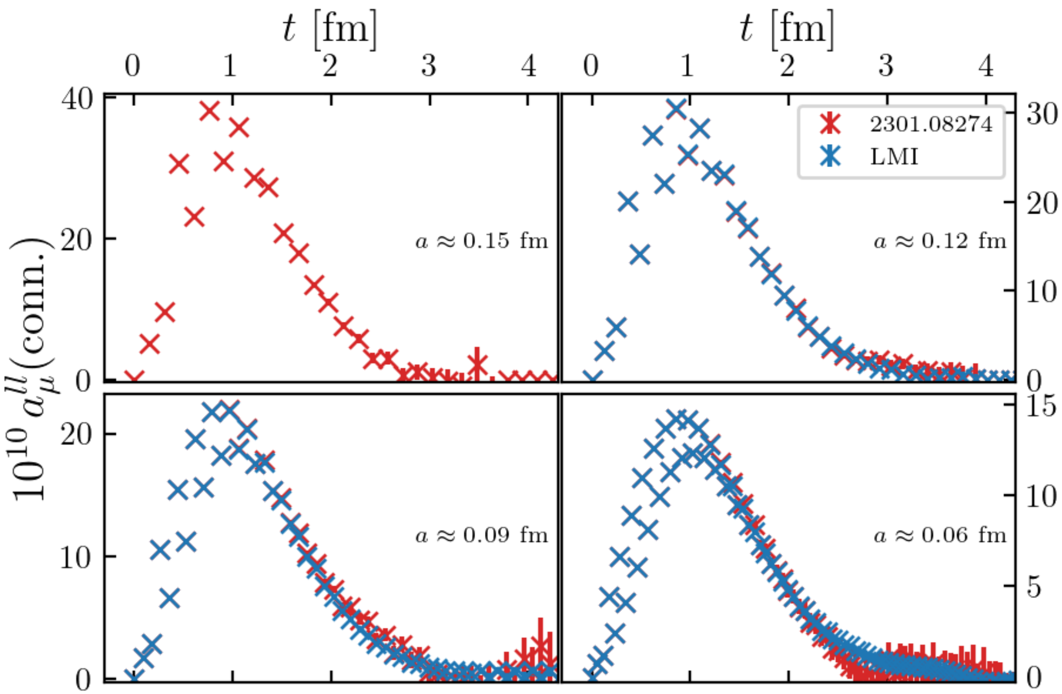
Dominant uncertainties:

- W: continuum extrap. (including TB)
- W2: statistical.

Source	$\delta a_\mu^{ll,\mathrm{W}}$(conn.) (%)	$\delta a_\mu^{ll,\mathrm{W2}}$(conn.) (%)
Monte Carlo statistics	0.19	2.44
Continuum extrapolation ($a \to 0$, Δ_{TB})	0.34	1.05
Finite-volume correction (Δ_{FV})	0.16	0.23
Pion-mass adjustment (Δ_{M_π})	0.06	0.96
Scale setting (w_0 (fm), w_0/a)	0.21	1.28
Current renormalization (Z_V)	0.17	0.16
Total	0.50%	3.18%

Correlator data has significant StN issues at larger times.

Low-mode improved light-quark data



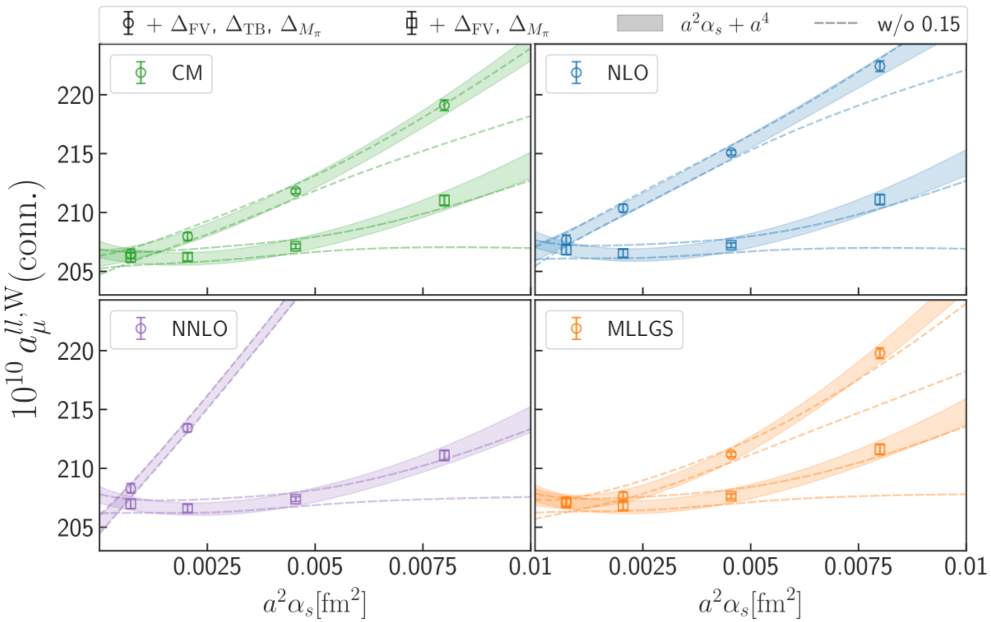
- LMI datasets at three finest ensembles.
- Re-tuned valence mass at 0.09 fm.
- Data generation ongoing at 0.06, 0.04, and retuned 0.09 fm ensemble.

$\approx a/\mathrm{fm}$	L/fm	$N_s^3 \times N_t$	$am_l^{\mathrm{sea}}/am_s^{\mathrm{sea}}/am_c^{\mathrm{sea}}$	M_{π_5}/MeV	N_{conf}	N_{eig}	N_{src}
0.15	4.85	$32^3 \times 48$	0.002426/0.0673/0.8447	134.73(71)	9362	0	48
0.12	5.83	$48^3 \times 64$	0.001907/0.05252/0.6382	134.86(71)	9637	0	64
0.12_{LMI}					1060	2000	
0.09	5.62	$64^3 \times 96$	0.00120/0.0363/0.432	128.34(68)	5384	0	48
0.09_{LMI}				135.07(71)	1000	2000	
0.06	5.46	$96^3 \times 128$	0.0008/0.022/0.260	134.95(72)	2621	0	24
0.06_{LMI}					508	2000	96

LMI code-base and data generation: Michael Lynch @ Software Development and Machines at 5pm

Connected light-quark analysis strategy

- (Software) blinded analysis.
- Incorporate different EFT-based correction schemes.
- Continuum extrapolation w/ and w/o taste-breaking corrections and powers of α_s.
- Systematics error estimates choices through Bayesian Model Averaging (BMA).



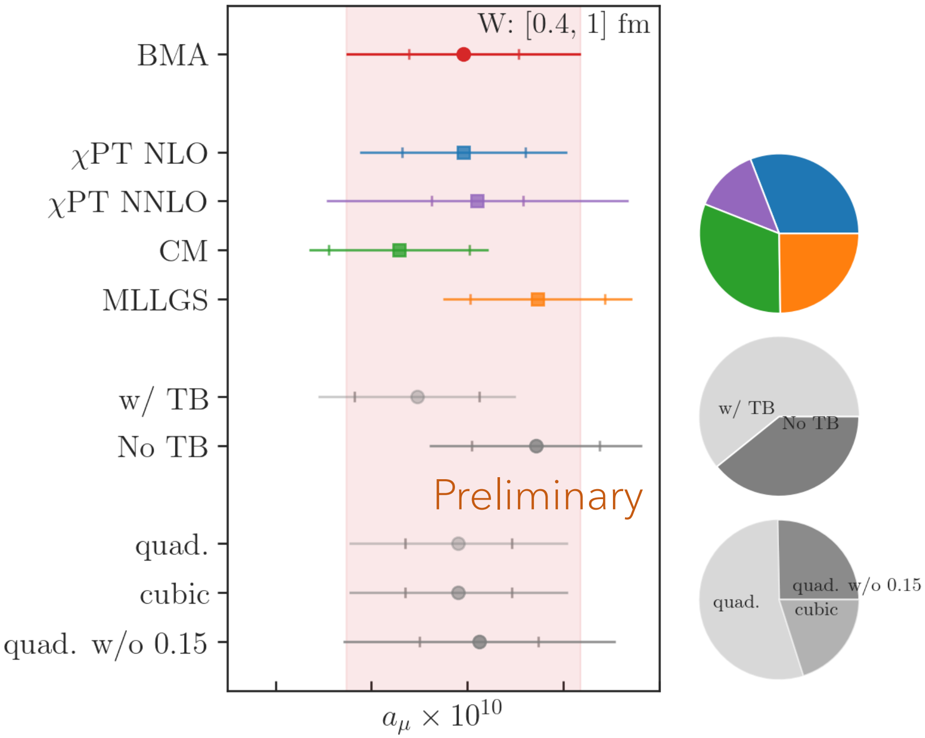
Each set of analysis choices constitutes a 'model'.

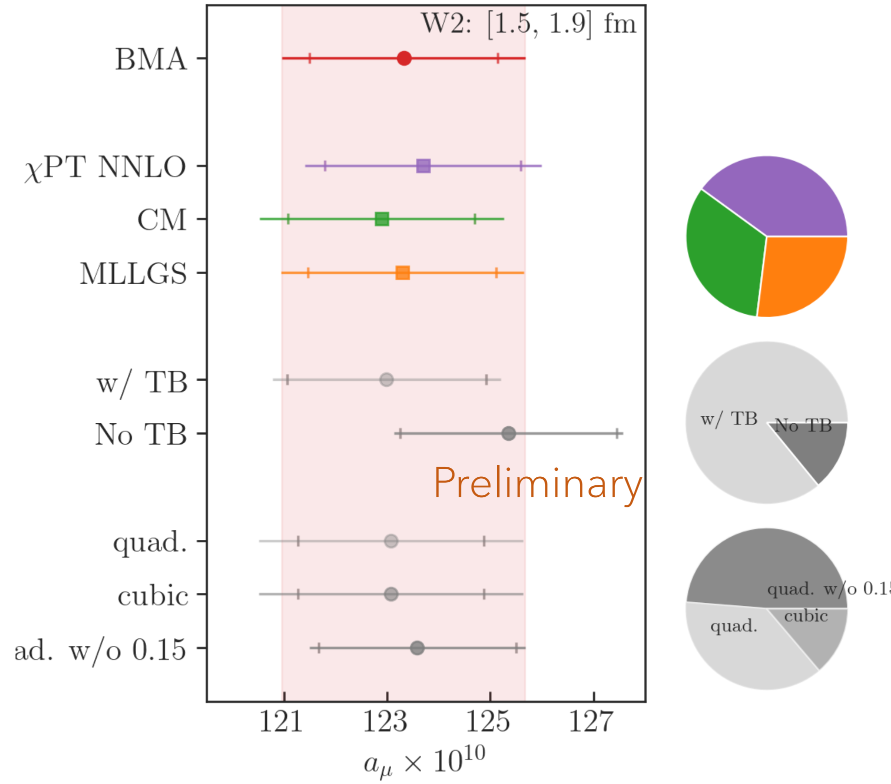
Bayesian Akaike information criterion

Model Probability: $$\mathrm{pr}(M \mid D) \equiv \mathrm{pr}(M) \exp\left[-\frac{1}{2}\left(\chi^2_{\rm data}(\mathbf{a}^\star) + 2k + 2N_{\rm cut}\right)\right]$$

Talk by Ethan Neil @ Algorithms and AI, Friday

Updated light-quark windows





Source	$\delta a_\mu^{ll,\mathrm{W}}$(conn.) (%)	$\delta a_\mu^{ll,\mathrm{W2}}$(conn.) (%)
Monte Carlo statistics	0.19 → 0.09	2.44 → 0.73
Continuum extrapolation ($a \to 0$, Δ_{TB})	0.34 → 0.27	0.34 → 0.9
Total	0.50 → 0.44	3.18 → 1.94

W: ~2x improvement in stat. uncertainty.

W2: ~3x improvement in stat. uncertainty.

$$10^{10} a_\mu^{\mathrm{HVP,LO}} = 699(15)_{u,d}(1)_{s,c,b}$$

Light connected:

Source	a_μ^{ll}(conn.) (%)
Lattice-spacing (a^{-1}) uncertainty	0.8
Monte Carlo statistics	0.7
Continuum ($a \to 0$) extrapolation	0.7
Finite-volume and discretization corrections	0.6
Current renormalization (Z_V)	0.1
Chiral (m_l) interpolation	0.1
Sea (m_s) adjustment	0.1
Total	1.4%

Light-quark noise reduction strategy

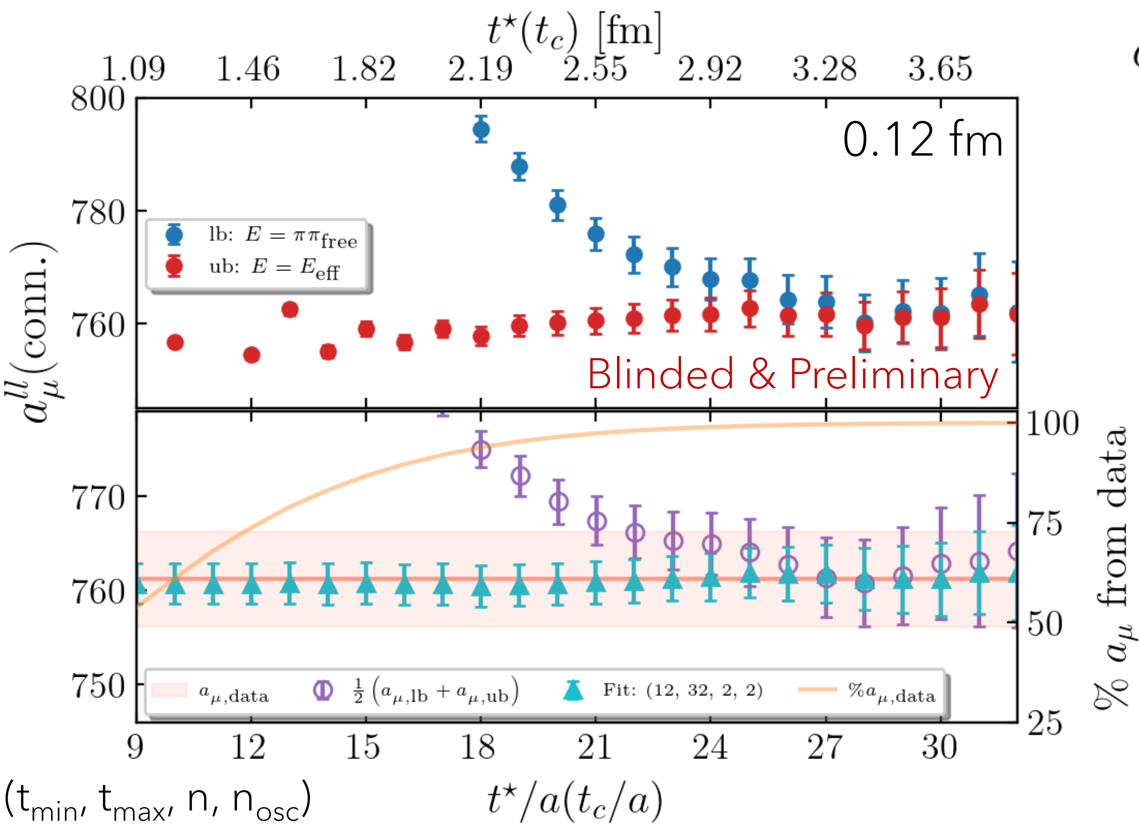
In absence of direct two-pion data, requires noise-reduction strategy:

- Bounding method: Bound energy-dependence of correlator after t_c with upper ($E_{eff.}$) and lower ($m_{\pi\pi}$) energies. a_μ determined when bounds meet.

$$E_{\text{eff.}} = \frac{1}{2}\text{arccosh}\left[\frac{C(t+2)+C(t-2)}{2C(t)}\right]$$

- Fit method: Fit correlator and replace after t* with fit reconstruction.

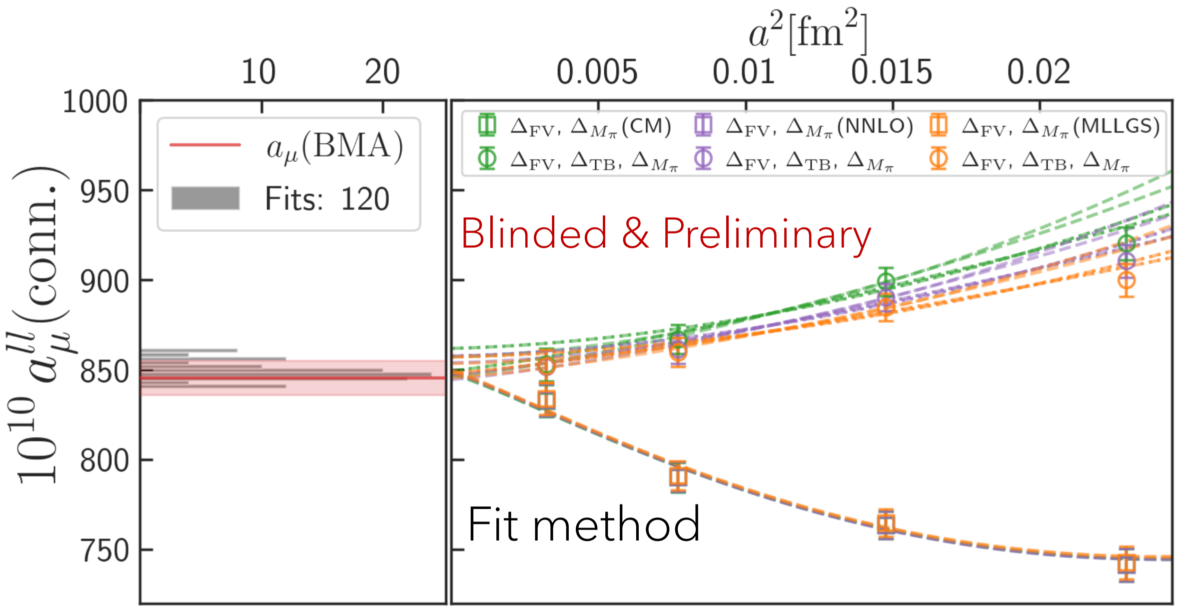
$$C_{\text{fit}}(t) = \sum_n^{N_{\text{states}}} \left[Z_n^2 e^{-E_n t} + (-1)^t Z_{n,\text{osc}}^2 e^{-E_{n,\text{osc}} t}\right]$$



(t_{min}, t_{max}, n, n_{osc})

Todo: Quantify noise reduction method systematics with two-pion data.

Full light-quark a_μ



- Statistical uncertainty reduced ~50% from 1902.04223v2
- Dominant uncertainty from w_0 in fermi.
  Scale-setting project: Alexei Bazavov, 2:30 @ SM parameters
- Ongoing: Further refine stat & cont. extrap. with more stats at 0.06 and new 0.042 fm data set.

Preliminary error budget

Source	δa_μ(Fit) (%)	δa_μ(Bound) (%)
Monte Carlo statistics	0.39	0.51
Continuum extrapolation ($a \to 0$, Δ_{TB})	0.48	0.51
Model correction (Δ_{FV}, Δ_{M_π})	0.17	0.15
Scale setting (w_0 (fm), w_0/a)	0.91	0.9
Current renormalization (Z_V)	0.09	0.09
Total w/o scale uncertainty	0.66%	0.75%
Total	1.12%	1.17%

Sub-leading contributions

Sub-leading contributions to W

Near-term goal: Complete calculation of W to compare with R-ratio.

$$
\begin{aligned}
a_\mu^W &= a_\mu^{ll,\,W}(\text{ conn. }) + a_\mu^{ss,\,W}(\text{ conn. }) + a_\mu^{cc,\,W}(\text{ conn. }) + \ldots \\
&+ a_\mu^{lsc\ldots,\,W}(\text{ disc. }) \\
&+ \Delta a_\mu^{ud,\,W}(\text{SIB}) + \Delta a_\mu^{ud,\,W}(\text{QED})
\end{aligned}
$$

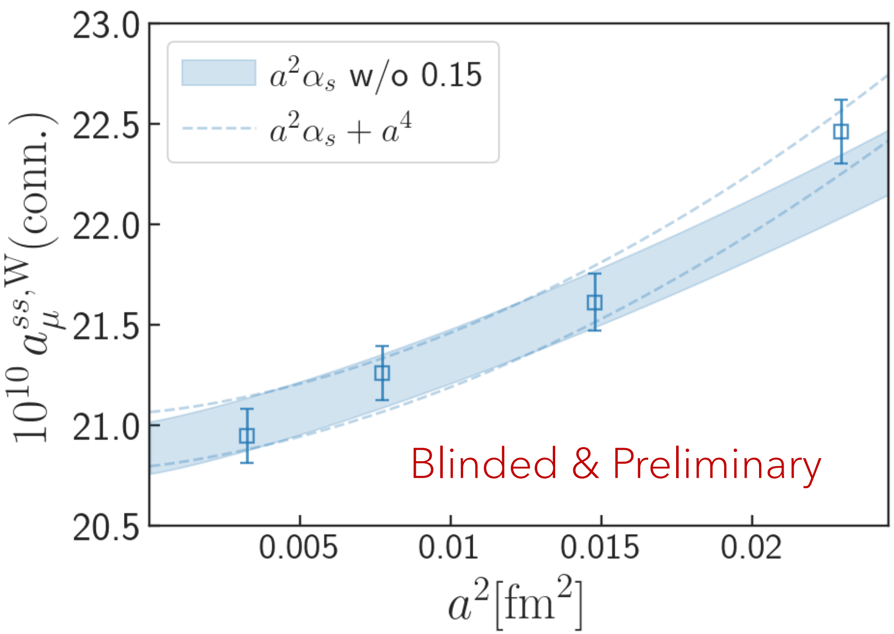
Isospin Breaking

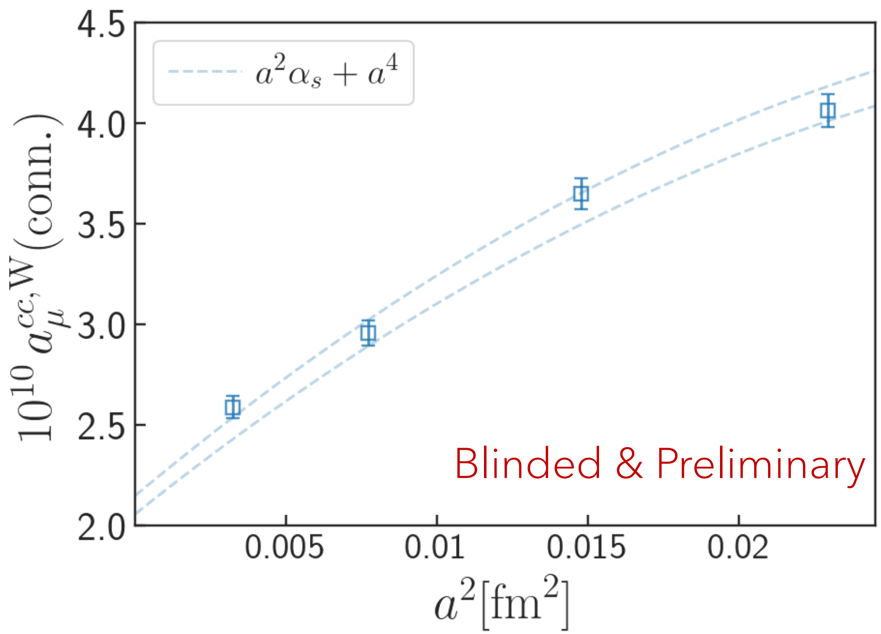
Direct lattice calculations of all contributions

W: Strange and Charm connected

HISQ physical mass ensembles w/ random-wall sources (tsm).

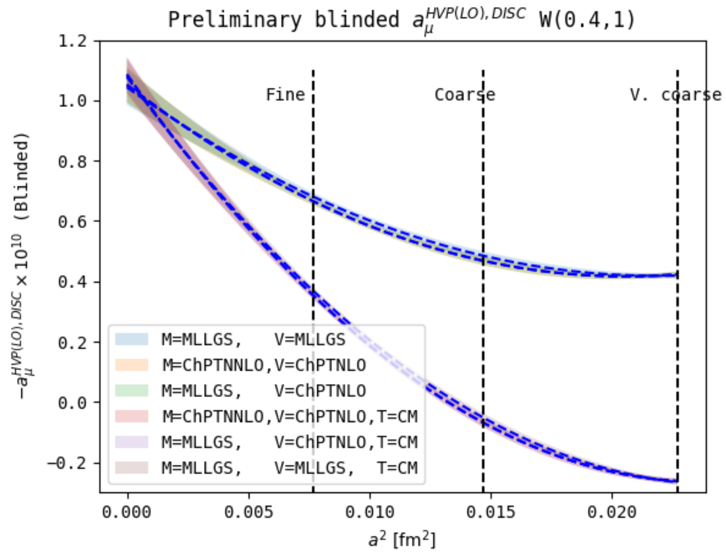
$\approx a/\mathrm{fm}$	$N_{\mathrm{conf,\ strange}}$	$N_{\mathrm{src,\ strange}}$	$N_{\mathrm{conf,\ charm}}$	$N_{\mathrm{src,\ charm}}$
0.15	10019	48	10019	48
0.12	2985	64	2985	64
0.09	241	16	565	8
0.06	1424	24	1424	24

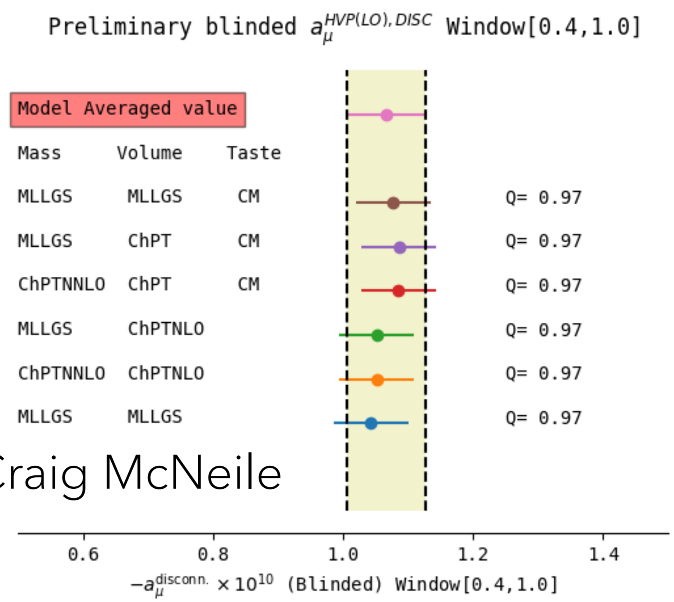




- Significant discretization effects in charm.
- Finalizing continuum extrap. but already at precision goals.

W: Disconnected Contribution





plots: Craig McNeile

$\approx a/\text{fm}$	$N_s^3 \times N_t$	N_{conf}
0.15	$32^3 \times 48$	1047
0.12	$48^3 \times 64$	562
0.09	$64^3 \times 96$	750

$$a_\mu\left(L_\infty, M_{\pi_{\text{phya}}}\right) = a_\mu\left(L_{\text{latt}}, M_{\pi_{\text{lat}},\xi_1}, \cdots, M_{\pi_{\text{lat}},\xi_{16}}\right) + \Delta_{\text{FV}} + \Delta_{M_\pi} + \Delta_{\text{TB}}$$

$$\Delta a_\mu(\text{disc.}) = -\frac{1}{10}\Delta a_\mu(\text{conn.})$$

- BMA approach used.
- Investigating NLO ChPT result of -1/10 using lattice data to construct ratio of connected to disconnected.

Conclusion / Outlook

Light-quark connected

- 2x reduction in statistical uncertainty on full a_μ with new data set, scale setting uncertainty now ~2x larger than all other sources. ~1.8 x δw_0
- Adding more statistics at 0.06 fm.
- Data generation @ 0.042 fm underway.
- Including second discretization (taste-singlet @ 4 lattice spacings) .
- FV study on HISQ @ 0.09 fm with large volume lattice, L_{large}=11.2, to address model correction systematic.
- Complete NNLO ChPT taste-vector result in progress.
- Ongoing two-pion project to improve control over long-time region.

Sub-leading contributions.

- Heavy flavor analysis already at precision goals.
- Disconnected, SIB (conn. + disc @ 3 lattice spacings) and QED analysis ongoing.

Thank you

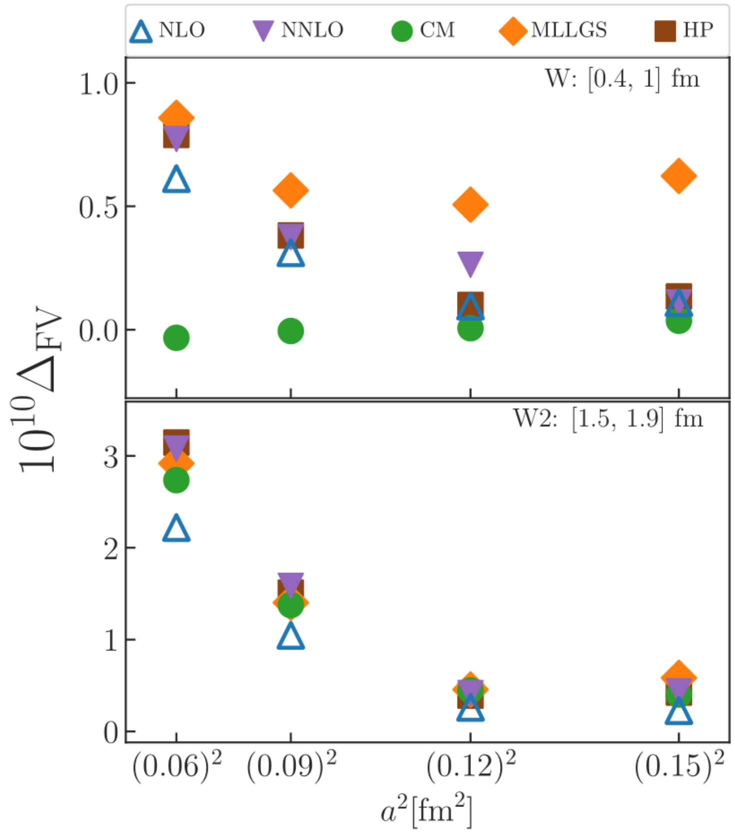
Finite Volume Effects

Lattice results corrected for FV, (TB) and pion mass mistuning before continuum extrapolation..

$$a_\mu\left(L_\infty, M_{\pi_{\rm phys}}\right) = a_\mu\left(L_{\rm latt}\,, M_{\pi_{{\rm lat}\,,\xi_1}}, \cdots, M_{\pi_{{\rm lat}\,,\xi_{16}}}\right) + \Delta_{\rm FV} + \Delta_{M_\pi}(+\Delta_{\rm TB})$$

FV effects from low energy two-pion physics.

- (N)NLO chiral perturbation theory.
  (Theory of pions + LECs) BMW, Aubin et. al.
- CM FHM, BMW, Aubin et. al.
  (ChPT + dynamical rho meson.)
- MLLGS ETMC, Mainz
  (IV scattering amplitude ⇔ FV energies and overlap amplitudes.)
- HP RBC/UKQCD
  (Non-perturbative re-summation of scalar QED)



Correction is 0.5% (3%) effect for W (W2).

Continuum Extrapolation

BMA continuum fit function:

$$a_\mu^{ll}(a, \{m_f\}) = a_\mu^{ll}\left(1 + F^{\text{disc.}}(a) + F^m(\{\delta m_f\})\right),$$

Variations:

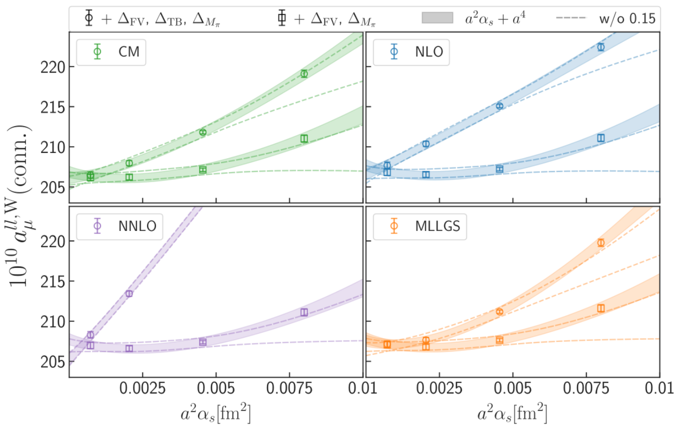
- Powers of the strong coupling
- Turning on/off the cubic term
- Turning on/off the mass term
- Dropping coarsest ensemble.

$$F^{\text{disc.}}(a) = C_{a^2,n}\left[(a\Lambda)^2\alpha_s^n\right] + C_{a^4}(a\Lambda)^4 + C_{a^6}(a\Lambda)^6$$

$$F^m(\{\delta m_f\}) = C_{\text{sea}} \sum_{f=l,l,s} \delta m_f/\Lambda.$$

Analysis Strategy

- (Software) blinded analysis.
- Considered all EFT-based correction schemes.
- Continuum extrapolation analysis including taste-breaking.
- Systematics from analysis choices through Bayesian Model Averaging (BMA).



Each set of analysis choices constitutes a 'model'.

Bayesian Akaike information criterion

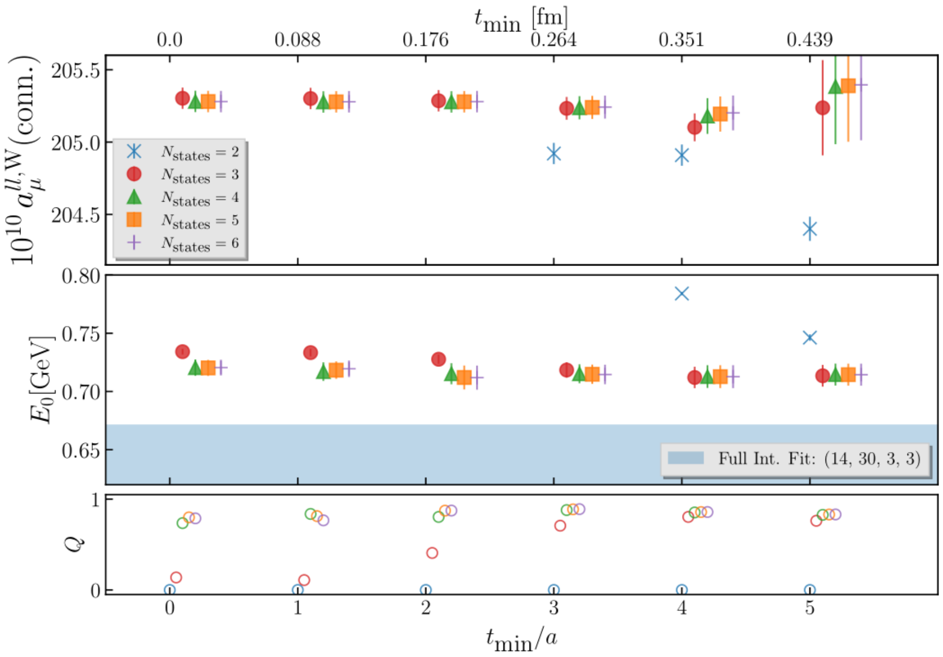
Model Probability: $$\operatorname{pr}(M \mid D) \equiv \operatorname{pr}(M) \exp\left[-\frac{1}{2}\left(\chi^2_{\text{data}}\left(\mathbf{a}^\star\right) + 2k + 2N_{\text{cut}}\right)\right]$$

2208.14983

BMA mean and variance.

$$\langle a_\mu \rangle = \sum_{i=1}^{N_M} \langle a_\mu \rangle_i \operatorname{pr}\left(M_i \mid D\right),$$

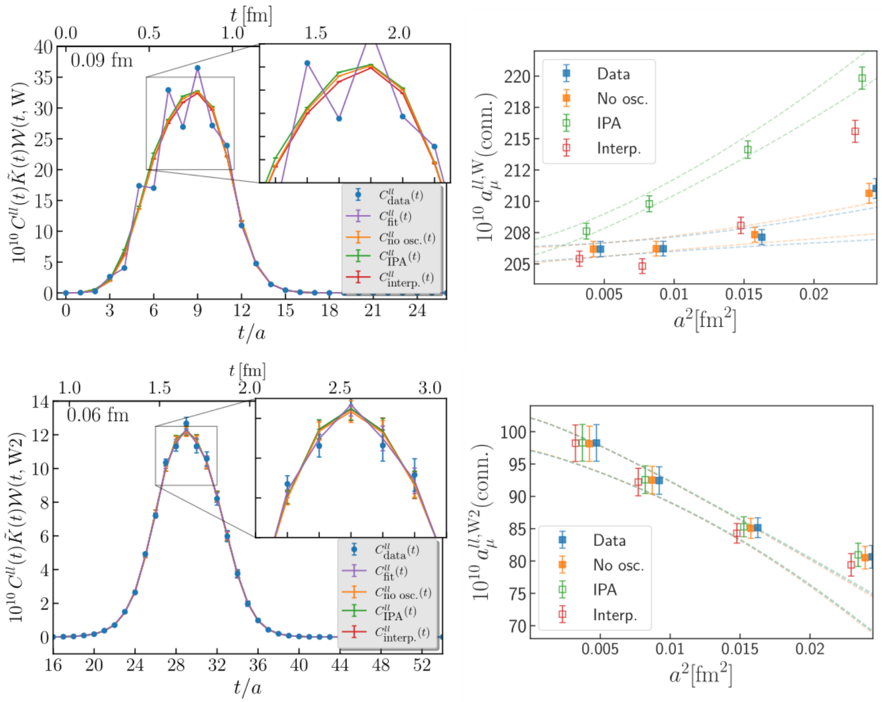
$$\sigma^2_{a_\mu} = \sum_{i=1}^{N_M} \sigma^2_{a_\mu, i} \operatorname{pr}\left(M_i \mid D\right) + \sum_{i=1}^{N_M} \langle a_\mu \rangle_i^2 \operatorname{pr}\left(M_i \mid D\right) - \left(\sum_{i=1}^{N_M} \langle a_\mu \rangle_i \operatorname{pr}\left(M_i \mid D\right)\right)^2.$$



$$C_{\mathrm{fit}}(t) = \sum_{n}^{N_{\mathrm{states}}} \left[Z_n^2 e^{-E_n t} + (-1)^t Z_{n,\mathrm{osc}}^2 e^{-E_{n,\mathrm{osc}} t} \right]$$

Table B.1: $a_\mu^{ll,\mathrm{W}}$(conn.) and $a_\mu^{ll,\mathrm{W2}}$(conn.) computed from the raw data (columns two and five), the fit reconstruction with oscillating states (columns three and six) and the correlated difference between them (columns four and seven).

$\approx a$	$a_\mu^{ll,\mathrm{W}}$(conn.)	$a_{\mu,\,\mathrm{fit}}^{ll,\mathrm{W}}$(conn.)	$\Delta a_\mu^{ll,\mathrm{W}}$(conn.)	$a_\mu^{ll,\mathrm{W2}}$(conn.)	$a_{\mu,\,\mathrm{fit}}^{ll,\mathrm{W2}}$(conn.)	$\Delta a_\mu^{ll,\mathrm{W2}}$(conn.)
0.15	211.01(79)	211.15(80)	−0.14(11)	80.3(1.7)	80.1(1.7)	0.20(19)
0.12	207.13(60)	207.16(60)	−0.025(29)	84.7(1.5)	84.6(1.5)	0.09(10)
0.09	206.56(55)	206.58(55)	−0.016(10)	92.7(1.8)	92.7(1.8)	−0.07(22)
0.06	206.22(61)	206.22(61)	0.003(61)	95.6(2.8)	95.5(2.7)	0.12(73)



$$C_{\text{no osc.}}(t) = \sum_n^{N_{\text{states}}} Z_n^2 e^{-E_n t}.$$

$$C_{\text{IPA}}(t) = \frac{e^{-m_\rho t}}{4}\left[\frac{C(t-1)}{e^{-m_\rho(t-1)}} + 2\frac{C(t)}{e^{-m_\rho(t)}} + \frac{C(t+1)}{e^{-m_\rho(t+1)}}\right]$$

$$C_{\text{interp}}(t) = \frac{1}{2}\left(C^{\text{even. interp}}(t) + C^{\text{odd. interp}}(t)\right)$$

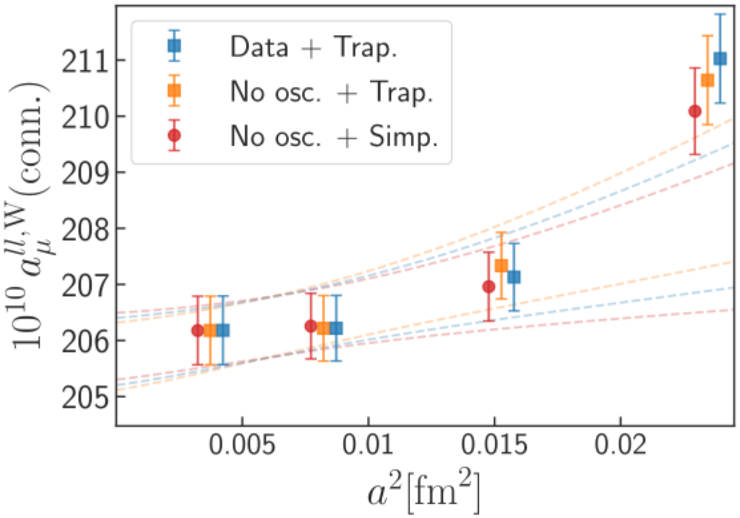
Table B.2: $a_\mu^{ll,\mathrm{W}}$(conn.) and $a_\mu^{ll,\mathrm{W2}}$(conn.) computed from the raw data (columns two and five), the fit reconstruction without oscillating states (columns three and six) and the correlated difference between them (columns four and seven).

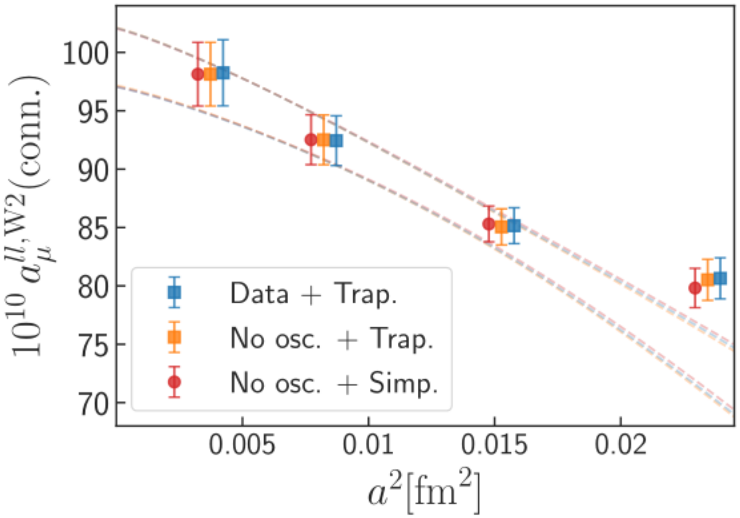
$\approx a$	$a_\mu^{ll,\mathrm{W}}$(conn.)	$a_{\mu,\text{ No osc.}}^{ll,\mathrm{W}}$(conn.)	$\Delta a_\mu^{ll,\mathrm{W}}$(conn.)	$a_\mu^{ll,\mathrm{W2}}$(conn.)	$a_{\mu,\text{ No osc.}}^{ll,\mathrm{W2}}$(conn.)	$\Delta a_\mu^{ll,\mathrm{W2}}$(conn.)
0.15	211.01(79)	210.62(79)	0.39(20)	80.3(1.7)	80.2(1.7)	0.13(19)
0.12	207.13(60)	207.34(59)	−0.204(34)	84.7(1.5)	84.6(1.5)	0.10(11)
0.09	206.56(55)	206.56(55)	0.001(10)	92.7(1.8)	92.7(1.8)	−0.07(22)
0.06	206.22(61)	206.22(61)	0.003(60)	95.6(2.8)	95.5(2.7)	0.12(73)

Trapezoidal versus Simpson's rule.

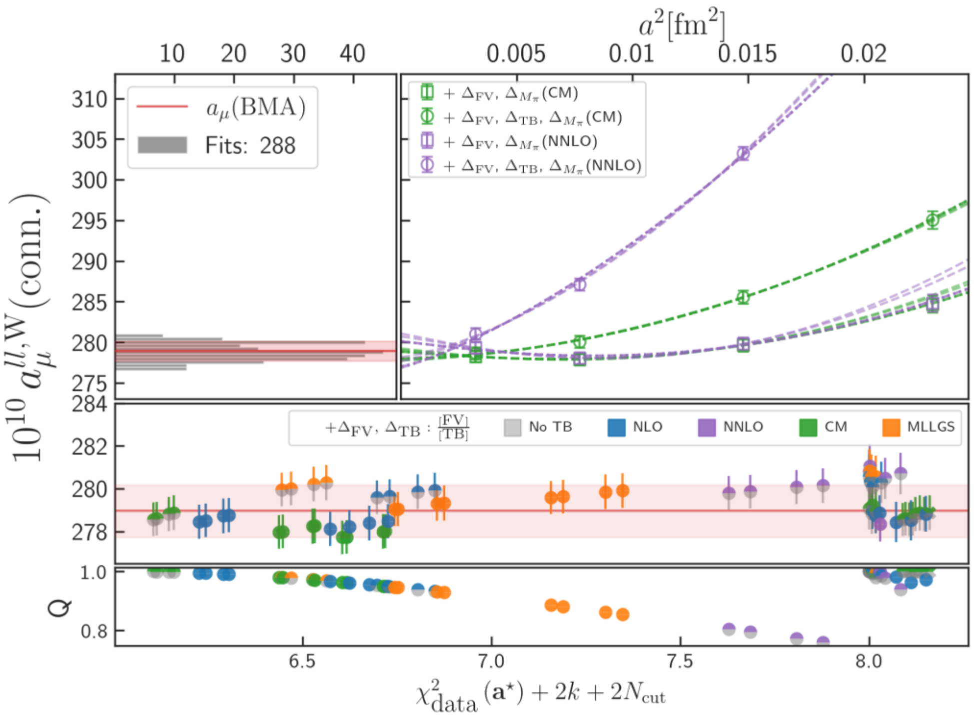
$$a_{\mu,\,\mathrm{Trap.}}^{\mathrm{win}(t_0,t_1,\Delta)} = 4\alpha^2 a \sum_{t=1}^{N_t/2-1} C(t)\tilde{K}(t)\mathcal{W}(t,t_0,t_1,\Delta), \qquad a_{\mu,\mathrm{Simp}}^{\mathrm{win}(t_0,t_1,\Delta)} = 4\alpha^2 \frac{a}{3}\left[\left(4\sum_{t\in\{t_{\mathrm{odd}}\}}^{N_t/2-1} + 2\sum_{t\in\{t_{\mathrm{even}}\}}^{N_t/2-1}\right) C(t)\tilde{K}(t)\mathcal{W}(t,t_0,t_1,\Delta)\right]$$

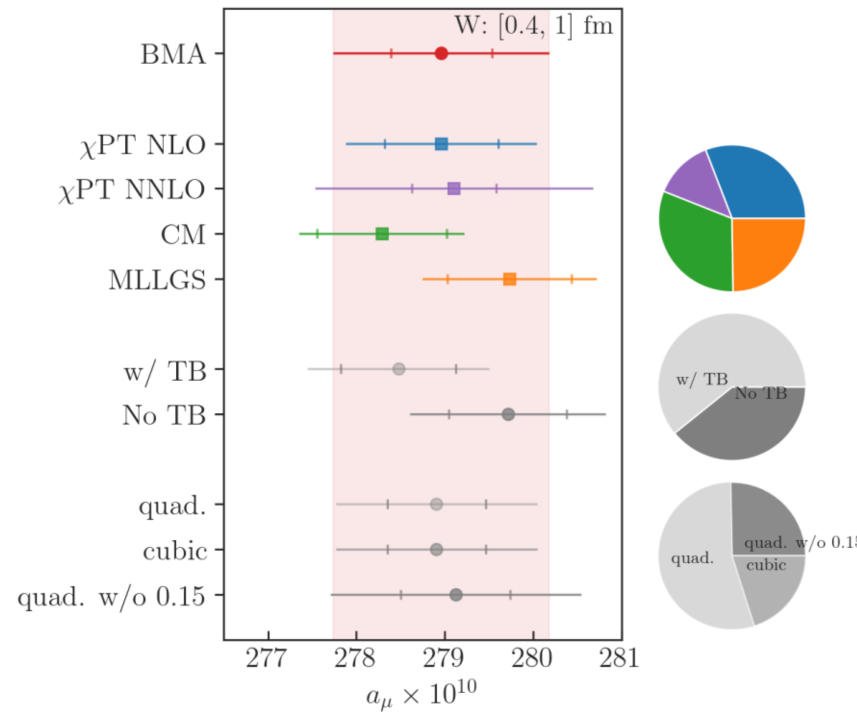
Simpsons rule is formally a higher order integration scheme.





- Small differences on coarse ensembles for W.
- Oscillatory effects have died out for W2.
- Statistically equivalent continuum results.

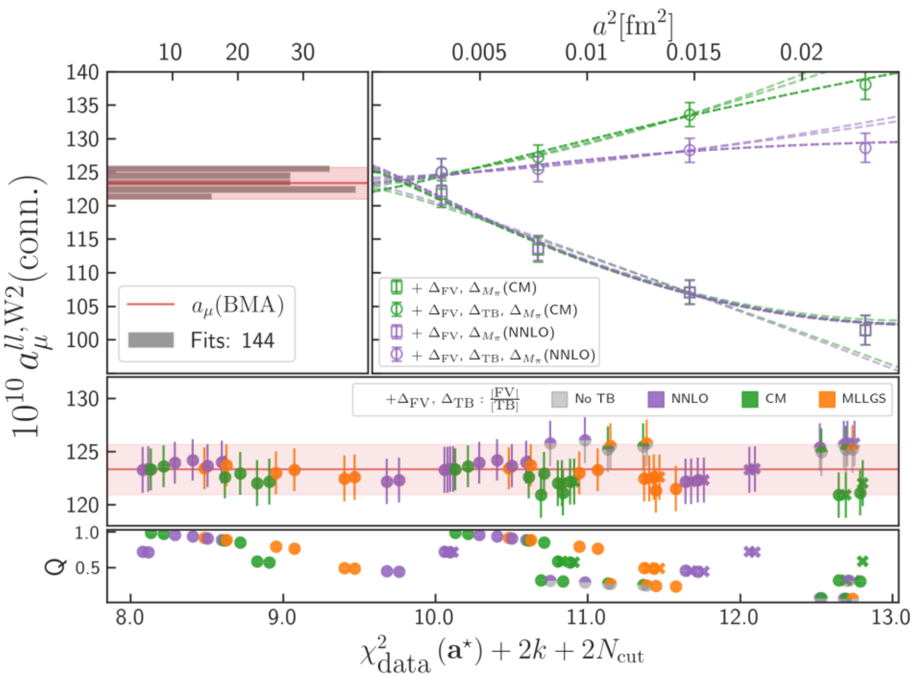


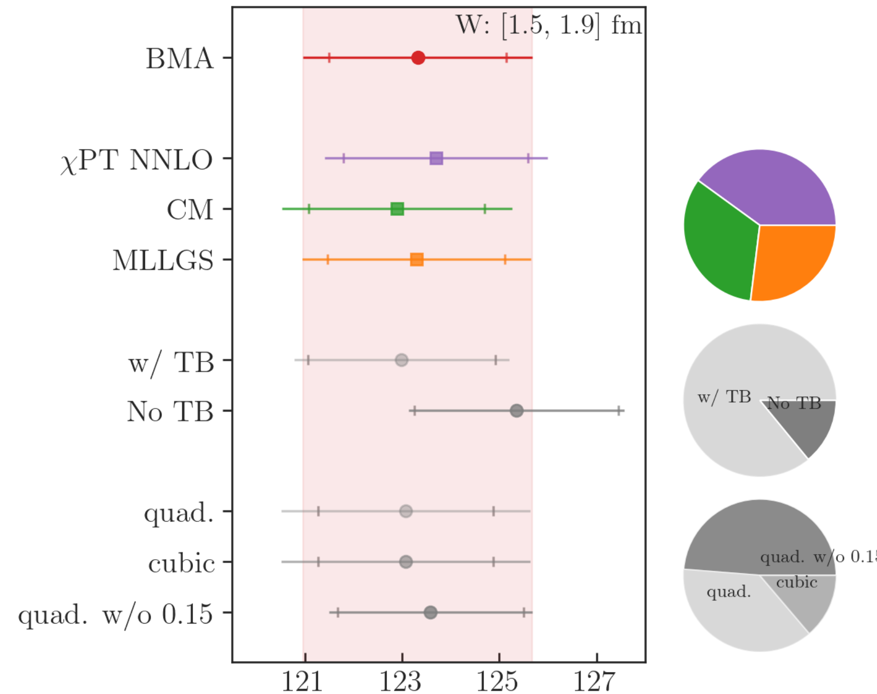


Source	$\delta a_\mu^{ll,W}$(conn.) (%)
Monte Carlo statistics	0.19 → 0.09
Continuum extrapolation ($a \to 0$, Δ_{TB})	0.34 → 0.27
Total	0.50 → 0.44

Factor of ~2 improvement in stat. uncertainty.

Updated W2 Results





Source	$\delta a_\mu^{ll,\mathrm{W2}}(\mathrm{conn.})$ (%)
Monte Carlo statistics	2.44 → 0.73
Continuum extrapolation ($a \to 0$, Δ_{TB})	0.34 → 0.9
Total	3.18 → 1.94

Over a factor of ~3 improvement in stat. uncertainty.