Determination of the gradient flow scale t_0 from a Mixed Action with Wilson Twisted Mass Valence Quarks.

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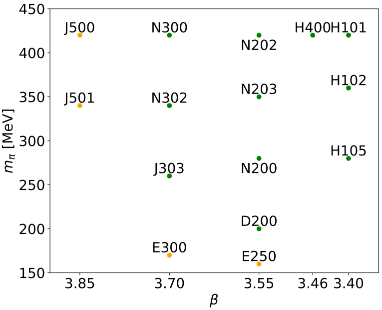
Mixed Action Setup

-Sea: CLS $N_f = 2 + 1$ ensembles

- Lüscher-Weisz gauge action
- Non-perturbatively $O(a)$ improved Wilson fermions
- Open boundary conditions in time (topology freezing)
- Chiral trajectory: $\mathrm{Tr}(\mathbf{m}_\mathrm{q}) = \mathrm{m_u} + \mathrm{m_d} + \mathrm{m_s} = \mathrm{cnst}$



[Lüscher and Schaefer, JHEP,1107 036; Bruno et al. JHEP 1502 043 - 1712.04884 - 2003.13359], [Lüscher and Schaefer,1206.2809]

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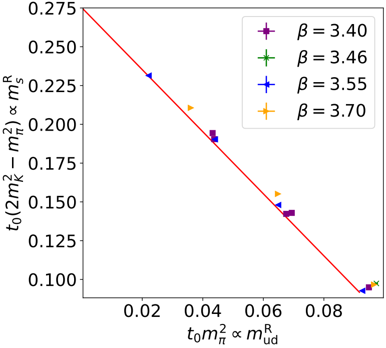
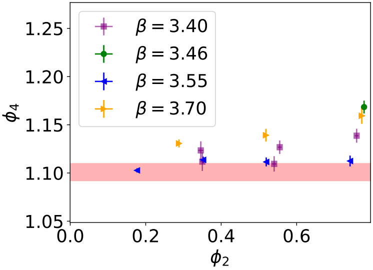


$$\phi_2 = 8t_0 m_\pi^2, \qquad \phi_4 = 8t_0\left(m_K^2 + \tfrac{1}{2}m_\pi^2\right) \propto \mathrm{Tr}(\mathbf{m}_\mathrm{R}) + \text{higher orders}$$

Mixed Action Setup

-Mass shift:

- Fix $\phi_4 = 8t_0\left(m_K^2 + \frac{1}{2}m_\pi^2\right) = \phi_4^{\mathrm{phys}}$
- Iterative procedure, keep correlations in ϕ_4^{guess}



$$\mathcal{O}(m') = \mathcal{O}(m) + \sum_q (m'_q - m_q)\frac{d\mathcal{O}}{dm_q},$$

$$\frac{d\mathcal{O}}{dm_q} = \sum_i \frac{\partial \mathcal{O}(P_i)}{\partial \langle P_i\rangle}\left[\left\langle \frac{\partial P_i}{\partial m_q}\right\rangle - \left\langle P_i \frac{\partial S}{\partial m_q}\right\rangle + \langle P_i\rangle \left\langle \frac{\partial S}{\partial m_q}\right\rangle\right]$$

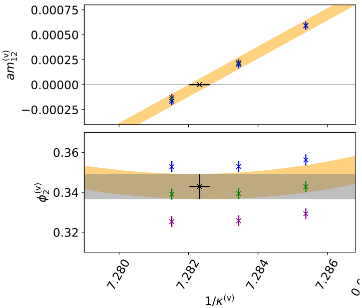
[Bruno, Korzec, Schaefer, 1608.08900], [Straßberger et al. 2112.06696]

Mixed Action Setup

[ALPHA, hep-lat/0101001; Frezzotti and Rossi hep-lat/0306014, Pena et al., hep-lat/0405028]

-Valence:

- Twisted mass valence quarks
- Automatic $O(a)$ improvement
  up to small $O(a\mathrm{Tr}(\mathbf{m}_\mathrm{q}))$ effects
- Match valence & sea, tune to maximal twist $\rightarrow (\kappa, \mu_l, \mu_s)^{(\mathrm{v})}$ grid



$$D = \frac{1}{2}\gamma_\mu(\nabla^*_\mu + \nabla_\mu) - \frac{a}{2}\nabla^*_\mu\nabla_\mu + \frac{i}{4}ac_{\mathrm{sw}}\sigma_{\mu\nu}\hat{F}_{\mu\nu} + am + i\gamma_5 a\mu$$

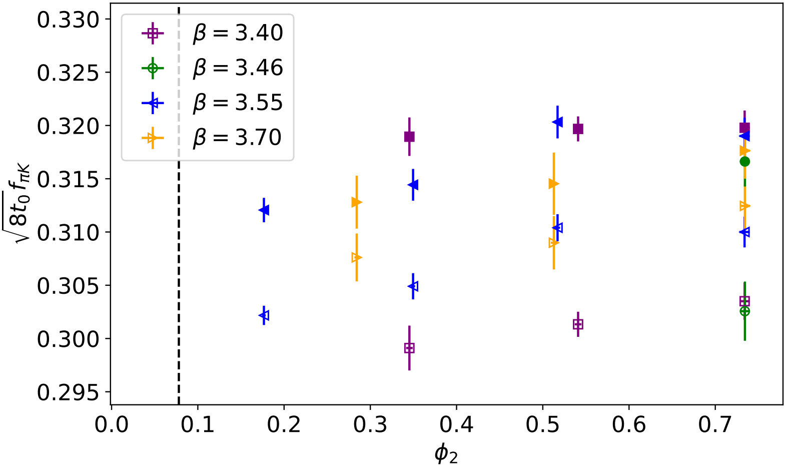
$$\phi_2 = 8t_0 m_\pi^2$$

$$\phi_4 = 8t_0 m_K^2 + \frac{1}{2}\phi_2$$

Measurements for $\sqrt{8t_0}f_{\pi K}$: Wilson & Wtm

$f_{\pi K} = \frac{2}{3}\left(f_k + \frac{1}{2}f_\pi\right)$

Filled points: Wtm   Empty points: Wilson

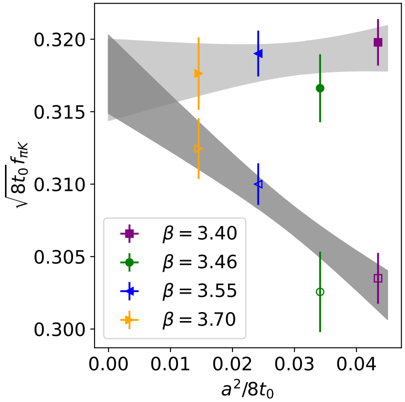


Continuum Limit Scaling: $\sqrt{8t_0} f_{\pi K}$

Filled points: Wtm Empty points: Wilson

$$f_{\pi K} = \frac{2}{3}\left(f_k + \frac{1}{2} f_\pi\right)$$

- Symmetric point ensembles
  $m_l = m_s$, $\phi_2 \approx 0.73$
- Wilson and Wtm data
- Continuum limit fit: linear in a^2



Chiral-Continuum Model Variation: $\sqrt{8t_0}f_{\pi K}$

-Models:

- Mass dependence
  - $SU(3)$ χPT
  - Taylor expansion in ϕ_2
- Cutoff dependence
  - $O(a^2)$ cutoff effects
  - $O(a^2) + O(a^2\phi_2)$
  - $O(a^2\alpha_s^{\hat{\Gamma}_i})$ [Husung, 2206.03536]
- Cuts in data
  - Cut $\beta \leq 3.40$
  - Cut $m_\pi \geq 420$ MeV
  - Cut $m_\pi L \leq 3.9$
  - Cut $m_\pi L \leq 4.1$

-Model average:

$$\mathrm{TIC} = \chi^2 - 2\left\langle \chi^2 \right\rangle,$$

$$\mathrm{W_M} \propto \exp\left(-\frac{1}{2}\mathrm{TIC_M}\right),$$

$$\langle t_0 \rangle = \sum_M t_0^{(M)} W_M,$$

$$\sigma^2_{\mathrm{syst}} = \left\langle t_0^2 \right\rangle - \langle t_0 \rangle^2$$

[J. Frison, 2302.06550], [Parallel talk by Julien Frison Friday 9:00 AM], [M. Bruno, R. Sommer, 2209.14188]

Chiral-Continuum Model Variation: $\sqrt{8t_0} f_{\pi K}$

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-Model average:

$$\mathrm{IC} = \chi^2 + 2n_{\mathrm{param}} + 2n_{\mathrm{cut}},$$

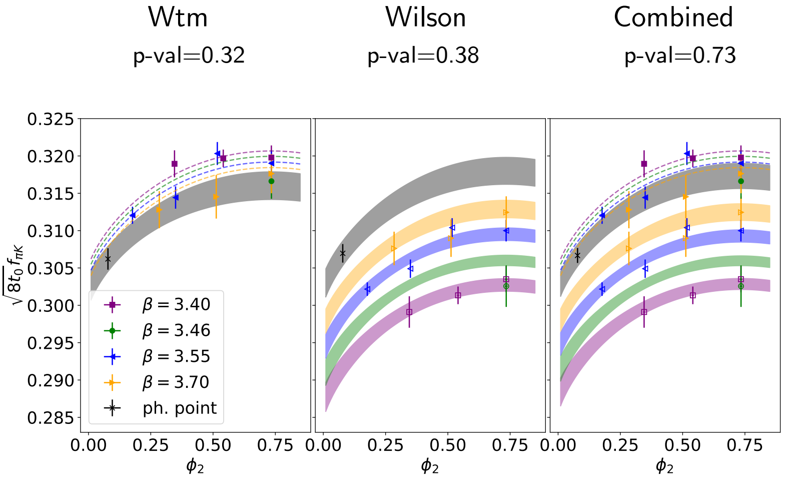
$$\mathrm{W_M} \propto \exp\left(-\frac{1}{2}\mathrm{IC_M}\right),$$

$$\langle t_0 \rangle = \sum_M t_0^{(M)} W_M,$$

$$\sigma^2_{\mathrm{syst}} = \langle t_0^2 \rangle - \langle t_0 \rangle^2$$

[E. Neil, 2008.01069], [Parallel talk by Ethan Neil Friday 9:20 AM], [M. Bruno, R. Sommer, 2209.14188]

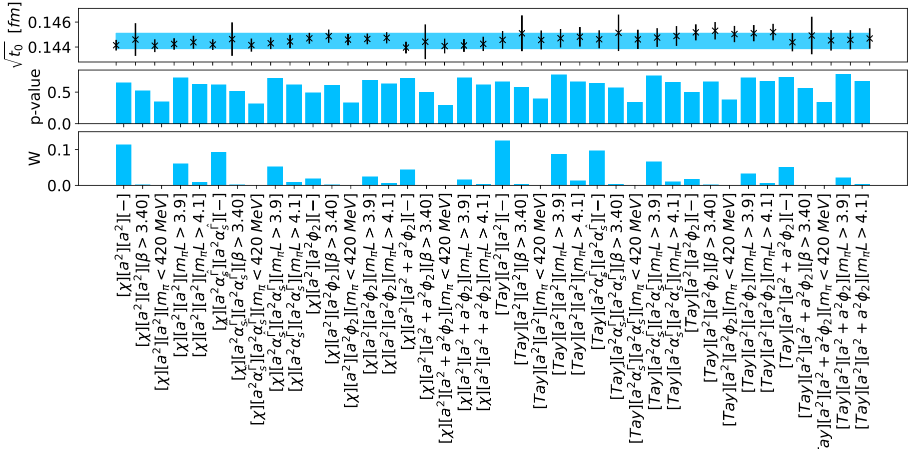
Chiral-Continuum Model Variation: $\sqrt{8t_0}f_{\pi K}$



- $SU(3)$ χPT, $O(a^2)$ cutoff effects, no cuts
- Correlations in y, neglect them in $x \to \langle \chi^2 \rangle \approx 0.97\mathrm{dof}$, FLAG21 for inputs

Model Average (Combined Data, TIC)

NB: the band covers all the points from individual models

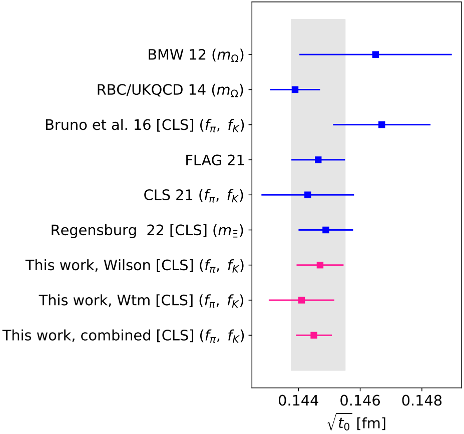


Results

[Y. Aoki et. al, 2111.09849]

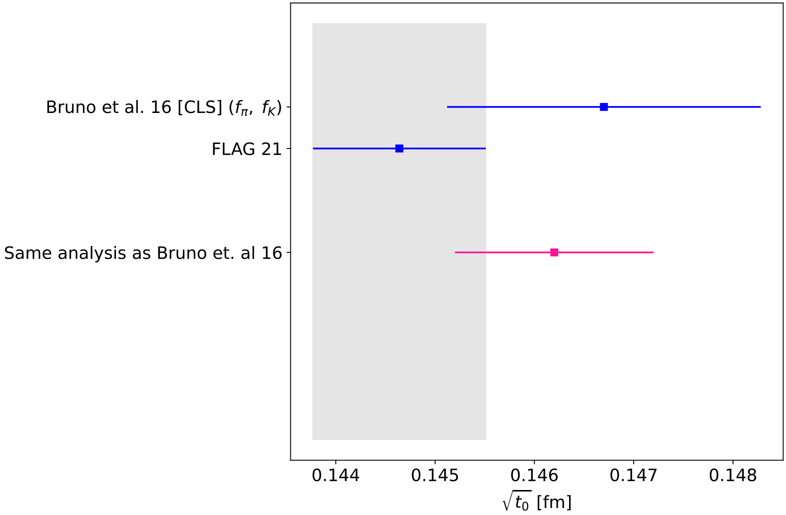
$$\sqrt{t_0^{\mathrm{Wilson}}} = 0.1447(7)(3)\ \mathrm{fm},$$
$$\sqrt{t_0^{\mathrm{Wtm}}} = 0.1442(7)(8)\ \mathrm{fm},$$
$$\sqrt{t_0^{\mathrm{Combined}}} = 0.1446(5)(3)\ \mathrm{fm}.$$

$$m_\pi = 134.9768(5)\ \mathrm{MeV},$$
$$m_K = 497.611(13)\ \mathrm{MeV},$$
$$f_\pi = 130.56(2)(13)(2)\ \mathrm{MeV},$$
$$f_K = 157.2(2)(2)(4)\ \mathrm{MeV}$$



$N_f = 2 + 1$

Results: Wilson



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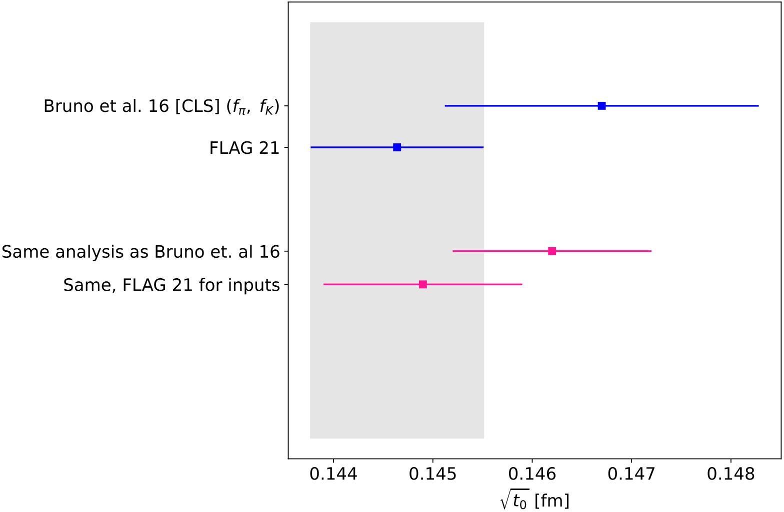
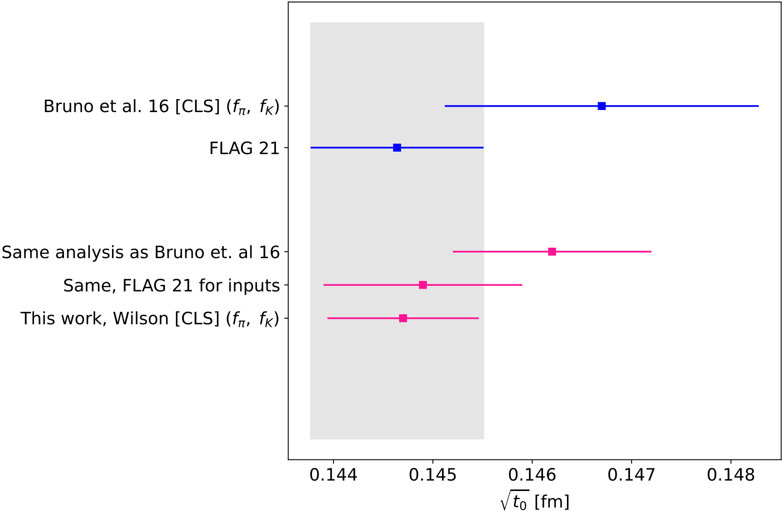


Results: Wilson



Results

FLAG16 [S. Aoki et. al, 1607.00299], [Review of Particle Physics, Chin. Phys. C38 (2014) 090001.]

$$m_\pi = 134.8(3)\ \mathrm{MeV},$$
$$m_K = 494.2(3)\ \mathrm{MeV},$$
$$f_\pi = 130.4(2)\ \mathrm{MeV},$$
$$f_K = 156.2(7)\ \mathrm{MeV}$$

FLAG21 [Y. Aoki et. al, 2111.09849], [Review of Particle Physics, PTEP 2020.8 (2020)]

$$m_\pi = 134.9768(5)\ \mathrm{MeV},$$
$$m_K = 497.611(13)\ \mathrm{MeV},$$
$$f_\pi = 130.56(2)(13)(2)\ \mathrm{MeV},$$
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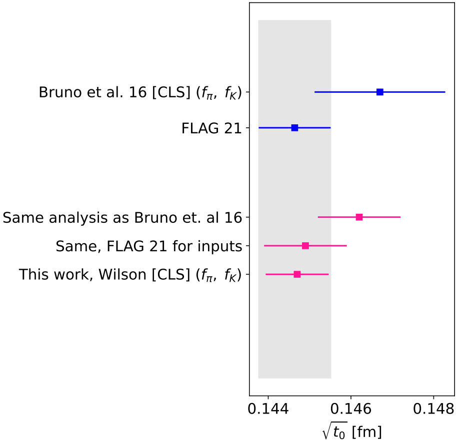

Conclusions

- FLAG21 vs. FLAG16
  - Brings value of $\sqrt{t_0}$ down, in line with recent determinations
- Wilson + Wtm data
  - Better p-values
  - Good contron of continuum limit
  - Improves precision: $\sim 0.4\%$ on $\sqrt{t_0}$
- Takeuchi's Information Criterion
  - Not fully correlated fits
  - Consistent, grounded estimation of average over models
- New ensembles
  - Analize E250, E300, J500 & J501
  - Explore further models $\rightarrow$ hidden systematics?

The End

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Backup

Model variation: $IC = \chi^2 + 2n_{param} + n_{cut}$

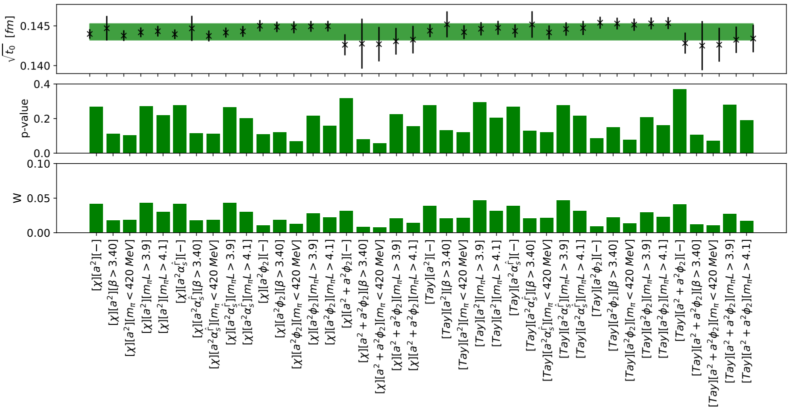


Figure: Model variation for t_0 (Wtm).

Model variation: $\mathrm{TIC} = \chi^2 - 2\left\langle \chi^2 \right\rangle$

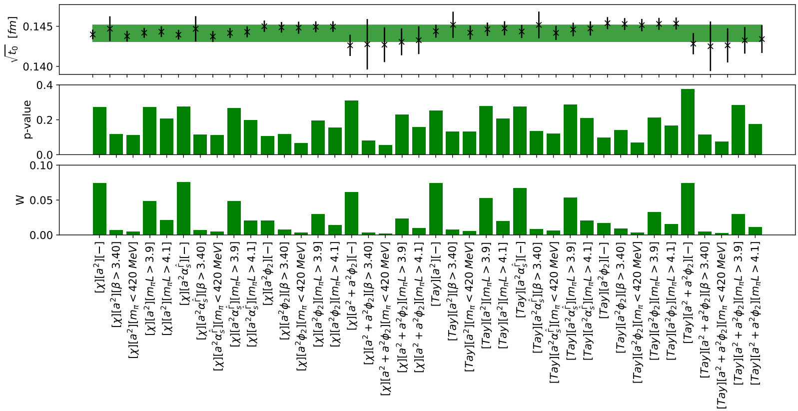


Figure: Model variation for t_0 (Wtm).

Model variation: $\mathrm{BMW-IC} = \chi^2 + 2n_{param} + n_{cut}$

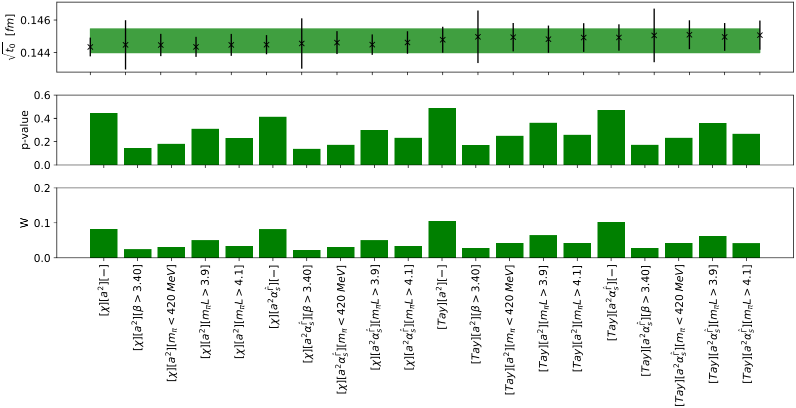
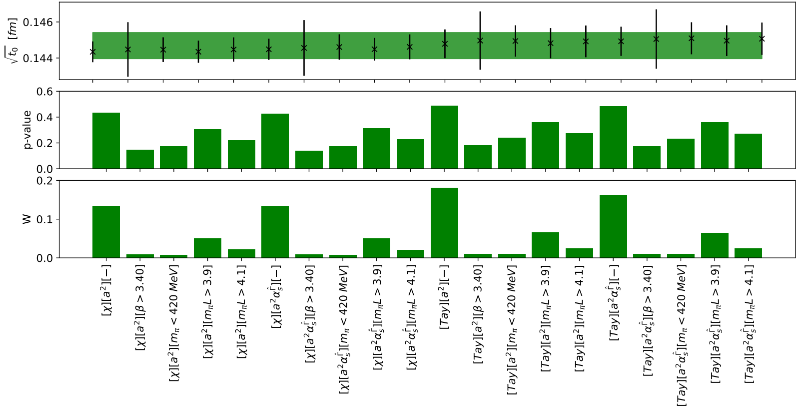


Figure: Model variation for t_0 (Wilson).

Model variation: $\mathrm{TIC} = \chi^2 - 2\left\langle\chi^2\right\rangle$



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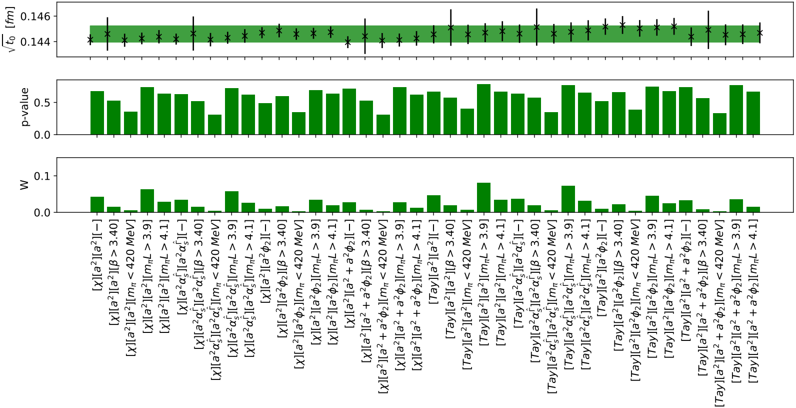


Figure: Model variation for t_0 (Combined).

Model variation: $\mathrm{TIC} = \chi^2 - 2\left\langle \chi^2 \right\rangle$

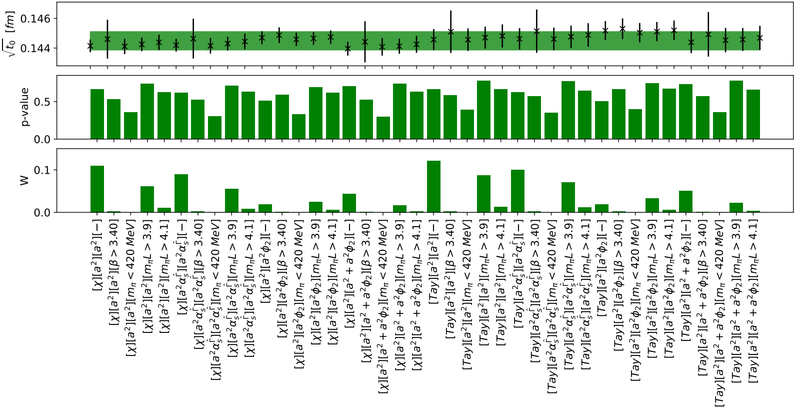


Figure: Model variation for t_0 (Combined).

Analysis comparisson to Bruno et. al (Wilson)

Ph. inputs	Mass shift	Chiral & cont. models	Model av.	$\sqrt{t_0}$ [fm] (Wilson)
FLAG16	δm_{uds}	SU(3) χPT, Tay, $O(a^2)$	CLS 21	0.1462(7)(7)
	δm_{uds}	SU(3) χPT, Tay, $O(a^2)$	TIC	0.1461(7)(2)
	δm_{uds}	Full set	CLS 21	0.1459(8)(4)
	δm_{uds}	Full set	TIC	0.1456(8)(2)
	δm_s	SU(3) χPT, Tay, $O(a^2)$	CLS 21	0.1461(7)(5)
	δm_s	SU(3) χPT, Tay, $O(a^2)$	TIC	0.1461(7)(2)
	δm_s	Full set	CLS 21	0.1459(8)(4)
	δm_s	Full set	TIC	0.1454(7)(2)
FLAG21	δm_{uds}	SU(3) χPT, Tay, $O(a^2)$	CLS 21	0.1449(9)(5)
	δm_{uds}	SU(3) χPT, Tay, $O(a^2)$	TIC	0.1449(9)(2)
	δm_{uds}	Full set	CLS 21	0.1448(10)(4)
	δm_{uds}	Full set	TIC	0.1445(9)(4)
	δm_s	SU(3) χPT, Tay, $O(a^2)$	CLS 21	0.1449(9)(3)
	δm_s	SU(3) χPT, Tay, $O(a^2)$	TIC	0.1449(8)(2)
	δm_s	Full set	CLS 21	0.1449(8)(4)
	δm_s	Full set	TIC	0.1447(7)(3)

BKS comparisson

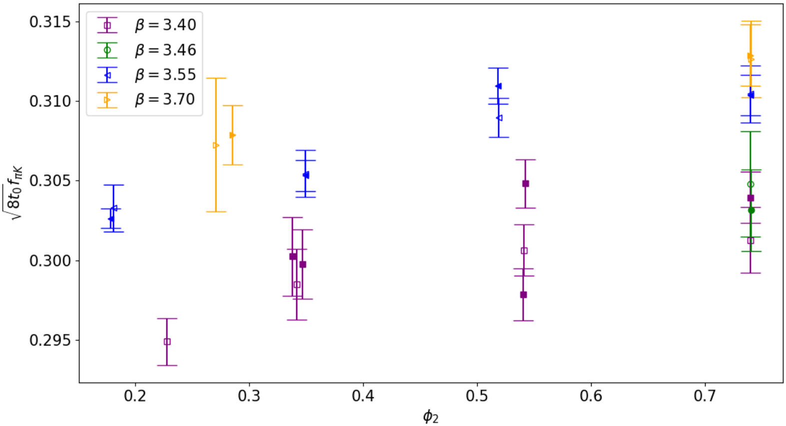
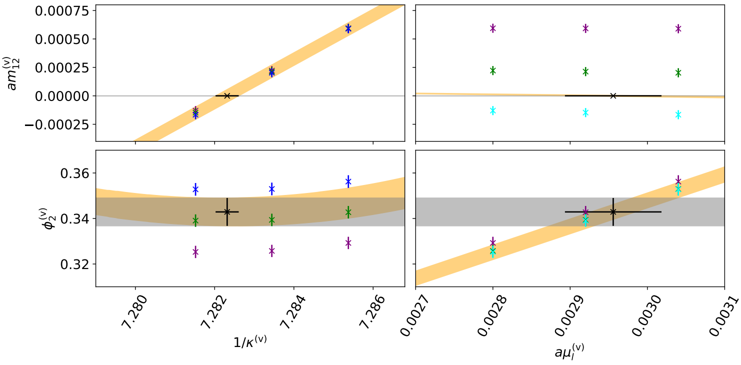


Figure: Empy points: Bruno et. al. measurements. Filled points: our measurements, shifting to the same ϕ_4 as in BKS.

Mixed Action Setup



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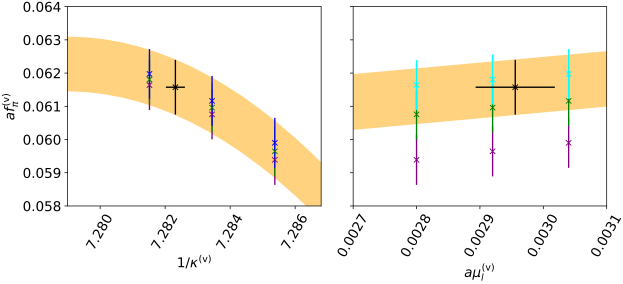

- After matching, need to interpolate $f_{\pi,K}$ along the grid

Results

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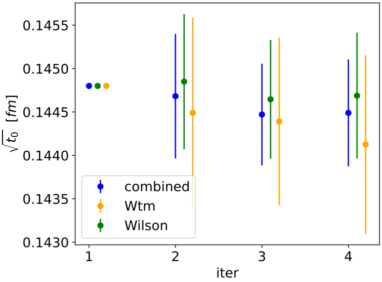

At each iterative step, we feed the analysis with the result t_0^{combined}

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