Hadron structure from lattice QCD

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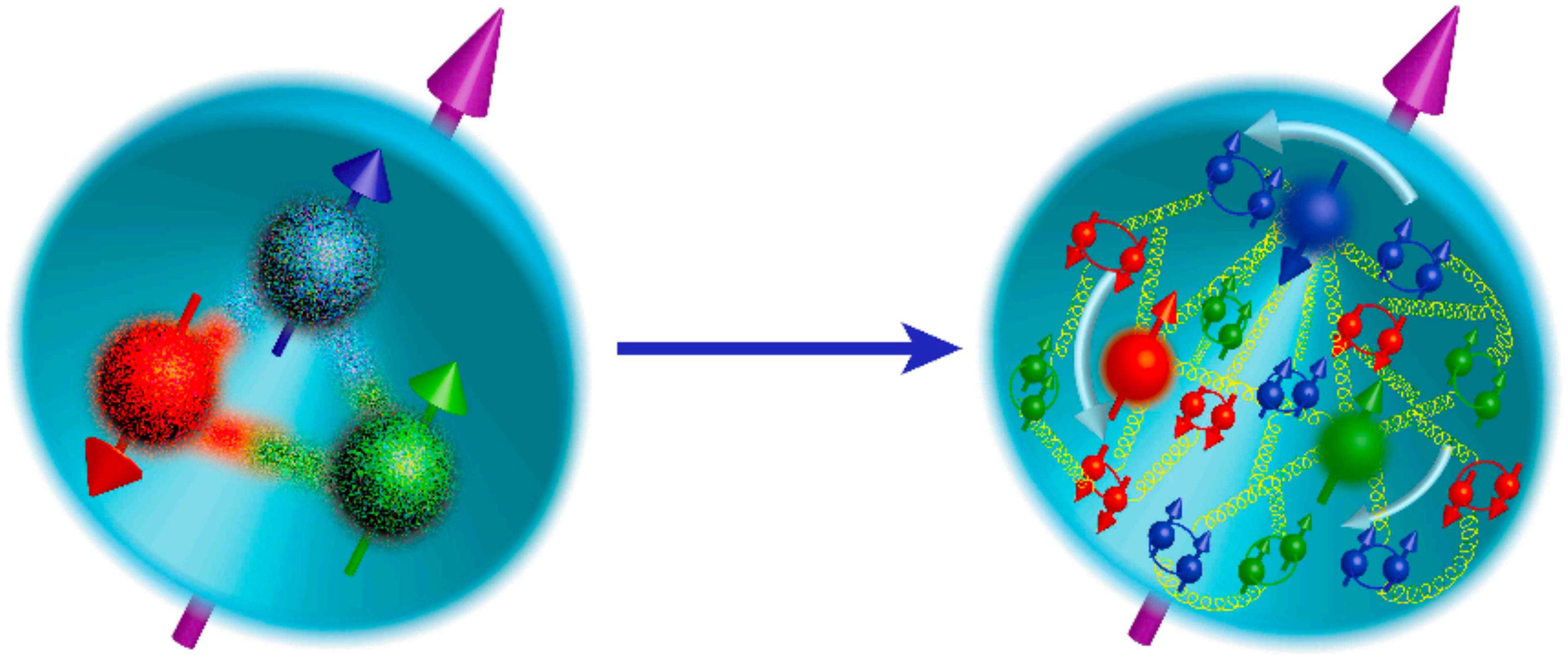
Argonne National Laboratory

2023 LATTICE

Lattice 2023, Fermilab, July 31- Aug. 4, 2023

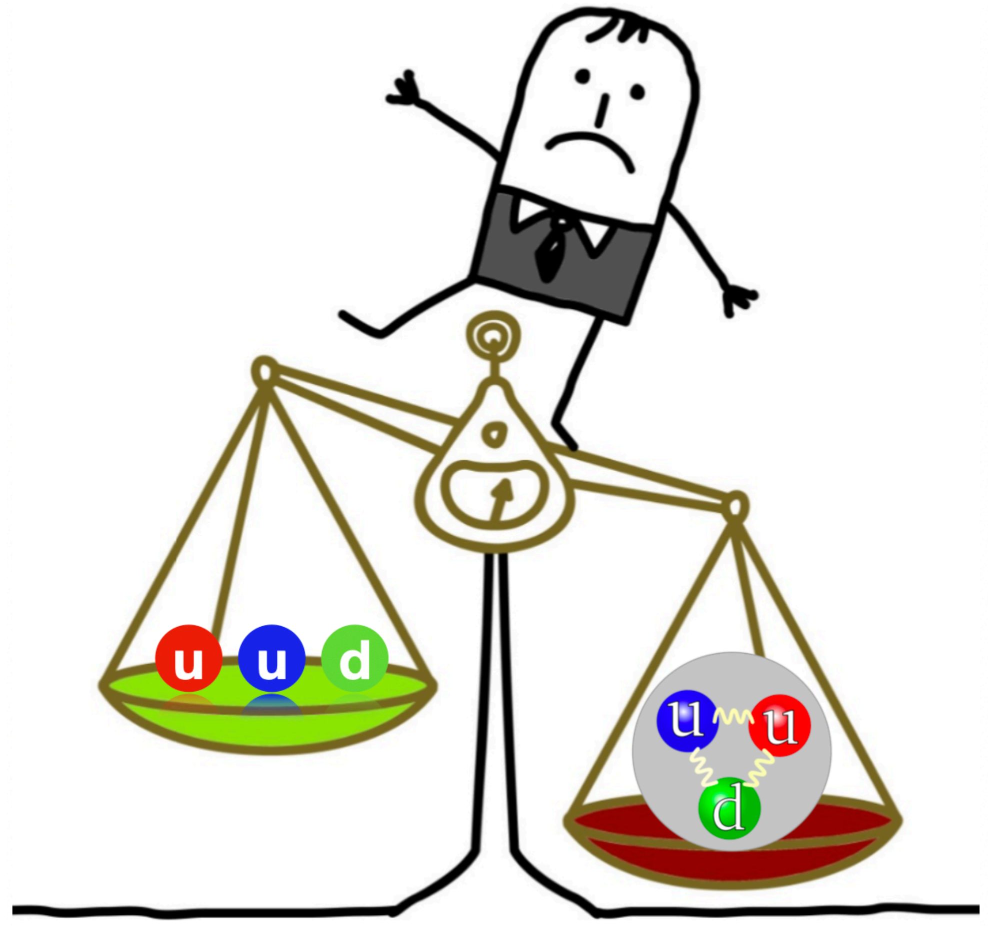
2 Hadron structure



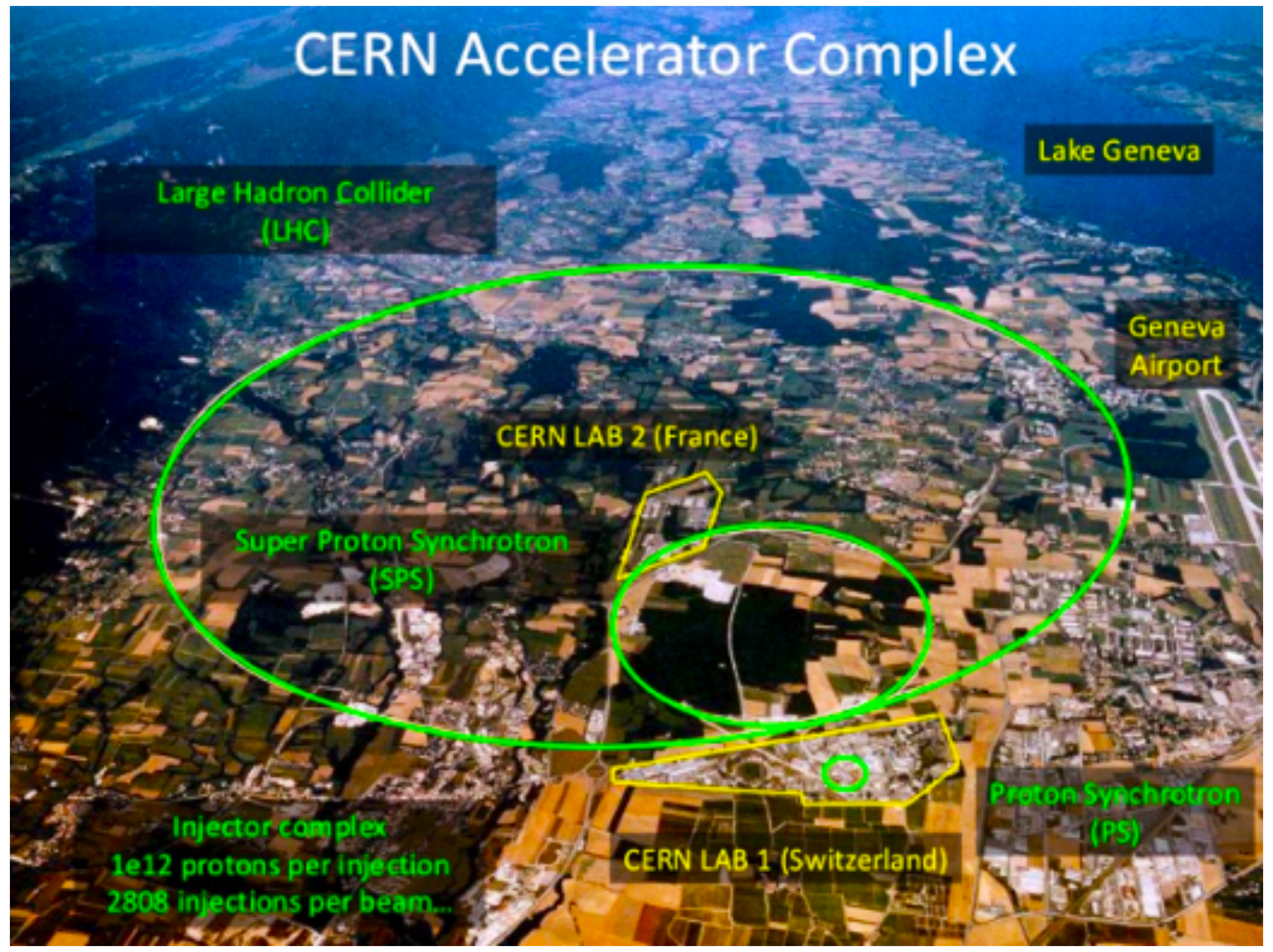
- The nucleons are fundamental building blocks of the visible universe.
- They are further composed of quarks and gluons.

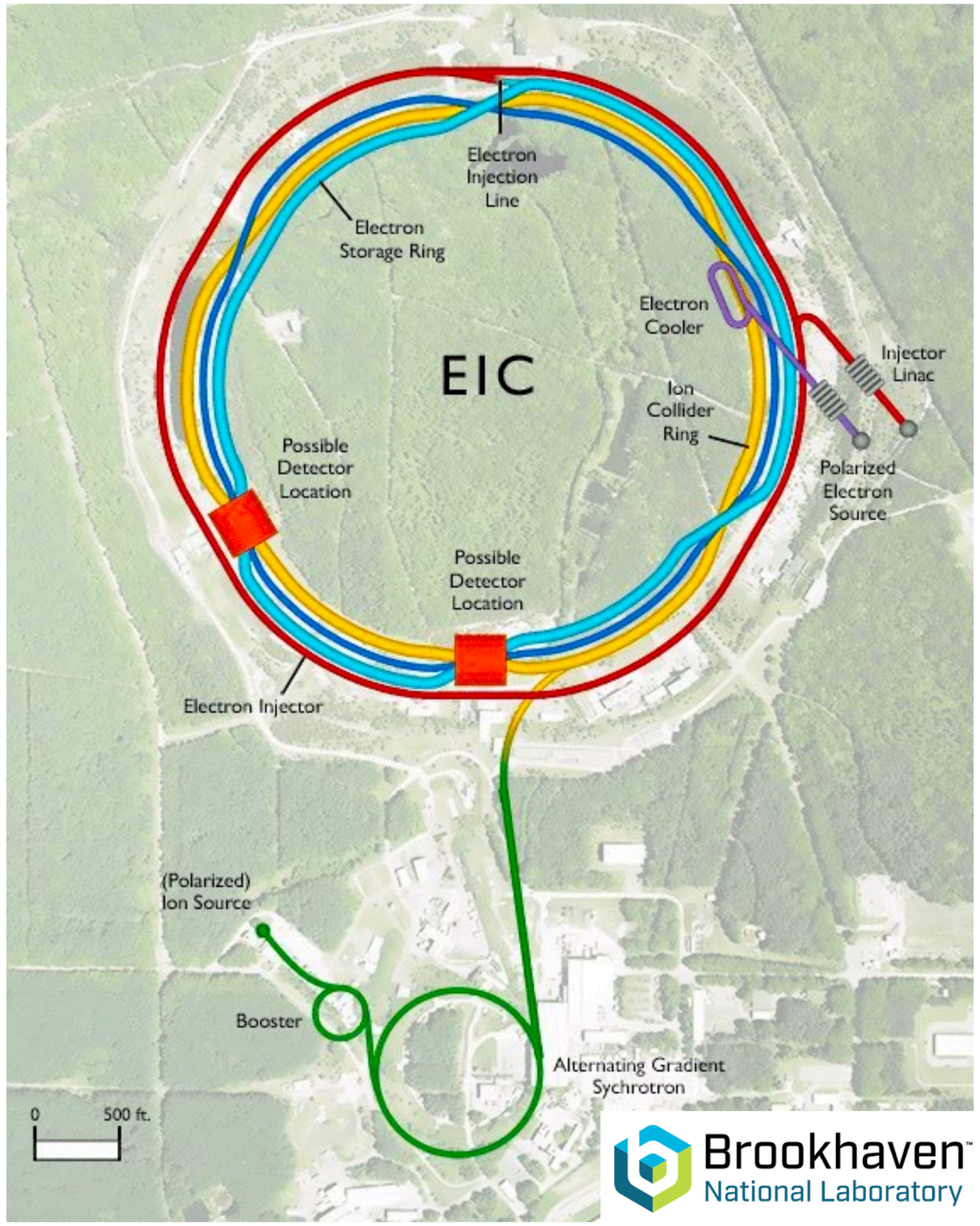
Hadron Structure and Tomography:

- The distribution of quarks, gluons, and their spins, inside the nucleon?
- The emergence of nucleon mass & spin from strong interactions?



Hadron structure

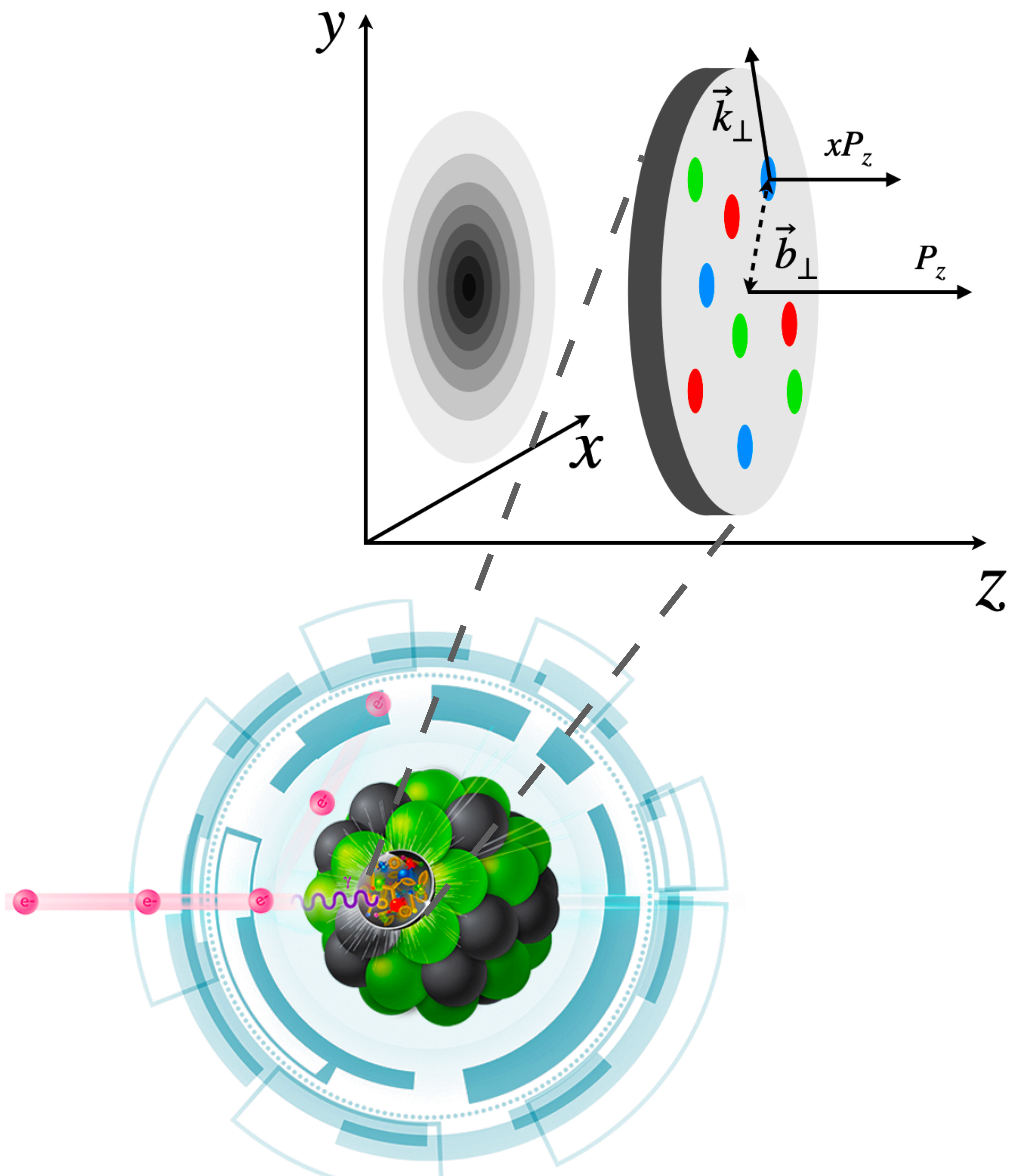




Ongoing and new experiments provide great opportunity for:

- Precision study of flavor and spin structure of hadrons.
- The three-dimensional structure of hadrons in coordinate and momentum space.

Hadron structure



- **The multi-dimensional image of nucleon**

$$W(x, b_T, k_T)$$

Wigner Distributions

$\int d^2 b_T$ $\qquad$ $\int d^2 k_T$

$$f(x, k_T)$$

Transverse momentum distributions (**TMDs**)

$$f(x, b_T)$$

Impact parameter distributions (**IPDs**)

$b_T \leftrightarrow \Delta$

$$F(x, \xi, \Delta^2)$$

Generalized Parton distributions (**GPDs**)

$\int d^2 k_T$ $\qquad$ $\int d^2 b_T$ $\qquad$ $\int dx x^{n-1}$

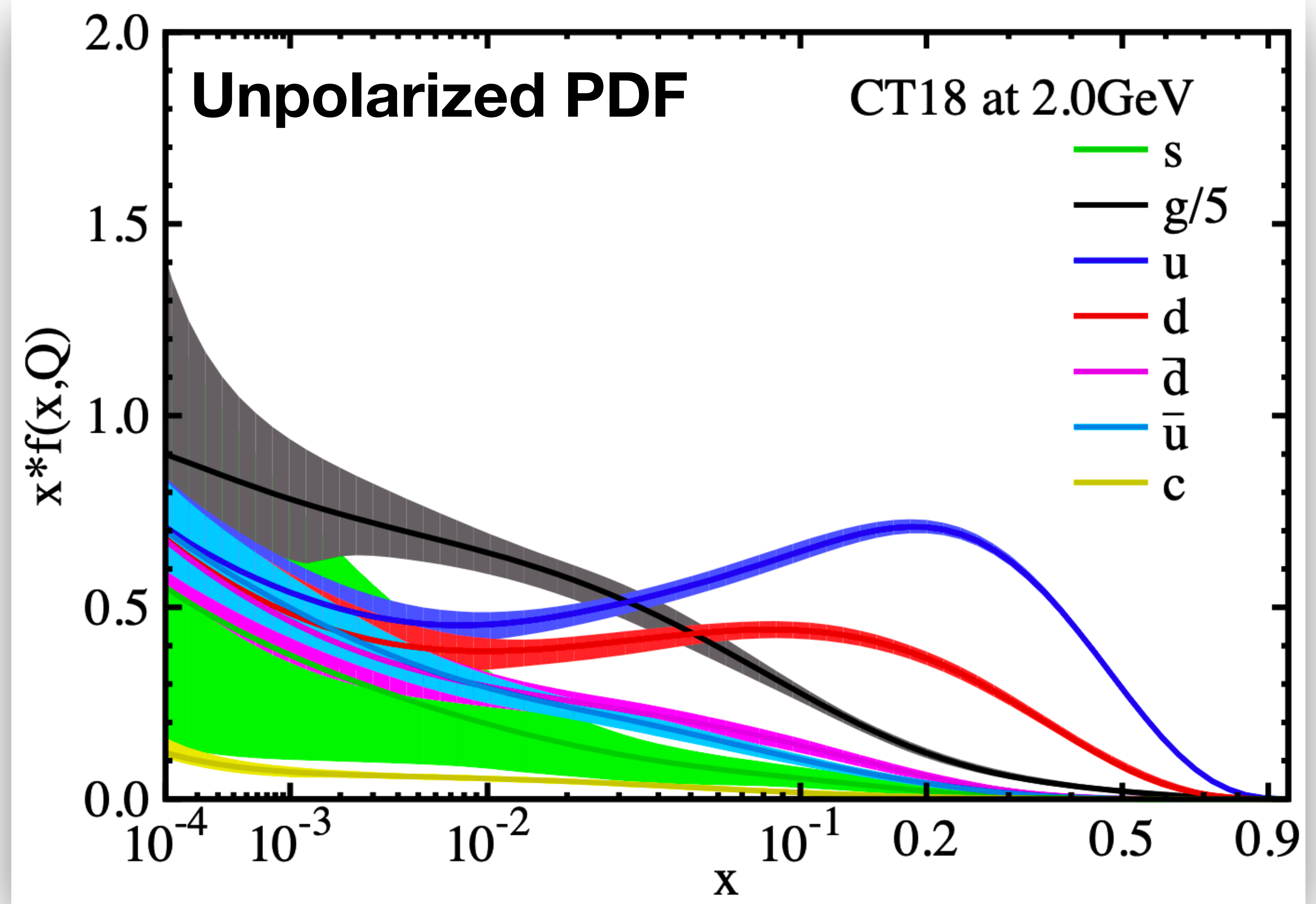
$$f(x)$$

Parton distribution functions (**PDFs**)

$$A_{n,0}(t) + 4\xi A_{n,2}(t) + \ldots$$

Form factors and (**FFs**) and generalized FFs (**GFFs**)

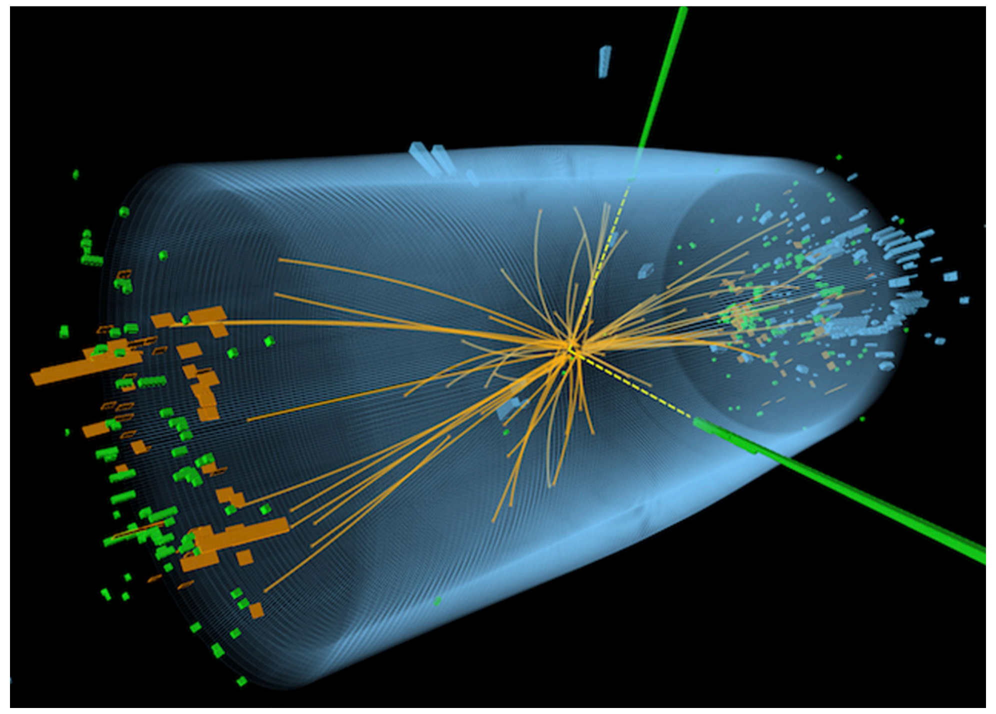
Parton distribution functions



CTEQ: Phys.Rev.D 103 (2021) 1, 014013

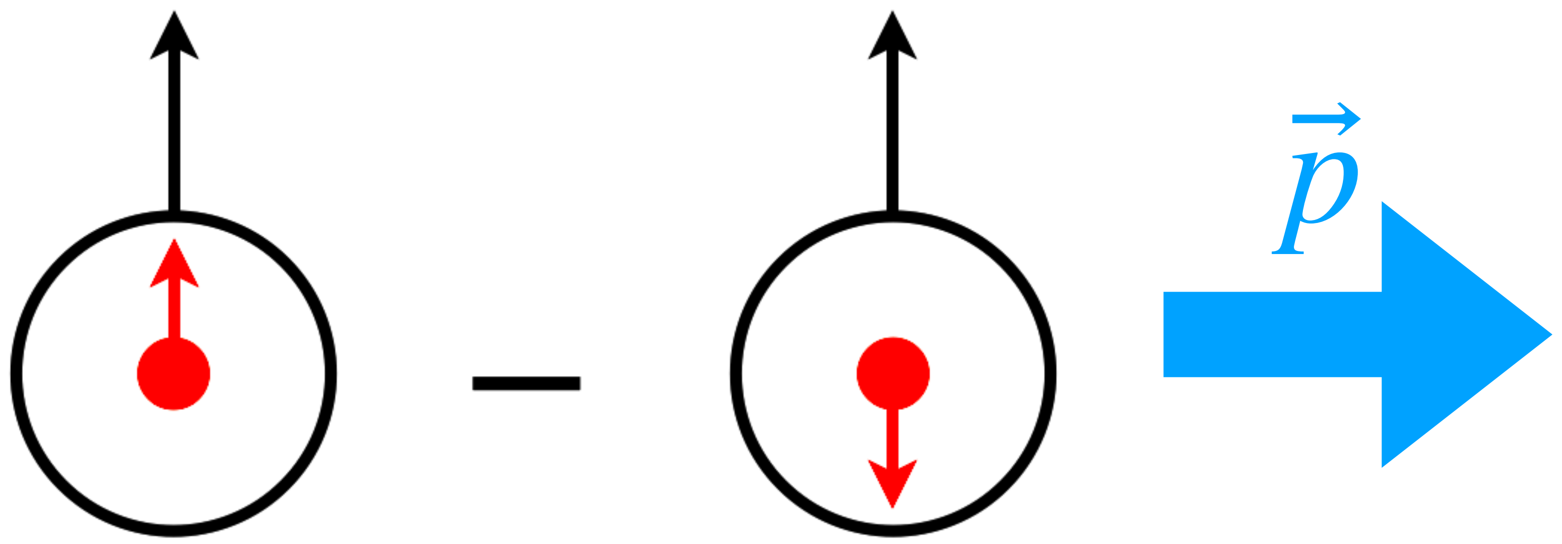
Global analysis: DIS, DY, vector boson production, single-inclusive jet production, W/Z bosons production, …

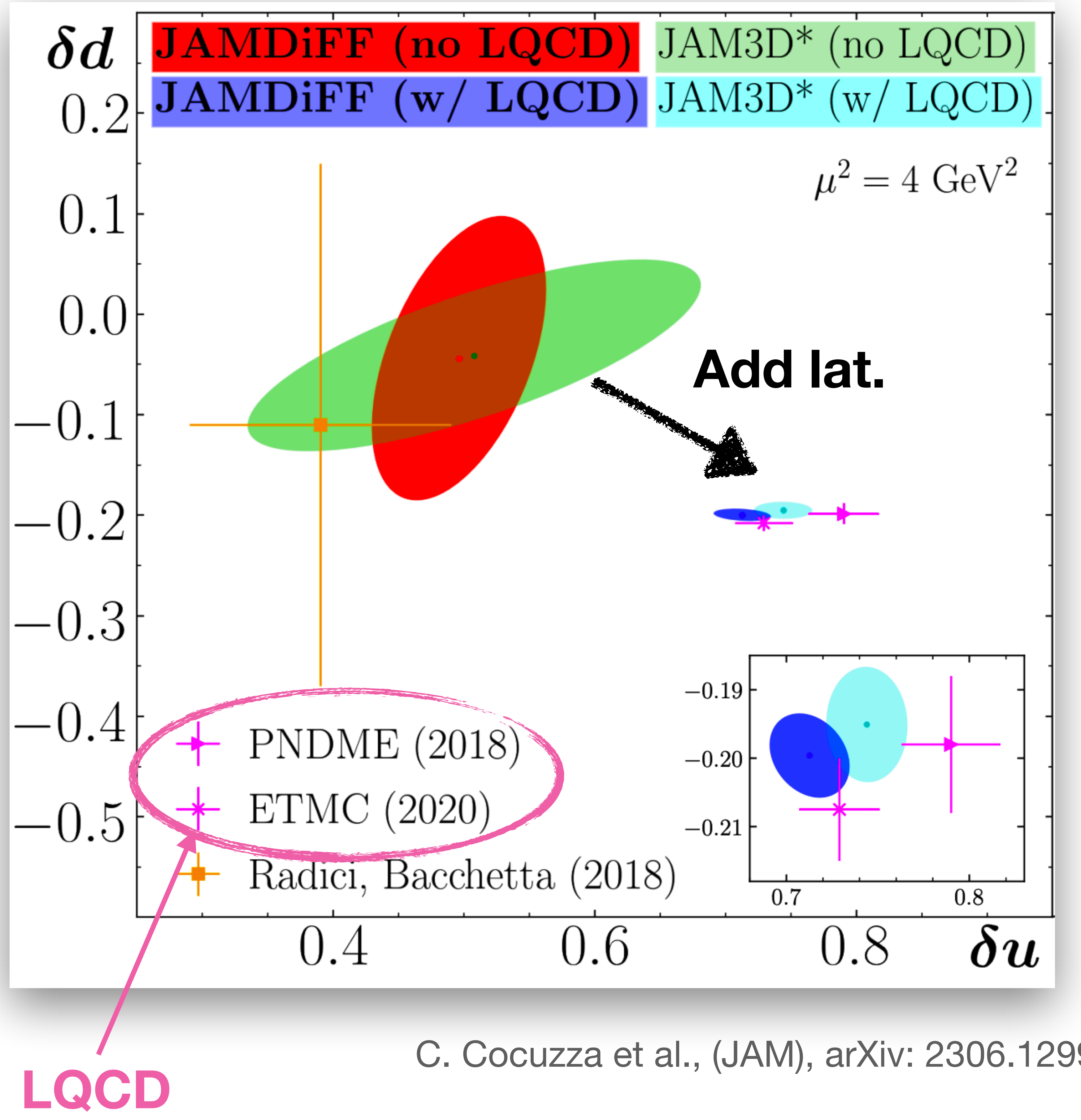
- During the last fifty years, significant progress has been made in understanding the nucleon's 1-D structure, e.g. nucleon unpolarized PDF.
- Towards understanding QCD and precision determination of SM background.



Parton distribution functions

- Polarized PDFs are critical to understand the nucleon spin structure.
- But less constraint from experiments, e.g. transversity PDF.

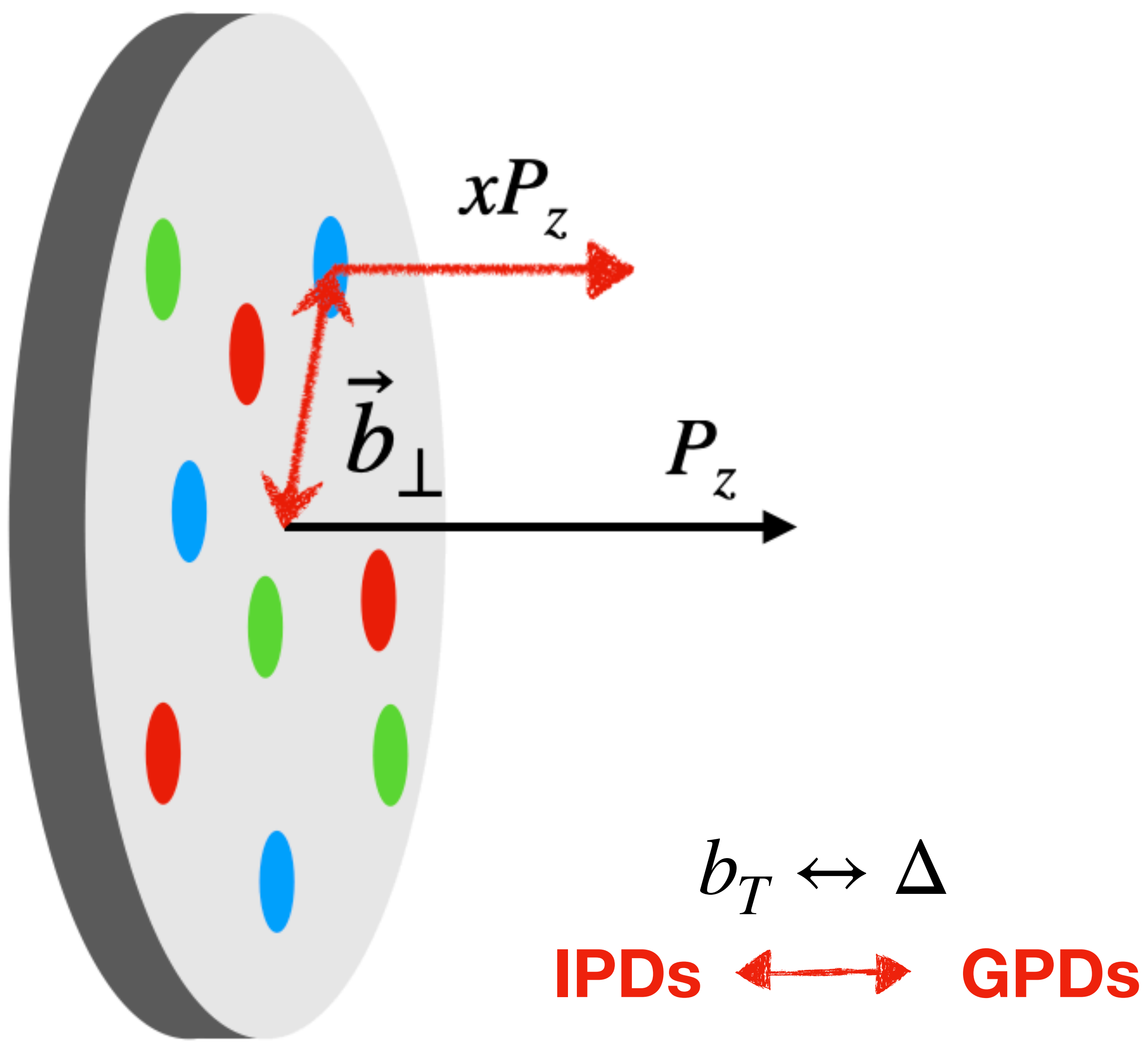




C. Cocuzza et al., (JAM), arXiv: 2306.12998

Generalized parton distributions

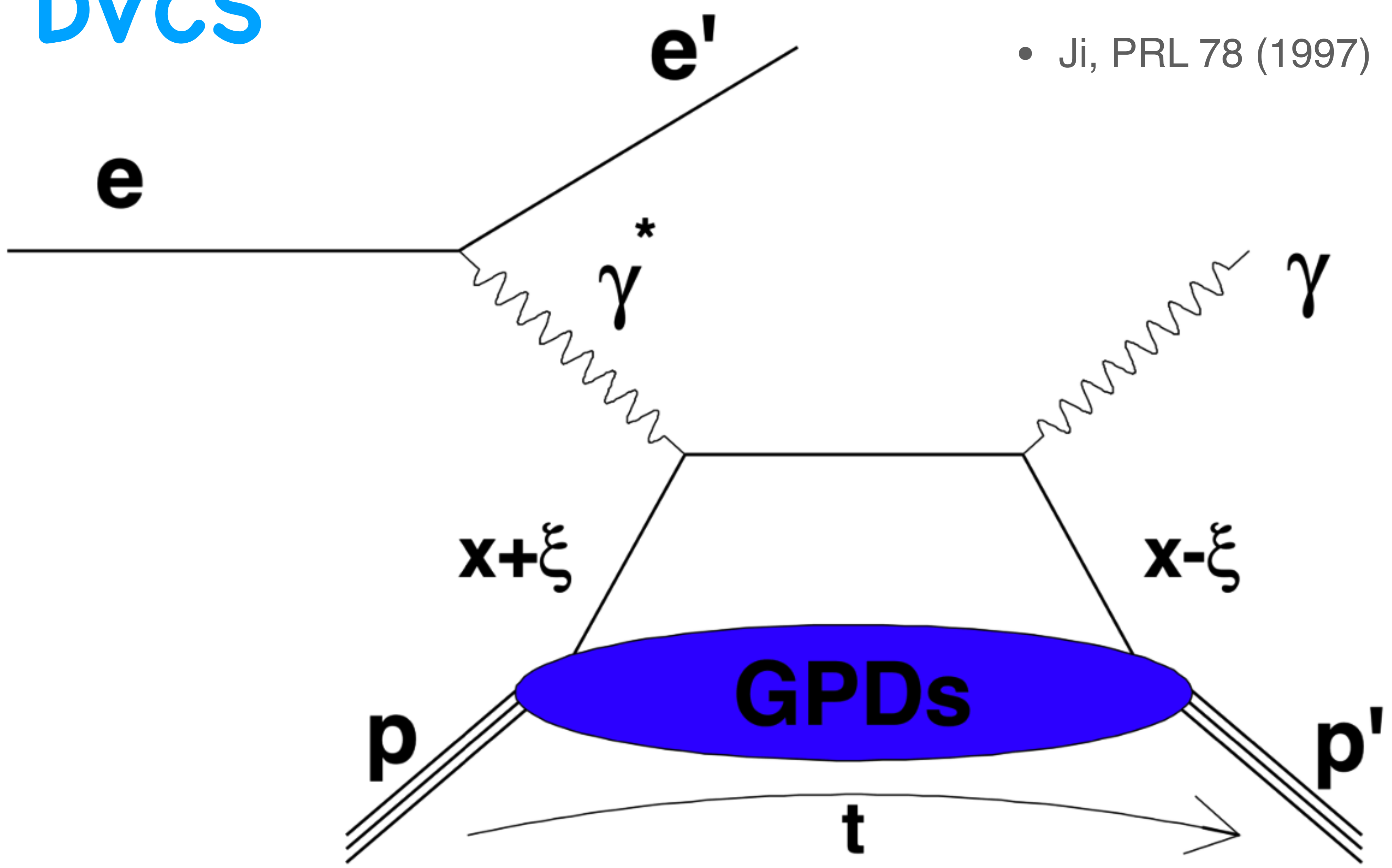
The GPDs provide a more comprehensive view of nucleon structure



- Offer insights into the 3D image of hadrons.
- Their 2nd moments $\langle x \rangle$ have a relation to the QCD EM tensor $T^{\mu\nu}$ and therefore the gravitational form factors (GFF).
- Provide information of the parton orbital motion and spin inside hadron, as well as the pressure and shear forces there.

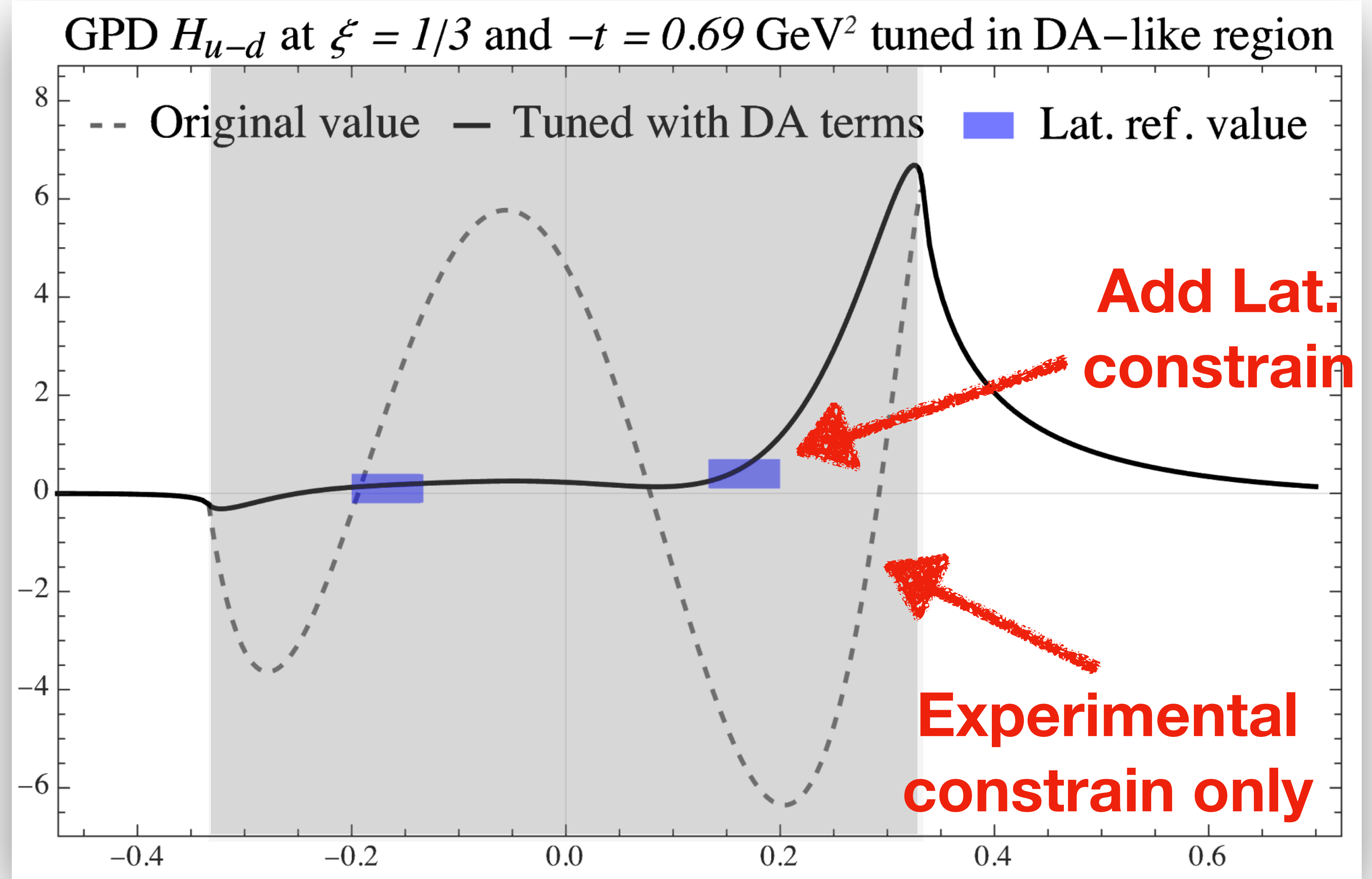
Generalized parton distributions

DVCS



• Ji, PRL 78 (1997)

• Y. Guo, et al., JHEP 05 (2023) 150



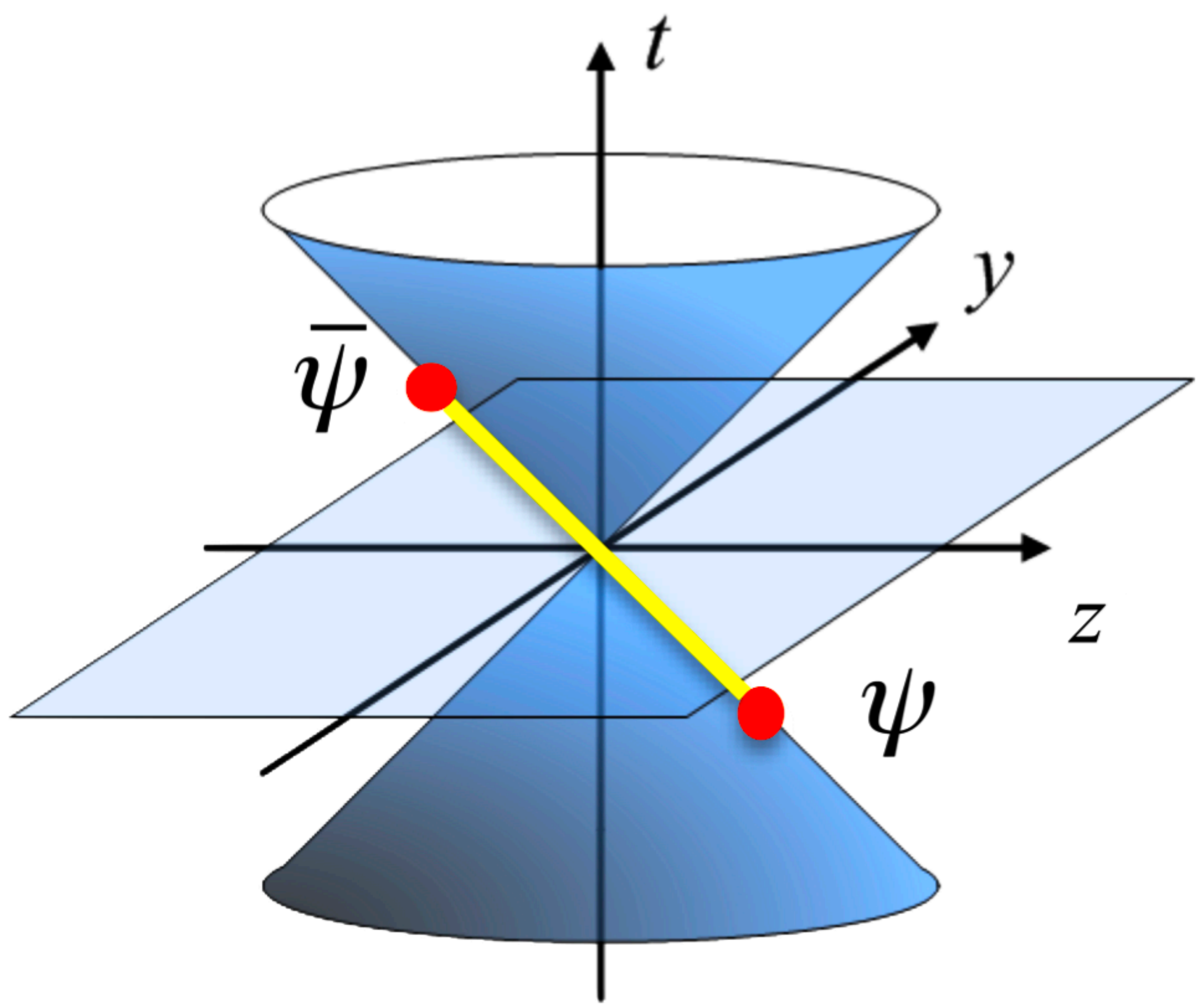
Challenging:

- Observables appear at the amplitude level.
- Multi-dimensionality $F(x, \xi, t)$.
- The momentum fraction x is integrated over (Compton Form Factors).

Hadron structure from lattice

- Moments from Local operator
  - Since the 1980s

$$\bar{q}\gamma^{\{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}q$$



$$\langle p_f|\,\bar{q}(-\frac{z^-}{2})\gamma^\mu \mathscr{W}(-\frac{z^-}{2},\frac{z^-}{2})q(\frac{z^-}{2})\,|p_i\rangle$$

Light-cone correlation: Cannot be calculated on the lattice

Moments from Local operator

Many significant progress such as the form factors (FF), …

- For review: FLAG Review Eur.Phys.J.C 82 (2022) 10, 869

- **Towards precision calculation of form factor ($Q^2 \lesssim 1\ \mathrm{GeV}^2$), magnetic moments and radius, …**

Konstantin Ottnad, Mon 2:50 PM

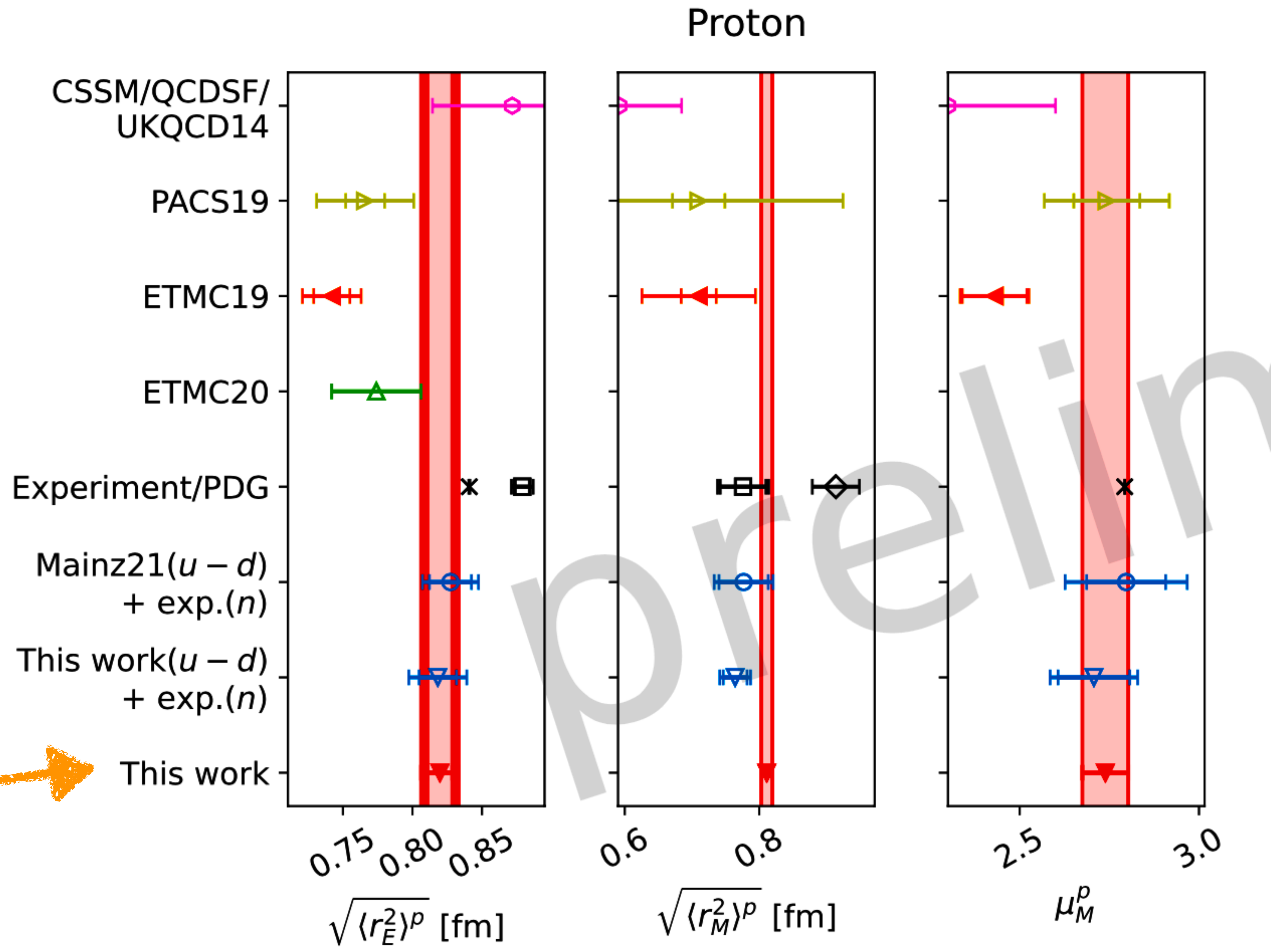
Kohei Sato, Mon 3:10 PM

Miguel Salg, Tue 1:30 PM

Ryutaro Tsuji, Tue 1:50 PM

Aaron Meyer, Tue 2:10 PM

Shigemi Ohta, Tue 2:30 PM



$1.5\ \%$ to $4\ \%$ precision

Moments from Local operator

Many significant progress such as the form factors (FF), …

- **Form factors at large momentum transfer $Q^2 \gg 1\ \mathrm{GeV}^2$.**

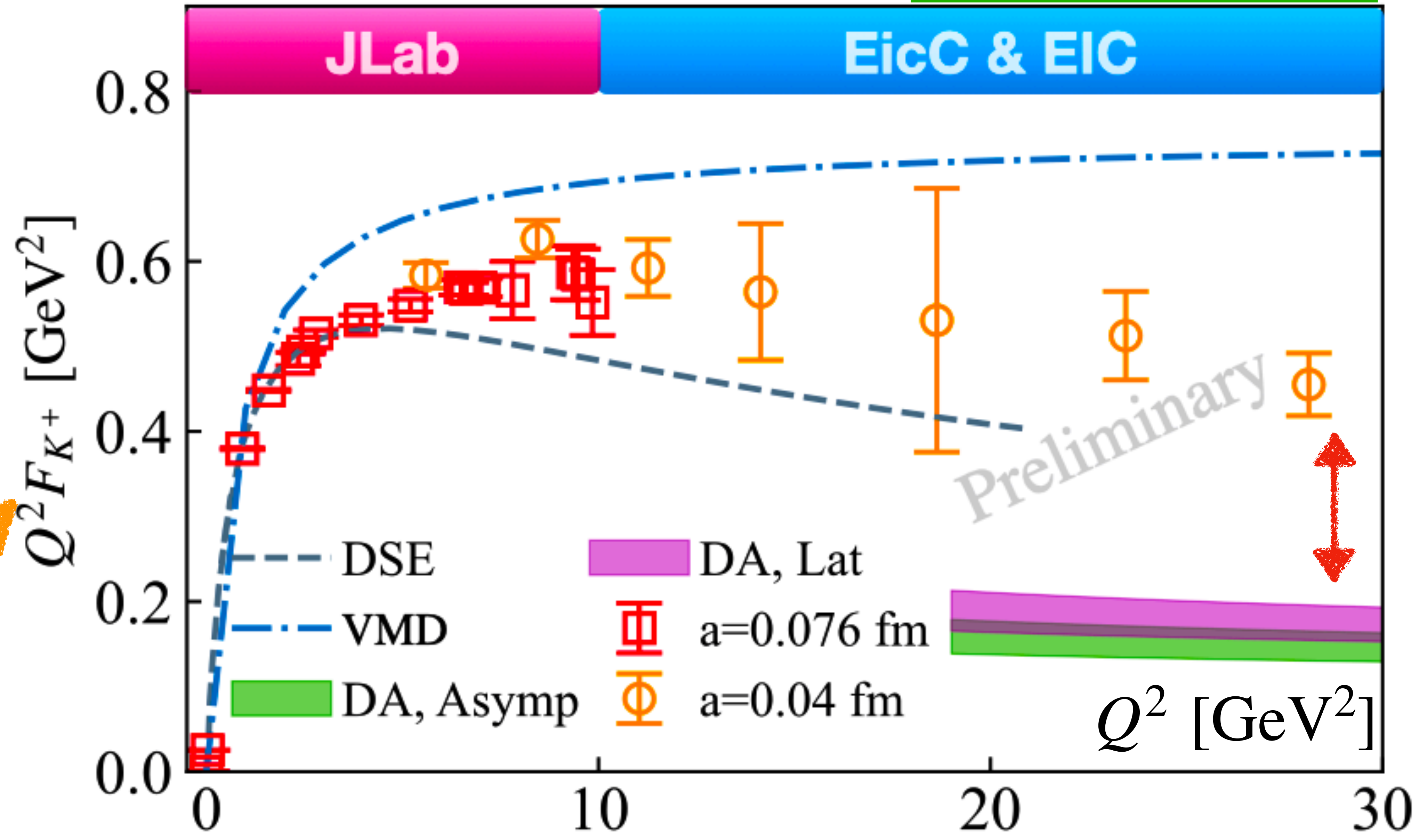
Sergey Syritsyn, Tue 2:50 PM

Qi Shi, Tue 3:10 PM

- ▶ **Provide the clearest opportunity to study the transition from non-perturbative to perturbative QCD.**

● Kaon FF

$m_\pi = 140$ MeV
$m_K = 495$ MeV



• Q. Shi, X. Gao, et al., (BNL+ANL), in preparation

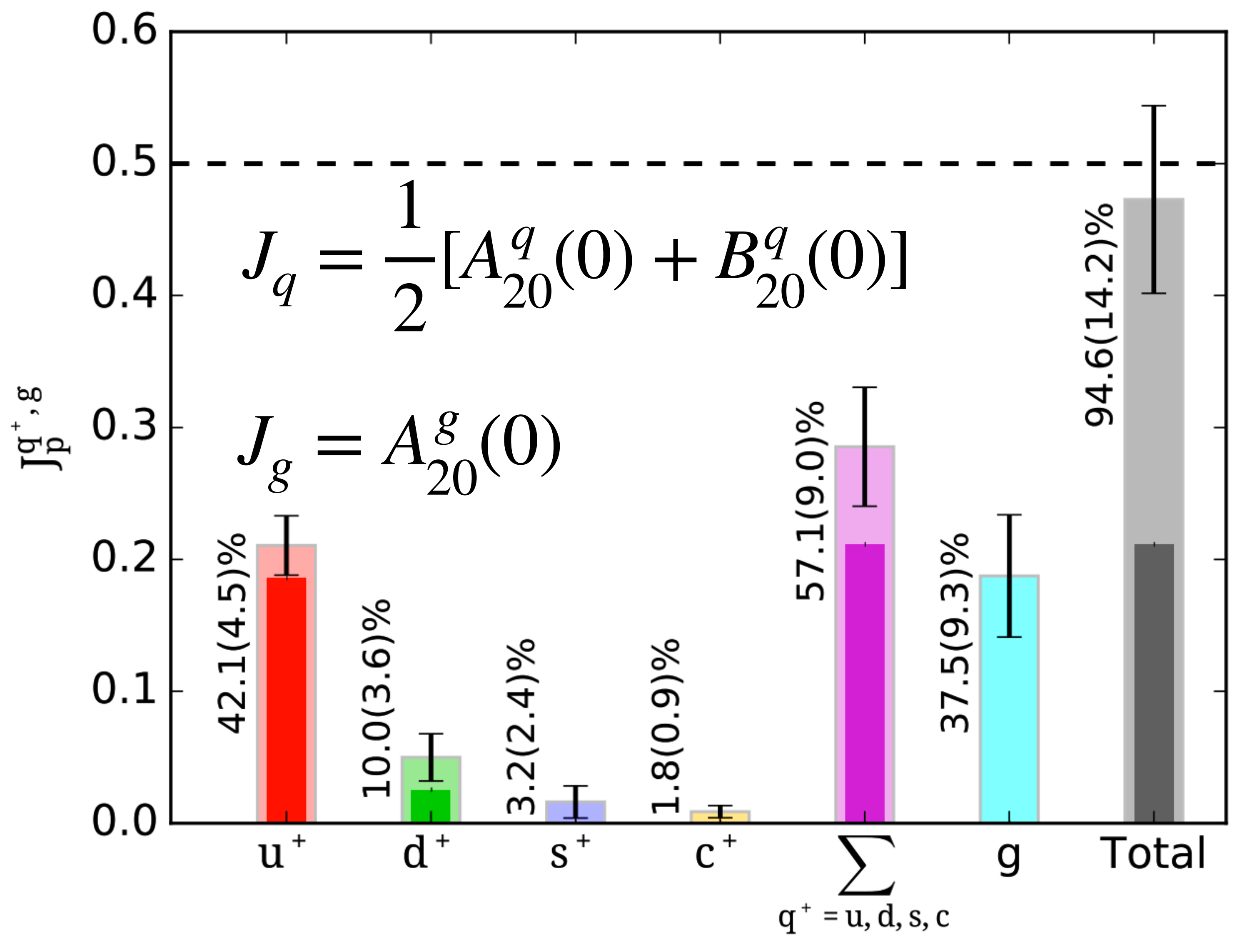
$$F_\pi(Q^2) \overset{Q^2 \to \infty}{=} \mathcal{N} \int_0^1 \int_0^1 dxdy\ \phi^*(v, \mu_F^2)$$

$$\times\, T_F^V(u, v, Q^2, \mu_R^2, \mu_F^2)\phi(u, \mu_F^2) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

Moments from Local operator

Progress also on 2nd moments, e.g., gravitational form factors (GFF)…

The nucleon spin decomposition from GFF



• ETMC, Phys.Rev.D 101 (2020)

- **GFF of π and K using twisted mass fermions**

Joseph Delmar, Wed 9:00 AM

- **GFF of π and nucleon with flavor decomposition.**

Dimitra Pefkou, Wed 9:20 AM

• D. Hackett, et al., (MIT), arXiv:2307.11707

- **Trace anomaly FF of pion and nucleon.**

Bigeng Wang, Wed 9:40 AM

For review:

- M. Constantinou, et al., Prog.Part.Nucl.Phys. 121 (2021) 103908
- V.D. Burkert, et al., arXiv:2303.08347

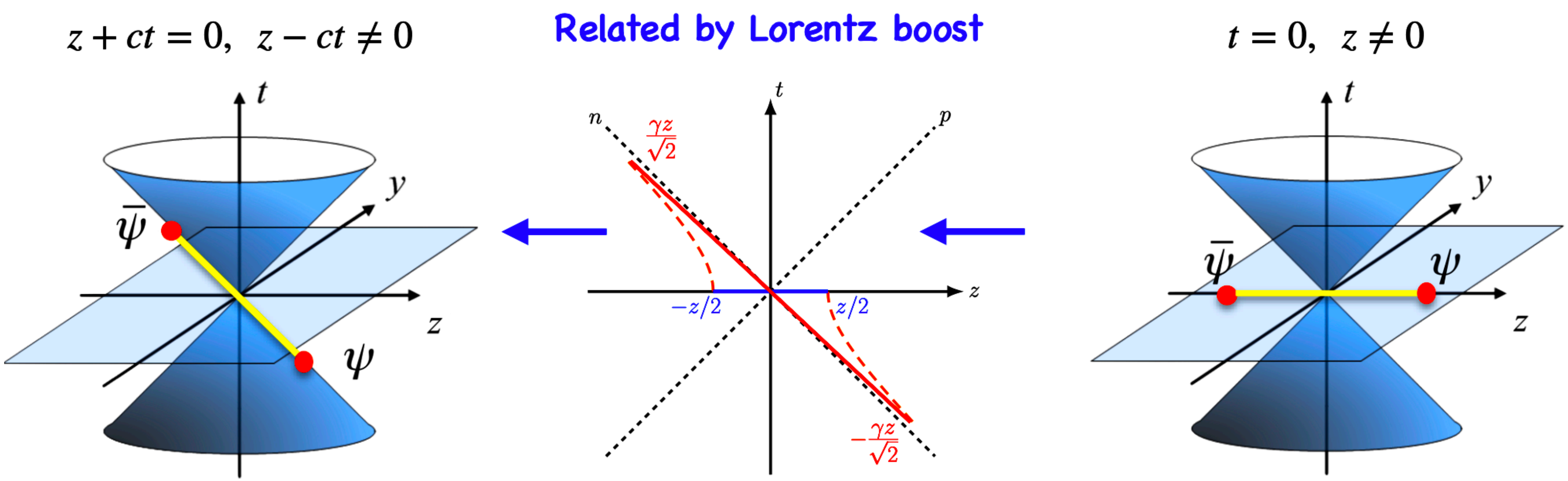
- Limited up to $\langle x^3 \rangle$ due to signal decay and power-divergent mixing under renormalization.

Large momentum effective theory

The quasi distribution from equal-time correlators,

- X. Ji, PRL 110 (2013); SCPMA57 (2014);

$$\tilde{q}(x, P_z, \mu) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P | \bar{q}(0) \Gamma \mathscr{W}(0,z) q(z) | P \rangle$$



Light-cone

$$\langle P | \bar{q}(0) \Gamma \mathscr{W}(0,z^-) q(z^-) | P \rangle$$

Quasi

$$\langle P | \bar{q}(0) \Gamma \mathscr{W}(0,z) q(z) | P \rangle$$

Large momentum effective theory

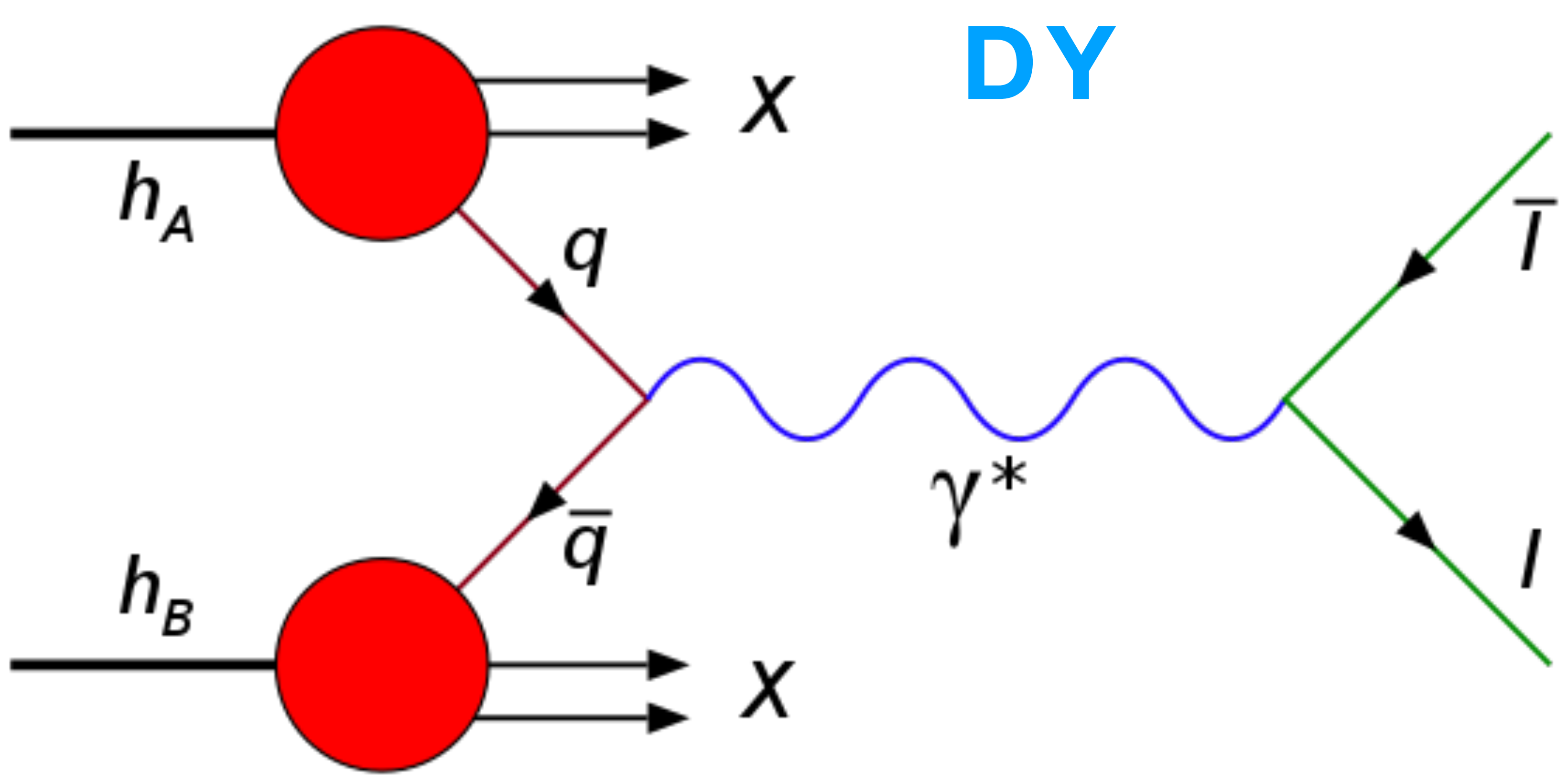
- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, 90 PRD (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).

Large P_z expansion of quasi distribution:

large P_z is the key

$$\tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

- Similar to DIS, DY, ..., LaMET has direct sensitivity to the local $x \in [x_{\min}, x_{\max}]$ dependence instead of the moments.



$$\frac{d\sigma}{dQ^2} = \sigma_0 \sum_{a,b} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} q_a^A(x_1, \mu^2)\ q_b^B(x_2, \mu^2) \omega_{ab}(z, \frac{\mu}{Q})$$

Short distance factorization

SDF/OPE in z: Ioffe-time pseudo distributions

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

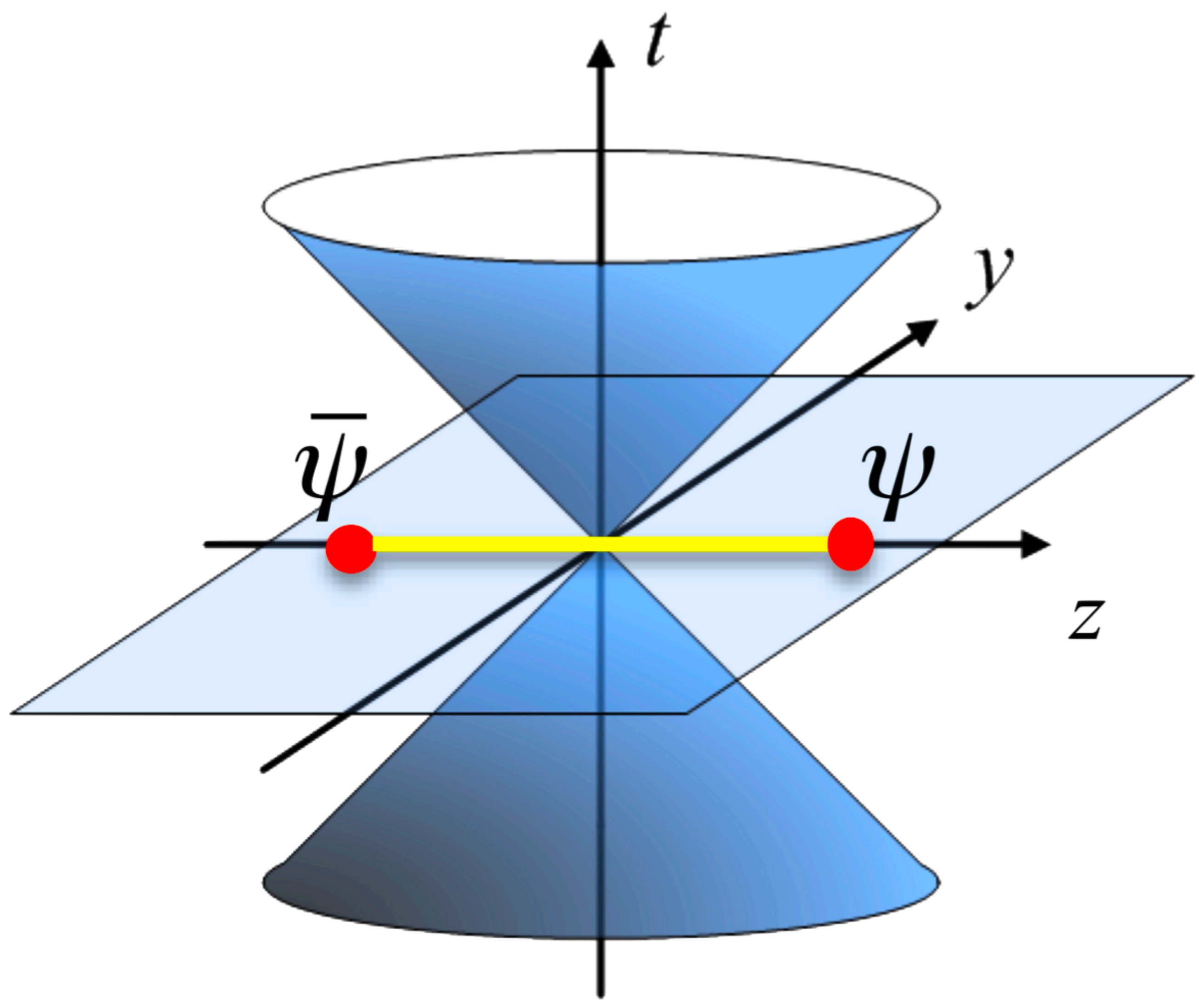
$t = 0, \quad z \neq 0$

$$\tilde{h}(z, P_z, \mu) = \tilde{h}(z^2, \lambda, \mu) \qquad \lambda = zP_z$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2)\langle x^n\rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$

Sum over the moments

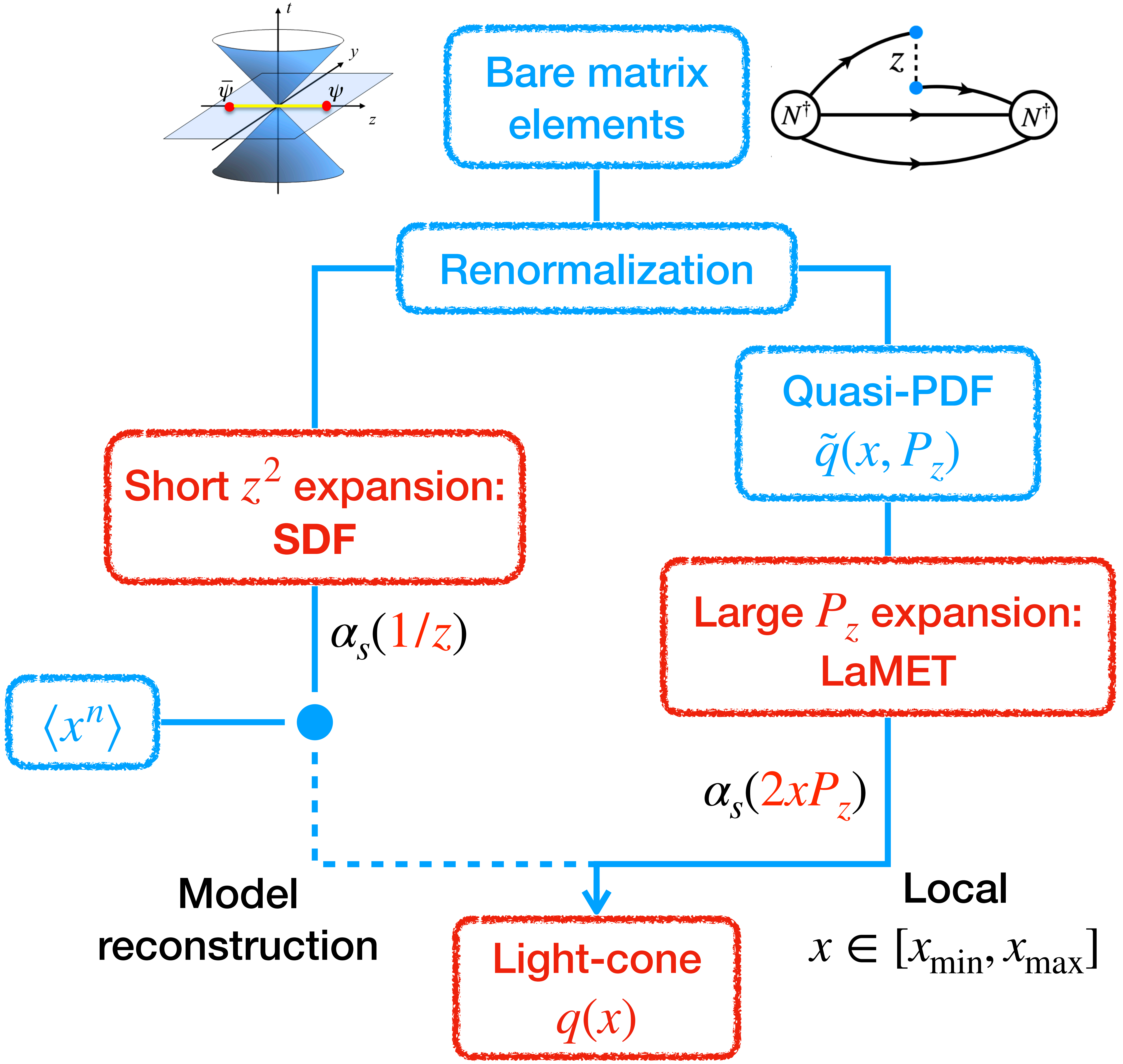
$$= \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu) + \mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$



$$\langle P | \bar{q}(0)\Gamma\mathscr{W}(0,z)q(z) | P\rangle$$

- In principle can get access to the higher moments without power divergent mixing.
- In practice the information is limited by the range of finite $\lambda = zP_z$, large P_z essential.

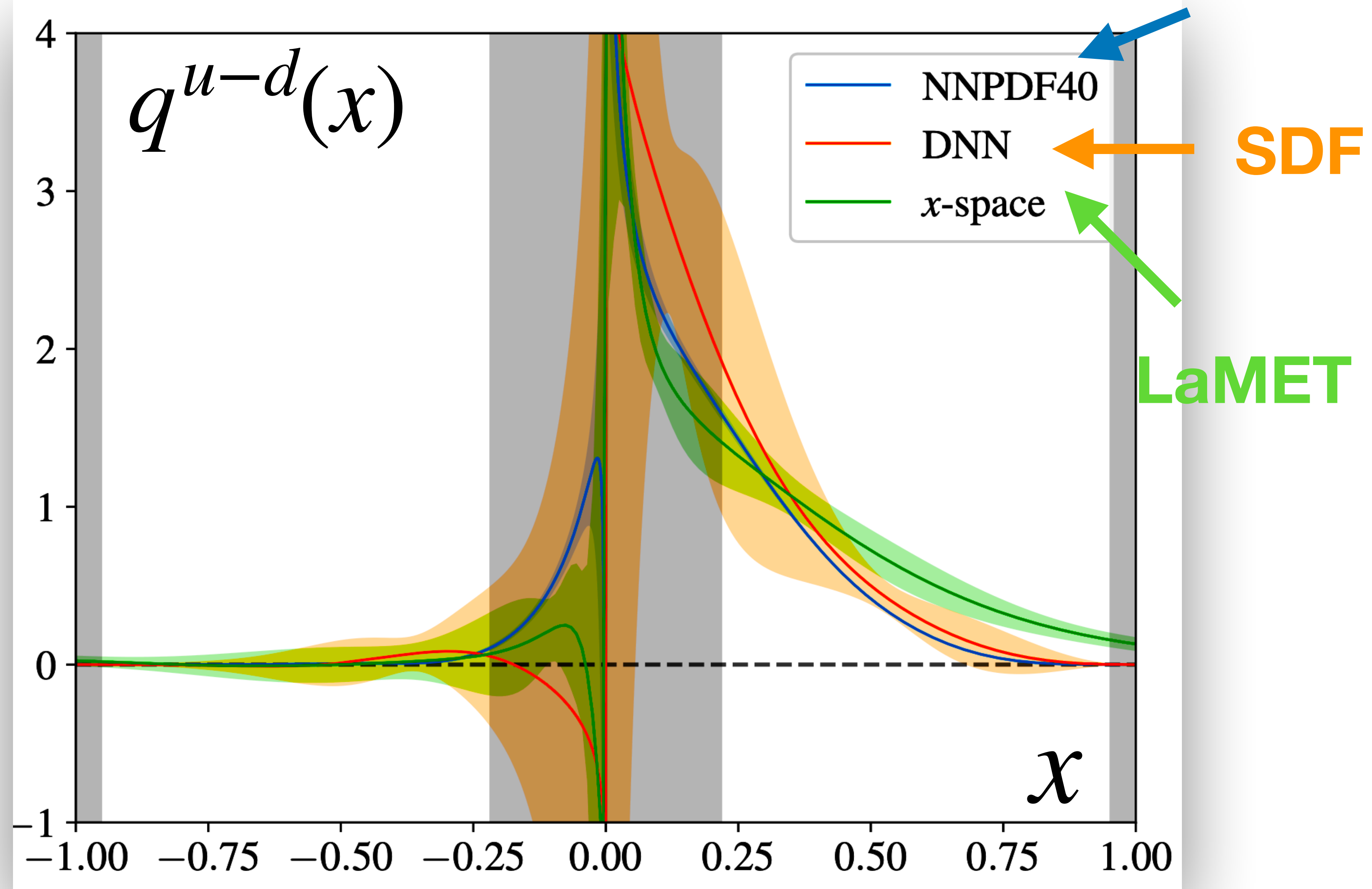
x-dependent parton distributions



- **What we can compute?**
  - PDFs, GPDs, TMDs …
- **How well we can compute?**
  - Data precision.
  - Theory/Analysis systematics.

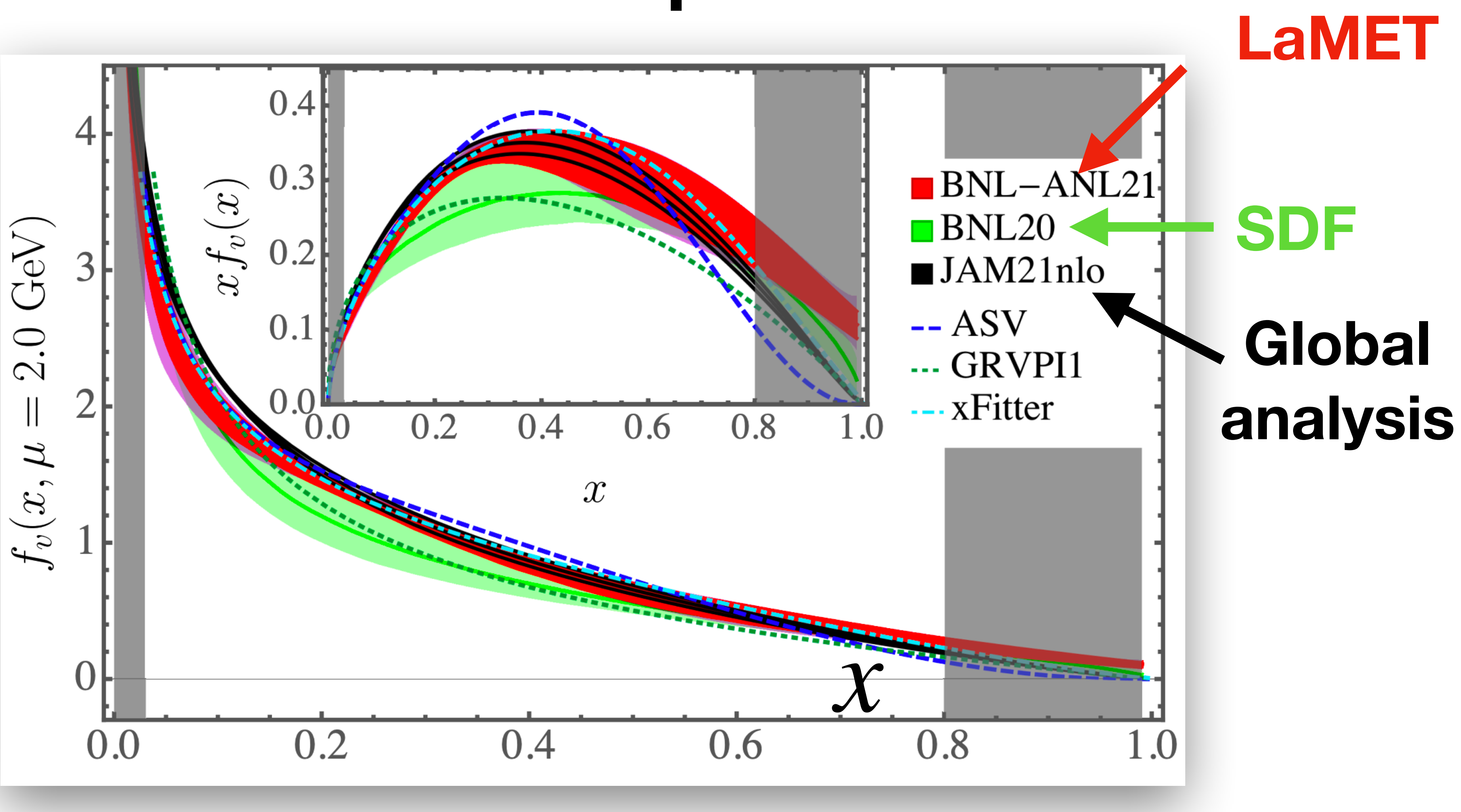
Unpolarized quark PDFs

Nucleon unpolarized PDF



- X. Gao, et al., (BNL+ANL), Phys.Rev.D 107 (2023), 074509

Pion valence quark PDF



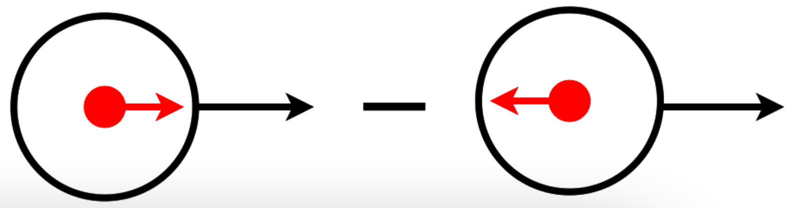
- X. Gao, et al., (BNL+ANL), Phys.Rev.Lett. 128 (2022), 142003

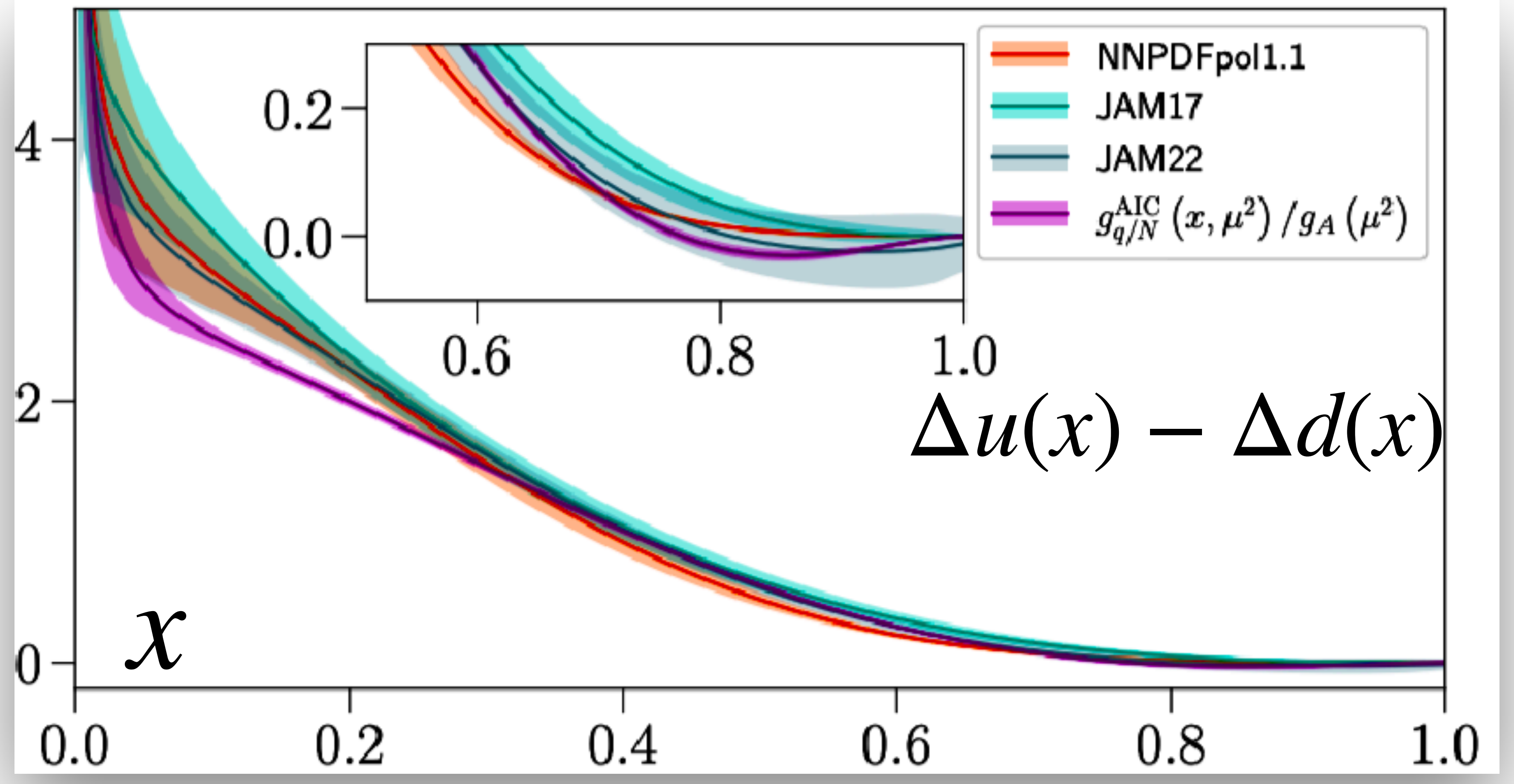
- Unpolarized PDF should be a benchmark.
- Though there are some agreement, lattice results show larger uncertainty and require more precise data and larger momentum.

- Pion has better signal and is easier to approach light-cone with light mass.
- Good agreement with global analysis.

Polarized quark PDFs

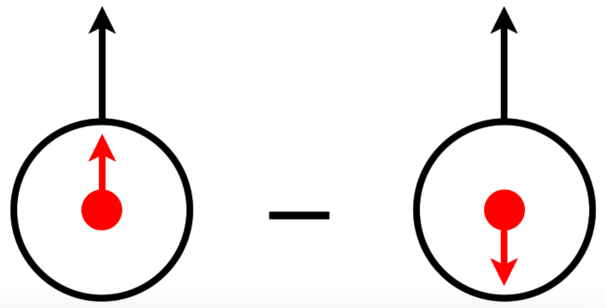
Helicity

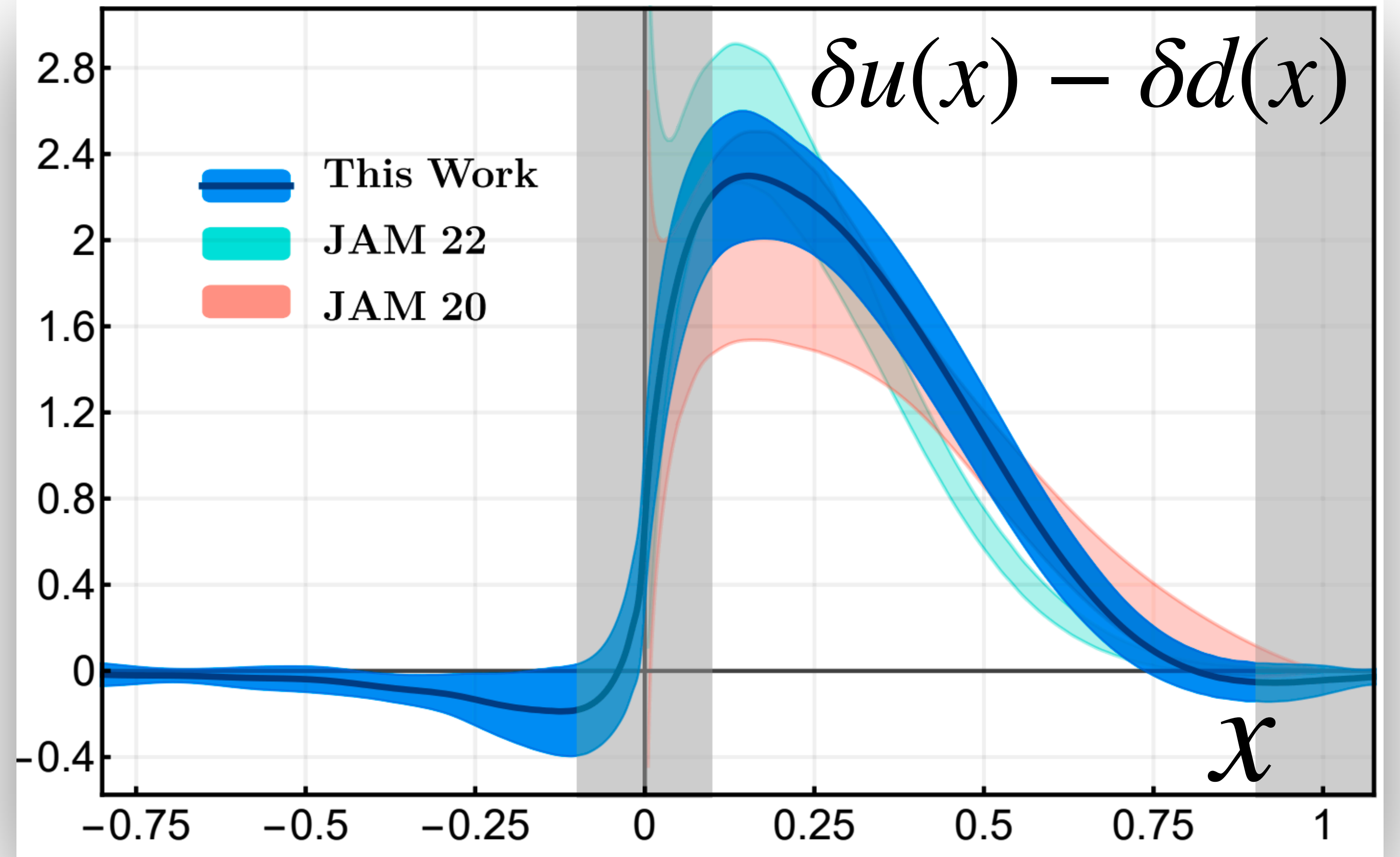




• R. Edwards., (HadStruc Collaboration), JHEP 03 (2023) 086

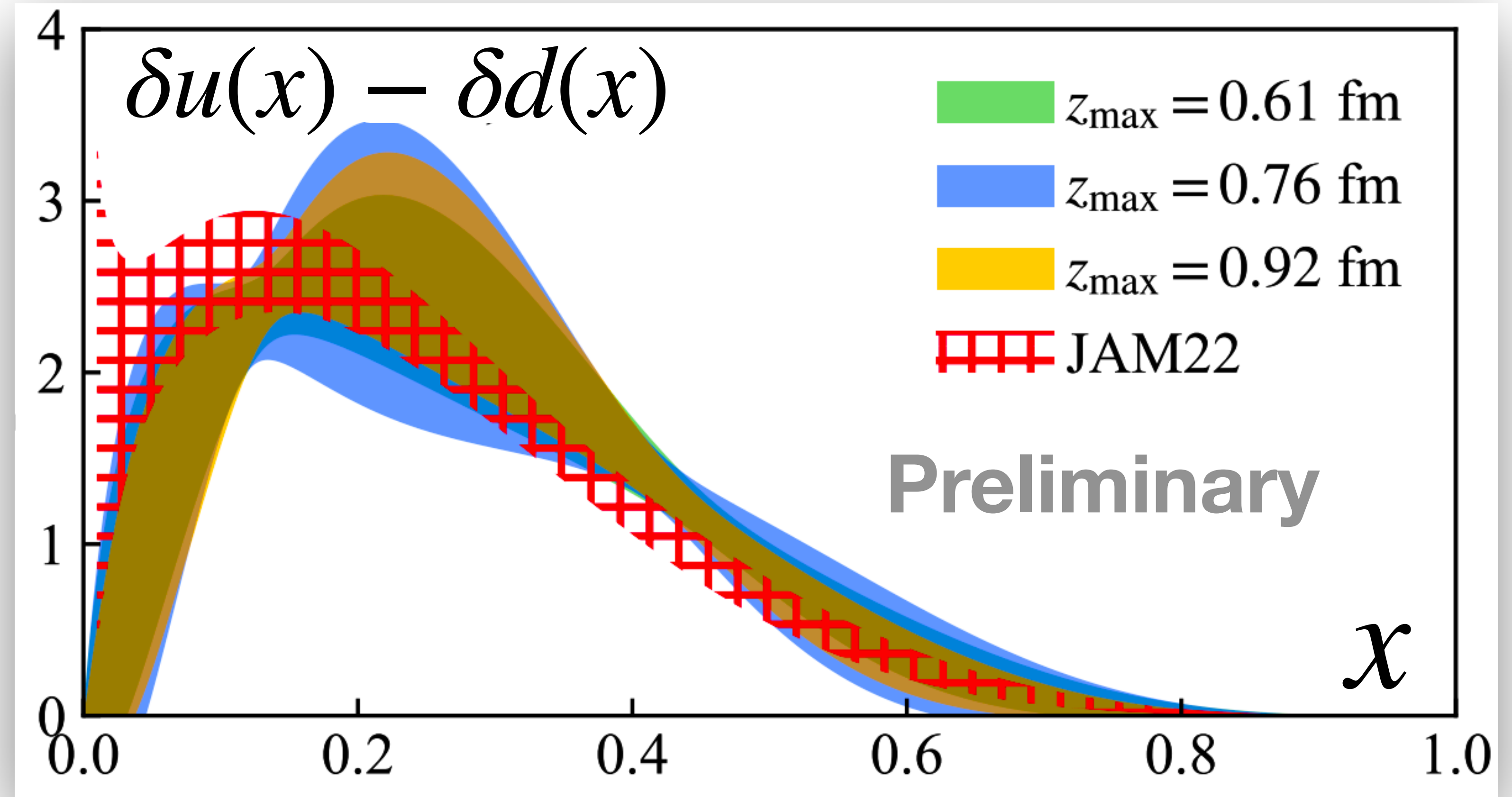
Transversity





• F. Yao, et al., (LPC), arXiv: 2208.08008

Transversity



• X. Gao, et al., (BNL+ANL), in preparation.

- Polarized PDFs (helicity, transversity) are also less constrained from experiments.
- Global analysis of transversity (JAM) used the tensor charge from lattice.
- New Lattice calculation could provide more complementary information.

Gluon distribution

- Gluons are the mediator bosons of the strong interaction, play a key role in the nucleon's mass and spin.
- Difficult to measure compared to quark, especially the gluon helicity.
- Pioneering attempts from lattice are in progress.

- **Renormalization of non-local gluon operator:**

  Demetrianos Gavriel, Thu 5:00 PM

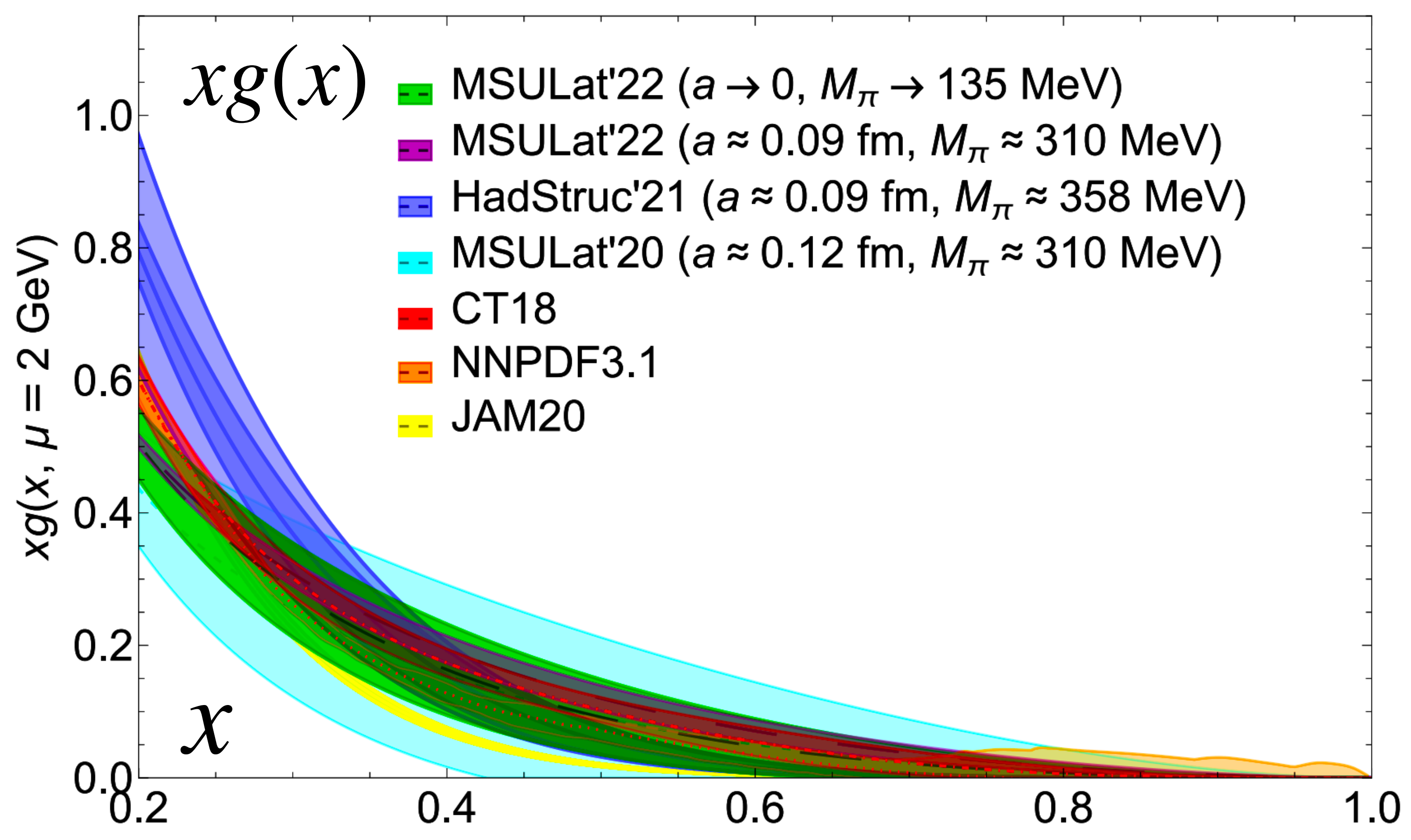
- **Updates on gluon distributions of π and K.**

  Huey-Wen Lin, Tue 4:20 PM

- **Effects of smearing on gluon matrix elements**

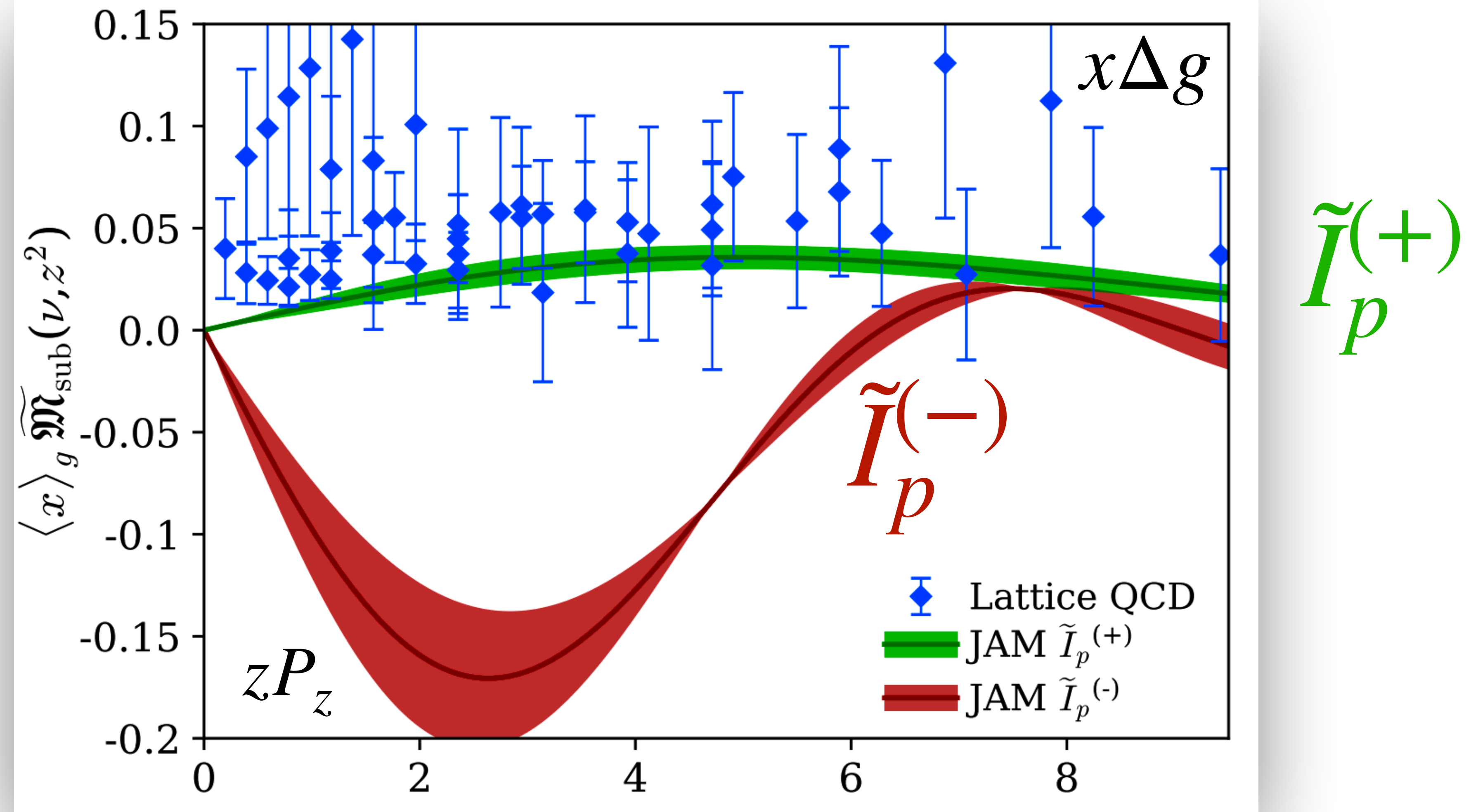
  William Good, Thu 5:20 PM

Unpolarized



- Z. Fan, et al., (MSU), Phys.Rev.D 108 (2023), 014508

Helicity



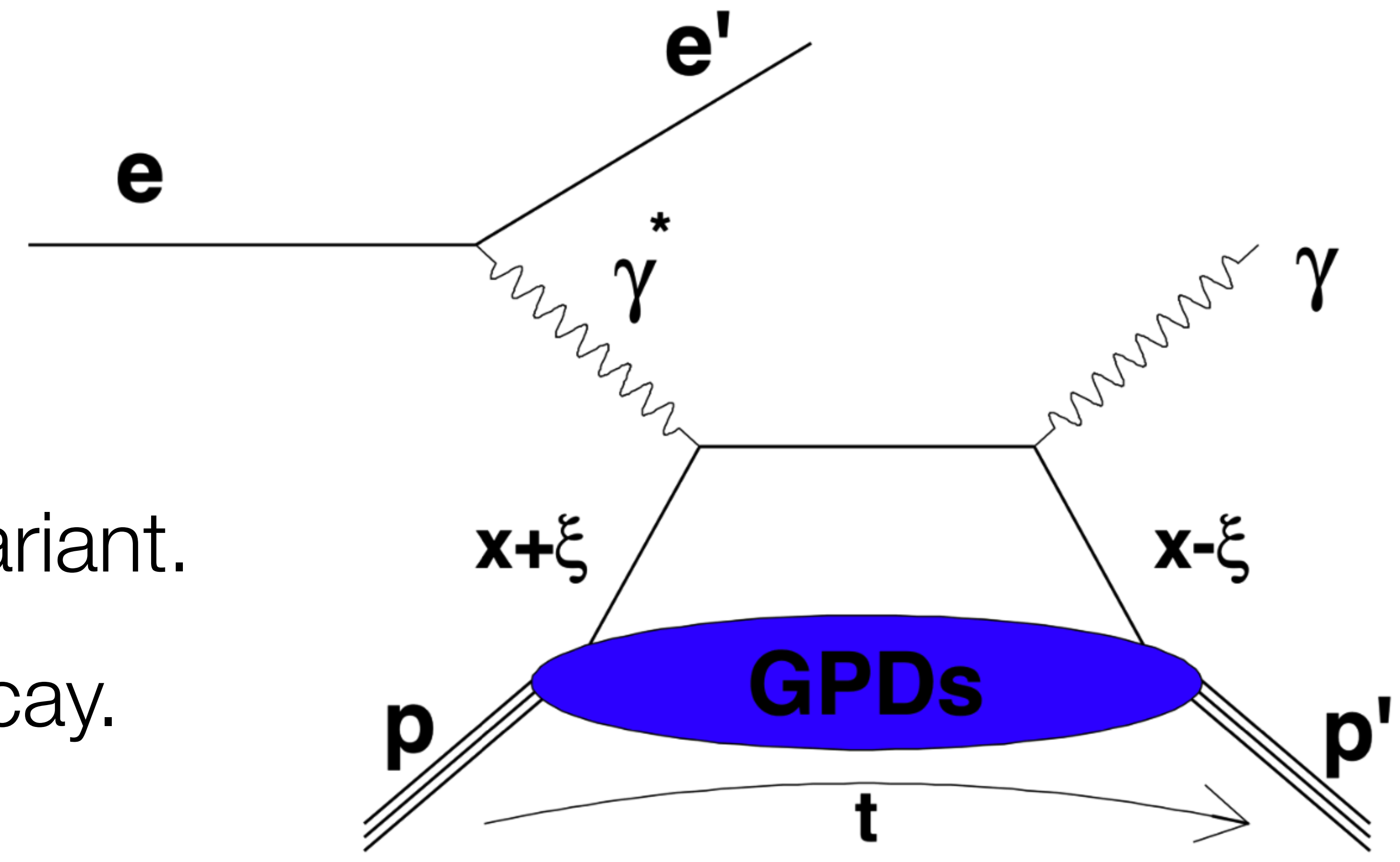
- C. Egerer, et al., (HadStruc Collaboration), Phys.Rev.D 106 (2022), 094511

Generalized parton distributions

- Quasi-GPDs are F.T. of off-forward matrix elements:

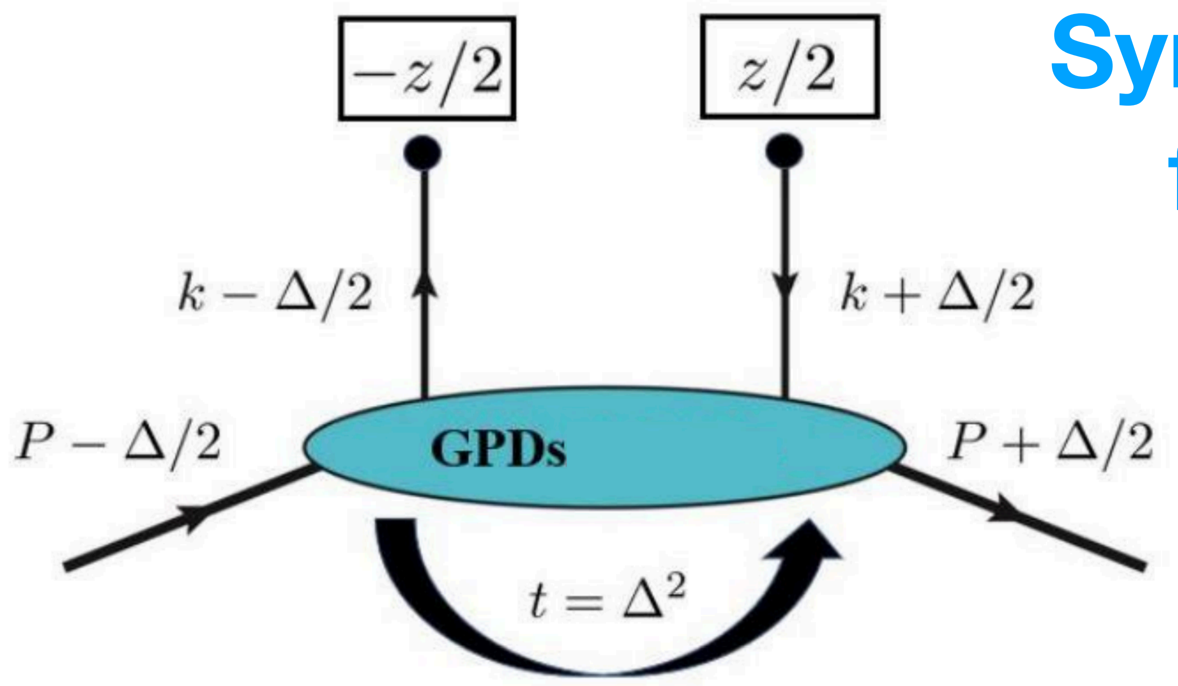
$$F_{q/g}(x,\xi,t)=\int\frac{dz}{4\pi}e^{-ixPz}\langle p_f|\,\bar{q}(-\tfrac{z}{2})\gamma^{\mu}\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})\,|p_i\rangle$$

- The qGPDs are frame dependent though light-cone GPDs are Lorentz invariant.
- Computationally expensive for multiple t in symmetric frame and signal decay.



$F_{q/g}(x,\xi,t)$

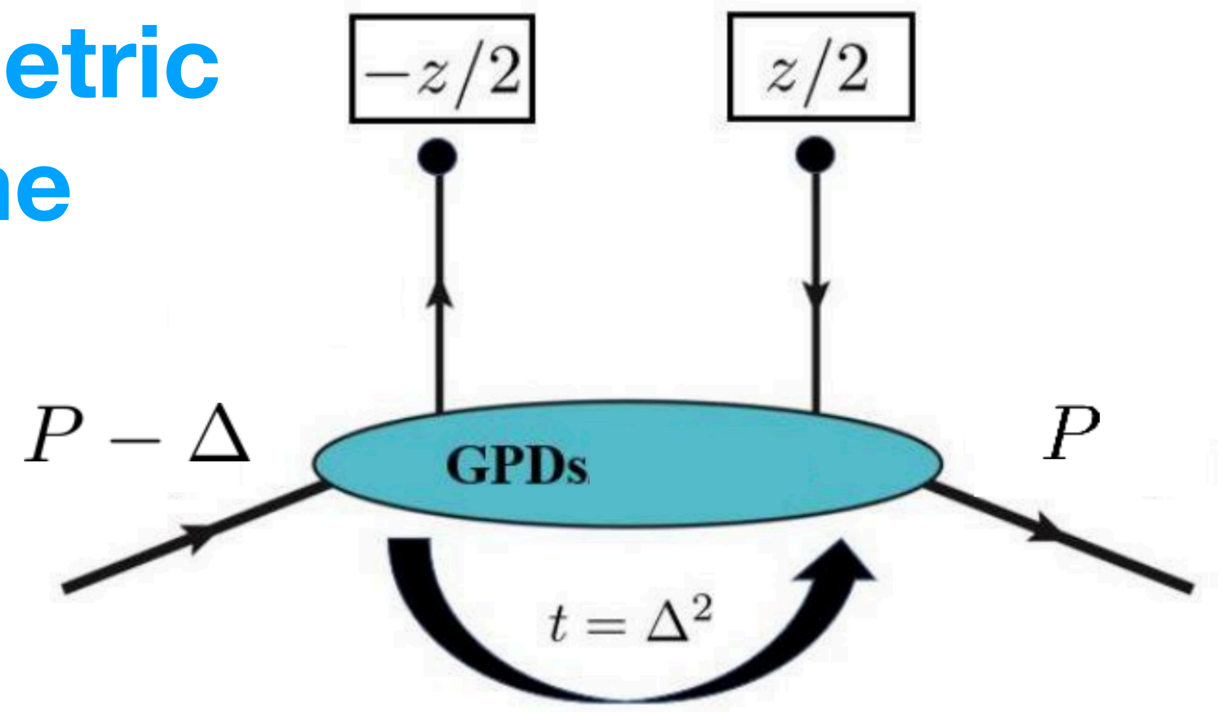
New development: **Shohini Bhattacharya, Thu 1:30 PM**



Symmetric frame

Lorentz transform

Asymmetric frame

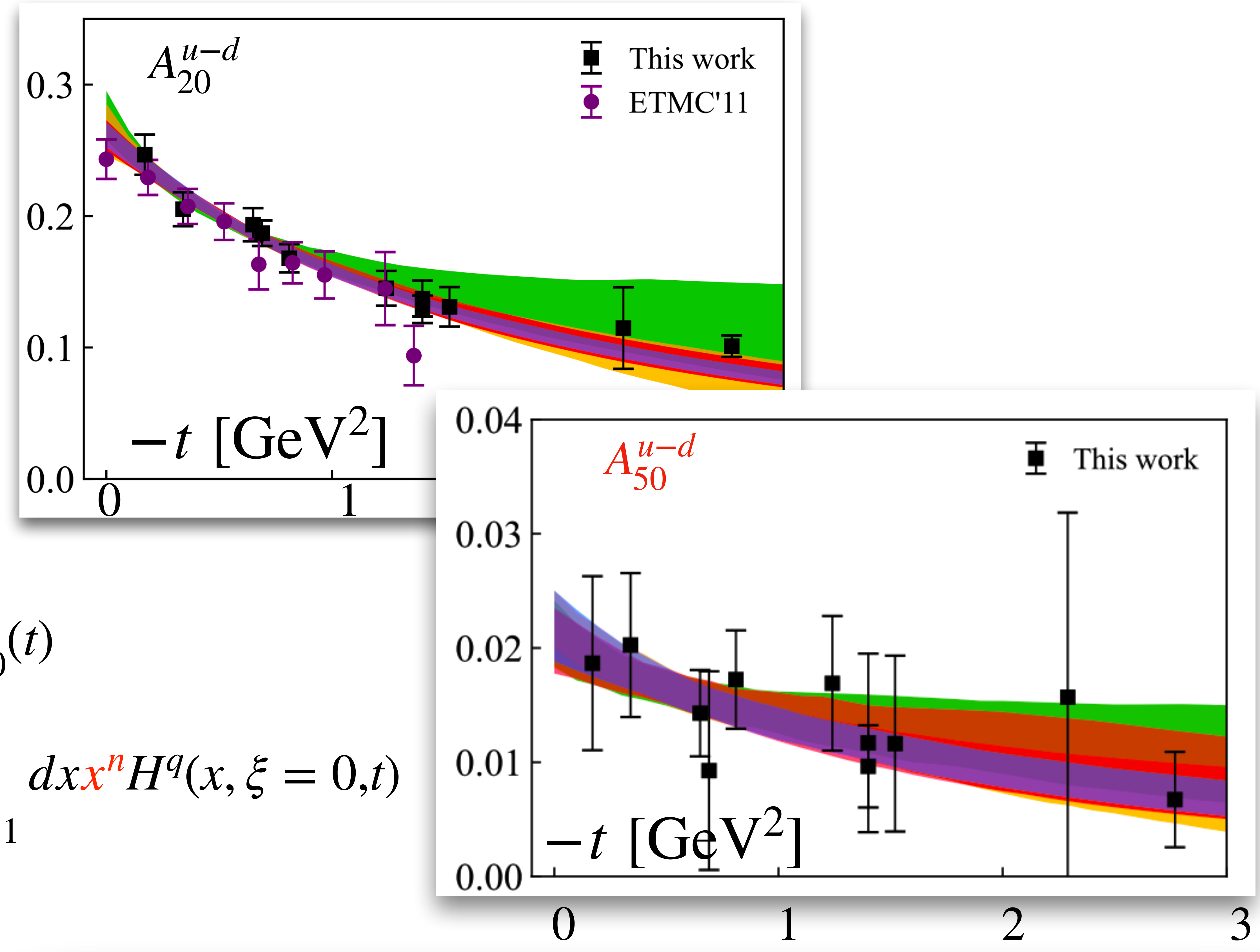


- Construct qGPD in symmetric frame from asymmetric calculation.
- Computational much cheaper for multiple $-t$, and possibly reducing the power corrections.

• S. Bhattacharya, X. Gao, et al., Phys.Rev.D 106 (2022), 114512

Moments of GPDs

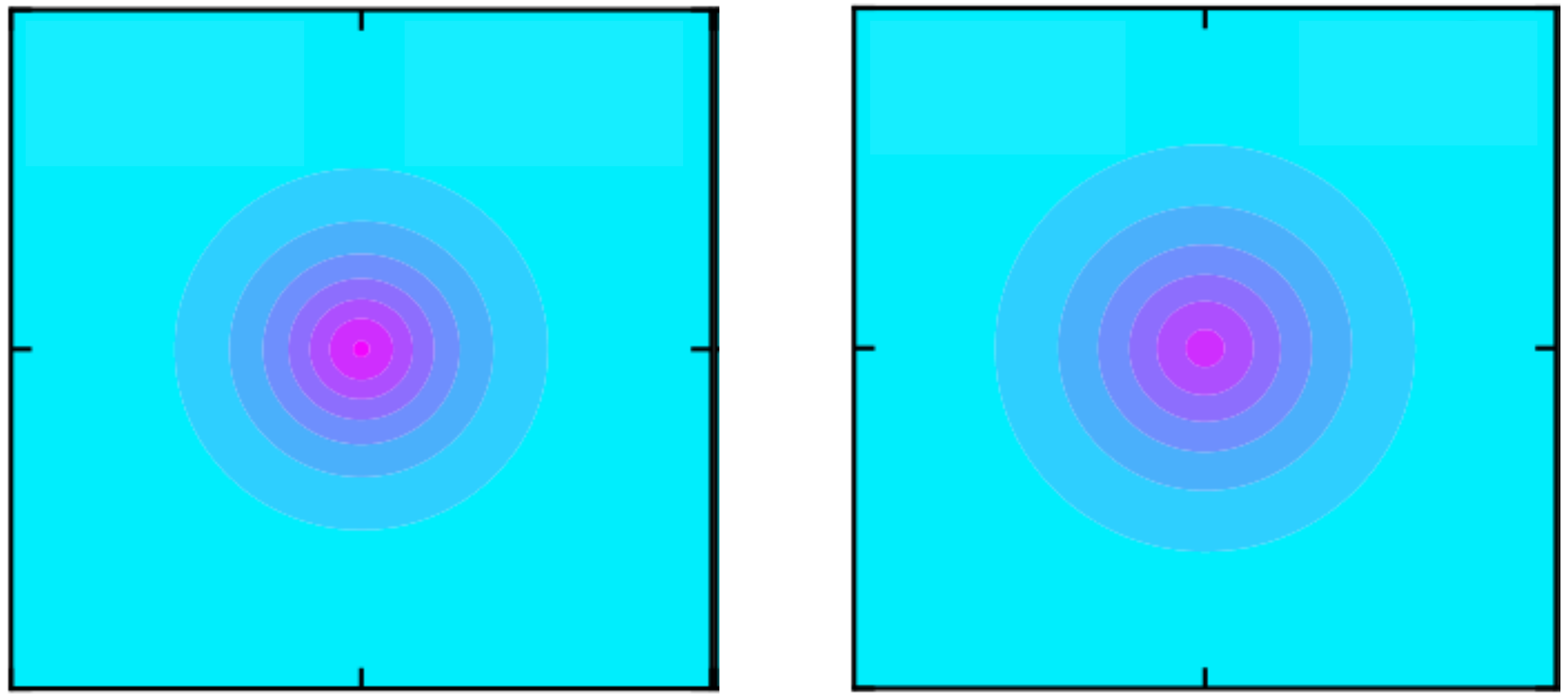
Moments as a function of t



$$A^q_{n+1,0}(t) = \int_{-1}^{1} dx x^n H^q(x, \xi = 0, t)$$

- 2nd moments from SDF are consistent with local operator calculations, and extend up to 5th moments for the first time.

Momentum density in transverse plane



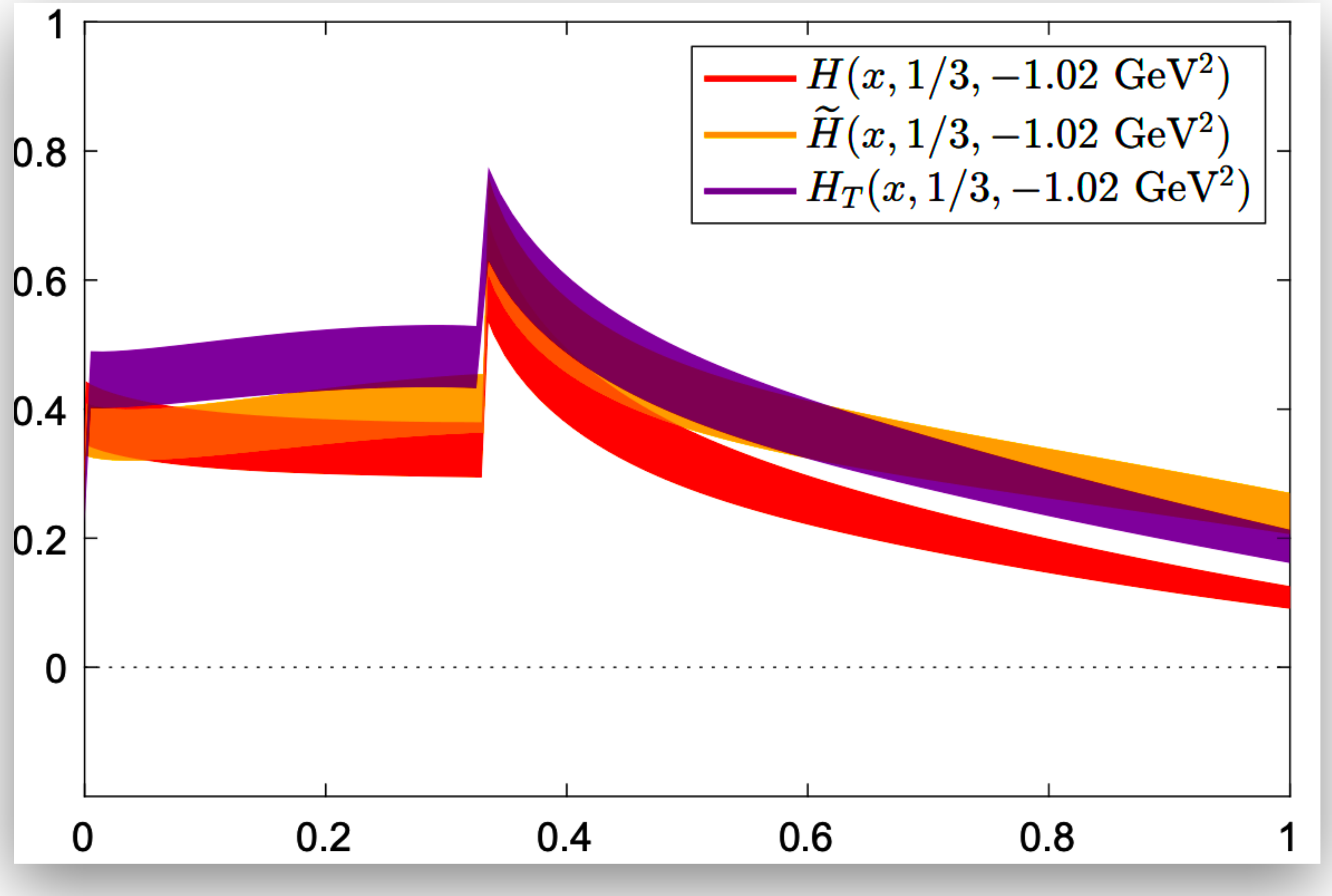
u quark $\qquad$ d quark

- F.T. over t provide the density of quarks in the impact parameter plane.

• S. Bhattacharya, X. Gao, et al., Phys.Rev.D 108 (2023) 1, 014507

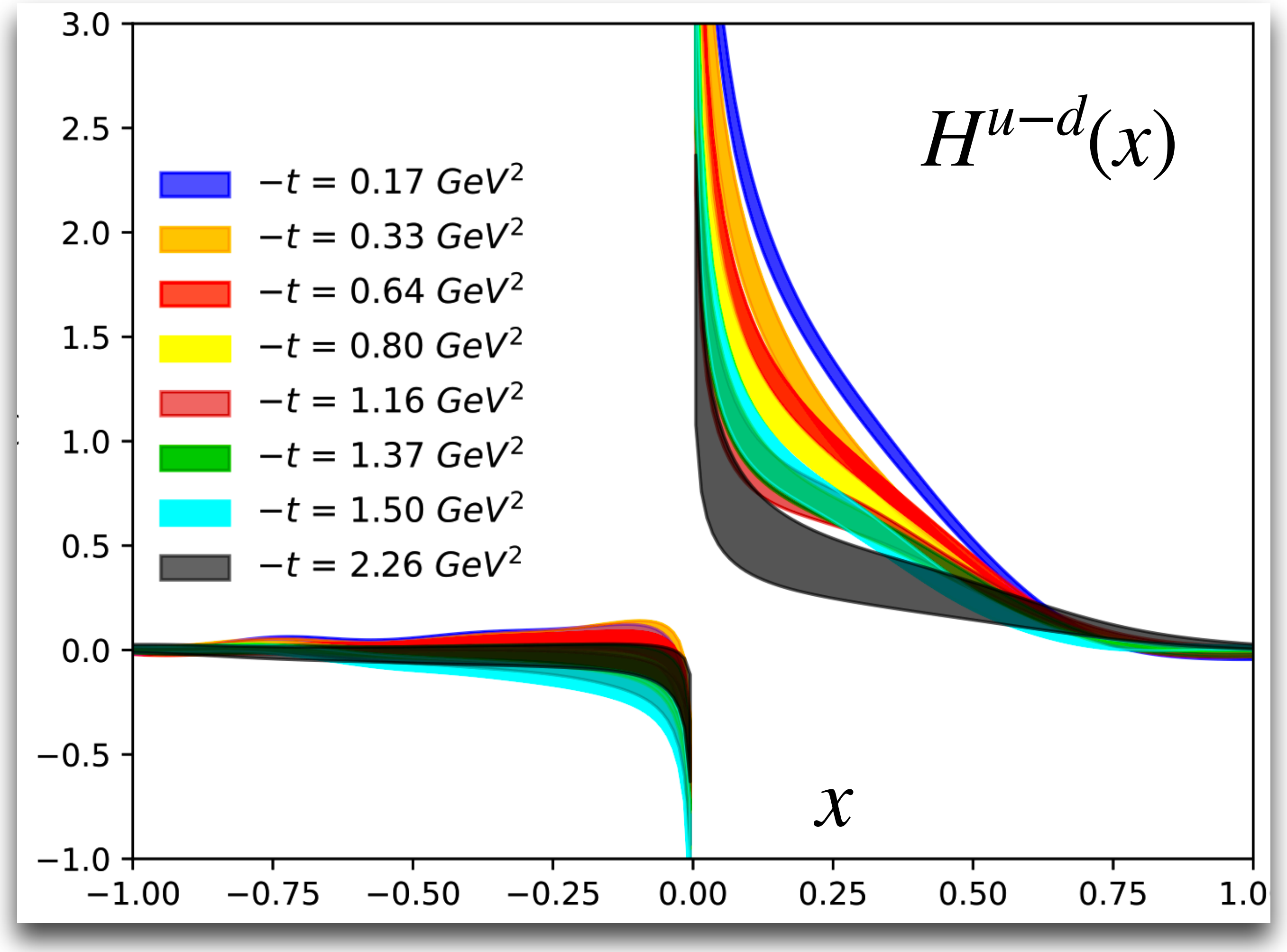
x dependent GPDs

GPDs from symmetric frame:
Unpolarized, helicity, transversity



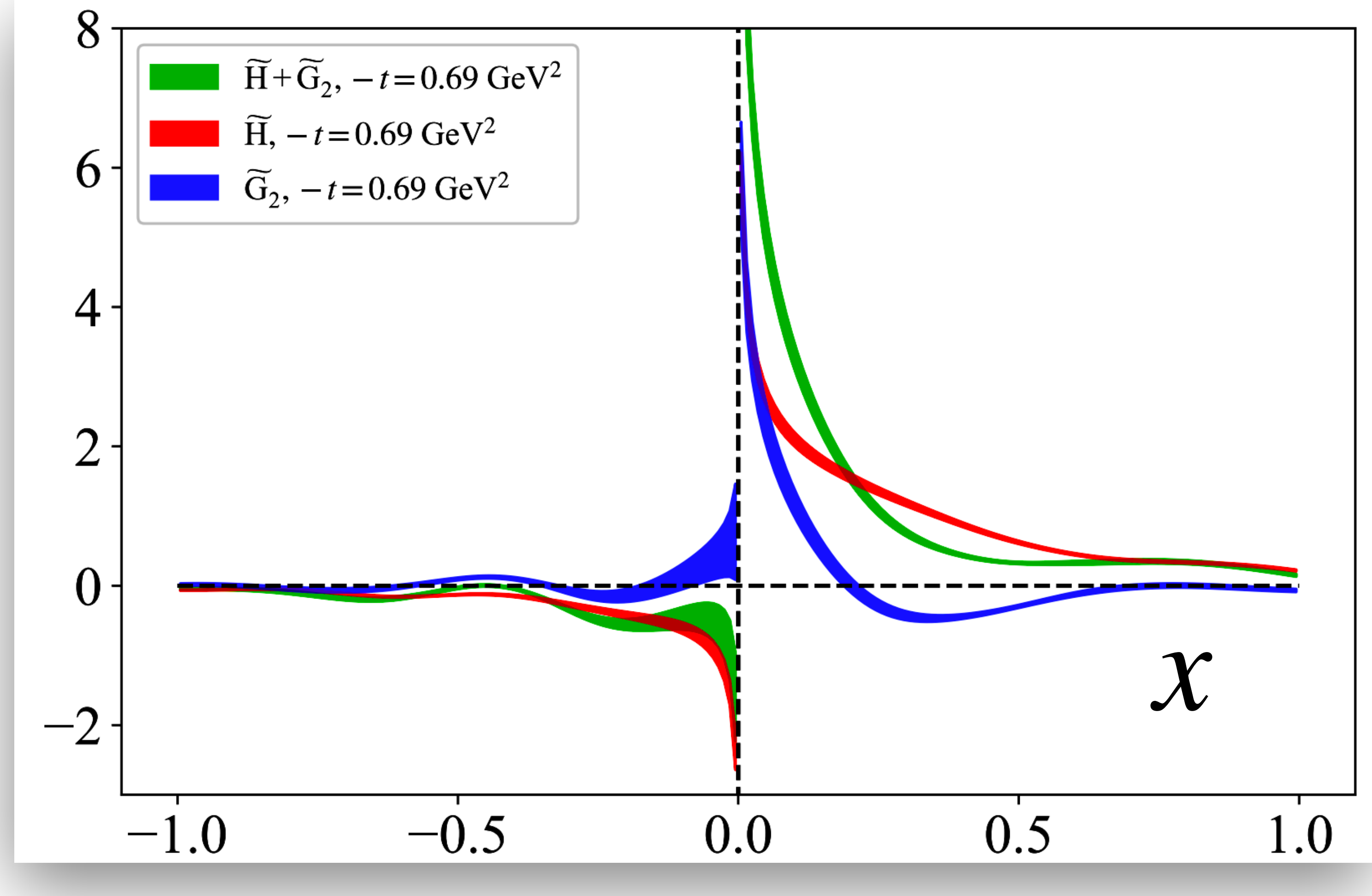
- C. Alexandrou, et al., (ETMC), , Phys.Rev.D 105 (2022) 3, 034501

GPD from asymmetric frame



Joshua Miller, Thu 1:50 PM

Chiral-even axial twist-3 GPDs



Martha Constantinou, Thu 3:10 PM

- S. Bhattacharya, et al., arXiv: 2306.05533

- **Encouraging results reported.**

Other new results

Wilson-line quasi operator

$$\bar{q}(0)\mathscr{W}(0,z)q(z)$$

- GPD and DA from **Pseudo-Distributions.**

Daniel Kovner, Tue 2:30 PM

Joseph Karpie, Thu 2:30 PM

Niilo Nurminen, Thu 2:50 PM

Current-current correlators

$$J^{\mu}(0)J^{\nu}(z)$$

- **Pion DA** from **HOPE** method.

Robert Perry, Tue 5:00 PM

- The **parity-odd structure function** of nucleon from **Compton amplitude.**

K. Utku Can, Fri 9:40 AM

- **Hadronic tensor** in lattice QCD.

Raza Sufian, Fri 10:00 AM

Towards precision

Data precision

- In progress: large P_z, continuum, physical point, …

Theory precision

Model bias

- Jacobi polynomial, DNN, …

• L. Debbio, et al., (NNPDF+Jlab), JHEP 02 (2021) 138
• C. Alexandrou, et al., (ETMC), Phys.Rev.D 103 (2021) 094512
• X. Gao et al., (BNL+ANL), Phys.Rev.D 107 (2023), 074509

Renormalization

- Ratio for SDF.
- RIMOM→Hybrid for LaMET.

• K. Orginos, et al., Phys.Rev.D 96 (2017), 094503
• J.-W. Chen, et al., Phys.Rev.D 97 (2018), 014505
• X. Ji, et al., Nucl.Phys.B 964 (2021) 115311

Perturbative matching in α_s

- NLO.
- NNLO for unpolarized iso-vector quark, …

• L.-B. Chen, et al., Phys. Rev. Lett. 126 (2021) 072002
• Z.-Y. Li, et al., Phys. Rev. Lett. 126 (2021) 072001

Large logs resummation

Power corrections

- Higher-twist, **renormalon**, …

Jianhui Zhang, Thu 11:00 AM

Leading renormalon resummation (LRR)

$$\cancel{\mathcal{O}(\Lambda_{\rm QCD}/P_z)} + \mathcal{O}(\Lambda_{\rm QCD}^2/P_z^2)$$

Yushan Su, Tue 4:00 PM

Large logarithms resummation: LaMET

$$\lim_{\xi\to 1} C\left(\xi, \frac{\mu}{P^z}\right) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left[2\frac{\ln|1-\xi|}{|1-\xi|} - \frac{2}{1-\xi}\ln\frac{\mu^2}{(2xP_z)^2}\theta(1-\xi) + \frac{3}{2|1-\xi|}\right] \qquad \xi = \frac{x}{y}$$

- Resum the DGLAP (RG) logs $\ln(\mu^2/(2xP_z)^2)$

$$C(\xi, \frac{\mu}{P_z}) \sim \left(\frac{\alpha_s(2xP_z)}{\alpha_s(\mu)}\right)^{\frac{\gamma_n}{\beta_0}}$$

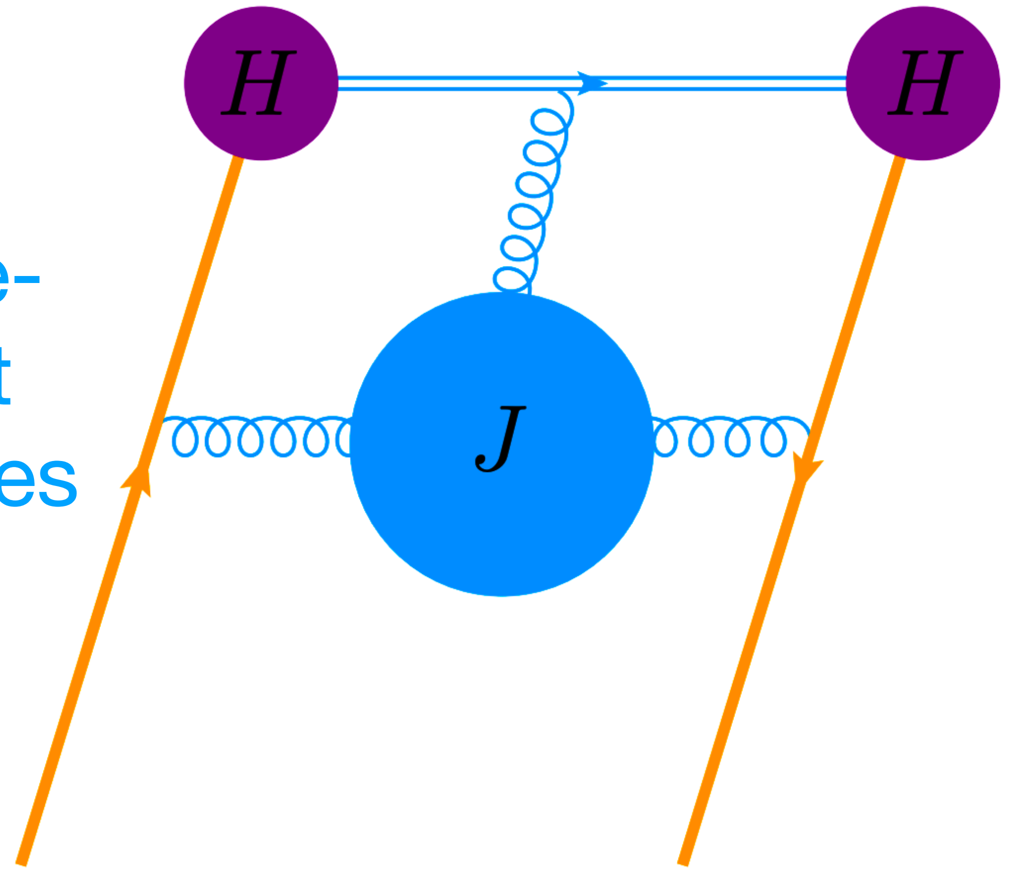
  - The perturbative matching is unreliable at small x ($2xP_z \approx \Lambda_{\rm QCD}$ or $\alpha_s \gtrsim 1$).
  - Enhances the accuracy of the expansion at moderate x ($2xP_z \gtrsim 0.7\ \text{GeV}$).

• Y.S. Su, et al., Nucl.Phys.B 991 (2023) 116201

- Resum large threshold logs $\ln^n|1-\xi|/(1-\xi)$

$$\mathcal{C}\left(\xi, \frac{p^z}{\mu}\right)\Big|_{\xi\to 1} = H\left(\frac{4p_z^2}{\mu^2}\right) p^z J_f\left(\frac{(1-\xi)p^z}{\mu}, \frac{4p_z^2}{\mu^2}\right)$$

Jet function re-sums the soft gluon exchanges



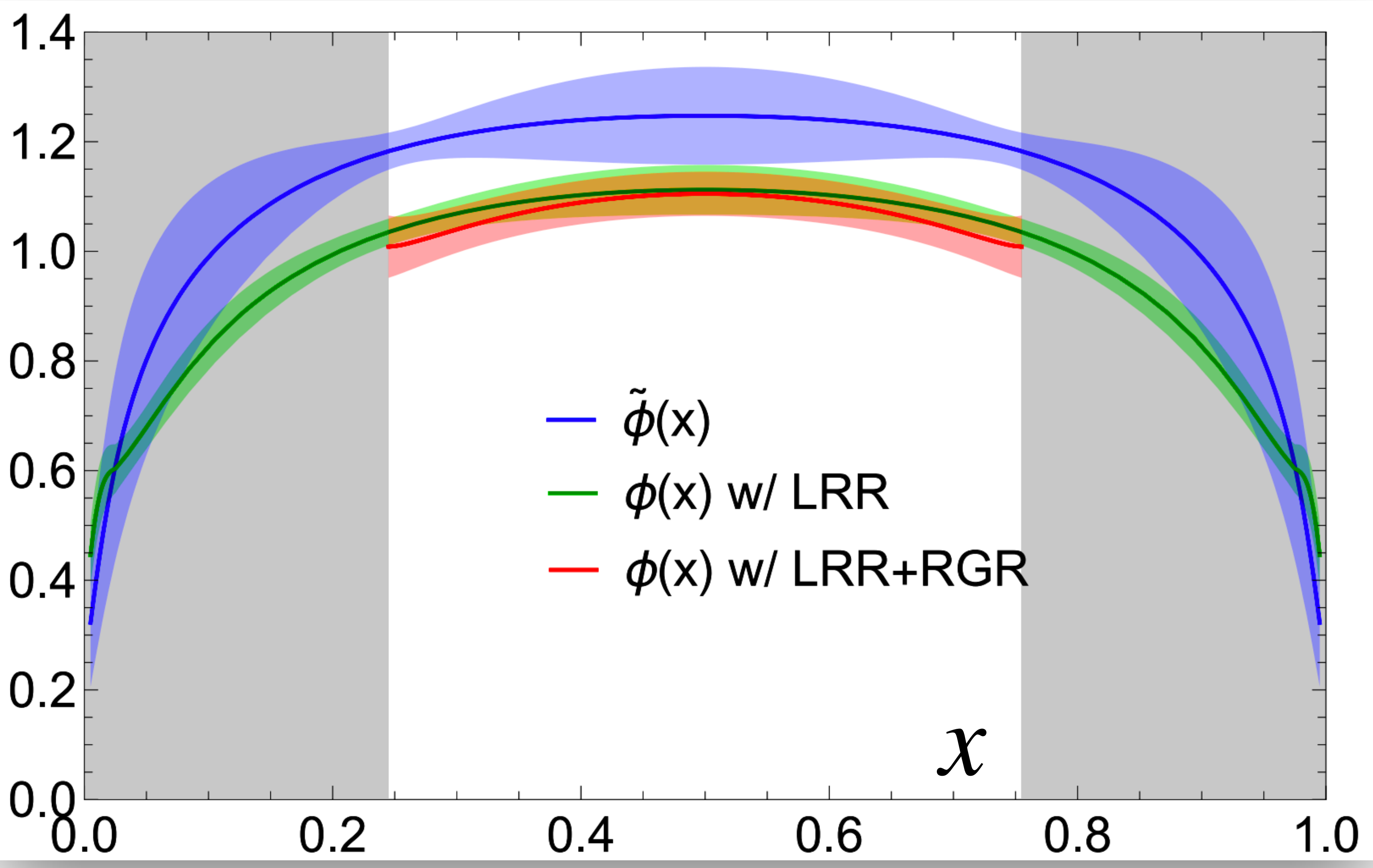
heavy-light Sudakov form factors

  - Enhances the accuracy at large x.

• X. Ji, et al., arXiv: 2305.04416.

Large logarithms resummation: LaMET

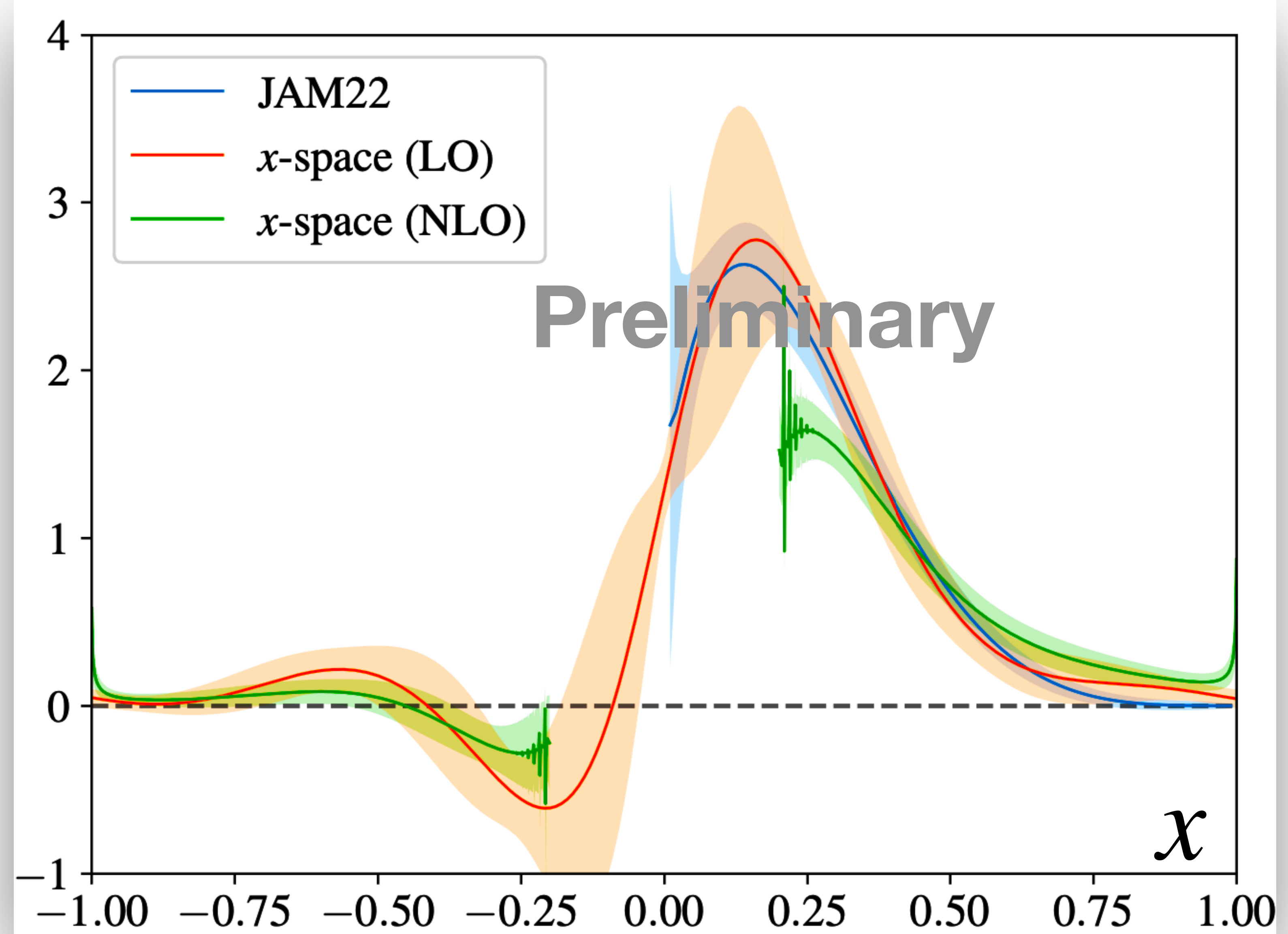
Pion DA



• J. Holligan, et al., Nucl.Phys.B 993 (2023) 116282

- Leading renormalon resummation (LRR) and leading DGLAP (RGR) evolution are included.
- Predict more reliable x dependence in the moderate region with reduced theoretical uncertainty.

Nucleon transversity PDF



• X. Gao, et al., (BNL+ANL), in preparation.

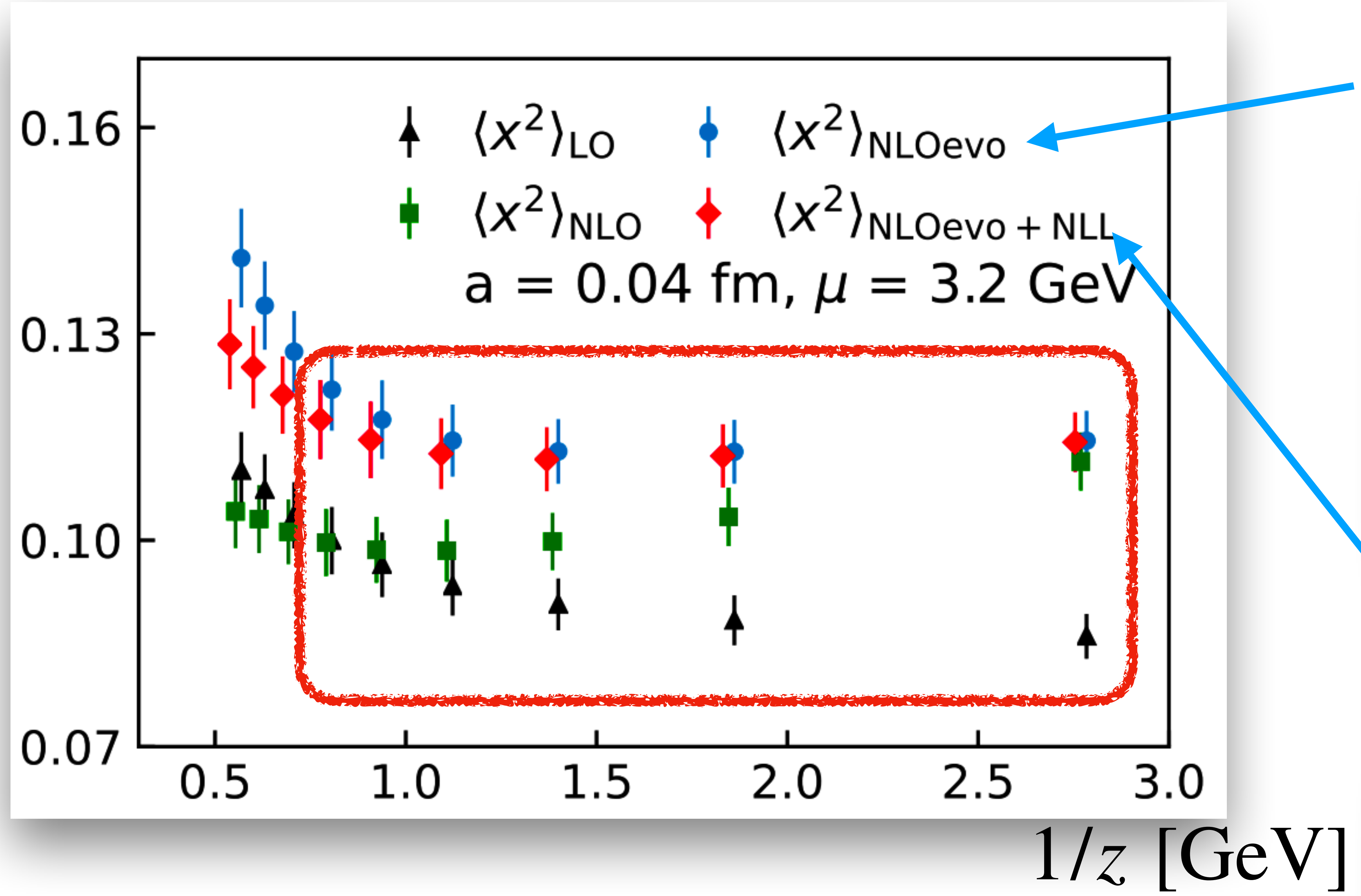
Toward a precision calculation of GPDs

Jack Holligan, Thu 2:10 PM

Large logarithms resummation: SDF

$$\lim_{n\to\infty} C_n = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left[2\ln n \ln(z^2\mu^2) - 2\ln^2 n + 2\ln n - \frac{\pi^2}{3}\right] + \mathcal{O}(\alpha_s^2)$$

$\langle x^2 \rangle_\pi^{val}$ **from NLO+Resummation**



- X. Gao et al. (BNL), Phys.Rev.D 103 (2021), 094504

- Resum the DGLAP (RG) logs $\ln(z^2\mu^2)$

$$C_n(z^2\mu^2) \sim \left(\frac{\alpha_s(z^{-1})}{\alpha_s(\mu)}\right)^{\frac{\gamma_n}{\beta_0}}$$

  - The perturbative matching is unreliable at large z ($\alpha_s \gtrsim 1$ or $z \gtrsim 0.4$ fm).
  - Enhances the accuracy of the expansion at small and moderate z.

- Resum large threshold logs $\alpha_s^m \ln^k n$
  - Little impact for low moments.
  - Enhance the accuracy of future high moments ($\ln n \gg 1/\alpha_s$) extraction.

Recent calculations from LaMET

Nucleon quark unpolarized distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Renorm.	Matching	Thres. resum.	RG	LRR
ETMC, 2021	1.65	0.093	260	RI-MOM	NLO	NO	NO	NO
BNL+ANL, 2023	1.53	0.076	140	Hybrid	NNLO	NO	NO	NO

- C. Alexandrou, et al., (ETMC), Phys.Rev.D 104 (2021), 054503
- X. Gao, et al., (BNL+ANL), Phys.Rev.D 107 (2023), 074509

Nucleon quark transversity distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Renorm.	Matching	Thres. resum.	RG	LRR
LPC, 2022	2.8	0.049~0.098	220~350	Hybrid	NLO	NO	NO	NO
BNL+ANL, 2023	1.53	0.076	140	Hybrid	NLO	NO	Yes	Yes

- F. Yao, et al., (LPC), arXiv, 2208.08008
- X. Gao, et al., (BNL+ANL), in preparation.

Nucleon quark helicity distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Renorm.	Matching	Thres. resum.	RG	LRR
ETMC, 2021	1.65	0.093	260	RI-MOM	NLO	NO	NO	NO

- C. Alexandrou et al., (ETMC), *Phys.Rev.Lett.* 126 (2021), 102003
- C. Alexandrou et al., (ETMC), Phys.Rev.D 104 (2021), 054503

Pion valence quark distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Renorm.	Matching	Thres. resum.	RG	LRR
BNL+ANL, 2021	2.42	0.04~0.06	300	Hybrid	NNLO	NO	NO	NO
BNL+ANL, 2022	2.42	0.04~0.076	140, 300	Hybrid	NNLO	NO	NO	NO
Y.-S. Su et al.	1.9	0.04	300	Hybrid	NNLO	NO	Yes	NO
R. Zhang et al.	1.9	0.04	300	Hybrid	NNLO	NO	NO	Yes

- X. Gao, et al., (BNL-ANL), Phys.Rev.Lett. 128 (2022), 142003
- X. Gao, et al., (BNL-ANL), Phys.Rev.D 106 (2022), 114510
- Y.-S. Su, et al., *Nucl.Phys.B* 991 (2023) 116201
- R. Zhang et al., arXiv: 2305.05212

Nucleon GPDs

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Renorm.	Matching	Thres. resum.	RG	LRR
H.-W. Lin, 2022	2.2	0.09	~140	RI-MOM	NLO	NO	NO	NO
ETMC, 2022	1.67	0.093	260	RI-MOM	NLO	NO	NO	NO

- H.-W. Lin, Phys.Lett.B 824 (2022) 136821
- C. Alexandrou, et al., (ETMC), , Phys.Rev.D 105 (2022) 3, 034501

For review:

- X.Ji, et, al., Rev.Mod.Phys. 93 (2021) 3, 035005
- M. Constantinou, et al., arXiv: 2202.07193
- H.-W. Lin, Few Body Syst. 64 (2023) 3, 58

Recent calculations from SDF

Nucleon quark unpolarized distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Model	Matching	RG	Thres. resum.
M. Bhat, et al., 2022	1.89	0.064~0.093	370	2-p model	NNLO	NO	NO
BNL+ANL, 2023	1.53	0.076	140	DNN	NNLO	NO	NO

- M. Bhat, et. al., Phys.Rev.D 106 (2022) 054504
- X. Gao, et al., (BNL+ANL), Phys.Rev.D 107 (2023), 074509

Nucleon quark transversity distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Model	Matching	RG	Thres. resum.
HadStruc, 2022	2.46	0.094	358	Jacobi Poly.	NLO	NO	NO
BNL+ANL, 2023	1.53	0.076	140	DNN	NLO	NO	NO

- C. Egerer, et al., (HadStruc Collaboration), Phys.Rev.D 105 (2022), 034507
- X. Gao, et al., (BNL+ANL), in preparation.

Nucleon quark helicity distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Method	Matching	RG	Thres. resum.
HadStruc, 2023	2.46	0.094	358	Jacobi Poly.	NLO	NO	NO

- R. Edwards., (HadStruc Collaboration), JHEP 03 (2023) 086

Nucleon gluon unpolarized distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Method	Matching	RG	Thres. resum.
HadStruc, 2022	2.46	0.094	358	Jacobi Poly.	NLO	NO	NO
MSU, 2022	3	0.09~0.15	220~690	2-p model	NLO	NO	NO

- T. Khan, et al., (HadStruc Collaboration), Phys.Rev.D 104 (2021), 094516
- Z. Fan, et al., (MSU), Phys.Rev.D 108 (2023), 014508

Nucleon gluon helicity distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Method	Matching	RG	Thres. resum.
HadStruc, 2022	2.46	0.094	358	Jacobi Poly.	NLO	NO	NO

- C. Egerer, et al., (HadStruc Collaboration), Phys.Rev.D 106 (2022), 094511

Pion valence quark distribution

	P_z^{max} [GeV]	a [fm]	m_π [MeV]	Method	Matching	RG	Thres. resum.
BNL, 2021	2.42	0.04	300	2-p model	NLO	Yes	Yes
BNL+ANL, 2023	2.42	0.04~0.76	140, 300	DNN	NNLO	NO	NO

- X. Gao, et al., (BNL), Phys.Rev.D 103 (2021), 094504
- X. Gao, et al., (BNL+ANL), Phys.Rev.D 106 (2022), 114510

For review:

- X.Ji, et, al., Rev.Mod.Phys. 93 (2021) 3, 035005
- M. Constantinou, et al., arXiv: 2202.07193
- H.-W. Lin, Few Body Syst. 64 (2023) 3, 58

Towards systematic control

LaMET

Systematics	Continuum	Physical points	$P_z^{\max}$ [GeV]	Renorm.	Matching	DGLAP/ RG	Threshold resummation
Up-to-date calculation	Yes	Yes	2~3 GeV	Hybrid+ LRR	NLO/ NNLO	Yes	In progress

SDF

Systematics	Continuum	Physical points	$P_z^{\max}$ [GeV]	Renorm.	Matching	DGLAP/ RG	Threshold resummation
Up-to-date calculation	Yes	Yes	2~3 GeV	Ratio	NLO/ NNLO	In progress	In progress

- **Data and theoretical precision in good progress.**

LaMET: new approach

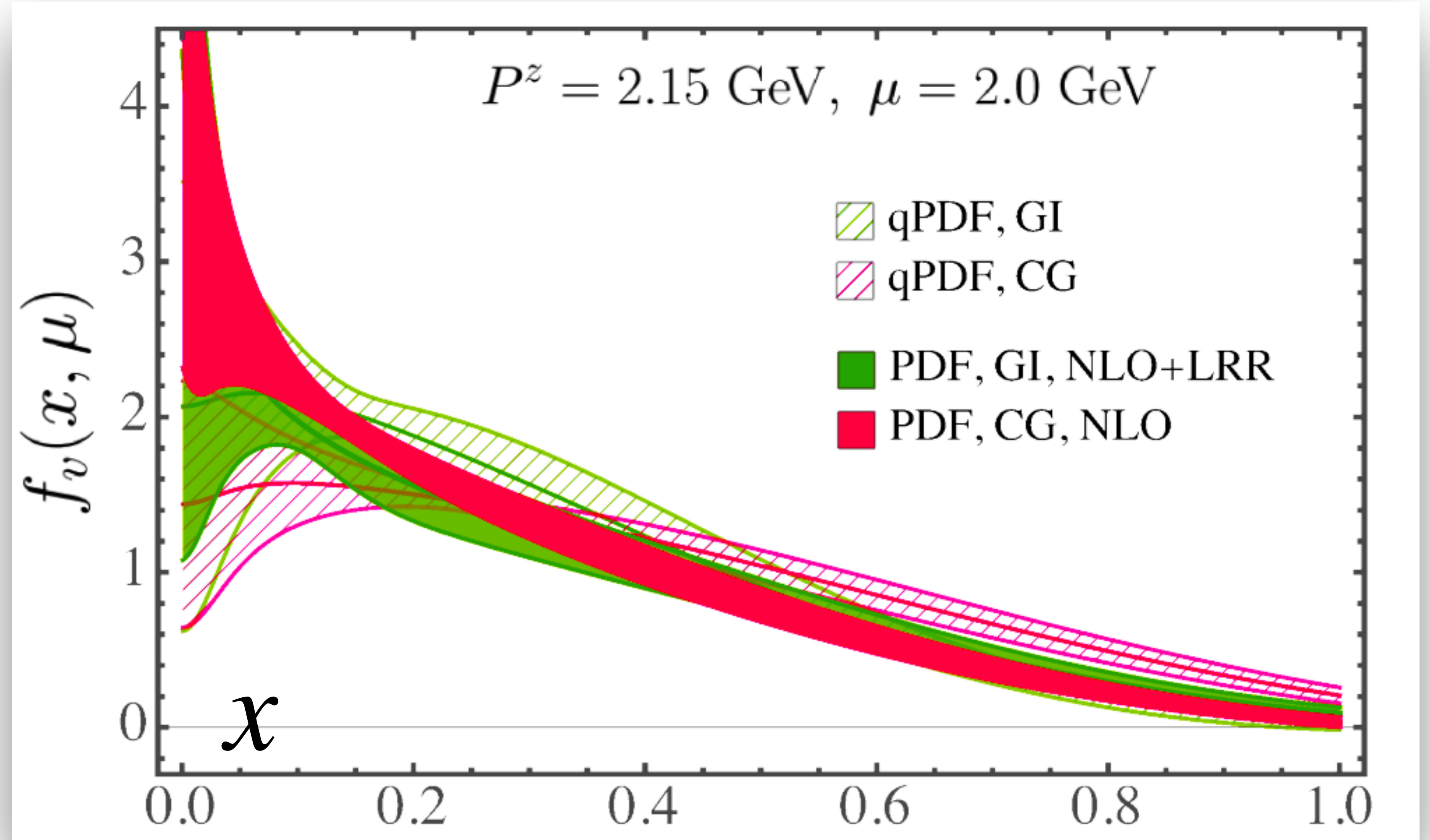
- **Quasi-PDF in Coulomb gauge (CG)**

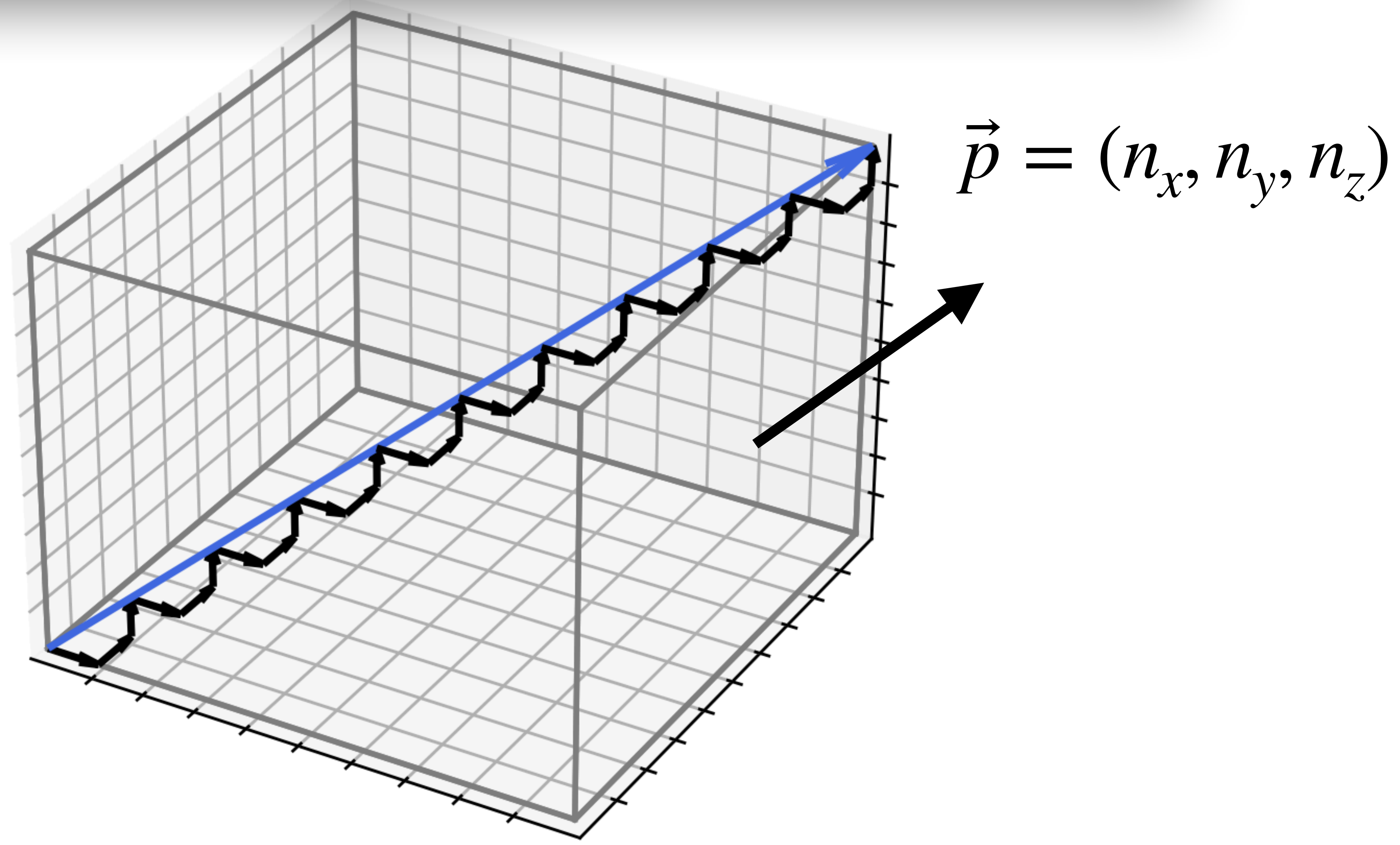
$$\tilde{f}(x, P^z, \mu) = P^z \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP^z z} \tilde{h}(z, P^z, \mu)$$

$$\tilde{h}(z, P^z, \mu) = \frac{1}{2P^t} \langle P | \bar{\psi}(z) \gamma^t \psi(0) \Big|_{\vec{\nabla}\cdot\vec{A}=0} | P \rangle$$

- No Wilson line, reduces to the light-cone gauge with infinite boost.
- Consistent with the gauge invariant (GI) calculation.
- Larger off-axis momenta: $\vec{p} \sim \sqrt{3} p_z$.
- Absence of linear divergence from Wilson line, free of leading renormalon.

Pion valence quark distribution



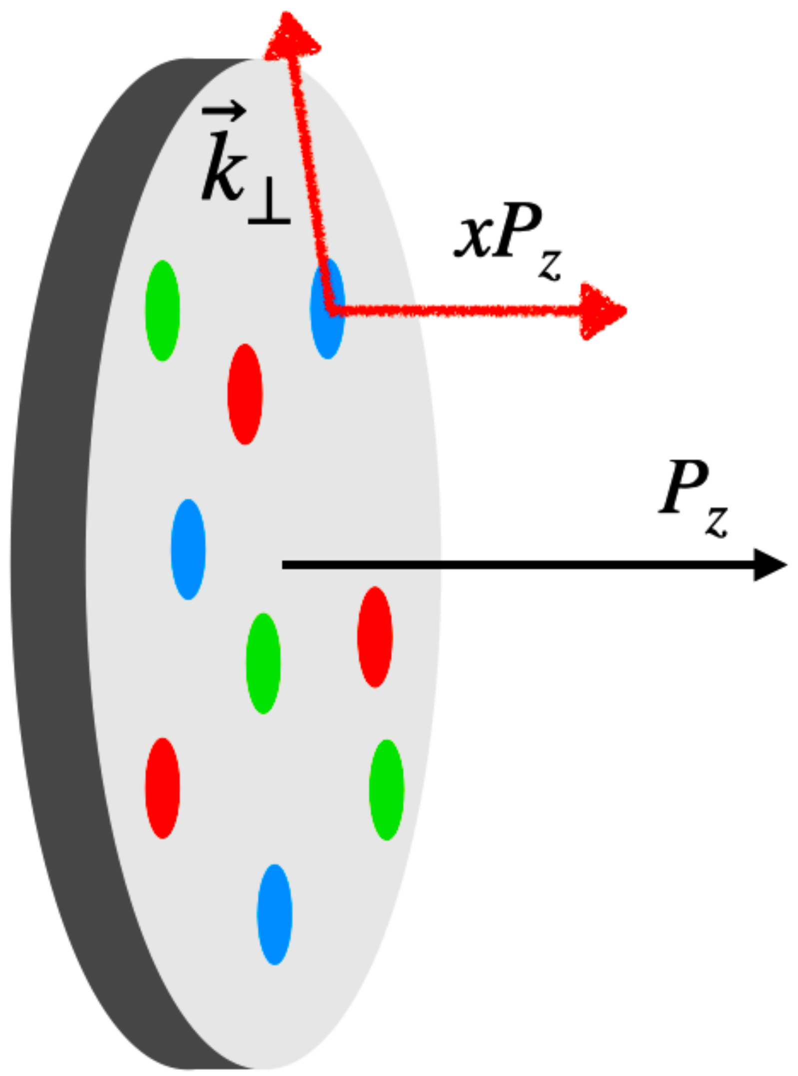


Yong Zhao, Thu 5:20 PM

- X. Gao, W.-Y. Liu, Y. Zhao, arXiv: 2306.14960

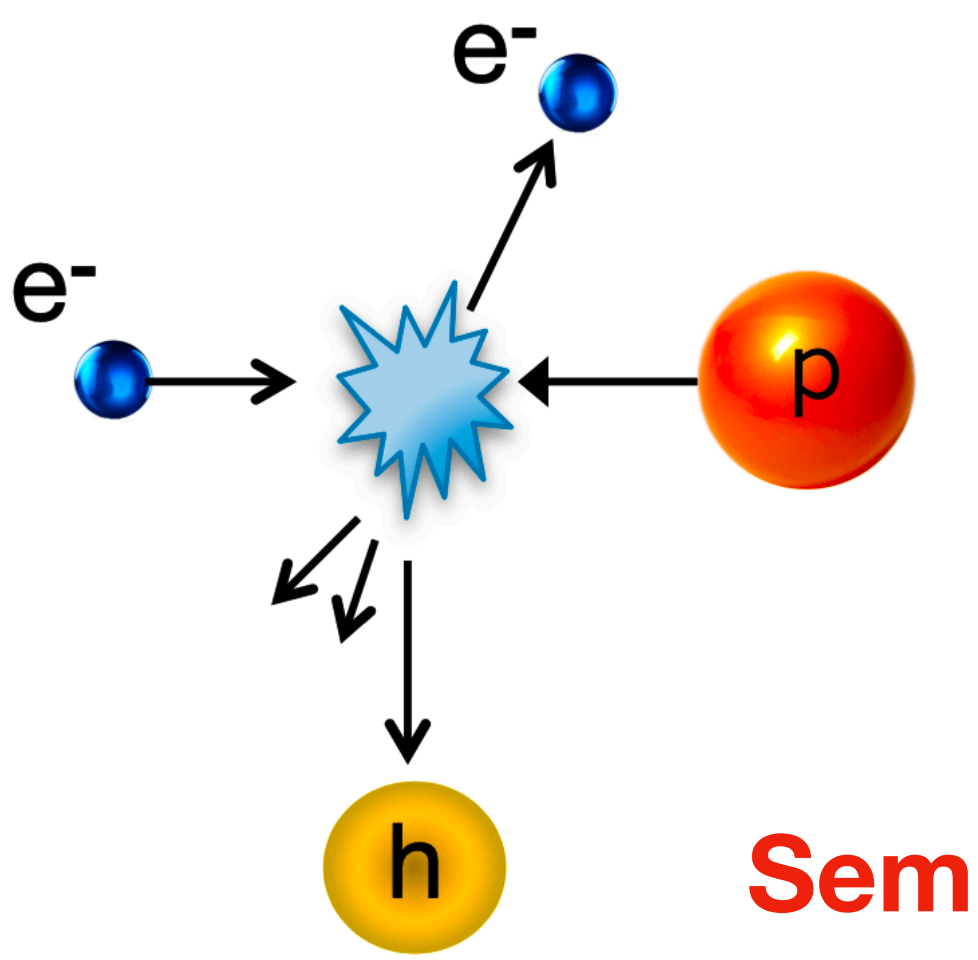
Transverse-momentum-dependent distributions

The polarization and k_T dependence of TMDs provide rich information of hadron structure.



$$f_q(x, \vec{k}_T, \mu, \zeta)$$

- 3D image: intrinsic (confined) motion in the transverse directions.
- Spin-orbit correlations: the coupling of the transverse momentum of quarks with the spin of nucleon (or quarks).



Semi-inclusive DIS

Transverse-momentum-dependent distributions

Collins-Soper scale:

$$\zeta \propto (xP^+)^2$$

Beam function

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0, \tau\to 0} Z_{\rm uv}(\epsilon, \mu, \zeta)\frac{B_q}{\sqrt{S_q}}$$

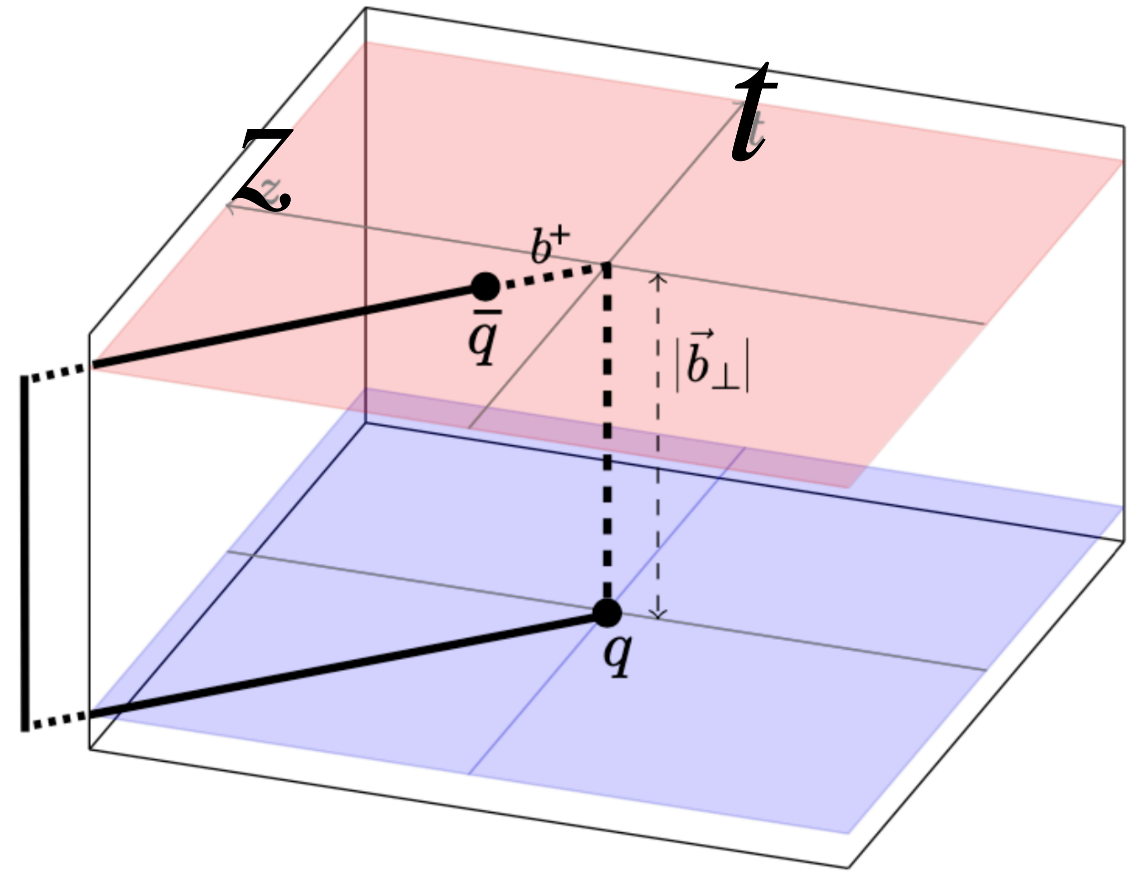
Soft function

$$\mu\frac{d}{d\mu}\ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma^q_\mu(\mu, \zeta)$$

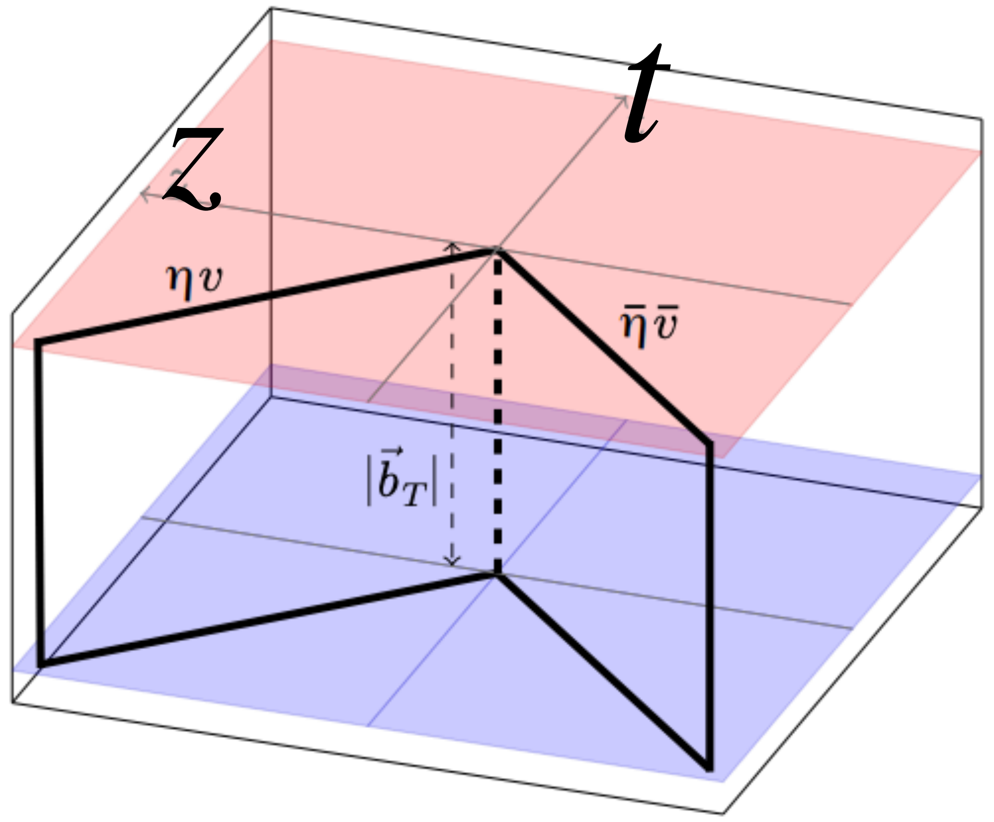
$$\mu\frac{d}{d\zeta}\ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma^q_\zeta(\mu, b_T)$$

Solve both equation to relate TMDs at different energy scales

Collin Soper kernel, become non-perturbative when $b_T^{-1} \sim \Lambda_{\rm QCD}$



Hadron matrix elements B_q



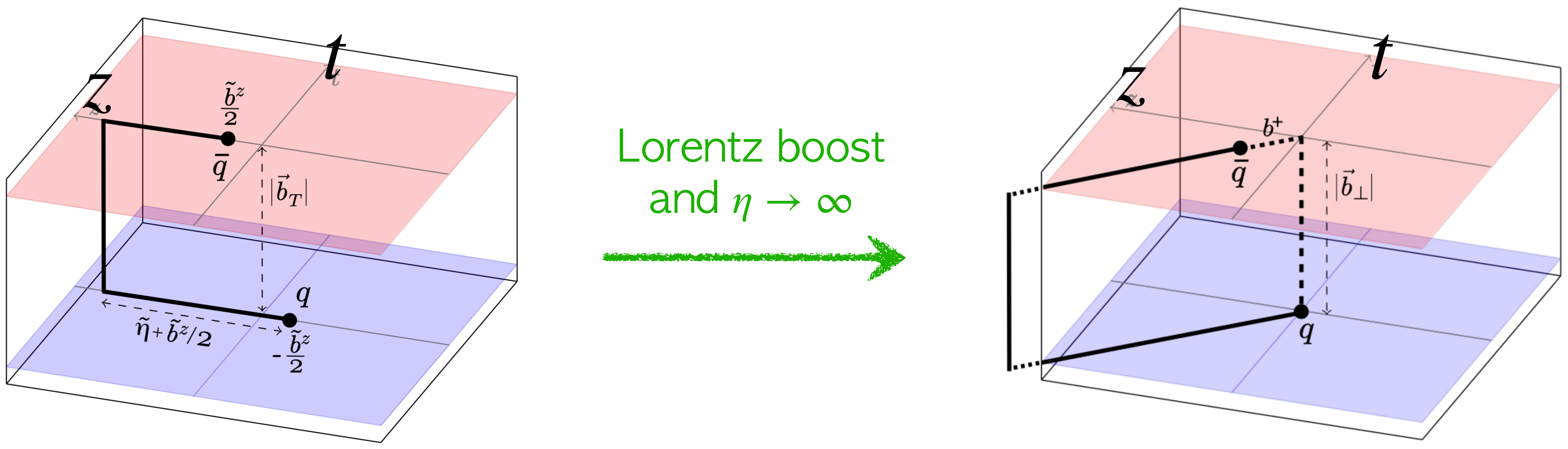
Vacuum matrix elements S_q

TMDs from LaMET

Quasi beam function **Collin Soper kernel** **Physical TMD**

$$\frac{\tilde{f}(x,\vec{b}_T,\mu,P_z)}{\sqrt{S_r(\vec{b}_T,\mu)}} = C(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda^2_{\rm QCD}}{(xP_z)^2}]\}$$

Reduced soft factor



Quasi beam function **Light-cone Beam function**

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- I. Stewart, Y. Zhao et al., JHEP 04 (2022) 178

Collin-Soper kernel

$$\gamma_\zeta(b_T,\mu) = \frac{d}{d\ln P_z}\ln\frac{\tilde f(x,\vec b_T,\mu,P_z)}{C(\mu,xP_z)}$$

Universal kernel, key to the TMD evolution.

- Quasi TMD beam functions.

  $$\langle \pi(P)\,|\,\bar q(z\hat n_z + b_\perp \hat n_\perp) W_\sqsupset q(0)\,|\,\pi(P)\rangle$$

  - P. Shanahan, et al., Phys.Rev.D 104 (2021), 114502

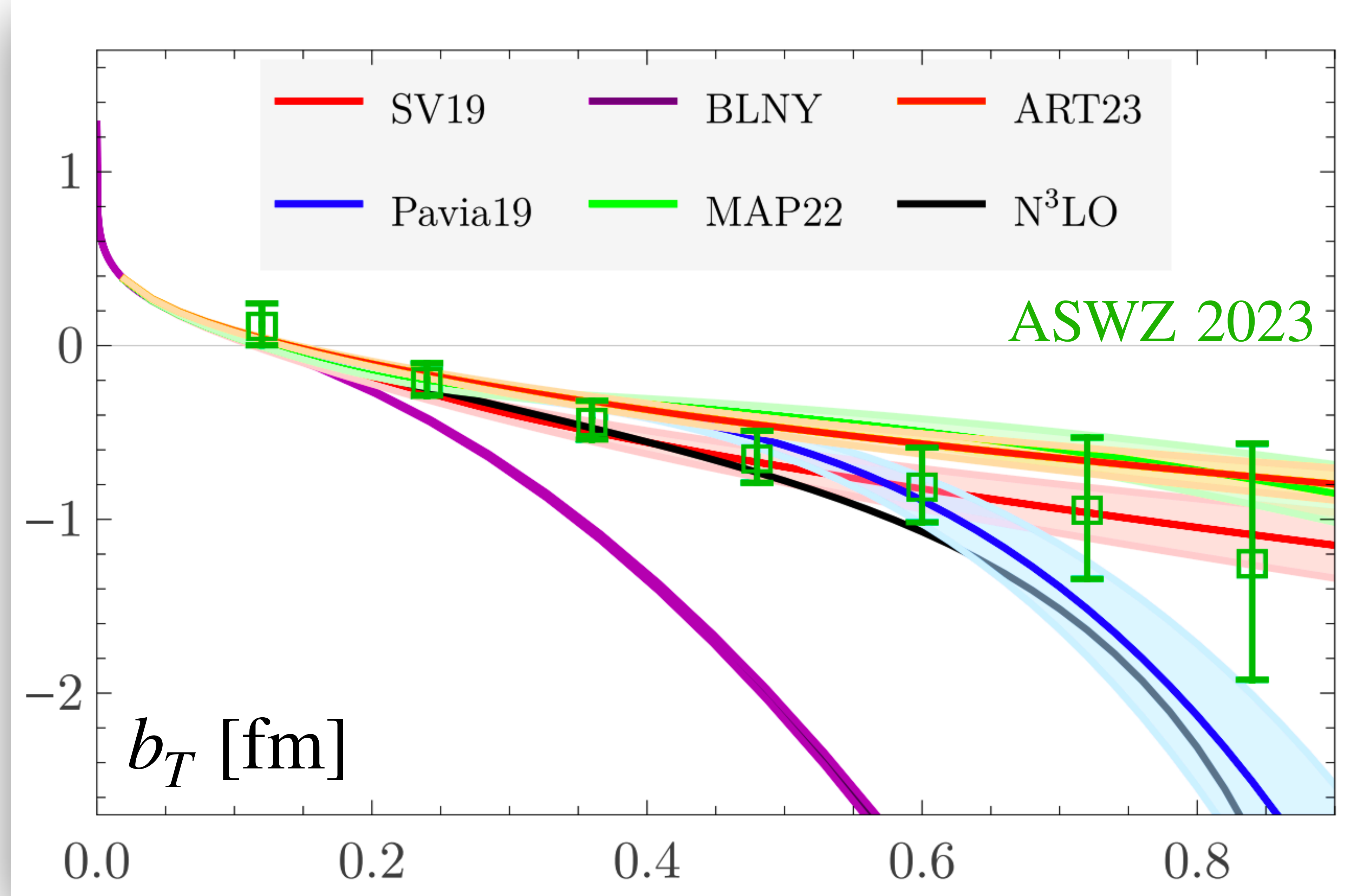
- Quasi TMD wave functions.

  $$\langle \Omega\,|\,\bar q(z\hat n_z + b_\perp \hat n_\perp) W_\sqsupset q(0)\,|\,\pi(P)\rangle$$

  - M.-H. Chu, et al., (LPC), Phys.Rev.D 106 (2022), 034509
  - Y. Li, et al., (ETMC/PKU), Phys.Rev.Lett. 128 (2022), 062002
  - M.-H. Chu, et al., (LPC), arXiv: 2306.06488
  - A. Avkhadiev, P. Shanahan, et al., (ASWZ), arXiv: 2307.12359

- Moments of quasi TMDs

  - M. Schlemmer, et al., (RQCD), JHEP 08 (2021) 004
  - H.-T. Shu, et al., (RQCD), arXiv: 2302.06502



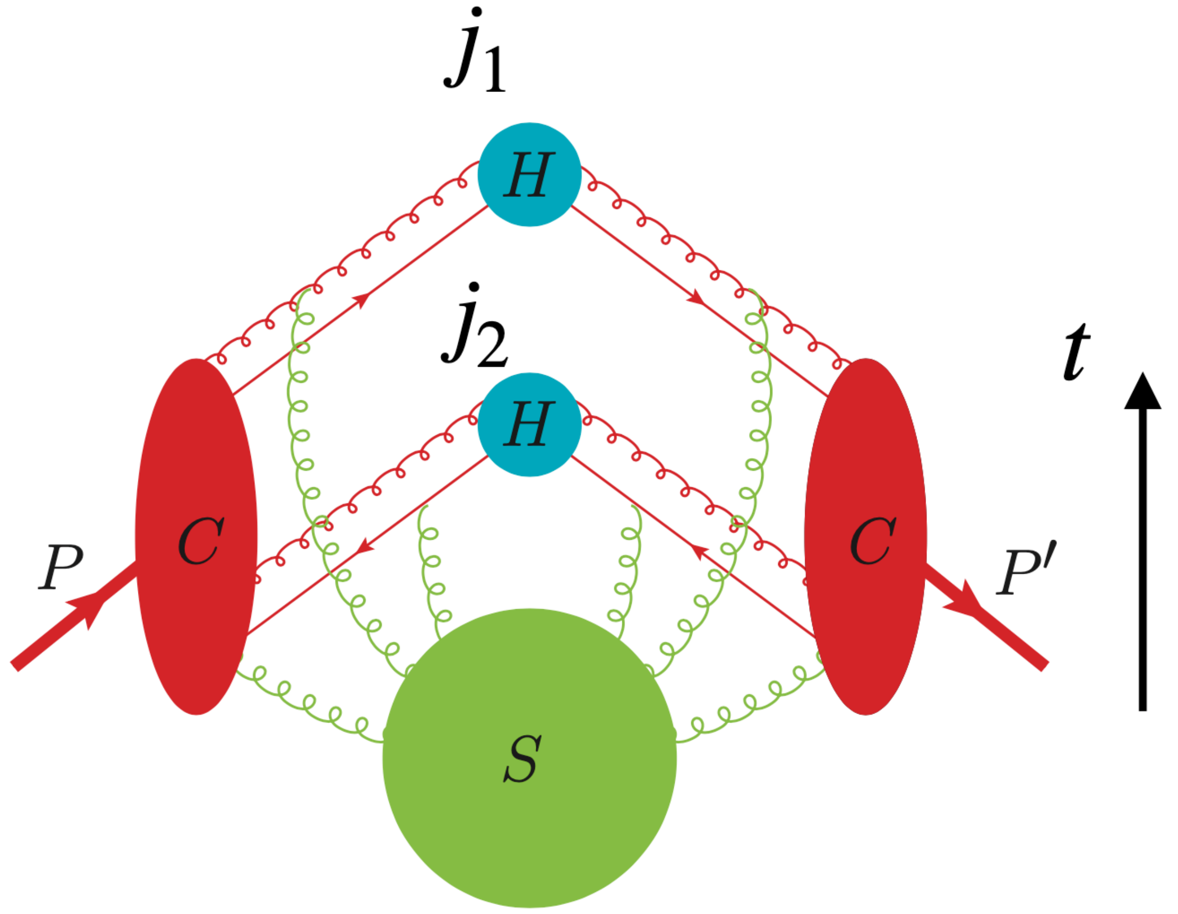
Artur Avkhadiev, Thu 4:20 PM

Towards precision control:

- Close to physical m_π
- Next-to-next-to-leading logarithmic (**NNLL**) matching.
- Well controlled Fourier transform.
- Inclusion of perturbative power corrections.
- Complete analysis of systematics from **operator mixing**.

Gregoris Spanoudes, Thu 4:40 PM

Soft factor



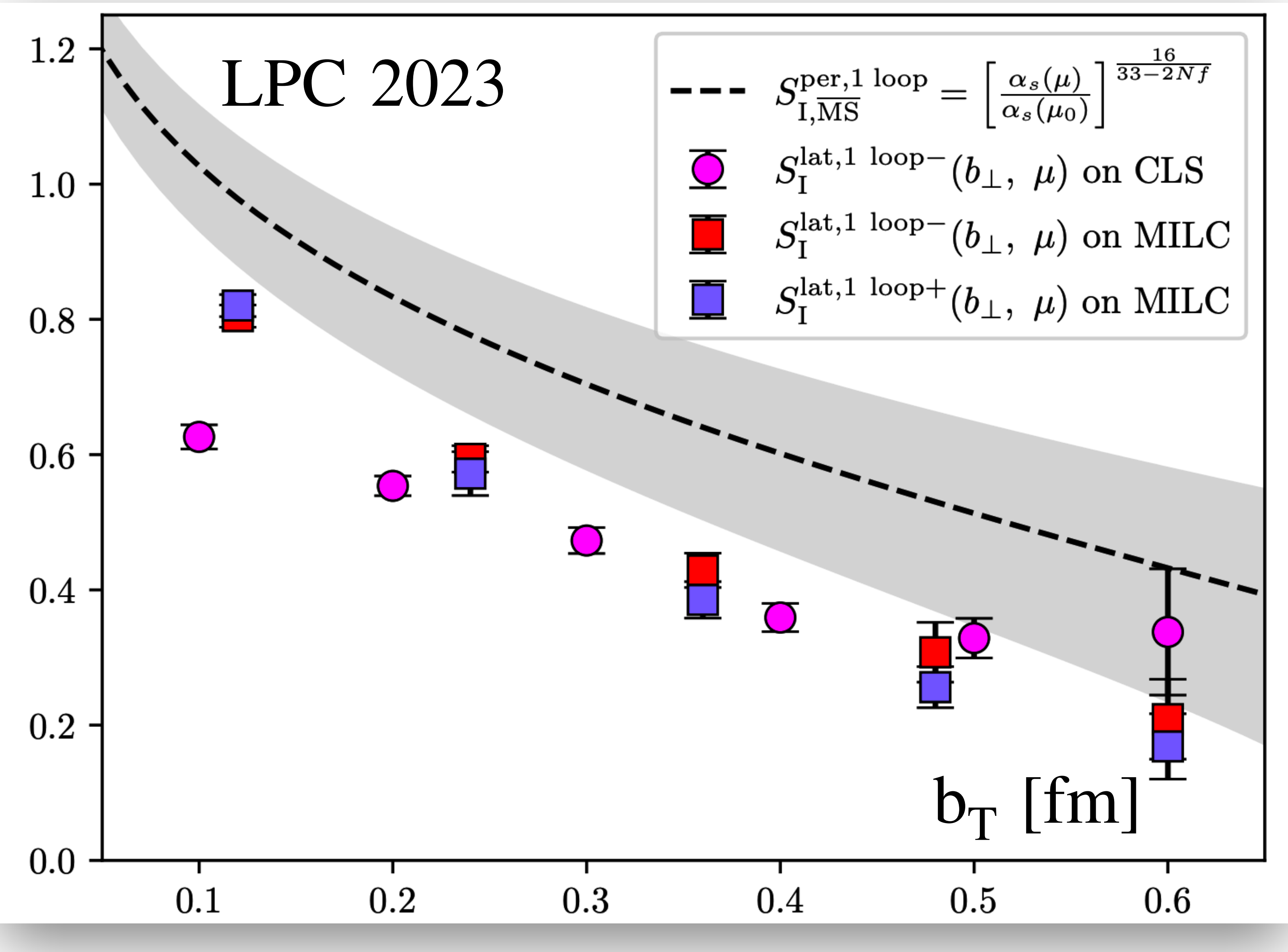
- Four-point correlators
- High boosted pion in opposite direction.

$$\langle \pi(-P) \,|\, j_1(b_T) j_2(0) \,|\, \pi(P) \rangle$$

quasi TMD wave function

$$= S_r(b_T, \mu) \int dx dx' H(x, x', \mu) \tilde{\Phi}^\dagger(x, b_T, P_z, \mu) \tilde{\Phi}(x', b_T, P_z, \mu)$$

- LO approximation $H(x, x', \mu) = 1 + \mathcal{O}(\alpha_s)$
  - X. Ji, et al., Nucl.Phys.B 955 (2020) 115054
  - Q.-A. Zhang, et al., (LPC), Phys.Rev.Lett. 125 (2020), 192001
  - Y. Li, et al., (ETMC/PKU), Phys.Rev.Lett. 128 (2022), 062002
- NLO matching
  - Z.-F. Deng, et al., JHEP 09 (2022) 046
  - M.-H. Chu, et al., (LPC), arXiv: 2306.06488



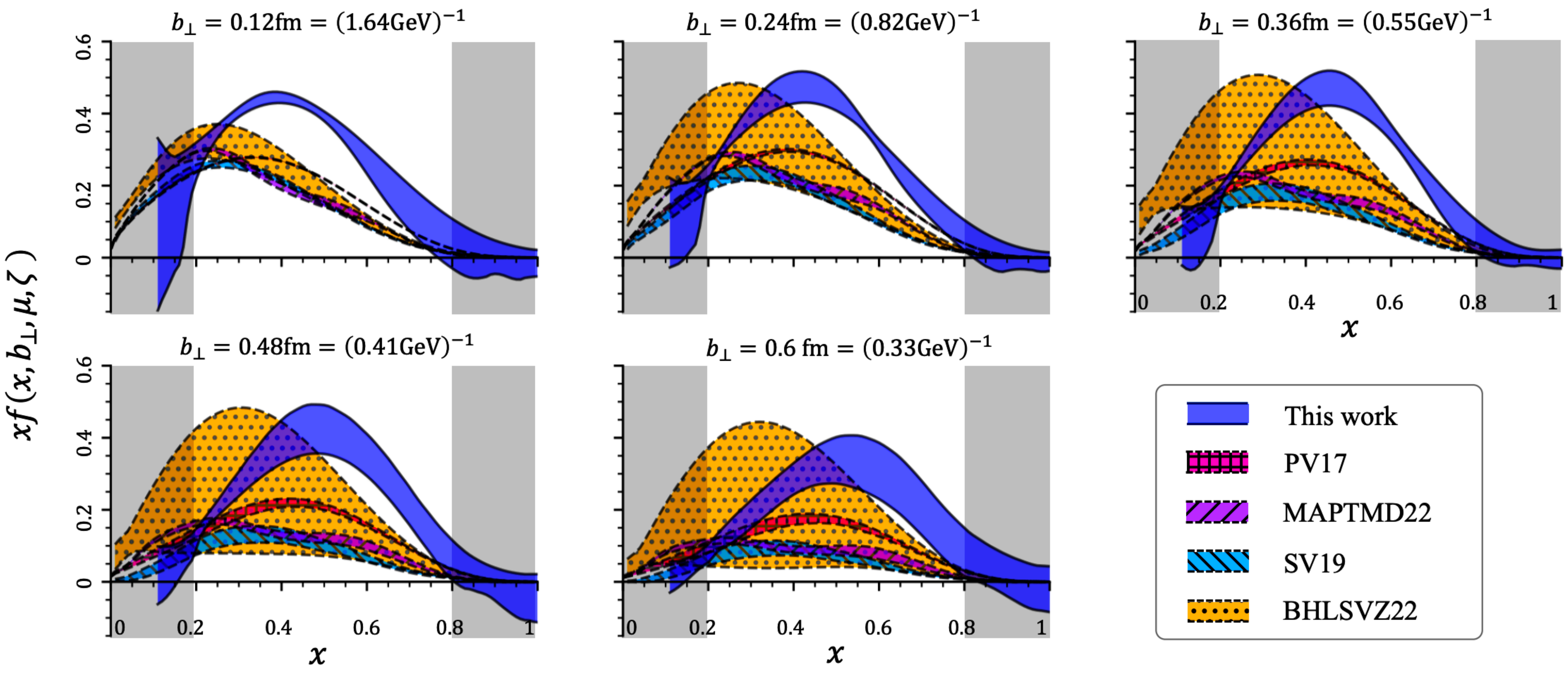
Clover/HISQ, $m_\pi \in [132{,}333]$ MeV,
$p \leq 3.16$ GeV, $a = 0.098,\ 0.121$ fm, NLO

TMDs: pioneering lattice calculations established

◉ Nucleon unpolarized TMD PDF

- J.-C. He, et al., (LPC), arXiv: 2211.02340

Clover on HISQ, $m_\pi = 220,\ 310\ \mathrm{MeV}$, $p \le 2.58\ \mathrm{GeV}$, $a = 0.12\ \mathrm{fm}$, NLO



◉ Pion TMD wave function

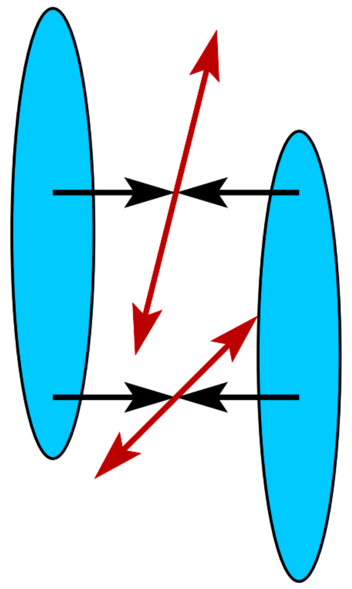
- M.-H. Chu, et al., (LPC), arXiv: 2302.09961

Renormalization of TMD PDF on the lattice

Kuan Zhang, Thu 4:40 PM

Summary and Outlook

- PDFs/GPDs:
  - ✓ Many development and calculations, systematics control in rapid progress.
  - ▸ More studies on gluon, sea quark distributions, and GPDs.
  - ▸ Towards physical point, continuum limit, and higher hadron boost.
- TMDs:
  - ✓ All key techniques exist, first results of TMD PDF and wave function established.
  - ▸ Keep improving the systematic control, e.g. small b_T power correction, …
  - ▸ Studies on spin dependent TMD, gluon TMD.
  - ▸ Towards larger b_T, physical point, continuum limit, and higher hadron boost.
- New observables:
  - Double parton distribution functions (DPD).
  - Wigner Distributions (GTMD).
  - Fragmentation?



DPD moments.

- C. Zimmermann, and D. Reitinger 2211.14151

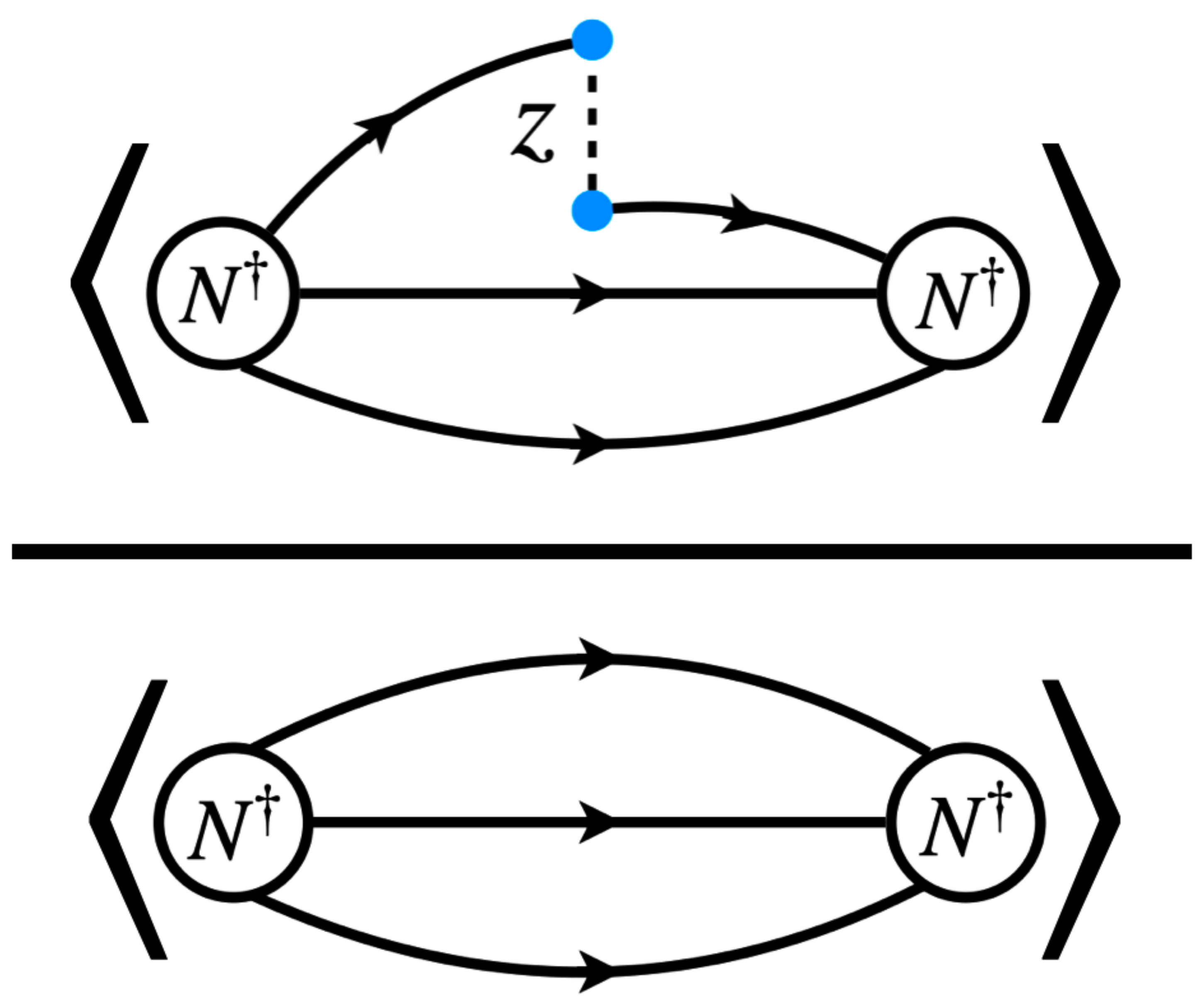
Quasi DPD proposed.

- J.-H. Zhang, 2304.12481
- M. Jaarsma et al., 2305.09716

Thanks for your attention!

Back up slides

Data precision



$$R(t_s,\tau) = \frac{\langle N(t_s)\mathcal{O}^f_\Gamma(\vec{z},\tau)\overline{N}(0)\rangle}{\langle N(t_s)\overline{N}(0)\rangle} \xrightarrow{t_s\to\infty} \tilde{h}^B(z,P_z,a)$$

Bare matrix elements

Towards precision calculation:

$$\tilde{h}^B(z,P_z,a) = \langle P\,|\,\bar{q}(0)\Gamma\mathscr{W}(0,z)q(z)\,|\,P\rangle$$

- Continuum limit, physical point and chiral fermion.
- Large momentum P_z / signal-to-noise ratio.
- Excited state contamination.

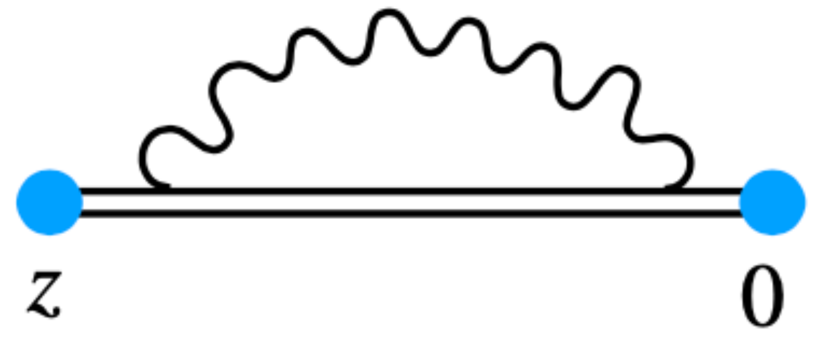
- Distillation from all time slices improves signal, …
- Smeared link: Wilson flow, HYP, …
- Smeared source: momentum smearing, …
- …

- M. Peardon, et al., (Hadron Spectrum), Phys.Rev.D 80 (2009) 054506
- C. Egerer, et al., Phys.Rev.D 103 (2021), 034502
- G. S. Bali, et al. (RQCD), Phys.Rev.D 93 (2016), 094515
- A. Hasenfratz, et al., Phys.Rev.D 64 (2001) 034504
- M. Lüscher, JHEP 08 (2010) 071, JHEP 03 (2014) 092

Renormalization

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)

Multiplicative renormalizable



$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m|z|}Z(a)[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

Frequently used ratio-type renormalization:

$$\tilde{h}(z, P_z, \mu) = \frac{h(z, P_z, a)}{Z_X(z, \tilde{\mu}, a)}$$

- RI-MOM: $Z_X = \langle q | O^\Gamma(z) | q \rangle$
- Hadron ratio: $Z_X = \langle P_z^0 | O^\Gamma(z) | P_z^0 \rangle$

- Good for SDF.
- Uncontrolled non-perturbative effect at long distance $z \gtrsim 1/\Lambda_{\mathrm{QCD}}$.

- J.-W. Chen, et al., Phys.Rev.D 97 (2018), 014505
- K. Orginos, et al., Phys.Rev.D 96 (2017), 094503

Hybrid renormalization scheme:

- X. Ji, et al., Nucl.Phys.B 964 (2021) 115311

- Long distance $z > z_s$:

$$h^R = e^{\delta m|z-z_s|}\frac{\tilde{h}(z, P_z, a)}{Z_X(z_s, \tilde{P}_z^0, a)}$$

δm can be extracted from e.g., Wilson loops.

- Good for F.T. to quasi-PDF and LaMET.
- $\delta m(a) = m_{-1}(a)/a + m_0$ has a renormalon ambiguity.

Leading renormalon resummation (LRR)

$$\cancel{\mathcal{O}(\Lambda_{\mathrm{QCD}}/P_z)} + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2/P_z^2)$$

Yushan Su, Tue 4:00 PM

Power corrections

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m|z|}Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

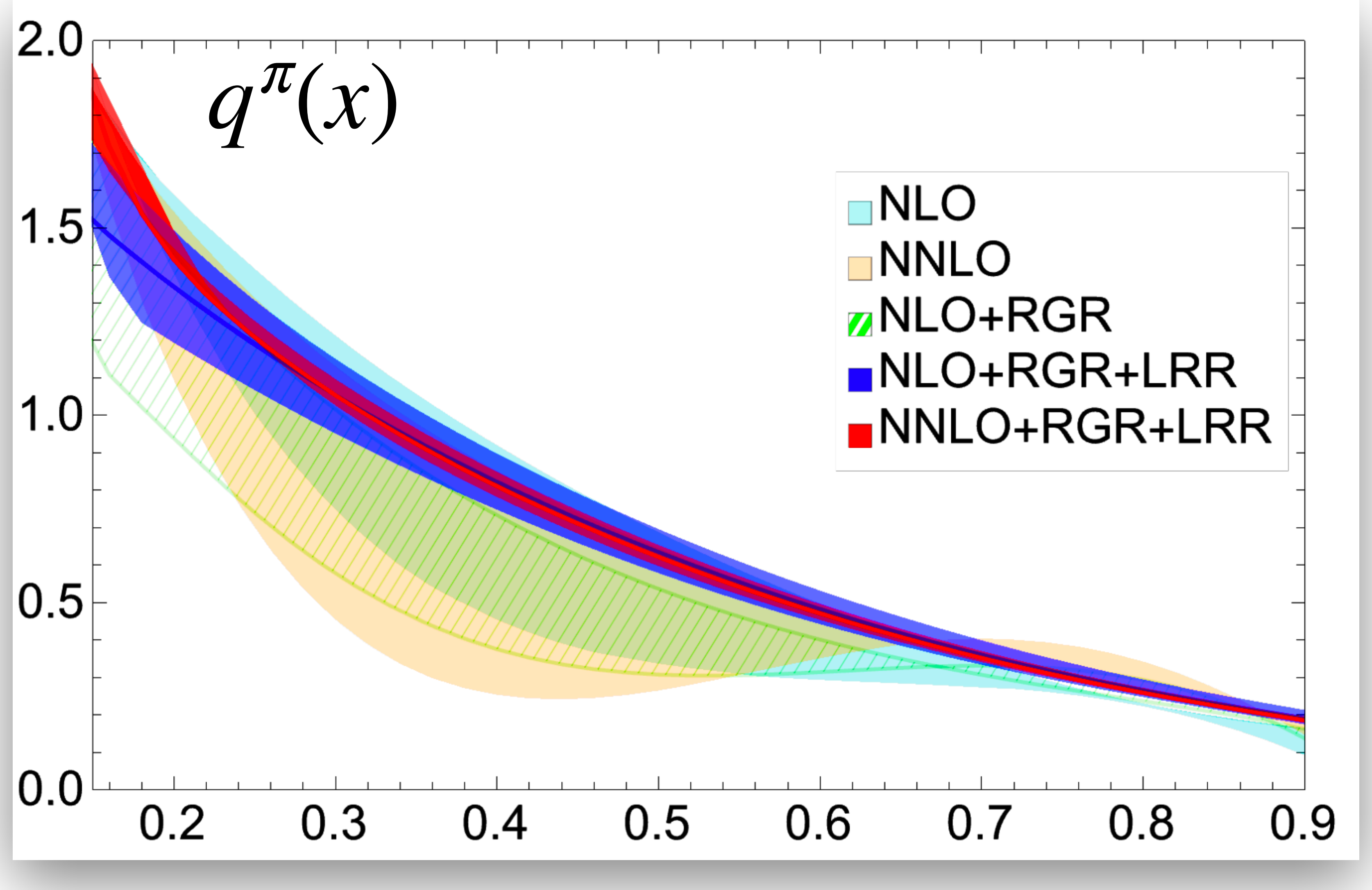
$$\delta m = \frac{1}{a}\sum \alpha_s^{n+1}(a) r_n$$

- The number of diagrams grows, $r_n \sim n!$
- Divergent for any α_s
- No well-defined sum

renormlon ambiguity

$$\delta m(a) = m_{-1}(a)/a + m_0$$

Leading renormalon resummation



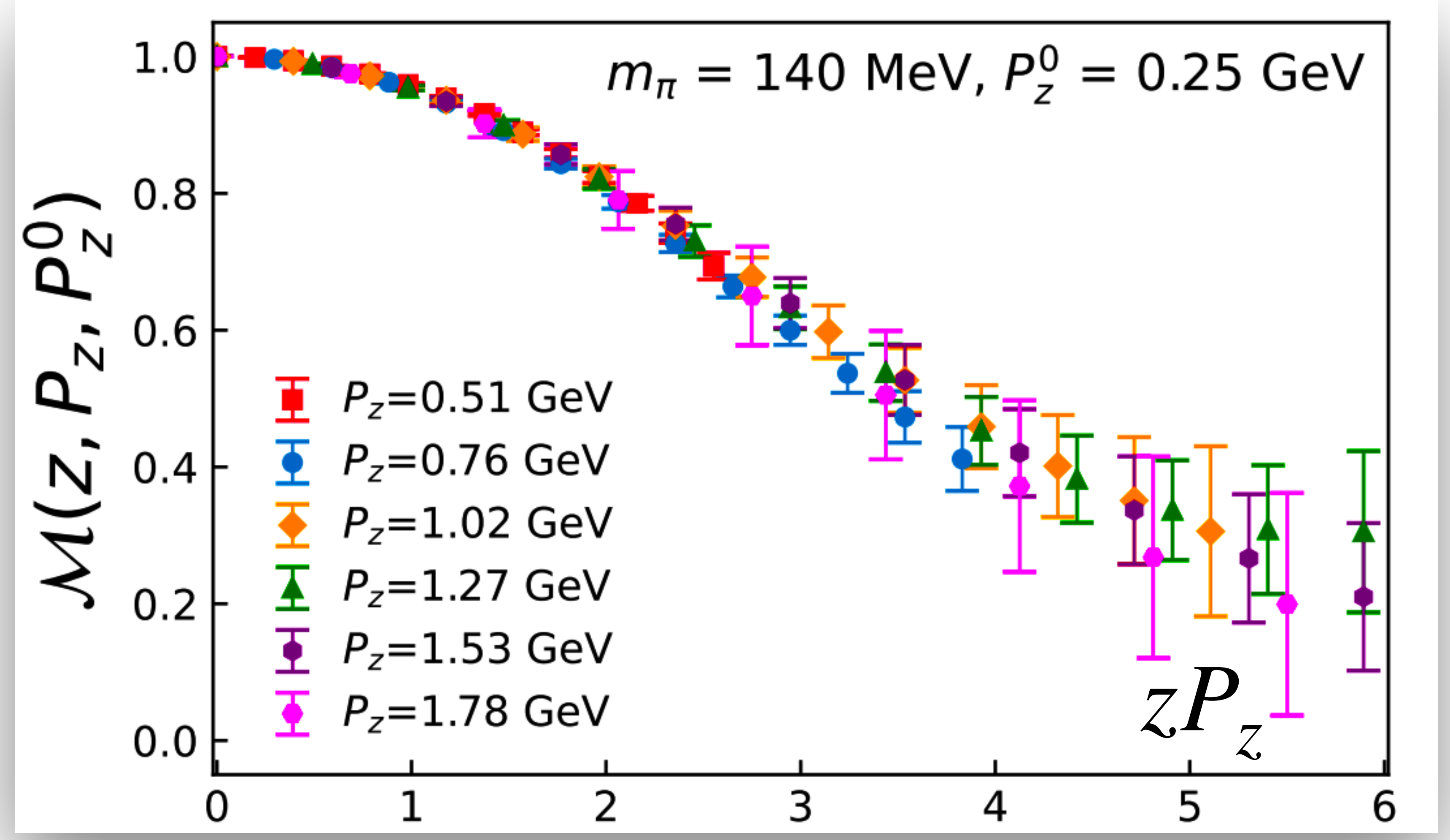
• R. Zhang, et al., Phys.Lett.B 844 (2023) 138081

Yushan Su, Tue 4:00 PM

- The leading renormalon cause a $\mathcal{O}(\Lambda_{\mathrm{QCD}}/P_z)$ uncertainty of qPDF.
- Eliminate the uncertainty by choosing the mass renormalization parameter consistently with the resummation scheme of the infrared-renormalon series.
- Significantly reduces the (theoretical) uncertainty from linear corrections.

Perturbative matching

Renormalized matrix elements of pion

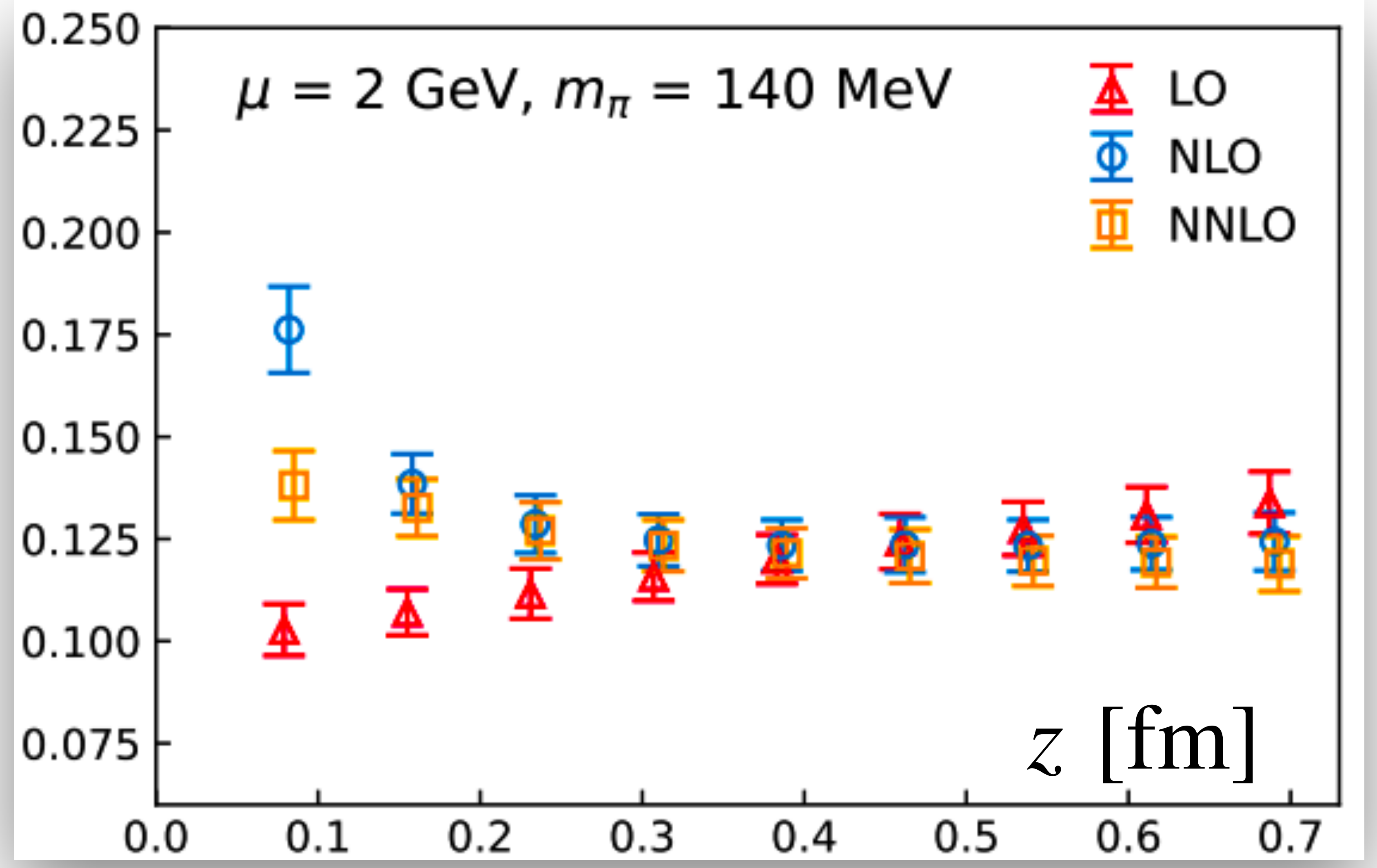


Clover on HISQ, $m_\pi = 140$ MeV,
$p \lesssim 1.78$ GeV, $a = 0.076$ fm

$$\tilde{h}(z, P_z, \mu) = \tilde{h}(z^2, \lambda, \mu)$$
$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2)\langle x^n \rangle(\mu) + \mathcal{O}(z^2\Lambda_{\mathrm{QCD}}^2)$$

- Moments $\langle x^n \rangle(\mu)$ from LO show clear z dependence.
- NLO correction compensate the z dependence well within current statistical errors.
- NNLO correction make improvement particularly when the nature scale $1/z \gg \mu$.

$\langle x^2 \rangle_\pi^{val}$ from LO, NLO, NNLO



- X. Gao et al., BNL+ANL, Phys.Rev.D 106 (2022), 114510

TMDs: perturbative matching and systematics

Quasi beam function — Collin Soper kernel — Physical TMD

$$\frac{\tilde{f}(x,\vec{b}_T,\mu,P_z)}{\sqrt{S_r(\vec{b}_T,\mu)}} = C(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda^2_{\text{QCD}}}{(xP_z)^2}]\}$$

Reduced soft factor

Matching coefficient $C(\mu, xP_z)$:

- **Independent of spin.**
- **No quark-gluon mixing.**

- A. Vladimirov, A. Schäfer Phys.Rev.D 101 (2020), 074517
- I. Stewart, Y. Zhao et al., JHEP 09 (2020) 099
- X. Ji et al., Phys.Rev.D 103 (2021) 7, 074005
- I. Stewart, Y. Zhao et al., JHEP 08 (2022) 084

Systematics:

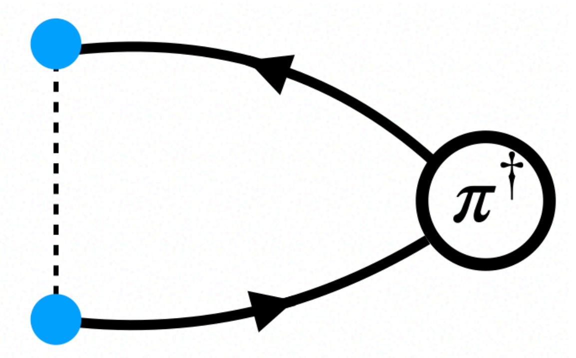
Kuan Zhang, Thu 4:40 PM

Gregoris Spanoudes, Thu 4:20 PM

- **Renormalization and operator mixing.**
- **Perturbative matching: NLO, NNLO, NNLL, ...**
- **Power corrections: Wilson line length η, small b_T, next-to-leading power $\sim q_T/Q$ TMD matching.**

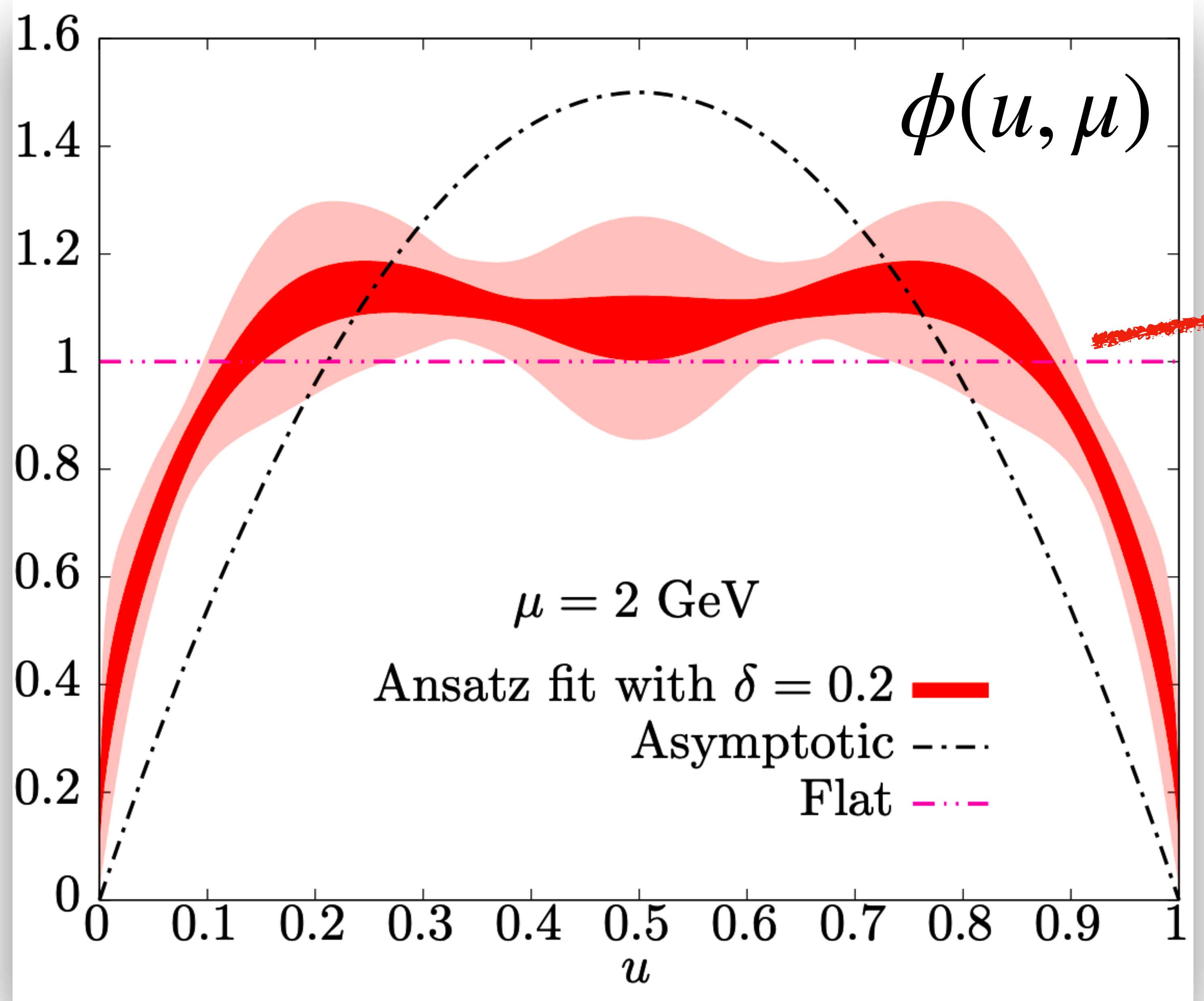
- X. Ji, Y.-Z. Liu, Y.-S. Su, arXiv: 2305.04416
- Ó. Río, A. Vladimirov, arXiv: 2304.14440
- P. Shanahan, et al., arXiv: 2307.12359
- A. Vladimirov, et al., JHEP 01 (2022) 110
- A. Vladimirov, et al., arXiv: 2211.04494

Distribution amplitudes



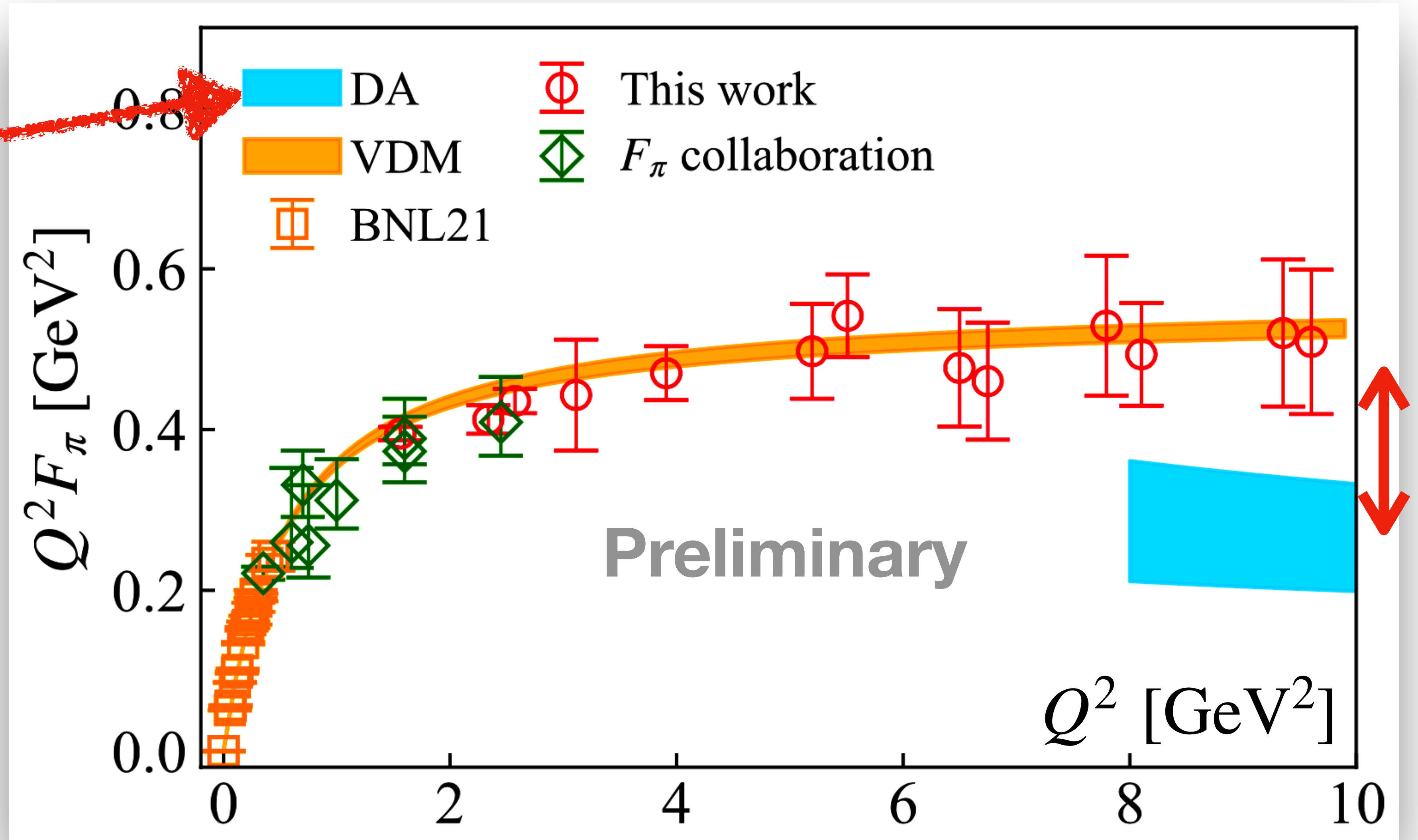
- Pion DAs captures the overlap of the pion with a state of two collinear valence quark carrying momentum fraction x and $(1-x)$.
- Key input to the collinear factorization for exclusive QCD processes, e.g. FFs.

$$F_\pi(Q^2) = \mathcal{N}\int_0^1\int_0^1 dxdy\ \phi^*(v,\mu_F^2)T_F^V(u,v,Q^2,\mu_R^2,\mu_F^2)\phi(u,\mu_F^2)+\mathcal{O}(\frac{1}{Q^2})$$



Same lattice

Clover on HISQ,
$m_\pi = 140$ MeV,
$a = 0.076$ fm



Not consistent

- X. Gao, et al., (BNL+ANL), Phys.Rev.D 106 (2022) 7, 074505
- Q. Shi, X. Gao, et al., (BNL+ANL), in preparation

Qi Shi, Tue 3:10 PM