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LATTICE CONFERENCE 02 AUGUST 2023

FINDING VORTICES IN THE BKT TRANSITION OF 2D FERMI GASES

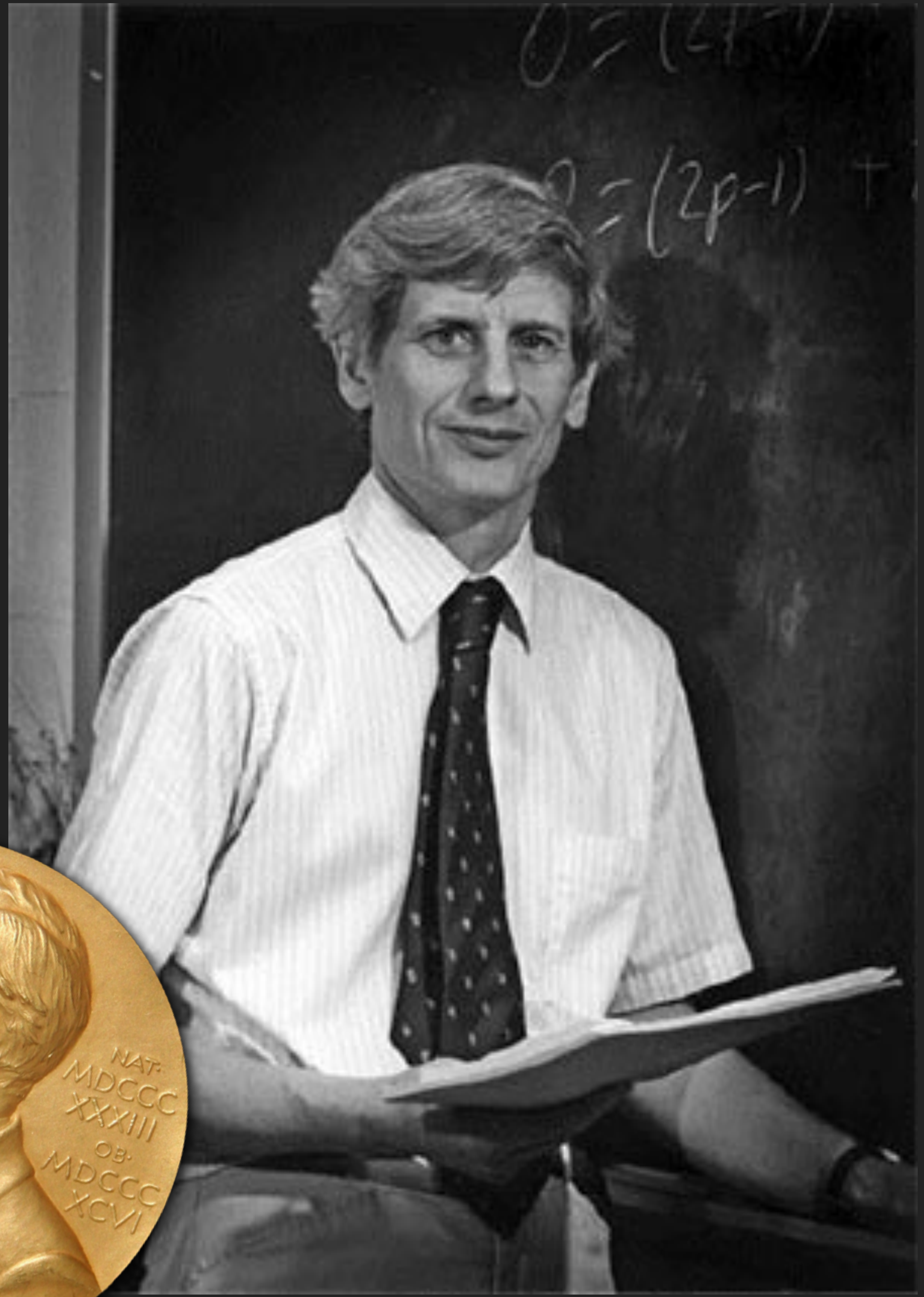
CODE AVAILABILITY

https://github.com/evanberkowitz/two-dimensional-gasses

https://two-dimensional-gasses.readthedocs.io/

THE BKT TRANSITION

A little about BKT:



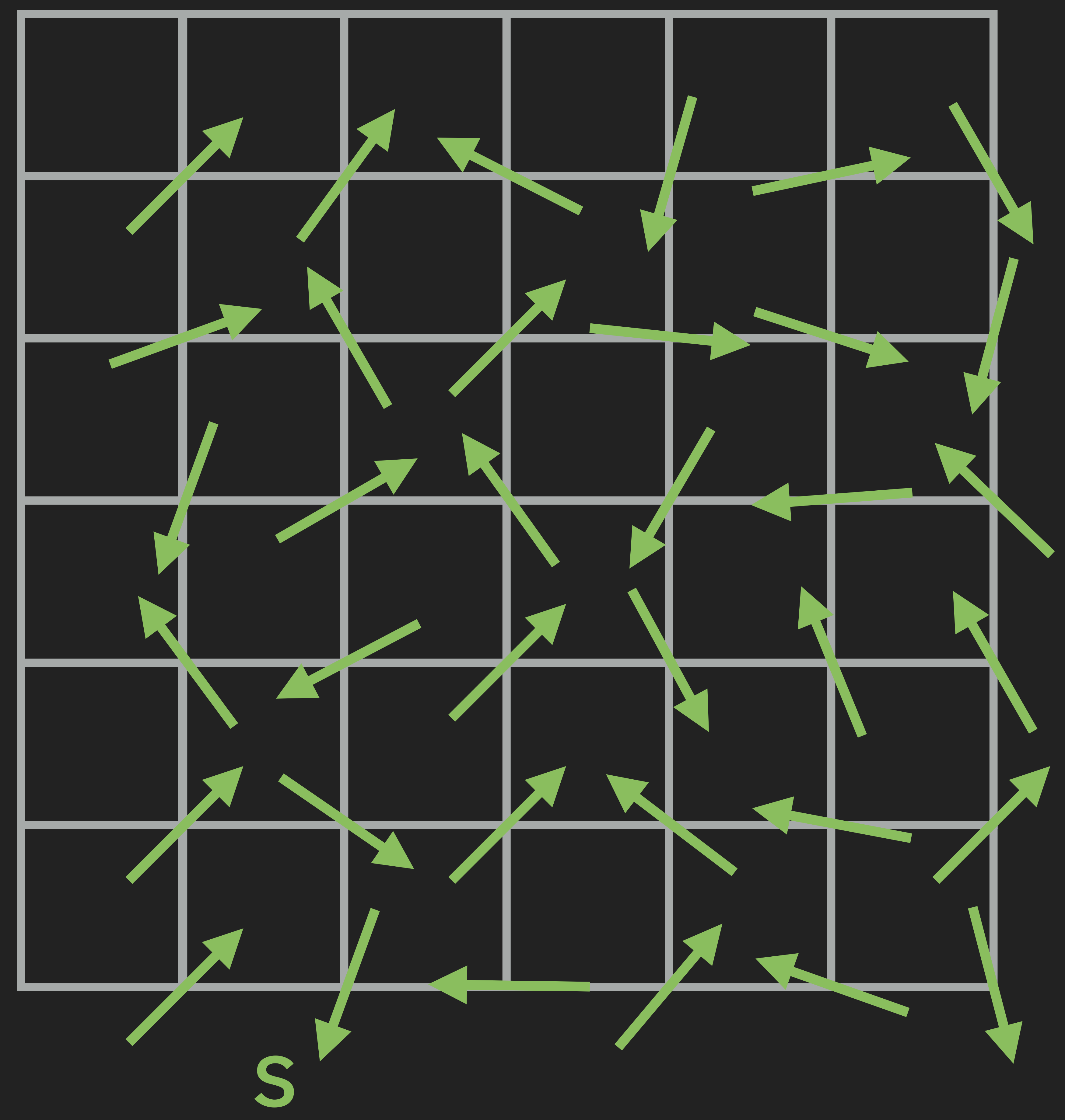
Vadim L'Vovich Berezinskii **J. Michael Kosterlitz** **David J. Thouless**

BASIC KNOWLEDGE TIME

Березинский ЖЭТФ (1970)
Kosterlitz + Thouless 10.1088/0022-3719/6/7/010. (1972)

- Finite-temperature phase transition in 2D
- Happens in the XY model, cold fermi gases, arrays of Josephson junctions, ...

$$H = -J\sum_{\langle ij\rangle} s_i \cdot s_j = -J\sum_{\langle ij\rangle}\cos(\theta_i - \theta_j)$$

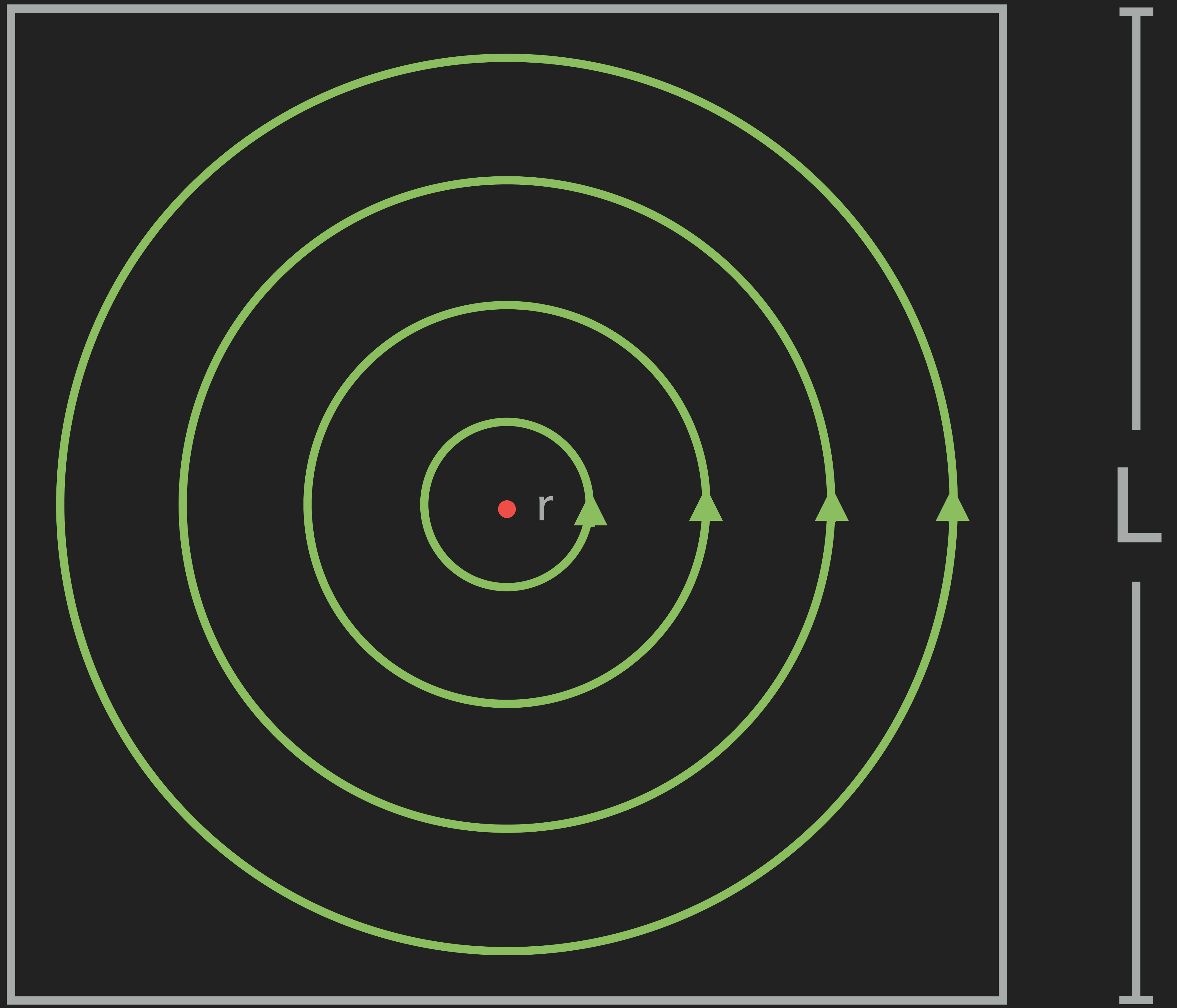


- Divergent critical exponents (!)
- No local order parameters!
- *Topological*

TURNING AND TURNING IN THE WIDENING GYRE

Березинский ЖЭТФ (1970)
Kosterlitz + Thouless 10.1088/0022-3719/6/7/010. (1972)

$$H = -J\sum_{\langle ij\rangle}\cos(\theta_i - \theta_j) \simeq H_0 + \frac{J}{2}\int d^2x \; |\nabla\theta|^2$$



Vortex: $\int d\vec{\ell}\cdot\nabla\theta = 2\pi n$

$$\nabla\theta \sim \frac{n}{r}$$

Energy: $E \sim J\pi n^2 \log(L/r)$

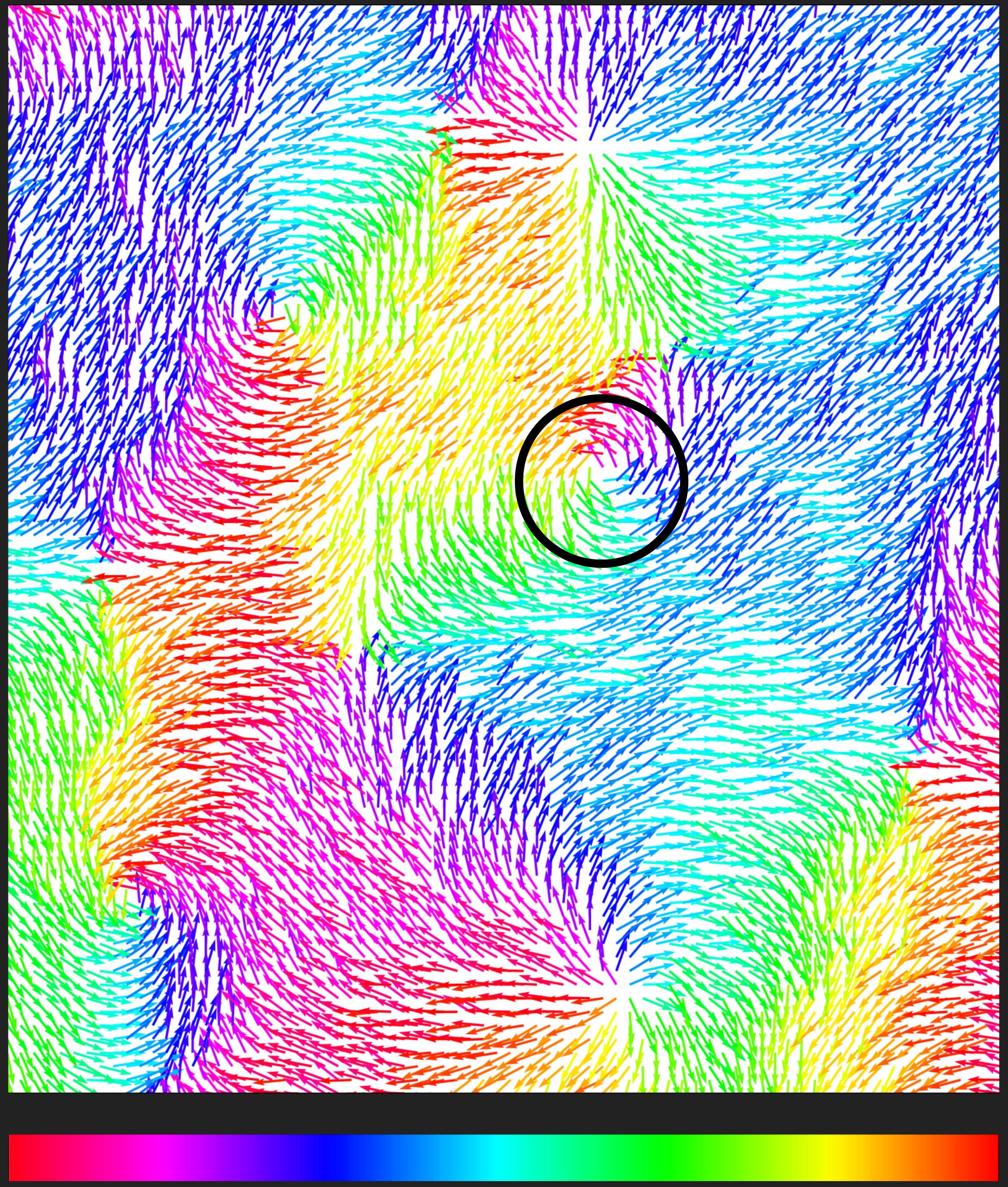
Entropy: $S = \log\Omega \sim \log(L^2/r^2)$

Free Energy: $F = E - TS$

No vortices at all — T/J — Vortices everywhere

TURNING AND TURNING IN THE WIDENING GYRE

T/J = 0.4 on 250^2



θ

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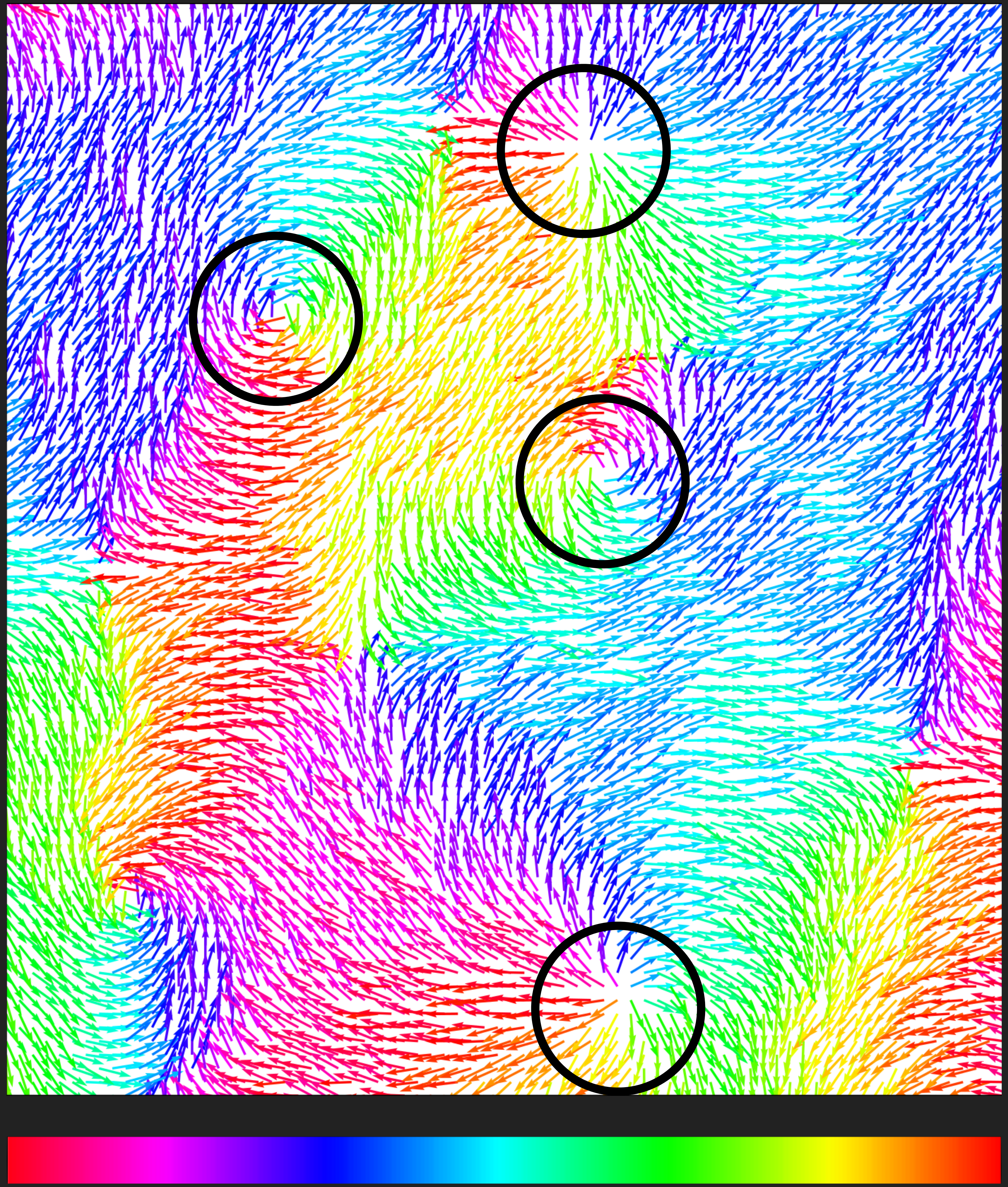
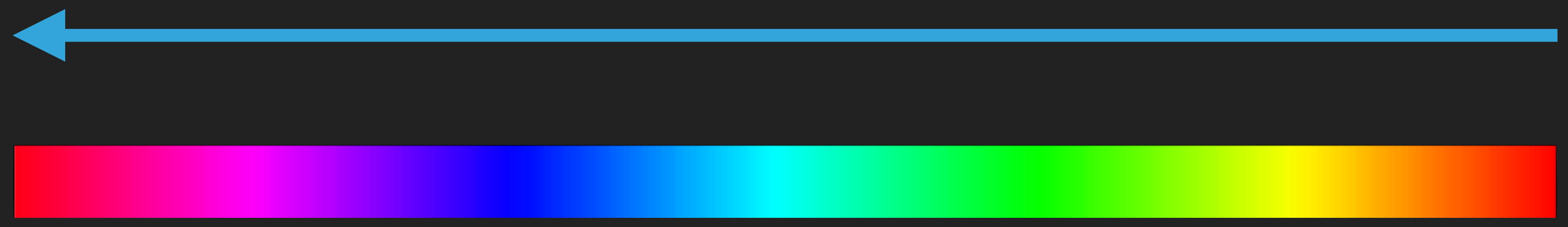


θ

vortex



$$\int d\vec{\ell} \cdot \nabla\theta \rightarrow \sum_i d\hat{x}_i \cdot (s_{i+\hat{x}} - s_i)$$

TURNING AND TURNING IN THE WIDENING GYRE

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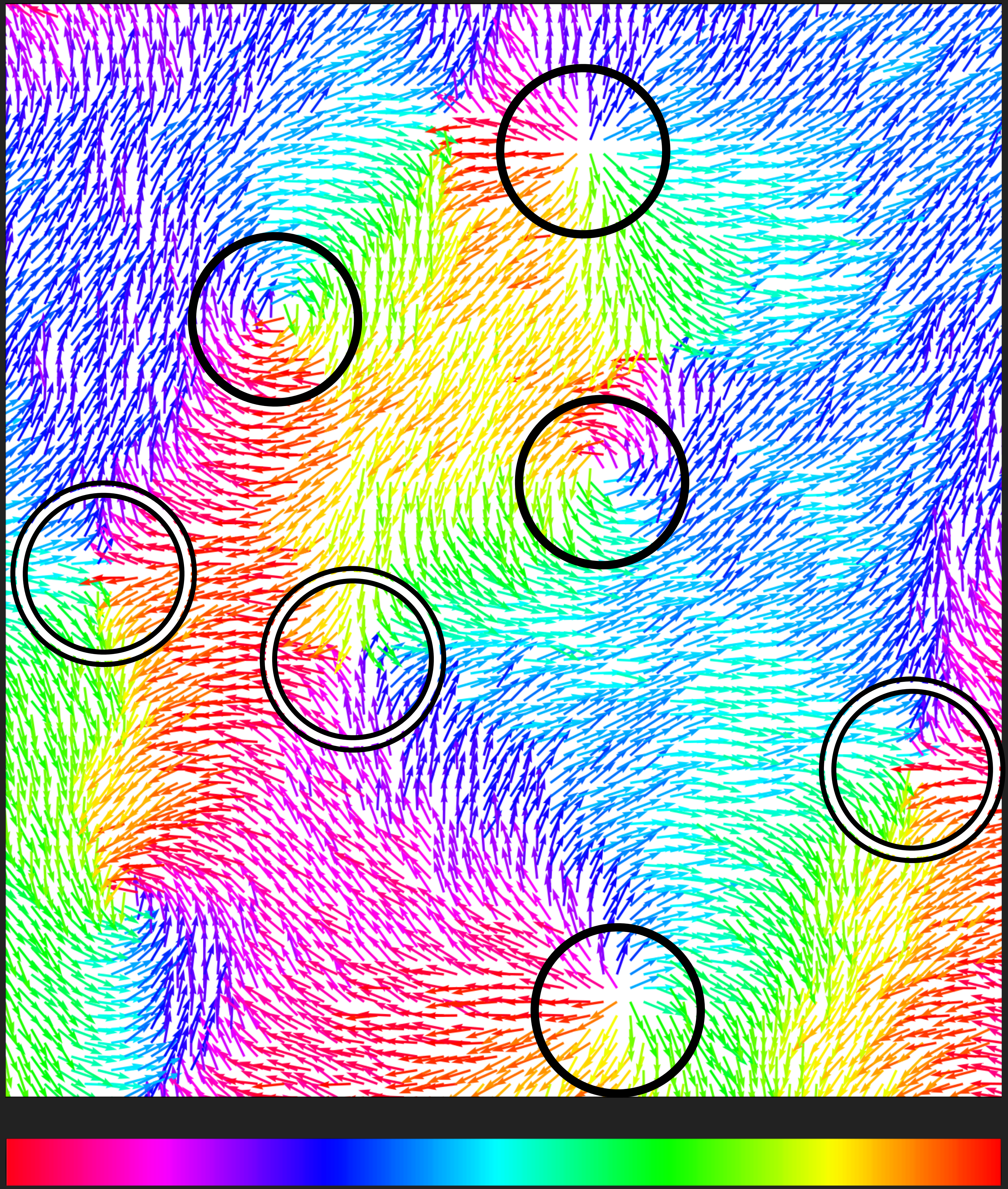


θ

- No local order parameters!
- *Topological*

$$\int d\vec{\ell} \cdot \nabla \theta \rightarrow \sum_i d\hat{x}_i \cdot (s_{i+\hat{x}} - s_i)$$

FERMI GASSES IN 2D

NONRELATIVISTIC 2D FERMI GASES

- $[\psi] = [L]^{-1}$
- $C_0 M$ is dimensionless

Kinetic　　Contact

$$H = \int d^2x \, \frac{(\nabla\psi)^\dagger \cdot (\nabla\psi)}{2M} + C_0(\psi^\dagger\psi)^2$$

- 2D two-particle scattering amplitude

$$T = \frac{4/M}{\cot\delta_0(k) - i}$$

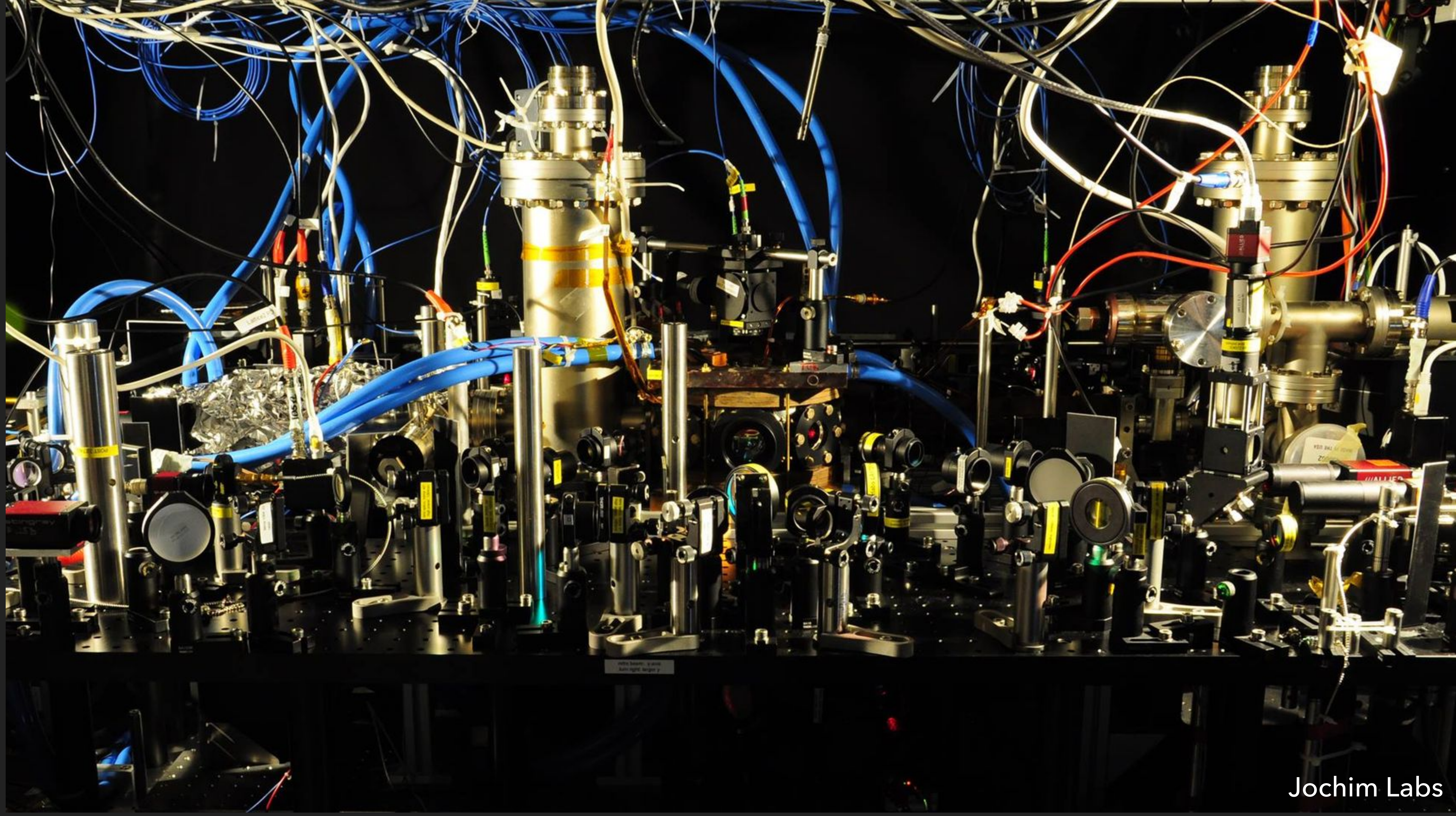
- Low-energy EFT; 'universal'
- But also: a UV fixed point!

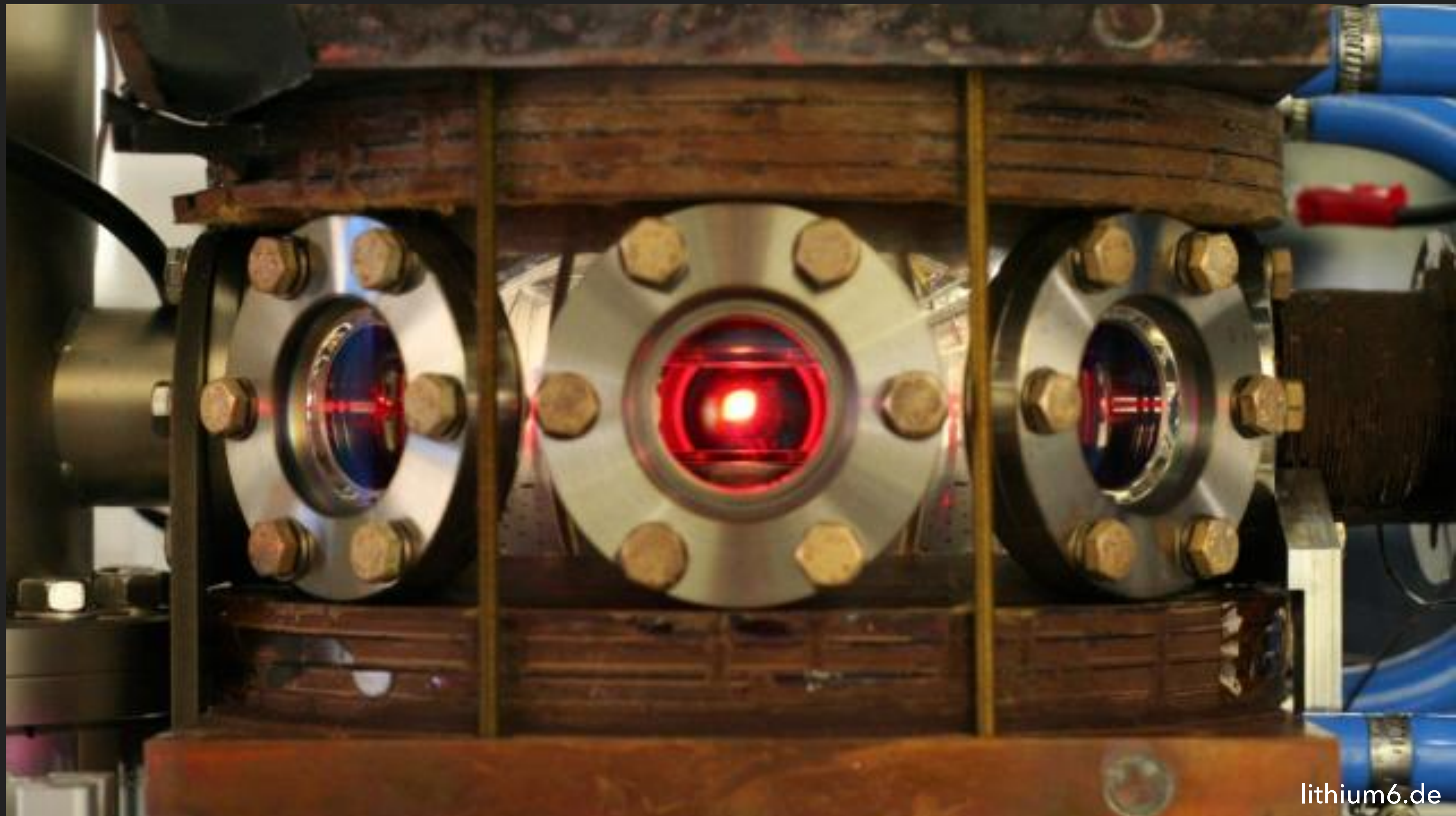
$$\cot\delta_0(k) = \frac{2}{\pi}\log ka + \mathcal{O}(k^2)$$

Warning: 2 common conventions

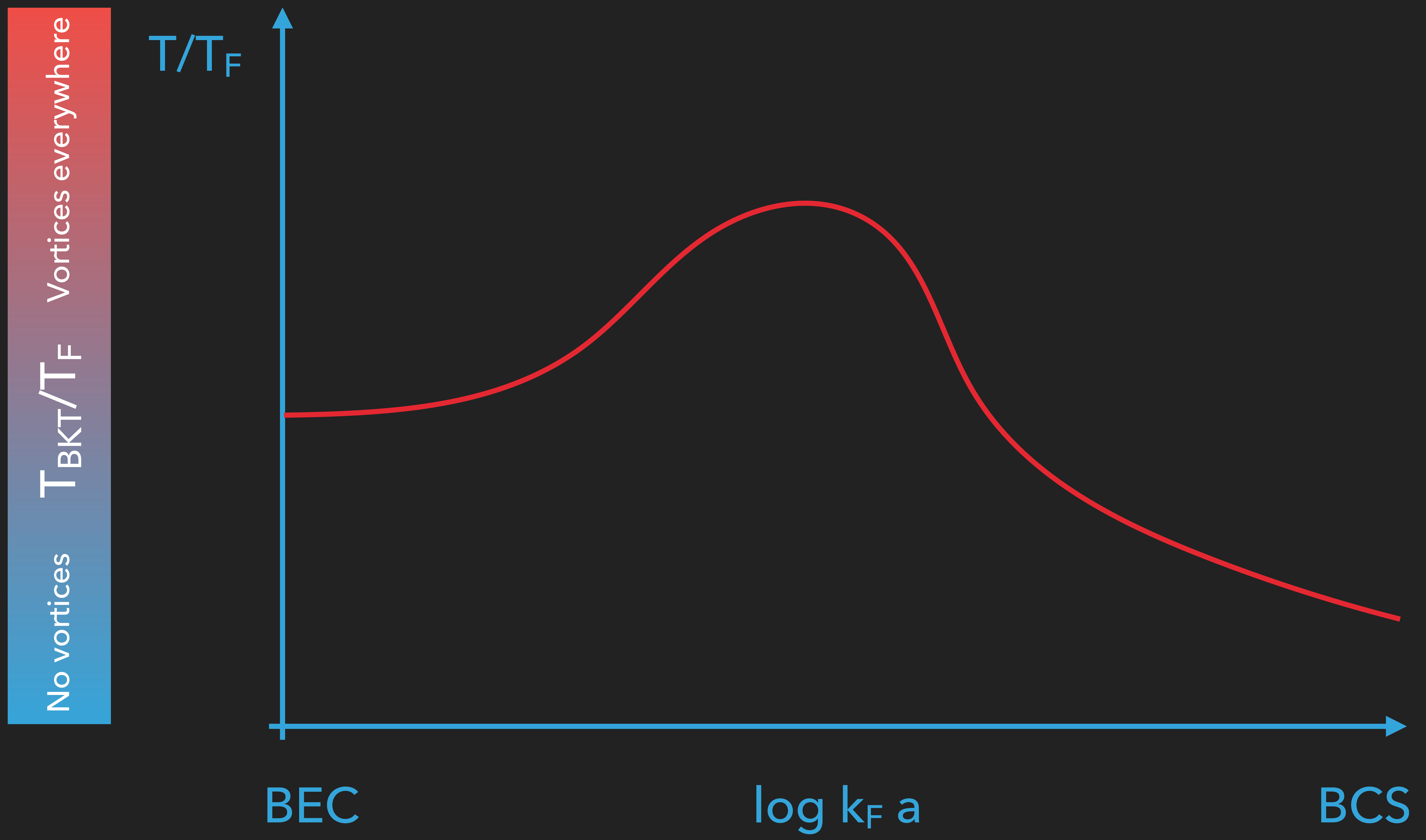
IT'S REAL!

- Trapping!
- Lasers!
- Feshbach Resonances!
- Ions!



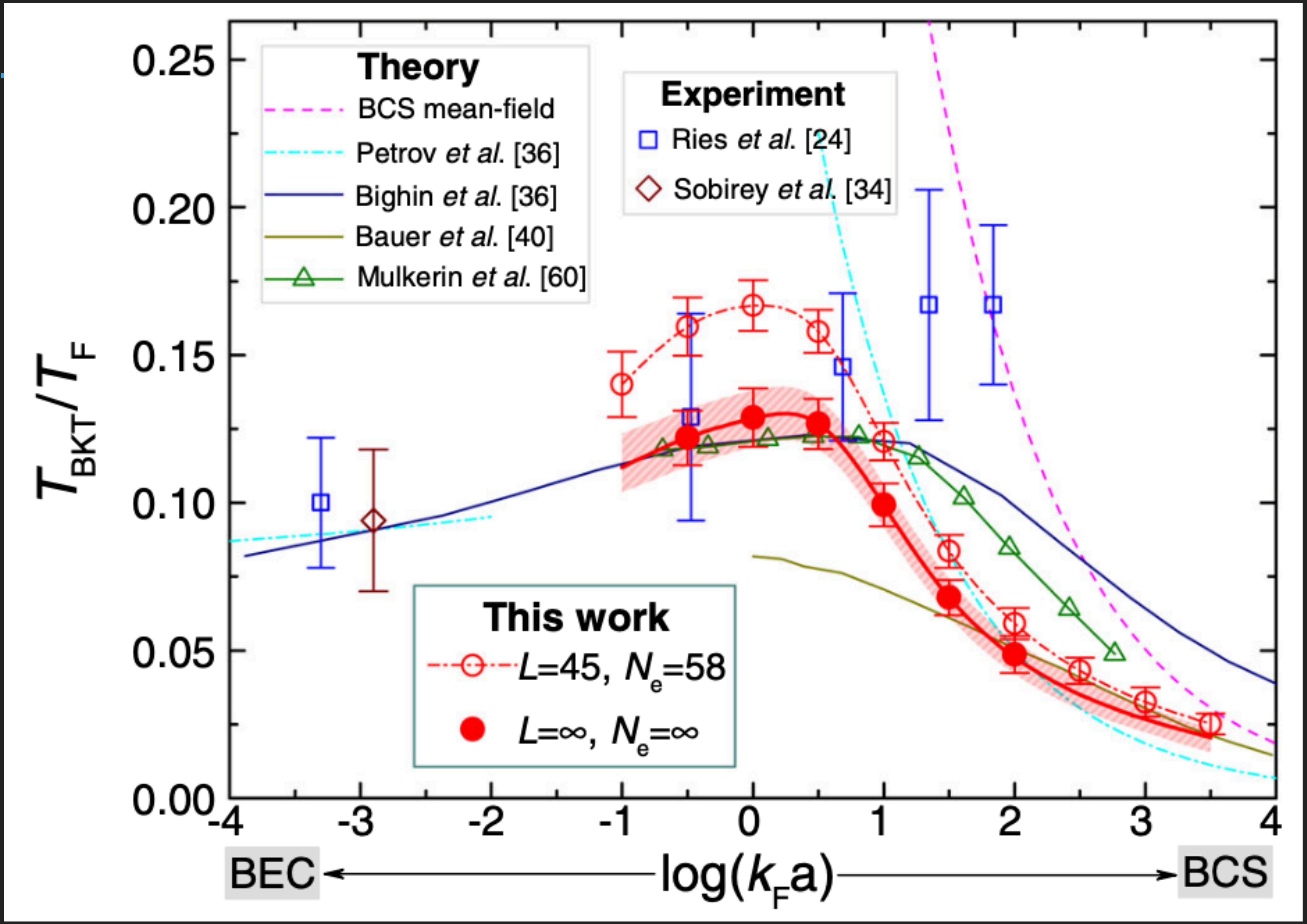


PHASE STRUCTURE



PHASE STRUCTURE

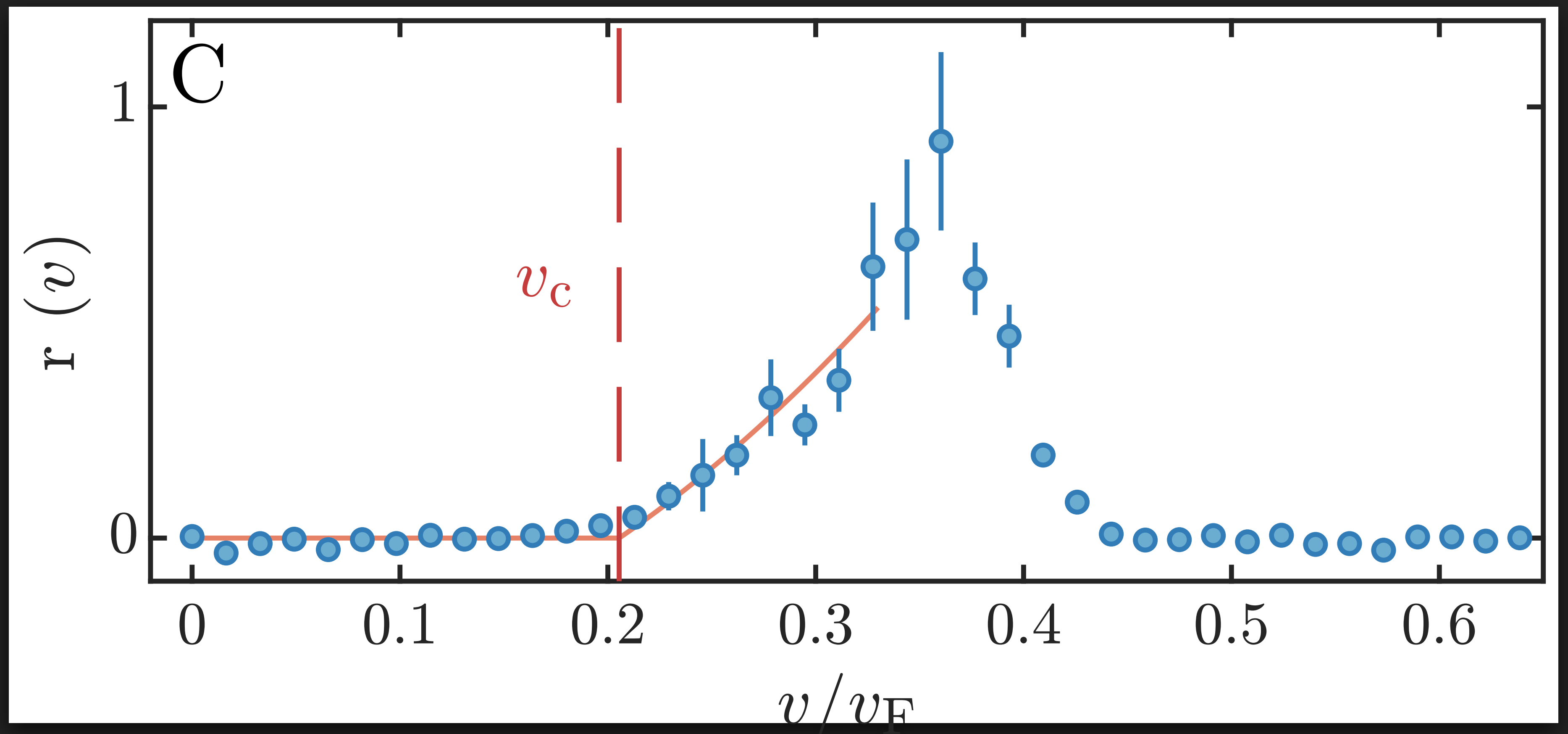
Zhang et. al. (2022) PRL 129, 076403



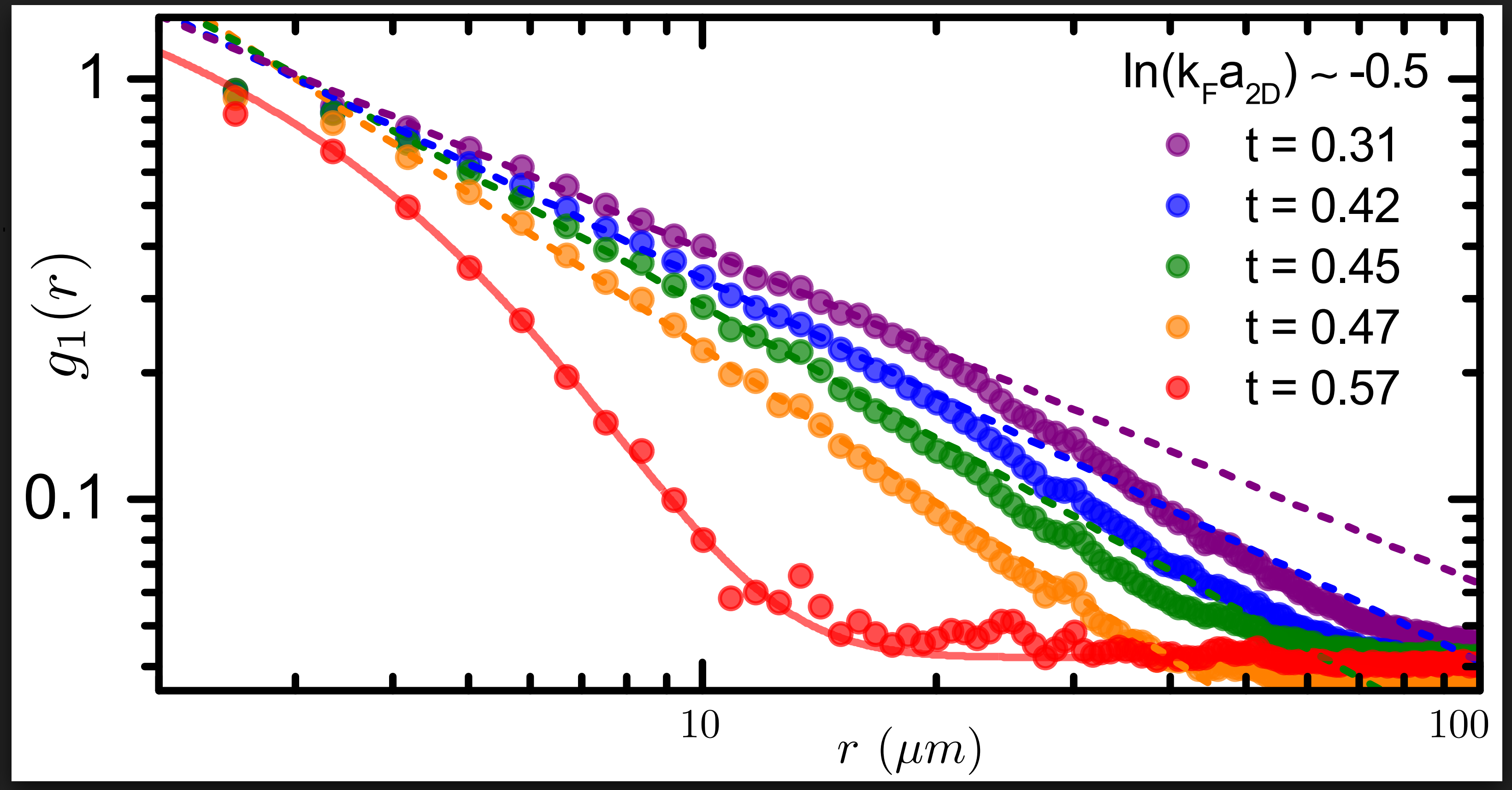
DETECTION STRATEGIES

Sobirey et al. (2021) Science 372
Jochim et. al. (2015) PRL 115, 010401
Zhang et. al. (2022) PRL 129, 076403

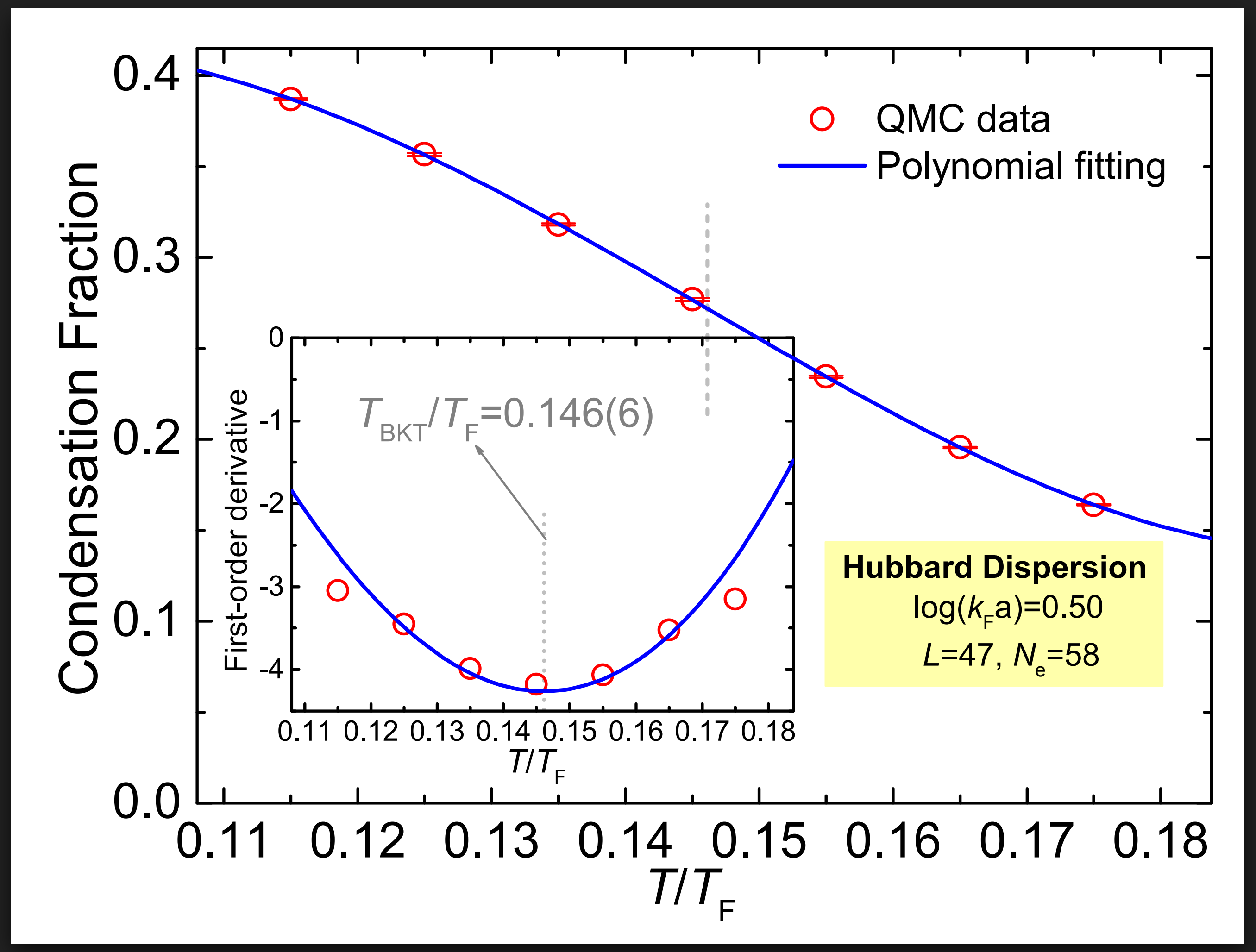
Critical velocity



2-point function



dn_c/dT



WHERE ARE THE VORTICES?

A PROBE OF VORTICITY

Berkowitz + Warrington, 2309.????
Could also examine spin current

- Idea: try to find the vortices!

- What are 'the arrows'?

$$j = -\frac{i}{2M}\left(\psi^\dagger \nabla \psi - \nabla \psi^\dagger \psi\right)$$

Lattice-exact Noether theorem

- What tracks the vortices?

$$\omega = \nabla \times j = -\frac{i}{M} \nabla \psi^\dagger \times \nabla \psi$$

- Example

$$|\chi\rangle = \int d^2x\, \chi(x) \,|\, x\rangle$$

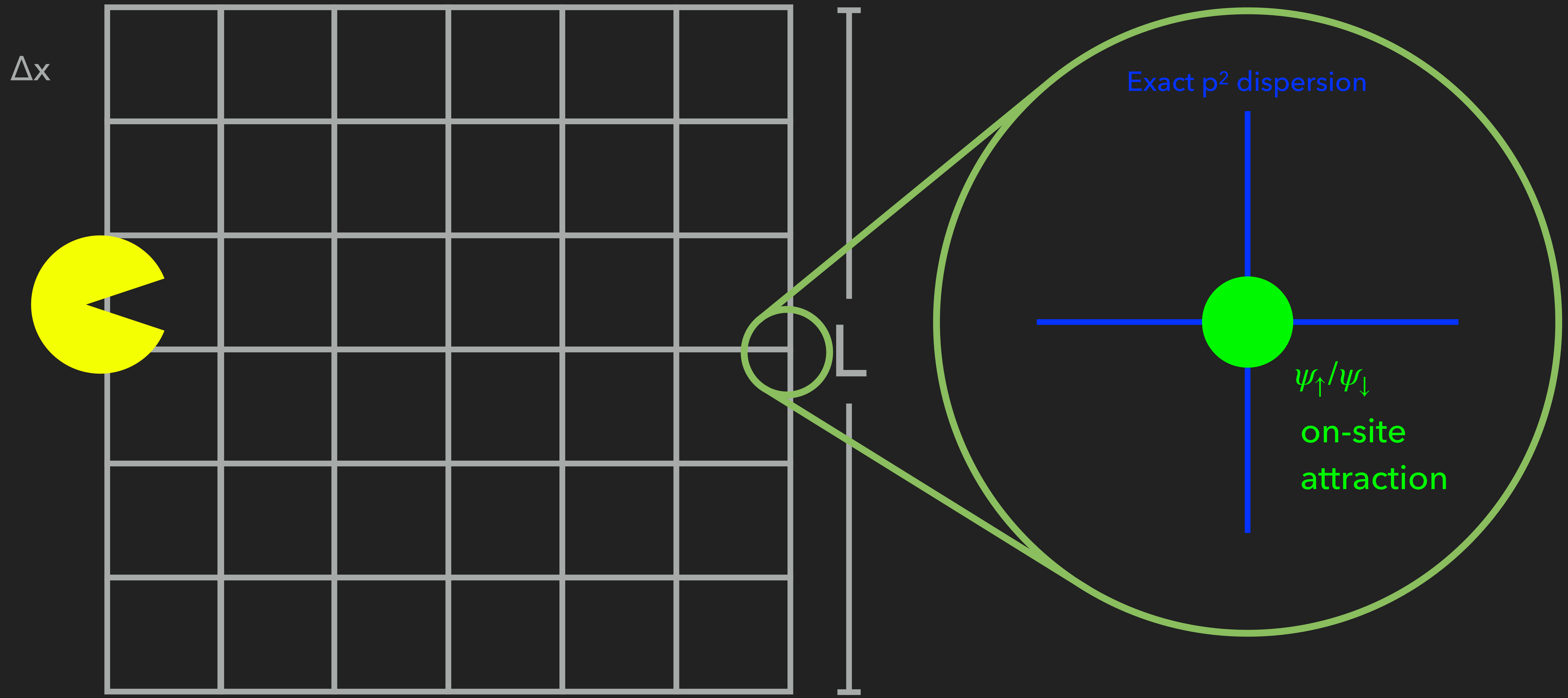
$$\chi(x) = e^{i\ell\theta} f(r)$$

$$\Longrightarrow \quad \langle \chi \,|\, \omega(x) \,|\, \chi \rangle \propto \ell$$

LATTICE APPROACH

TOOLS OF LATTICE FIELD THEORY

$$H = \int d^2x \, \frac{(\nabla \psi)^\dagger \cdot (\nabla \psi)}{2M} + C_0(\psi^\dagger \psi)^2 \rightarrow \sum \Delta x^2 \, \frac{(\nabla \psi)^\dagger \cdot (\nabla \psi)}{2M} + C_0(\psi^\dagger \psi)^2$$



TOOLS OF LATTICE FIELD THEORY

Duane, Kennedy, Pendleton, and Roweth, PLB 195 216-222 (1987)
10.1016/0370-2693(87)91197-X

$$H = \int d^2x \, \frac{(\nabla\psi)^\dagger \cdot (\nabla\psi)}{2M} + C_0(\psi^\dagger\psi)^2 \;\; \rightarrow \sum \Delta x^2 \frac{(\nabla\psi)^\dagger \cdot (\nabla\psi)}{2M} + C_0(\psi^\dagger\psi)^2$$

- Work in the grand-canonical ensemble
- Trotterize + control time discretization
- Eliminate fermions for auxiliary field path integral
- Markov-Chain Monte Carlo / HMC

$$Z = \mathrm{tr}\left[e^{-\beta(H-\mu N)}\right] = Z(\Delta t) + \mathcal{O}(\Delta t^2)$$

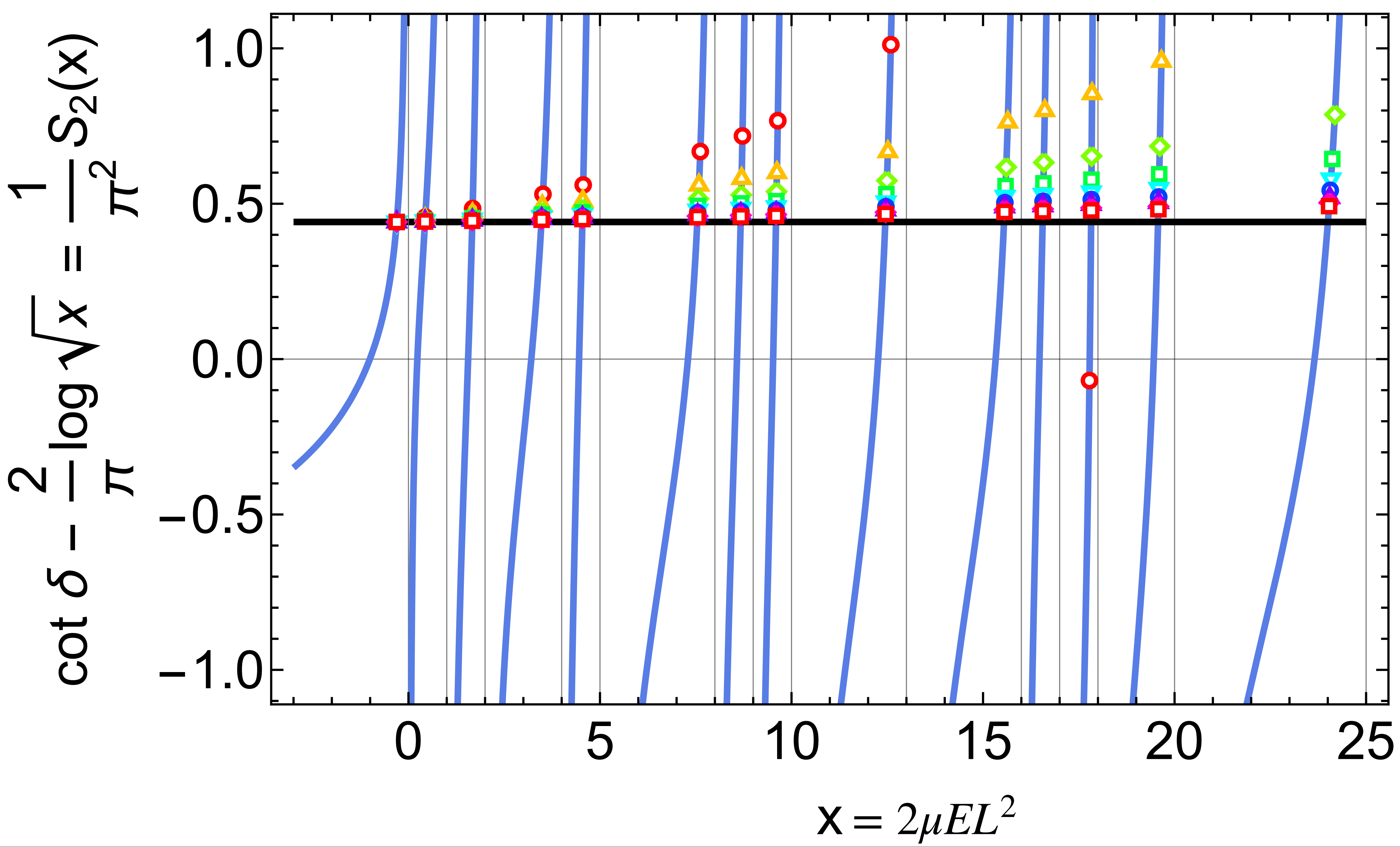
$$Z(\Delta t) = \int \mathcal{D}\phi \; e^{-S(\phi)}$$

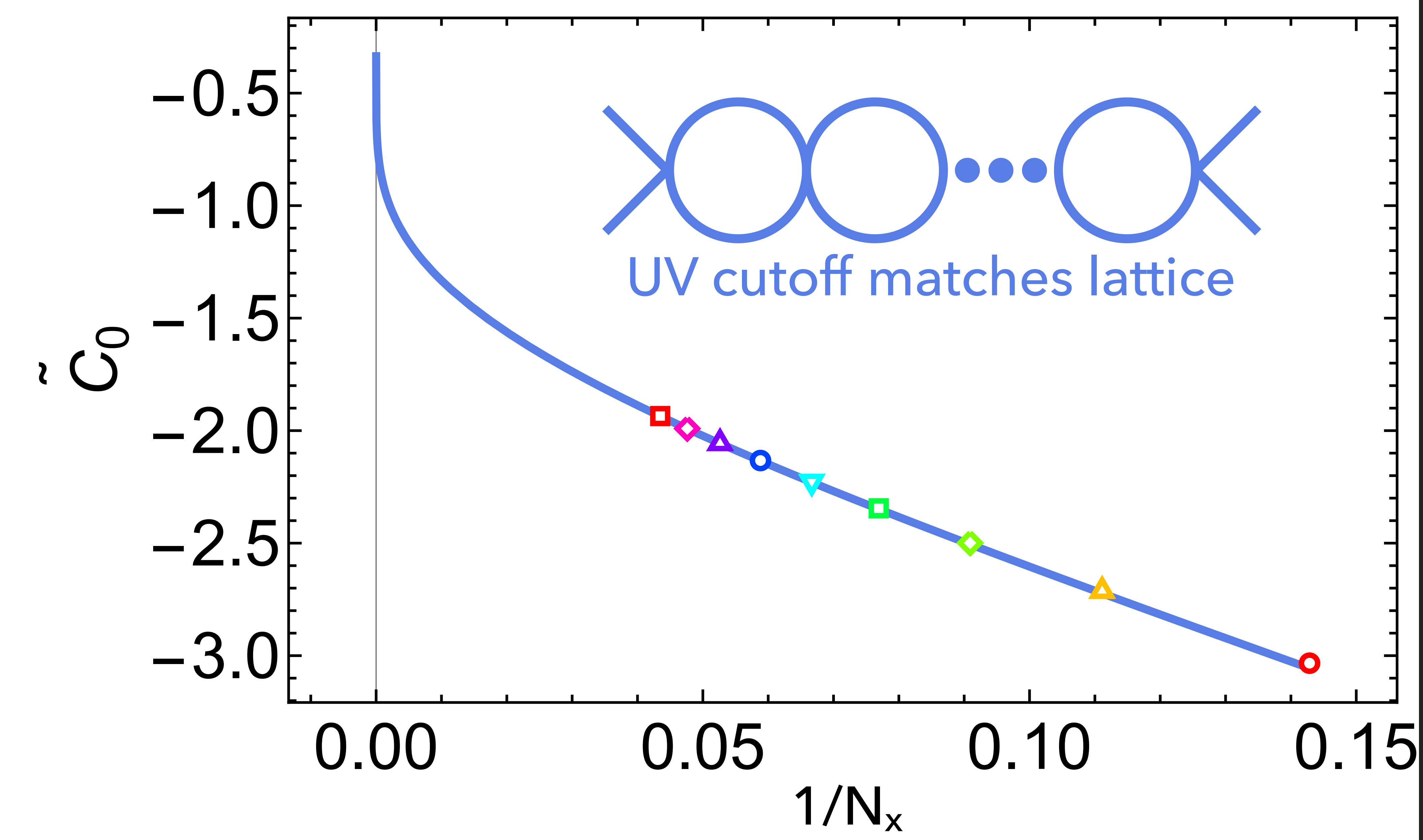
A real equal sign!

LINE OF CONSTANT PHYSICS AND LÜSCHER'S METHOD

Lüscher 10.1007/BF01211589 (1986)
and 10.1007/BF01211097 (1986)
Körber, Berkowitz, Luu 1912.04425

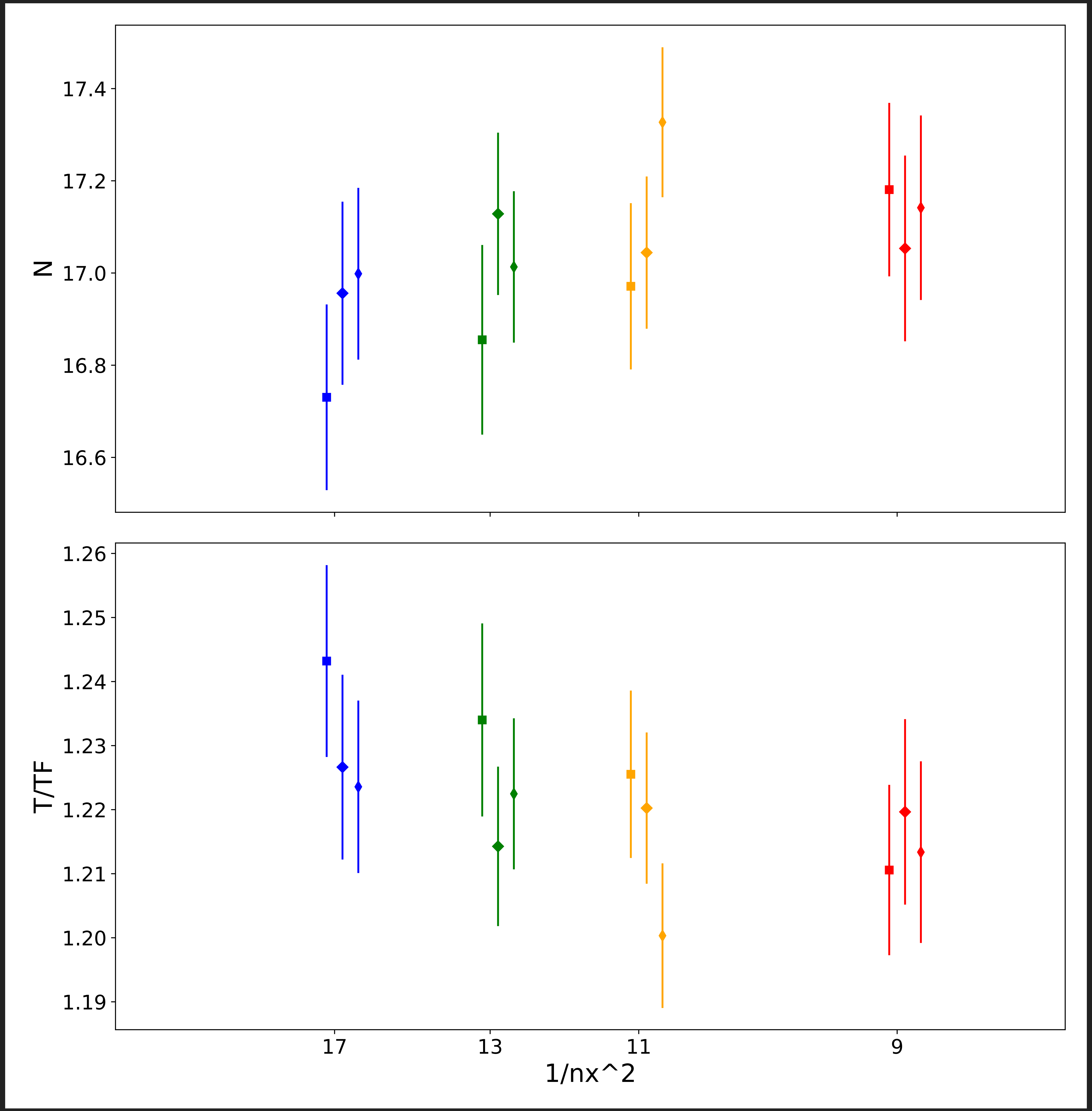
Finite volume 2-body energies $\Longleftrightarrow$ $\cot\delta_0(k) = \frac{2}{\pi}\log ka + \mathcal{O}(k^2)$





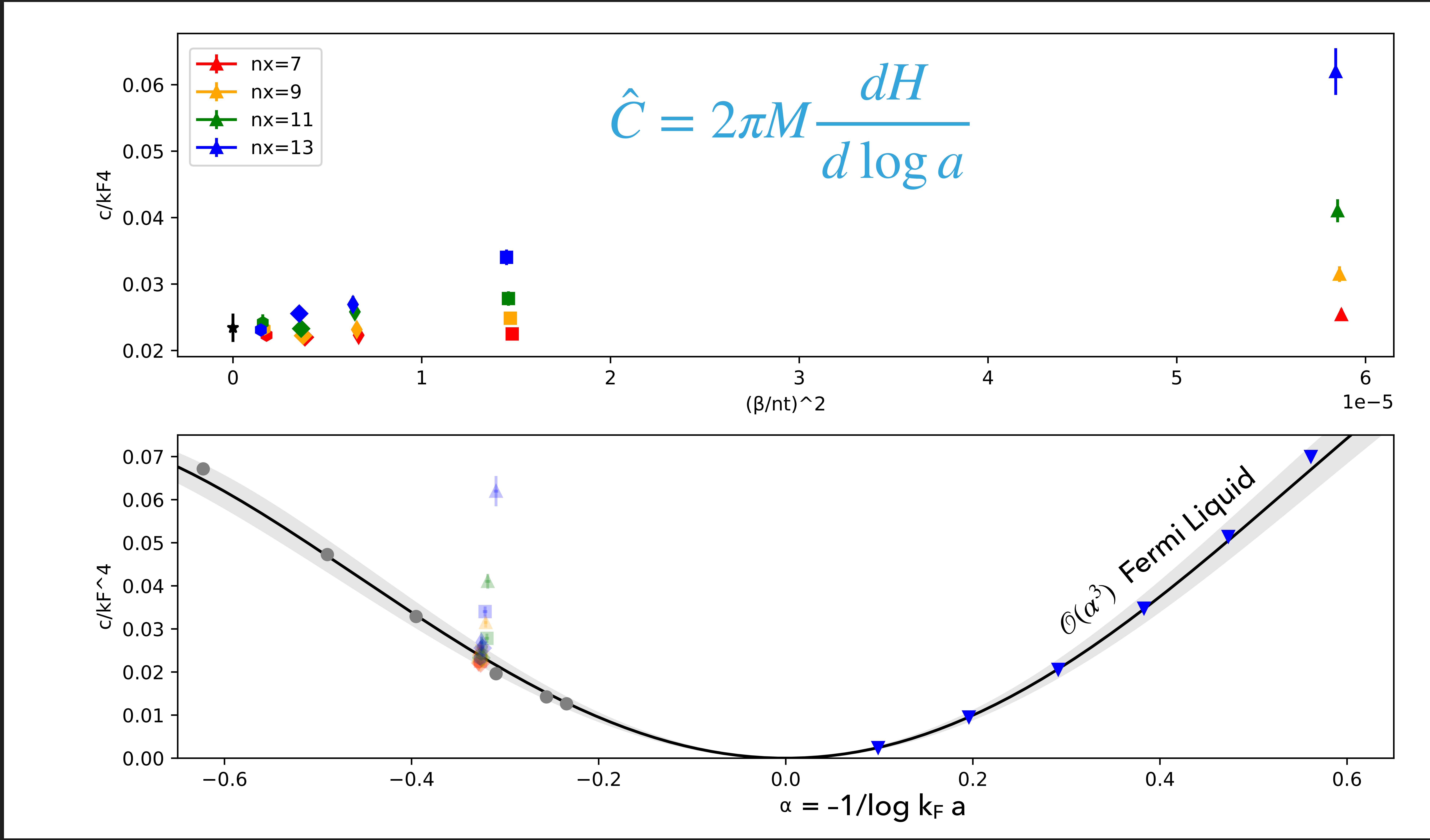
Energies converge with Δx^2

EXAMPLE CONTINUUM BEHAVIOR



SANITY CHECK: TAN'S CONTACT

Bertaina+Giorgini, PRL 106, 110403 (2011)
Bertaina, EPJ Special Topics, 217(1):153–162, (2013)
Beane et. al., PRA 107 043314 (2023)



A PROBE OF VORTICITY

Berkowitz + Warrington, 2309.????
Could also examine spin current

- What tracks the vortices?
- ω(x) is local
- Need ⟨ ω(x) ω(y) ⟩ correlations

  Caution: OPE analysis shows it diverges as $\partial^2\delta(x-y)$

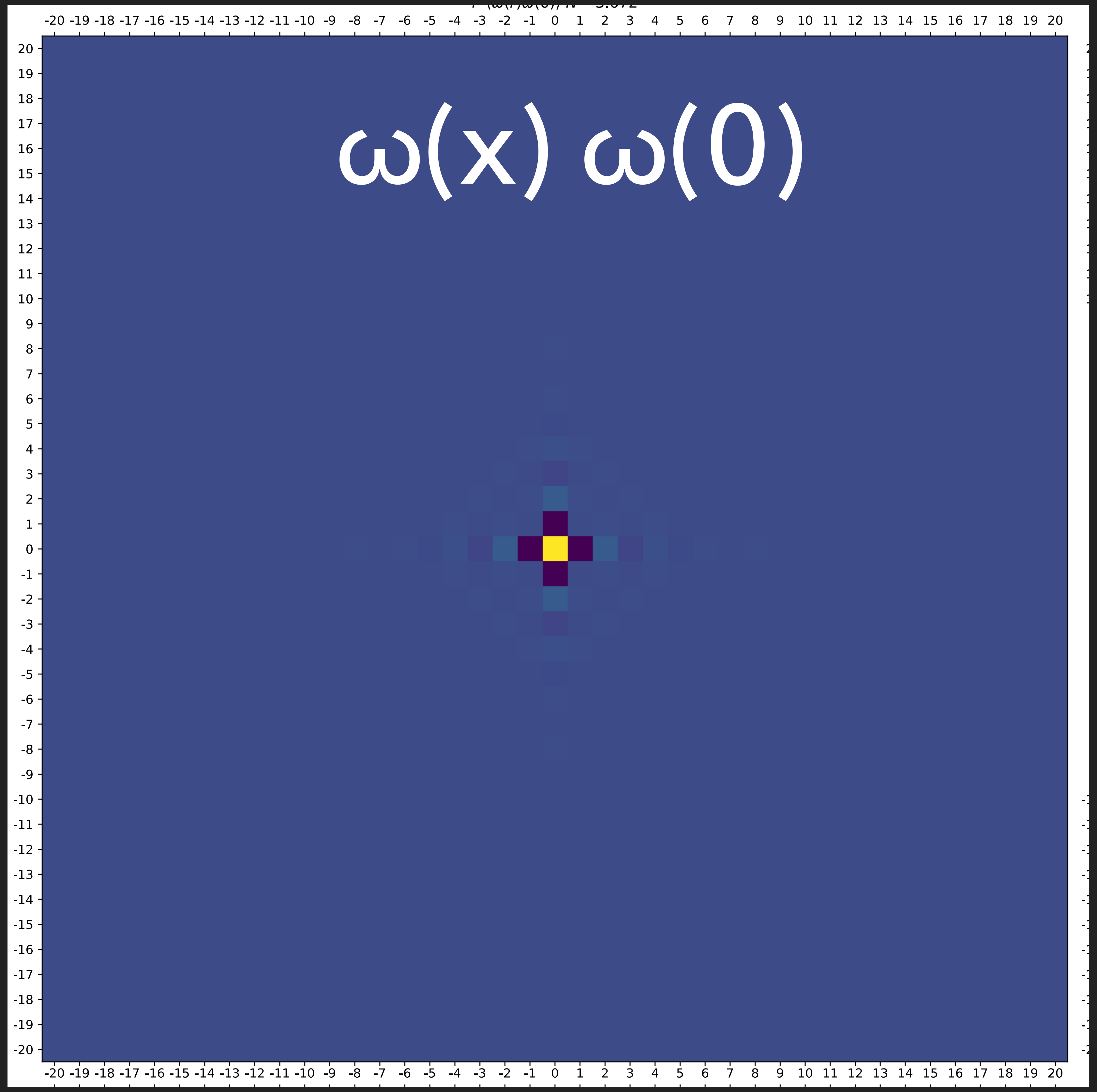
- $$B_n = \int d^2x\, |x|^n \langle \omega(x)\ \omega(0) \rangle$$

  intensive

  n≥2 well-behaved in the continuum

$$\omega = \nabla \times j = -\frac{i}{M} \nabla \psi^\dagger \times \nabla \psi$$

$$\sum_x \Delta x^2 \omega(x) = 0$$ with periodic boundary conditions



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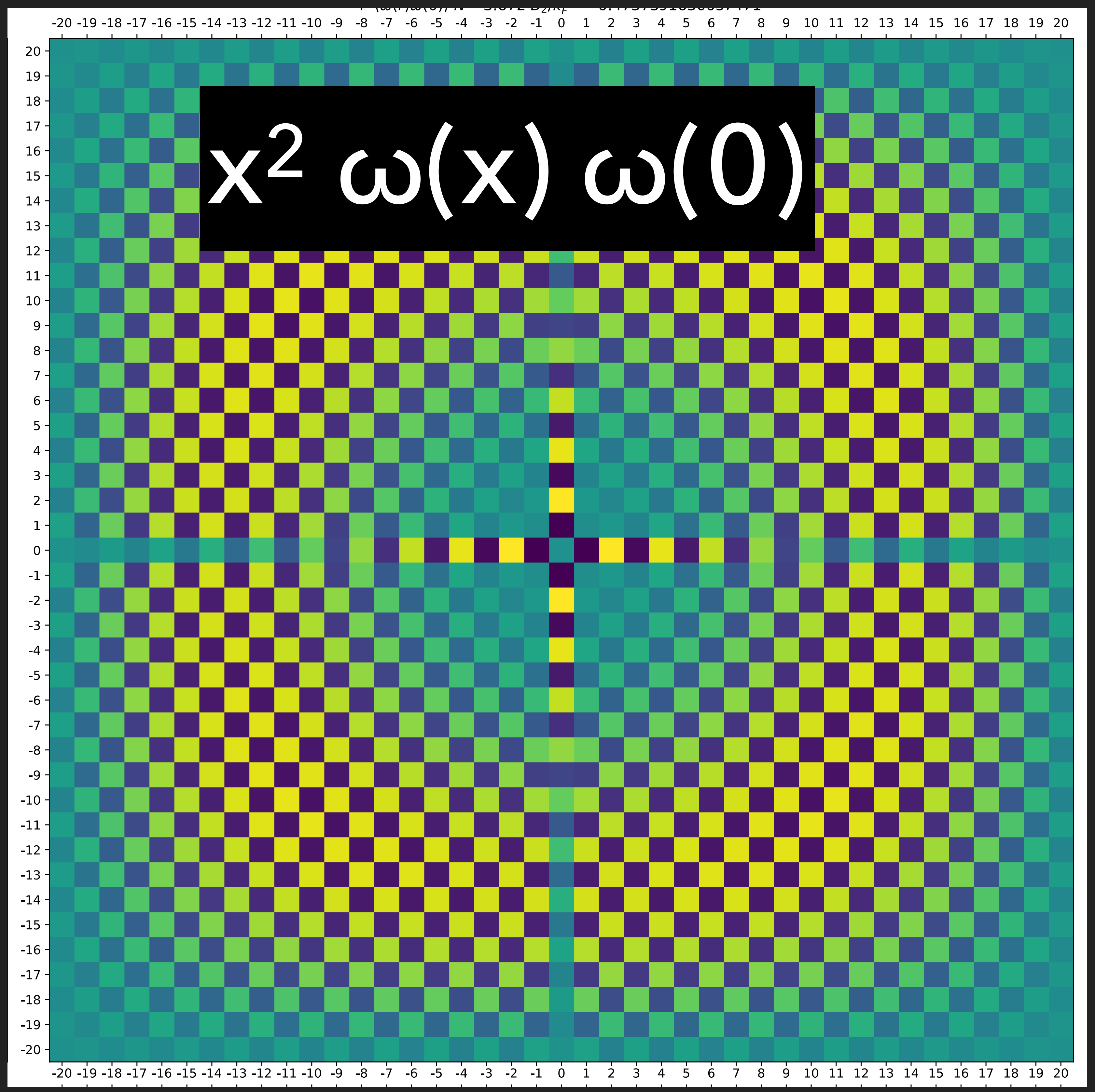
intensive
$n \geq 2$ well-behaved in the continuum



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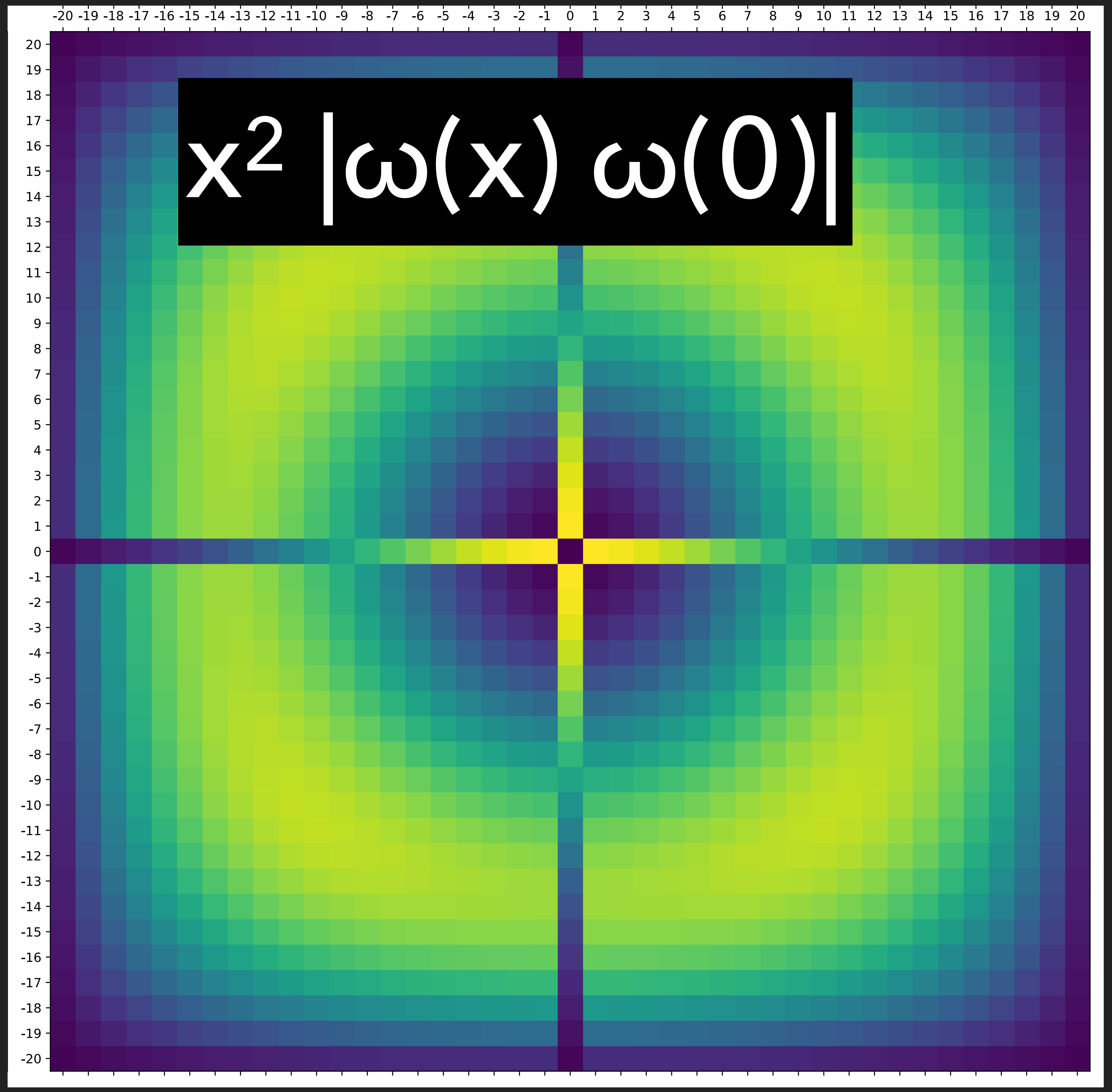
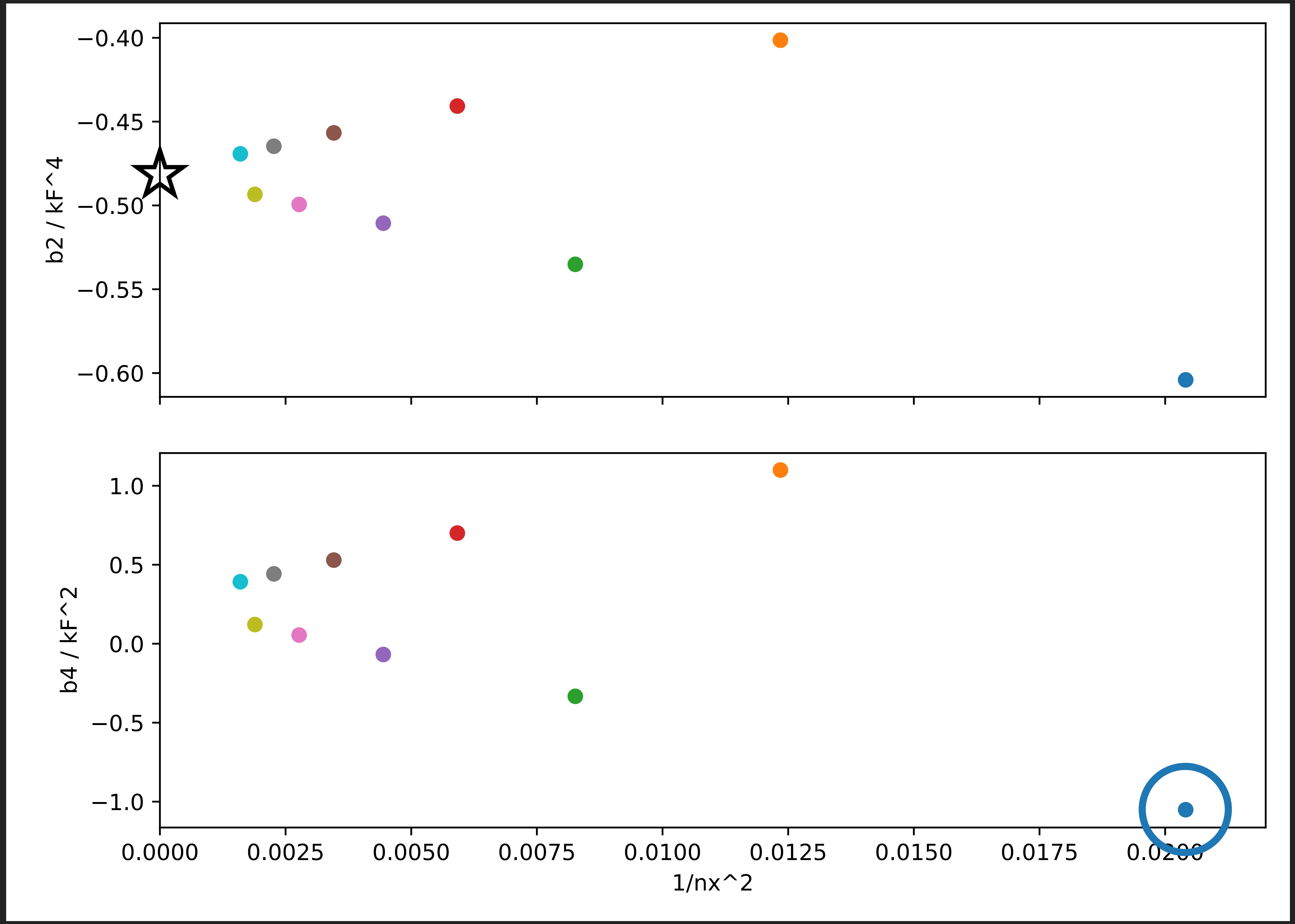


LIMITS OF VORTICITY PROBES

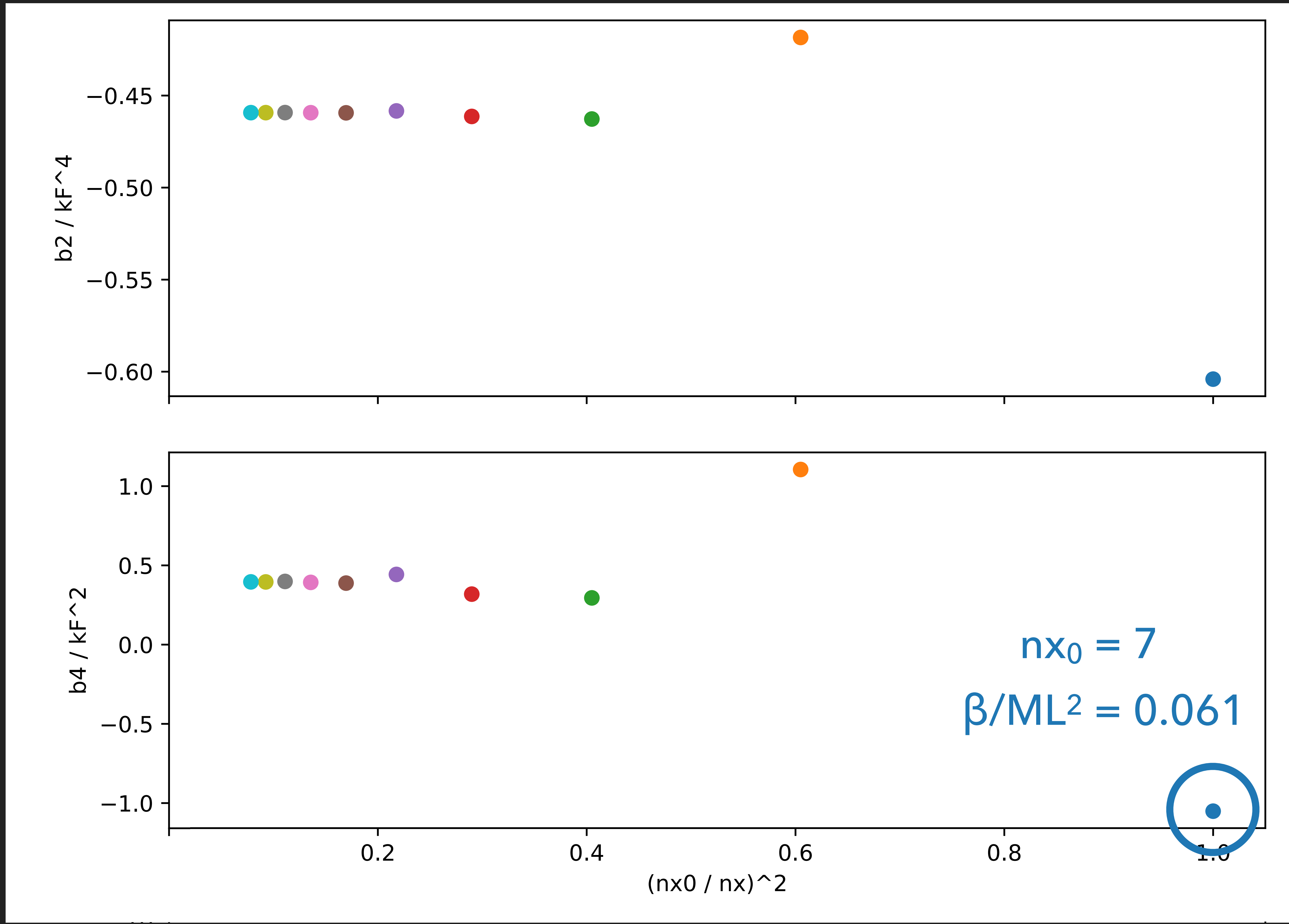
Berkowitz + Warrington 2309.????

Free theory, μ=0, $\beta/ML^2 = 0.061$



Spatial continuum

Free theory, μ=0



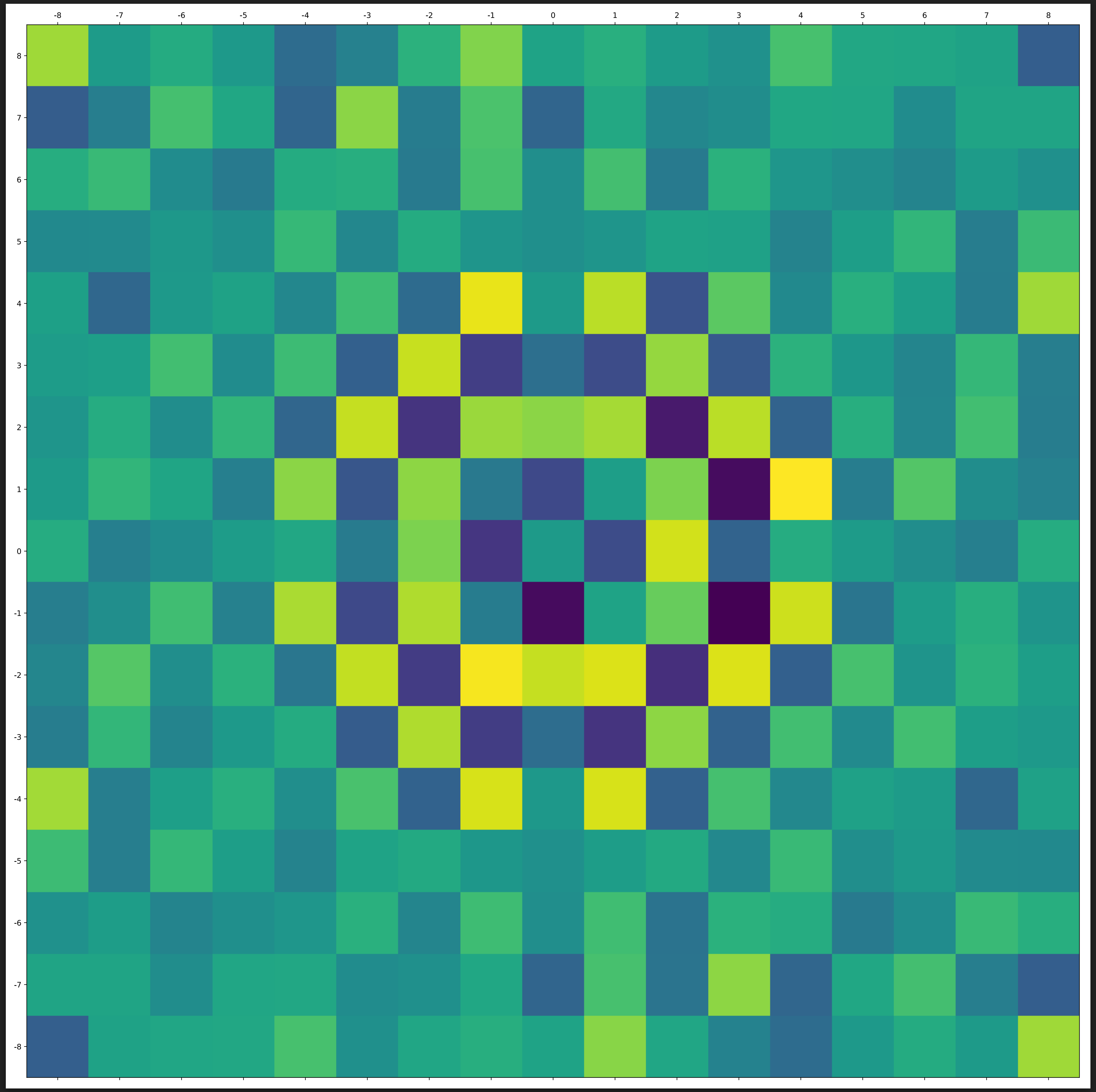
Fixed lattice spacing
Infinite volume

FIRST RESULTS!

VORTICITY CORRELATIONS

Berkowitz + Warrington 2309.????

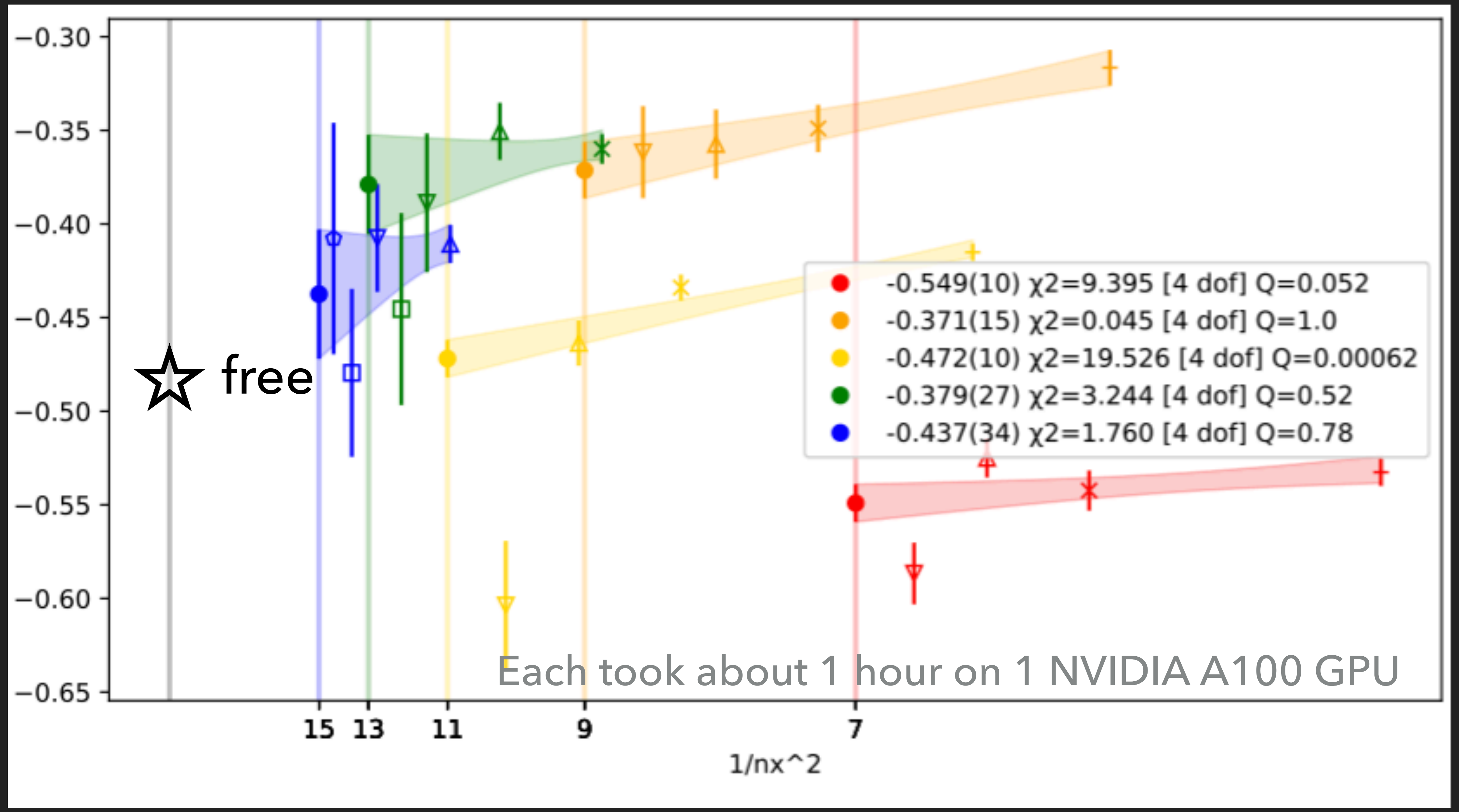
$x^2 \langle \omega(x)\,\omega(0) \rangle$

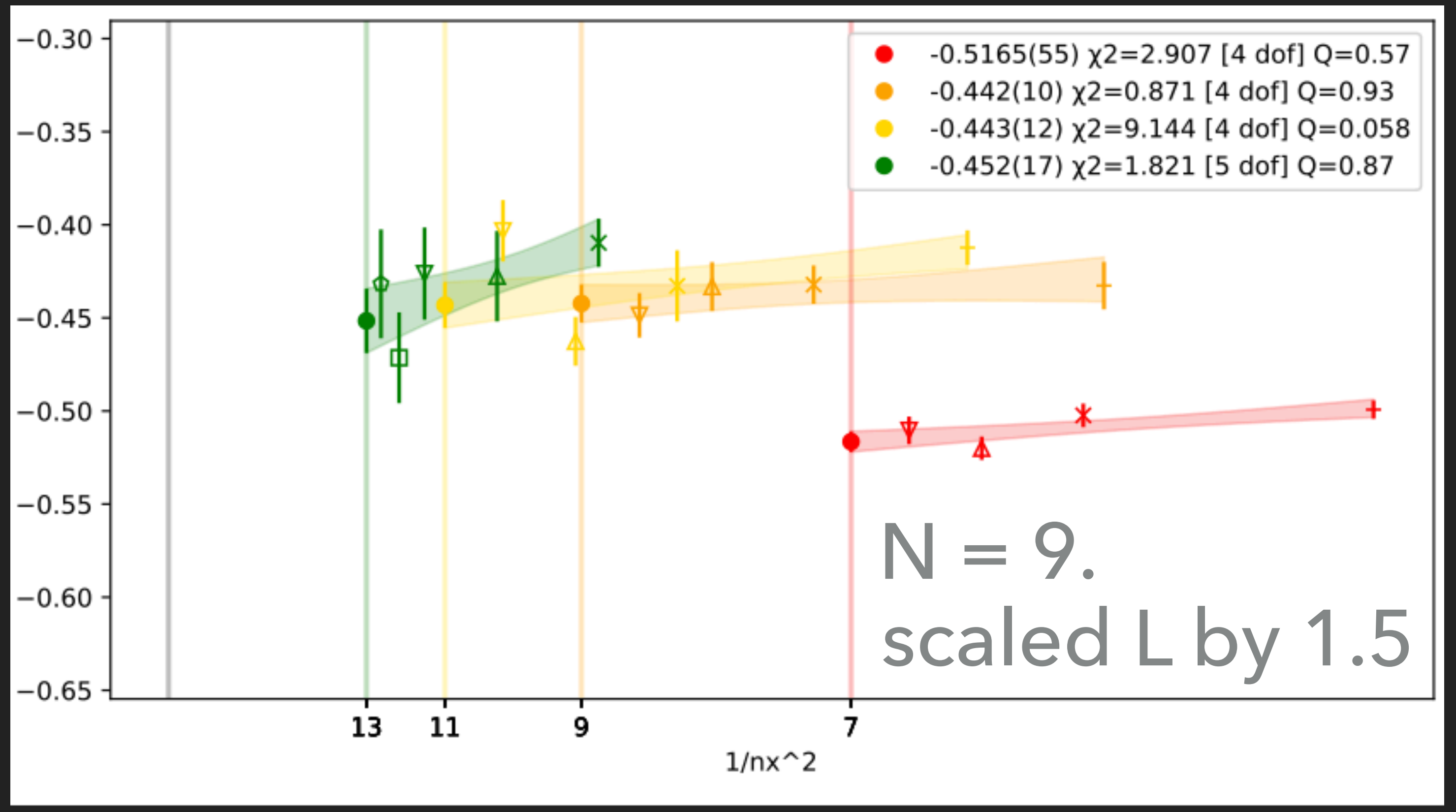


log k_F a = 3.3 T/T_F = 1.2 N = 4.

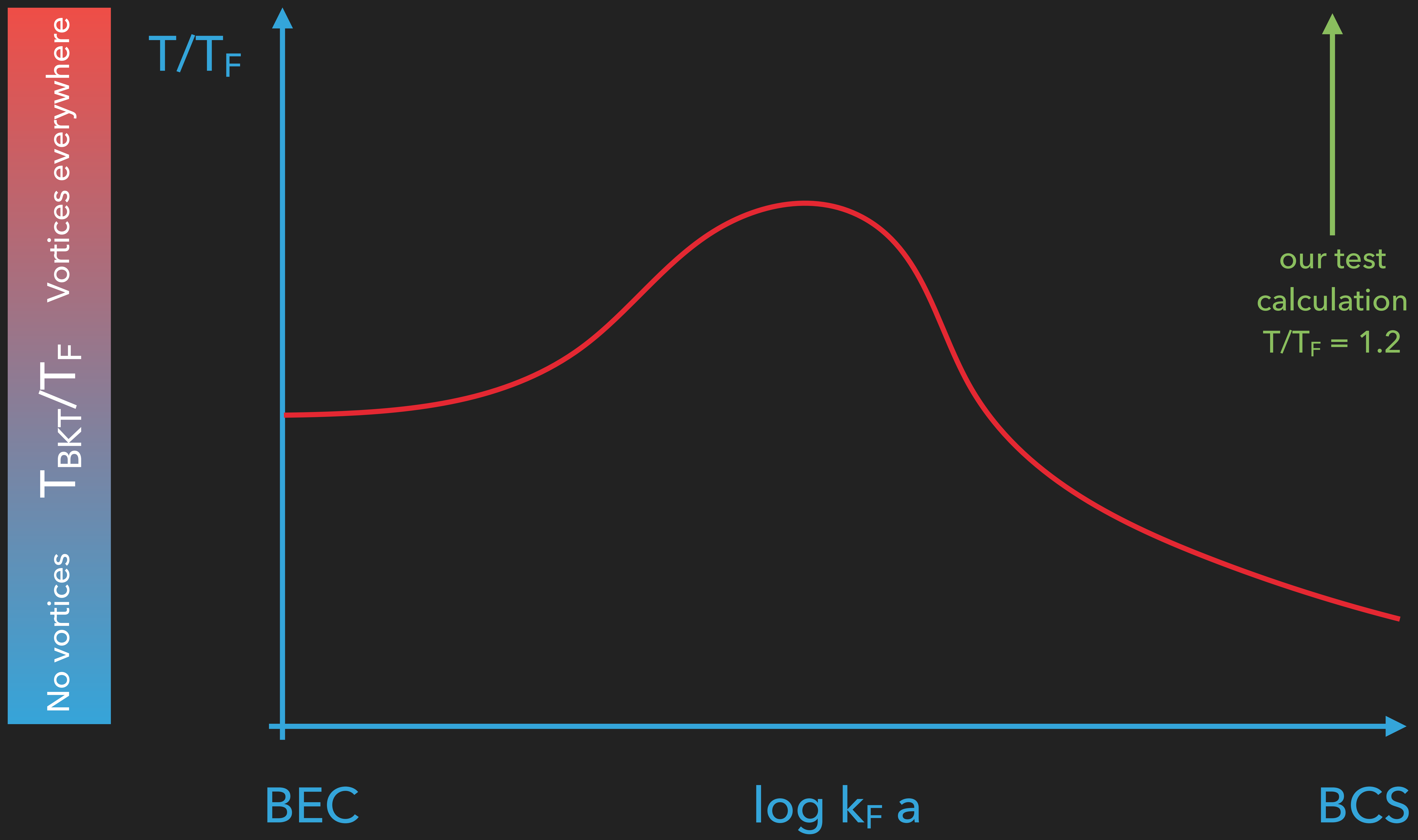
Same parameters as the contact check

$\frac{MB_2}{k_F^4}$



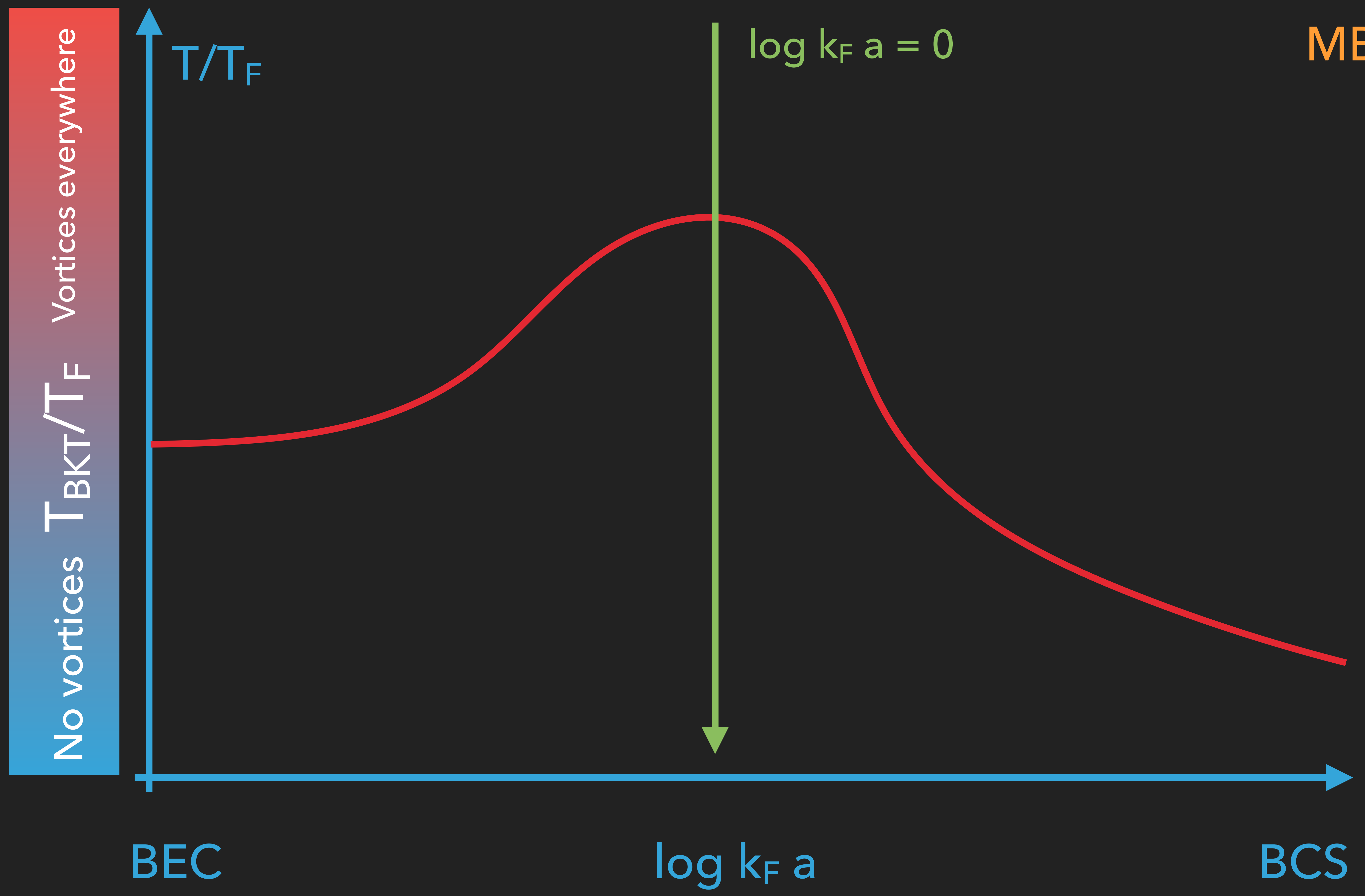


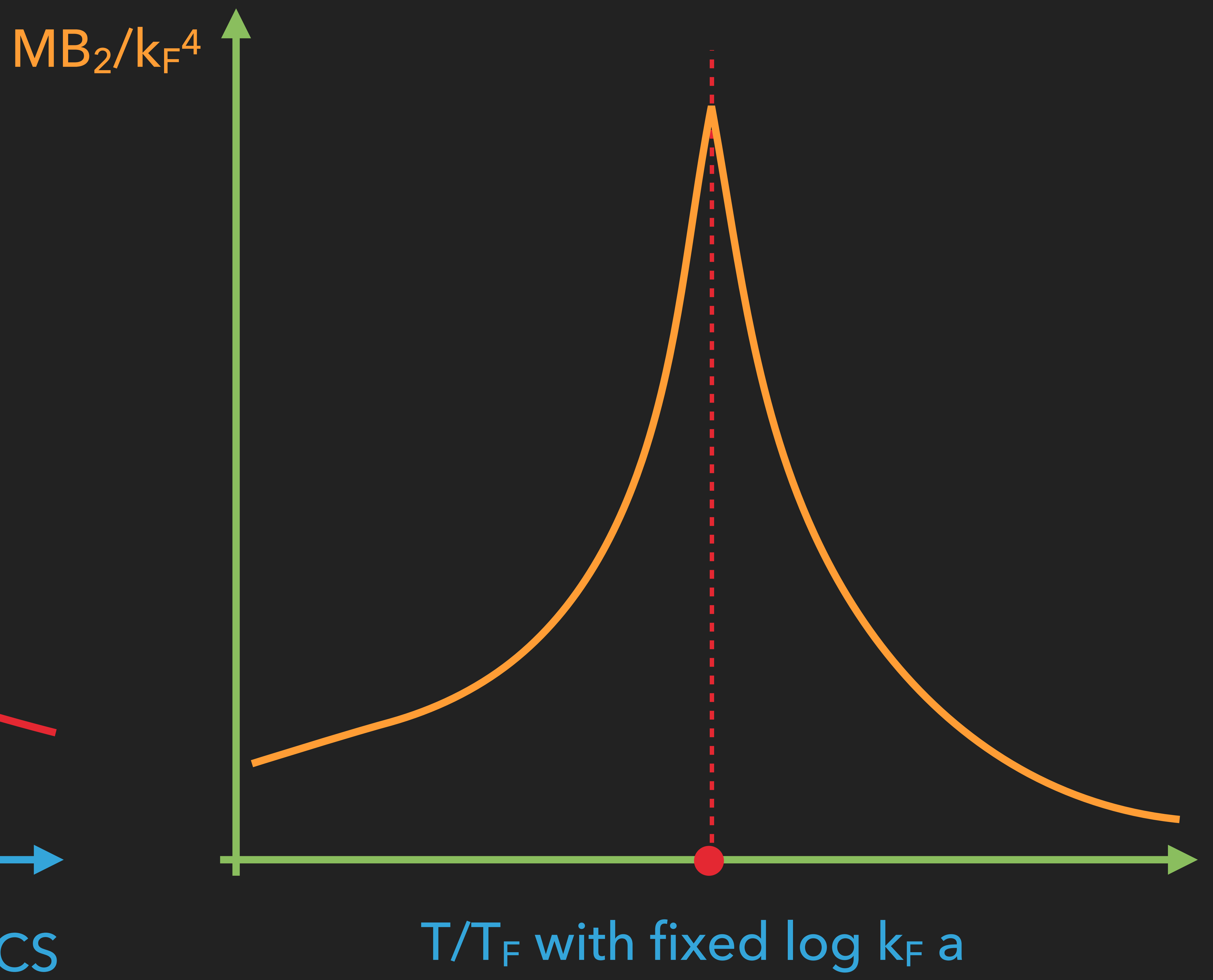
PHASE STRUCTURE



CURRENT CAMPAIGN

Berkowitz + Warrington 2309.????





BACKUP SLIDES

CODE AVAILABILITY

WITH NEILL WARRINGTON
INSTITUTE FOR NUCLEAR THEORY, AND NOW MIT

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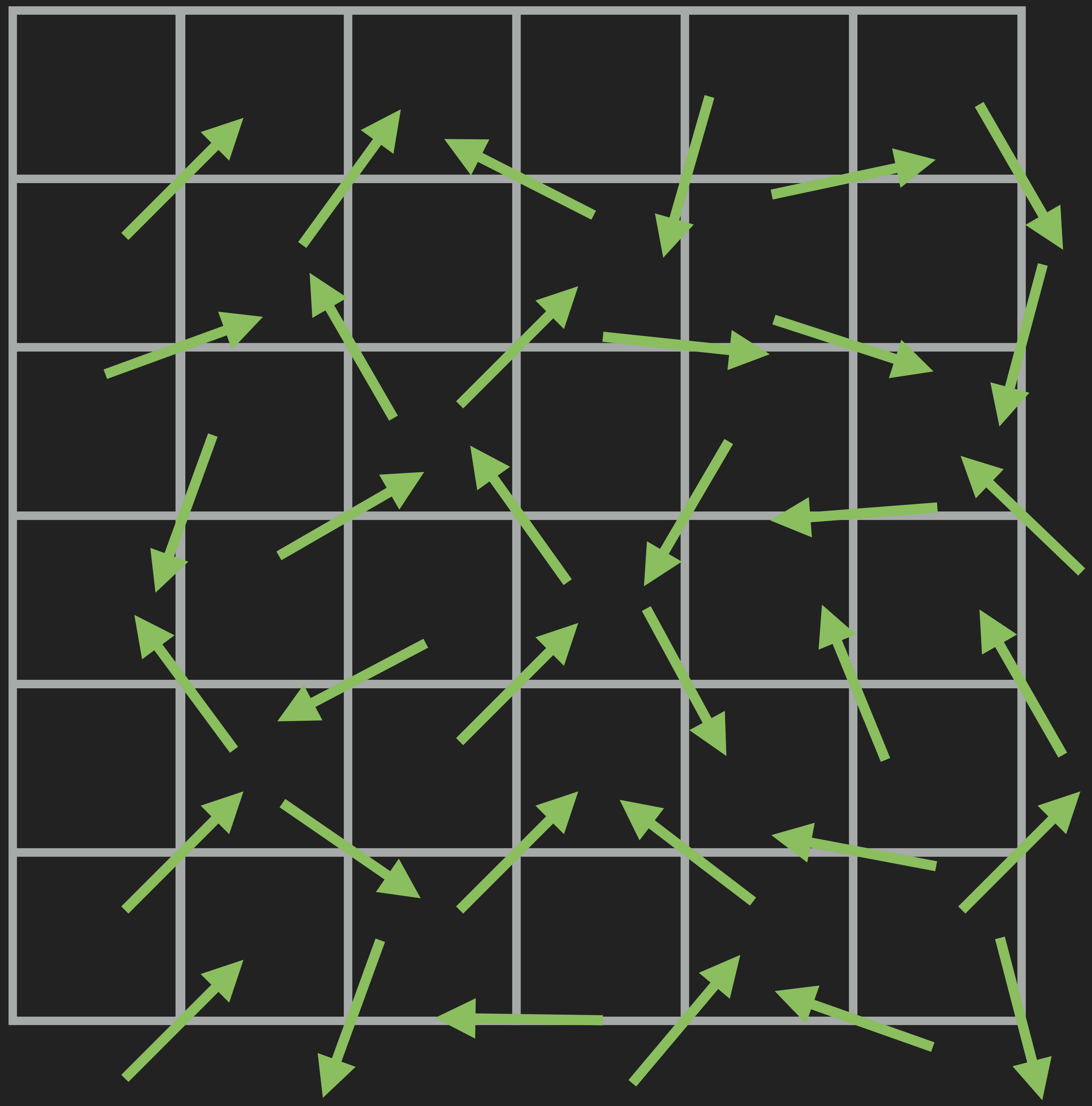
https://two-dimensional-gasses.readthedocs.io/

BUT KNOWN THEOREM?

Mermin+Wagner, PRL 17 (22) 1133-1136 (1966)
Coleman, Commun. Math. Phys., **31** (4): 259–264

- Mermin-Wagner(-Berezinskii-Coleman): continuous symmetries don't spontaneously break in 2D
- Therefore, correlation functions don't go to a finite constant at long distance.
- BKT: OK, but if they decay with a power law?

$$H = -J \sum_{\langle ij \rangle} s_i \cdot s_j \qquad O(2)$$
$$= -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$



TOWARDS TOPOLOGY

Березинский ЖЭТФ (1970)
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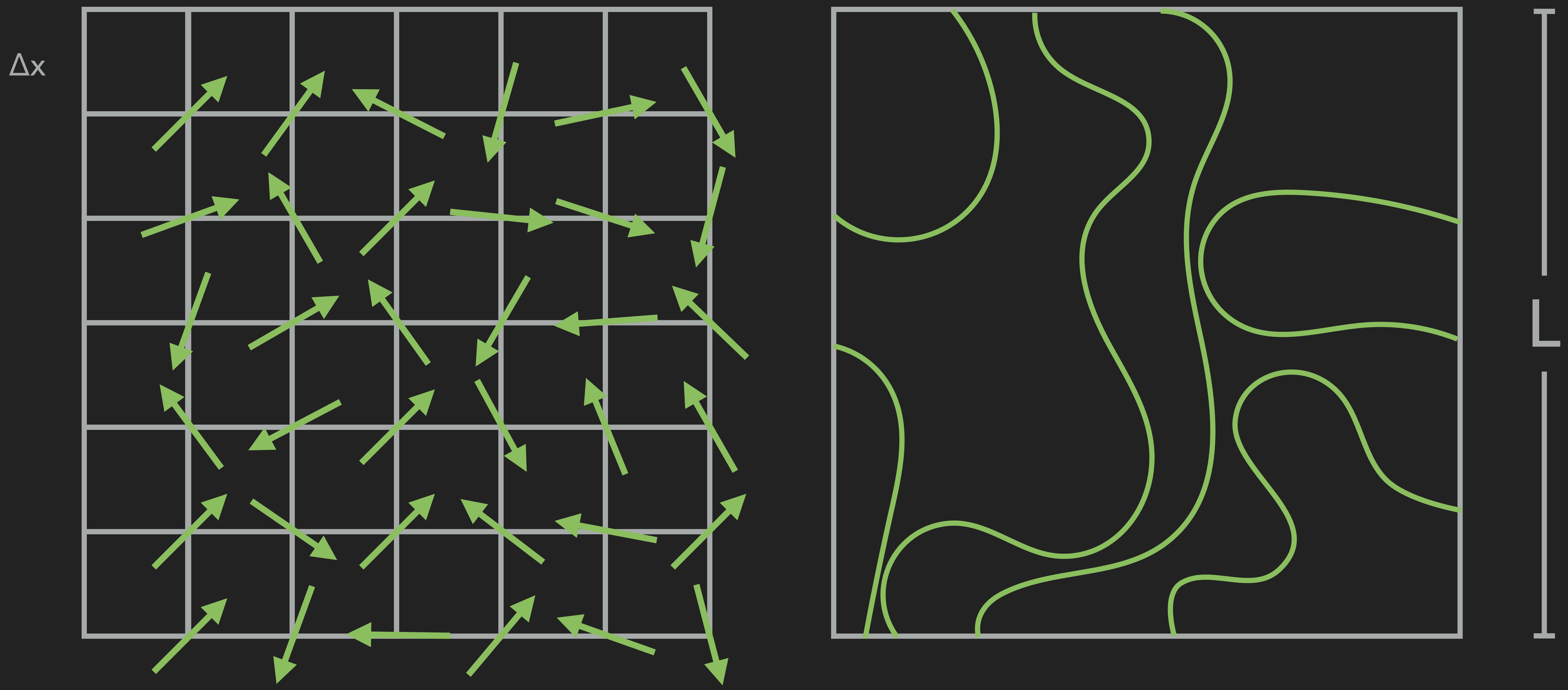
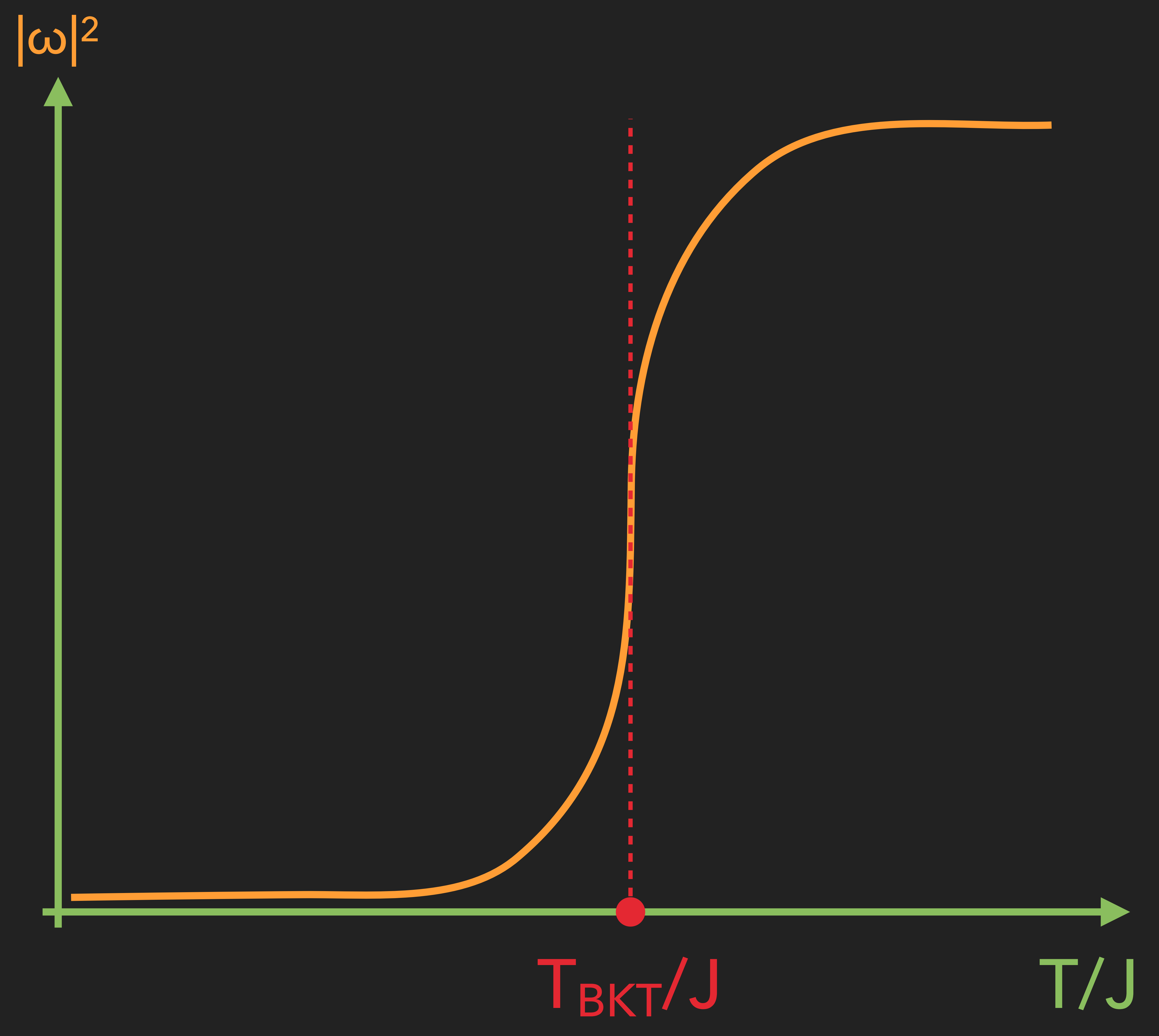
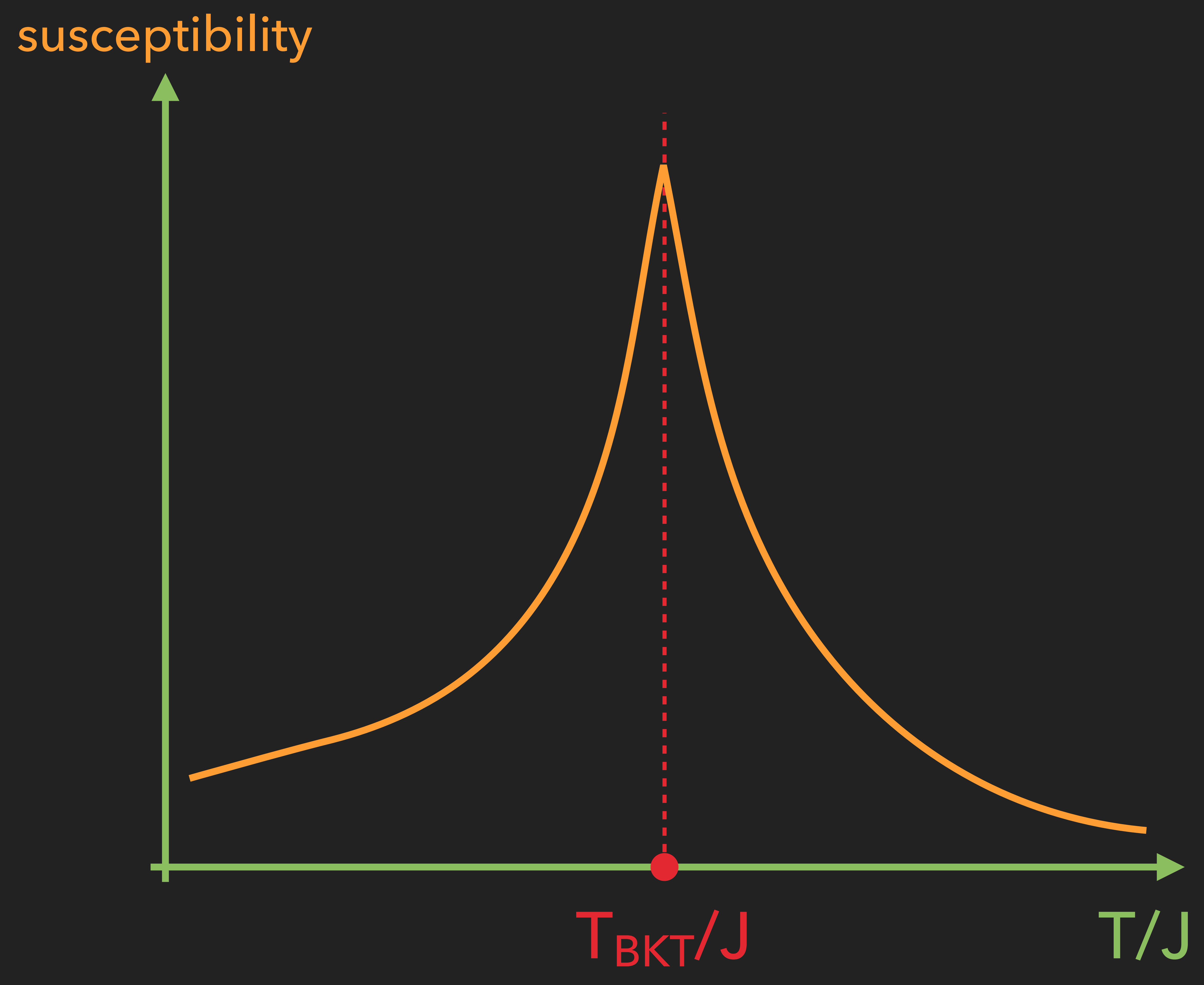


EXPECTATION VALUES AND SUSCEPTIBILITIES





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Berkowitz + Warrington, 2309.????
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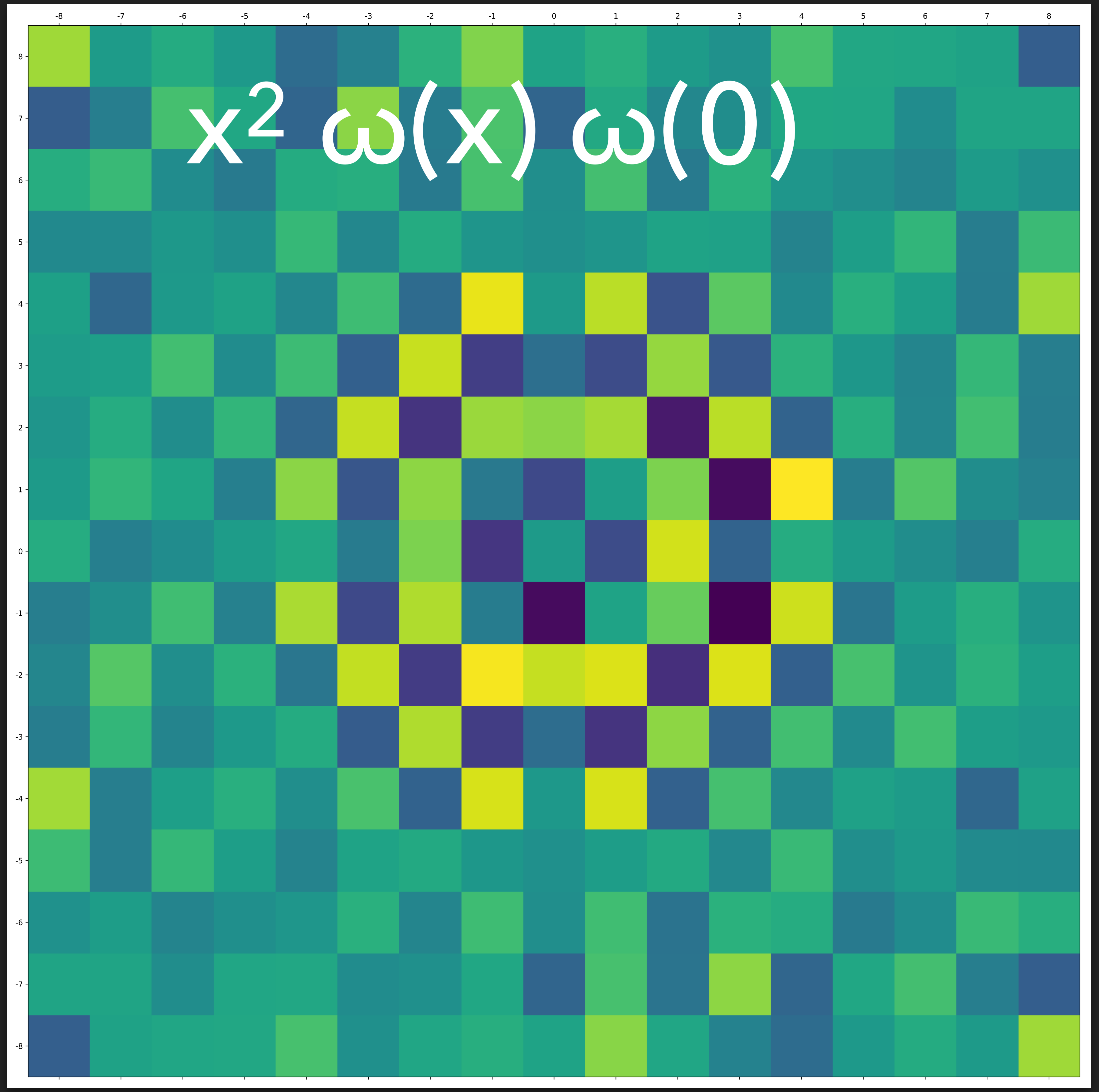

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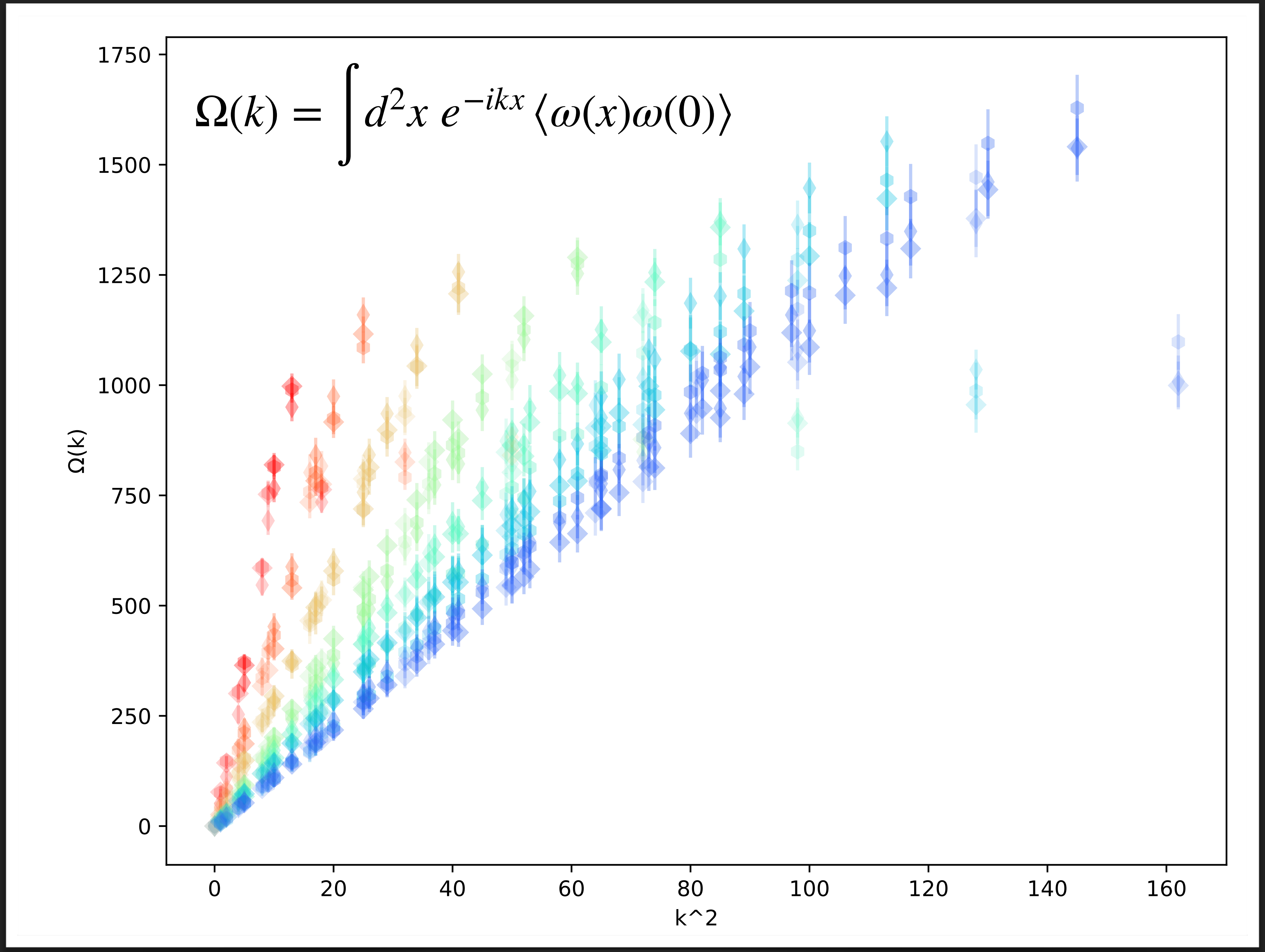

ANTICIPATED BEHAVIOR

Berkowitz + Warrington 2309.????