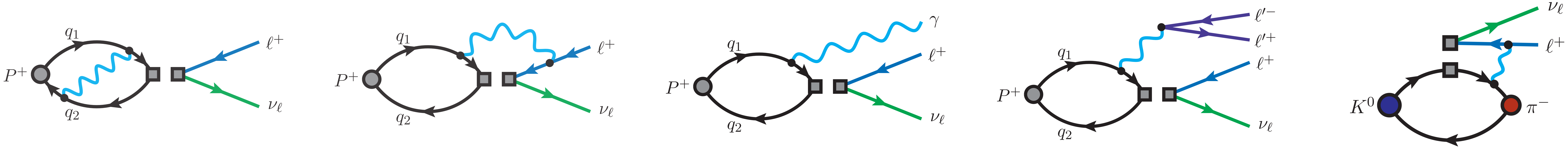

Isospin-breaking and electromagnetic corrections to weak decays

Matteo Di Carlo

3rd August 2023

Centre for Theoretical Physics Higgs
THE UNIVERSITY of EDINBURGH

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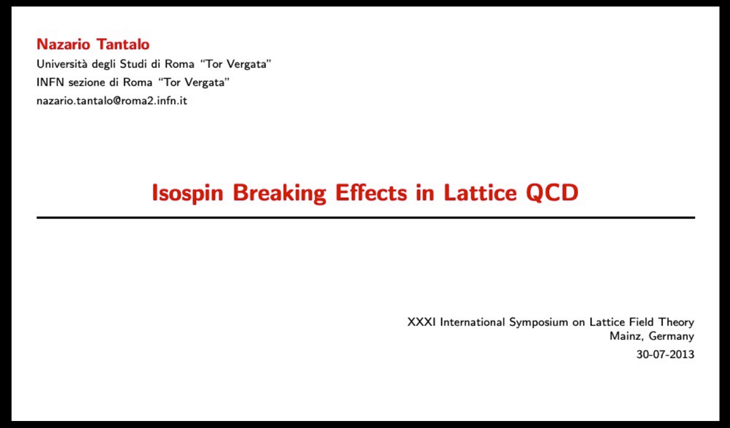
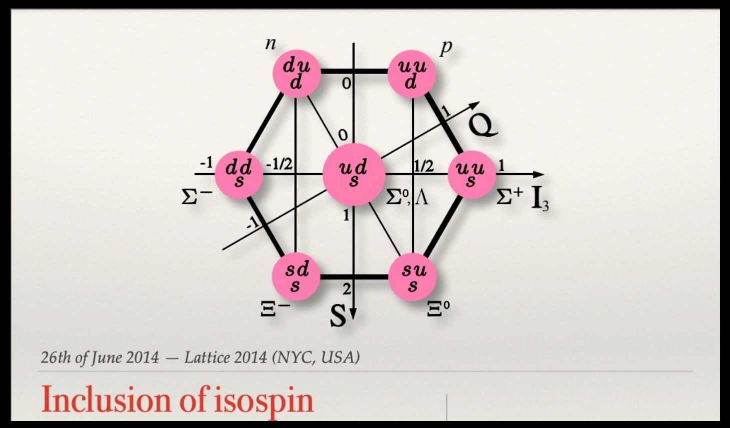


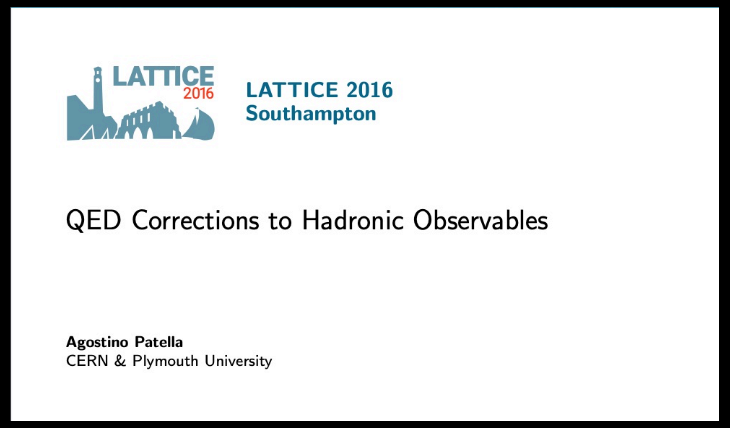
N. Tantalo (2013)

Isospin symmetry is not exact and the corrections to the isosymmetric limit are, in general, at the percent level. For gold plated quantities, such as pseudoscalar meson masses or the kaon leptonic and semileptonic decay rates, these effects are of the same order …



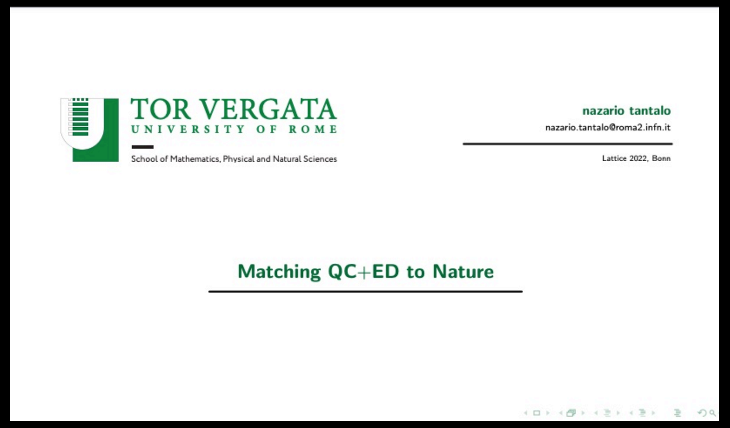
A. Portelli (2014)

Isospin symmetry is explicitly broken in the Standard Model by the mass and electric charge of the up and down quarks. These effects represent a perturbation of hadronic amplitudes at the percent level. Although these contributions are small, they play a crucial role in hadronic …



A. Patella (2016)

When aiming at a percent precision in hadronic quantities calculated by means of lattice simulations, isospin breaking effects become relevant. These are of two kinds: up/down mass splitting and electromagnetic corrections. In order to properly account for the latter …



N. Tantalo (2022)

The first step in any QFT calculation of a phenomenological observable is the matching of the theory to Nature. The matching procedure fixes the parameters of the theory in terms of an equal number of external inputs that, if the theory is expected to reproduce …

OVERVIEW EPISODES TRAILERS & MORE MORE LIKE THIS DETAILS

Outline of this talk

1. **Why** are isospin-breaking and QED corrections **relevant**?

2. **How** are these effects **included in lattice calculations**?

3. **Which** **weak decays** have been / are being / can be studied?

1. Why

Indirect searches of new physics using CKM matrix unitarity constraints

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

in the Standard Model:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

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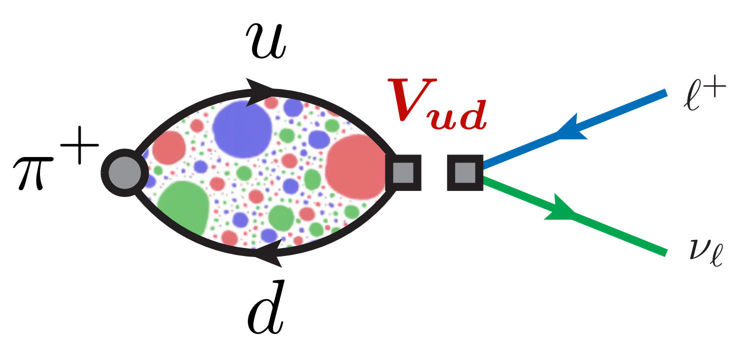
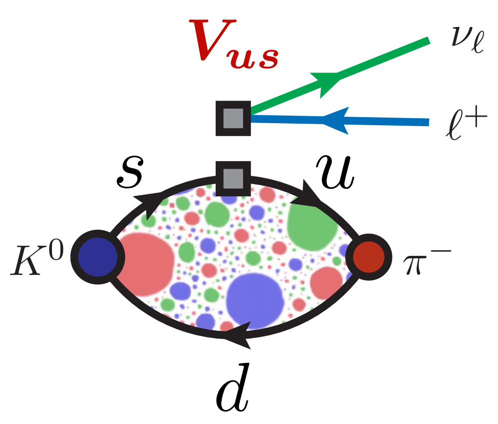
Matrix elements can be extracted e.g. from leptonic and semileptonic decays of mesons



$$\underbrace{\frac{\Gamma\left[K \to \ell\nu_\ell(\gamma)\right]}{\Gamma\left[\pi \to \ell\nu_\ell(\gamma)\right]}}_{\text{experiments}} \propto \boxed{\left|\frac{V_{us}}{V_{ud}}\right|^2} \underbrace{\left(\frac{f_K}{f_\pi}\right)^2}_{\text{QCD}}$$



$$\underbrace{\Gamma\left[K \to \pi\ell\nu_\ell(\gamma)\right]}_{\text{experiments}} \propto \boxed{|V_{us}|^2} \underbrace{|f_+^{K\pi}(0)|^2}_{\text{QCD}}$$

1. Why

Indirect searches of new physics using CKM matrix unitarity constraints

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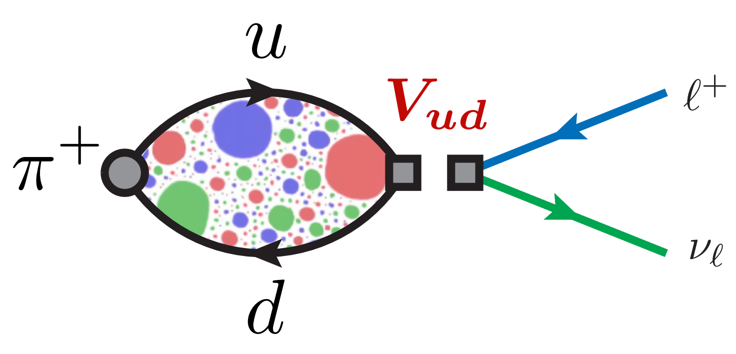

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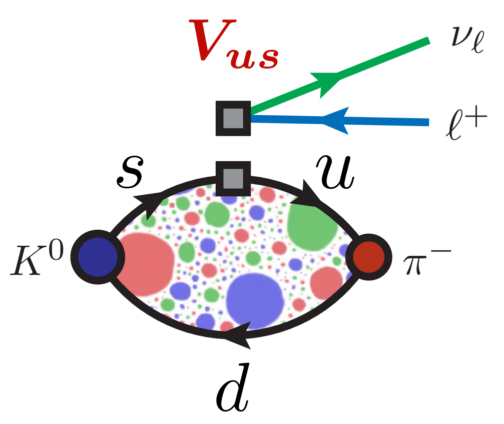

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FLAG 2021 Flavour Lattice Averaging Group

$$f_{K^\pm}/f_{\pi^\pm} = 1.1934\,(19)$$
$$f_+^{K\pi}(0) = 0.9698\,(17)$$

sub percent precision!

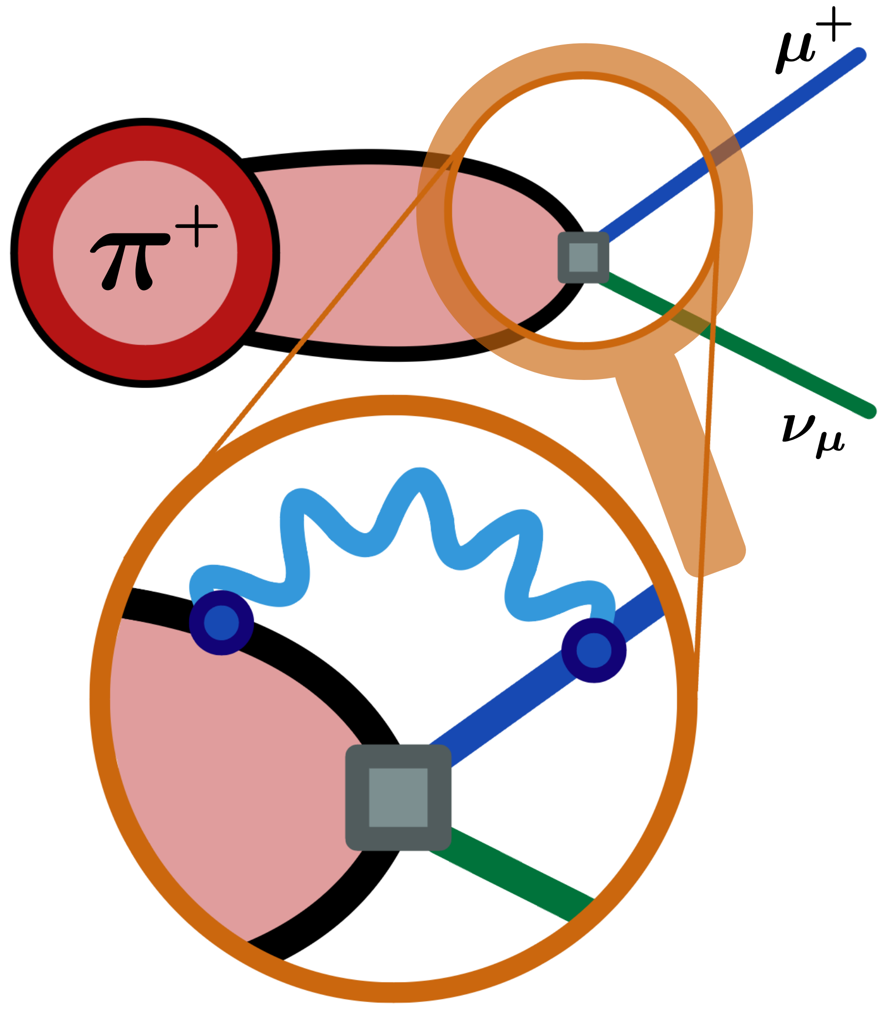
FLAG Review 2021. EPJC **82**, 869 (2022)

QED and isospin-breaking effects

Current level of precision requires the inclusion of isospin-breaking corrections due to

- strong effects $[m_u - m_d]_{\rm QCD} \neq 0$
- electromagnetic effects $\alpha \neq 0$

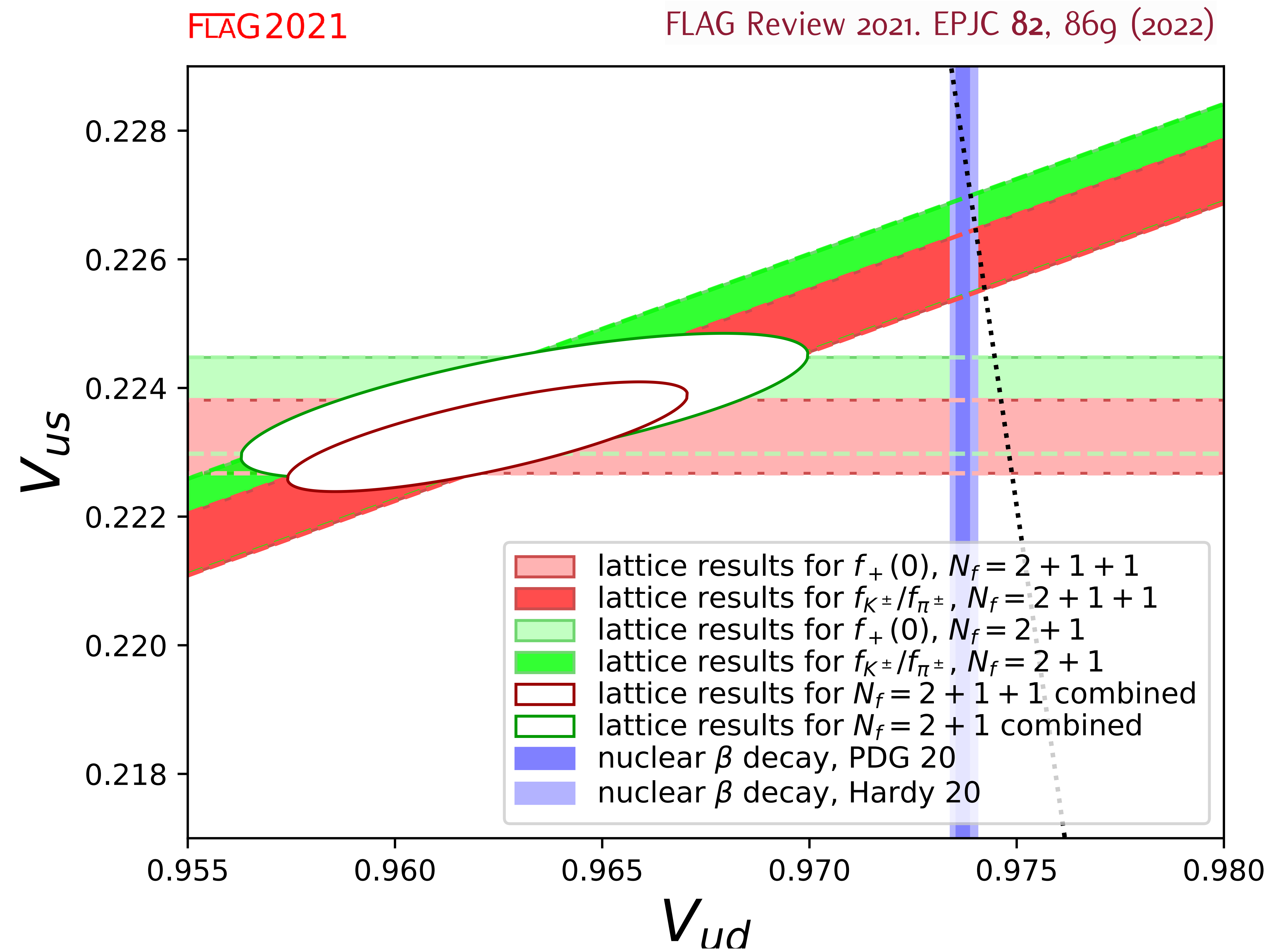
$\sim \mathcal{O}(1\%)$



$$\frac{\Gamma(K \to \ell\nu_\ell)}{\Gamma(\pi \to \ell\nu_\ell)} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2 (1 + \delta R_{K\pi}) \qquad \Gamma(K \to \pi\ell\nu_\ell) \propto |V_{us}|^2 \, |f_+^{K\pi}(0)|^2 \, (1 + \delta R^\ell_{K\pi})$$

- results from χPT currently quoted in the PDG — V.Cirigliano & H.Neufeld, PLB **700** (2011)
- these are fully **non-perturbative** (structure dependent)
- first-principle **lattice calculations** are possible!

Tests of the Standard Model



Different **tensions** in the V_{us}-V_{ud} plane:

$$|V_u|^2 - 1 = 2.8\sigma$$

$$|V_u|^2 - 1 = 5.6\sigma \qquad |V_u|^2 - 1 = 3.3\sigma$$

$$|V_u|^2 - 1 = 3.1\sigma \qquad |V_u|^2 - 1 = 1.7\sigma$$

Experimental and **theoretical** control of these quantities is of crucial importance to solve the issue

2. How

Computing **QED corrections on a finite-sized lattice** is challenging:

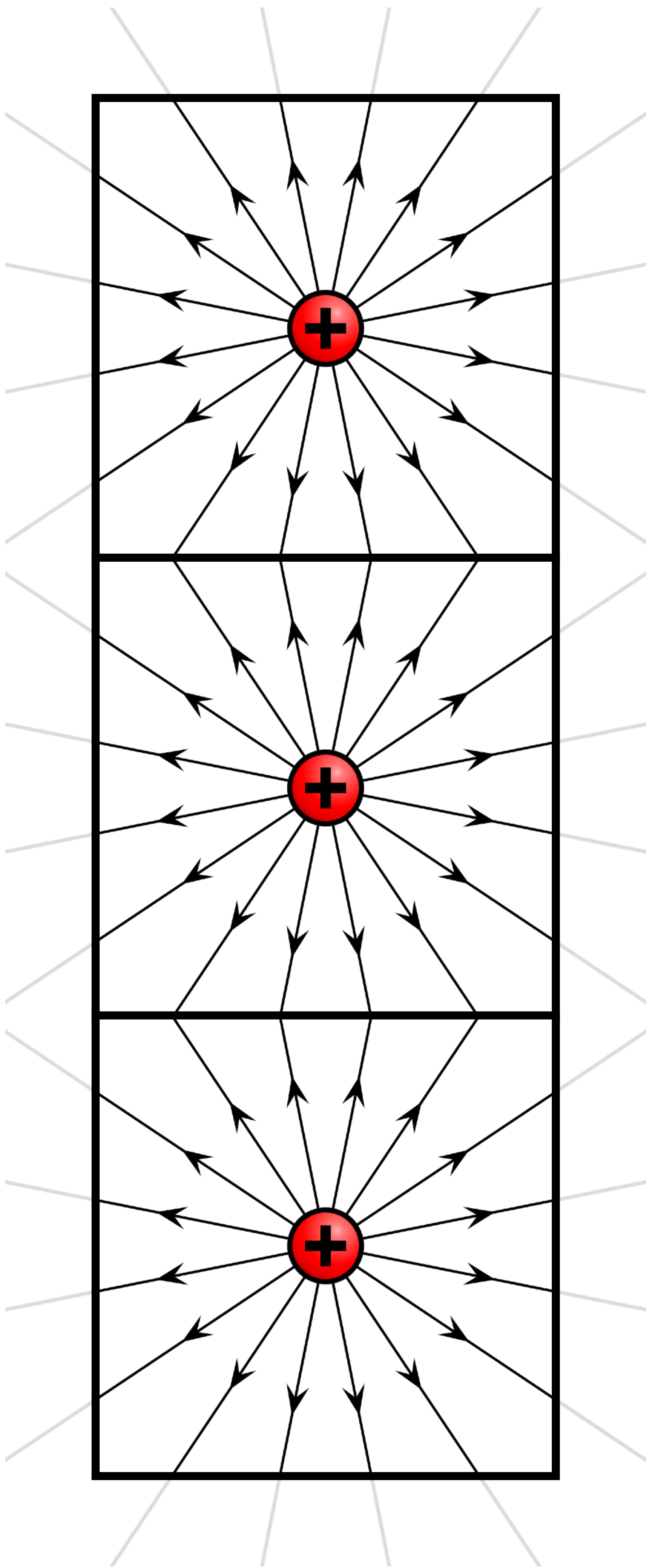
- **long-range interactions** don't like **finite volumes** with **periodic boundary conditions**

- **finite-volume effects** can be sizeable and power-like

  M.Hayakawa & S.Uno, PTP **120** (2008) / Z.Davoudi & M.Savage, PRD **90** (2014) / S.Borsanyi et al., Science **347** (2015)

- logarithmic **infrared divergences** arise in virtual/real decay rates

  V.Lubicz et al., PRD **95** (2017)



There are also recent proposals to compute radiative corrections as **convolutions** of hadronic correlators with **infinite-volume** QED kernels

N.Asmussen et al., [1609.08454] / T.Blum et al., PRD **96** (2017) / X.Feng & L.Jin, PRD **100** (2019) / N.Christ et al., [2304.08026]

Charged states in a finite box

Gauss law: only zero net charge is allowed in a finite volume with periodic boundary conditions

$$Q = \int_{\mathrm{p.b.c.}} \mathrm{d}^3\mathbf{x}\ j_0(t,\mathbf{x}) = \int_{\mathrm{p.b.c.}} \mathrm{d}^3\mathbf{x}\ \boldsymbol{\nabla} \cdot \boldsymbol{E}(t,\mathbf{x}) = 0$$

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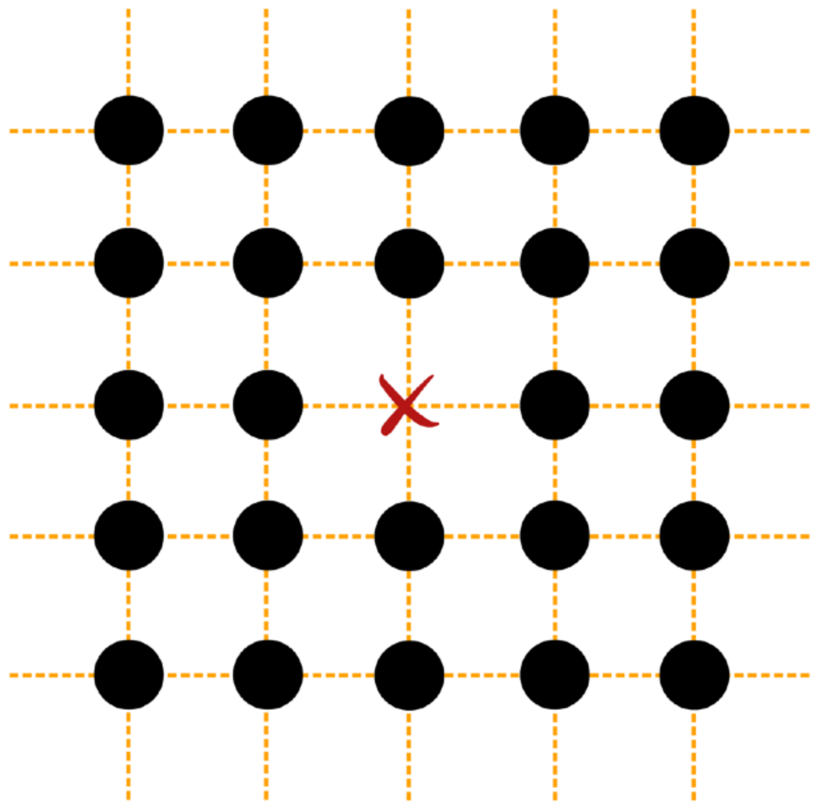
Possible solutions:

$\mathrm{QED_L}$

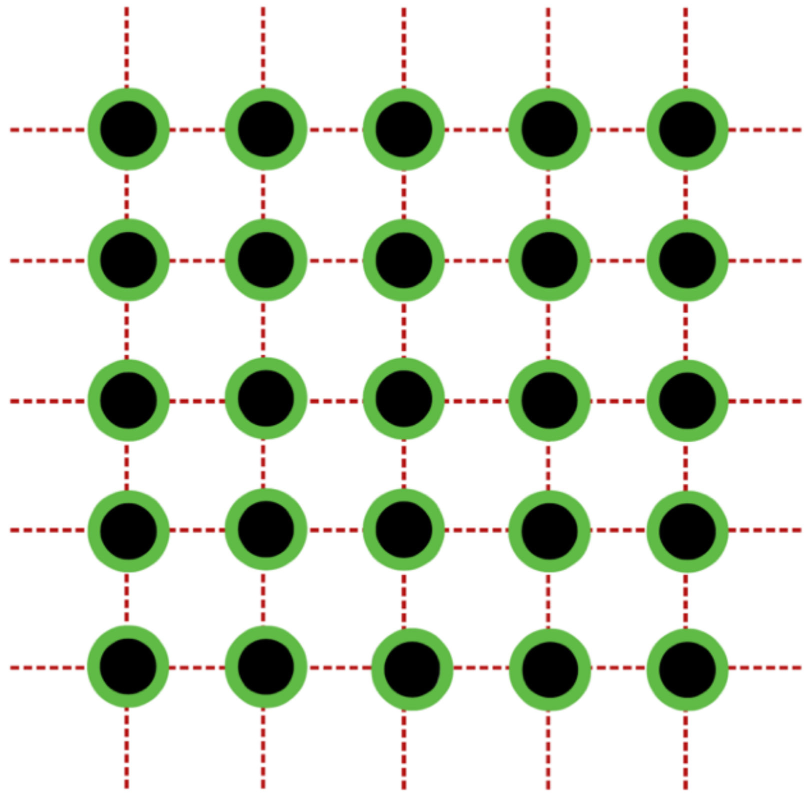


$\Omega_3 = 2\pi\mathbb{Z}^3/L$

remove spatial zero-mode of the photon field

M.Hayakawa & S.Uno, PTP **120** (2008)

$\mathrm{QED_m}$

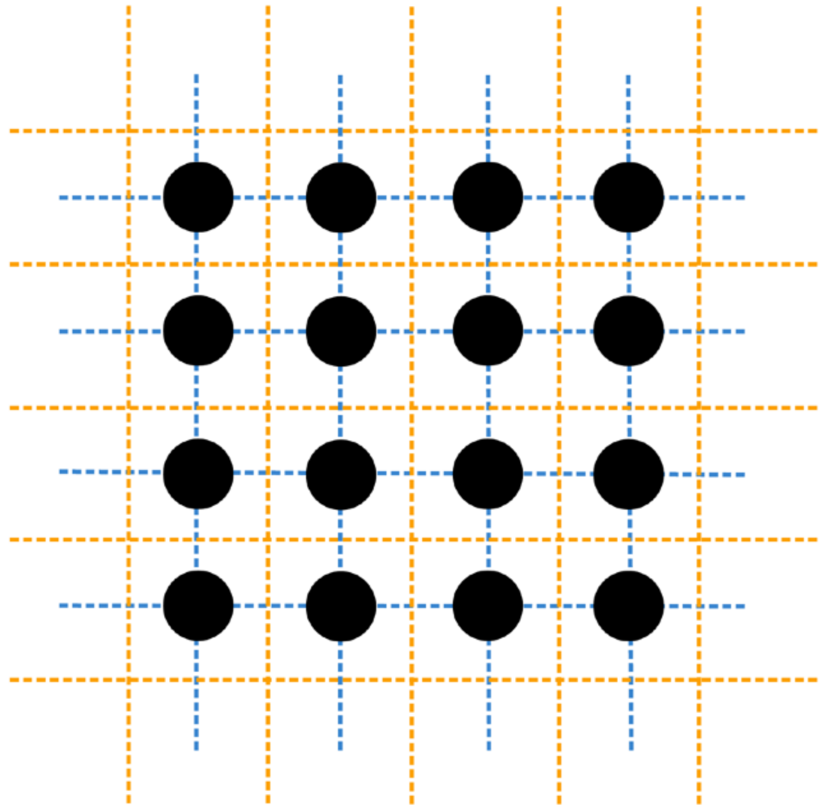


$\Omega_4 = 2\pi\{\mathbb{Z}^3/L, \mathbb{Z}/T\}$

use massive photon m_γ

M.G.Endres et al., [1507.08916]

$\mathrm{QED_{C^*}}$



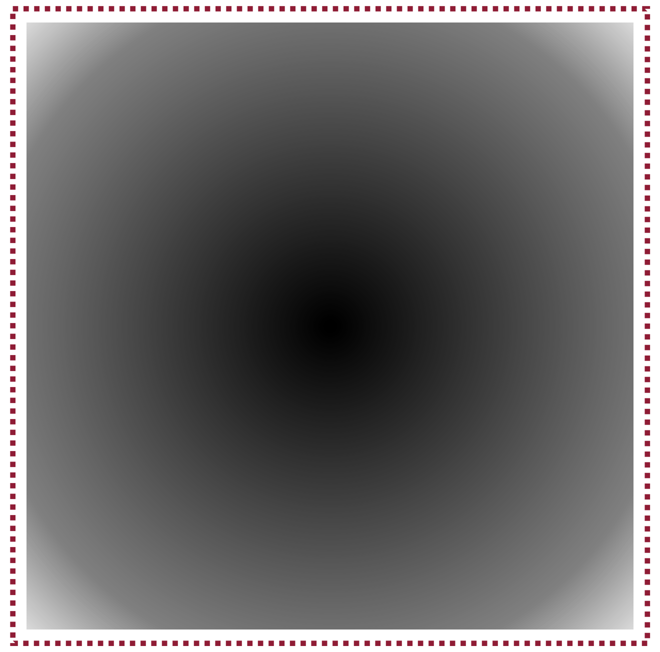
$\Omega_3 = 2\pi\mathbb{Z}^3/L$ $\quad \Omega_3' = (2\,\mathbb{Z}^3 + \bar{\mathbf{n}})\pi/L$

employ C* boundary conditions

A.S.Kronfeld & U.-J.Wiese, NPB **357** (1991)

B.Lucini et al., JHEP **02** (2016)

$\mathrm{QED_\infty}$



$\Omega_4 = \mathbb{R}^4$

infinite-volume reconstruction

X.Feng & L.Jin, PRD **100** (2019)

N.Christ et al., [2304.08026]

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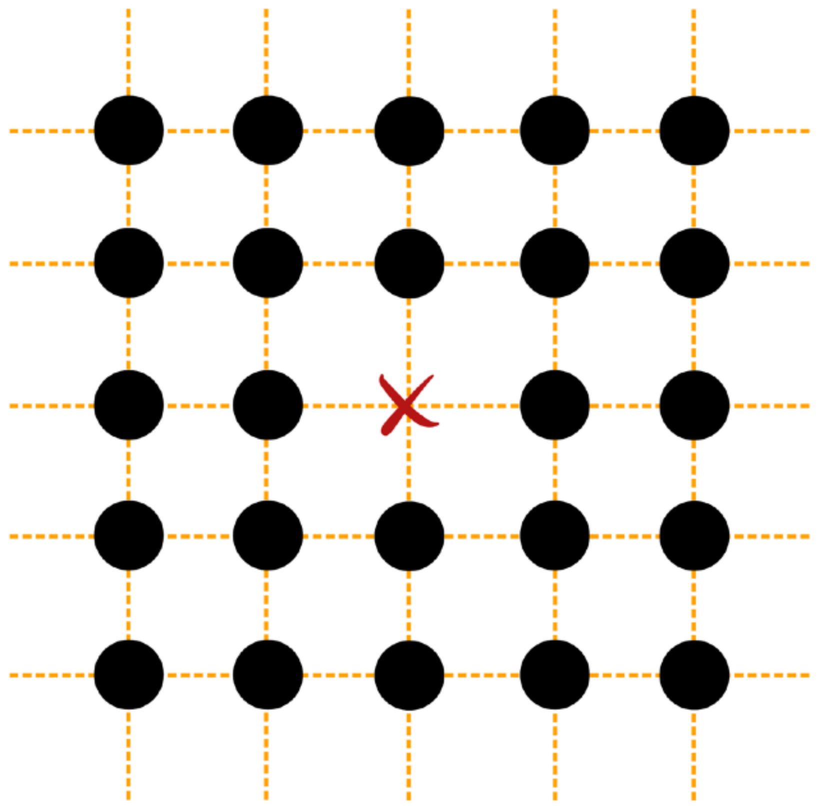

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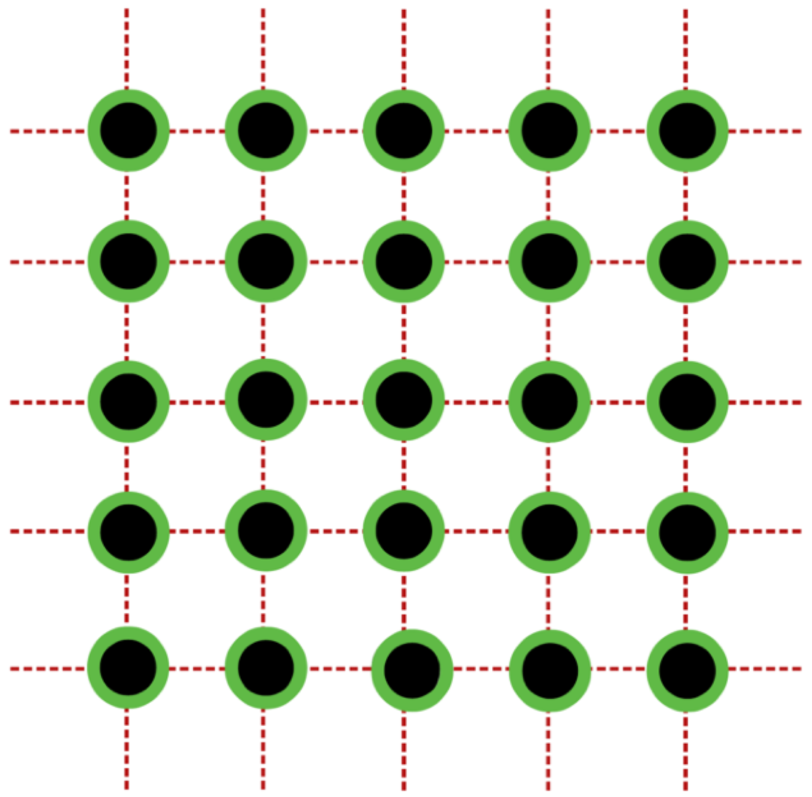

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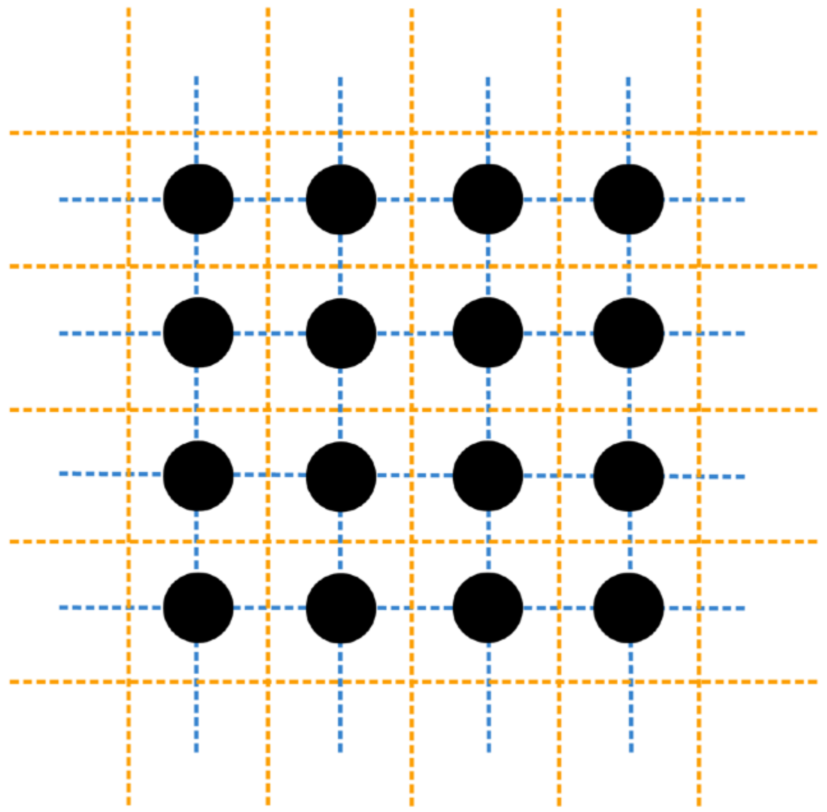
QED$_{\mathrm{C}^*}$



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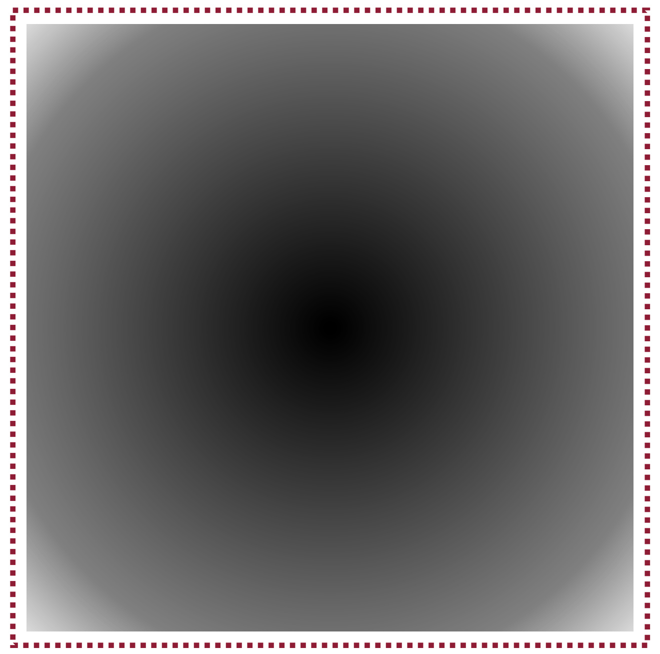

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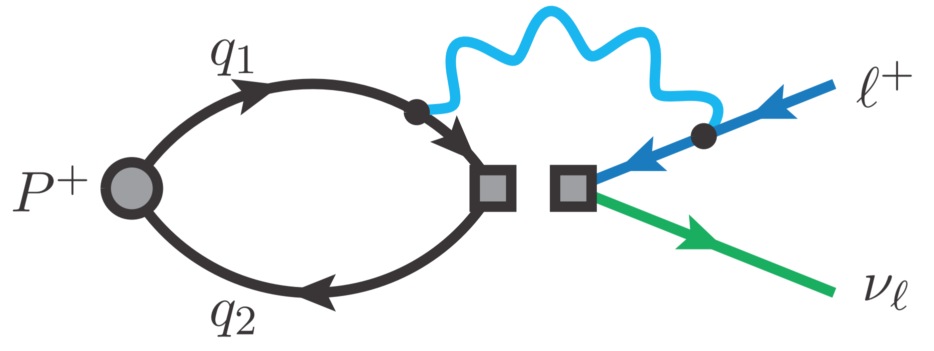
N.Christ et al., [2304.08026]

3. Which

I will mainly focus on virtual corrections to **leptonic decay rates:**



- RM123S calculation (QED$_L$) — D.Giusti et al., PRL **120** (2018); MDC et al., PRD **100** (2019)
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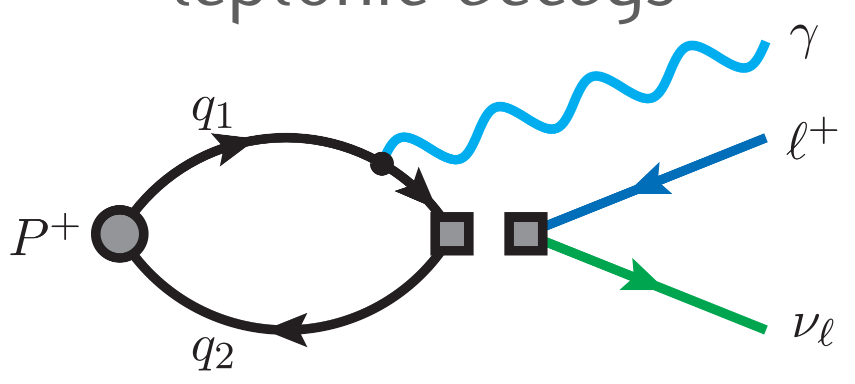
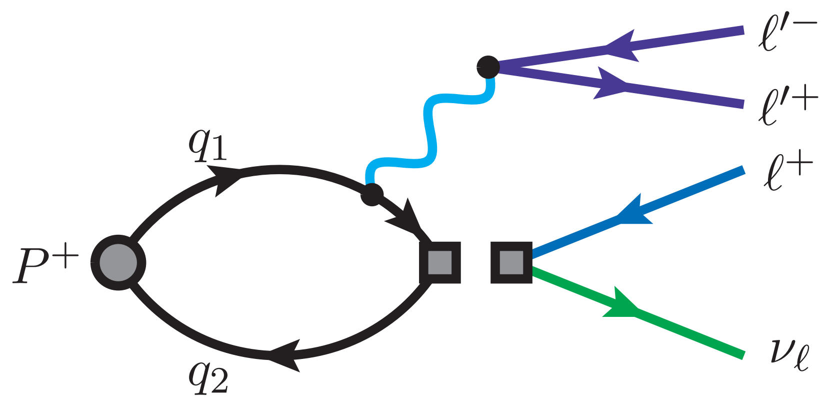
But there's intense activity and nice progress also on **other weak processes**:

real photon emission
leptonic decays



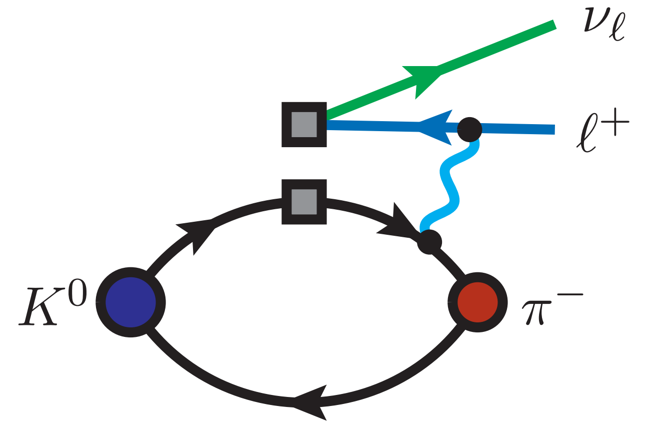
D.Giusti (Thursday 3, h 17.20)

virtual photon emission
leptonic decays



R.Frezzotti et al., [2306.07228]

semileptonic decays



N.Christ (Friday 4, h 9.00)

3. Which

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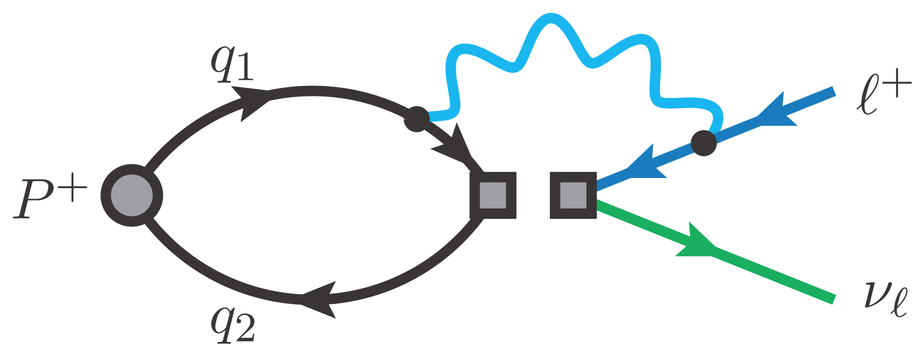

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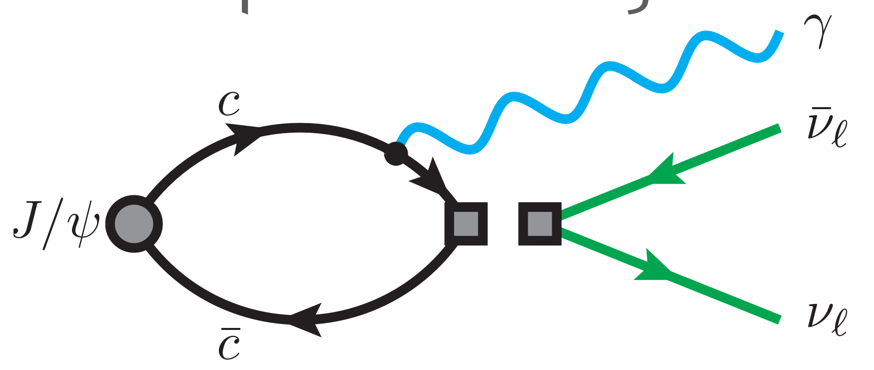
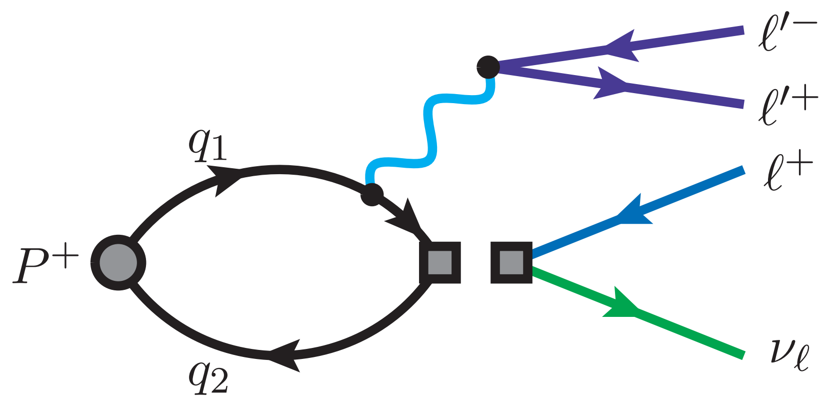


D.Giusti (Thursday 3, h 17.20)
Y.Meng (Tuesday 1, h 10.00)

virtual photon emission
leptonic decays



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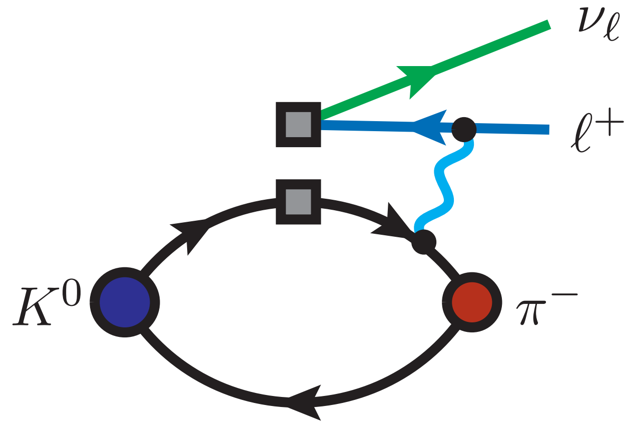

N.Christ (Friday 4, h 9.00)

- $\Gamma(K_{\mu 2})$ and $\Gamma(\pi_{\mu 2})$ separately
- Twisted Mass fermions
- multiple volumes and 3 lattice spacings
- unphysical pion masses ($\gtrsim$ 230 MeV)

1904.08731

PHYSICAL REVIEW D **100,** 034514 (2019)

Editors' Suggestion

Light-meson leptonic decay rates in lattice QCD + QED

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Dipartimento di Fisica and INFN, Università di Roma "Tor Vergata," Via della Ricerca Scientifica 1, I-00133 Roma, Italy

JHEP

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Isospin-breaking corrections to light-meson leptonic decays from lattice simulations at physical quark masses

Peter Boyle,[a,b] Matteo Di Carlo,[b] Felix Erben,[b] Vera Gülpers,[b] Maxwell T. Hansen,[b] Tim Harris,[b] Nils Hermansson-Truedsson,[c,d] Raoul Hodgson,[b] Andreas Jüttner,[e,f] Fionn Ó hÓgáin,[b] Antonin Portelli,[b] James Richings[b,e,g] and Andrew Zhen Ning Yong[b]

2211.12865

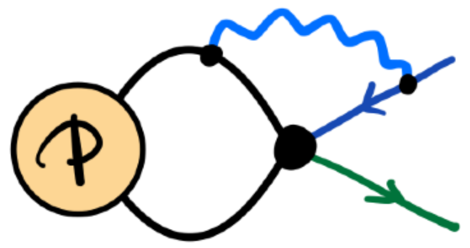
- ratio $\Gamma(K_{\mu 2})$ / $\Gamma(\pi_{\mu 2})$
- Domain Wall fermions
- single volume and lattice spacing
- physical quark masses

Decay rate at $\mathcal{O}(\alpha)$

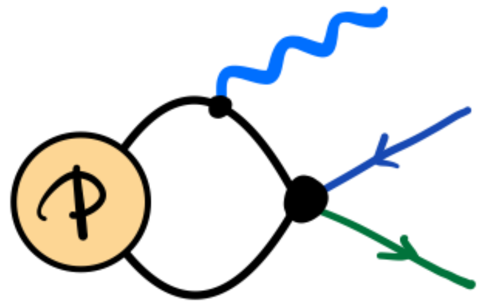
F. Bloch & A. Nordsieck, PR **52** (1937)

The RM123+Soton recipe

$$\Gamma(P_{\ell 2}) = \lim_{\Lambda_{\rm IR}\to 0} \Bigg\{$$



$$+$$



$$\Bigg\}$$

IR finite — IR divergent — IR divergent

Decay rate at $\mathcal{O}(\alpha)$

F. Bloch & A. Nordsieck, PR **52** (1937)
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V. Lubicz et al., PRD **95** (2017)
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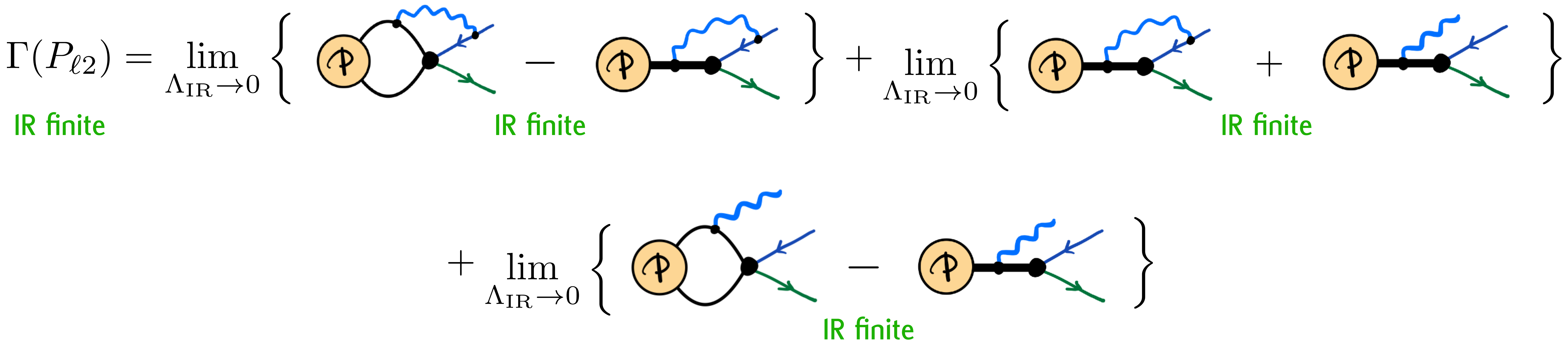
MDC et al., PRD **100** (2019)
P.Boyle, MDC et al., JHEP **02** (2023)

The RM123+Soton recipe



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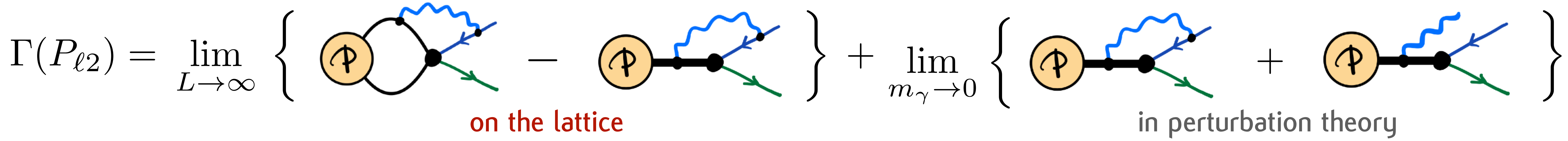

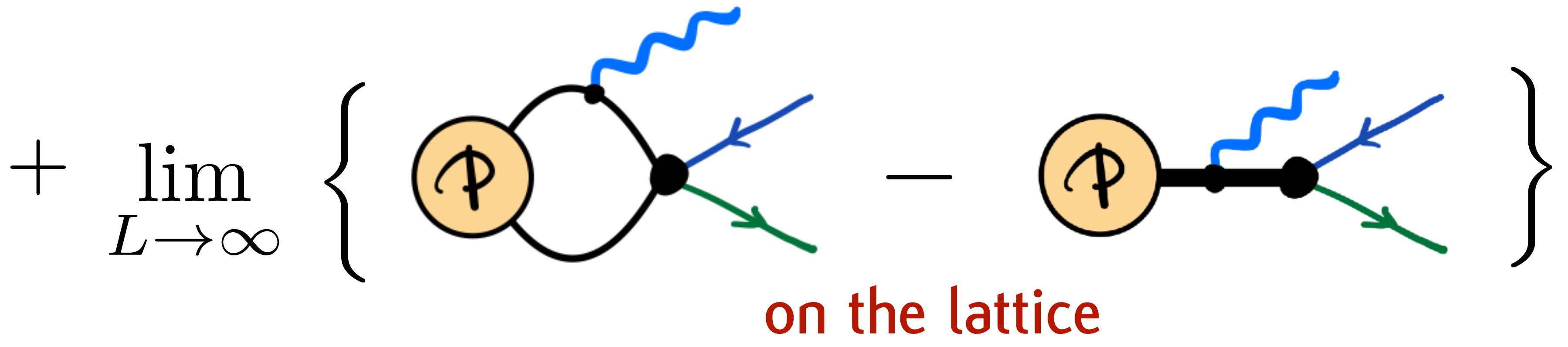

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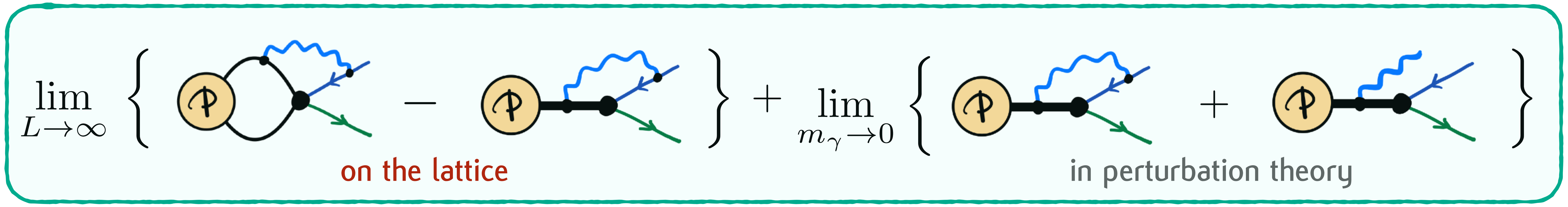
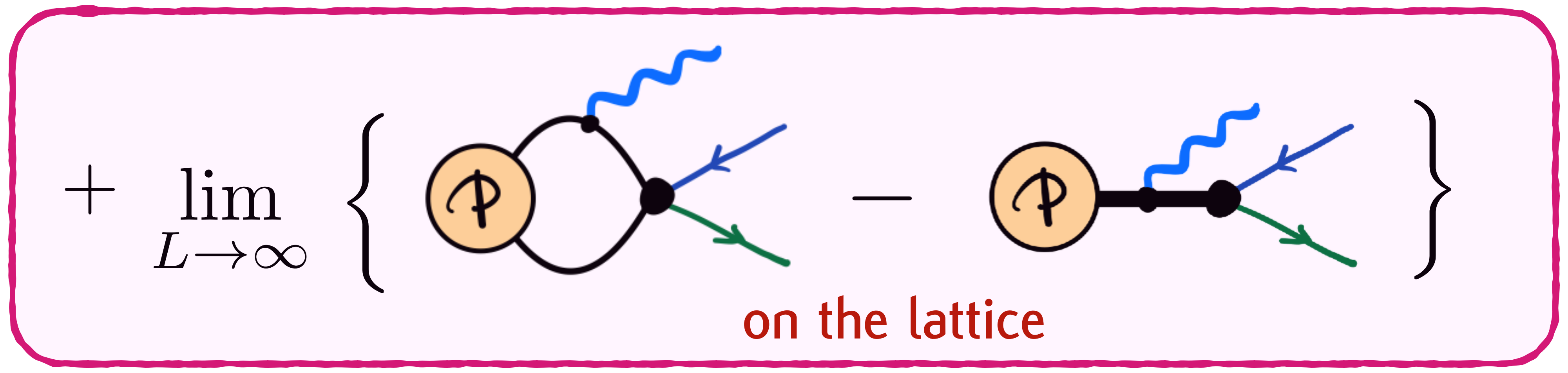
The RM123+Soton recipe

$\Gamma(P_{\ell 2}) =$





enough for $K_{\mu 2}$ and $\pi_{\mu 2}$

finite-volume scaling well studied

V.Lubicz et al., PRD **95** (2017) N.Tantalo et al., [1612.00199v2]
MDC et al., PRD **105** (2022)

N.Hermansson-Truedsson (Friday 4, h 9.20)

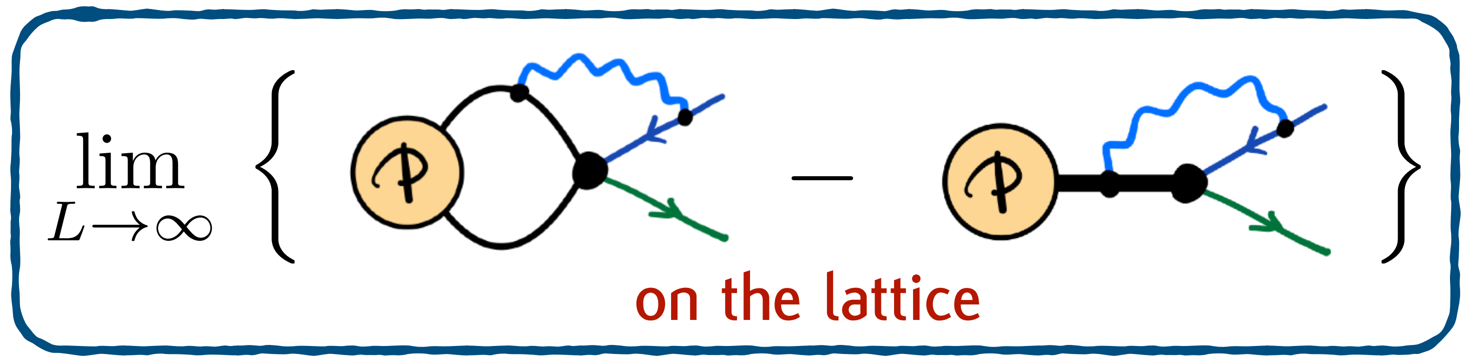
relevant for K_{e2} and π_{e2}
& decays of heavier mesons

G.M. de Divitiis et al., [1908.10160]
R. Frezzotti et al., PRD **103** (2021)
A. Desiderio et al., PRD **102** (2021)
C. Kane et al., [1907.00279 & 2110.13196]
D. Giusti et al., [2302.01298]
R.Frezzotti et al., [2306.05904]

D.Giusti (Thursday 3, h 17.20)

Decay rate at $\mathcal{O}(\alpha)$

Virtual decay rate



$$\Gamma(P_{\ell 2}) = \Gamma_P^{\text{tree}}(1+\delta R_P) \quad \blacktriangleright \quad \Gamma_P^{\text{tree}} = \frac{G_F^2}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 m_P \left[f_{P,0}\right]^2 \quad \blacktriangleright \quad \delta R_P = 2\left(\frac{\delta\mathcal{A}_P}{\mathcal{A}_{P,0}} - \frac{\delta m_P}{m_{P,0}} + \frac{\delta\mathcal{Z}}{\mathcal{Z}_0}\right)$$

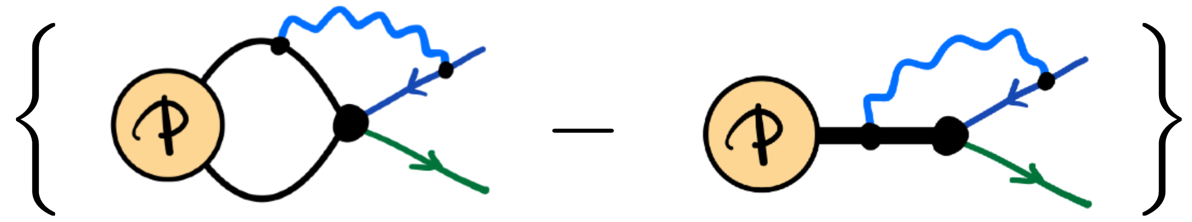
PDG convention

- $\boldsymbol{\delta\mathcal{A}_P}$ from the correction to the (bare) matrix element $\mathcal{M}_P^{rs}(\mathbf{p}_\ell) = \langle \ell^+, r, \mathbf{p}_\ell;\, \nu_\ell, s, \mathbf{p}_\nu |\, O_W\, | P^+, \mathbf{0}\rangle$
- $\boldsymbol{\delta m_P}$ correction to the meson mass
- $\boldsymbol{\delta\mathcal{Z}}$ correction to the renormalization of the weak operator O_W

MDC et al., PRD **100** (2019) / MDC @Lattice2019

Decay rate at $\mathcal{O}(\alpha)$

Virtual decay rate

$\lim_{L\to\infty}$ 

on the lattice

$$\Gamma(P_{\ell 2}) = \Gamma_P^{\text{tree}}(1+\delta R_P) \quad \blacktriangleright \quad \Gamma_P^{\text{tree}} = \frac{G_F^2}{8\pi} m_\ell^2 \left(1-\frac{m_\ell^2}{m_P^2}\right)^2 m_P \left[f_{P,0}\right]^2 \quad \blacktriangleright \quad \delta R_P = 2\left(\frac{\delta \mathcal{A}_P}{\mathcal{A}_{P,0}} - \frac{\delta m_P}{m_{P,0}} + \frac{\delta \mathcal{Z}}{\mathcal{Z}_0}\right)$$

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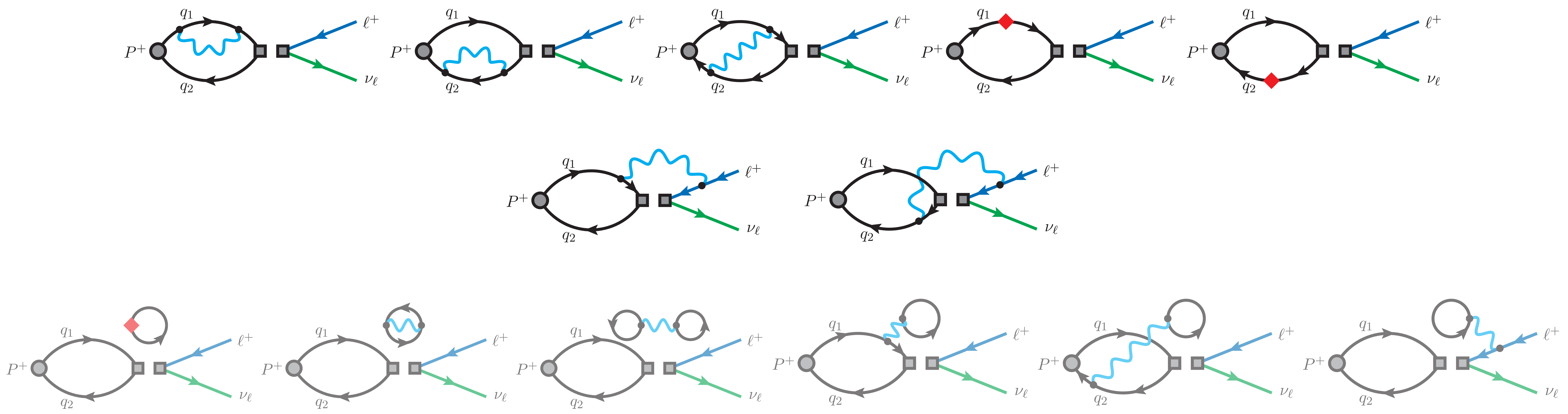
- $\boldsymbol{\delta m_P}$ correction to the meson mass
- $\boldsymbol{\delta \mathcal{Z}}$ correction to the renormalization of the weak operator O_W MDC et al., PRD 100 (2019) / MDC @Lattice2019

$$\boxed{\frac{\Gamma(K_{\ell 2})}{\Gamma(\pi_{\ell 2})}} \longrightarrow \boxed{\delta R_{K\pi} = 2\left(\frac{\delta \mathcal{A}_K}{\mathcal{A}_{K,0}} - \frac{\delta m_K}{m_{K,0}}\right) - 2\left(\frac{\delta \mathcal{A}_\pi}{\mathcal{A}_{\pi,0}} - \frac{\delta m_\pi}{m_{\pi,0}}\right)}$$

IB corrections to the decay amplitude

G.M.de Divitiis et al. [RM123], PRD **87** (2013)

RM123 perturbative method: expand lattice path-integral around isosymmetric point $\alpha = m_{\mathrm{u}} - m_{\mathrm{d}} = 0$

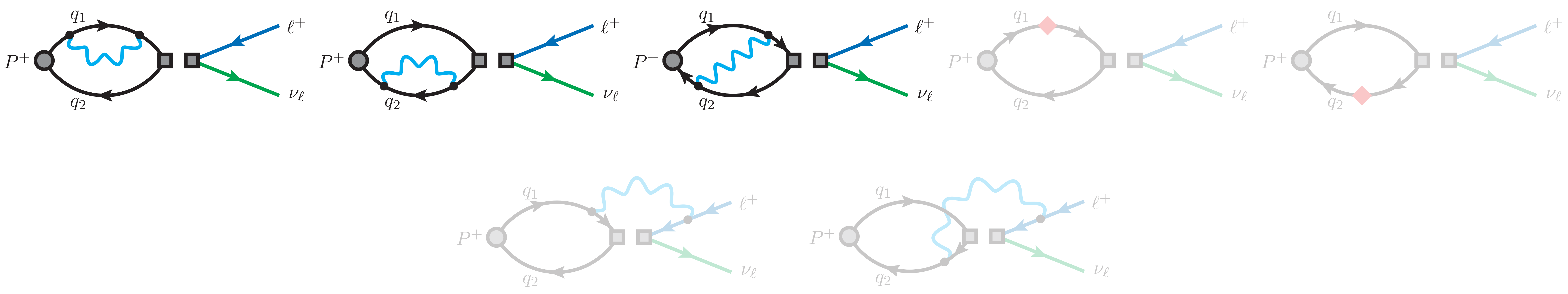


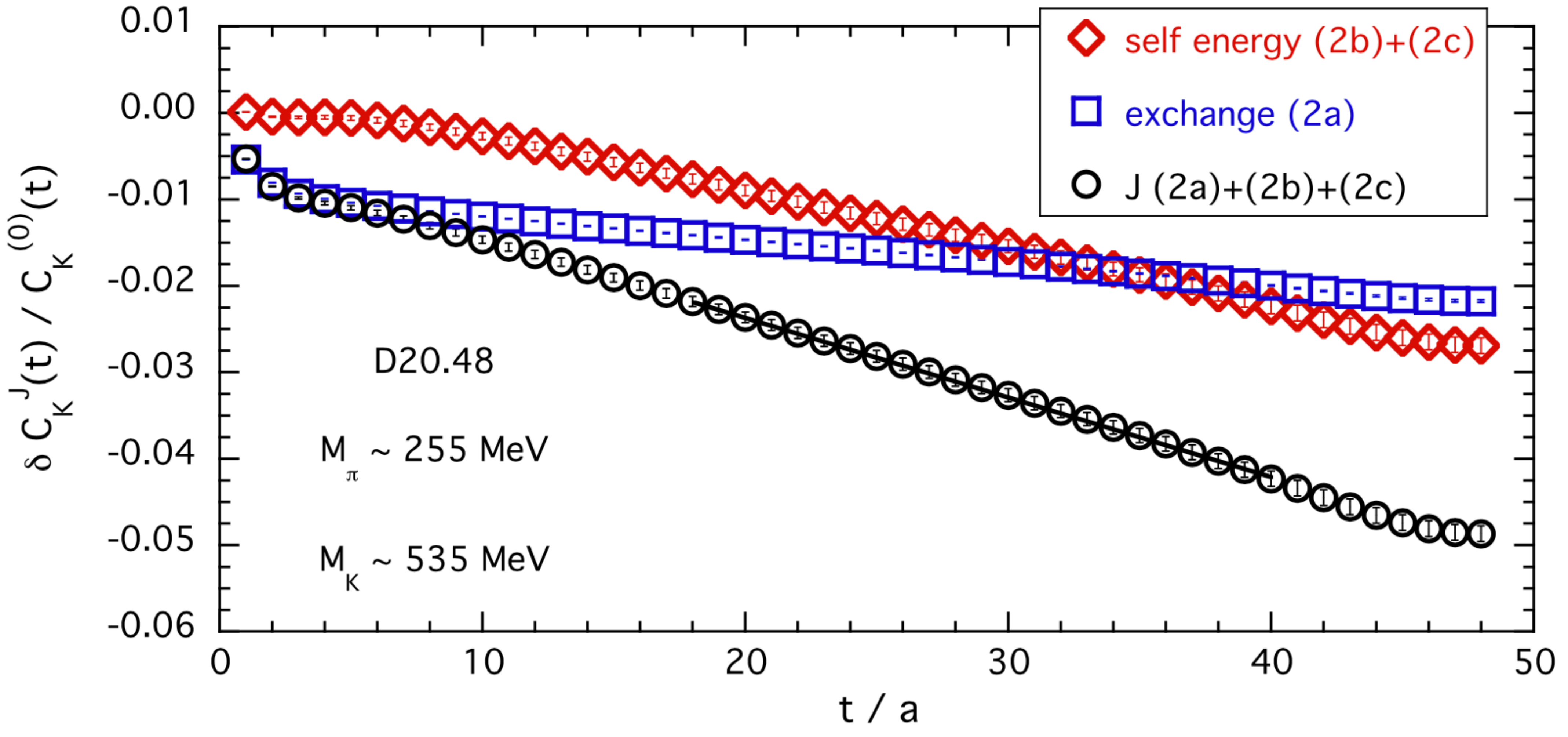
Both RM123S and RBC-UKQCD calculations are performed in the **electro-quenched approximation: sea quarks electrically neutral**

IB corrections to the decay amplitude

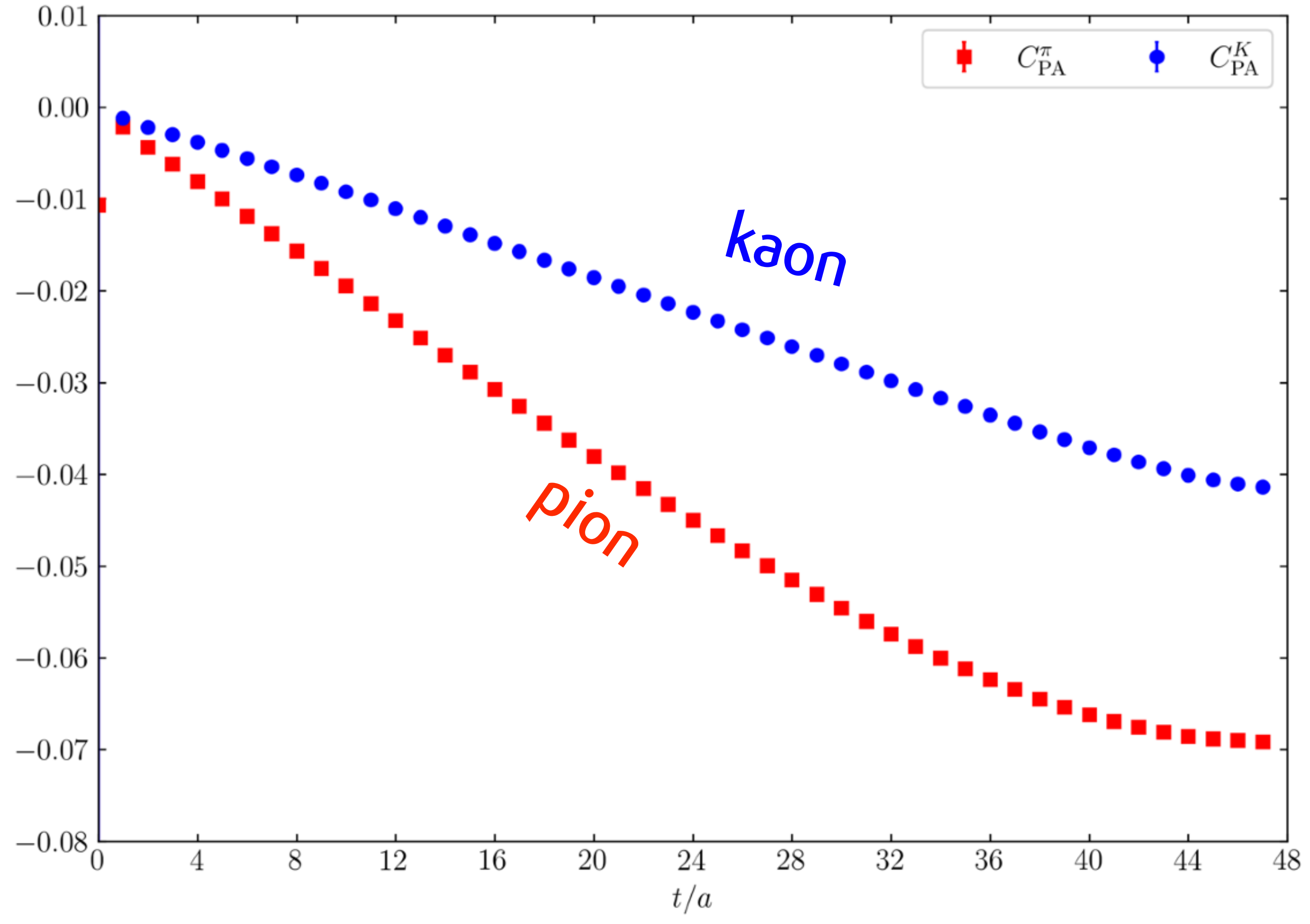
G.M.de Divitiis et al. [RM123], PRD 87 (2013)

RM123 perturbative method: expand lattice path-integral around isosymmetric point $\alpha = m_{\rm u} - m_{\rm d} = 0$





MDC et al., PRD 100 (2019)



P.Boyle, MDC et al., JHEP 02 (2023)

IB corrections to the decay amplitude

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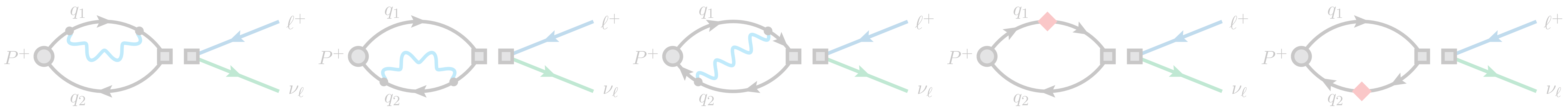
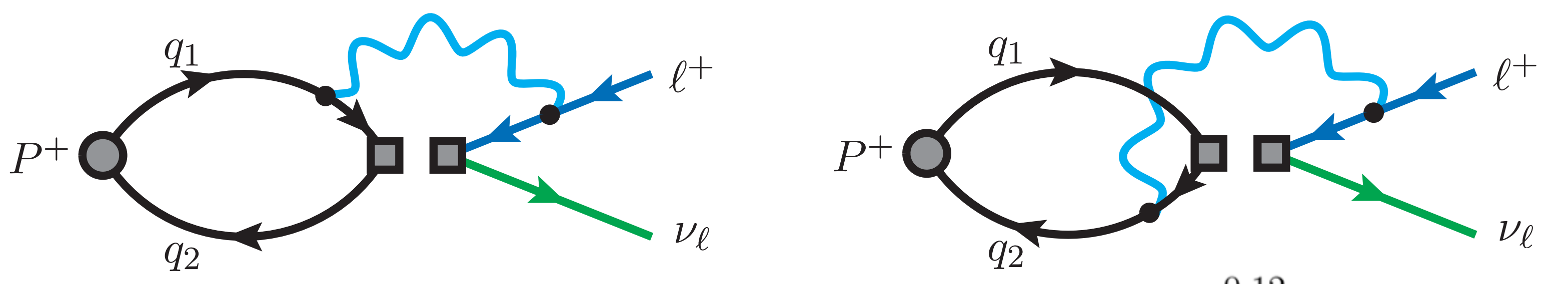
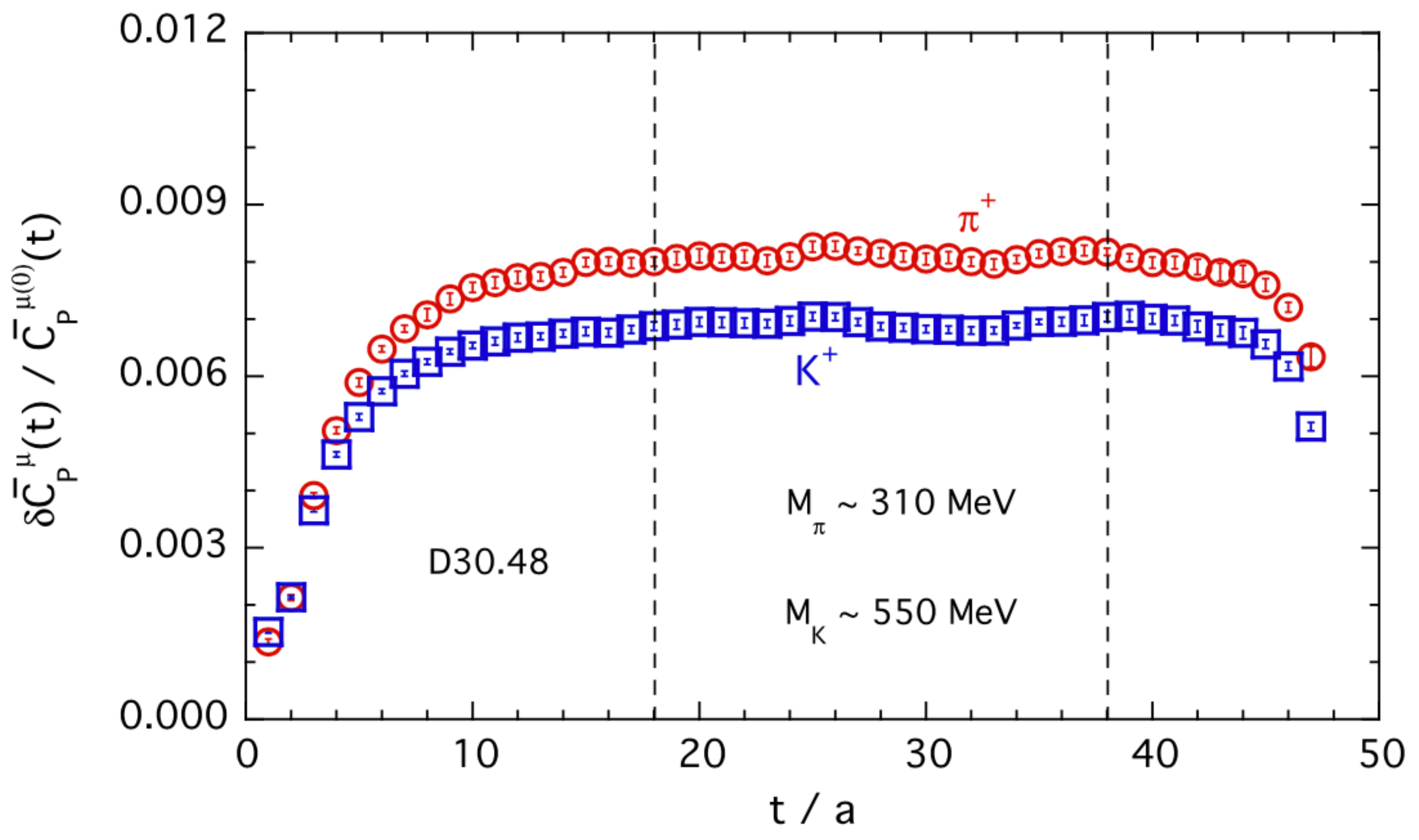
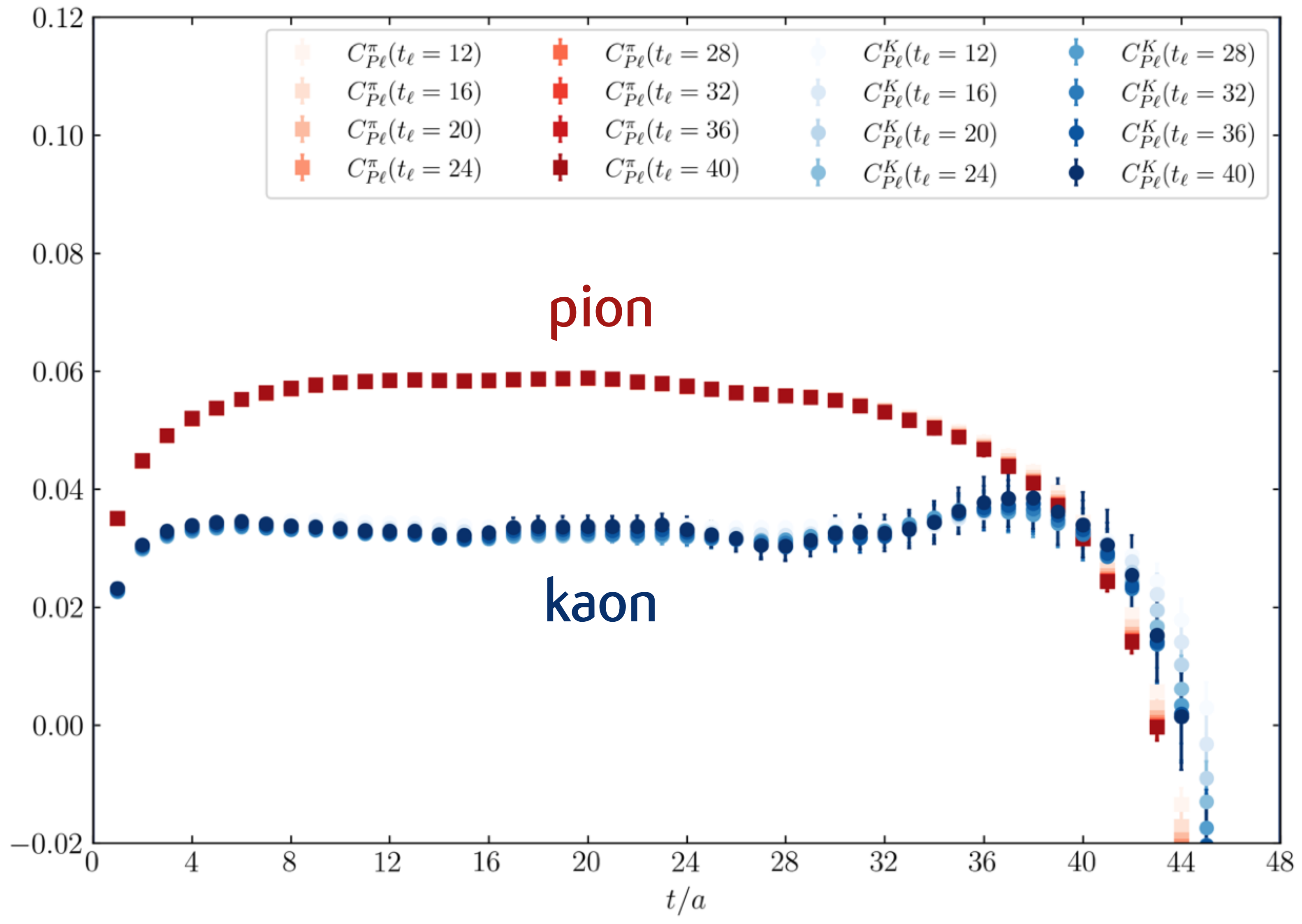






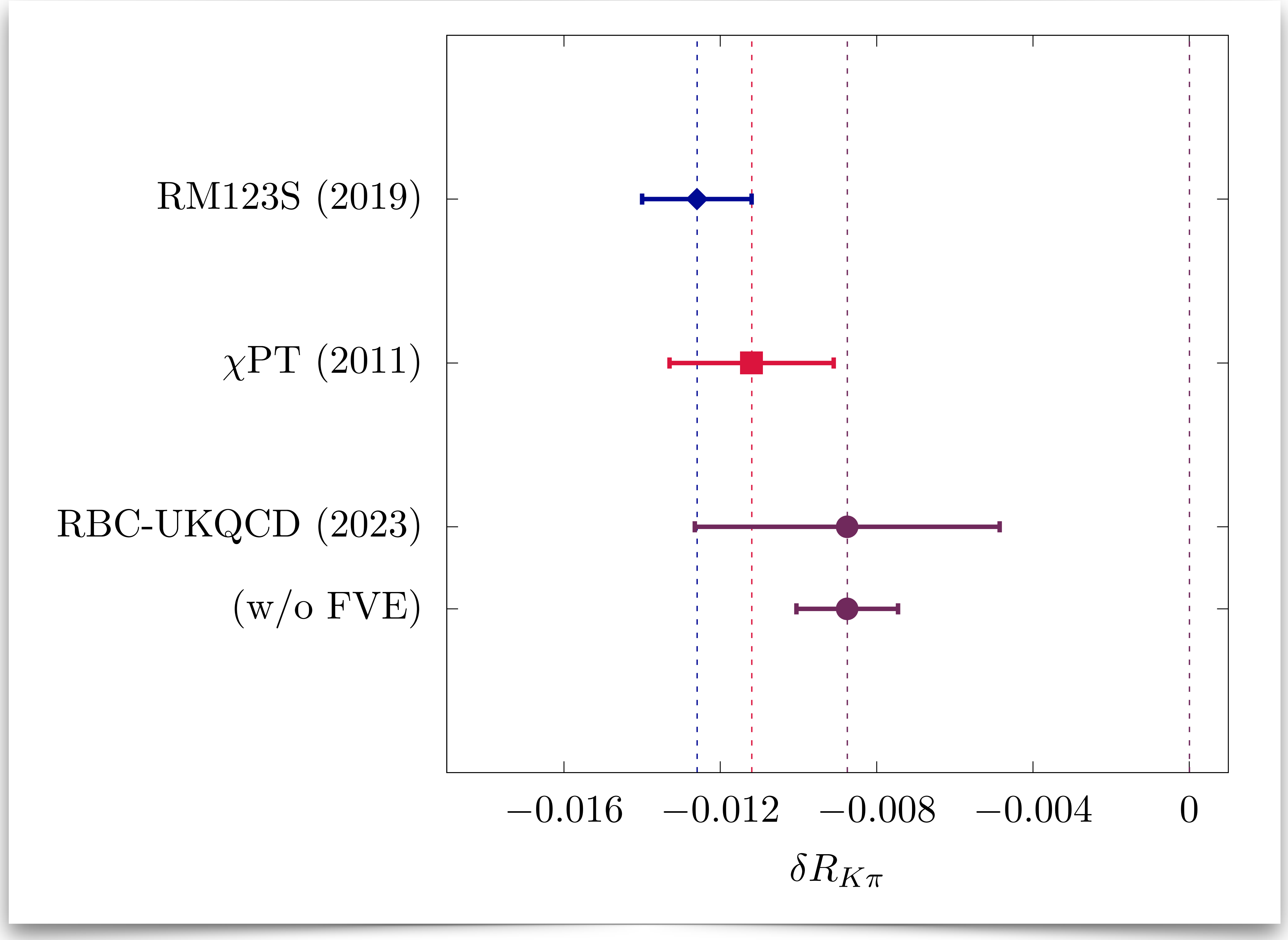
MDC et al., PRD 100 (2019)



P.Boyle, MDC et al., JHEP 02 (2023)

Results for $\delta R_{K\pi}$

V. Cirigliano et al., PLB **700** (2011)
MDC et al., PRD **100** (2019)
P.Boyle, MDC et al., JHEP **02** (2023)



RBC-UKQCD:

$$\delta R_{K\pi} = -0.0086\,(3)_{\text{stat.}}\,(^{+11}_{-4})_{\text{fit}}\,(5)_{\text{disc.}}\,(5)_{\text{quench.}}\,(39)_{\text{vol.}}$$

RM123S: $\delta R_{K\pi} = -0.0126\,(14)$ χPT: $\delta R_{K\pi} = -0.0112\,(21)$

- Our recent result is **compatible** with previous lattice calculation (RM123S) and with χPT
- The error is dominated by a large systematic uncertainty related to **finite-volume effects**

Solid evidence that $\delta R_{K\pi}$ can be computed from first principles non-perturbatively on the lattice!

Finite-volume effects in QED$_L$

V. Lubicz et al., PRD **95** (2017)
N. Tantalo et al., [1612.00199v2]
MDC et al., PRD **105** (2022)

Leptonic decay rate

$$\Gamma_0(L) = \Gamma_0^{\mathrm{tree}} \left\{ 1 + 2\,\frac{\alpha}{4\pi}\,Y(L) \right\}$$

$$Y(L) - Y(\infty) = Y_{\log}(L) + Y_0 + \frac{1}{m_P L}\,Y_1 + \frac{1}{(m_P L)^2}\,Y_2 + \frac{1}{(m_P L)^3}\,Y_3 + \mathrm{O}(1/L^4) + \mathrm{O}(\mathrm{e}^{-\alpha L})$$

Finite-volume effects in QED$_L$

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$$Y(L) - Y(\infty) = \underbrace{\boxed{Y_{\log}(L) + Y_0 + \frac{1}{m_P L}\,Y_1}}_{\approx\, -3.96} + \frac{1}{(m_P L)^2}\,Y_2 + \frac{1}{(m_P L)^3}\,Y_3 + \mathrm{O}(1/L^4) + \mathrm{O}(\mathrm{e}^{-\alpha L})$$

$m_\pi L \approx 3.9$

- structure independent ("**universal**") terms ✔

Finite-volume effects in $\mathrm{QED_L}$

Leptonic decay rate

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MDC et al., PRD **105** (2022)

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- **structure dependent** contribution at $\mathrm{O}(1/L^2)$ ✔

Finite-volume effects in $\mathrm{QED_L}$

Leptonic decay rate

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$$Y(L) - Y(\infty) = \boxed{Y_{\log}(L) + Y_0 + \frac{1}{m_P L}\,Y_1} + \boxed{\frac{1}{(m_P L)^2}\,Y_2} + \boxed{\frac{1}{(m_P L)^3}\,Y_3^{\text{pt}}} + \frac{1}{(m_P L)^3}\,Y_3^{\text{SD}} + \mathrm{O}(1/L^4) + \mathrm{O}(e^{-\alpha L})$$

$m_\pi L \approx 3.9$ $\quad\approx -3.96 \quad \approx -2.24 \quad \approx 3.37 \quad ?$

- structure independent ("**universal**") terms ✔
- **structure dependent** contribution at $O(1/L^2)$ ✔
- **sizeable pointlike** contribution at $O(1/L^3)$ ✔
- higher order effects ✘

N.Hermansson-Truedsson (Friday 4, h 9.20)

Current status

Finite volume effects produce large **systematic uncertainty**

$$\delta R_{K\pi} = -0.0086\,(13)(39)_{\text{vol.}}$$

repeat the calculation on **multiple volumes** & take **infinite volume limit**

$$\frac{1}{(m_P L)^3}\left[\textbf{structure-dependent}\right]$$

compute **missing effects** at $\mathcal{O}(1/L^3)$

$$\left(\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right)$$

adopt or **develop QED formulations** with **reduced** finite volume effects

Current status

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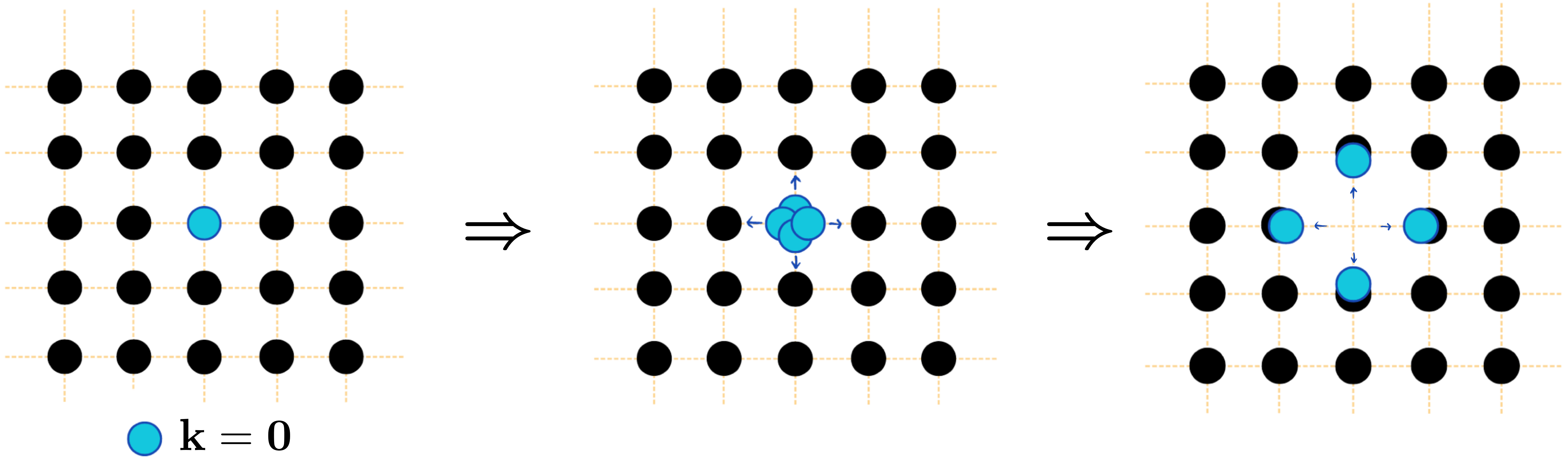
adopt or **develop QED formulations** with **reduced** finite volume effects

... can we formulate QED on a finite-volume <u>without</u> corrections at $O(1/L^3)$?

A new idea under investigation

Z.Davoudi et al., PRD 99 (2019)

Special case of IR-improvement

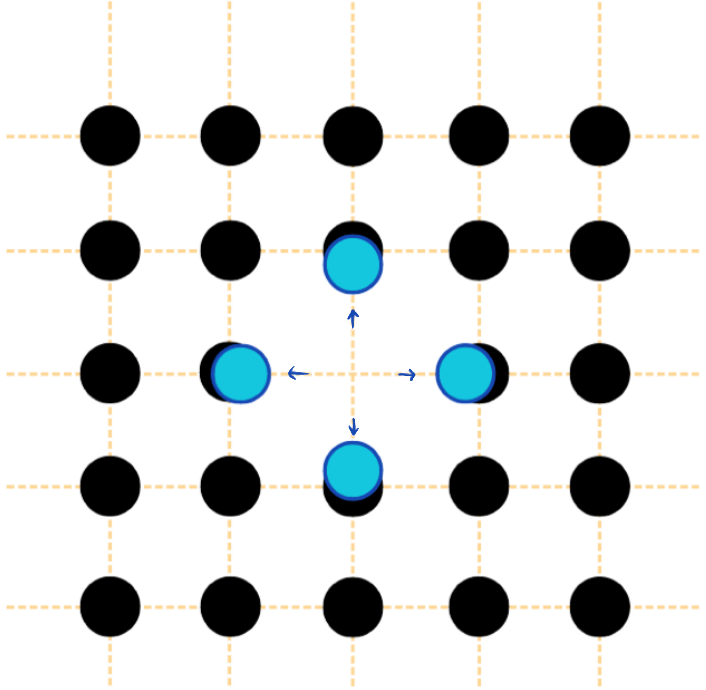


The spatial zero mode is not removed but redistributed over the neighbouring modes on a shell of radius $|\mathbf{p}| = \frac{2\pi}{L}|\mathbf{r}| \quad (\mathbf{r} \in \mathbb{Z}^3)$

QED$_L$: $D_L^{\mu\nu}(k_0,\mathbf{k}) = \delta^{\mu\nu}\frac{1-\delta_{\mathbf{k},\mathbf{0}}}{k_0^2+\mathbf{k}^2} \quad \Rightarrow \quad$ **QED$_r$:** $D_{\mathbf{p}}^{\mu\nu}(k_0,\mathbf{k}) = \delta^{\mu\nu}\frac{1-\delta_{\mathbf{k},\mathbf{0}}}{k_0^2+\mathbf{k}^2} + \frac{\delta_{\mathbf{k}^2,\mathbf{p}^2}}{n(\mathbf{p}^2)}\frac{\delta^{\mu\nu}}{k_0^2+\mathbf{p}^2}$

Finite-volume effects in QED$_r$

Zero net velocity



1. In systems with **zero net velocity**, the corrections at $O(1/L^3)$ due to the removal of the zero mode are <u>absent</u>!

Hadron masses:

notation from
B.Lucini et al., JHEP **02** (2016)

$$\Delta m^2(L)\big|_{\mathrm{QED_r}} = \underbrace{\left[\frac{1}{L^3}\sum_{\mathbf{k}\neq\mathbf{0}} - \int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right]\frac{M^{\mu\mu}(-|\mathbf{k}|,\mathbf{k})}{2|\mathbf{k}|}}_{\mathrm{QED_L}} + \frac{1}{L^3}\,\frac{M^{\mu\mu}(-|\mathbf{p}|,\mathbf{p})}{2|\mathbf{p}|} \qquad |\mathbf{p}| = \tfrac{2\pi}{L}|\mathbf{r}| \quad (\mathbf{r}\in\mathbb{Z}^3)$$

$$= \cdots + \frac{\mathcal{M}'(0)}{2L^3}\left\{\underbrace{\left[\sum_{\mathbf{n}\neq\mathbf{0}} - \int \mathrm{d}^3\mathbf{n}\right]1}_{\mathrm{QED_L}} + 1\right\} = \cdots + \frac{\mathcal{M}'(0)}{2L^3}\left\{-1 + 1\right\} = \cdots + 0$$

The redistributed modes "reproduce" the zero mode in the ∞-volume limit!

Finite-volume effects in QED$_r$

Non-zero net velocity

2. For systems with non-zero velocity, the cancellation of $O(1/L^3)$ is less straightforward.

In the case of leptonic decay rates, the coefficient at $O(1/L^3)$ can depend on lepton velocity $\mathbf{v}$:

QED$_r$:

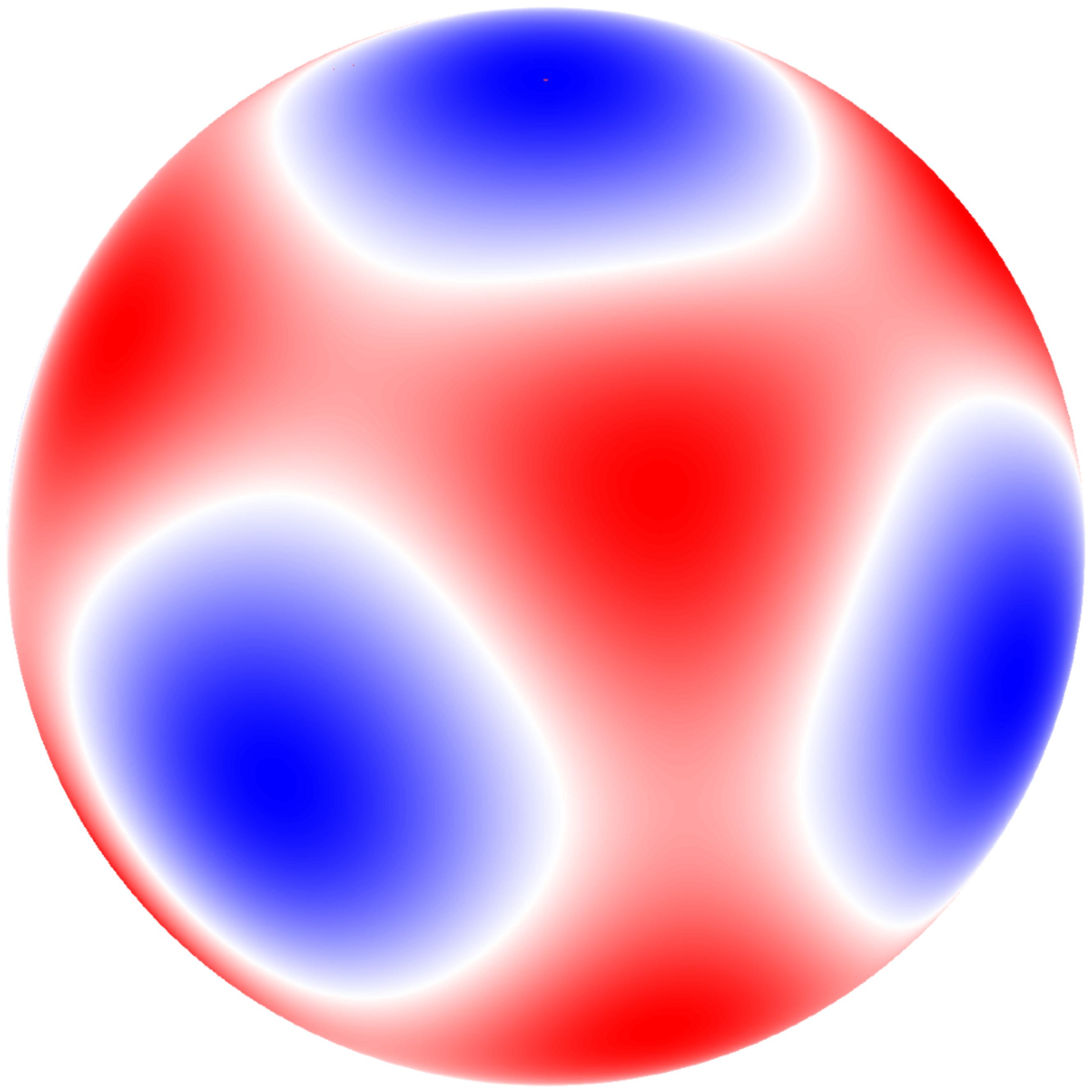
$$\boxed{\bar{c}_0(\mathbf{v}) = \left[\sum_{\mathbf{n}\neq\mathbf{0}} - \int \mathrm{d}^3\mathbf{n}\right] \frac{1}{1-\mathbf{v}\cdot\hat{\mathbf{n}}} + \frac{1}{6}\sum_{|\mathbf{r}|^2=1} \frac{1}{1-\mathbf{v}\cdot\hat{\mathbf{r}}}} \qquad |\mathbf{v}| = \frac{m^2 - m_\ell^2}{m^2 + m_\ell^2}$$

- **Collinear** divergent terms as $|\mathbf{v}| \to 1$ and $\mathbf{v} \parallel \mathbf{k}$
- Dependence on the direction $\hat{\mathbf{v}}$ due to **rotational symmetry breaking** in a finite volume

Finite-volume effects in QED$_r$

Non-zero net velocity

2. For systems with non-zero velocity, the cancellation of $O(1/L^3)$ is less straightforward.



$\boxed{\bar{c}_0(\mathbf{v})}$

$\max \bar{c}_0(\mathbf{v}) = 0.0171$

$\min \bar{c}_0(\mathbf{v}) = -0.0114$

Finite-volume effects in QED$_r$

Non-zero net velocity

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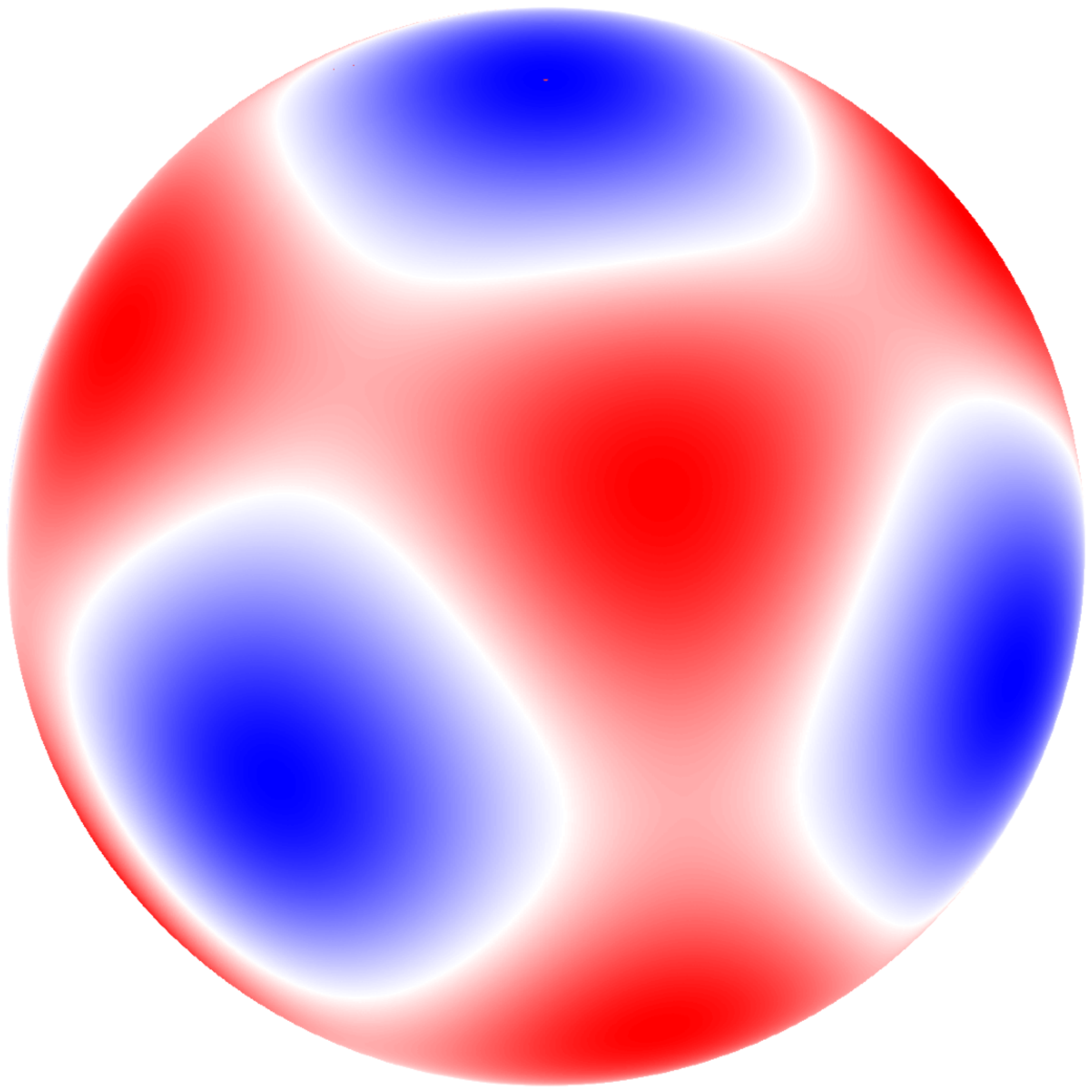
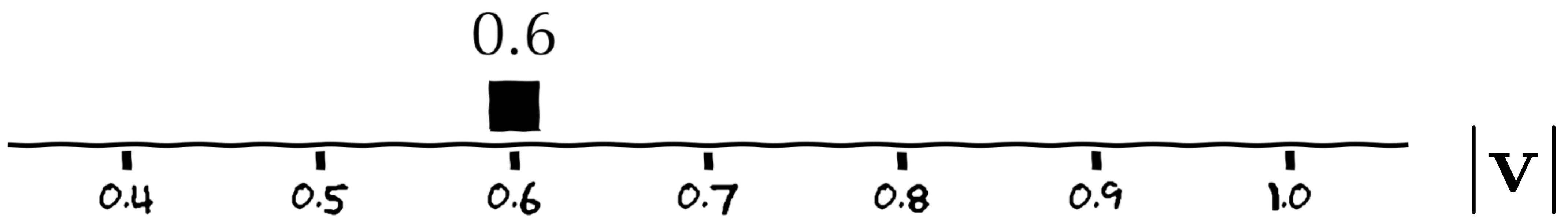


$\boxed{\bar{c}_0(\mathbf{v})}$

$$\max \bar{c}_0(\mathbf{v}) = 0.1199$$

$$\min \bar{c}_0(\mathbf{v}) = -0.0747$$



Finite-volume effects in QED$_r$

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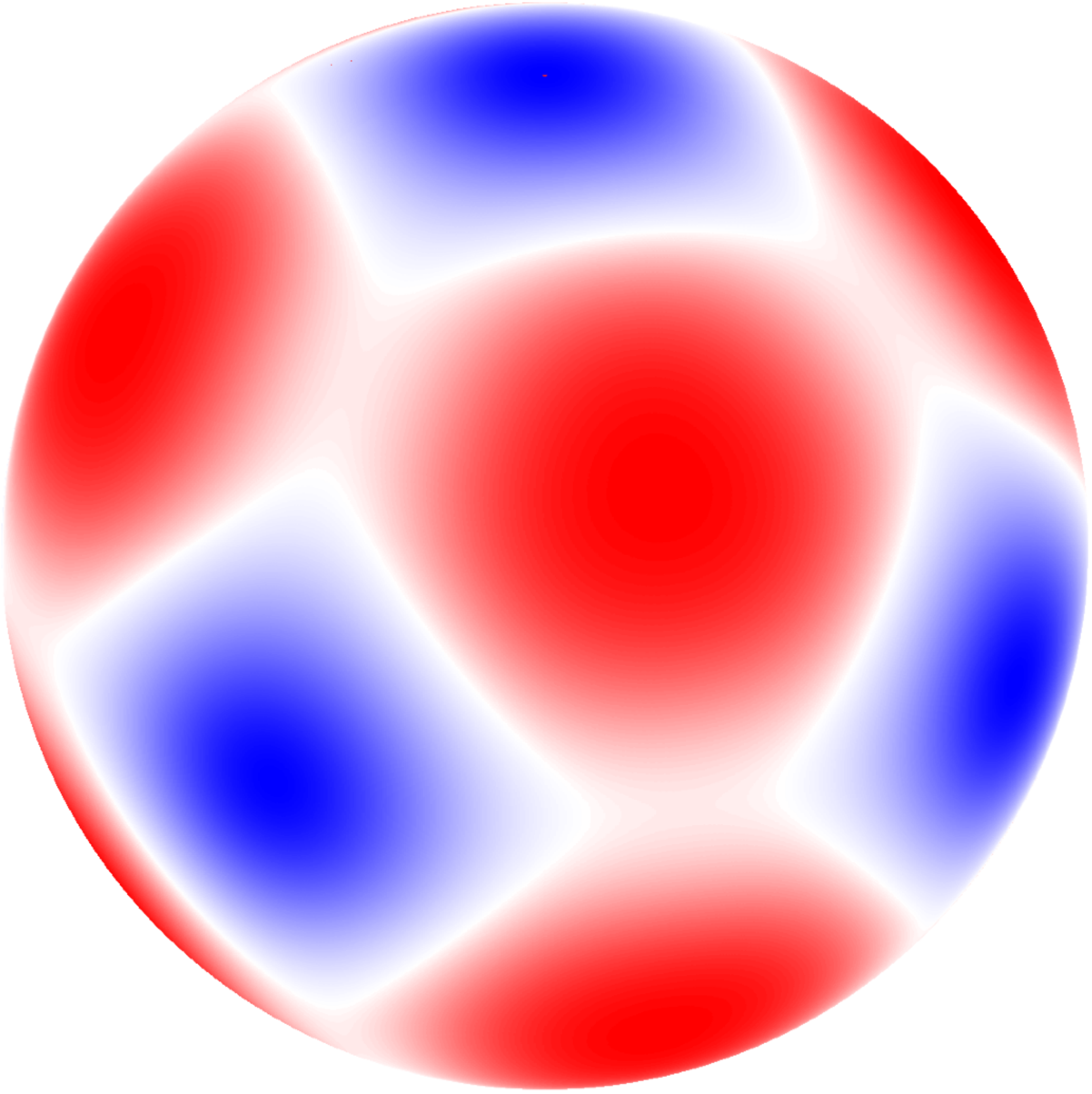
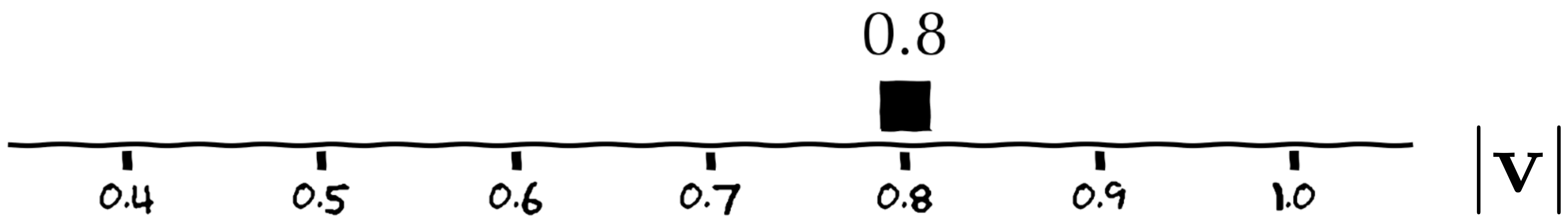


$\boxed{\bar{c}_0(\mathbf{v})}$

$$\max \bar{c}_0(\mathbf{v}) = 0.9446$$

$$\min \bar{c}_0(\mathbf{v}) = \text{-}0.4115$$



Finite-volume effects in QED_r

Non-zero net velocity

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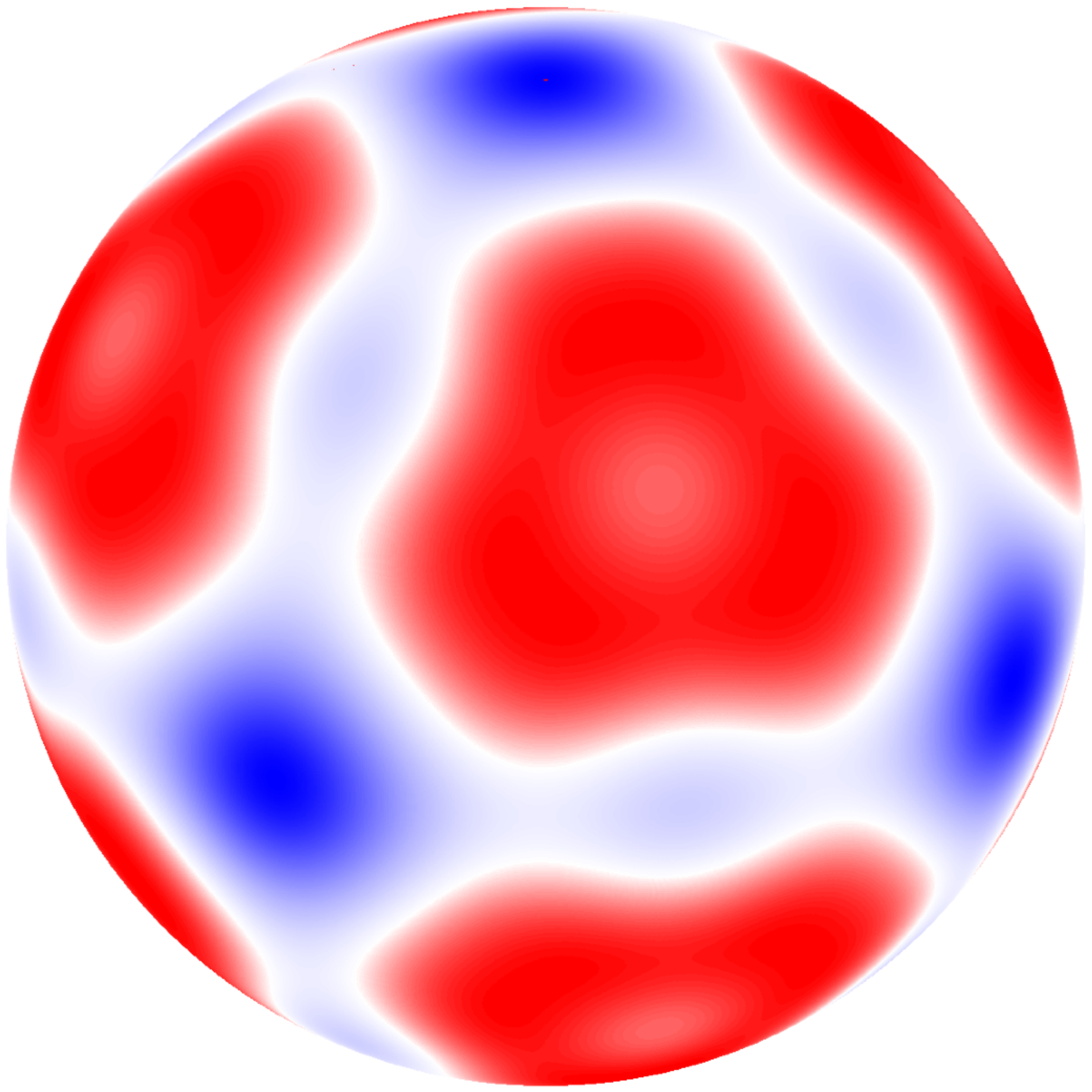
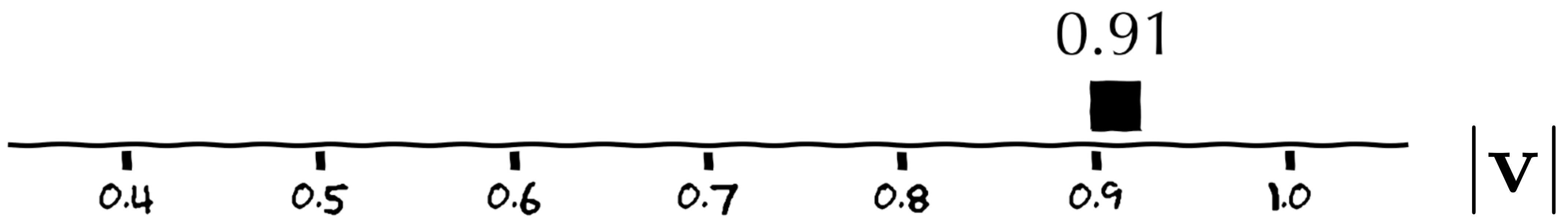


$$\boxed{\bar{c}_0(\mathbf{v})}$$

$$\max \bar{c}_0(\mathbf{v}) = 5.2059$$

$$\min \bar{c}_0(\mathbf{v}) = -1.2689$$



Finite-volume effects in QED$_r$

Non-zero net velocity

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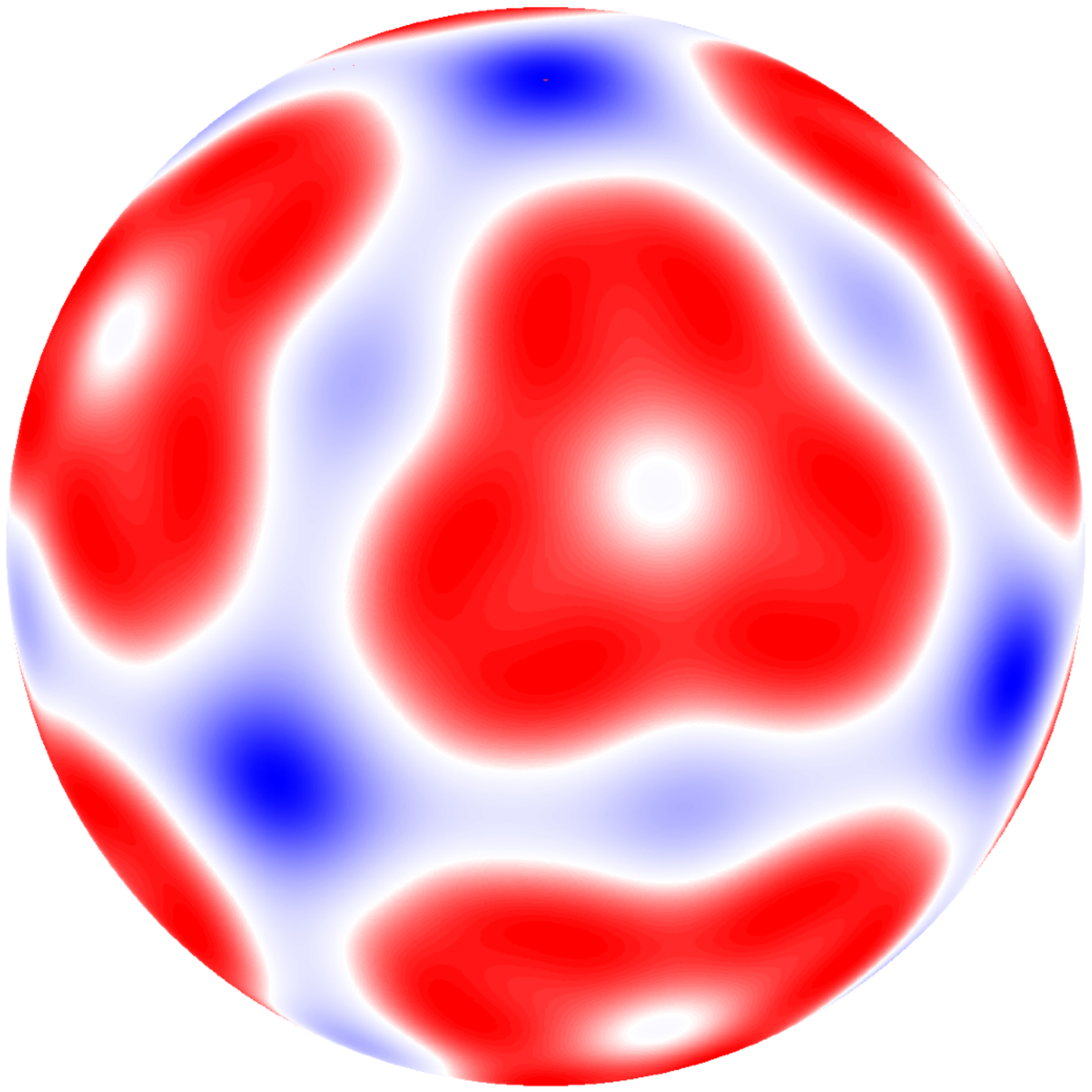
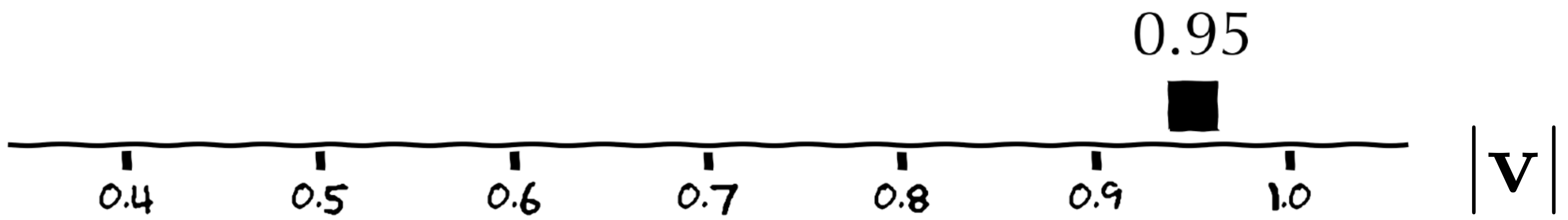


$\boxed{\bar{c}_0(\mathbf{v})}$

$\max \bar{c}_0(\mathbf{v}) = 15.2832$

$\min \bar{c}_0(\mathbf{v}) = -2.8258$



Finite-volume effects in QED_r

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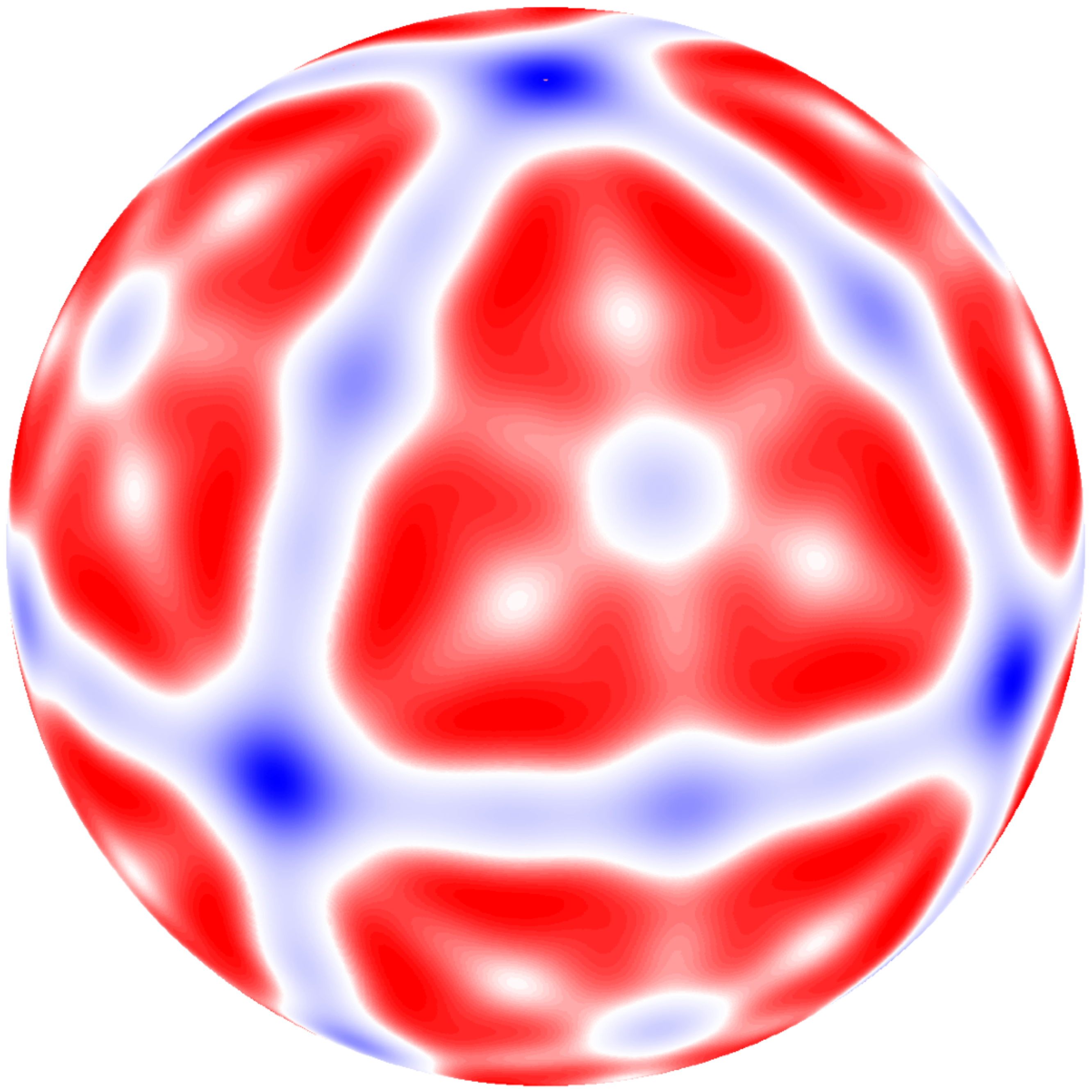
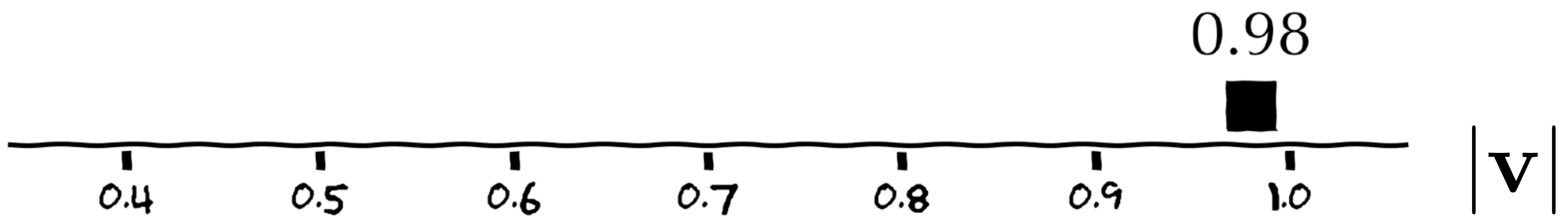


$$\boxed{\bar{c}_0(\mathbf{v})}$$

$$\max \bar{c}_0(\mathbf{v}) = 71.5812$$

$$\min \bar{c}_0(\mathbf{v}) = -10.0290$$



Finite-volume effects in QED$_r$

Non-zero net velocity

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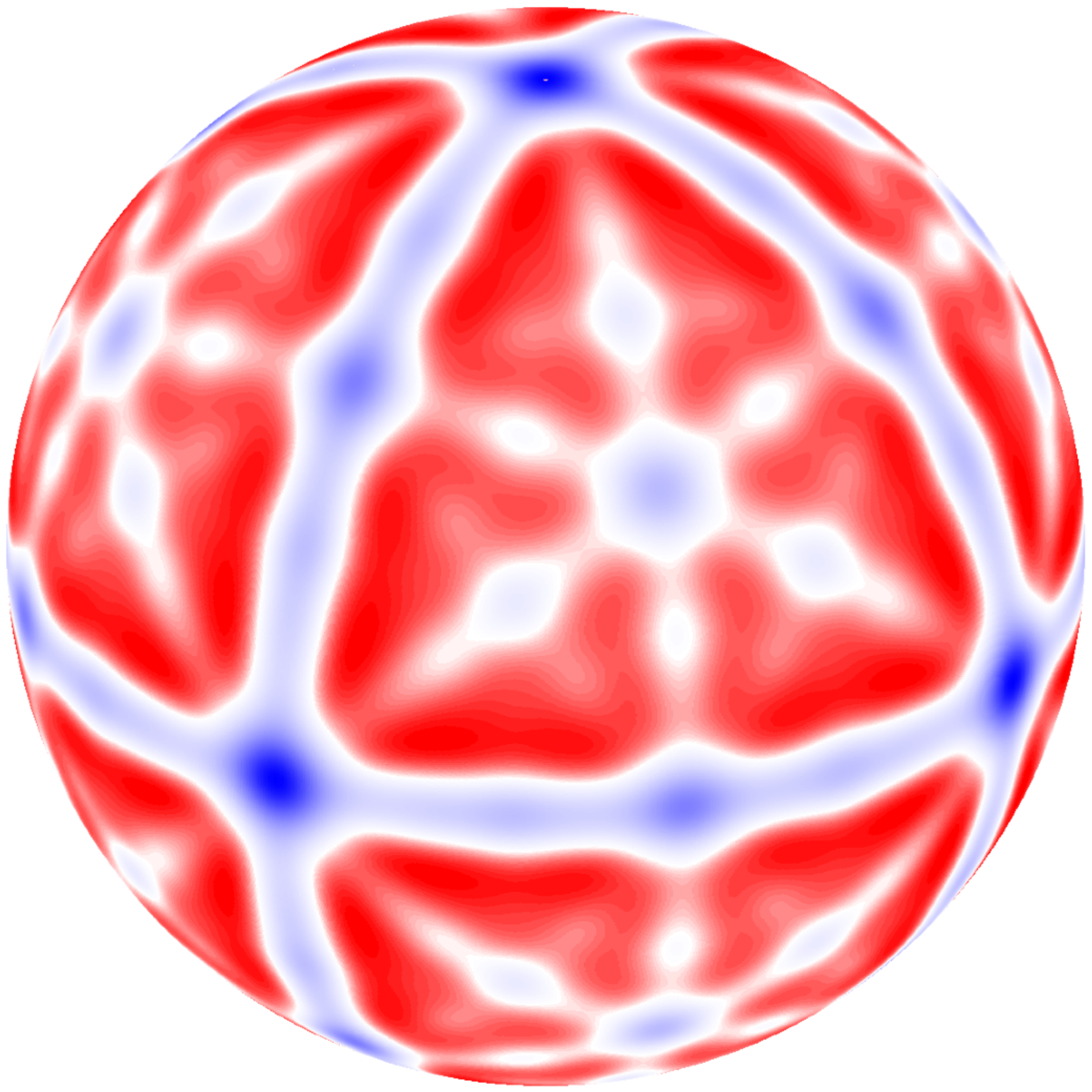
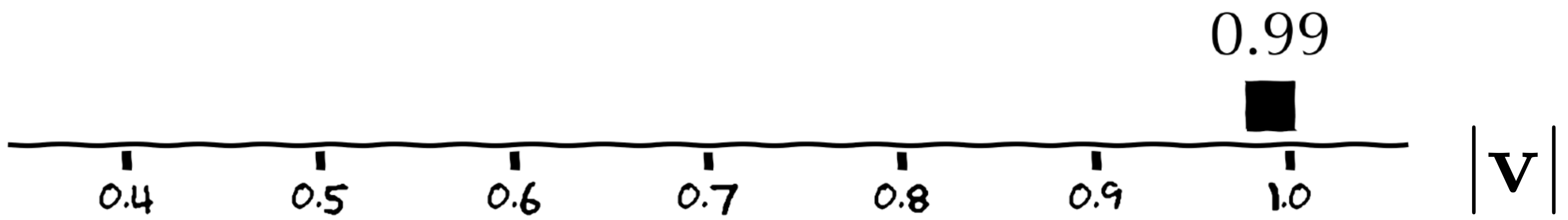


$$\boxed{\bar{c}_0(\mathbf{v})}$$

$$\max \bar{c}_0(\mathbf{v}) = 215.7470$$

$$\min \bar{c}_0(\mathbf{v}) = -25.4646$$



Finite-volume effects in QED$_r$

Non-zero net velocity

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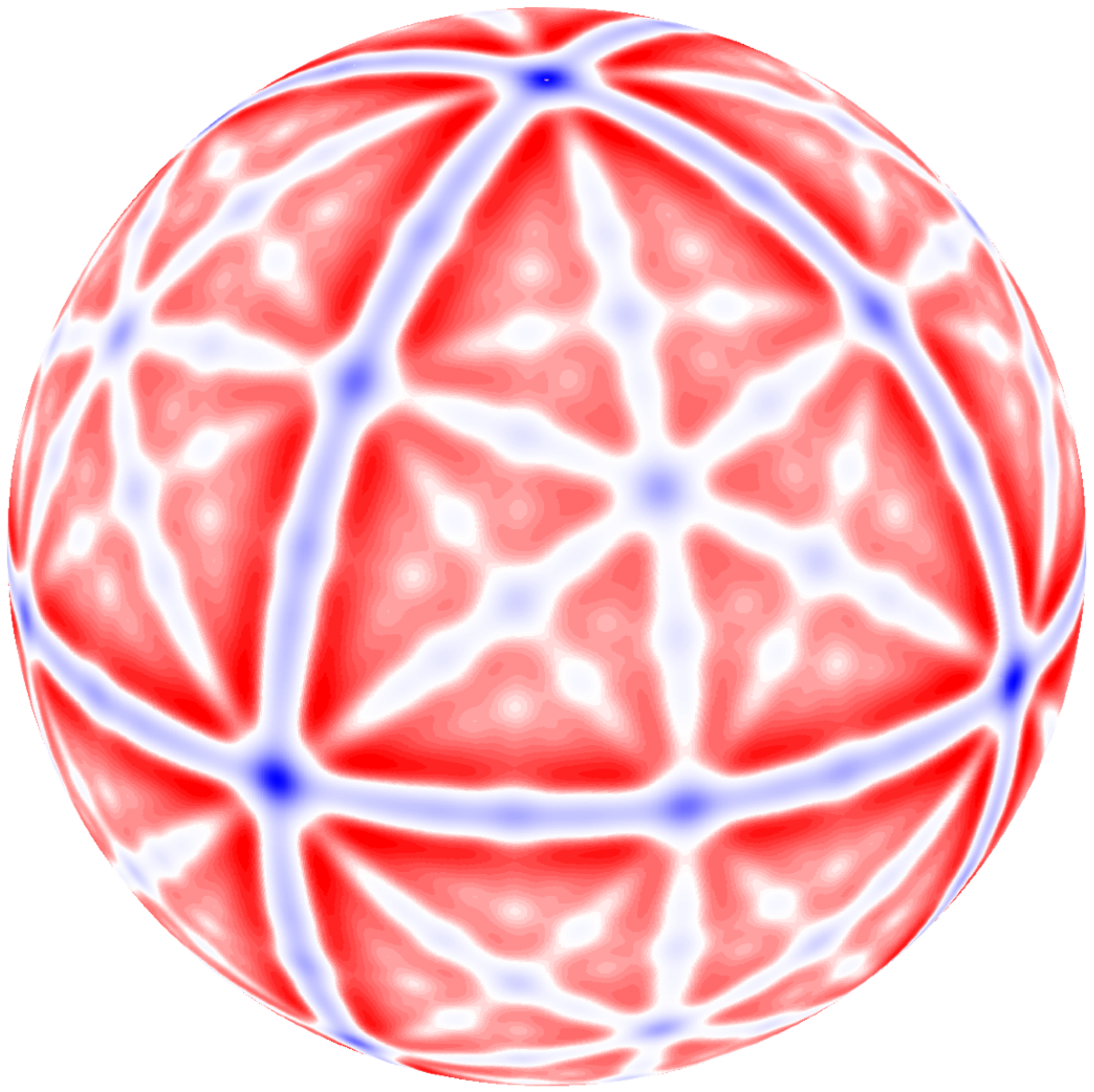
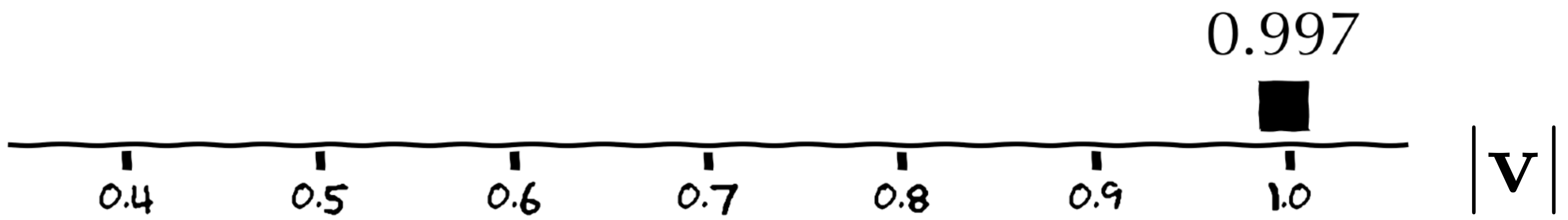


$$\boxed{\bar{c}_0(\mathbf{v})}$$

$$\max \bar{c}_0(\mathbf{v}) = 1445.9149$$

$$\min \bar{c}_0(\mathbf{v}) = -142.3143$$



Finite-volume effects in QED$_{\mathrm{r}}$

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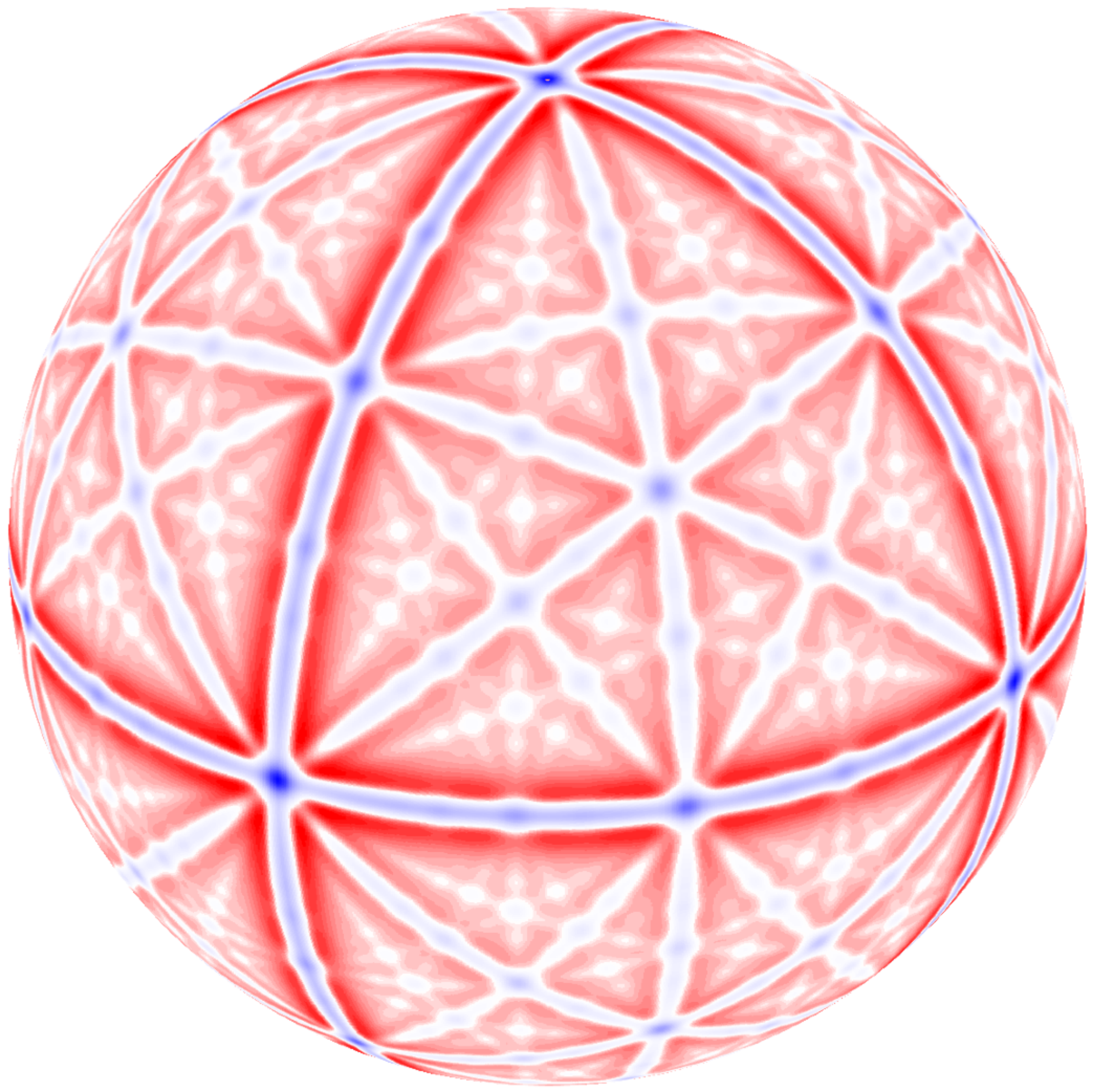
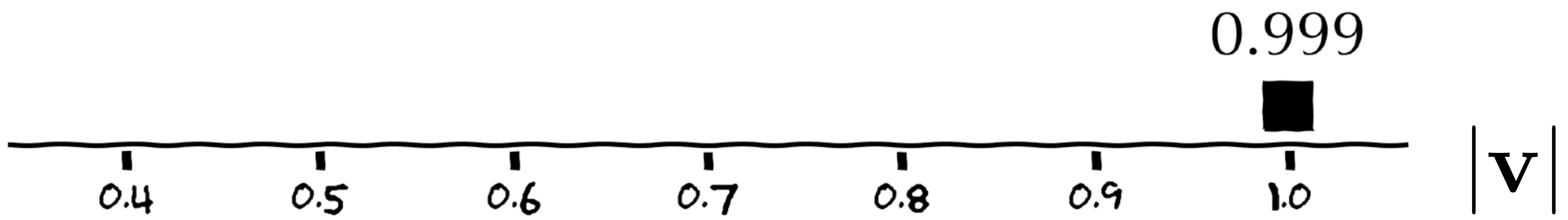


$$\boxed{\bar{c}_0(\mathbf{v})}$$

$$\max \bar{c}_0(\mathbf{v}) = 9002.2317$$

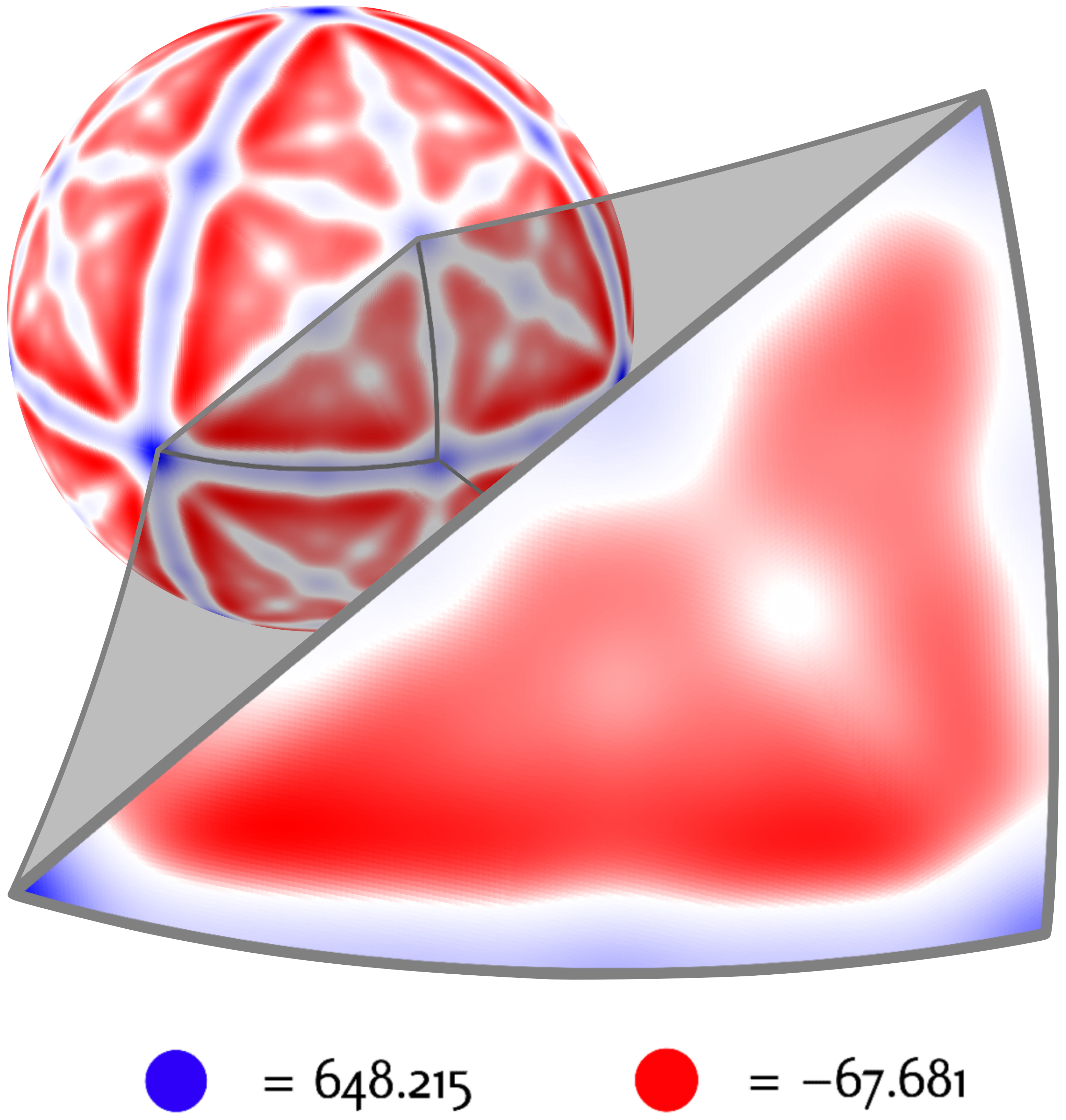
$$\min \bar{c}_0(\mathbf{v}) = -807.4018$$

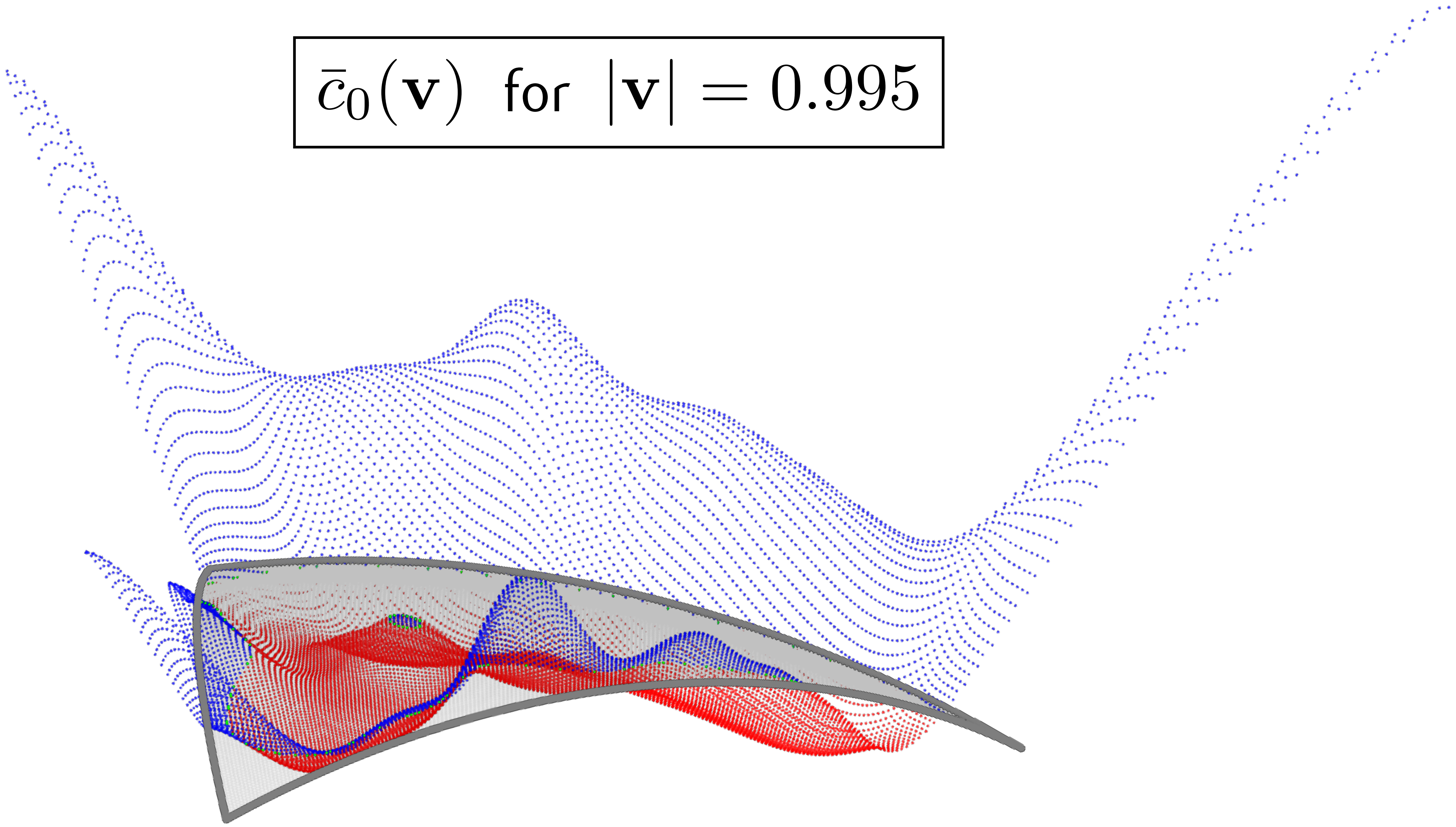


Finite-volume effects in QED$_r$

Non-zero net velocity

2. For systems with non-zero velocity, the cancellation of $O(1/L^3)$ is less straightforward.





Finite-volume effects in $\mathrm{QED_r}$

Non-zero net velocity

2. For systems with non-zero velocity, the cancellation of $O(1/L^3)$ is less straightforward.

$$\boxed{\bar{c}_0(\mathbf{v}) = \left[\sum_{\mathbf{n}\neq\mathbf{0}} - \int d^3\mathbf{n}\right] \frac{1}{1-\mathbf{v}\cdot\hat{\mathbf{n}}} + \frac{1}{6}\sum_{|\mathbf{r}|^2=1} \frac{1}{1-\mathbf{v}\cdot\hat{\mathbf{r}}}}$$

Two fundamental properties in $\mathrm{QED_r}$:

A. For any value of $|\mathbf{v}|$, there **always exists** a **direction** $\hat{\mathbf{v}}^\star$ such that $\bar{c}_0(\mathbf{v}^\star) = 0$

B. The **average** over the **solid angle** gives $\displaystyle\frac{1}{4\pi}\int d\Omega_{\mathbf{v}}\, \bar{c}_0(\mathbf{v}) = 0$

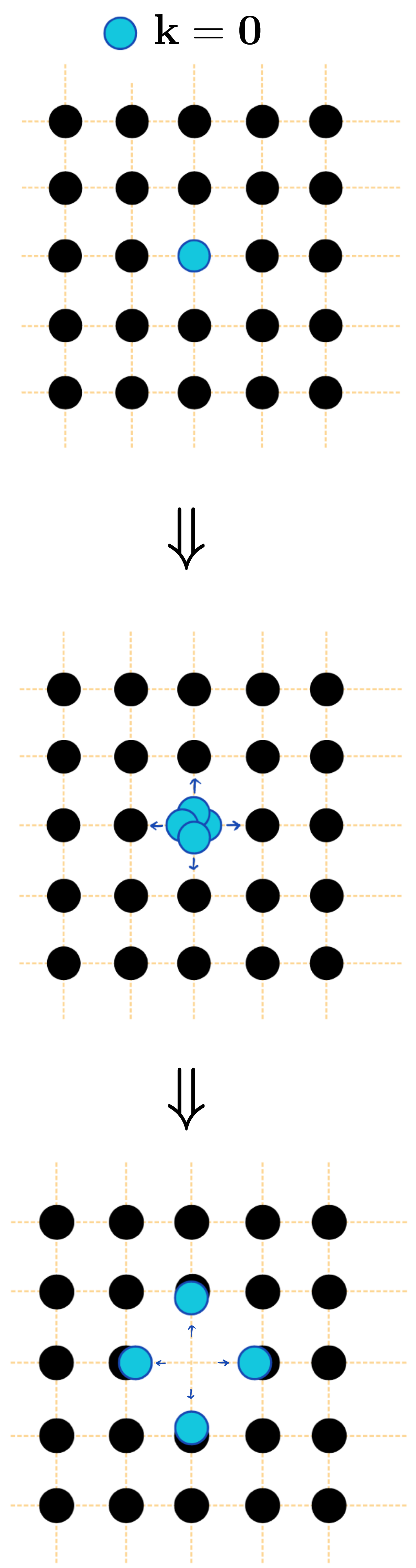
see A.Portelli (Friday 4, h 9.40) for a more detailed discussion!

QED$_r$ summary

- **Infrared improvement of QED$_L$:** redistribution of the spatial zero-mode
- **Free from** (problematic) **$O(1/L^3)$ effects**:
  - absent by construction for **zero-velocity** systems
  - improvement less straightforward for **velocity-dependent observables**, due to non-trivial collinear divergences A.Portelli (Friday 4, h 9.40)

Ongoing **numerical tests** of QED$_r$:

- finite-volume study on **new gauge ensembles** with 4 **different volumes**
- investigation of π, K, D and D_s decays



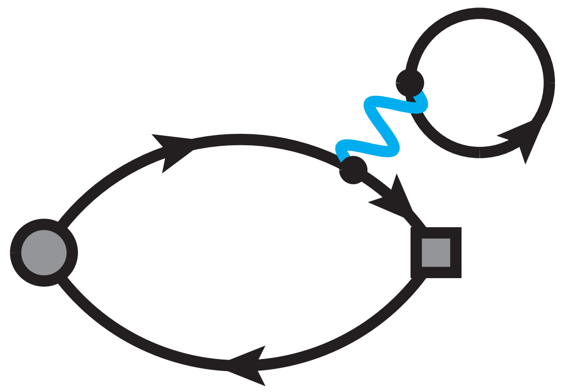
4. Where are we ...

- Current **tensions** in CKM unitarity require a combined effort of **theory** and **experiments**
- **Two lattice calculations** of IB and QED corrections to **light-meson leptonic decay rates**
- **Finite volume QED effects** have to be carefully studied (interesting proposal with QED$_\infty$!)

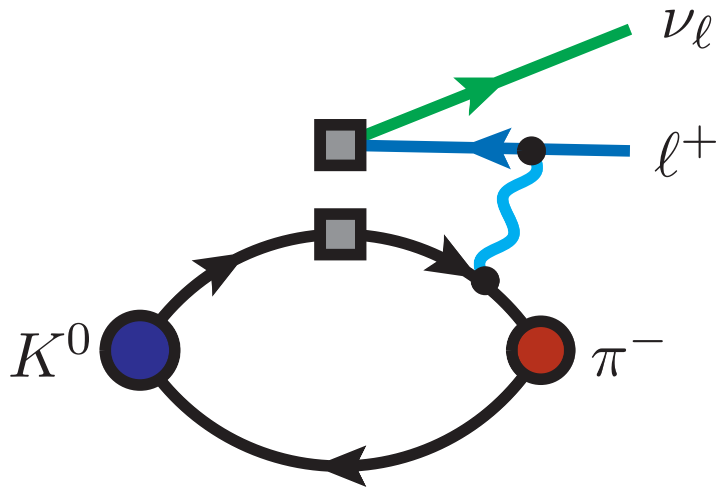
... and where to go?

$$\left(\frac{1}{L^3} \sum_{\mathbf{k} \neq \mathbf{0}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \right)$$

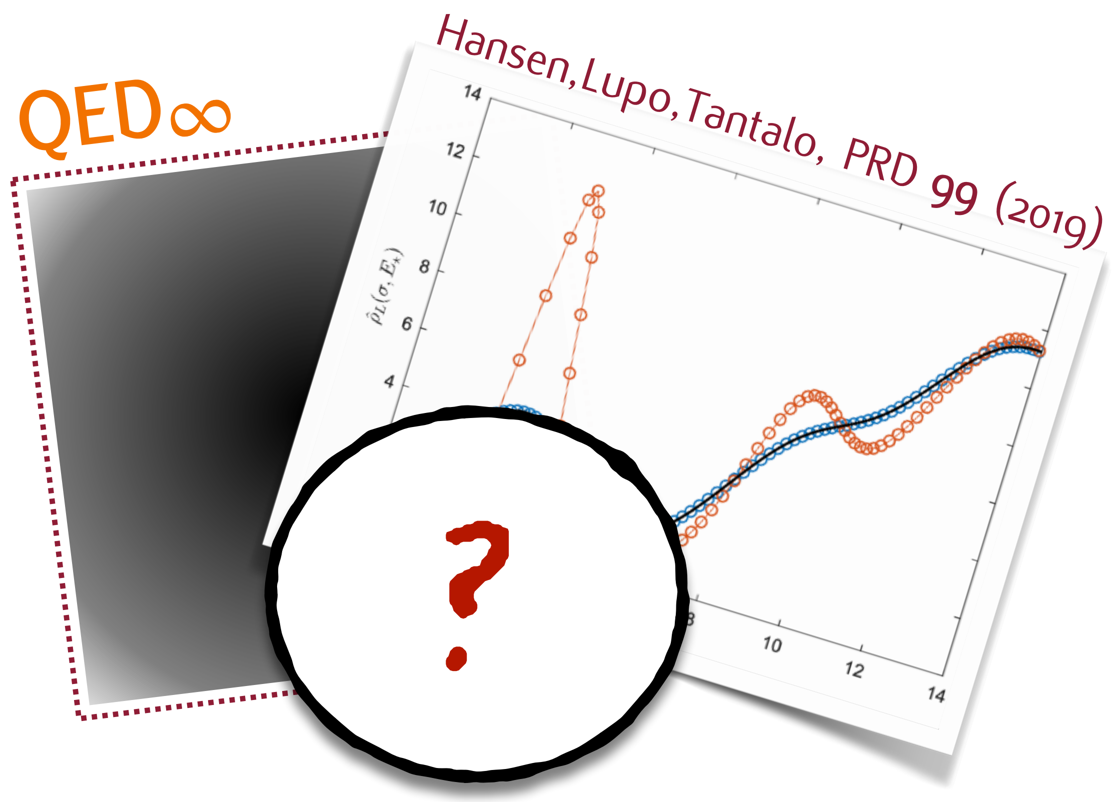
investigate & tame finite-volume effects



move to unquenched calculations



study different weak processes



develop and apply new techniques

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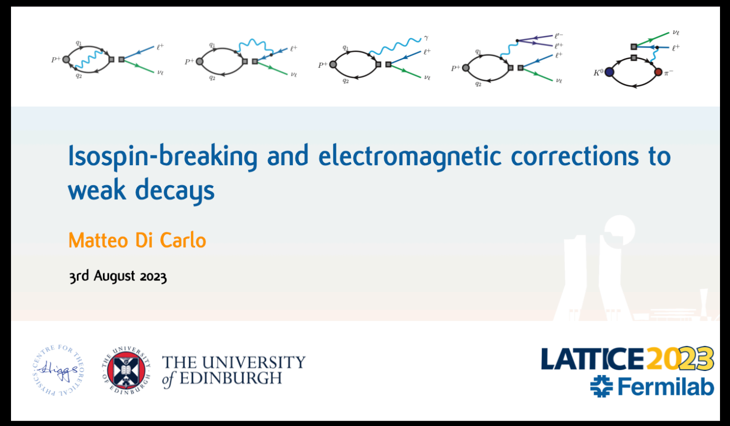
Plenary talks

2023 - 2032



M. Di Carlo (2023)

At the level of precision reached by the recent lattice calculations of decay constants and form factors, it becomes necessary to include the effects of QED interactions and those due to the up-down quark mass splitting in the calculation of weak decay rates of hadrons…

Looking forward to the new episodes!

OVERVIEW EPISODES TRAILERS & MORE MORE LIKE THIS DETAILS

Monday 31

- 16.20: J.Swaim — QED Corrections to meson and bare quark masses
- 16.40: A.Segner — Precision determination of baryon masses including isospin breaking

Thursday 3

- 17.20: D.Giusti — Structure-dependent form factors in radiative leptonic decays of the D_s meson with domain wall fermions

Friday 4

- 09.00: N.Christ — Lattice calculation of electromagnetic corrections to K_{l3} decay
- 09.20: N.Hermansson-Truedsson — Structure-dependent electromagnetic finite-volume effects through order $1/L^3$
- 09.40: A.Portelli — Finite-volume collinear divergences in radiative corrections to meson leptonic decays
- 10.00: J-S.Yoo — Radiative electroweak box correction to pion, kaon and nucleon β decay

Thank you

This work has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreements No 757646 & 813942

Backup slides

Velocity-dependent finite-volume coefficients

$$\boxed{c_j(\mathbf{v}) = \Delta'_{\mathbf{n}} \frac{1}{|\mathbf{n}|^j (1-\mathbf{v}\cdot\hat{\mathbf{n}})} = \left[\sum_{\mathbf{n}\neq\mathbf{0}} - \int \mathrm{d}^3\mathbf{n}\right] \frac{1}{|\mathbf{n}|^j (1-\mathbf{v}\cdot\hat{\mathbf{n}})}}$$

For $j=0$ we have:

Z.Davoudi et al. PRD99 (2019)

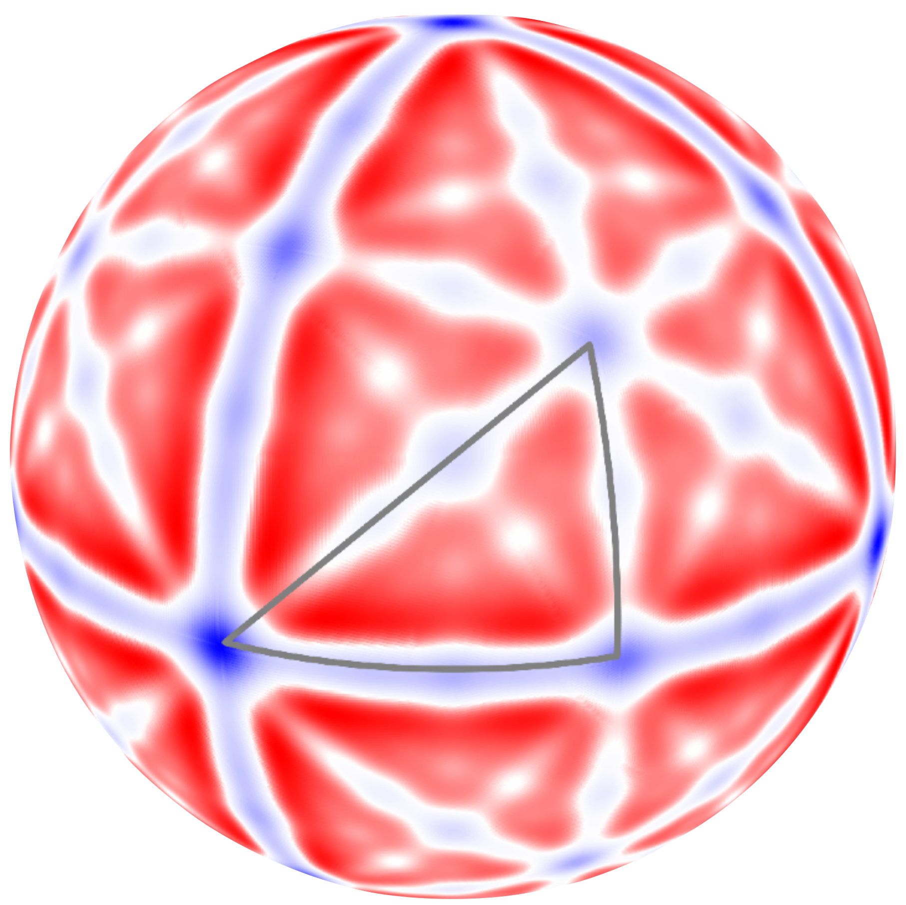
$$\frac{1}{1-\mathbf{v}\cdot\hat{\mathbf{n}}} = \frac{\mathrm{arctanh}(|\mathbf{v}|)}{|\mathbf{v}|} + \sum_{s=0}^{\infty}\sum_{l=1}^{s} p_{sl}\, P_l(\hat{\mathbf{v}}\cdot\hat{\mathbf{n}})\,|\mathbf{v}|^s$$

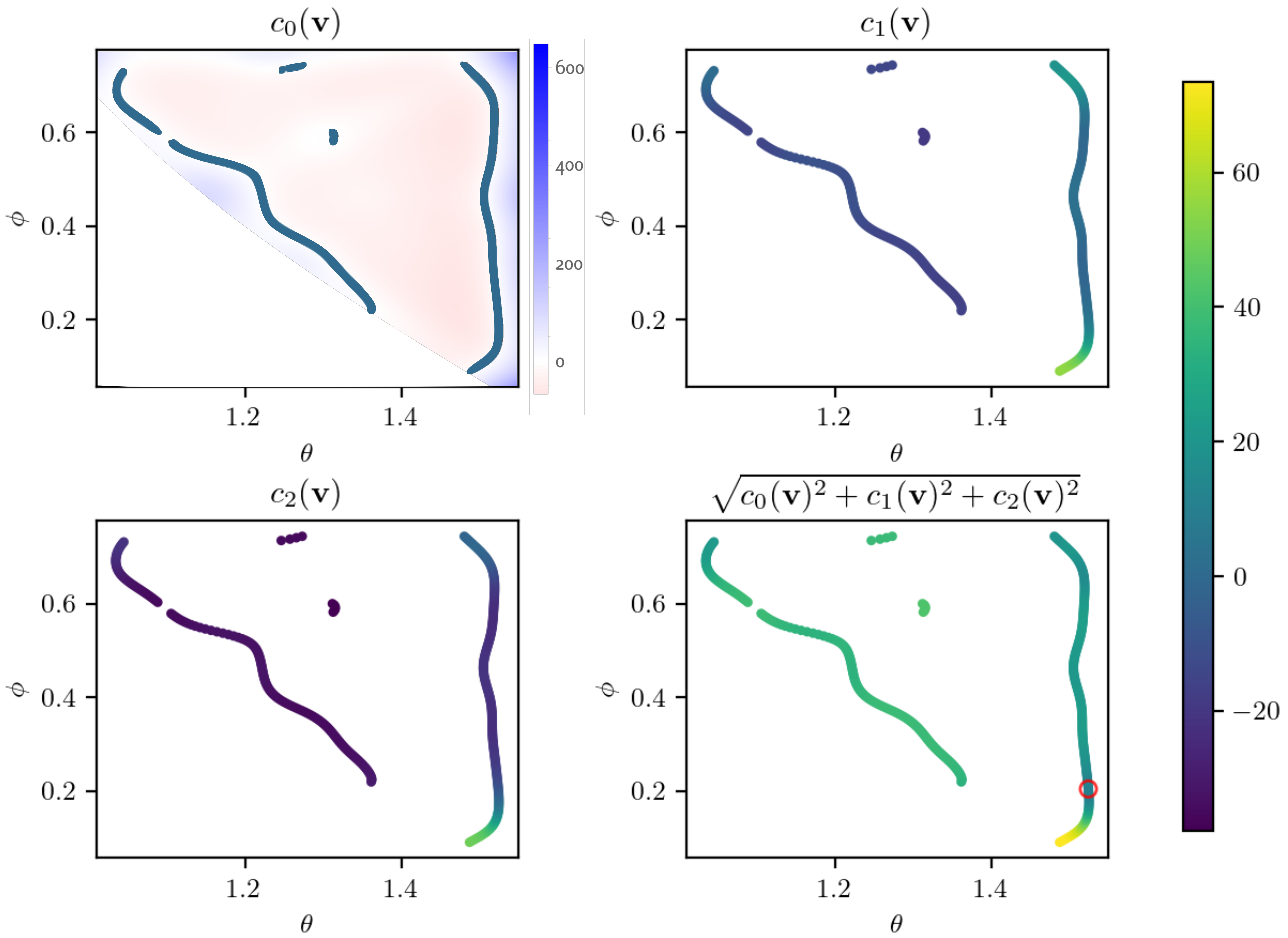
Therefore, in **QED$_r$** we get:

$$\bar{c}_0(\mathbf{v}) = \cancel{\frac{\mathrm{arctanh}(|\mathbf{v}|)}{|\mathbf{v}|}}\,\bar{c}_0 + \sum_{s=0}^{\infty}\sum_{l=1}^{s} p_{sl}\left[\Delta'_{\mathbf{n}} P_l(\hat{\mathbf{v}}\cdot\hat{\mathbf{n}}) + \frac{1}{6}\sum_{\mathbf{r}} P_l(\hat{\mathbf{v}}\cdot\hat{\mathbf{r}})\right]|\mathbf{v}|^s$$

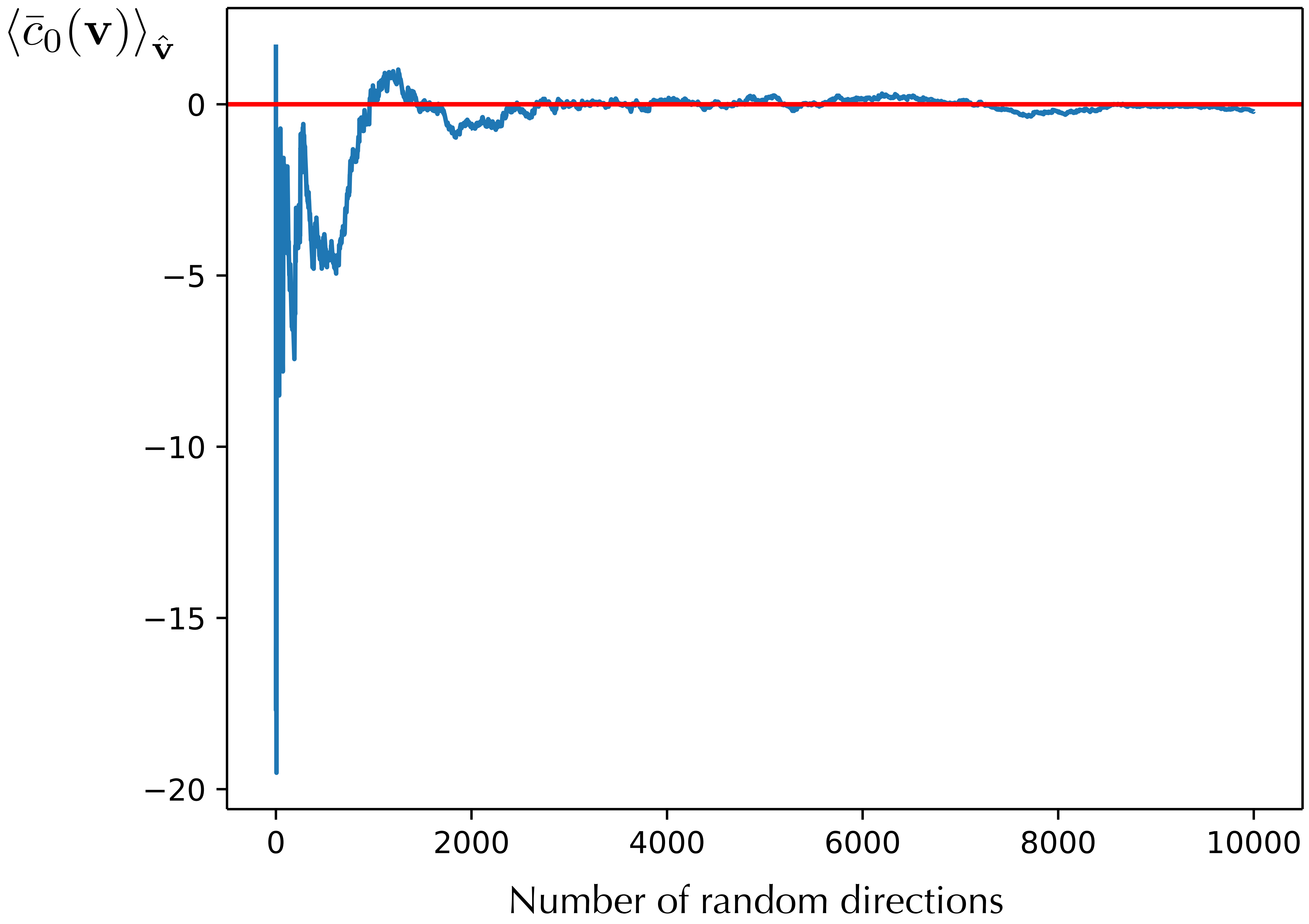
Finding "magic angles"

$\bar{c}_0(\mathbf{v})$ at $|\mathbf{v}| = 0.995$





Stochastic average over solid angle



A.Portelli (Friday 4, h 9.40)

$$\frac{1}{4\pi}\int d\Omega_{\mathbf{v}}\, \bar{c}_0(\mathbf{v}) = 0$$

$$\big\downarrow$$

$$\langle \bar{c}_0(\mathbf{v}) \rangle_{\hat{\mathbf{v}}} = \frac{1}{N}\sum_{n=1}^{N} \bar{c}_0(\mathbf{v}_n)$$

Infinite volume reconstruction

X.Feng & L.Jin, PRD **100** (2019)

QED$_\infty$

- An alternative approach is to compute radiative corrections as a **convolution** of **hadronic correlators** with **infinite-volume QED kernels**

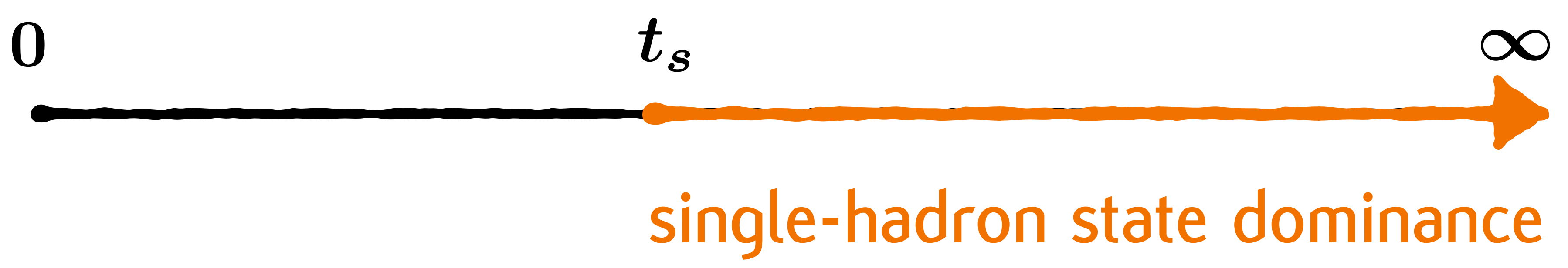
$$\Delta\mathcal{O} = \int \mathrm{d}t \int \mathrm{d}^3\mathbf{x}\ \mathcal{H}(t,\mathbf{x})\, f_{\mathrm{QED}}(t,\mathbf{x}) = \Delta\mathcal{O}^{(s)} + \Delta\mathcal{O}^{(l)}$$

Separate correlator into **short** and **long** distance parts:

Joshua Swaim (Mon 31, h 16.20)

$$\Delta\mathcal{O}^{(s)} \approx \frac{1}{2}\int_{-t_s}^{t_s} \mathrm{d}t \int_{L^3} \mathrm{d}^3\mathbf{x}\ \mathcal{H}^L(t,\mathbf{x}) f_{\mathrm{QED}}(t,\mathbf{x})$$

$$\Delta\mathcal{O}^{(l)} \approx \int_{L^3} \mathrm{d}^3\mathbf{x}\ \mathcal{H}^L(t_s,\mathbf{x})\, \mathcal{F}_{\mathrm{QED}}(t_s,\mathbf{x})$$

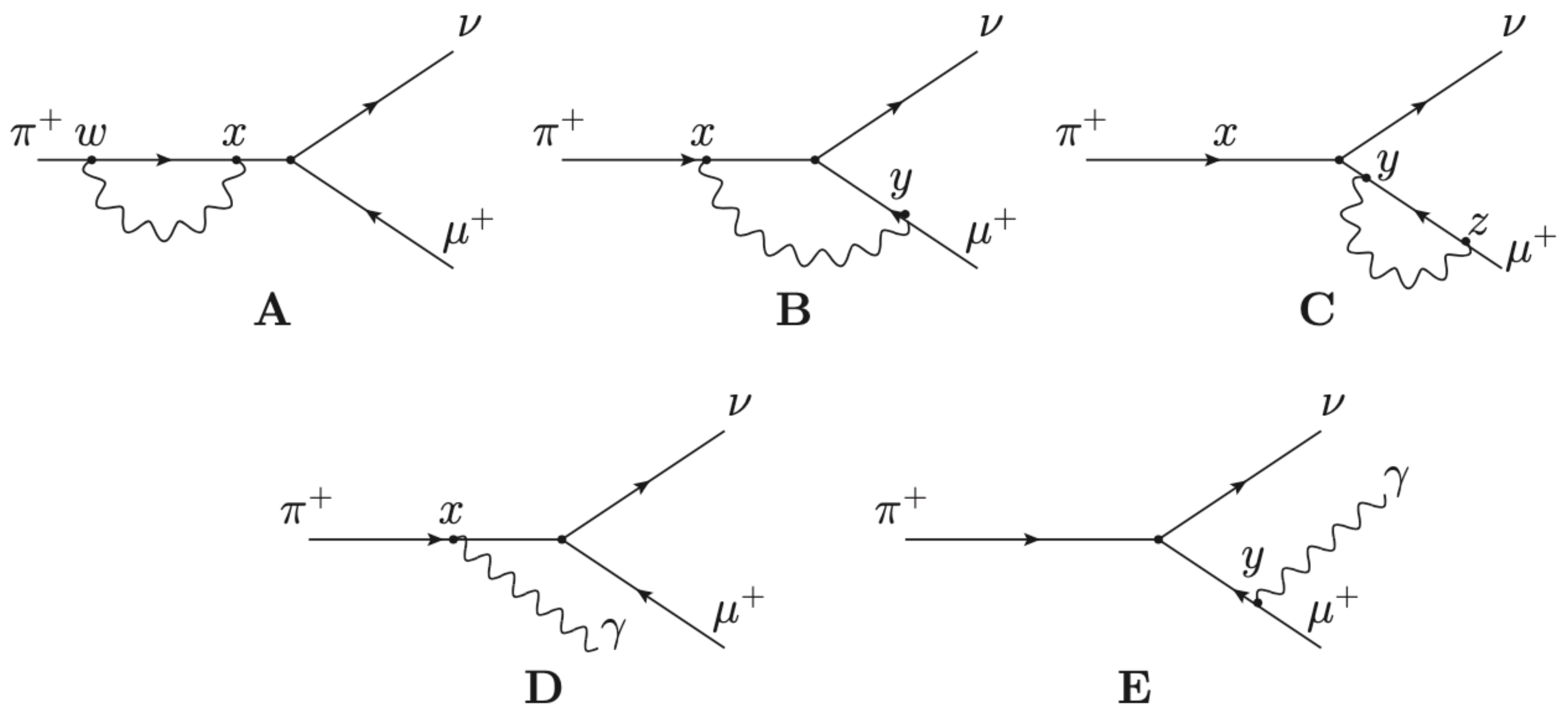


Exponentially suppressed (a) finite-volume effects (b) contributions of states with higher energy

Infinite volume reconstruction

N.Christ et al., [2304.08026]

QED$_\infty$



- Diagram A:

$$H^{(2)}_{\mu,\rho,\sigma}(t_1,t_2,\vec{x}) \;=\; \int d^3\vec{w}\,\langle 0|T\{J^W_\mu(0)J^{\rm EM}_\rho(t_1,\vec{w}+\vec{x})J^{\rm EM}_\sigma(t_2,\vec{w})\}|\pi(\vec{0})\rangle$$

- Diagram B and D:

$$H^{(1)}_{\mu,\rho}(x) = H^{(1)}_{\mu,\rho}(x_t,\vec{x}) \;=\; \langle 0|T\{J^W_\mu(0)J^{\rm EM}_\rho(x)\}|\pi(\vec{0})\rangle$$

- Diagram C and E ($f_\pi \approx 130$ MeV):

$$H^{(0)}_\mu = H^{(0)}_t\delta_{\mu,t} \;=\; \langle 0|J^W_\mu(0)|\pi(\vec{0})\rangle = -im_\pi f_\pi \delta_{\mu,t}$$

from Luchang Jin's talk @ Edinburgh May 30, 2023

Strategy applied to **leptonic decay rates**:

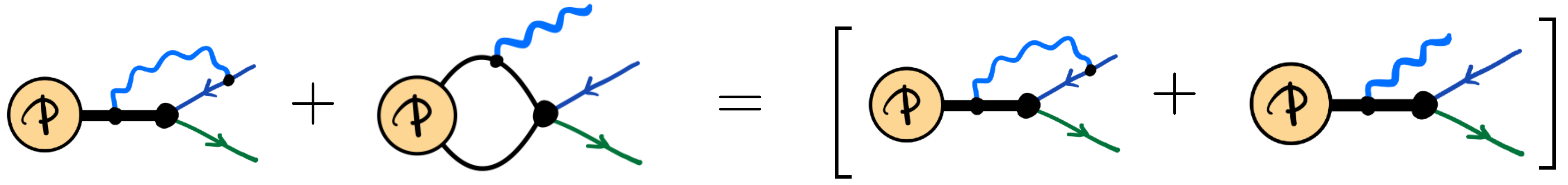
- **Logarithmic IR divergences** appear
- **BUT they cancel analytically** between diagrams
- numerical calculation still **ongoing**...

The method is **appealing**, but it has to be studied **case by case**.

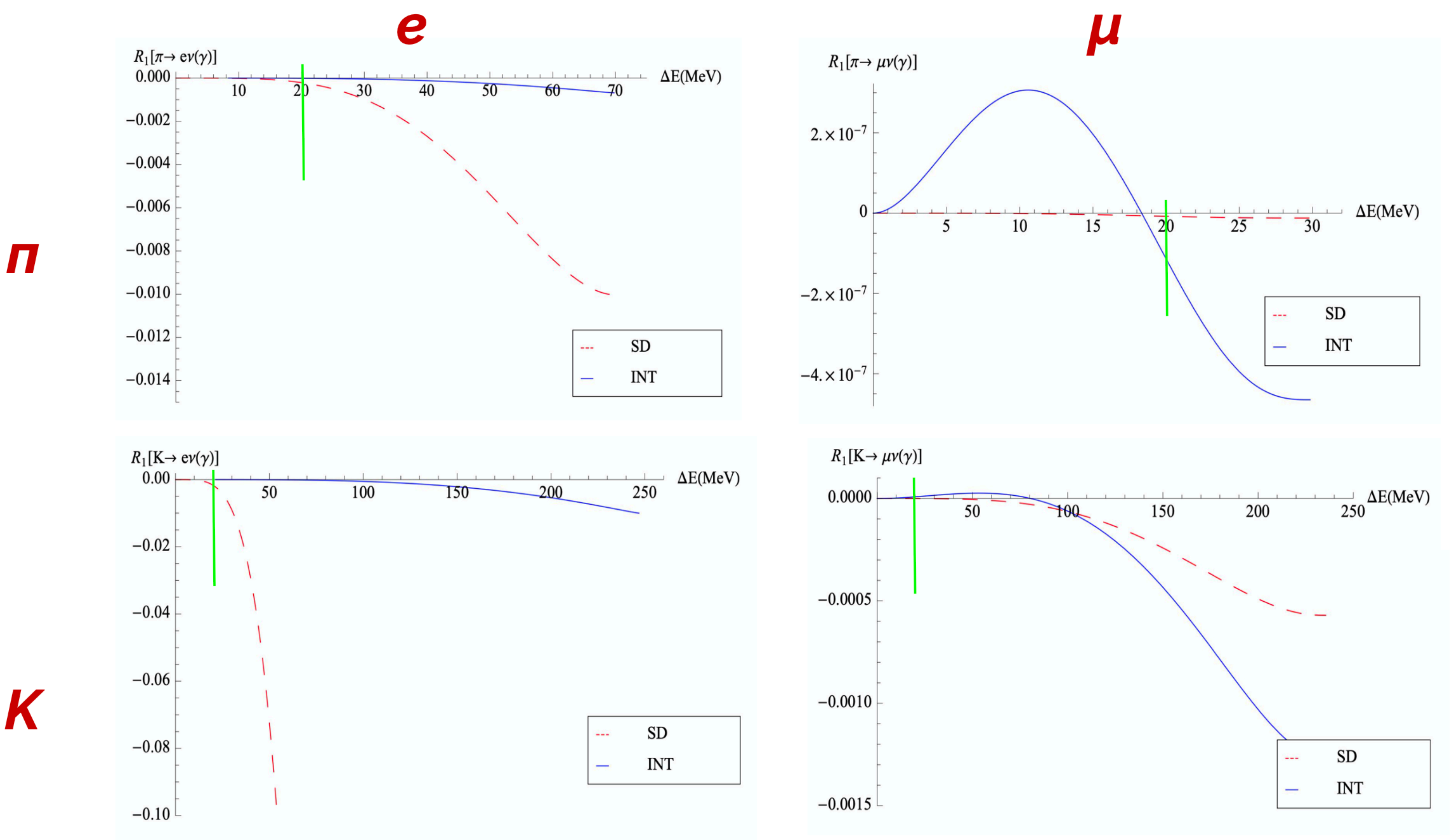
... systematics under control?

N.Christ (Friday 4, h 9.00)

Real photon emission and structure dependence



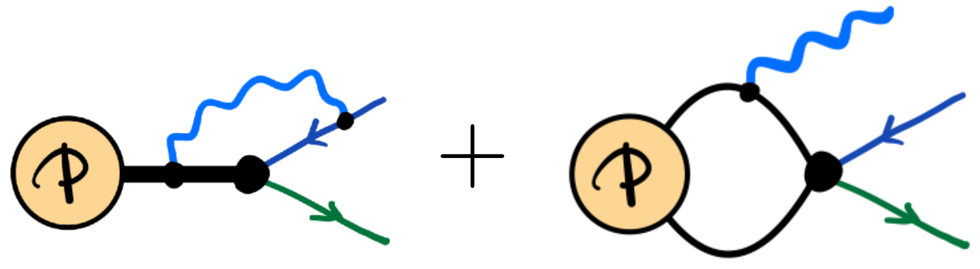
$$\left(1 + \underline{R_1^{\mathrm{SD}}(\Delta E)} + \underline{R_1^{\mathrm{INT}}(\Delta E)}\right)$$



Calculation at O(p4) in χPT

N. Carrasco et al., PRD 91 (2015)

Real photon emission and structure dependence



$$= \delta R_{\rm pt}(\Delta E) + \delta R_1^{\rm SD}(\Delta E) + \delta R_1^{\rm INT}(\Delta E)$$

	$\pi_{e2[\gamma]}$	$\pi_{\mu 2[\gamma]}$	$K_{e2[\gamma]}$	$K_{\mu 2[\gamma]}$
δR_0	$^{(*)}$	$0.0411\ (19)$	$^{(*)}$	$0.0341\ (10)$
$\delta R_{\rm pt}(\Delta E_\gamma^{max})$	-0.0651	-0.0258	-0.0695	-0.0317
$\delta R_1^{\rm SD}(\Delta E_\gamma^{max})$	$5.4\ (1.0) \times 10^{-4}$	$2.6\ (5) \times 10^{-10}$	$1.19\ (14)$	$2.2\ (3) \times 10^{-5}$
$\delta R_1^{\rm INT}(\Delta E_\gamma^{max})$	$-4.1\ (1.0) \times 10^{-5}$	$-1.3\ (1.5) \times 10^{-8}$	$-9.2\ (1.3) \times 10^{-4}$	$-6.1\ (1.1) \times 10^{-5}$
ΔE_γ^{max} (MeV)	69.8	29.8	246.8	235.5

$^{(*)}$ *Not yet evaluated by numerical lattice QCD+QED simulations.*

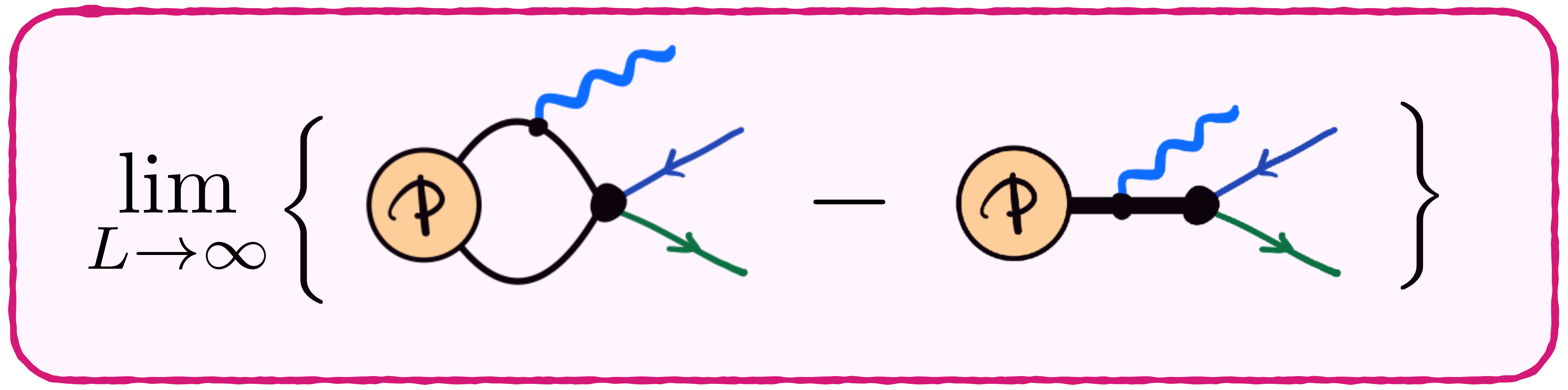
Confirmed by numerical lattice calculation

A. Desiderio et al., PRD 102 (2021)

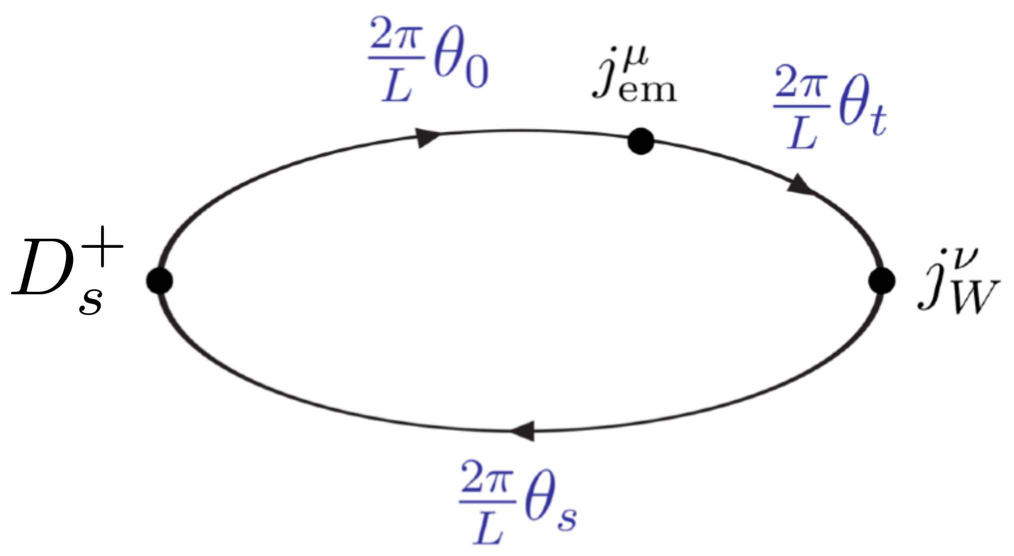
R. Frezzotti et al., PRD 103 (2021)

Real photon emission

Hadronic matrix element and form factors



$$H_W^{\mu\nu}(k,\mathbf{p}) = \int \mathrm{d}^4 y\, e^{ik\cdot y}\, \langle 0|\mathrm{T}\big[j_W^{\nu}(0) j_{\mathrm{em}}^{\mu}(y)\big]|D_s^+(\mathbf{p})\rangle = H_{\mathrm{SD}}^{\mu\nu}(k,\mathbf{p}) + H_{\mathrm{pt}}^{\mu\nu}(k,\mathbf{p})$$



$$H_{\mathrm{SD}}^{\mu\nu}(k,\mathbf{p}) = \frac{H_1(p\cdot k,k^2)}{m_{D_s}}\big[k^2 g^{\mu\nu} - k^\mu k^\nu\big] + \frac{H_2(p\cdot k,k^2)}{m_{D_s}}\frac{\big[(p\cdot k - k^2)k^\mu - k^2(p-k)^\mu\big]}{(p-k)^2 - m_{D_s}^2}(p-k)^\nu$$
$$- i\frac{\boldsymbol{F_V}(p\cdot k,k^2)}{m_{D_s}}\epsilon^{\mu\nu\alpha\beta}k_\alpha p_\beta + \frac{\boldsymbol{F_A}(p\cdot k,k^2)}{m_{D_s}}\big[(p\cdot k - k^2)g^{\mu\nu} - (p-k)^\mu k^\nu\big]$$

$$H_{\mathrm{pt}}^{\mu\nu}(k,\mathbf{p}) = f_{D_s}\left[g^{\mu\nu} + \frac{(2p-k)^\mu (p-k)^\nu}{2p\cdot k - k^2}\right]$$

Goal: extract form factors F_A and F_V from Euclidean three-point functions

D.Giusti (Thursday 3, h 17.20)

PRD **103**, 014502 (2021)

First lattice calculation of radiative leptonic decay rates of pseudoscalar mesons

A. Desiderio,[1] R. Frezzotti,[1] M. Garofalo,[2] D. Giusti,[3,4] M. Hansen,[5] V. Lubicz,[2] G. Martinelli,[6] C. T. Sachrajda,[7] F. Sanfilippo,[4] S. Simula,[4] and N. Tantalo[1]

- first calculation of $P^+ \to \ell^+\nu\gamma$ for pion and kaon + D_s in part of the kinematical range ($E_\gamma \lesssim 0.4$ GeV)

PRD **103**, 053005 (2021)

Comparison of lattice QCD + QED predictions for radiative leptonic decays of light mesons with experimental data

R. Frezzotti,[1] M. Garofalo,[2,3] V. Lubicz,[2] G. Martinelli,[4] C. T. Sachrajda,[5] F. Sanfilippo,[6] S. Simula,[6] and N. Tantalo[1]

- comparison of lattice results with experimental measurements
- good agreement with KLOE on $K \to e\nu_e\gamma$
- 3-4σ tensions on $K \to \mu\nu_\mu\gamma$ (also among experiments)

PRD **107**, 074507 (2023)

Methods for high-precision determinations of radiative-leptonic decay form factors using lattice QCD

Davide Giusti,[1] Christopher F. Kane,[2] Christoph Lehner,[1] Stefan Meinel,[2] and Amarjit Soni[3]

- study of $D_s^+ \to \ell^+\nu\gamma$ with different "3d" method
- improved control of systematic uncertainties
- but single lattice spacing

arXiv:2306.05904

Lattice calculation of the D_s meson radiative form factors over the full kinematical range

R. Frezzotti and N. Tantalo
Dipartimento di Fisica and INFN, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Roma, Italy

G. Gagliardi, F. Sanfilippo, and S. Simula
Istituto Nazionale di Fisica Nucleare, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy

V. Lubicz and F. Mazzetti
Dipartimento di Matematica e Fisica, Università Roma Tre and INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy

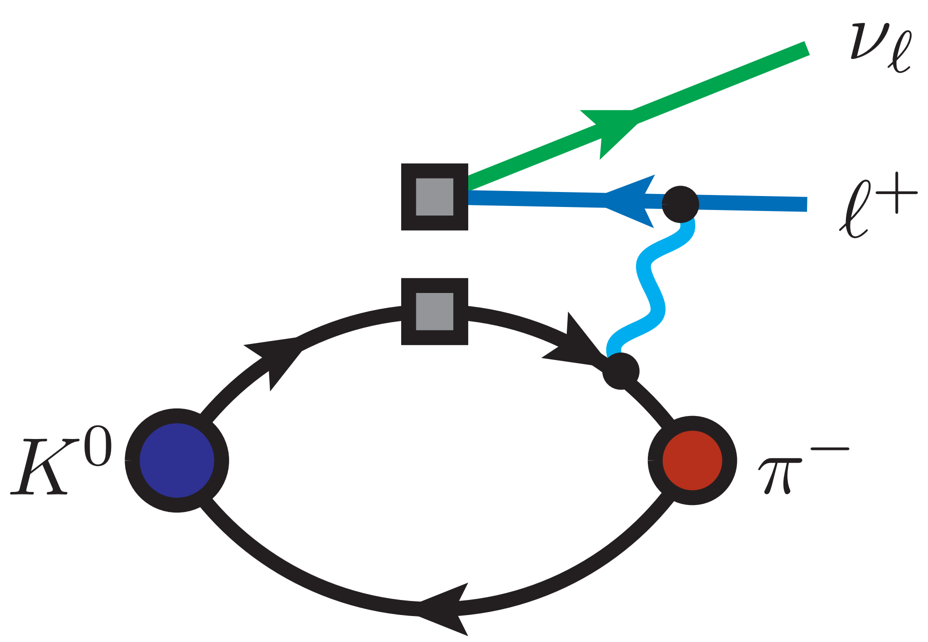
G. Martinelli
Physics Department and INFN Sezione di Roma La Sapienza, Piazzale Aldo Moro 5, 00185 Roma, Italy

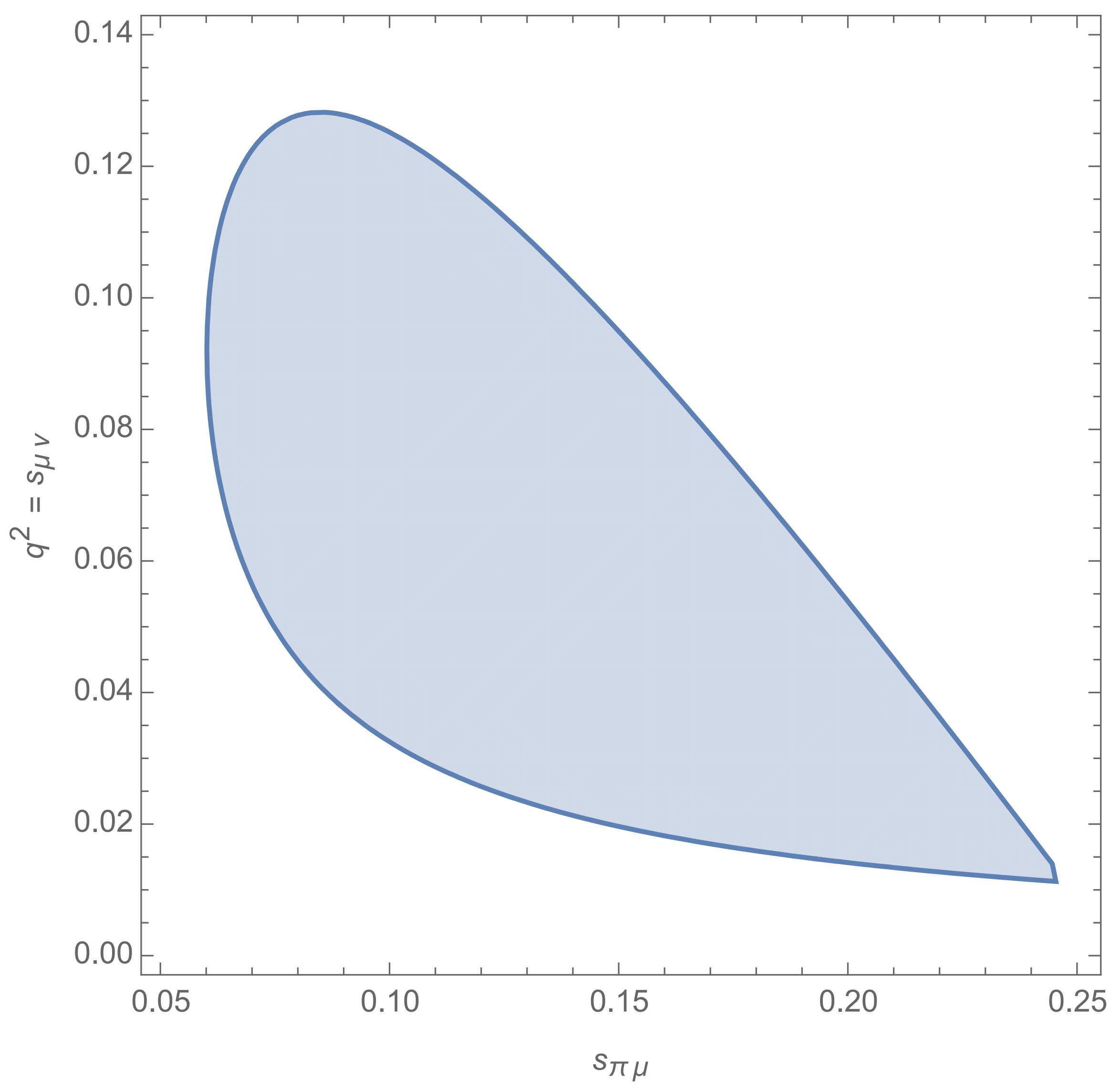
C.T. Sachrajda
Department of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK

- new calculation of $D_s^+ \to \ell^+\nu\gamma$ on full kinematical range

QED corrections to semileptonic decays

N.Christ (Friday 4, h 9.00)





Additional **difficulties** arise compared to **leptonic decays**:

- integration over **three-body phase-space**
- problems of **analytical continuation** when intermediate states lighter than external ones go on shell:

$$e^{-(\omega_{\pi\ell}^{\text{int}} - \omega_{\pi\ell}^{\text{ext}})(t_{\pi\ell} - t_{\text{H}})}$$

› growing exponentials if $\omega_{\pi\ell}^{\text{int}} < \omega_{\pi\ell}^{\text{ext}}$ C.Sachrajda @Lattice2019

These states should be **identified** and **subtracted**.

All becomes more problematic for decays of heavy mesons!

... spectral reconstruction? ... infinite-volume QED?

N.Christ et al., [2304.08026]