Quantum Simulations of Lattice Field Theories

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They/Them

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Simulations of the Standard Model

Studying the properties of strongly coupled theories from first principles is necessary to fully understand the Standard Model

- Classical lattice computations have made incredible advancements in our understanding of the non-perturbative properties of Quantum Chromodynamics

Many interesting properties of strongly-coupled theories, including real-time dynamics, seem incredibly challenging to study on classical machines

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Quantum computers have a fundamentally different computational strategy and will provide novel probes of fundamental questions in particle and nuclear physics

- The last decade has seen the rapid evolution of real-world quantum computers, with increasing size and decreasing noise
- It is imperative to begin exploratory studies of the applicability of this emerging technology

Goals of This Talk

I had two main goals in preparing this talk

1) Provide a general introduction to some of the challenges and hurdles that we must overcome before we can implement 3+1 dimensional QCD onto a quantum computer

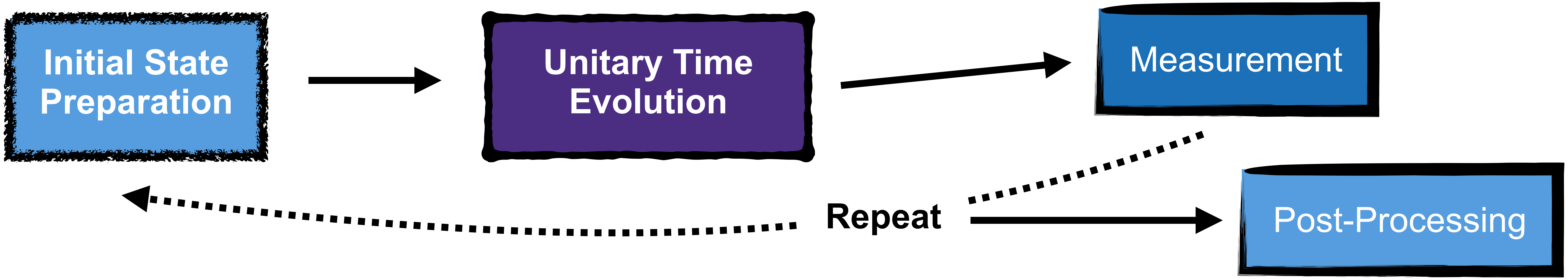
2) Inspire those who are not working on Hamiltonian lattice methods to attend the many great talks occurring in the parallel sessions

Main Take Away Message

We are a young vibrant field with many interesting theoretical and algorithmic challenges ahead!

What is Quantum Simulation?

Working Definition: Protocol to manipulate quantum degrees of freedom plus an experimental platform that utilizes collective properties for calculation



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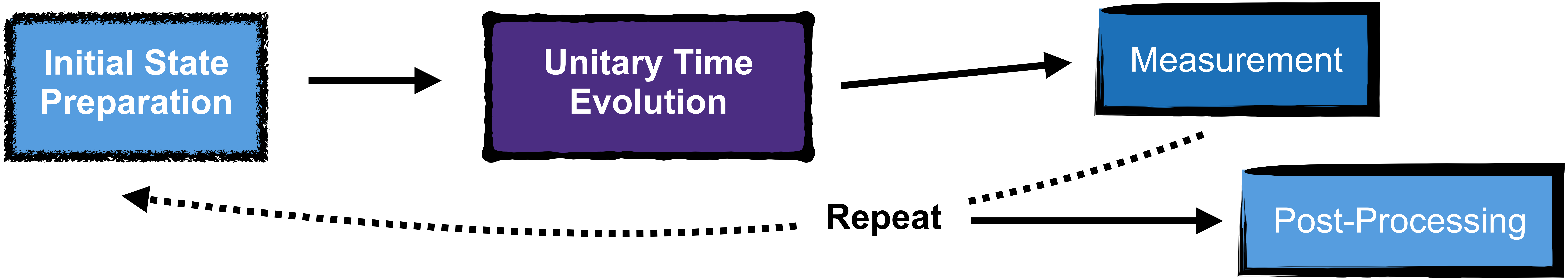


Two Different Approaches

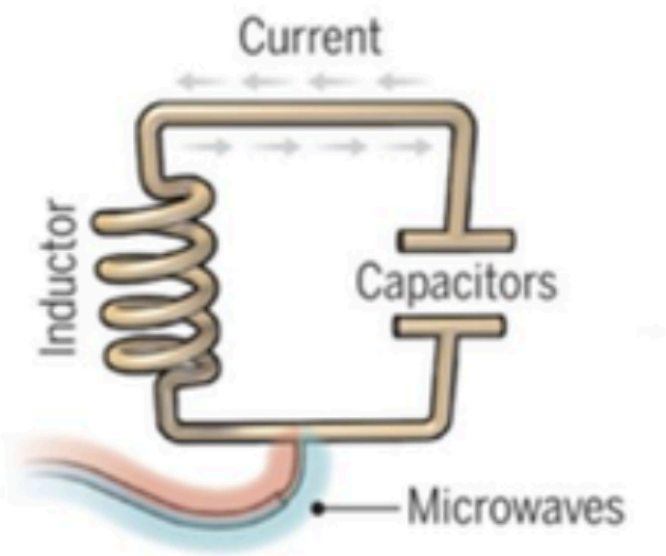
Digital	**Analog**
"Re-write" theory into quantum circuit formulation that runs in reasonable amount of time	*Construct quantum system that is "close" to target theory and let the system evolve in time*

Digital Quantum Computers

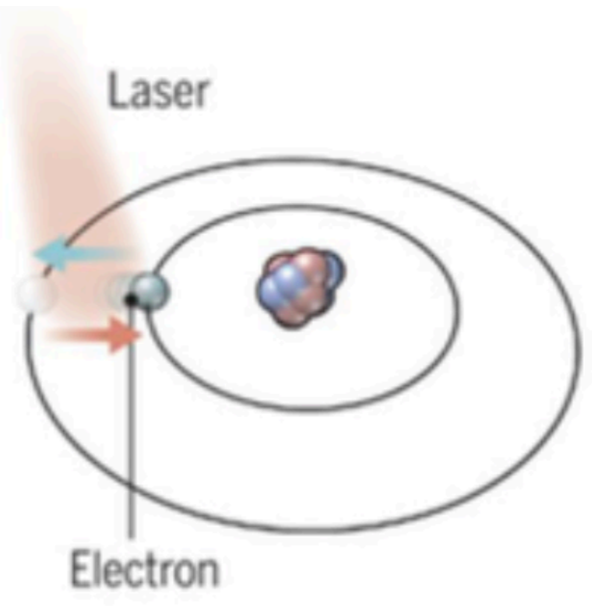
Computational Strategy: Quantum circuit is created by acting on collection of qubits with gates

- Any two-state system can be used as a qubit (in theory)
- Gates are unitary operations that usually act on one or two qubits
- Discrete time evolution

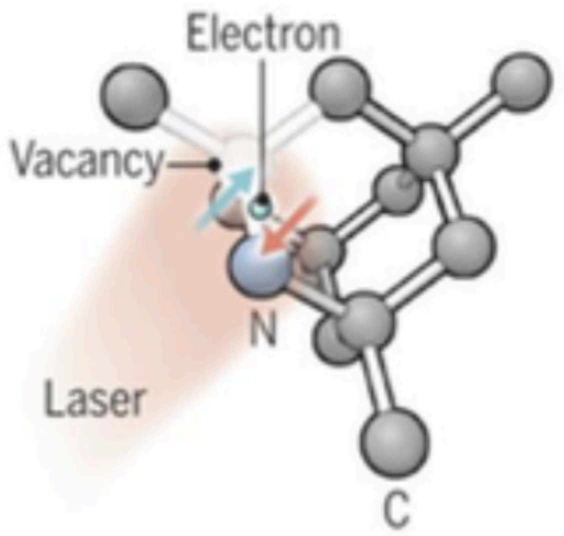
- Superconducting loops



- Trapped ions



- Diamond vacancies



Graphics by C. Bickle, Science Data by Gabriel Popkin

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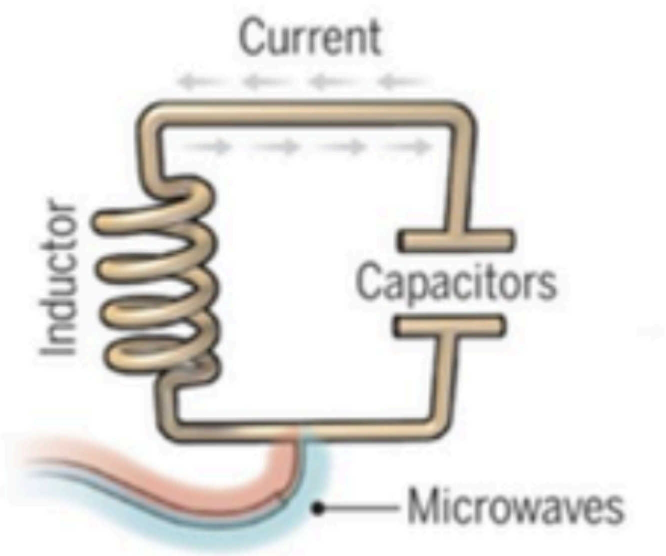

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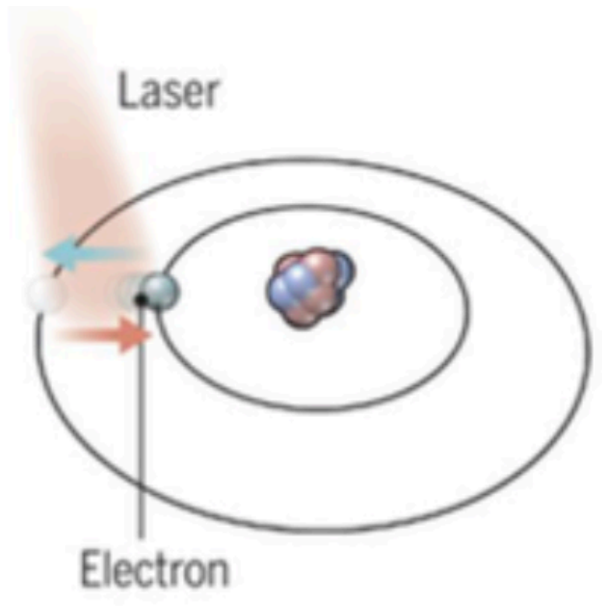

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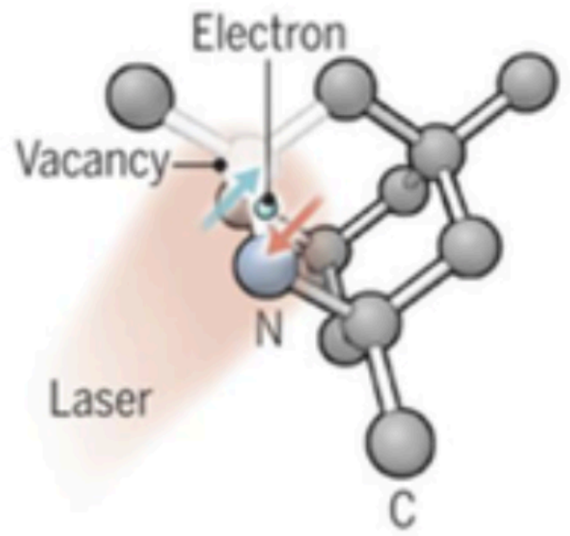

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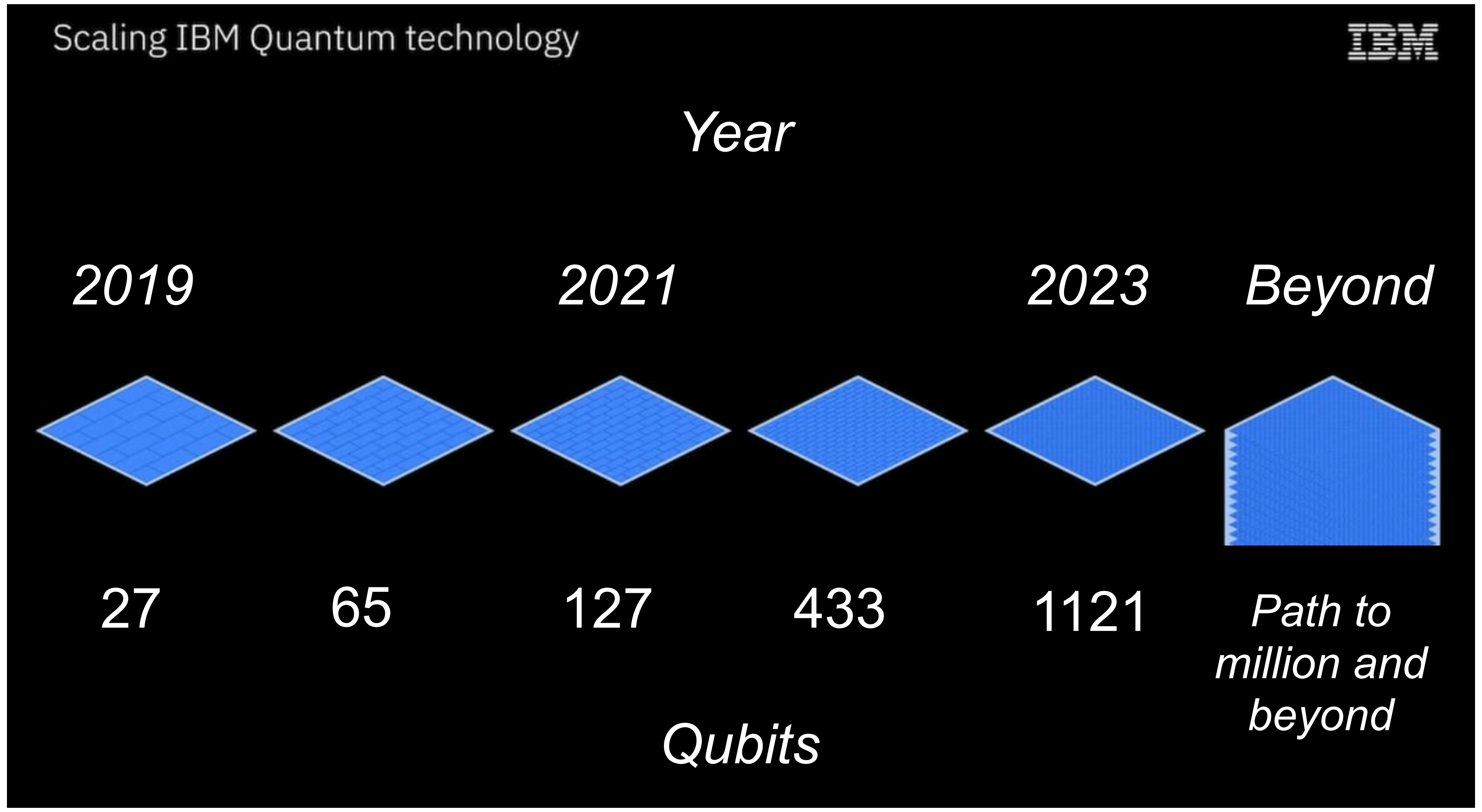
Currently in ***Noisy Intermediate-Scale Quantum*** (NISQ)-era

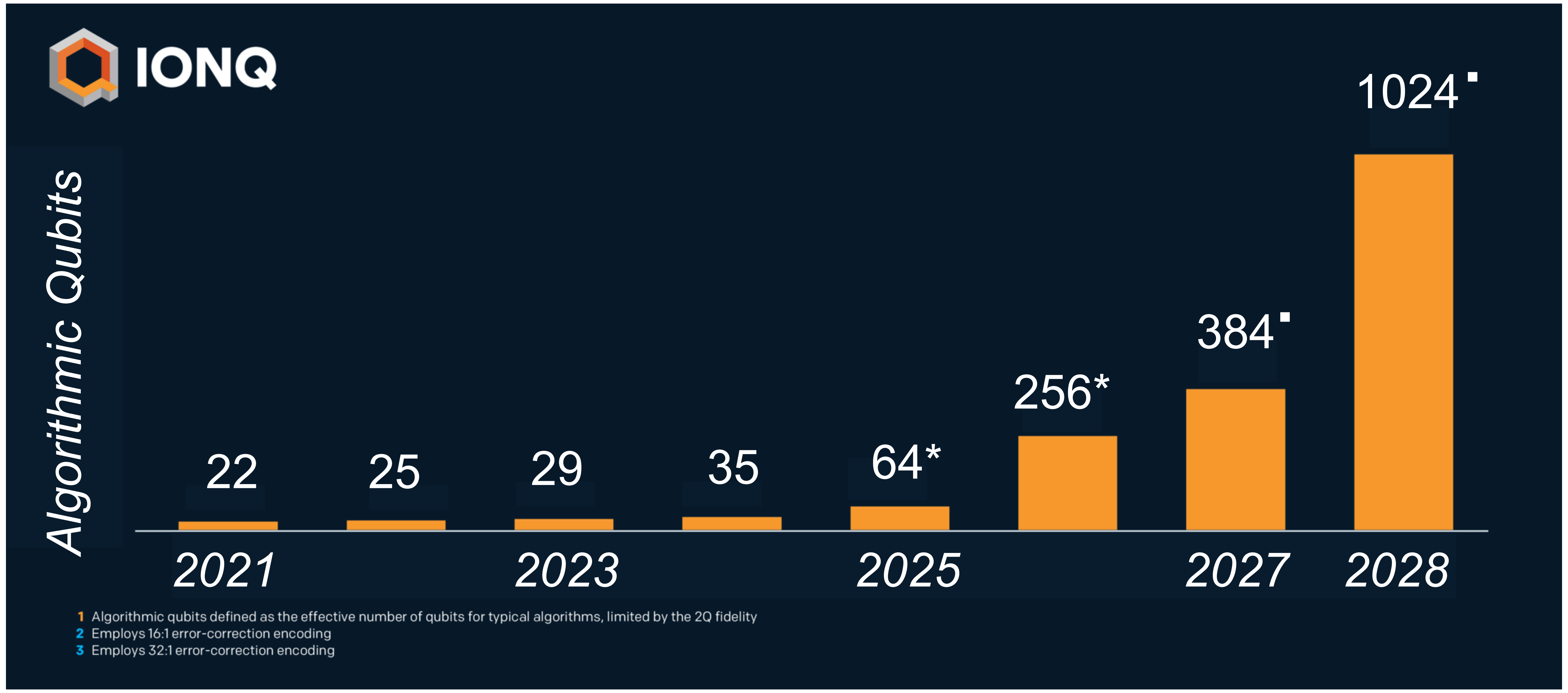
- Machines contain $\mathcal{O}(100)$ noisy qubits without error corrections
- Sensitive to various sources of noise, including decoherence and dephasing

Real-World Digital Computing Hardware

Many "commercial" computers are networking together ever-growing number of qubits



IBM Quantum Roadmap, 2020
Superconducting Qubits



IonQ Roadmap, 2020
Trapped Ion

Gate noise is currently at the 10^{-3} level, with ideas for how to decrease it further

Analog Quantum Computers

Computational Strategy: "Tweak" the natural degrees of freedom of your experiment to mimic a target model

Example: A simple toy model

- Naturally implemented in quantum simulator
- Shows some version of an interesting phenomenon

1+1 Ising mode

Trapped Ions, Rydberg atoms, etc

Confinement

Analog Quantum Computers

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Observation: Gauge theories emerge from simple condensed matter systems once local constraints are imposed

See parallel talk about scalar QED in Rydberg atoms!

Phys Rev D 11, 395 (1975)

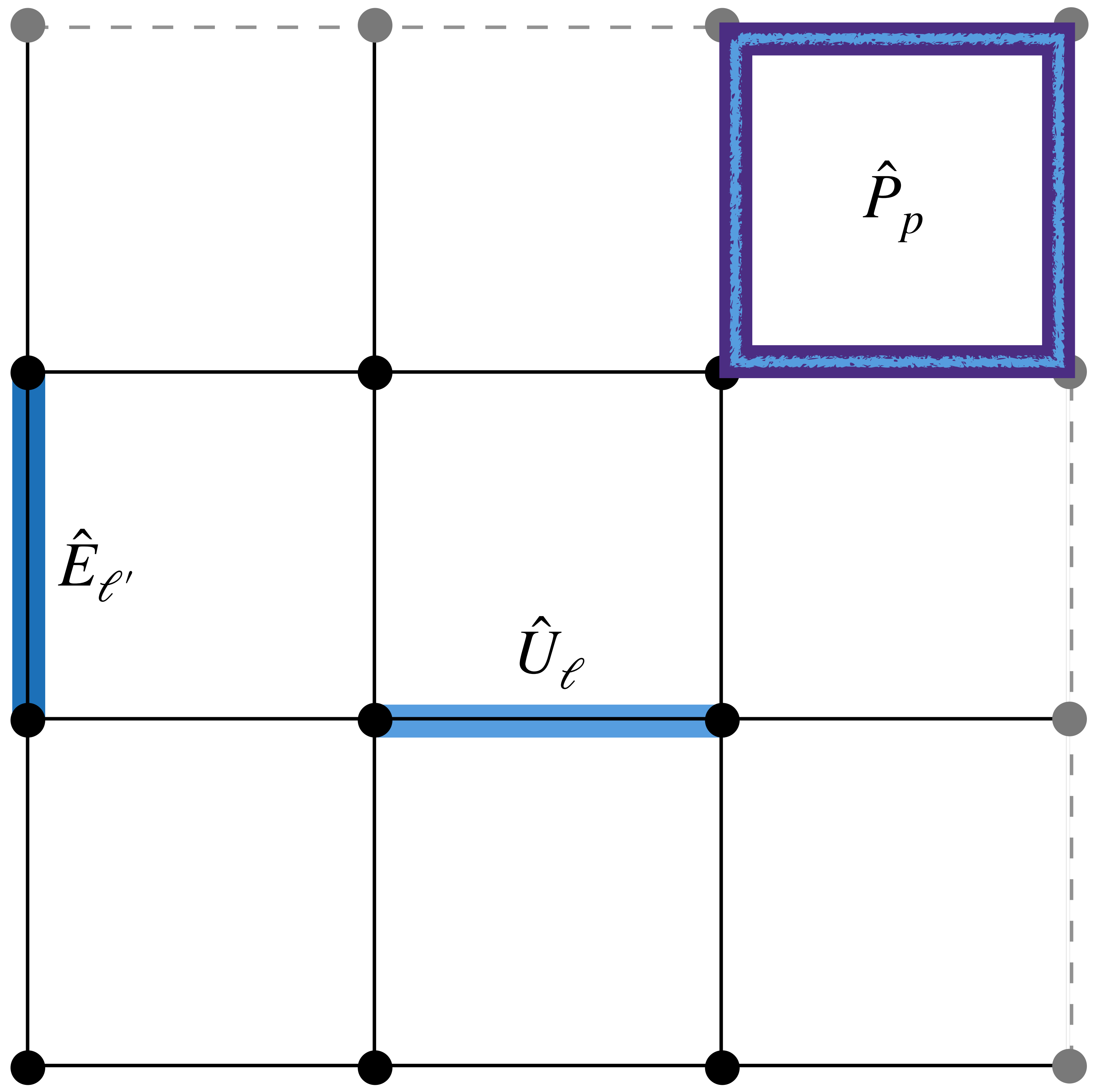
Hamiltonian Lattice Gauge Theory, Abelian

Quantum simulations utilize Hamiltonian formulations

- Continuous time, but discrete space
- Use Weyl Gauge ($A_0 = 0$)

Kogut-Susskind Hamiltonian

$$H = \frac{1}{2a}\left[g^2 \sum_{\ell \in links} E_\ell E_\ell + \frac{1}{g^2} \sum_{p \in plaquettes} \text{Tr}\left(2I - P_p - P_p^\dagger\right)\right]$$



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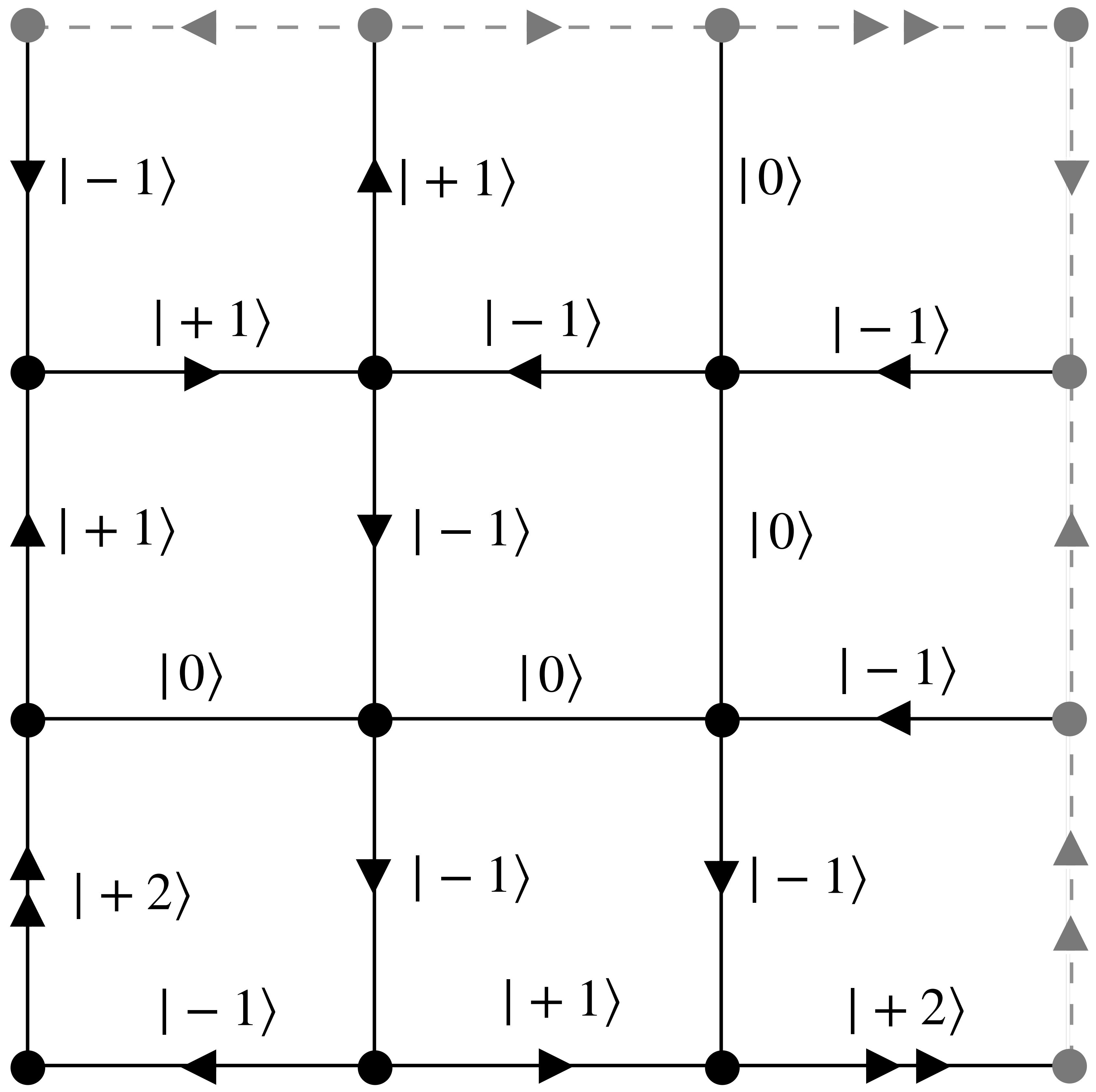
- Commutation relations inform how operators map onto qubits

$$\left[\hat{E}_\ell, \hat{U}_{\ell'}\right] = \hat{U}_\ell \delta_{\ell\ell'} \qquad \hat{E} = \sum_\epsilon \epsilon\,|\epsilon\rangle\langle\epsilon| \qquad \hat{U} = \sum_\epsilon |\epsilon + 1\rangle\langle\epsilon|$$

Operators defined in the electric basis



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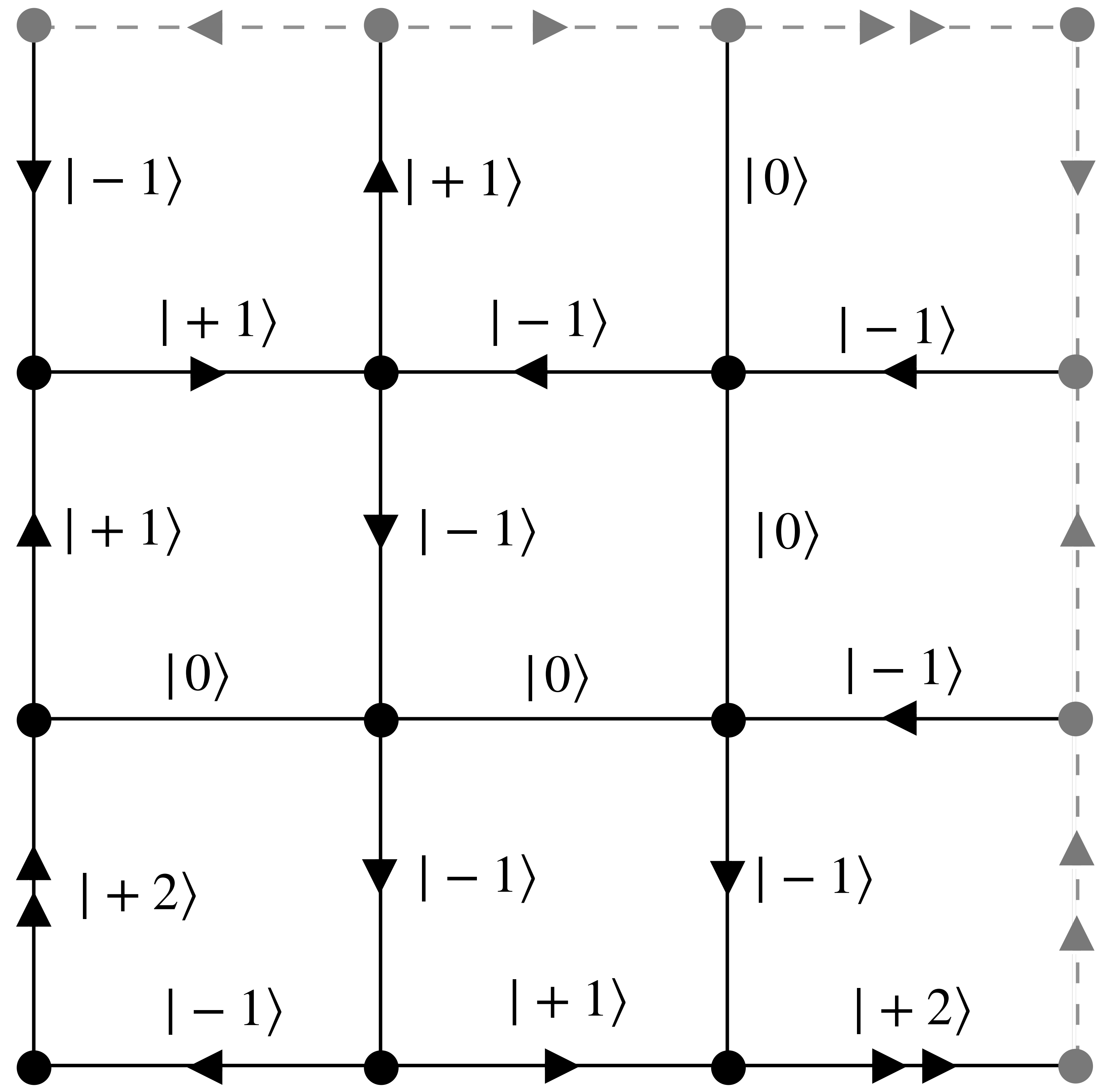

Is this the end of the story?

Theoretical Challenges of Lattice Gauge Theories

Three fundamental hurdles have to be overcome on the quest for quantum simulation of Hamiltonian lattice field theories

A) Infinite-dimensional Hamiltonian must be truncated

Construct finite-dimensional Hermitian matrix that faithfully captures desired physics

B) Phenomenologically-relevant gauge groups are continuous

Construct "sampling" method to capture gauge phenomena with finite number of samples

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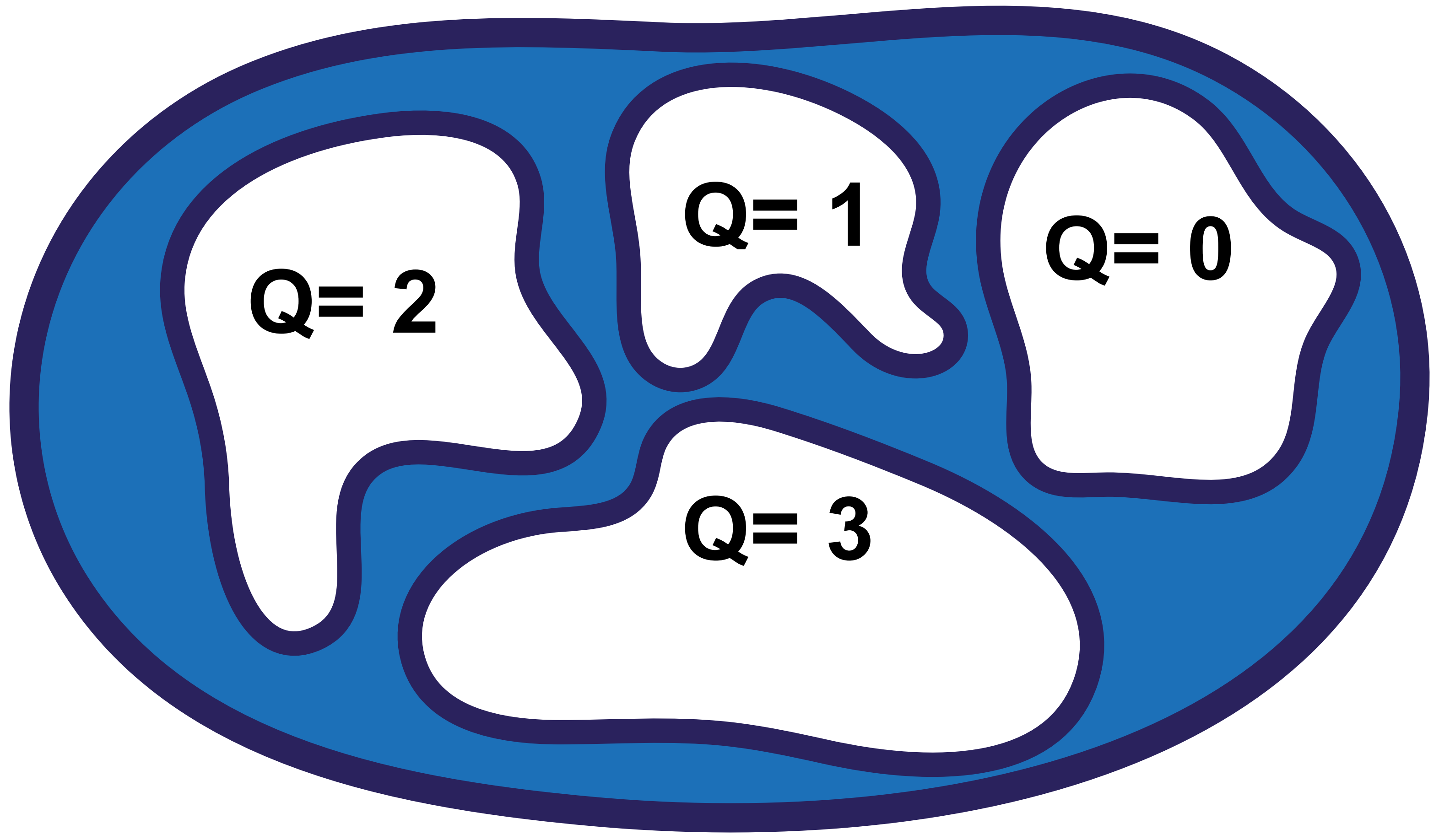
C) Gauss Law is not automatically satisfied

Gauss's law is the constraint associated with the A_0 Lagrange multiplier

Naive Hilbert space is tensor product of different charge sectors



Hamiltonian Lattice Gauge Theory, SU(N) Version

General Idea: Similar to Abelian, but electric and gauge link operators carry color indices

$$H = \frac{1}{2a}\left[g^2 \sum_{\ell \in links} E_\ell^a E_\ell^a + \frac{1}{g^2} \sum_{p \in plaquettes} \mathrm{Tr}\left(2I - P_p - P_p^\dagger\right)\right]$$

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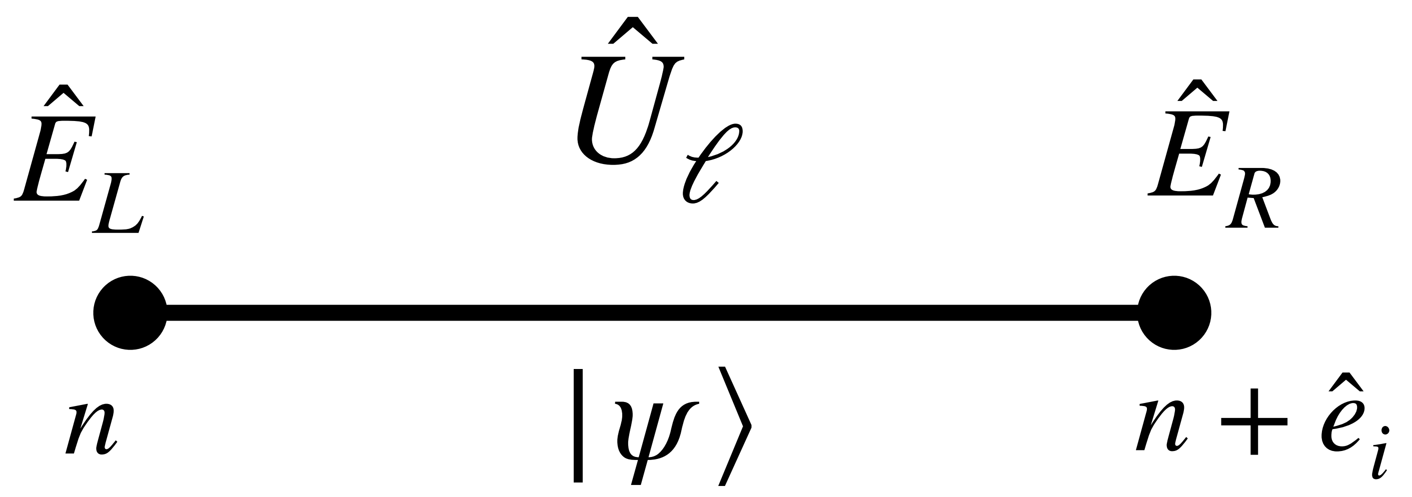
- Theory now contains both left and right electric operators



- Rotations of gauge link from left and right are generated by left and right electric fields

$$\hat{U}(n, e_i) \longmapsto \Omega(n)\, \hat{U}(n, e_i)\, \Omega(n + e_i)^\dagger$$

- Each electric field has their own Lie algebra and commutation relations

$$\left[\hat{E}^a_L, \hat{U}^j_{mn}\right] = T^{ja}_{mm'} \hat{U}^j_{m'n}$$

$$\left[\hat{E}^a_R, \hat{U}^j_{mn}\right] = \hat{U}^j_{mn'} T^{ja}_{n'n}$$

$$\left[\hat{E}^a_L, \hat{E}^b_L\right] = -\, i f^{abc} \hat{E}^c_L$$

$$\left[\hat{E}^a_R, \hat{E}^b_R\right] = i f^{abc} \hat{E}^c_R$$

$$\left[\hat{E}^a_L, \hat{E}^b_R\right] = 0$$

Gauge Fixing and Gauss Law

Key Issue: Weyl gauge is an incomplete gauge-fixing procedure. Gauge transformations with only spatial dependence still allowed and Gauss law becomes a constraint

SU(N) Gauss Law:

$$D \cdot E^a = 0$$

Continuum

$$\hat{G}^a(n) = \sum_{i=1}^{d} \left[\hat{E}^a_R(n - e_i, e_i) - \hat{E}^a_L(n, e_i) \right]$$

Lattice

Fact: Hamiltonian ***does*** commute with Gauss law operators and so charge is conserved

Gauge Fixing and Gauss Law

Gauss Law ⟷ Gauge Invariance
Charge Conservation

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Option One: No Additional Gauge Fixing

- No transitions between different charge sectors for noiseless simulations
- "Energy penalty" term can be added to Hamiltonian for noisy simulations

Option Two: Additional Gauge Fixing

- Fully gauge-fixed Hamiltonian spans only one charge sector
- Expect increase in non-locality due to imposition of Gauss law constraints

Halimeh, J.C. and Hauke, P. Phys. Rev. Lett. 125, 030503 (2020)

Coupling Strength and Basis Choices

Starting Point: Theory has fundamentally different properties at large and small (bare) gauge coupling

$$H = \frac{1}{2a}\left[g^2 \sum_{\ell \in links} E_\ell^a E_\ell^a + \frac{1}{g^2} \sum_{p \in plaquettes} \mathrm{Tr}\left(2I - P_p - P_p^\dagger\right)\right]$$

Strong Coupling (Irrep Basis)

Electric component of Hamiltonian dominates

Basis: $|j, m_L, m_R\rangle$

- States naturally discretized
- Gauss's law is function of electric fields
- Natural UV truncation

GOOD

- Not well-suited for "close to continuum" physics

BAD

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BAD

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Weak Coupling (Group Element Basis)

Magnetic component of Hamiltonian dominates

Basis: $|\mathfrak{g}\rangle$

GOOD

- Gauge links diagonal
- Well-suited for "close to continuum" physics

BAD

- Electric fields are more complicated
- Digitization/truncation of gauge links must be done carefully

Examples of Abelian & Non-Abelian Formulations + Basis

Kogut-Susskind formulation
– Irrep/"angular momentum" basis *Byrnes, Yamamoto, Zohar, Burrello, et al.*
– Group-element basis *Zohar, NuQS collab., et al.*

Gauge magnets/quantum link models: *Wiese, Chandrasekharan, et al.*

Tensor lattice field theory: *Meurice, Sakai, Unmuth-Yockey, et al.*

Dual/rotor formulations: *Kaplan, Stryker, Haase, Dellantonio, et al., Bauer,* **DMG** *Kane*

Casimir variables / "local-multiplet basis": *Klco, Savage, Stryker, Ciavarella*

Purely fermionic formulations (1+1D & OBC):
Muschik, Atas, Zhang, IQuS@UW group, Powell, et al.

Prepotential/Schwinger boson formulations: *Mathur, Anishetty, Raychowdhury, et al.*

Loop-string-hadron formulation: *Raychowdhury, Stryker, Davoudi, Shaw, Dasgupta, Kadam*

Light-front formulation: *Kreshchuk, Kirby, Love, Yao, et al.*

Qubit models: *Chandrasekharan, Singh, et al.*

q-deformed Kogut-Susskind: *Zache, González-Cuadra, Zoller*

Stryker, https://indico.ph.tum.de/event/7112/contributions/6917/

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Parallel Talk

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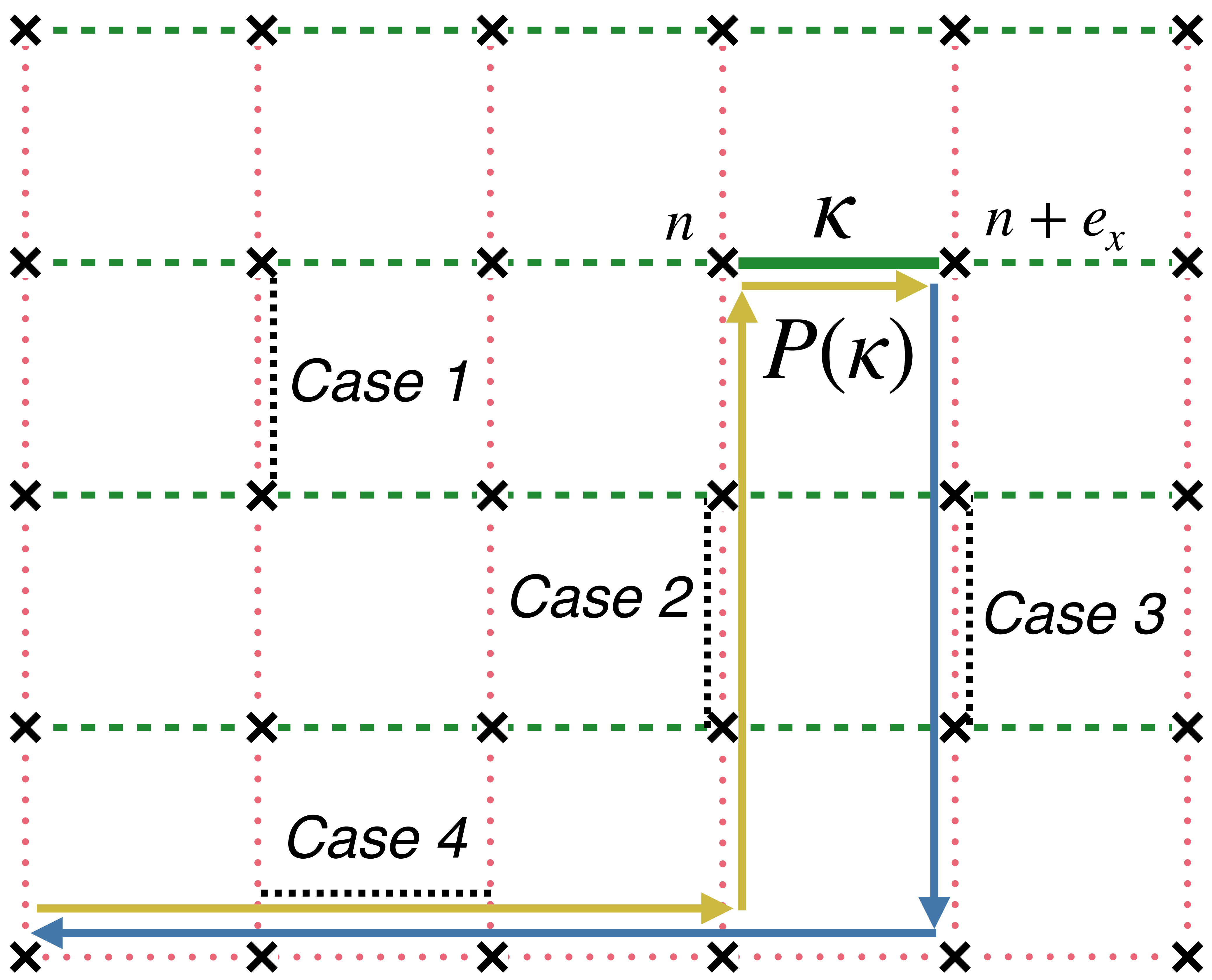
Mixed-Basis Approach to Digitizing Group Element Basis

General Idea: Gauge fixing allows us to do "importance sampling" on gauge variables

Step One: Gauge fix using maximal-tree gauge fixing procedure

- Use residual gauge transformations to set each link on the maximal tree to the identity



Bauer, D'Andrea, Freytsis and ***DMG****, arXiv: 2307.11829*

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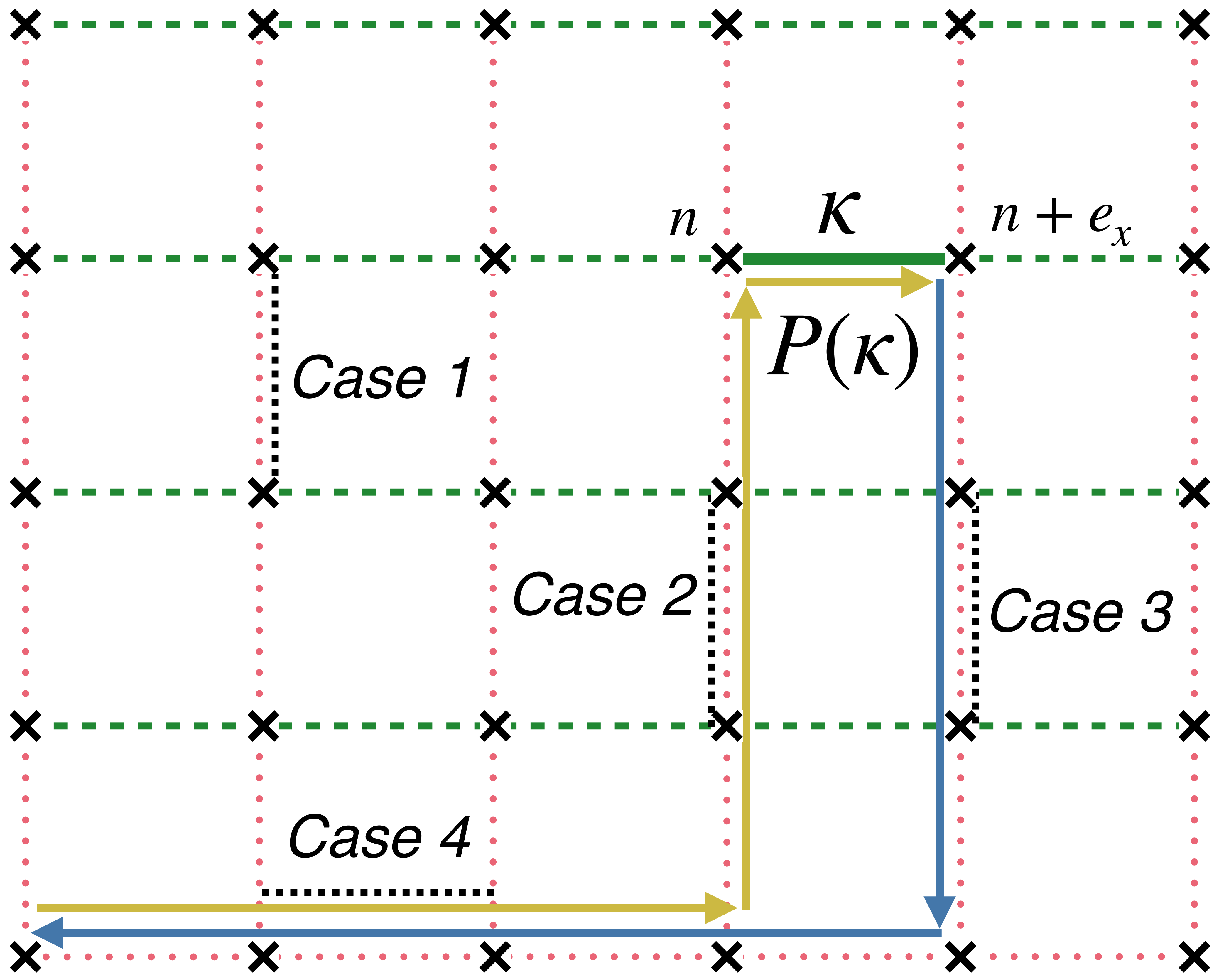
$$H_E = \frac{g^2}{2a} \sum_{\ell} \left(\sum_{\kappa \in t_+(\ell)} \hat{\mathscr{E}}^a_{L\kappa} - \sum_{\kappa \in t_-(\ell)} \hat{\mathscr{E}}^a_{R\kappa} \right)^2$$

- Must pay careful attention to commutation rules

$$[\hat{\mathscr{E}}^a_L(\kappa), \hat{X}(\kappa')] = T^a \hat{X}(\kappa)\delta_{\kappa,\kappa'} \qquad [\hat{\mathscr{E}}^a_R(\kappa), \hat{X}(\kappa')] = \hat{X}(\kappa) T^a \delta_{\kappa,\kappa'}$$



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Step Two: Utilize axis-angle coordinates to parameterize gauge links and electric links of SU(2)

- Each gauge link is given by

$$U = \begin{pmatrix} \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta & -i\sin\frac{\omega}{2}\sin\theta e^{-i\phi} \\ -i\sin\frac{\omega}{2}\sin\theta e^{i\phi} & \cos\frac{\omega}{2} + i\sin\frac{\omega}{2}\cos\theta \end{pmatrix}$$

- Electric operators are differential operators

$$E_R^z = i\left(\cos\theta\frac{\partial}{\partial\omega} - i\cot\frac{\omega}{2}\sin\theta\frac{\partial}{\partial\theta} - \frac{1}{2}\frac{\partial}{\partial\phi}\right)$$

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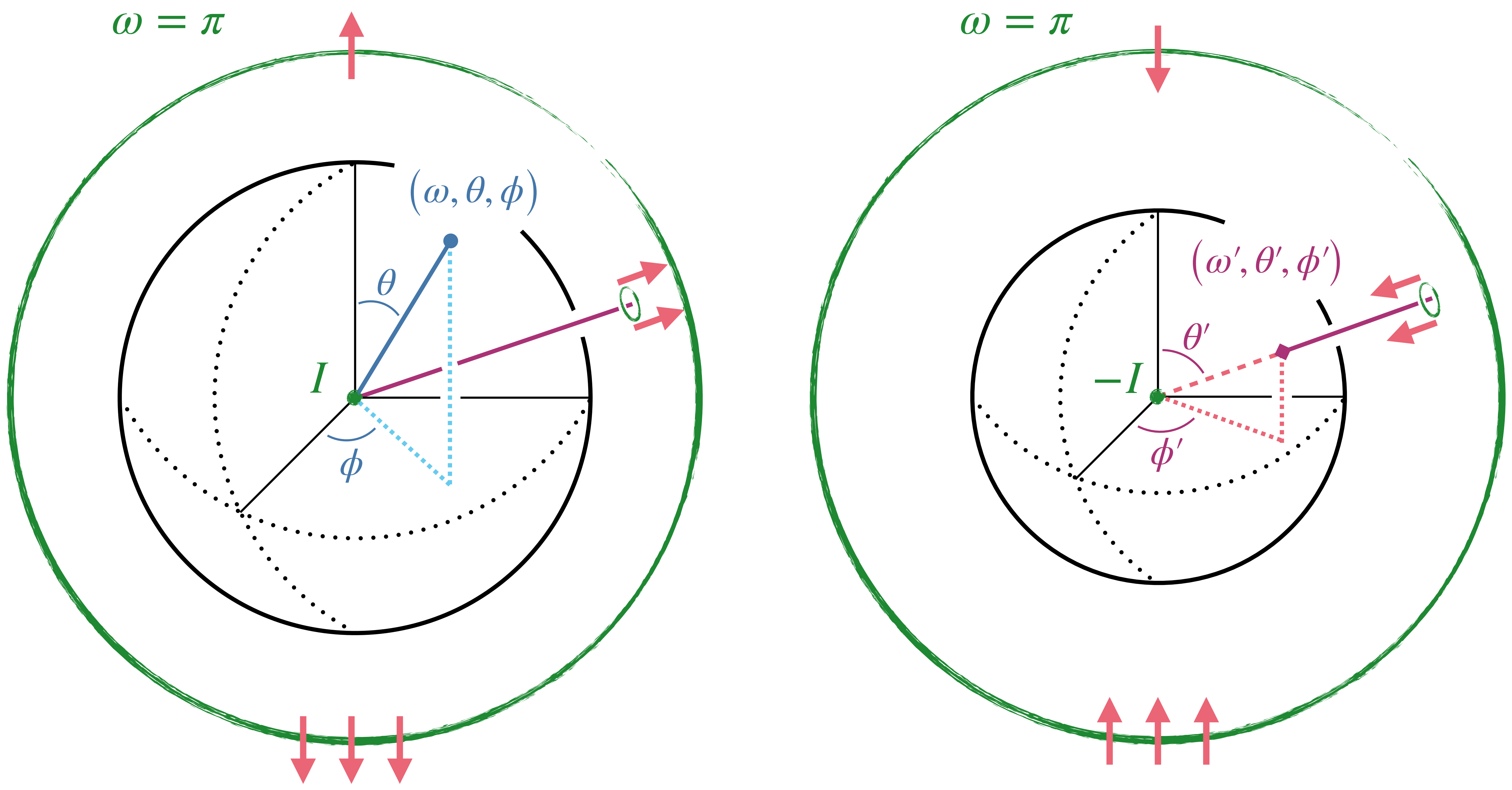
- Axis-angle coordinates are also hyperspherical coordinates of the double cover of S^3



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Step Three: Digitize in $\left(\omega_i, \theta_i, \phi_i\right) \rightarrow \left(\omega_i, L_i, m_i\right)$

- Variable ω_i acts like a radial coordinate and can be easily digitized using previously developed methods
- Variables (θ_i, ϕ_i) are angular coordinates and can be digitized via truncations on spherical harmonics
- Utilize Discrete fourier transformation to move between electric and magnetic basis

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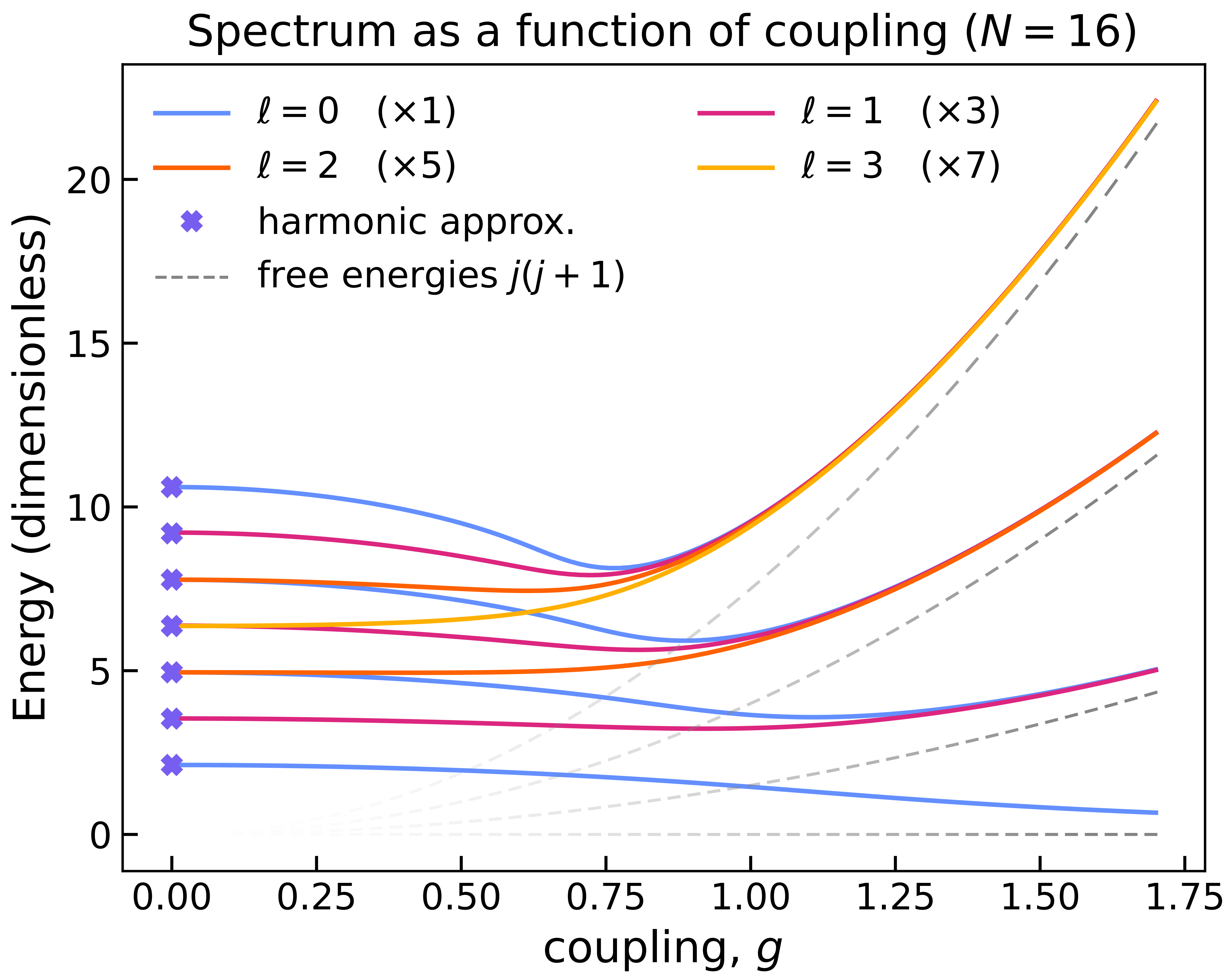
Example: One plaquette, open boundary conditions

$$H_{[1]} = \frac{2g^2}{a}\frac{\hat{L}^2}{4\sin^2\frac{\omega}{2}} - \frac{\partial^2}{\partial^2\omega} - \cot\frac{\omega}{2}\frac{\partial}{\partial\omega} + \frac{2}{g^2 a}\left(1 - \cos\frac{\omega}{2}\right)$$



Tasting Platter of QC and QI Ideas and Talks

We are in an incredibly vibrant and exciting time for this field - new ideas abound!

Initial State Preparation

How do you initialize a simulation when you do not know the eigenstates of the target theory

Finite-Temperature Simulations

How do you simulate finite-temperature systems (mixed states) on a computer that does only pure states?

Alternative Computational Approaches

What are alternatives to the quantum circuit qubit approach for digital quantum computers?

Scale Setting, Improvement Hamiltonians and Renormalization

How do you extract physically meaningful information from lattice Hamiltonian simulations?

Variational Quantum Methods

Can we use variational approaches to learn about QFTs on NISQ-era hardware?

Error Mitigation and Error Correction

How can we mitigate and correct quantum error and noise on the path towards fault-tolerant quantum computers?

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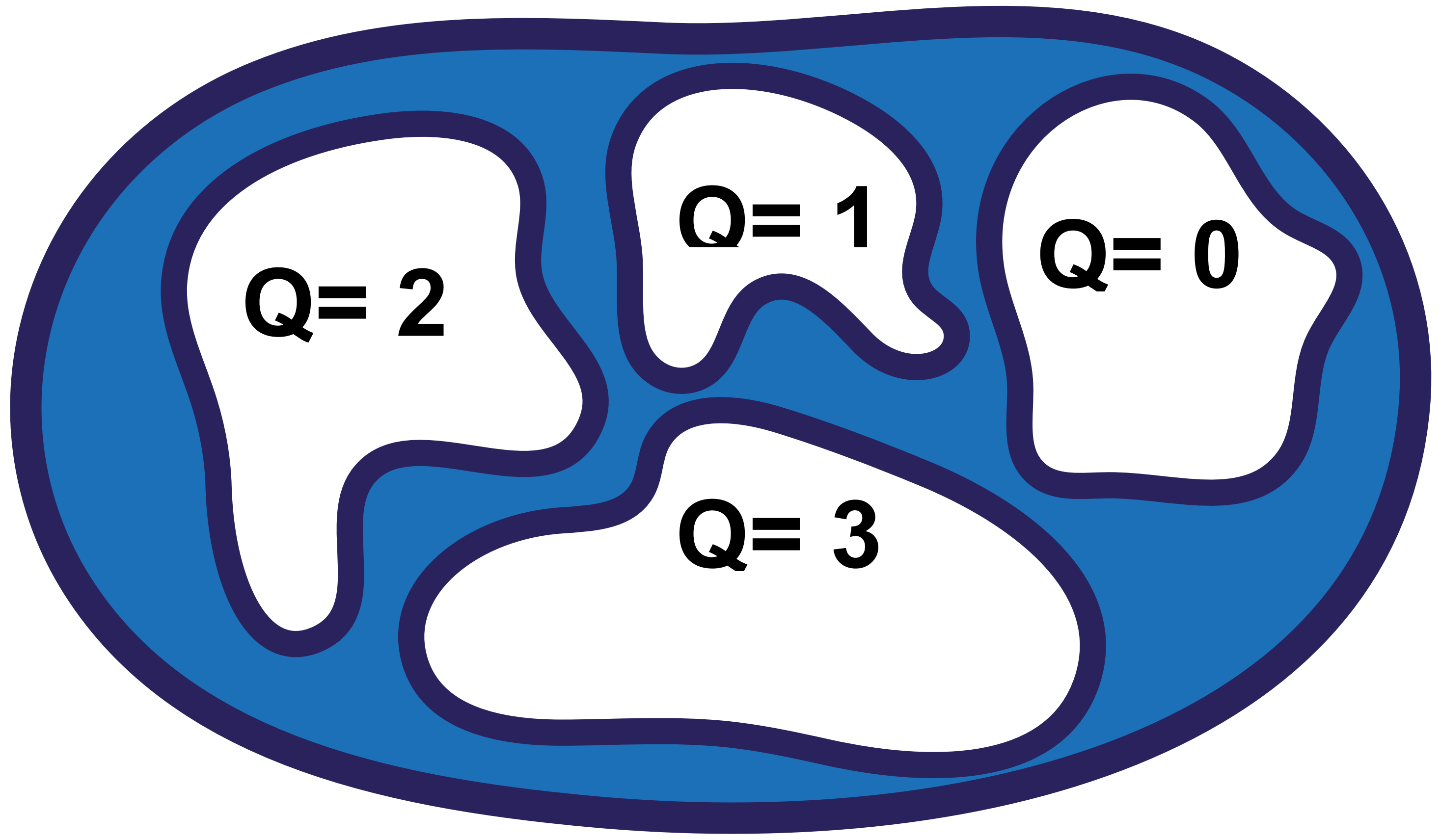
Conclusions

Quantum computers have a fundamentally different computational strategy and will provide novel probes of fundamental questions in particle and nuclear physics

We are well on our way to overcoming the many theoretical challenges and hurdles for implementing 3+1 dimensional QCD onto a quantum computer

- Truncate infinite dimension Hamiltonian
- Carefully sample continuous gauge groups
- Ensure charge is appropriately conserved during simulation



There is much exciting work that I could not discuss here but am excited to hear about in the parallel sessions!