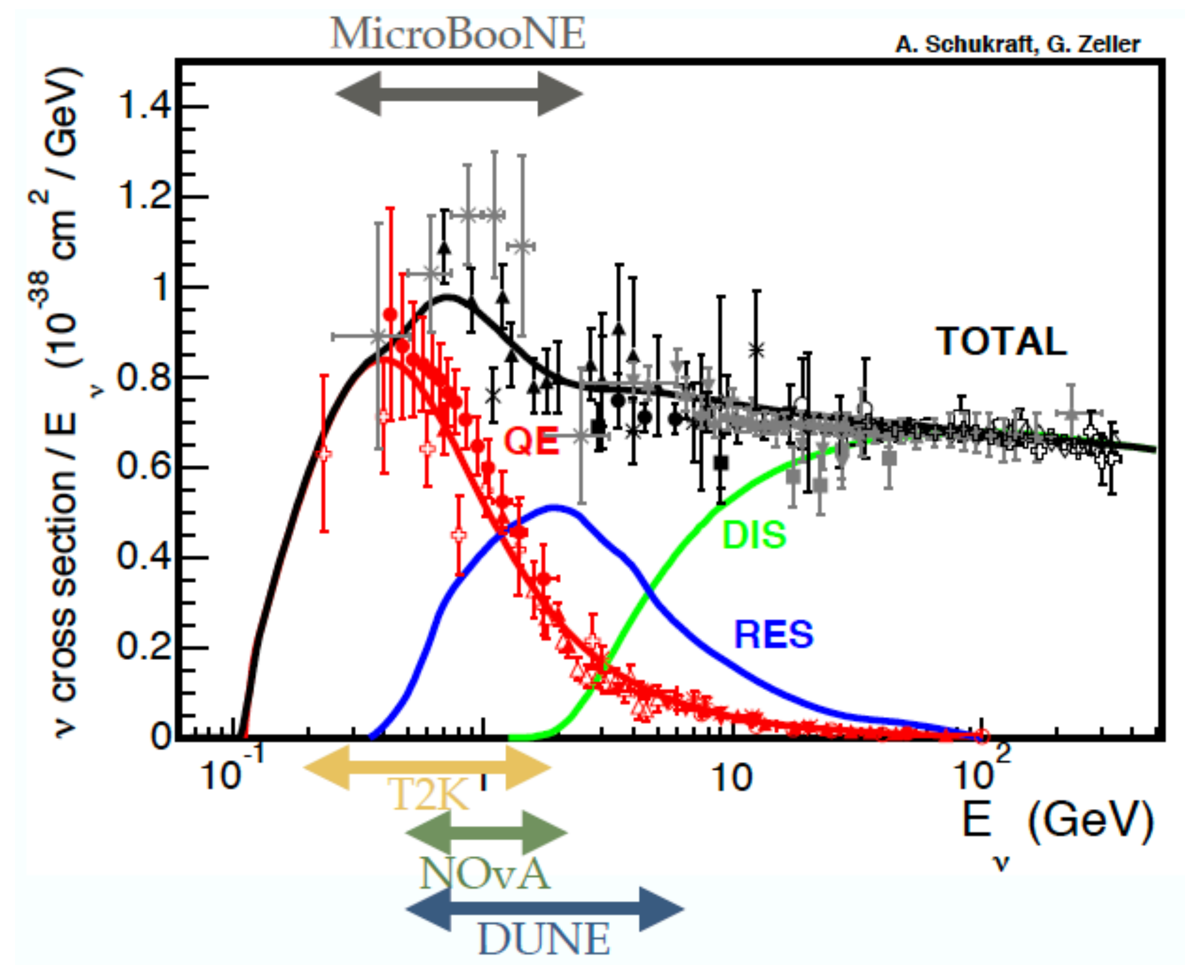
Theory needs of neutrino experiments

Noemi Rocco

Lattice 2023
Fermilab— July 31- August 4, 2023

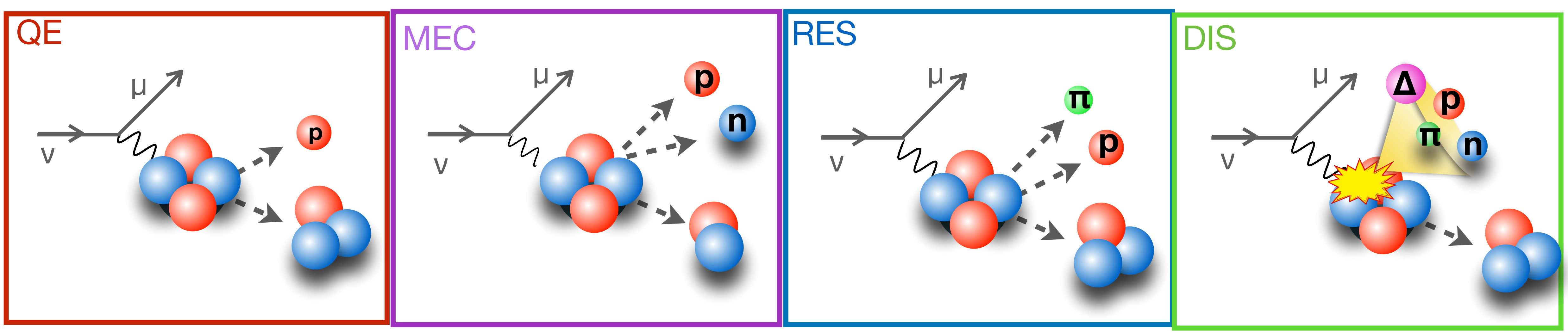
Inputs for the nuclear model

Unprecedented accuracy in the determination of **neutrino-argon cross section** is required to achieve design sensitivity to CP violation at DUNE



More than 60% of the interactions at DUNE are non-quasielastic

Theoretical tools for neutrino scattering, Contribution to: 2022 Snowmass Summer Study



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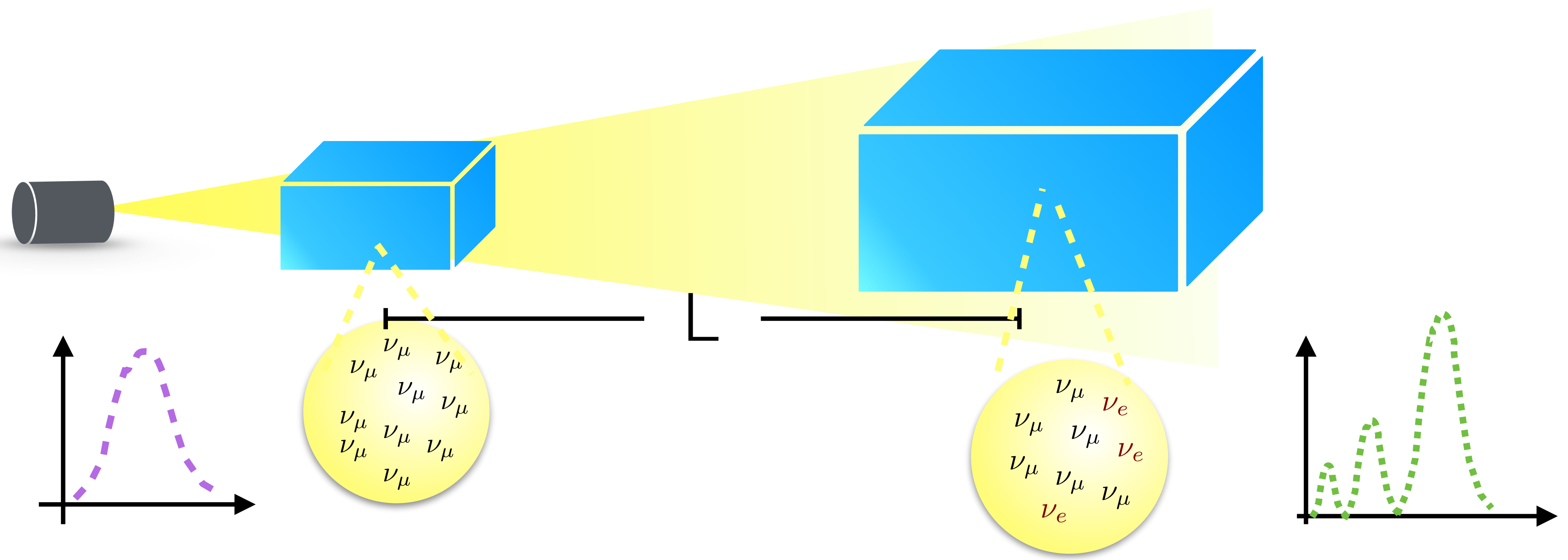
Why do we need more precision?

$$P(\nu_\mu \to \nu_e, E_\nu, L) = \frac{\Phi(E_\nu, L)}{\Phi_\mu(E_\nu, 0)} = \frac{N_e(E_\nu, L)/\sigma_e(E_\nu)}{N_\mu(E_\nu, L)/\sigma_\mu(E_\nu)}$$

Theory

Experiment

Detectors measure the **neutrino interaction rate:**



A precise determination of σ(E) is crucial to extract ν oscillation parameters. Nuclear effects at near and far detector **do not** cancel

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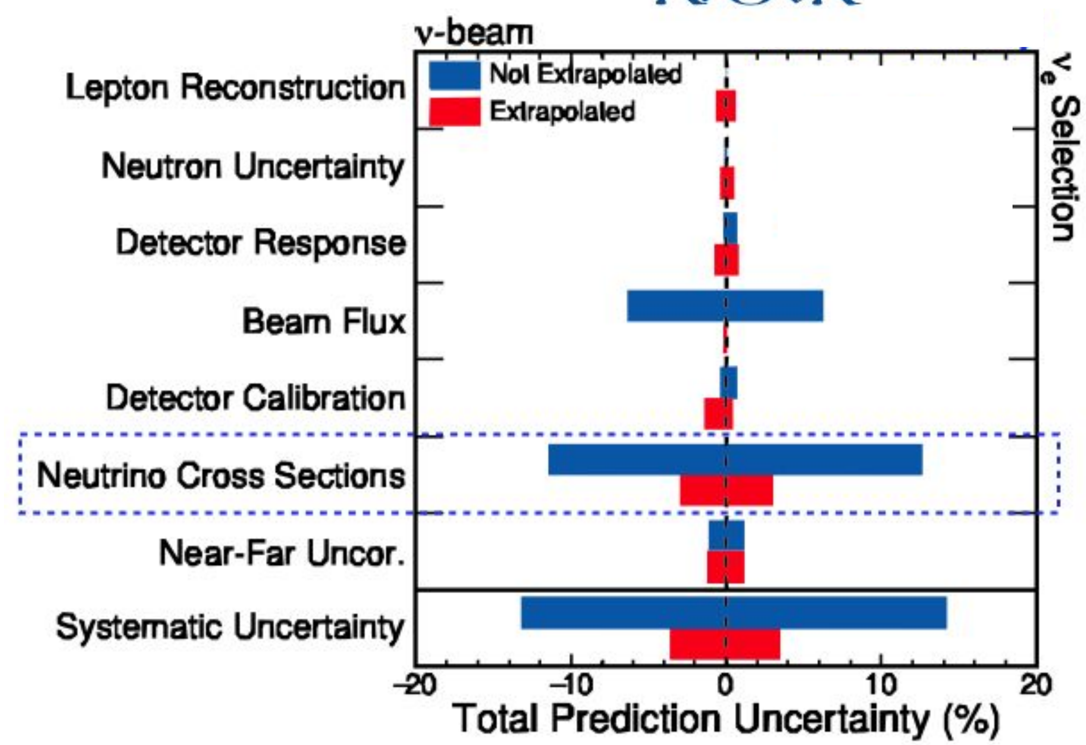
Neutrino-nucleus cross section systematics

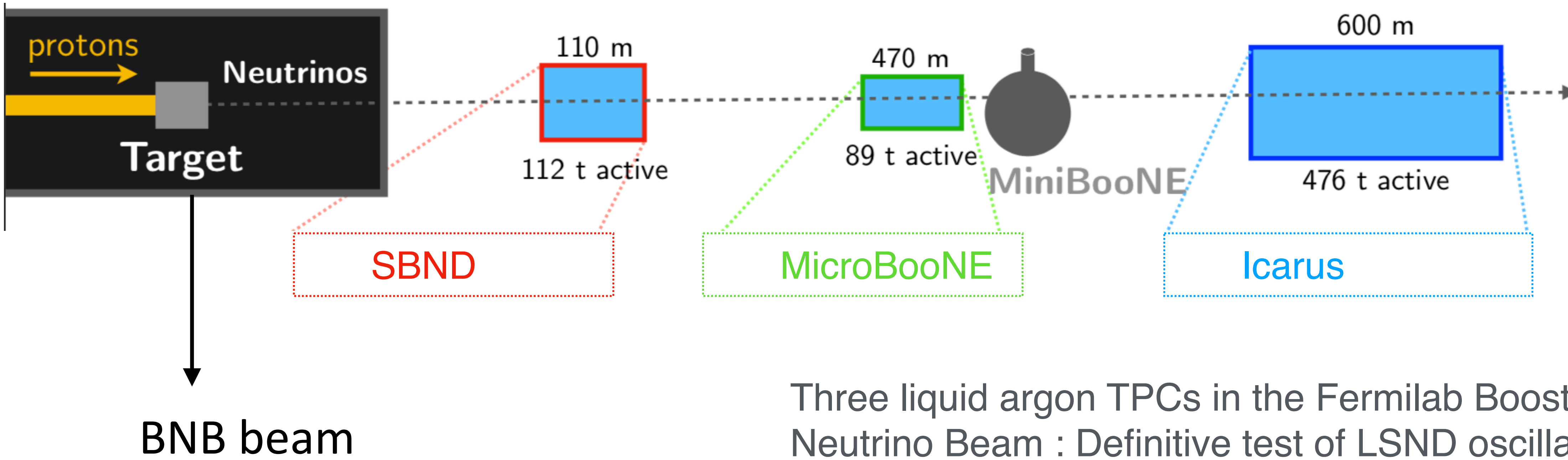
Current oscillation experiments report **large systematic uncertainties** associated with neutrino-nucleus interaction models.

Error source	ν_e FHC	$\bar{\nu}_e$ RHC	$\nu_e / \bar{\nu}_e$ FHC/RHC
Flux and (ND unconstrained)	15.1	12.2	1.2
cross section (ND constrained)	3.2	3.1	2.7
SK detector	2.8	3.8	1.5
SK FSI + SI + PN	3.0	2.3	1.6
Nucleon removal energy	7.1	3.7	3.6
$\sigma(\nu_e)/\sigma(\bar{\nu}_e)$	2.6	1.5	3.0
NC1γ	1.1	2.6	1.5
NC other	0.2	0.3	0.2
$\sin^2\theta_{23}$ and Δm^2_{21}	0.5	0.3	2.0
$\sin^2\theta_{13}$ PDG2018	2.6	2.4	1.1
All systematics	8.8	7.1	6.0

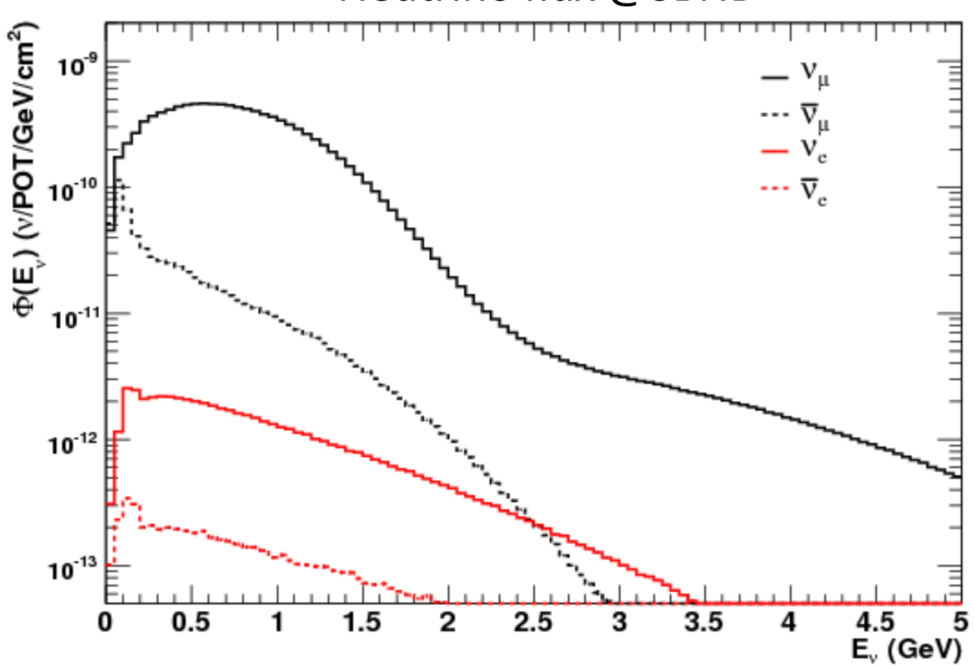


T2K, Phys. Rev. D 103, 112008 (2021)

Short Baseline Neutrino program



Neutrino flux @SBND



Three liquid argon TPCs in the Fermilab Booster Neutrino Beam : Definitive test of LSND oscillations using three baselines

For BNB and T2K the dominant reaction mechanisms are quasi-elastic scattering

The contribution of π-production channels is ~ 25 %

For the sub-GeV experiments the Delta is the only relevant resonance

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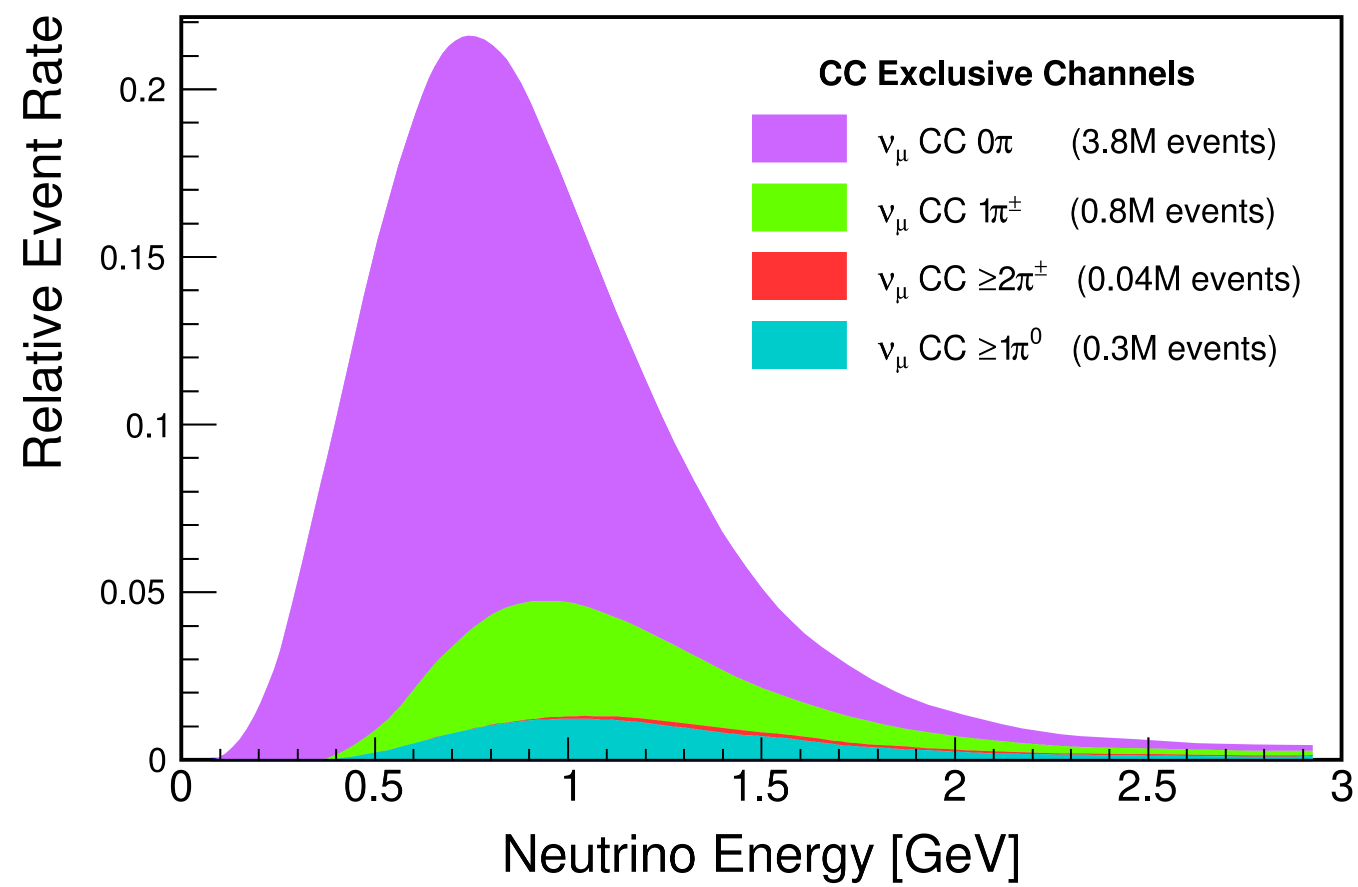
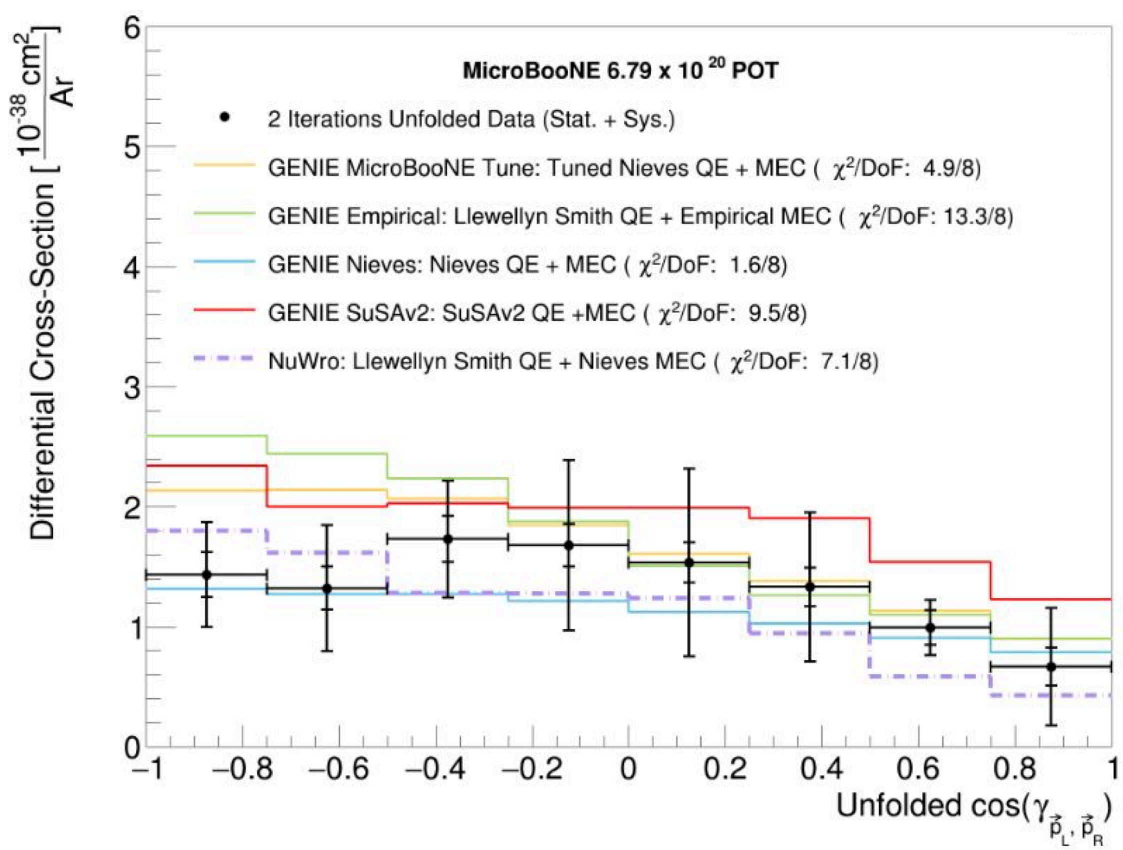
Short Baseline Neutrino program

P. Machado et al, 1903.04608 (2019)

SBND will provide the world's highest statistics cross section measurements in LAr: 2 million events for ν_μ per year for the next 3 years





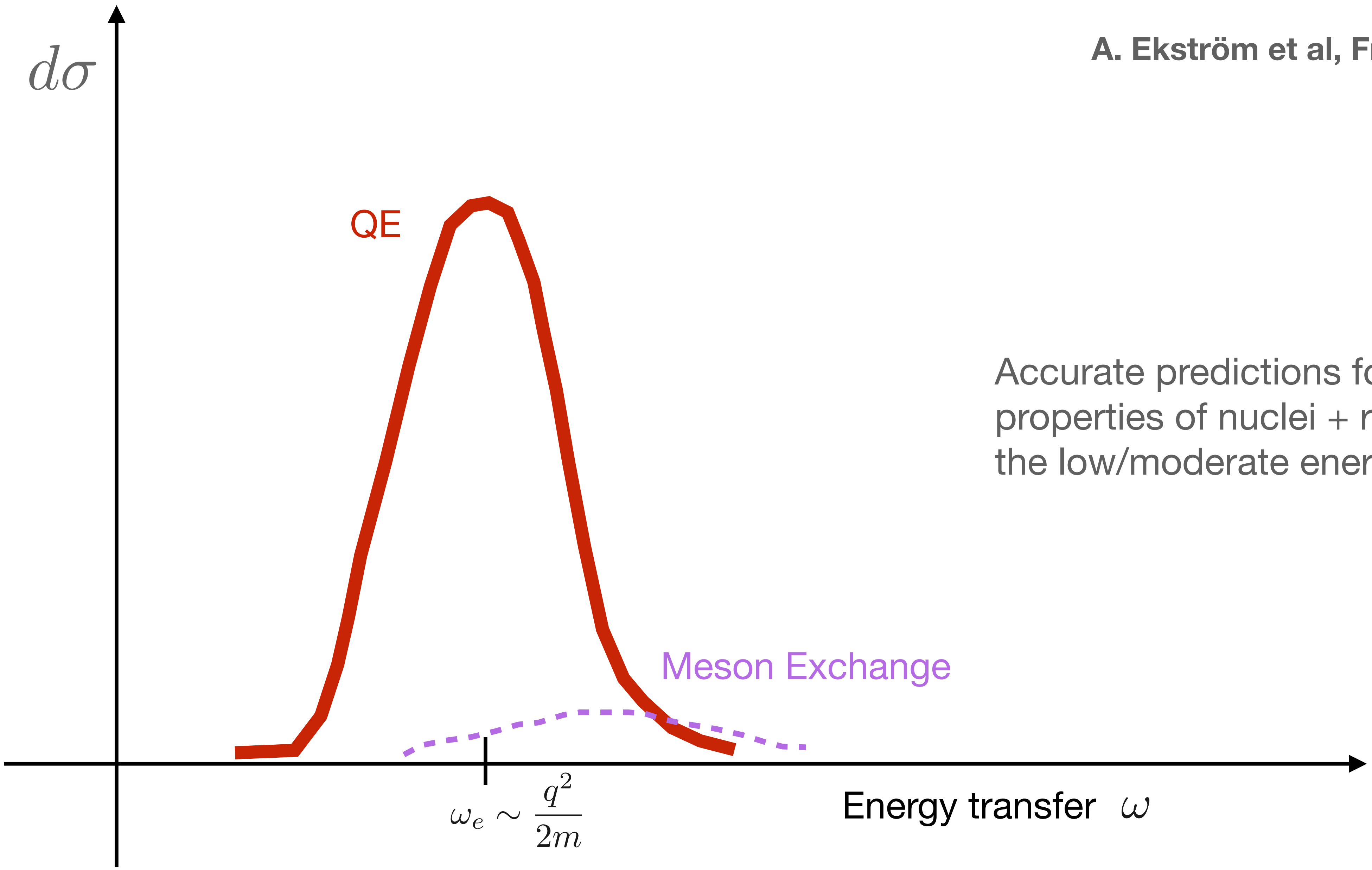
A. Papadopoulou W&C seminar June 2023

MicroBooNE provided first **two-proton knockout** single-differential **cross section on argon 2211.03734**

Ab initio Methods

Ab-initio methods (CC, IMSRG, SCGF, QMC, etc) are systematically improvable many-body approaches.

A. Ekström et al, Front. Phys.11 (2023) 29094



Accurate predictions for ground state properties of nuclei + response functions in the low/moderate energy region

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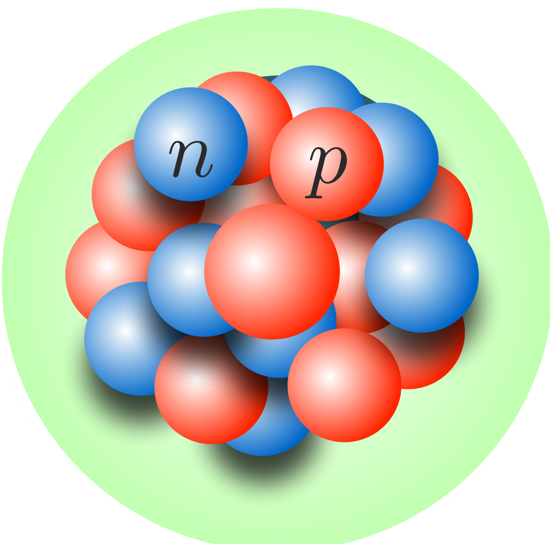
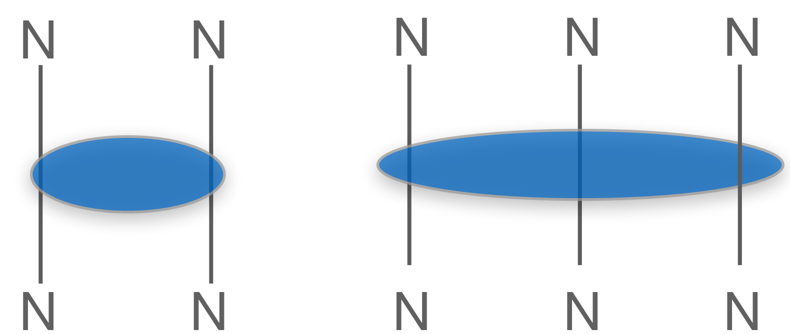
Hamiltonian and Currents

At low energy, the effective degrees of freedom are pions and nucleons:



$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

1-body **2-body** **3-body**



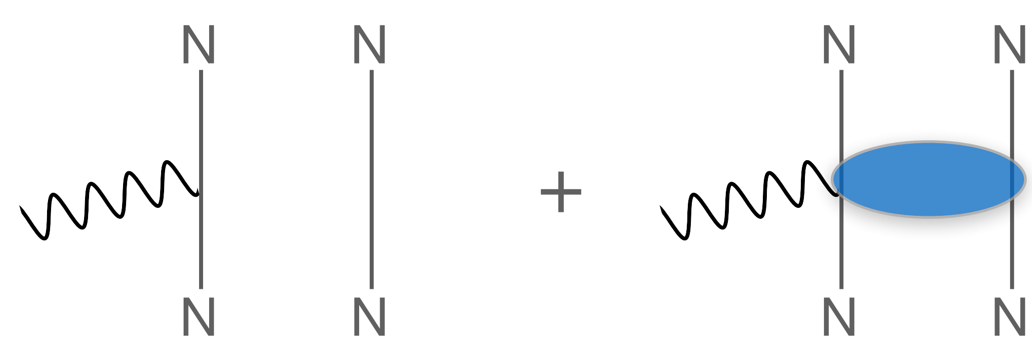
- AV18+IL7
- chiral interactions

The electromagnetic current is constrained by the Hamiltonian through the **continuity equation**

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0 \qquad [v_{ij}, j^0_i] \neq 0$$

The above equation implies that the current operator includes one and two-body contributions

$$J^\mu(q) = \sum_i j^\mu_i + \sum_{i<j} j^\mu_{ij} + \ldots$$



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Chiral effective field theory

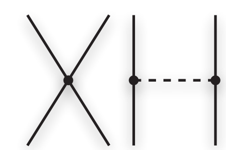
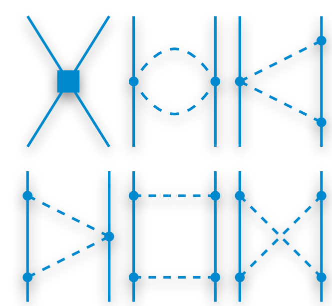
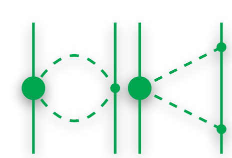
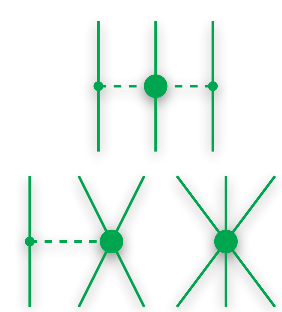
Chiral Hamiltonians exploits the (approximate) broken chiral symmetry of QCD

Identify the soft and hard scale of the problem $\mathcal{L}^{(n)} \sim \left(\frac{q}{\Lambda_b}\right)^n$ ~ 100 MeV soft scale, ~ 1 GeV hard scale

Design an organizational scheme that can distinguish between more and less important terms:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)}$$

H. Hergert, Front. in Phys. 8, 379 (2020)

	NN	3N
LO $(Q/\Lambda_\chi)^0$		
NLO $(Q/\Lambda_\chi)^2$		
NNLO $(Q/\Lambda_\chi)^3$		

Contact interactions lead to LEC:

Short range two-nucleon interaction fit to deuteron and NN scattering

Three nucleon interactions fitted on light nuclei

Long-range LEC are determined from π-nucleon scattering

Formulate statistical models for uncertainties: Bayesian estimates of EFT errors

S. Wesolowski, et al, PRC 104, 064001 (2021)

Green’s Function Monte Carlo

Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

$$|\Psi_V\rangle = \sum_n c_n |\Psi_n\rangle \qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

GFMC uses a projection technique to **enhance the true ground-state component** of a starting wave function.

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty} \sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

The direct calculation of the imaginary-time propagator for strongly-interacting systems involves prohibitive difficulties

J. Carlson , et al. Rev. Mod. Phys. 87 (2015) 1067

The imaginary-time evolution is broken into N small imaginary-time steps, and complete sets of states are inserted

$$e^{-(H-E_0)\tau}|\Psi_V\rangle = \int dR_1 \ldots dR_N |R_N\rangle\langle R_N|e^{-(H-E_0)\Delta\tau}|R_{N-1}\rangle \ldots \langle R_2|e^{-(H-E_0)\Delta\tau}|R_1\rangle\Psi_V(R_1)$$

Short Time Propagator

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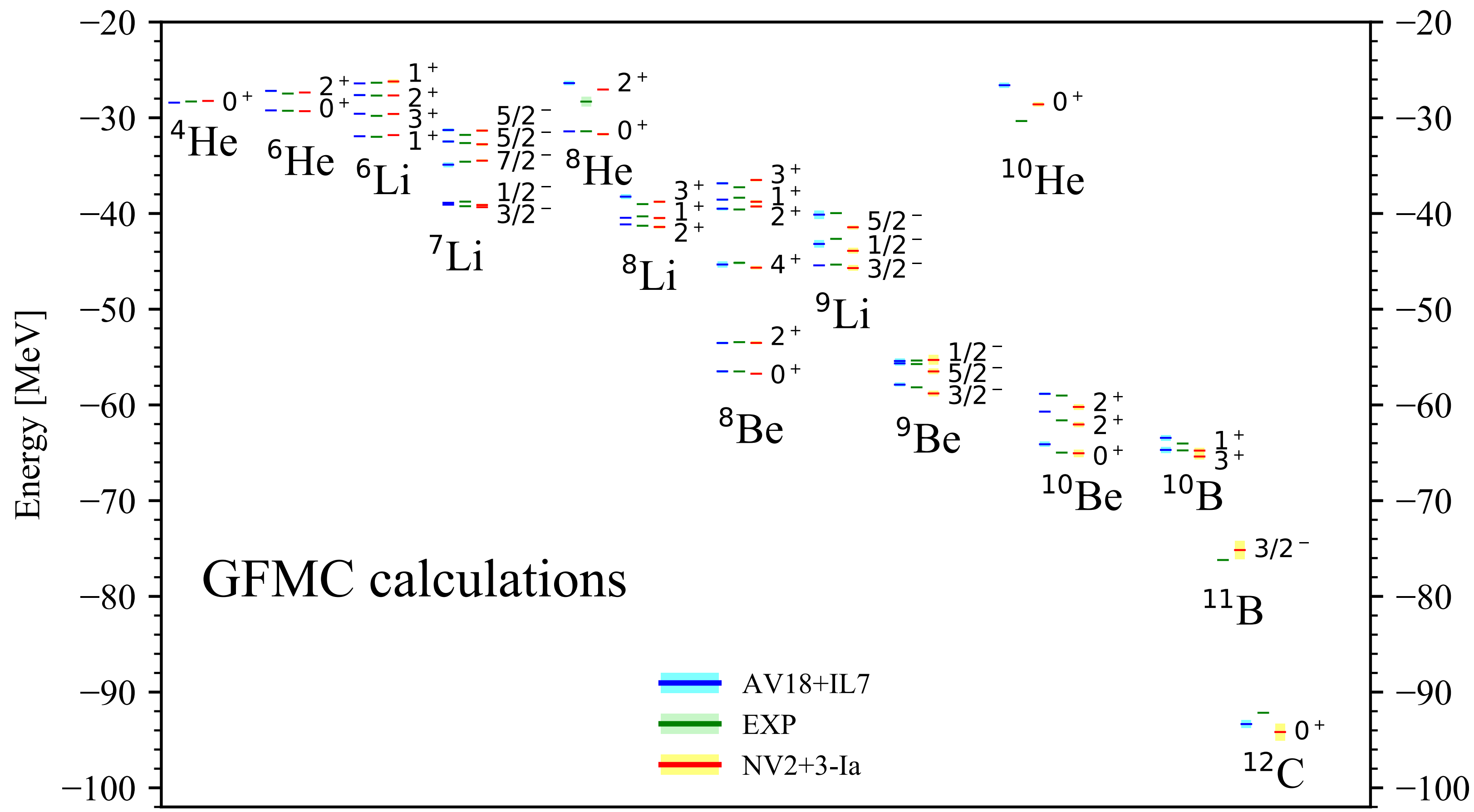
Solve the Many Body Nuclear problem

A sum over all the many-body spin-isospin states is performed

$$\sum_{SS'} \langle S' | e^{-[V-E_0]\delta\tau} | S \rangle \simeq \sum_{SS'} \langle S' | \prod_{i<j} e^{-V_{ij}\delta\tau} | S \rangle e^{E_0 \delta\tau}$$

GFMC is extremely accurate but limited to A < 13 nuclei. Semi-phenomenological, or chiral potentials can be used, excellent agreement with energy spectral of different targets



M. Piarulli, et al. Phys.Rev.Lett. 120 (2018) 5, 052503

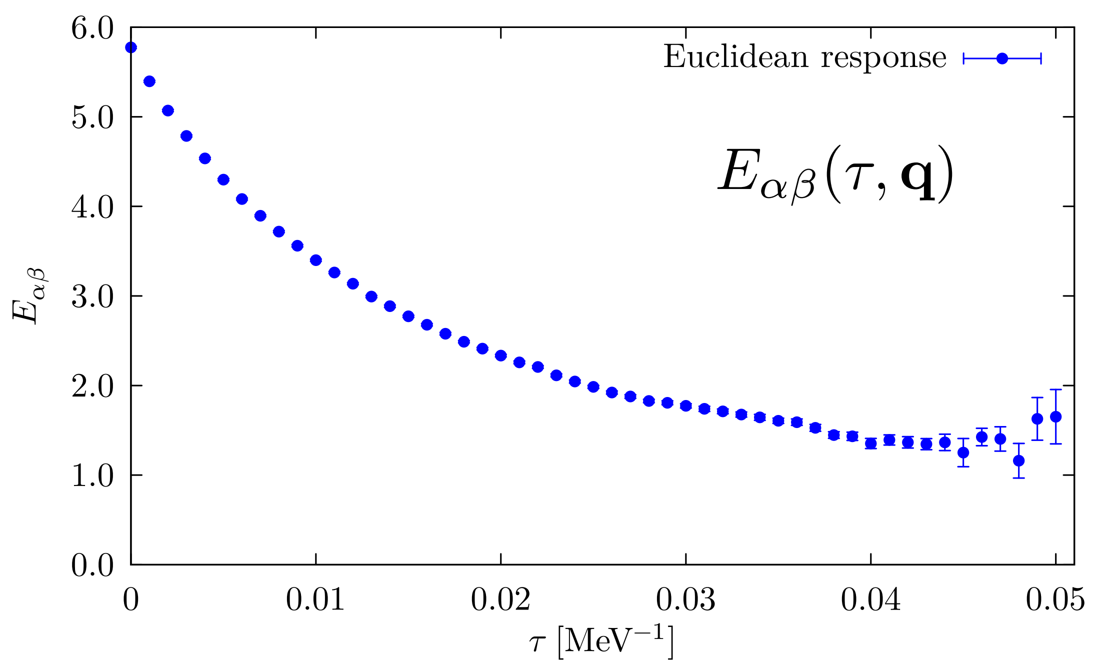
Green’s Function Monte Carlo

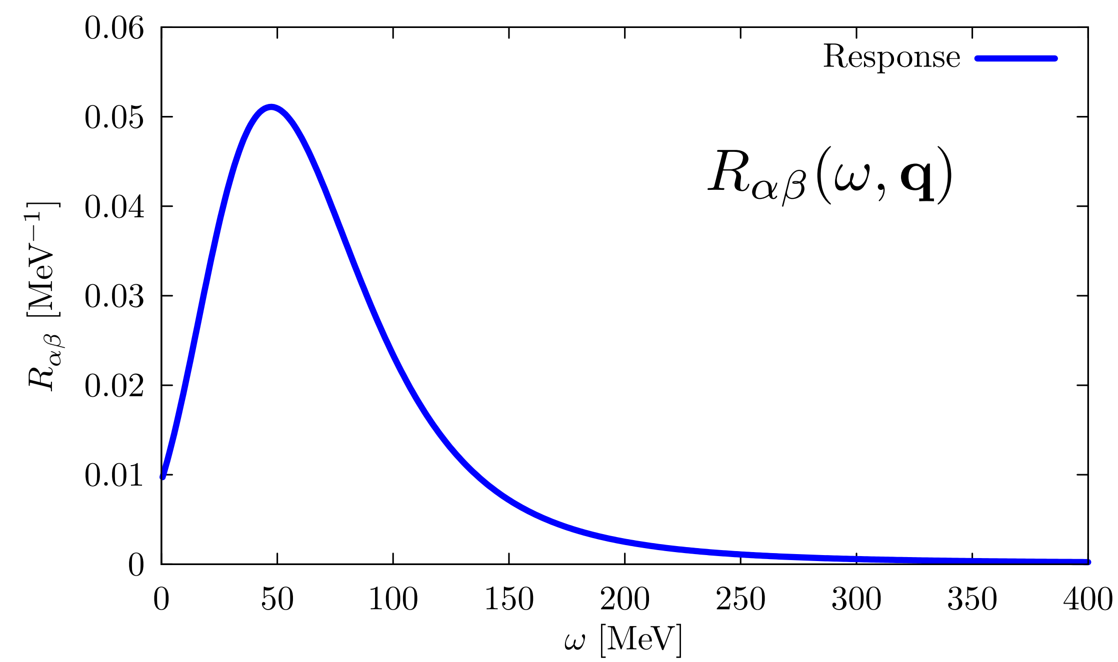
Nuclear response function involves evaluating a number of transition amplitudes.
Valuable information can be obtained from the **integral transform of the response function**

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \int d\omega K(\sigma,\omega) R_{\alpha\beta}(\omega,\mathbf{q}) = \langle\psi_0|J^\dagger_\alpha(\mathbf{q}) K(\sigma, H-E_0) J_\beta(\mathbf{q})|\psi_0\rangle$$

Inverting the integral transform is a complicated problem

A. Lovato et al, PRL117 (2016), 082501, PRC97 (2018), 022502





Same problem applies to different realm physics for example lattice QCD

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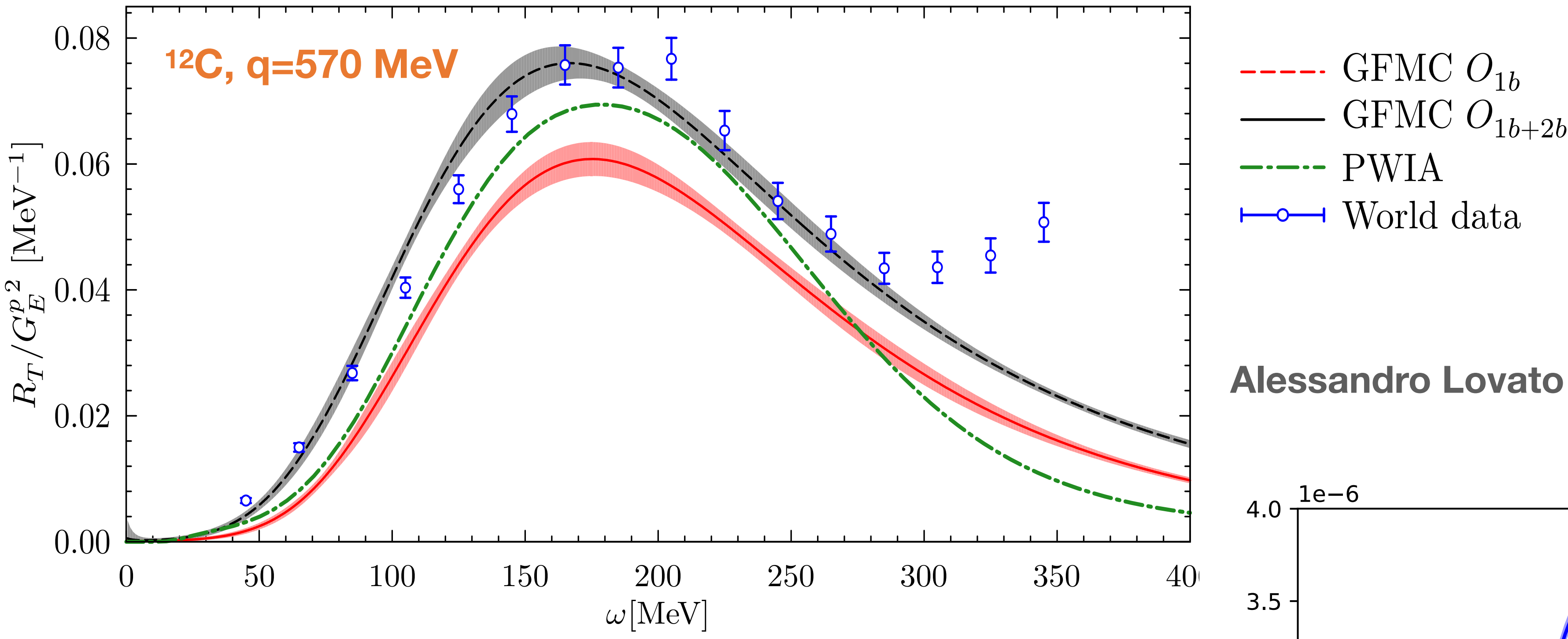
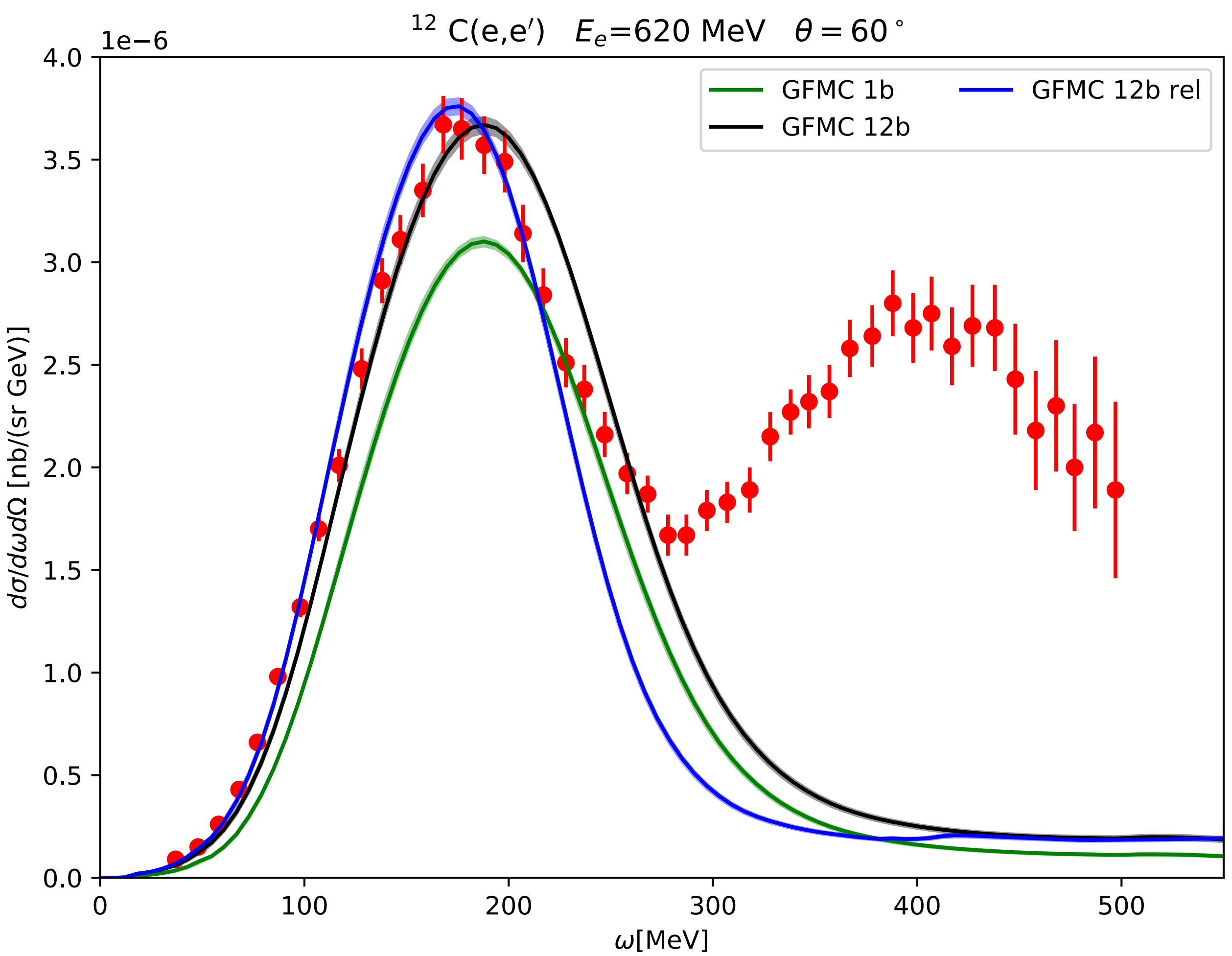
Cross sections: Green's Function Monte Carlo



Alessandro Lovato et al. PRL 117 082501 (2016)



A.Lovato, NR, et al, *submitted to Universe*

Limitations:

Medium mass nuclei A < 13

Inclusive results which are virtually correct in the QE

Relies on non-relativistic treatment of the kinematics

Can not handle explicit pion degrees of freedom

Axial form factor determination

- The axial form-factor has been fit to the dipole form

$$F_A(q^2) = \frac{g_A}{(1 - q^2/m_A^2)^2}$$

- The intercept g_A=-1.2723 is known from neutron β decay

- Different values of m_A from experiments
  - m_A =1.02 GeV q.e. scattering from deuterium
  - m_A =1.35 GeV @ MiniBooNE

- Alternative derivation based on **z-expansion** —model independent parametrization

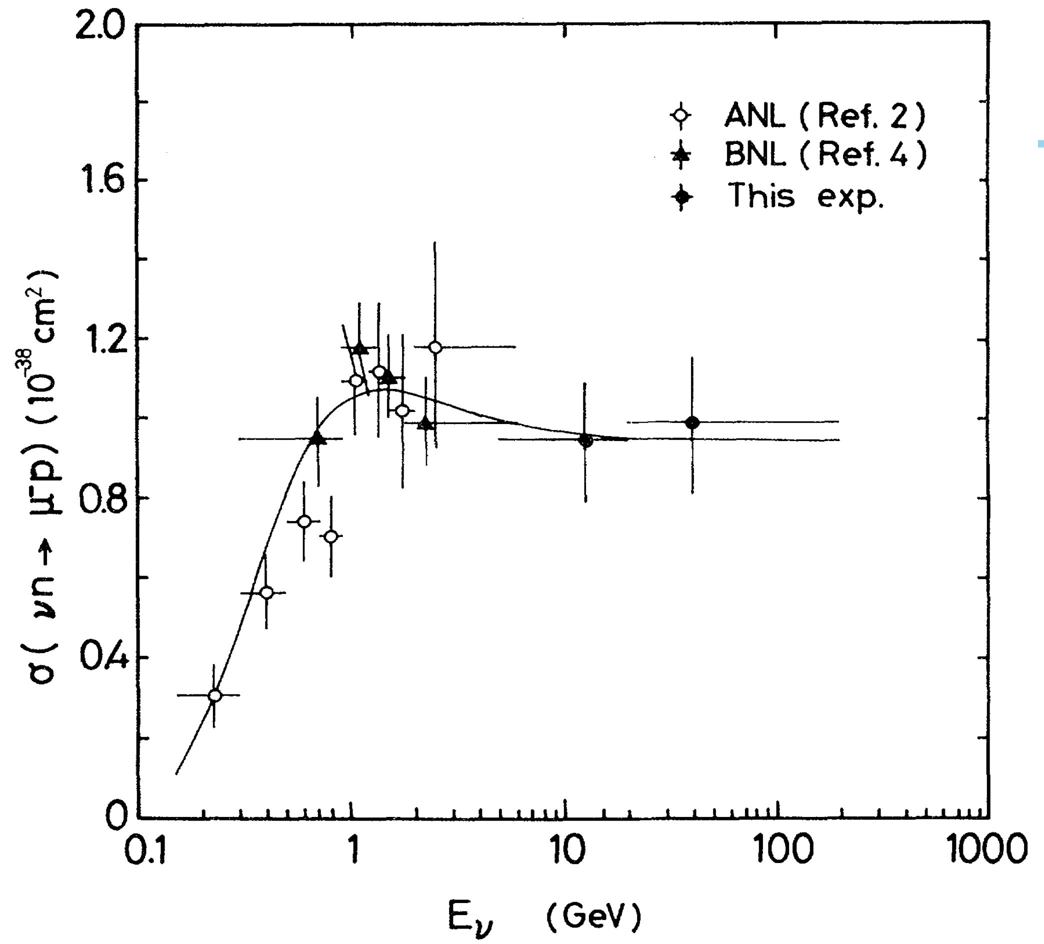
$$F_A(q^2) = \sum_{k=0}^{k_{\max}} a_k z(q^2)^k,$$

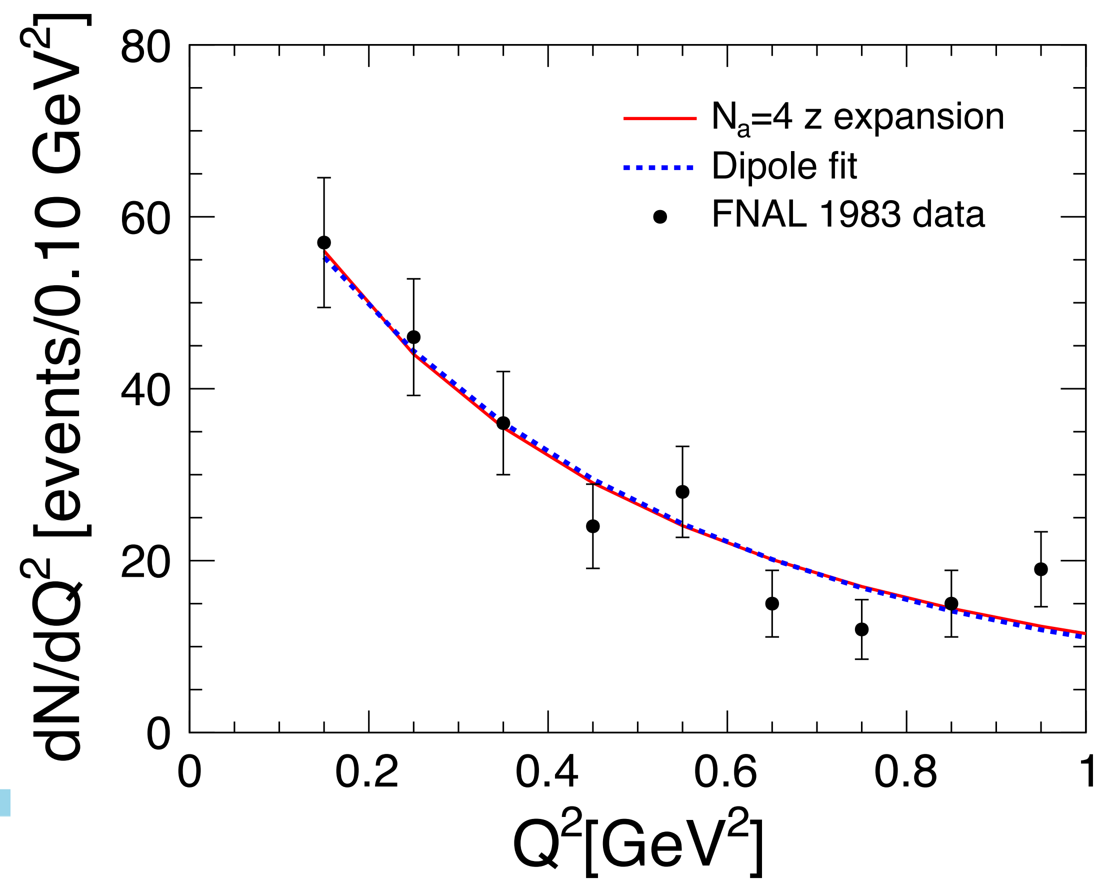
known functions

free parameters

Bhattacharya, Hill, and Paz PRD 84 (2011) 073006

A.S.Meyer et al, Phys.Rev.D 93 (2016) 11, 113015





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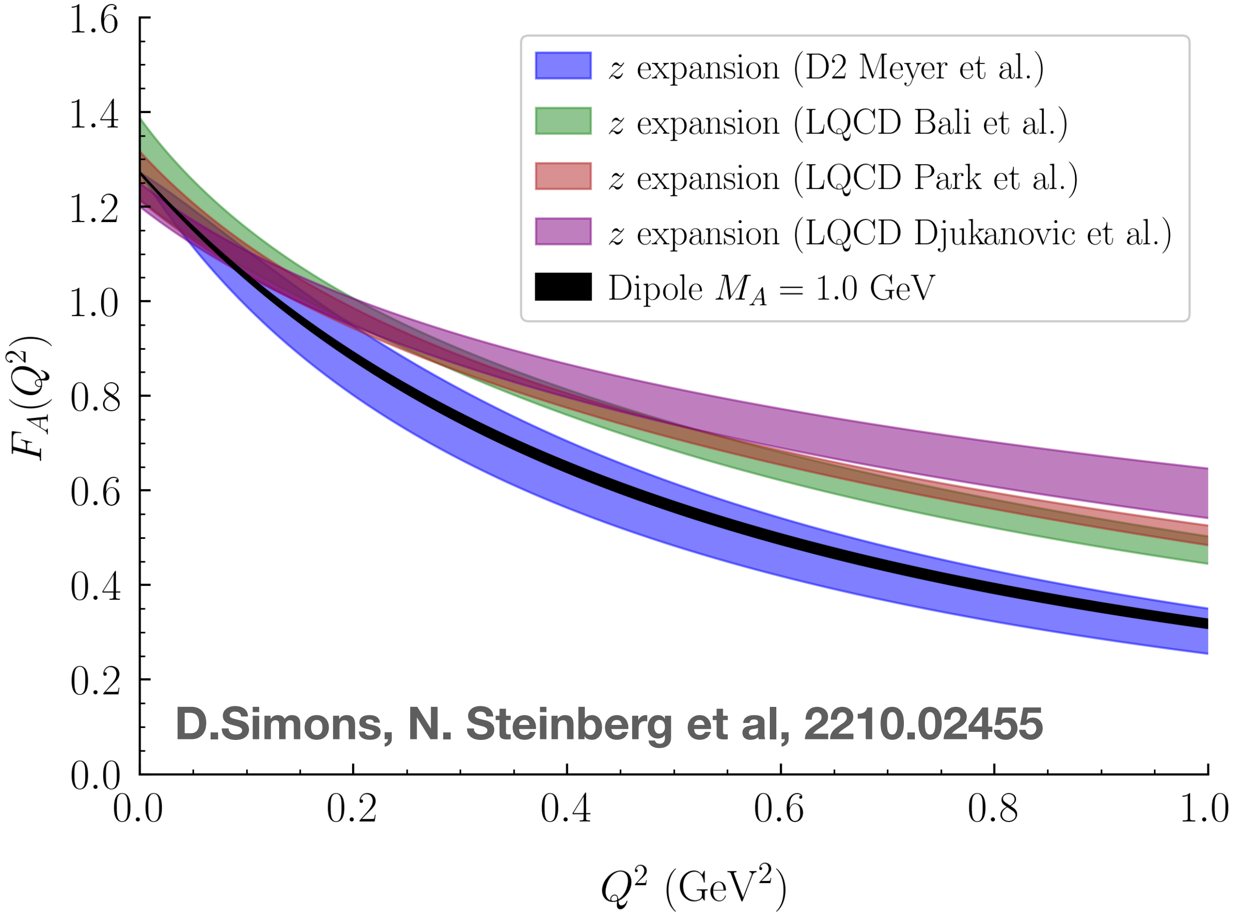
Axial form factor determination



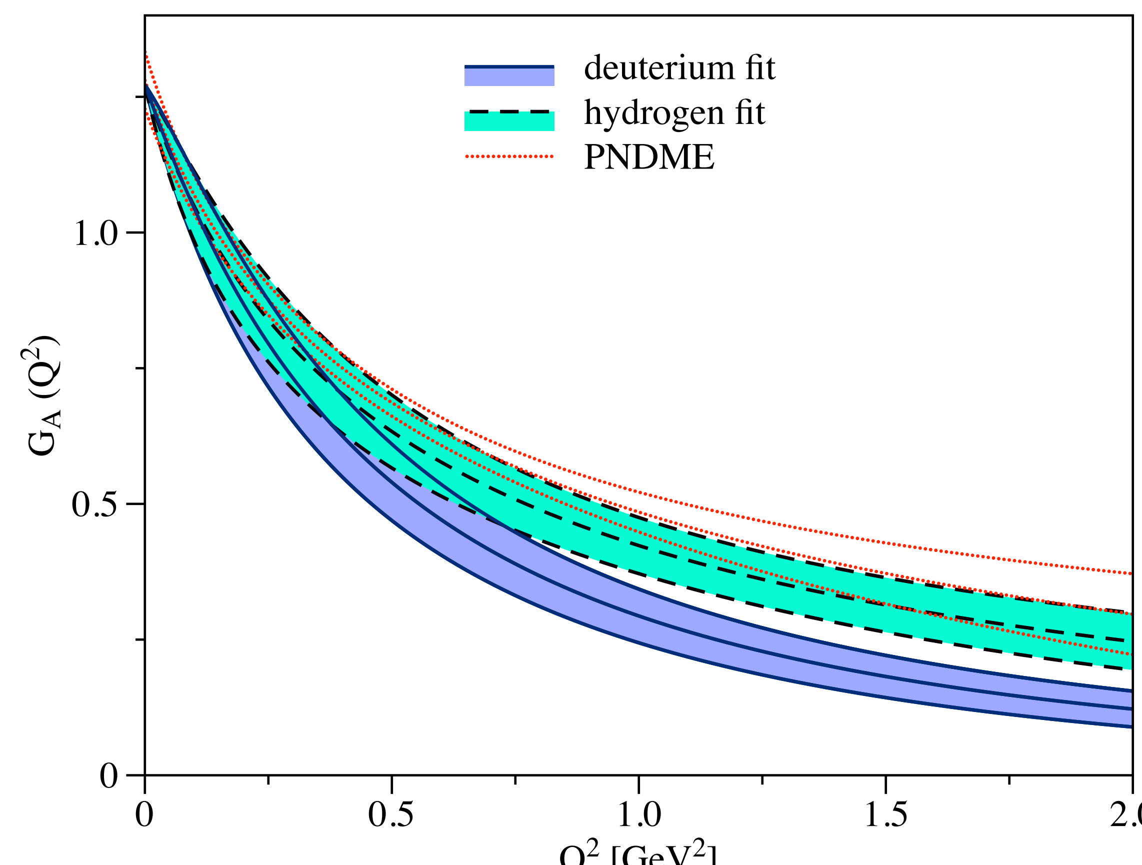
A. Meyer, A. Walker-Loud, C. Wilkinson, 2201.01839

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

LQCD results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV^2

O. Tomalak, R. Gupta, T. Battacharaya, 2307.14920



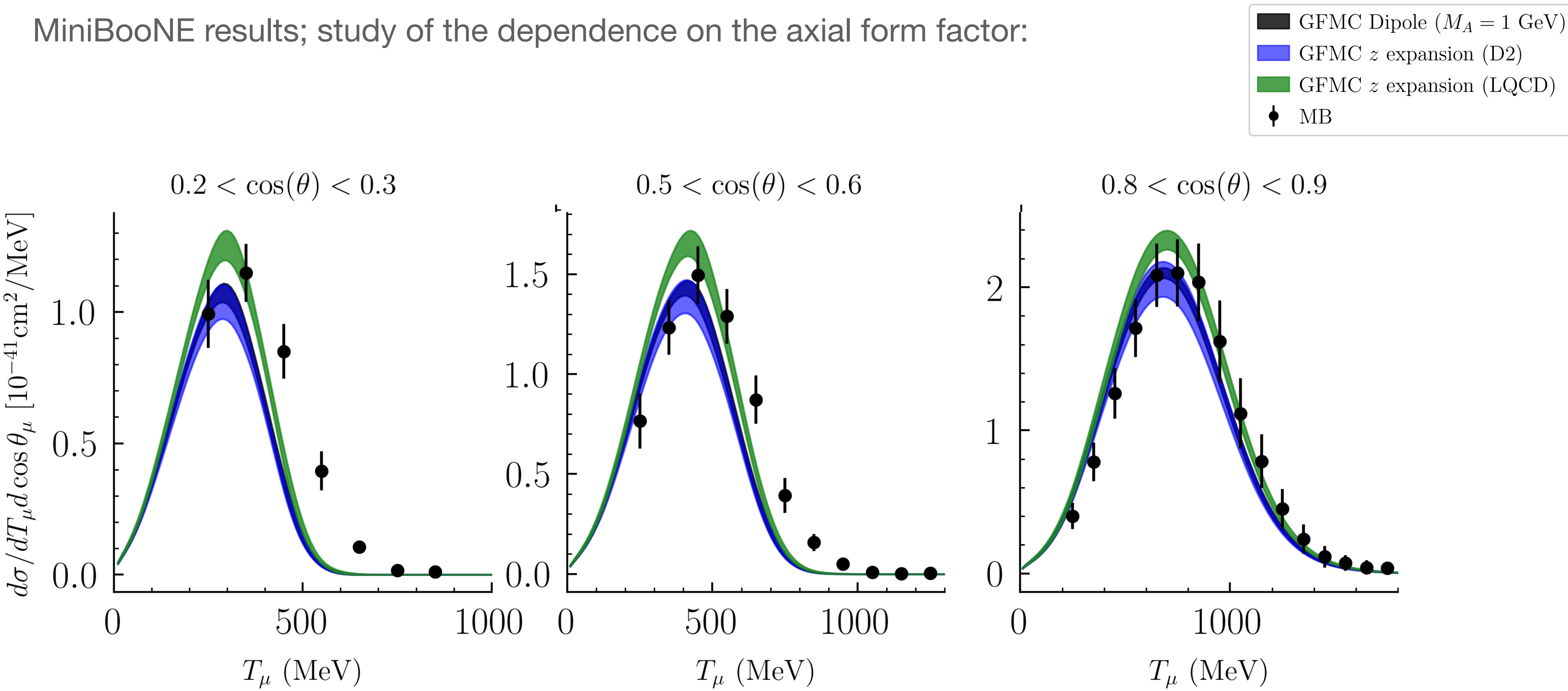
Comparison with recent MINERvA antineutrino-hydrogen charged-current measurements

1-2σ agreement with MINERvA data and LQCD prediction by PNDME Collaboration

Novel methods are needed to remove excited-state contributions and discretization errors

Study of model dependence in neutrino predictions

MiniBooNE results; study of the dependence on the axial form factor:



D.Simons, N. Steinberg et al, 2210.02455

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
GFMC Difference in $d\sigma_{\mathrm{peak}}$ (%)	18.6	17.1	12.2

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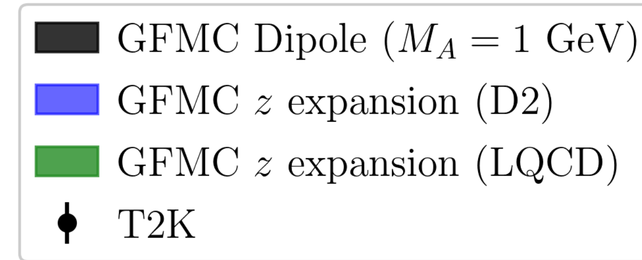
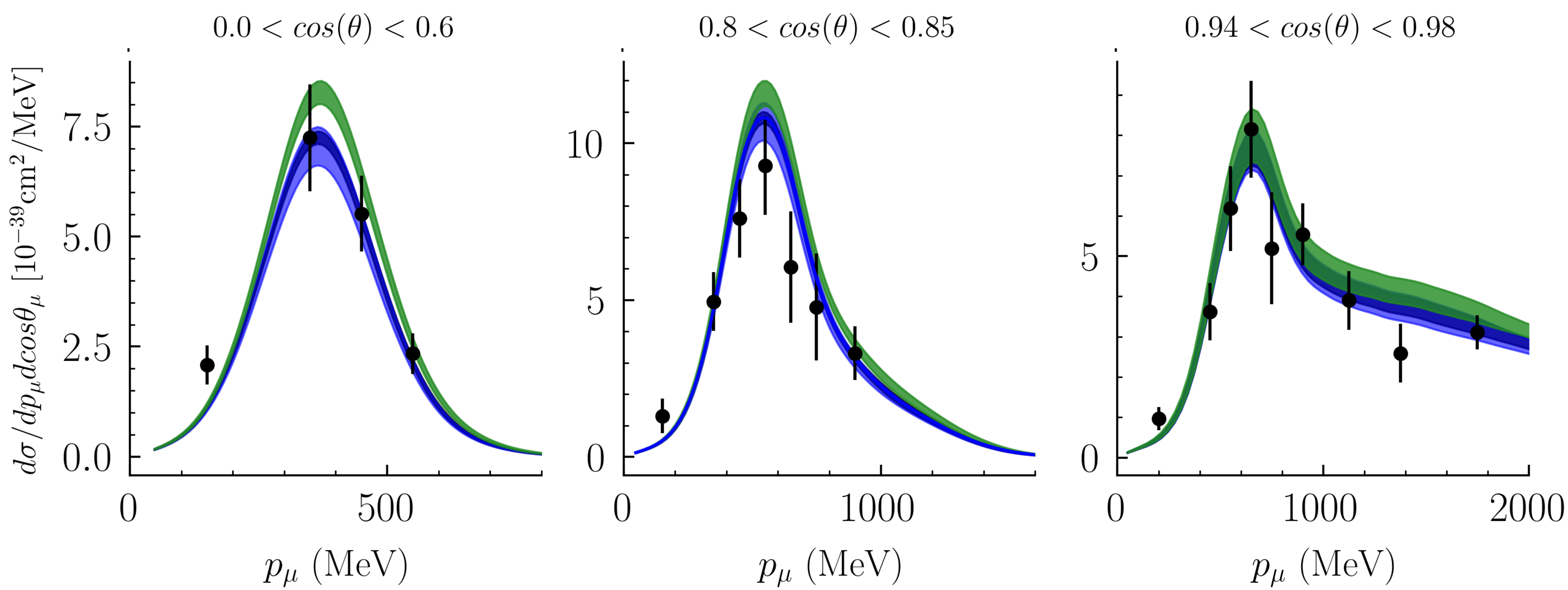
Study of model dependence in neutrino predictions

T2K results; study of the dependence on the axial form factor:





D.Simons, N. Steinberg et al, 2210.02455

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
GFMC difference in $d\sigma_{\text{peak}}$ (%)	15.8	8.0	4.6

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Coupled Cluster Method

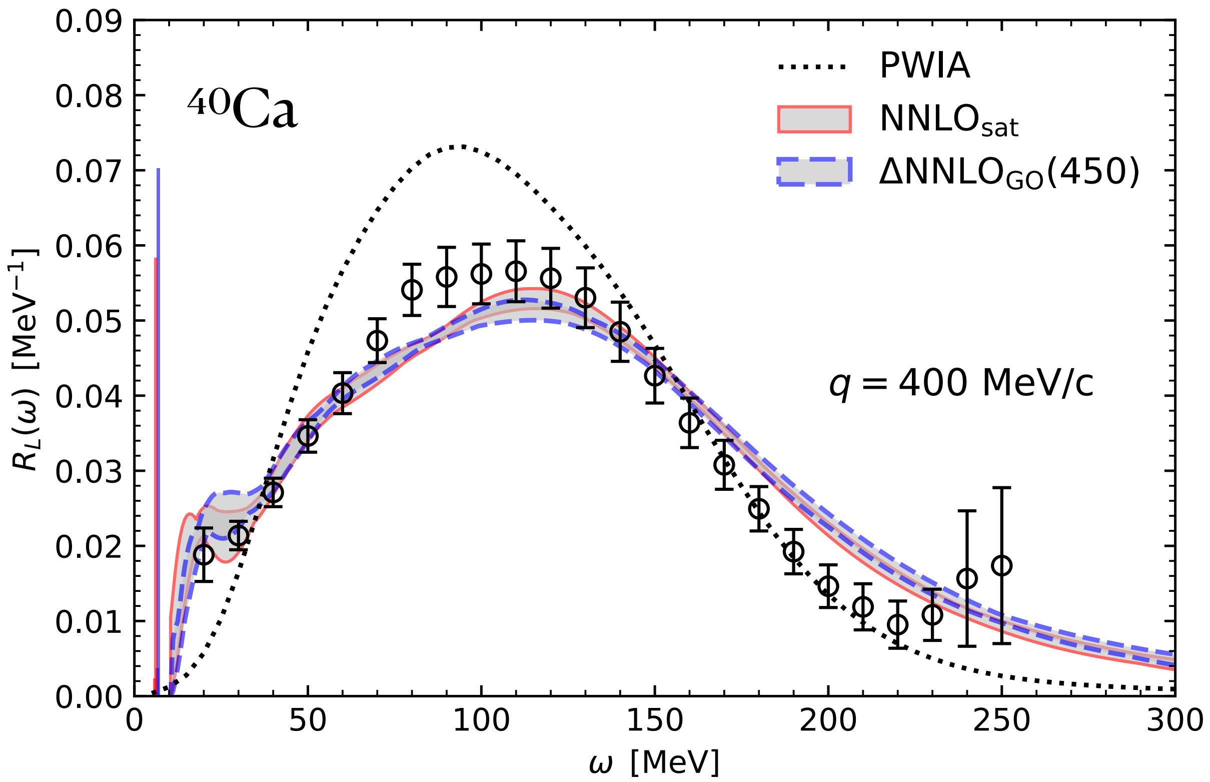
Reference state Hartree Fock: $|\Psi\rangle$

Include correlations through e^T operator

Similarity transformed Hamiltonian $e^{-T}He^{T}|\Psi\rangle = \bar{H}|\Psi\rangle = E|\Psi\rangle$

Expansion in second quantization single + doubles:

$$T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \dots$$



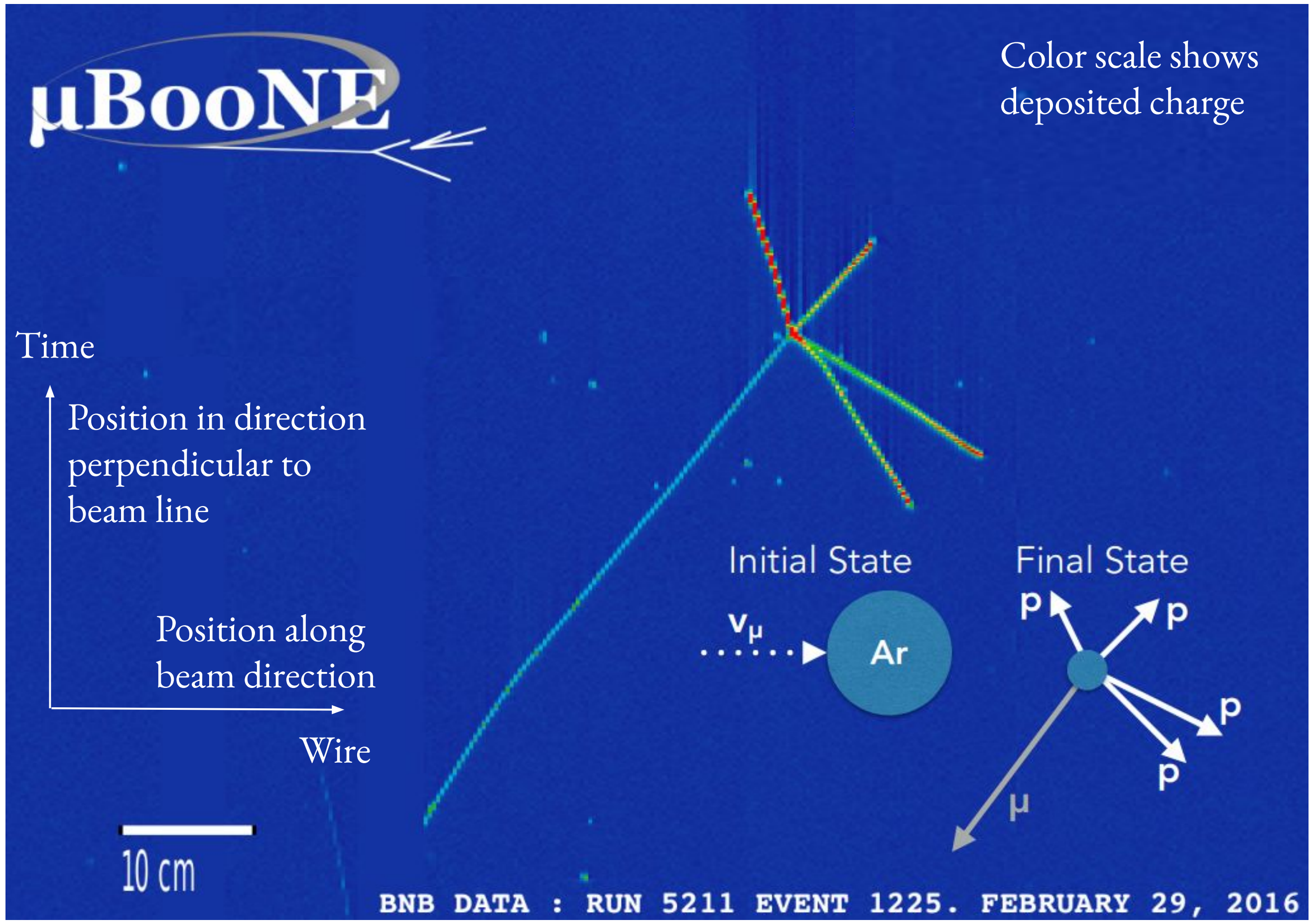
Polynomial scaling with the number of nucleons (predictions for ^{132}Sn and ^{208}Pb)

Electroweak response functions obtained using **LIT**

$$K_\Gamma(\omega,\sigma) = \frac{1}{\pi}\frac{\Gamma}{\Gamma^2 + (\omega-\sigma)^2}$$

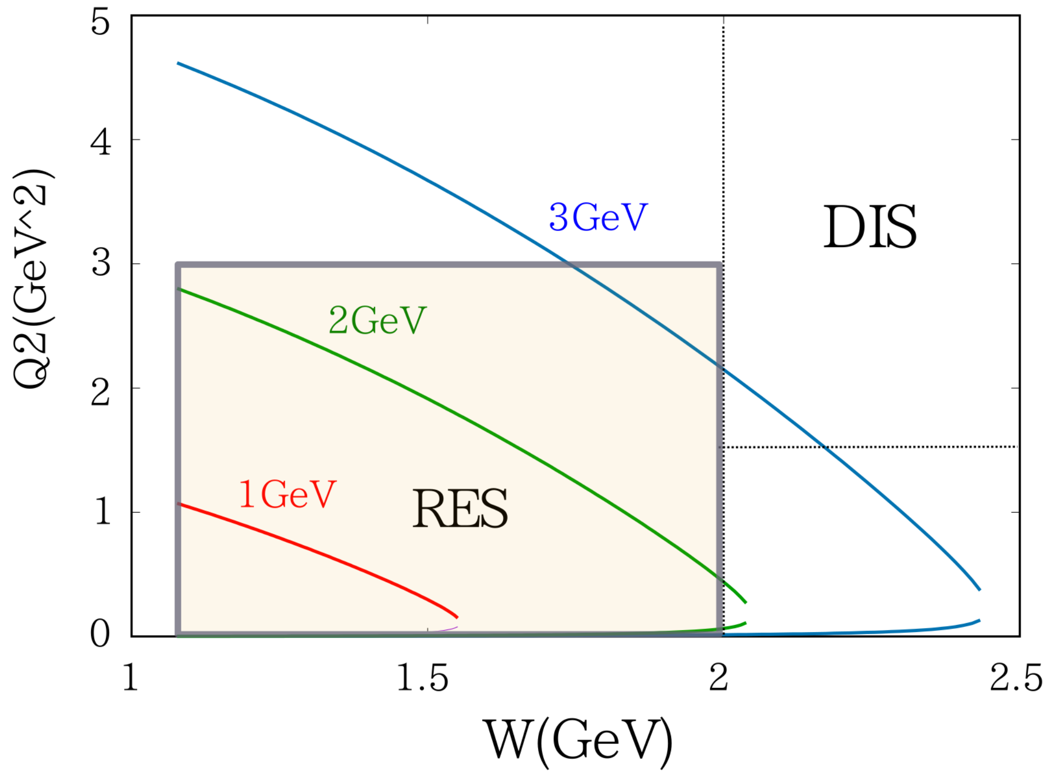
JES, B. Acharya, S. Bacca, G. Hagen; PRL 127 (2021) 7, 072501

Address new experimental capabilities



A. Papadopoulou W&C seminar June 2023

T.Sato talks @ NuSTEC Workshop on Neutrino-Nucleus Pion Production in the Resonance Region



- Excellent spatial resolution
- Precise calorimetric information
- Powerful particle identification

$$W = \sqrt{(p+q)^2}, Q^2 = -q^2 = -(p_\nu - p_l)^2$$

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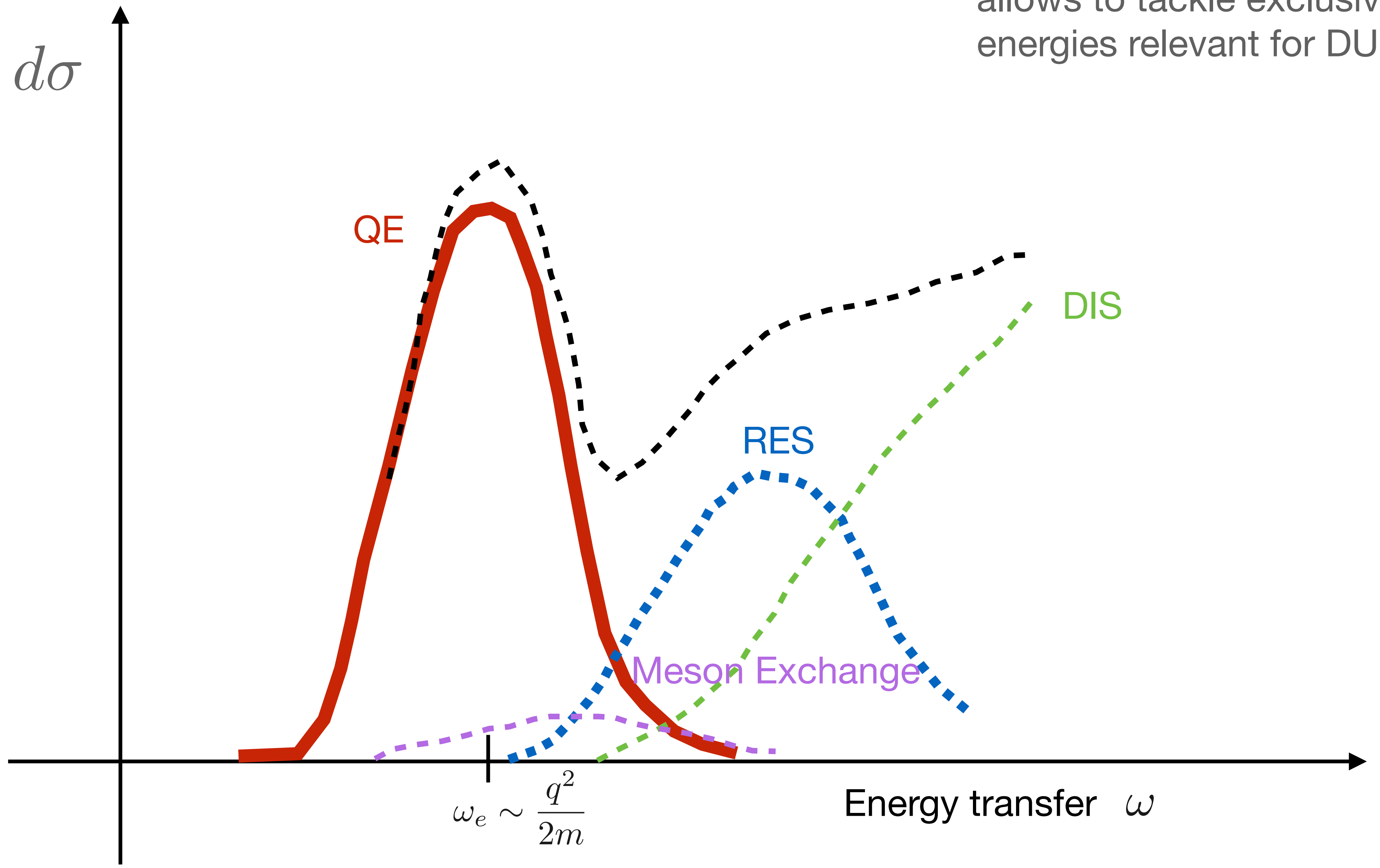
Factorization Based Approaches

Factorization of the hadronic final states: allows to tackle exclusive channels + higher energies relevant for DUNE



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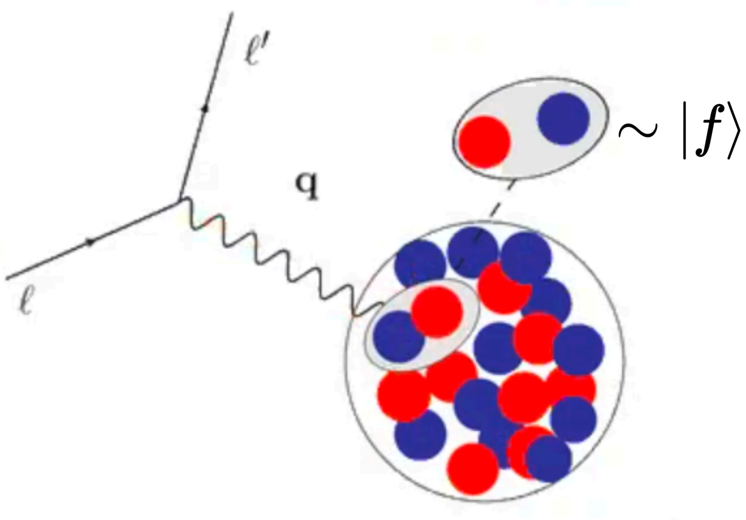
Short-Time Approximation

Response functions are given by the **scattering from pairs of fully interacting nucleons** that **propagate** into a **correlated pair** of nucleons



The sum over all final states is replaced by a two nucleon propagator

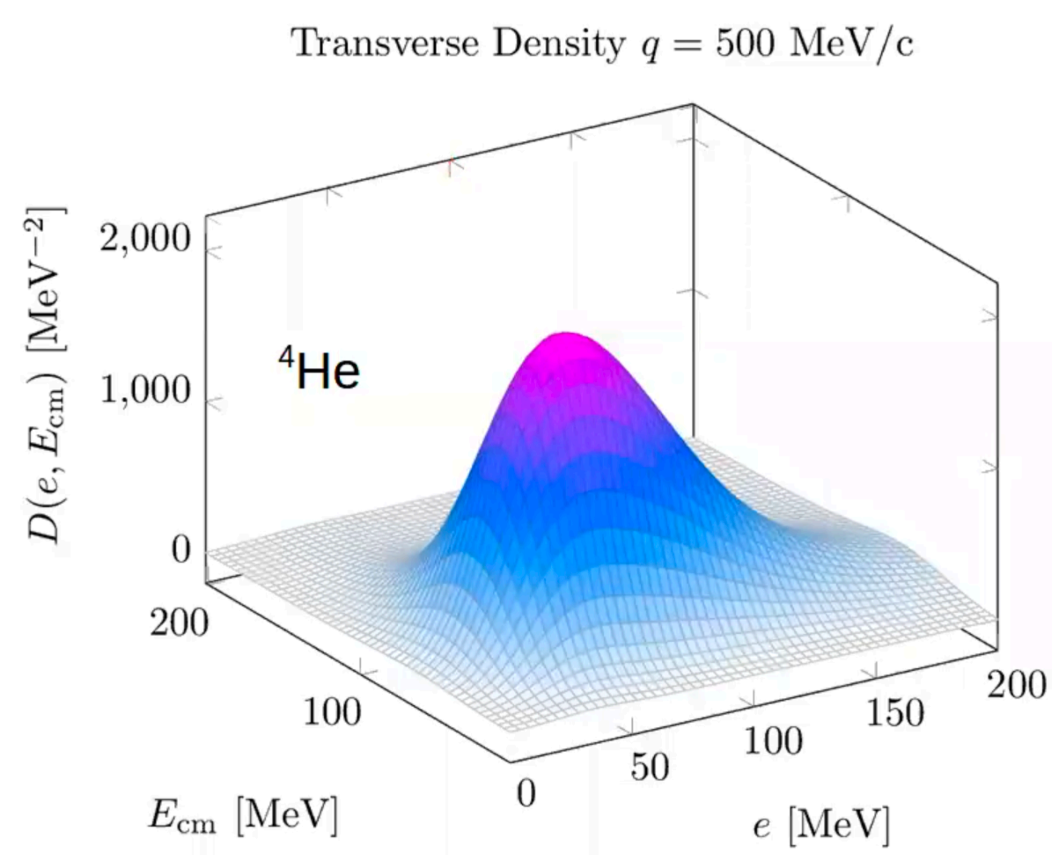
$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities as functions of (E,e)

$$R^{\rm STA}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Pastore et al. PRC101(2020)044612

L. Andreoli, NR, et al. PRC 105, 014002 (2022)



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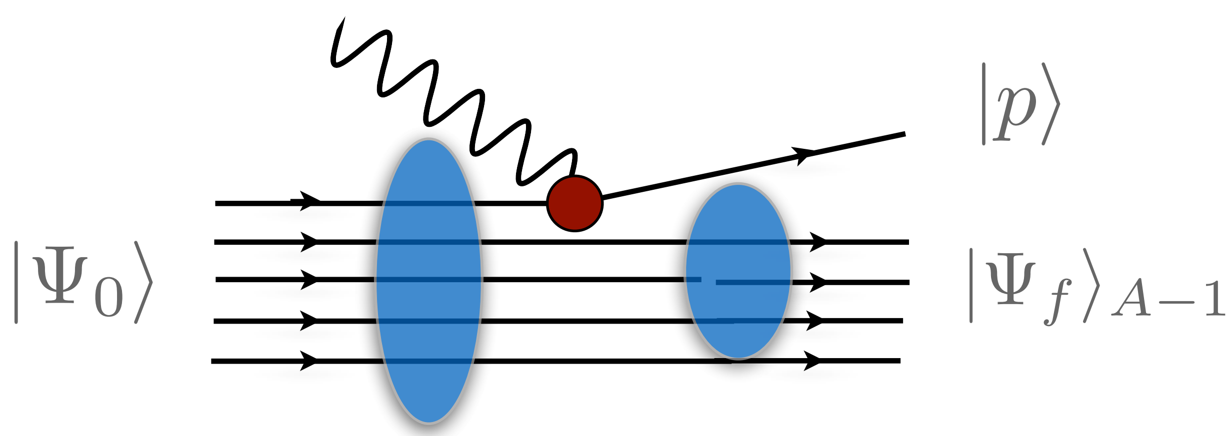
Spectral function approach

At large momentum transfer, the scattering reduces to the sum of individual terms

$$J_\alpha = \sum_i j^i_\alpha \qquad\qquad |\Psi_f\rangle \to |p\rangle \otimes |\Psi_f\rangle_{A-1}$$

The incoherent contribution of the one-body response reads

$$R_{\alpha\beta} \simeq \int \frac{d^3k}{(2\pi)^3} dE P_h(\mathbf{k}, E) \sum_i \langle k|{j^i_\alpha}^\dagger|k+q\rangle\langle k+q|j^i_\beta|k\rangle \delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$



The Spectral Function is the imaginary part of the two point Green's Function

Different many-body methods can be adopted to determine it

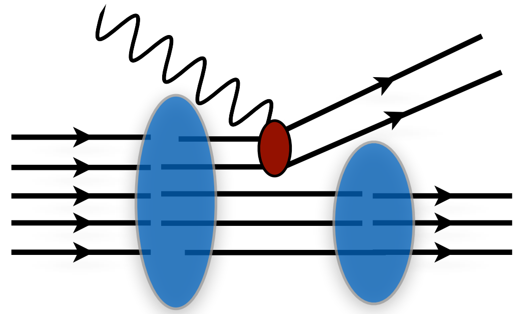
I. Korover, et al Phys.Rev.C 107 (2023) 6, L061301

L. Andreoli, NR, et al. PRC 105, 014002 (2022)

***NR,* Frontiers in Phys. 8 (2020) 116**

Spectral function approach

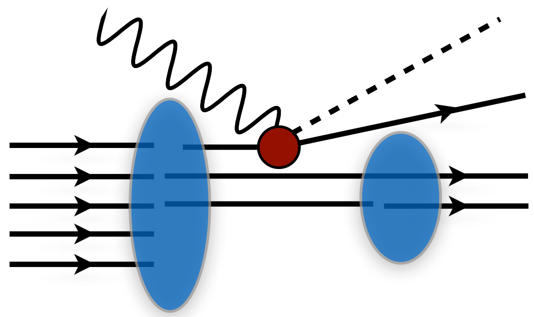
$$|f\rangle \rightarrow |pp'\rangle_a \otimes |f_{A-2}\rangle$$



The hadronic tensor for two-body current factorizes as

$$R^{\mu\nu}_{2b}(\mathbf{q},\omega) \propto \int dEd^3kd^3k' P_{2b}(\mathbf{k},\mathbf{k}',E) \times d^3pd^3p' |\langle kk'|j^\mu_{2b}|pp'\rangle|^2$$

$$|f\rangle \rightarrow |p_\pi p\rangle \otimes |f_{A-1}\rangle$$

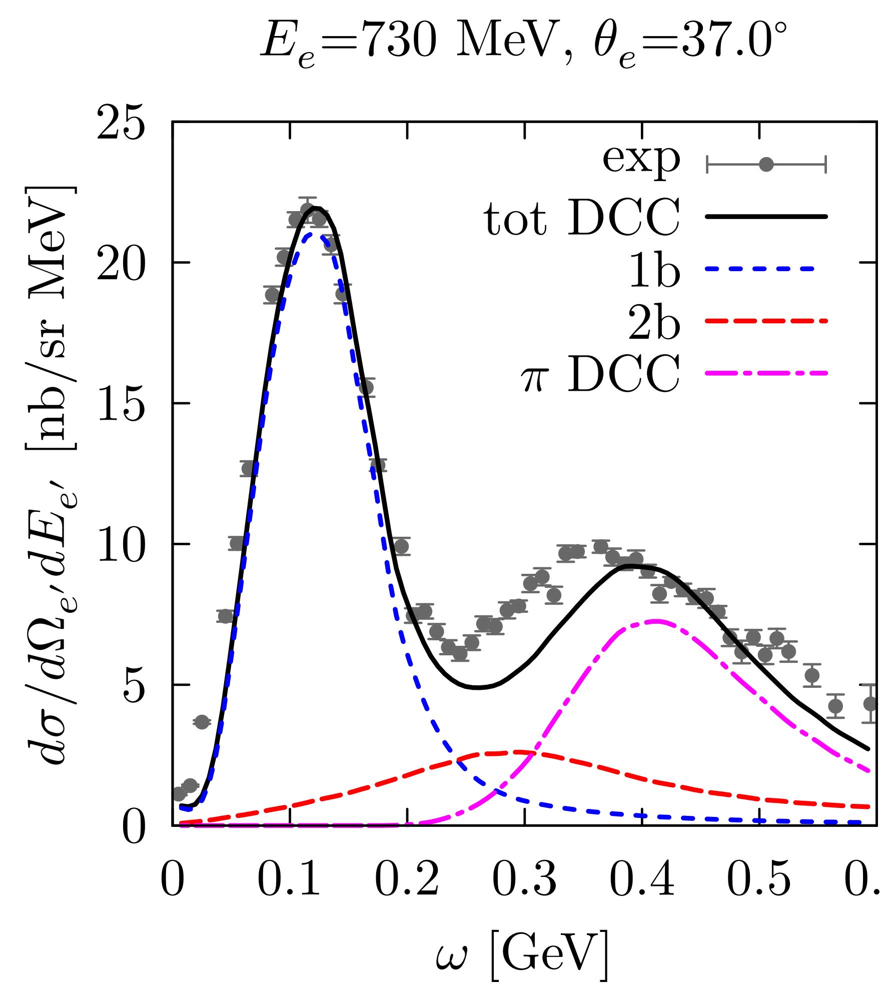


Production of real π in the final state

$$R^{\mu\nu}_{1b\pi}(\mathbf{q},\omega) \propto \int dEd^3kP_{1b}(\mathbf{k},E) \times d^3pd^3k_\pi |\langle k|j^\mu|pk_\pi\rangle|^2$$

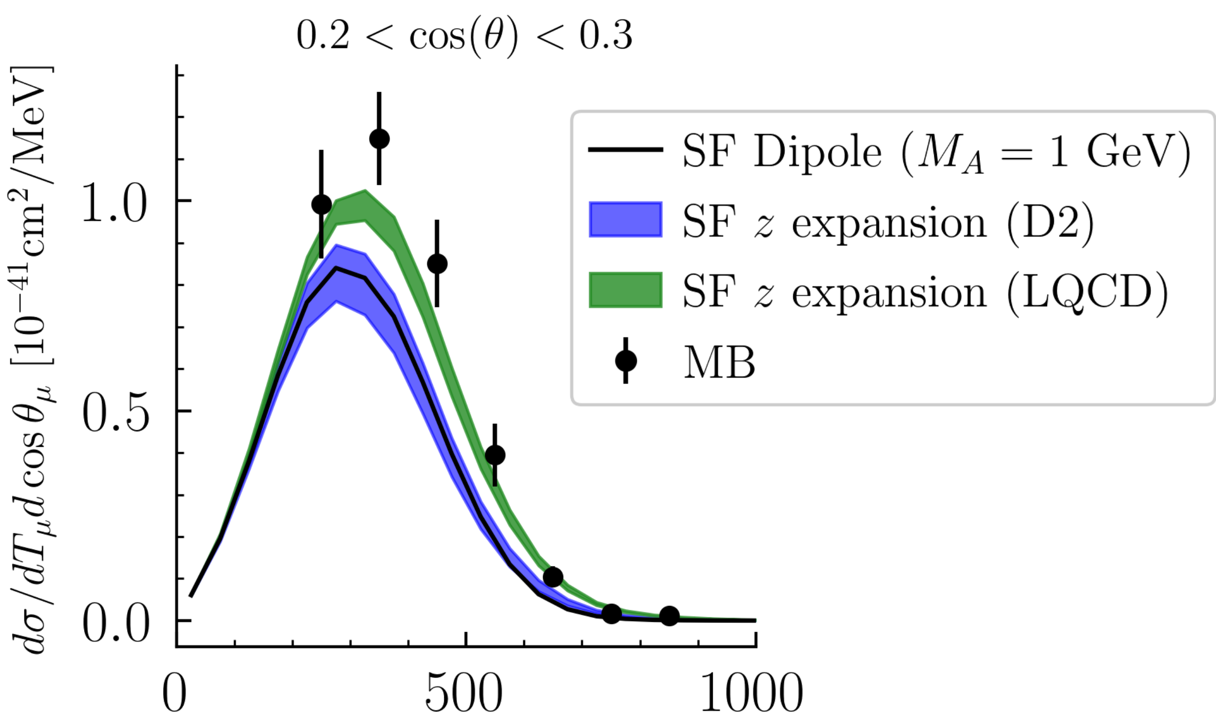
* Pion production elementary amplitudes currently derived within the extremely sophisticated **Dynamic Couple Chanel approach**;

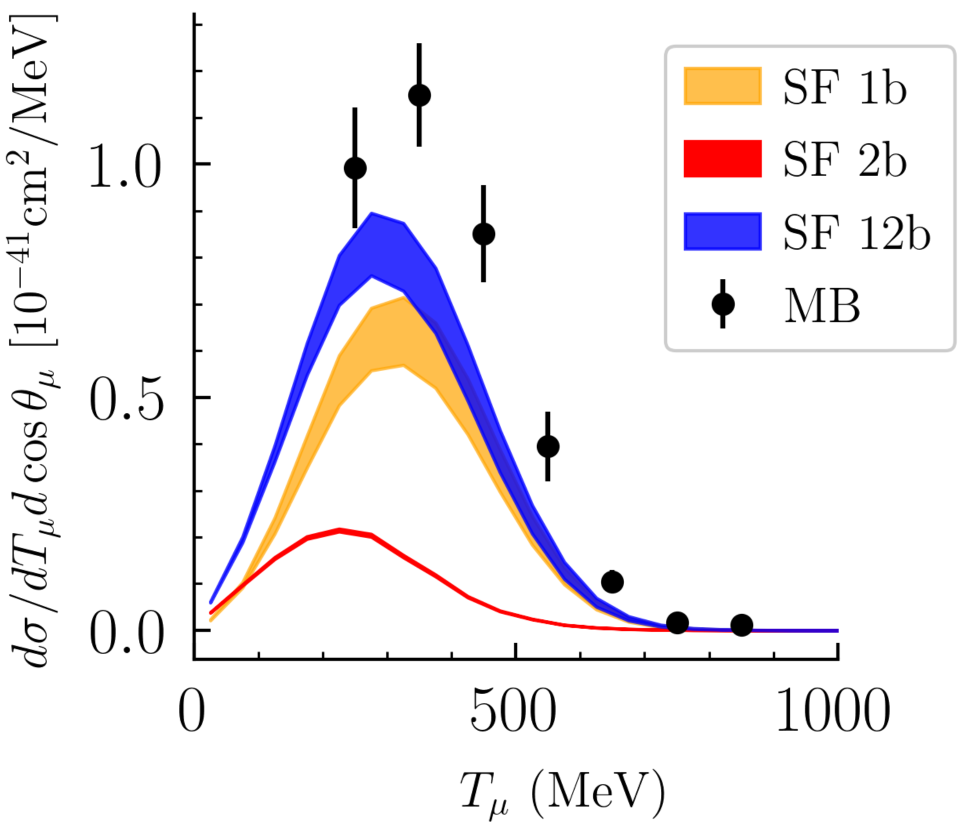
S.X.Nakamura, et al PRD92(2015)
T. Sato, et al PRC67(2003)



***NR*, Frontiers in Phys. 8 (2020) 116**

Axial Form Factors Uncertainty needs





D.Simons, N. Steinberg et al, 2210.02455

✳ Axial form factor dependence:

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$
SF Difference in $d\sigma_{\text{peak}}$ (%)	16.3

✳ Many-body method dependence:

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	22.8

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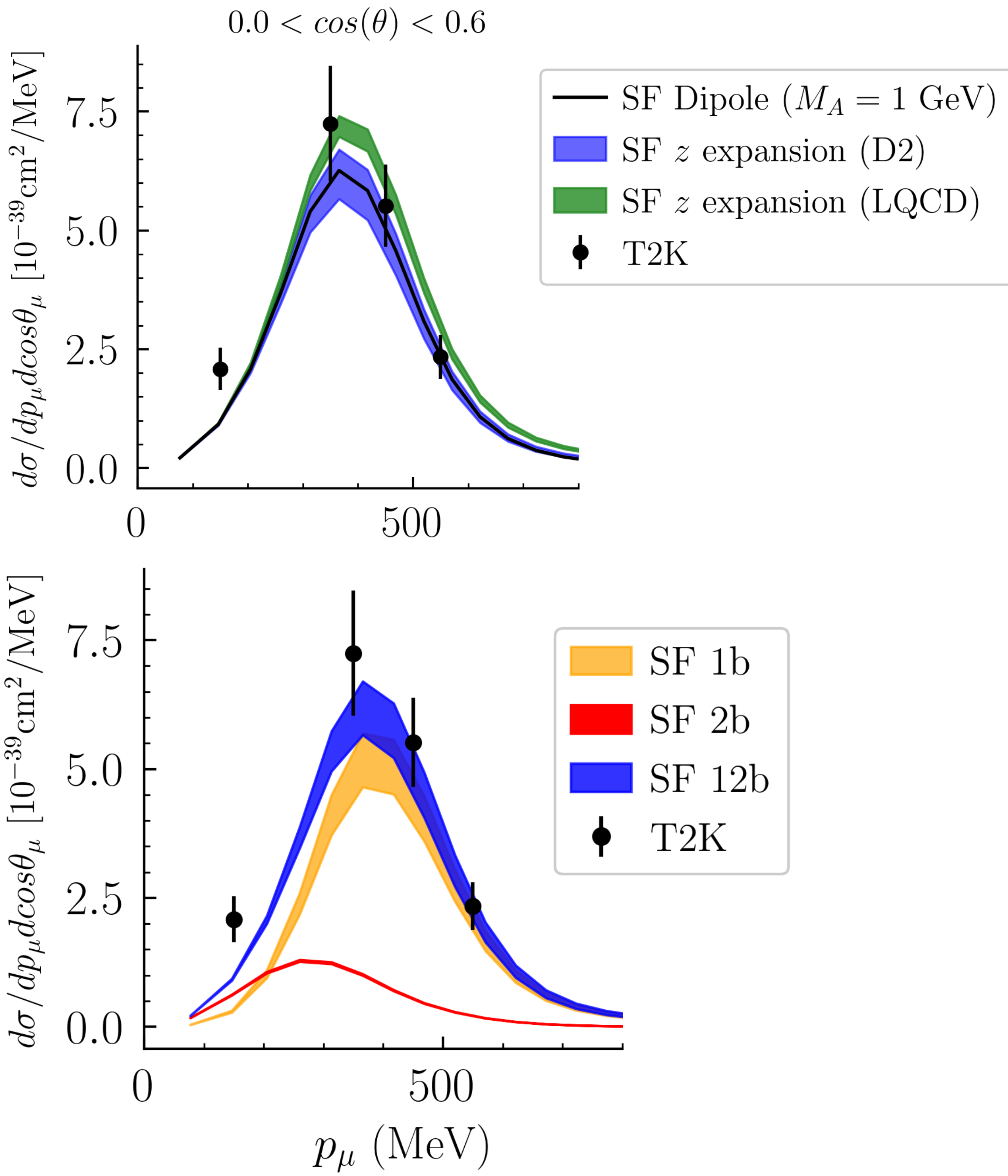
Axial Form Factors Uncertainty needs



D.Simons, N. Steinberg et al, 2210.02455

✳ Axial form factor dependence:

T2K	$0.0 < \cos\theta_\mu < 0.6$
SF difference in $d\sigma_{\text{peak}}$ (%)	15.3

✳ Many-body method dependence:

T2K	$0.0 < \cos\theta_\mu < 0.6$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	13.4

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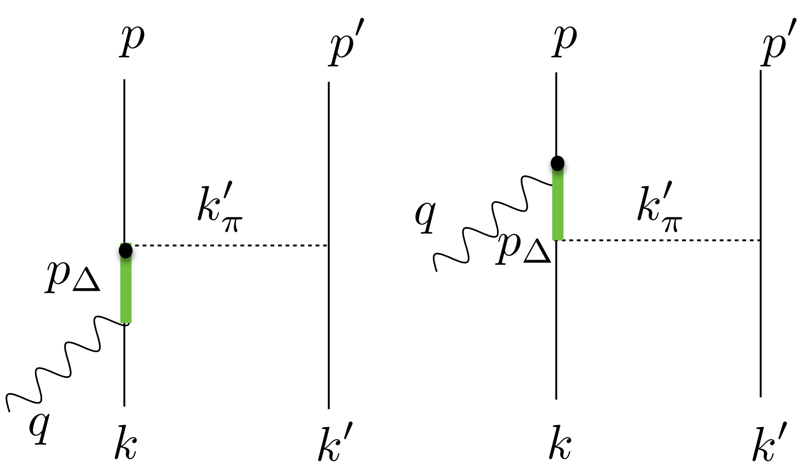
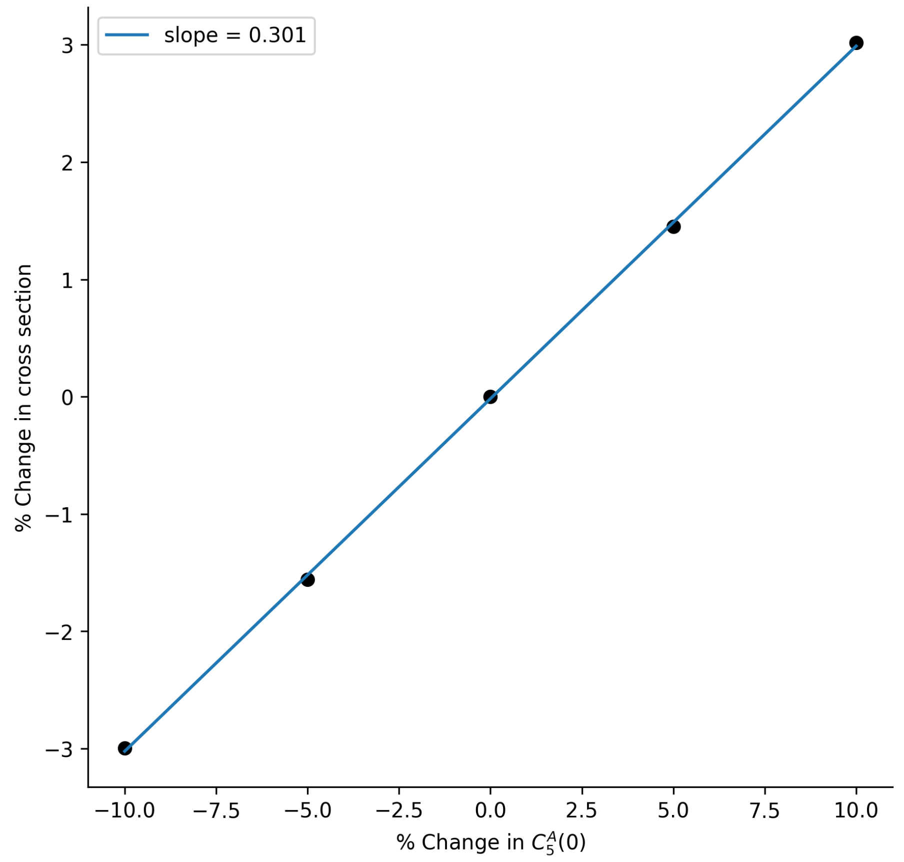
Resonance Uncertainty needs

The largest contributions to two-body currents arise from resonant $N \rightarrow \Delta$ transitions yielding pion production





D.Simons, N. Steinberg et al, 2210.02455

The normalization of the dominant $N \rightarrow \Delta$ transition form factor needs be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics

State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions)

Hernandez et al, PRD 81 (2010)

Further constraints on $N \rightarrow \Delta$ transition relevant for two-body currents and π production will be necessary to achieve few-percent cross-section precision

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Conclusions

- ✳ Assessing the overall uncertainty of theory calculations requires evaluating uncertainties:

  Nuclear Hamiltonians: different efforts in place to provide UQ in chiral EFT

  Form factors: one- and two-body currents, resonance/π production

  Error of factorizing the hard interaction vertex / using a non relativistic approach

- ✳ Address neutrino precision goals requires studying relations between cross section uncertainties and input parameter uncertainties

- ✳ Additional constraints on few-nucleon inputs from experiment and lattice QCD will be crucial

- ✳ Factorized approaches ideally suited to incorporate elementary amplitudes - nucleon hadron tensor

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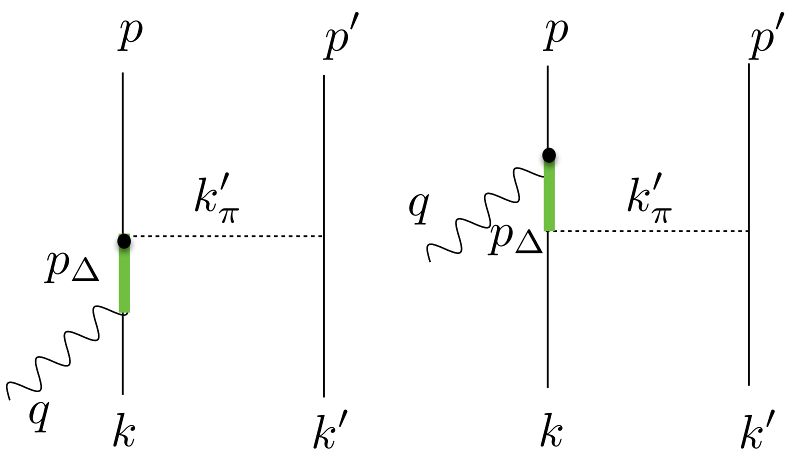
Thank you for your attention!

Delta contribution to MEC



Diagrams including the Delta current depend on many parameters.

$$(j_a^\mu)_A = (k'_\pi)^\alpha G_{\alpha\beta}(p_\Delta)\Big[\frac{C_3^A}{m_N}\Big(g^{\beta\mu}\not{q} - q^\beta\gamma^\mu\Big) + \frac{C_4^A}{m_N^2}\Big(g^{\beta\mu}q\cdot p_\Delta - q^\beta p_\Delta^\mu\Big) + C_5^A g^{\beta\mu} + \frac{C_6^A}{m_N^2}q^\mu q^\alpha\Big],$$

Parametrization chosen for the vector ff:

$$C_5^A = \frac{1.2}{(1-q^2/M_{A\Delta})^2} \times \frac{1}{1-q^2/(3M_{A\Delta})^2)},$$

Current extractions of C_A^5 (0) rely on single pion production data from deuterium bubble chamber experiments; estimated uncertainty ~ 15 %

Delta decay width:

$$\Gamma(p_\Delta) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2}\frac{|\mathbf{d}|^3}{\sqrt{s}}(m_N + E_d)R(\mathbf{r}^2)$$

$$R(\mathbf{r}^2) = \left(\frac{\Lambda_R^2}{\Lambda_R^2 - \mathbf{r}^2}\right)$$

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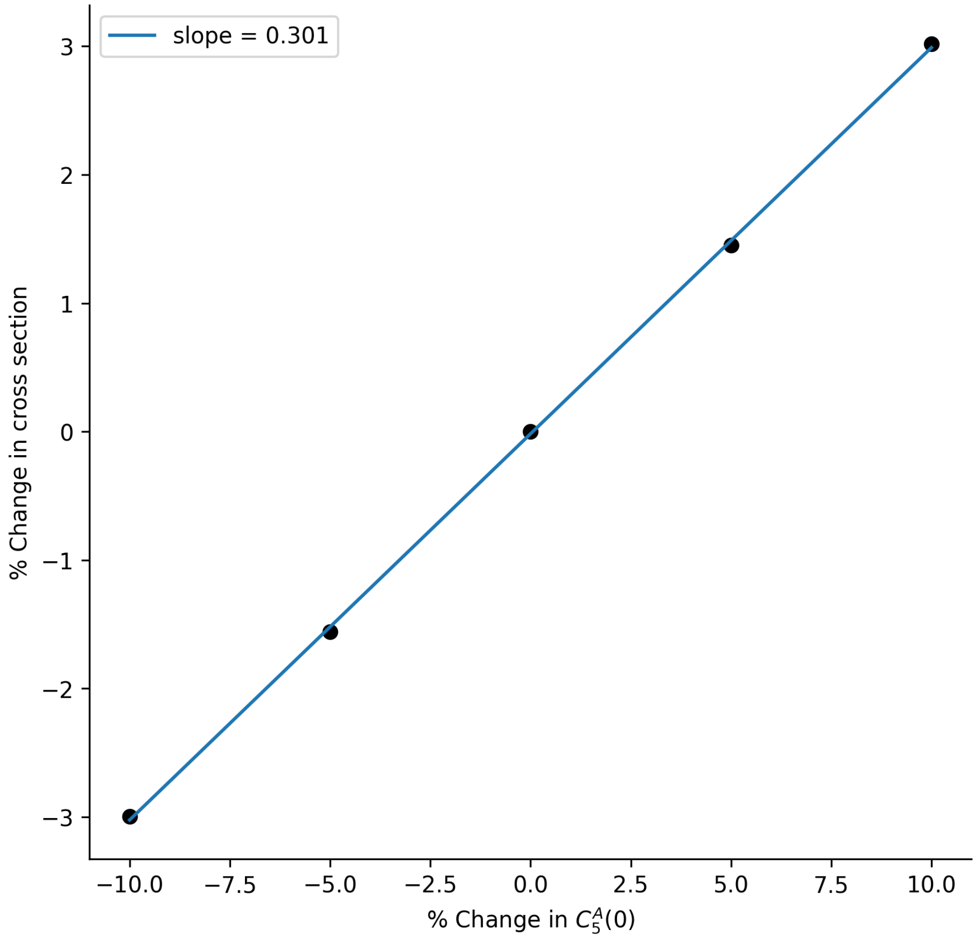
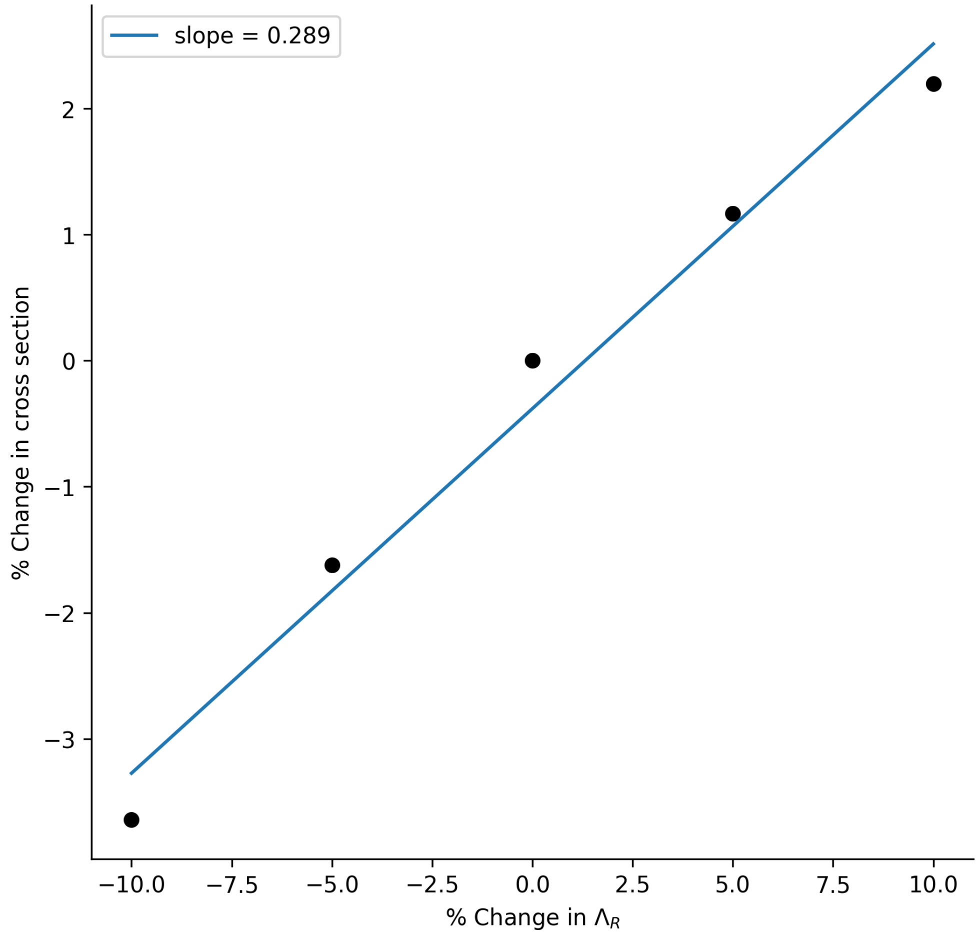
Study of model dependence in neutrino predictions

Percent change in the MiniBooNE cross section versus the percent change in the two-body current parameters for 0.5 < cos θμ < 0.6, Tμ = 325 MeV





A 15% variation in either $C_5^A(0)$ or Λ_R changes the flux-averaged cross section by roughly 5%

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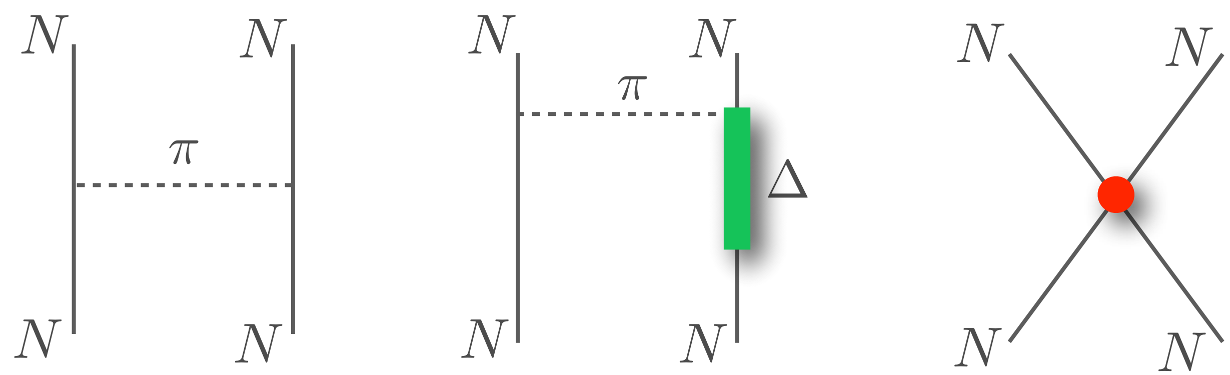
Phenomenological potential: av18 + IL7

Phenomenological potentials explicitly include the **long-range one-pion exchange interaction** and a set of **intermediate- and short-range phenomenological terms**

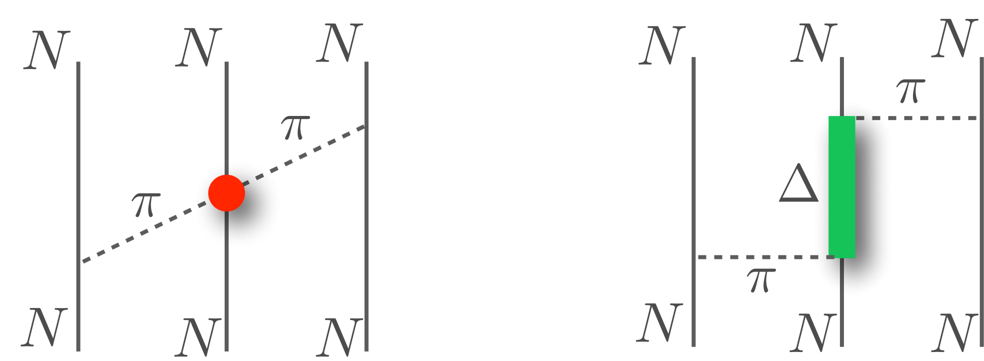
- **Argonne v18** is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O_{ij}^p$$



- Phenomenological three-nucleon interactions, like the **Illinois 7**, effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects

$$V_{ijk}^{3\mathrm{N}} = A_{2\pi}^{PW} O_{ijk}^{2\pi,PW} + A_{2\pi}^{SW} O_{ijk}^{2\pi,SW} + A_{3\pi}^{\Delta R} O_{ijk}^{3\pi,\Delta R} + A^R O_{ijk}^R$$



The parameters of the AV18 + IL7 are fit to properties of **exactly solvable light nuclear systems**.

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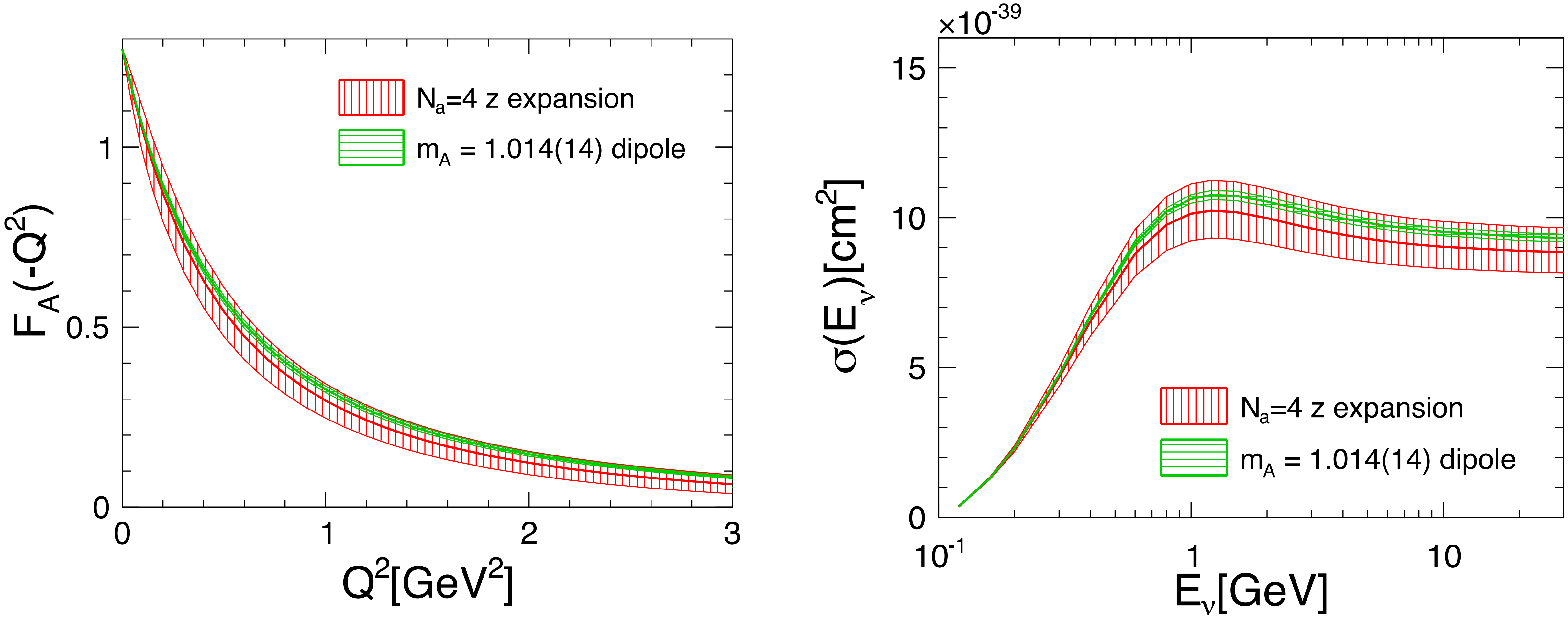
Neutrino-Nucleon scattering

- Sum rule can be enforced ensuring that the form factor falls smoothly to zero at large Q^2

$$\sum_{k=n}^{\infty} k(k-1)\cdots(k-n+1)a_k = 0\,, \quad n = 0, 1, 2, 3$$

Fit deuteron data replacing dipole axial form factor with z-expansion, enforce the sum rule constraints



A.S.Meyer, Phys.Rev.D 93 (2016) 11, 113015