2023 LATTICE

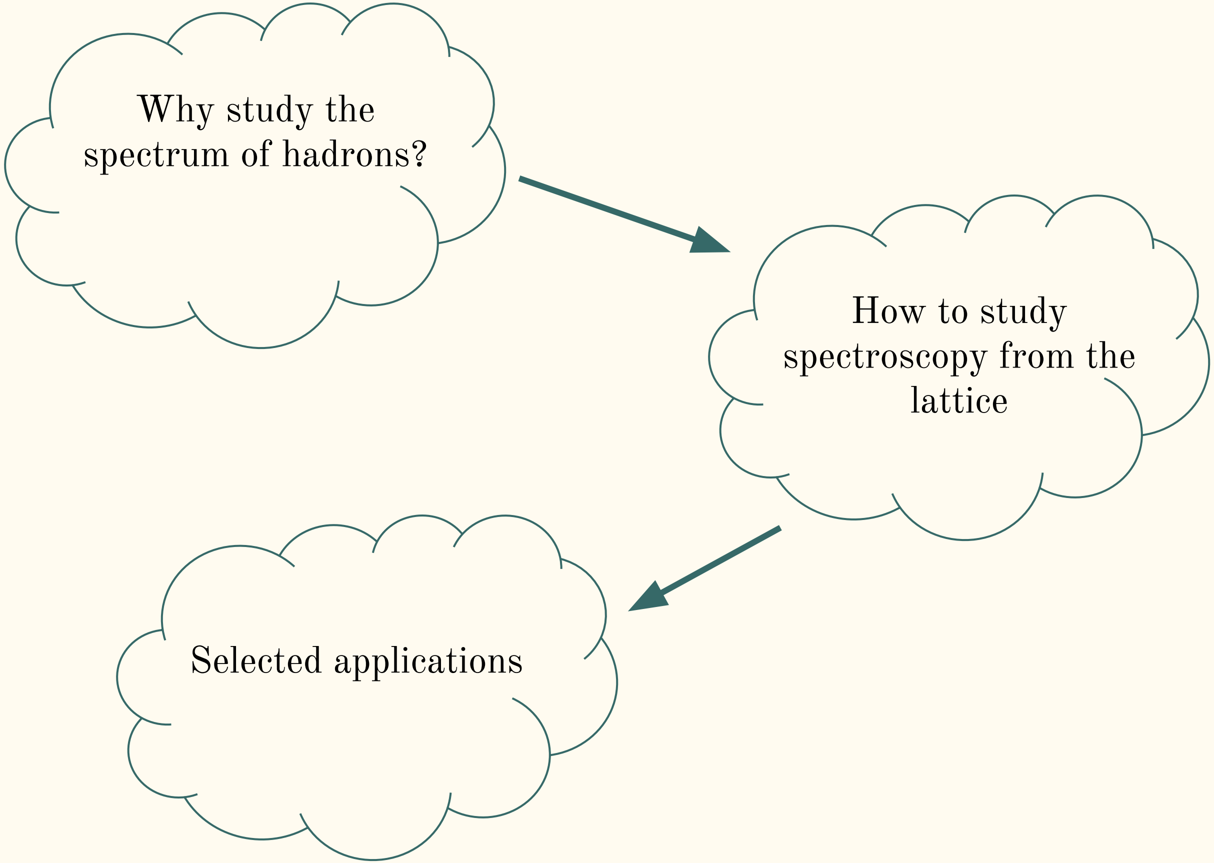
Hadron spectroscopy and few-body dynamics

Andrew Hanlon
Brookhaven National Laboratory

The 40th Lattice Conference
Fermilab
Aug. 3, 2023

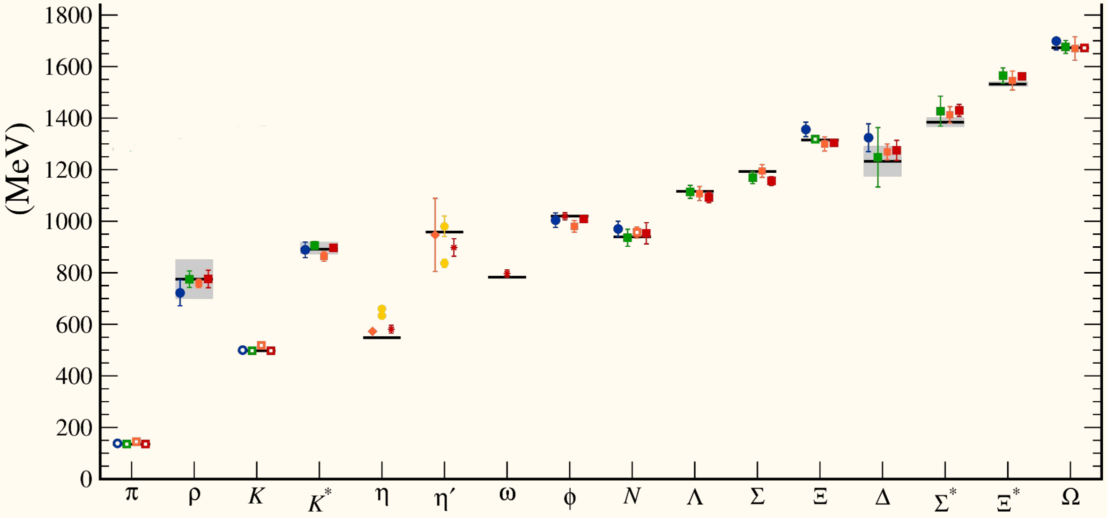
Brookhaven National Laboratory

Outline


Why study the spectrum of hadrons?
How to study spectroscopy from the lattice
Selected applications

Spectroscopy from the lattice

Spectrum of QCD-'stable' states agrees well with experiment



[A. Kronfeld 1203.1204]

<u>However</u>:

- Bound states near threshold can have significant finite-volume effects

- Further, most hadrons are *resonances*, i.e. unstable states

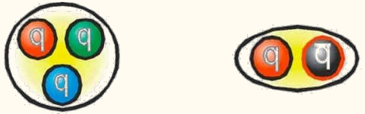
<u>How to proceed?</u>:

- Robust extraction of resonance information from multi-hadron states on the lattice can be (and has been) achieved!
  [Lüscher '86, '91; generalizations]

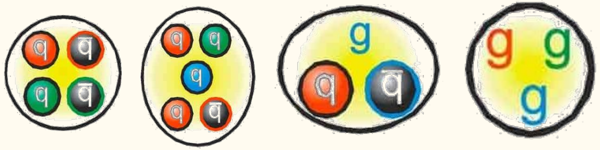
The limits of the quark model

The quark model was mostly successful at describing a wide range of the observed resonances, but is clearly incomplete

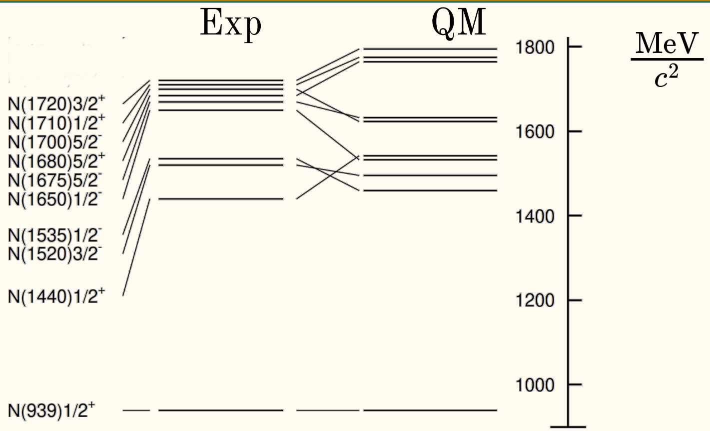
- It was puzzling that up until ~ 2000, all hadrons could be described by conventional baryons/mesons



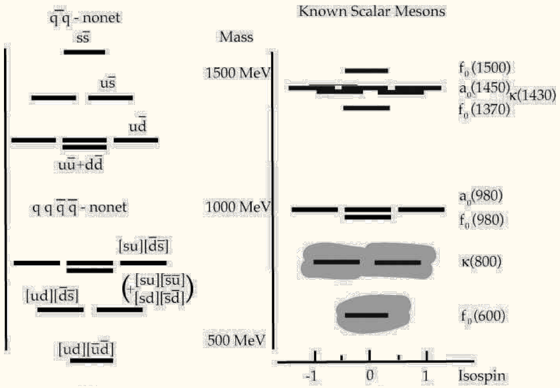
- Exotic hadrons possible



- However, many questions
    - why is the Roper, N(1440) $1/2^{+}$, so light?
    - why is the $\Lambda(1405)$ lighter than its nucleon counterpart, i.e. the N(1535) $1/2^{-}$?
    - is the nonet of light scalar mesons better described as tetraquarks?
    - and lots more...



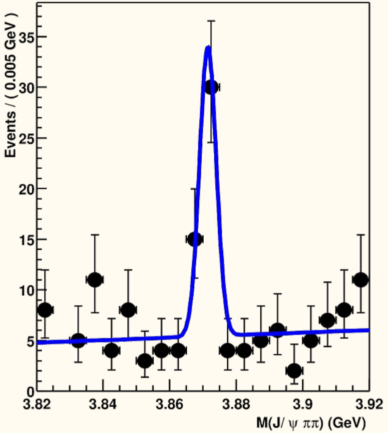
[Particle Data Group, *PTEP* 2022 083C01]



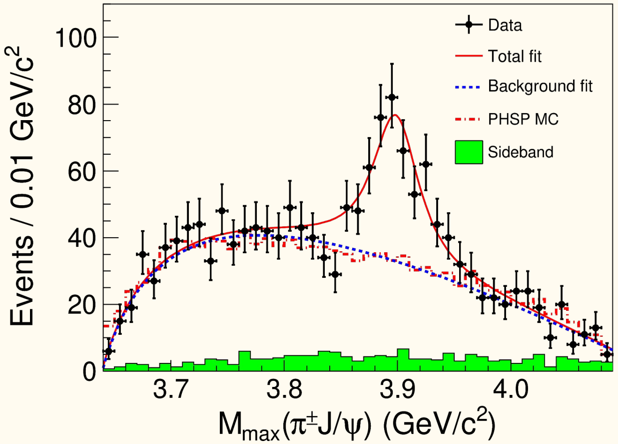
[R.L. Jaffe hep-ph/0409065]

Exotic candidates

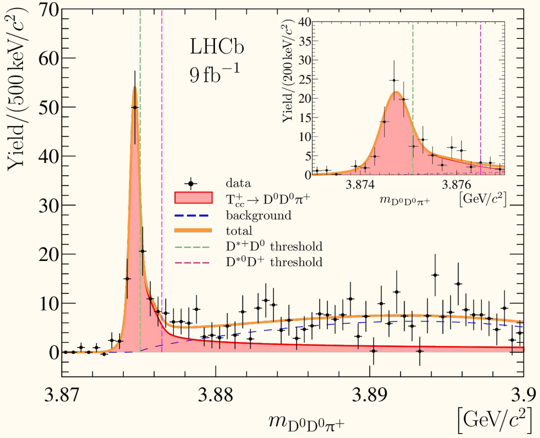
Since the early 2000s, several resonances that do not fit in the quark model have appeared



[Belle hep-ex/0309032]



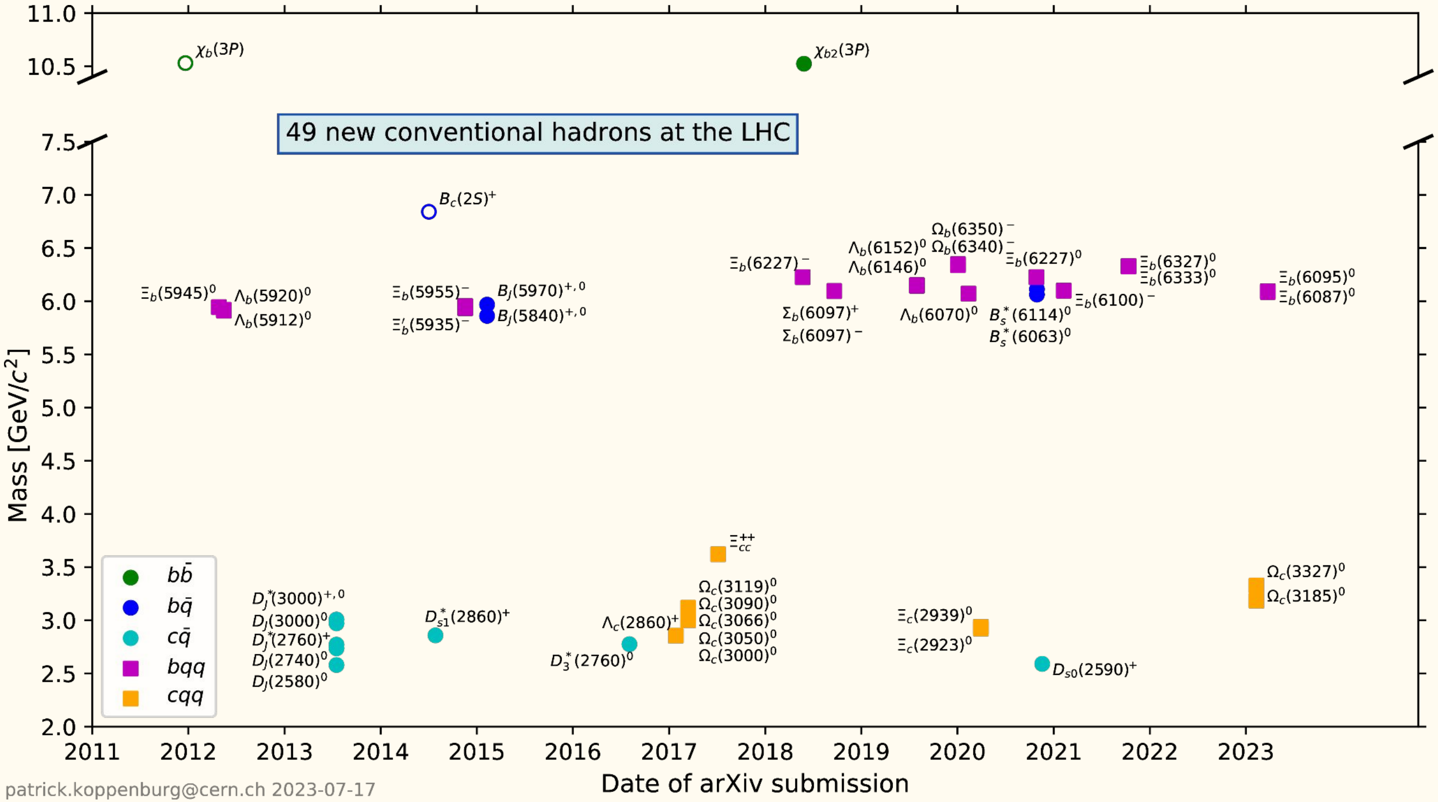
[BESIII 1303.5949]



[LHCb 2109.01056]

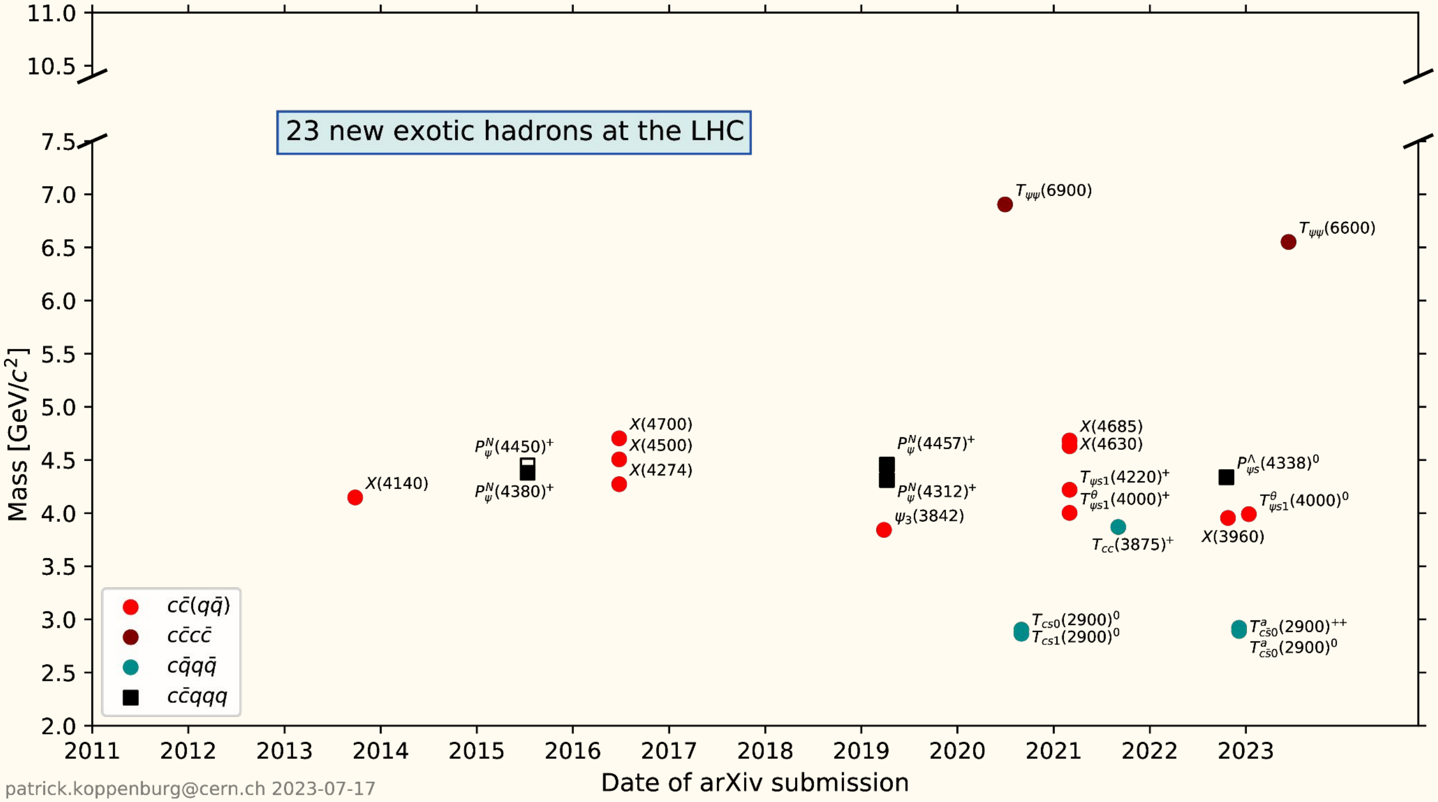
- $X(3872)$ - could be the $\chi_{c1}(2P)$, but very different from expected behavior
- $Z_c(3900)^+$ - charged charmonium-like state, must be more than $c\bar{c}$
- $T_{cc}(3875)^+$ - two charm and overall integer spin, must be exotic to be a color singlet

New hadrons at the LHC



[Image from https://www.nikhef.nl/~pkoppenb/particles.html]

New hadrons at the LHC



[Image from https://www.nikhef.nl/~pkoppenb/particles.html]

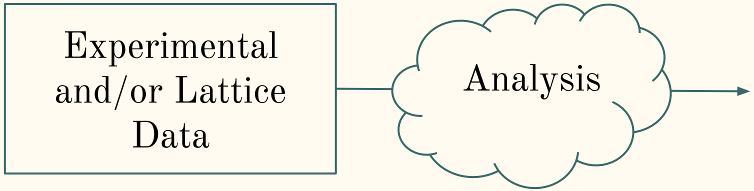
Extracting resonances from data

- Resonances appear as poles *off* the real-axis on the *unphysical* Riemann sheets of T_{fi}

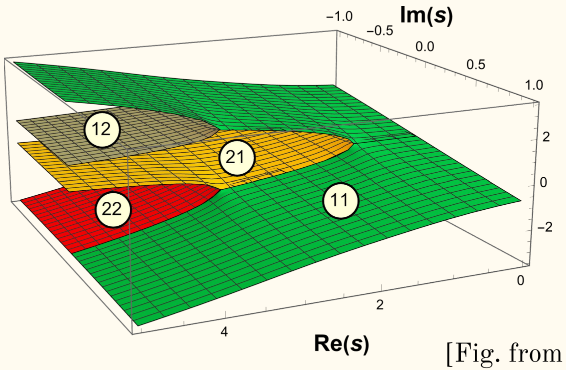
$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4 (p^f - p^i) T_{fi}$$

- Each scattering threshold doubles the number of Riemann sheets in which the scattering amplitude lives

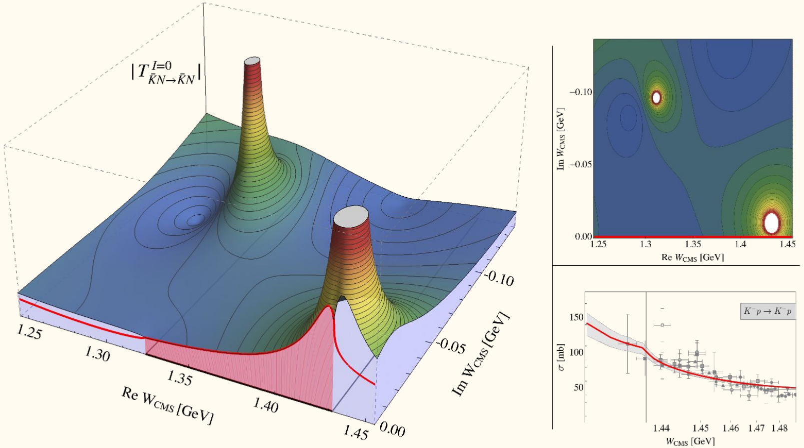
- Must infer poles from data on real axis



Goal: Reliably determine resonance properties by robustly extracting pole position and residue!



[Fig. from 2212.07856]



[M. Mai 2010.00056]

Lüscher two-particle formalism

Compact formula for quantization condition

$$\det\left[F(E_2,\mathbf{P},L)^{-1}+\mathcal{K}_2(E_2^*)\right]=0$$

E - finite-volume energies

$\mathcal{K}_2$- 2-to-2 K-matrix

F - known geometric function

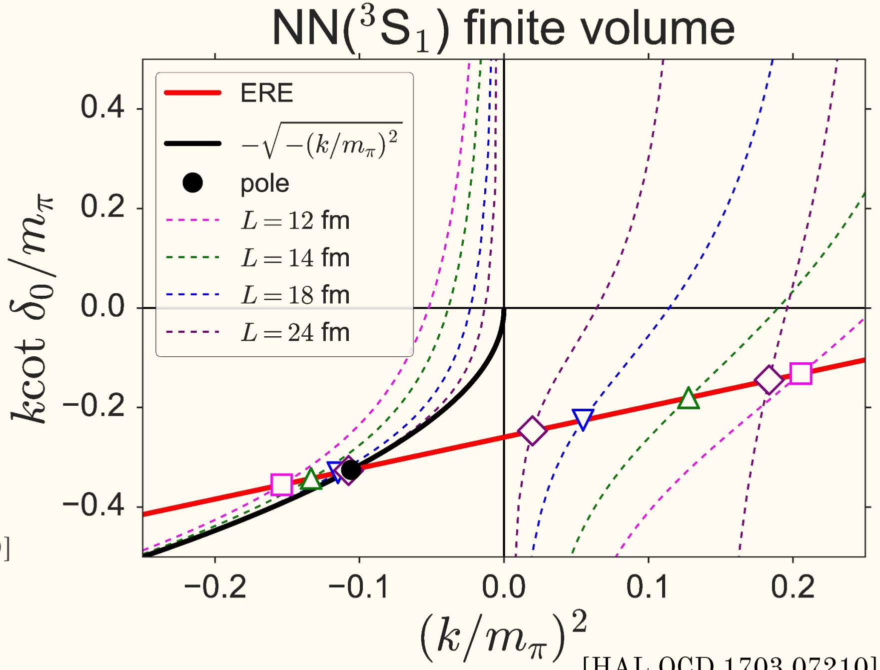
Caveats:

- truncated at some max ℓ
- only valid above left-hand cut and below 3 (or 4) particle threshold
- assumes continuum energies
- ignores exponentially small contributions

Alternatives:

- HAL QCD potential method [S. Aoki & T. Doi 2003.10730]
- Spectral functions from Euclidean correlators [J. Bulava & M. Hansen 1903.11735]

Truncated at $\ell_{\max}=0 \Rightarrow$ one-to-one mapping



[HAL QCD 1703.07210]

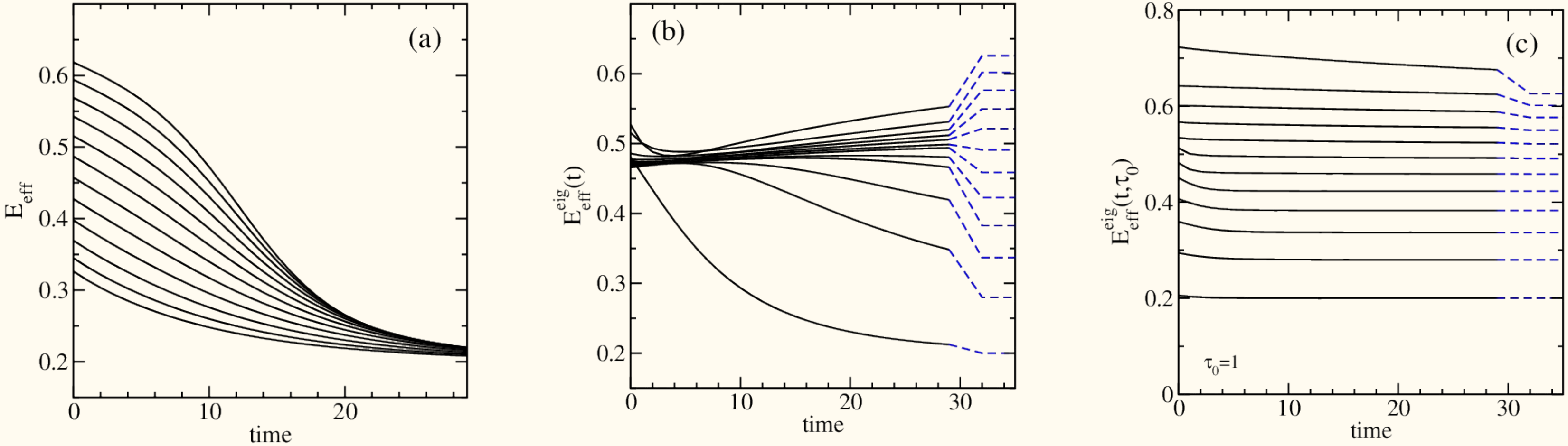
Finite-volume spectrum from correlator matrix

[Lüscher & Wolff *Nucl.Phys.B* 339, 222 (1990)]
[ALPHA 0902.1265]

$$C_{ij}(t) = \langle \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0) \rangle = \sum_{n=0}^{\infty} Z_i^{(n)}\, Z_j^{(n)*}\, e^{-E_n t}\,, \quad Z_j^{(n)} = \langle 0 | O_j | n \rangle$$

<u>Toy Model:</u>

$$E_n = E_{n-1} + \frac{0.08}{\sqrt{n}}, \quad n = 1, 2, \ldots, 199\,, \quad E_0 = 0.20, \quad Z_j^{(n)} = \frac{(-1)^{j+n}}{1 + 0.05(j-n)^2}$$



Diagonal elements of $C(t)$ — Eigenvalues of $C(t)$ — Generalized eigenvalues of $C(t)$

[Plots courtesy of Colin Morningstar]

The $\Lambda(1405)$ - Two poles or one?

An $I=0$, $J^P = 1/2^-$ resonance, with possible lower mass partner $\Lambda(1380)$ not predicted by quark model

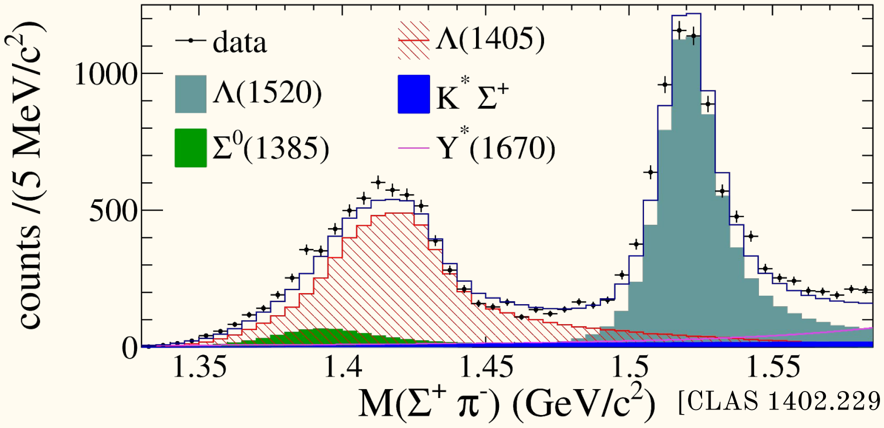
Experiment tends to be mixed regarding this

- CLAS, BGOOD, ALICE, GlueX suggest two poles
- J-PARC consistent with one pole
- Global analysis says one
  [Anisovich et al, EPJA 2020]

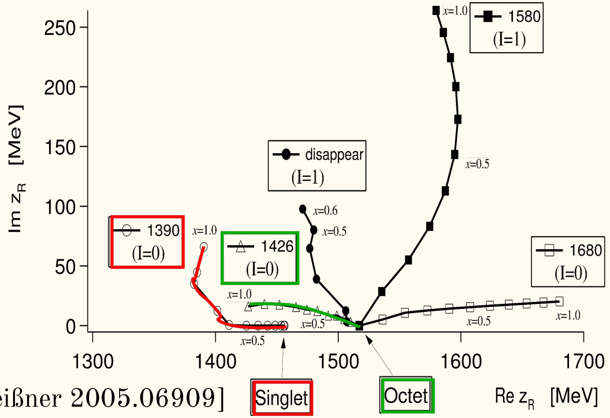
Unitarized Chiral EFT predicts two poles

Reference	high-mass pole [MeV]	low-mass pole [MeV]
arXiv:1109.3005 & 1201.6549	$1424^{+7}_{-23} - i26^{+3}_{-14}$	$1381^{+18}_{-6} - i81^{+19}_{-8}$
arXiv:1210.3485	$1421^{+3}_{-2} - i19^{+8}_{-5}$	$1388^{+9}_{-9} - i114^{+24}_{-25}$
arXiv:1411.7884, sol. #2	$1434^{+2}_{-2} - i\,10^{+2}_{-1}$	$1330^{+4}_{-5} - i\,56^{+17}_{-11}$
arXiv:1411.7884, sol. #4	$1429^{+8}_{-7} - i\,12^{+2}_{-3}$	$1325^{+15}_{-15} - i\,90^{+12}_{-18}$

[Particle Data Group, *Chpt.* 83, PTEP 2022 083C01]



[CLAS 1402.2296]



[Meißner 2005.06909]

What does QCD on the lattice say?

Recent determination of the $I=0$ coupled channel $\pi\Sigma$-$\bar{K}N$ scattering amplitudes with $m_\pi \sim 200$ MeV

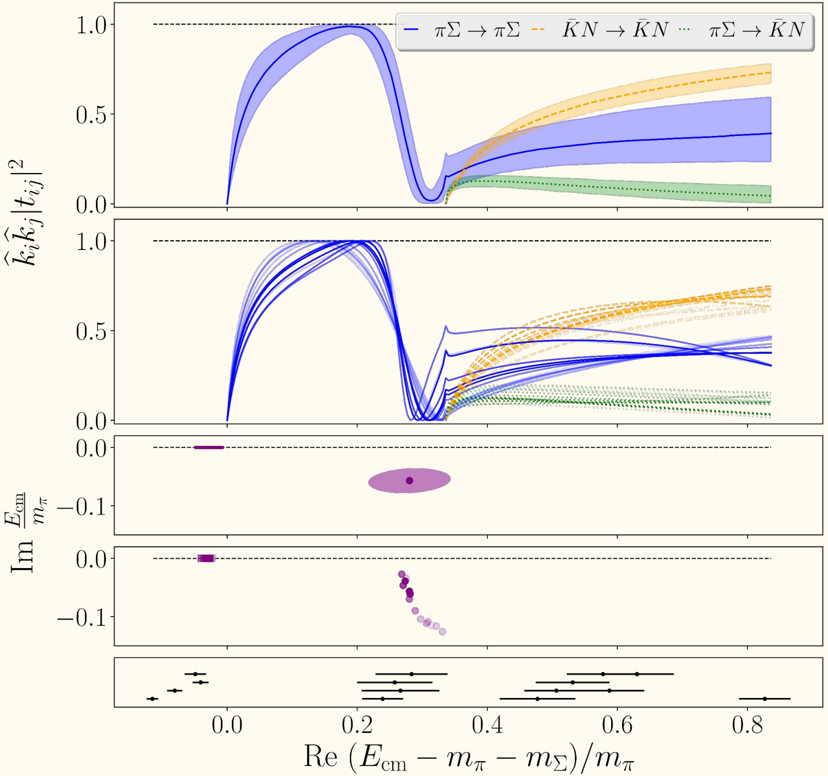
Several parameterizations considered, with effective-range expansion as preferred fit

$$\frac{E_{\rm cm}}{m_\pi}\widetilde{K}_{ij} = A_{ij} + B_{ij}\Delta_{\pi\Sigma}$$

with $B_{00} = B_{11} = 0$, and where $\Delta_{\pi\Sigma} = \frac{E_{\rm cm}^2-(m_\pi+m_\Sigma)^2}{(m_\pi+m_\Sigma)^2}$

Final estimate for pole positions consistent with results from UChPT

$$\begin{aligned} E_1 &= 1392(9)_{\rm stat}(2)_{\rm model}(16)_a \text{ MeV}, \\ E_2 &= [1455(13)_{\rm stat}(2)_{\rm model}(17)_a \\ &\quad - i \times 11.5(4.4)_{\rm stat}(4)_{\rm model}(0.1)_a] \text{ MeV}. \end{aligned}$$



[J. Bulava, ..., **ADH** *et al.* 2307.10413, 2307.13471]

See talks by:

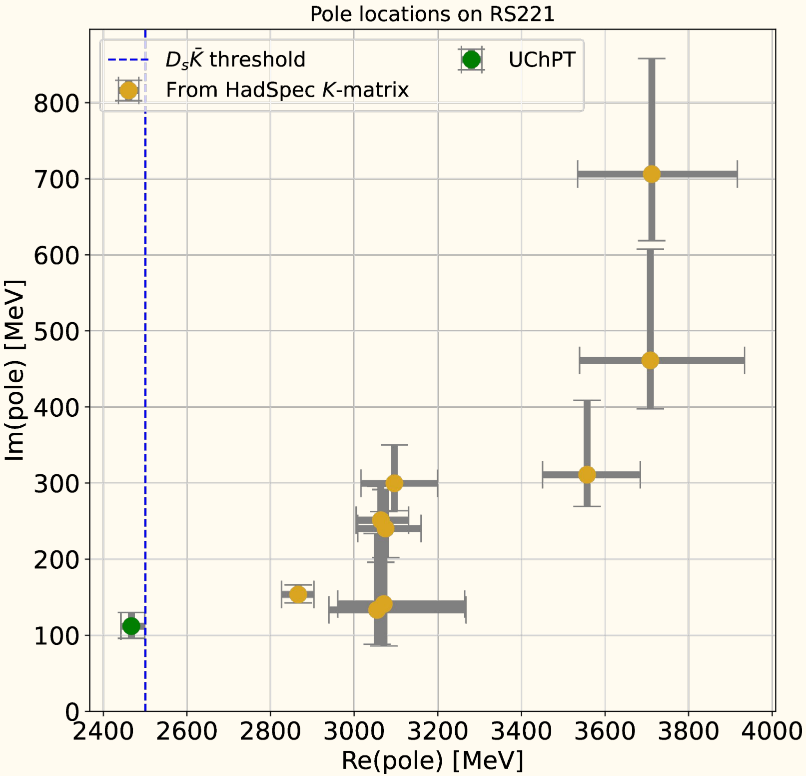
- Fernando Romero-López (this work), Jul. 31 at 5:00 pm
- Kotaro Murakami (HAL QCD), Jul. 31 at 5:20 pm

Two-pole structure for other resonances?

Pole extractions can be unstable if located on sheets not directly connected to the physical one

Lattice results for coupled $D\pi$-$D\eta$-$D_s\bar{K}$ showed strong dependence for second $D_0^*(2300)$ resonance pole on the scattering amplitude parameterization, thus was considered 'unreliable'

Poles with a small residue close to threshold look similar to poles with a large residue far from threshold



[A. Asokan *et al.* 2212.07856]

[Data: HadSpec 1607.07093]

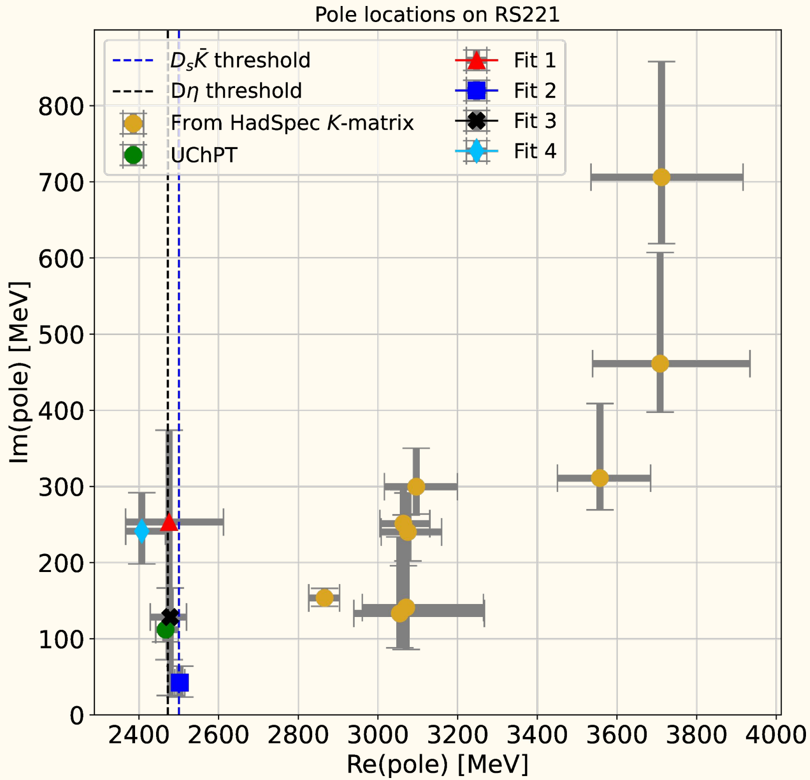
Two-pole structure for other resonances?

Pole extractions can be unstable if located on sheets not directly connected to the physical one

Lattice results for coupled $D\pi$-$D\eta$-$D_s\bar{K}$ showed strong dependence for second $D_0^*(2300)$ resonance pole on the scattering amplitude parameterization, thus was considered ‘unreliable’

Poles with a small residue close to threshold look similar to poles with a large residue far from threshold

Using constraints from ChPT for the parameterizations, stable pole positions could be extracted!



[A. Asokan *et al.* 2212.07856]

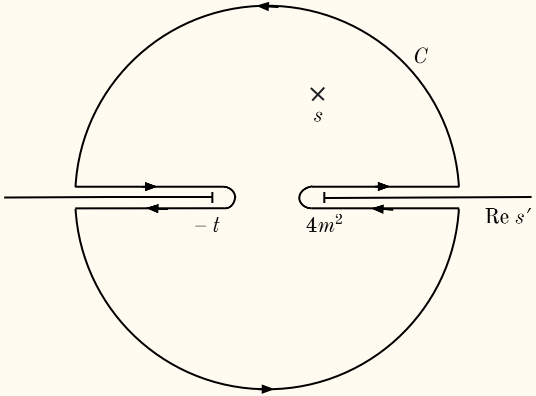
[Data: HadSpec 1607.07093]

See related talk by Haobo Yan ($D^{(*)}\pi$ scattering), Aug. 1 at 2;50 pm

General structure of the scattering amplitude

In cases where the resonance pole lies deep in the complex plane, the full structure should be taken into account

- Unitarity ($SS^\dagger = 1$, K-matrix enforces this)
- Crossing (relation between scattering channels)
- Analyticity in Mandelstam variables (from causality)

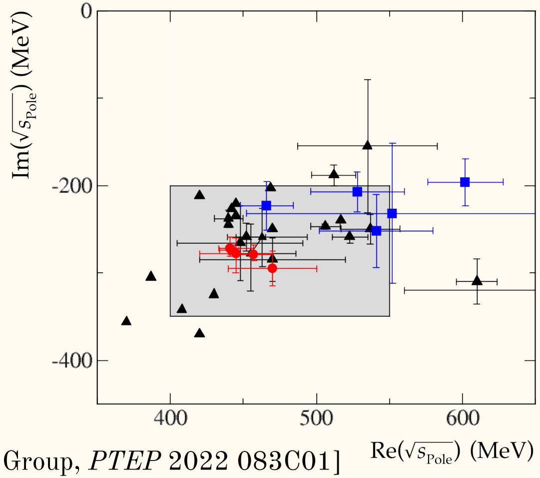


Analytic properties can be imposed via dispersion relations

$$T(s,t,u) = \underbrace{\frac{1}{\pi}\int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\, T(s',t,u)}{s'-(s+i\varepsilon)}}_{\text{threshold cut}} + \underbrace{\frac{1}{\pi}\int_{-\infty}^{-t} ds' \frac{\mathrm{Im}\, T(s',t,u)}{s'-(s+i\varepsilon)}}_{\text{left-hand cut}}$$

*May require subtractions in order to ignore contour at ∞

Leads to much more stable pole determination! (red points)



[Particle Data Group, *PTEP* 2022 083C01]

σ resonance from lattice with dispersion relations

Dispersion relation techniques can also be used with lattice data

In general, the partial-wave projected amplitudes

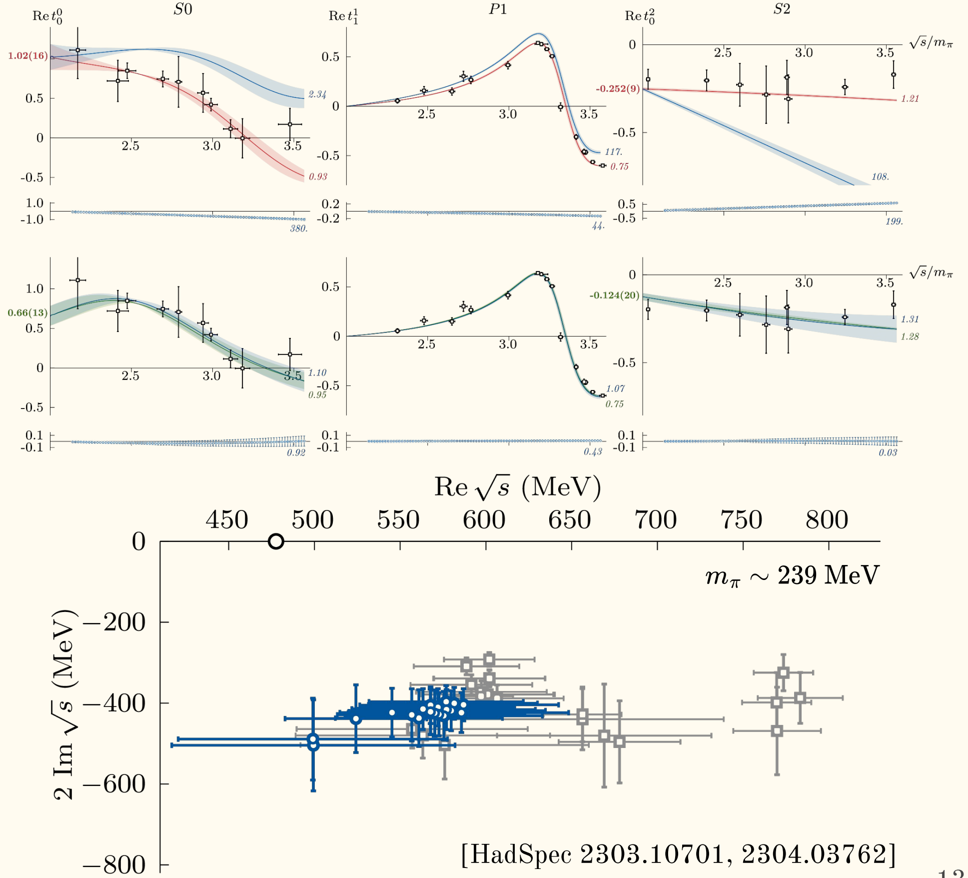
$$t_\ell(s) = \frac{1}{64\pi} \int \mathrm{d}\cos\theta_s \, T(s,t,u) \, P_\ell(\cos\theta_s)$$

constrained from lattice data do not respect crossing symmetry. However, those that do will respect the dispersion relation

$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s) \, \mathrm{Im}\, t^{I'}_{\ell'}(s') \,,$$

i.e. they will result in $\tilde{t}^I_\ell(s) = t^I_\ell(s)$

Again leads to more stable pole positions!



[HadSpec 2303.10701, 2304.03762]

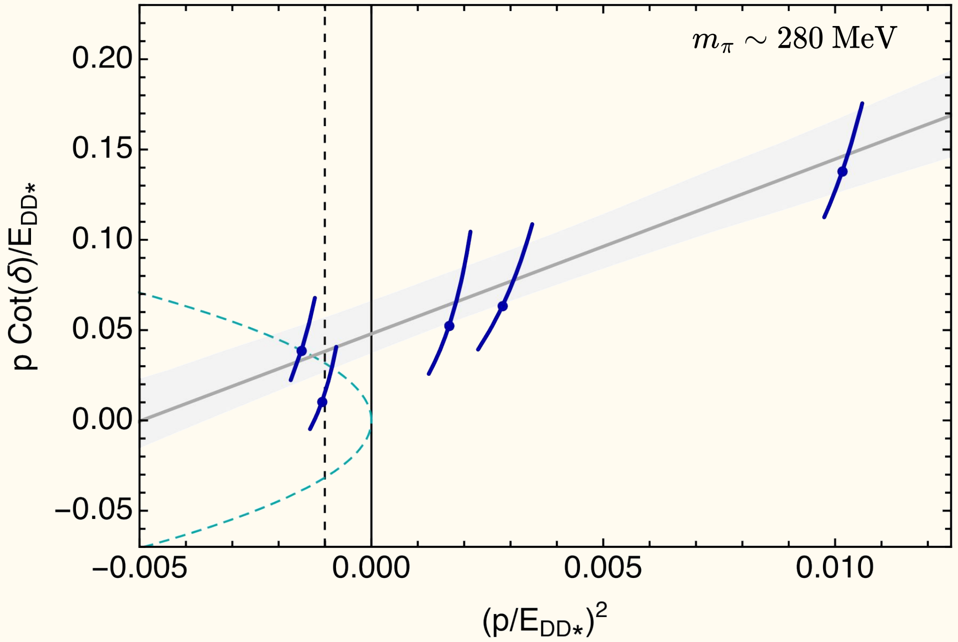
Speaking of left-hand cuts...

The T_{cc}^+ has been studied from the lattice using pion masses in which the D^*-meson is stable

Lüscher quantization condition not valid on the left-hand cut

- For DD^* system, this cut can lie close to threshold, spoiling the conclusions drawn

Work towards extending the quantization condition is underway [Raposo and Hansen 2301.03981]



[M. Padmanath & S. Prelovsek 2202.10110]

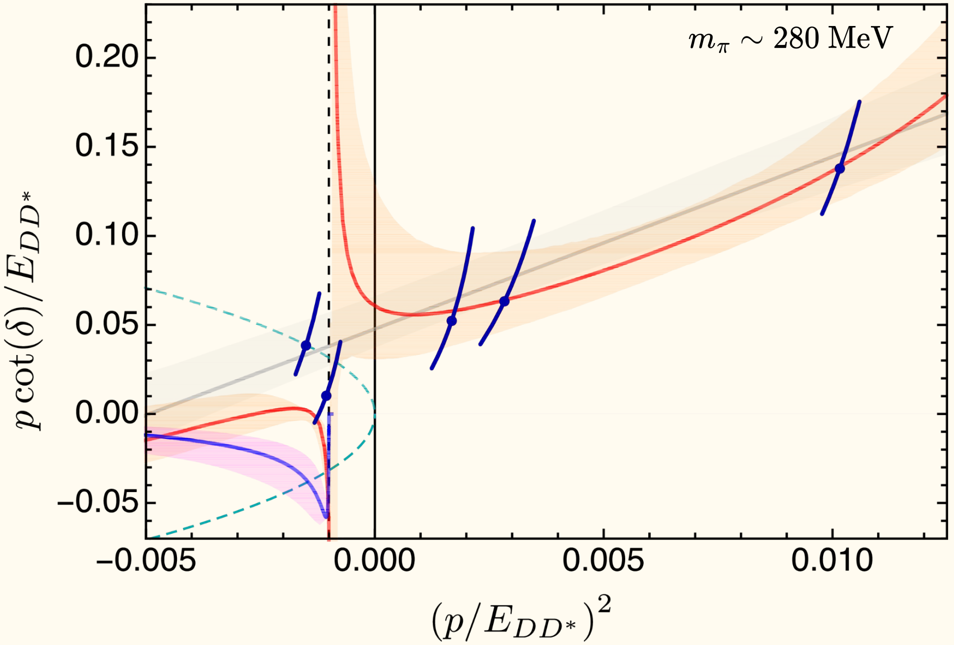
Speaking of left-hand cuts...

The T_{cc}^{+} has been studied from the lattice using pion masses in which the D^{*}-meson is stable

Lüscher quantization condition not valid on the left-hand cut

- For DD^{*} system, this cut can lie close to threshold, spoiling the conclusions drawn

Work towards extending the quantization condition is underway [Raposo and Hansen 2301.03981]



[M.-L. Du *et al.* 2303.09441]

[Data: M. Padmanath & S. Prelovsek 2202.10110]

Speaking of left-hand cuts...

The T_{cc}^{+} has been studied from the lattice using pion masses in which the D^{*}-meson is stable

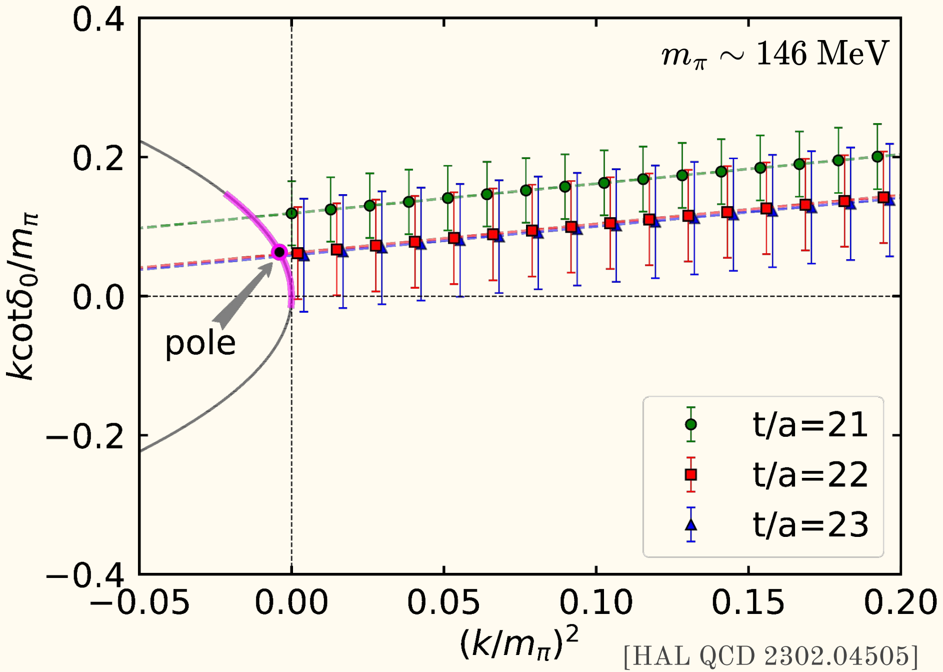
Lüscher quantization condition not valid on the left-hand cut

- For DD^{*} system, this cut can lie close to threshold, spoiling the conclusions drawn

Work towards extending the quantization condition is underway [Raposo and Hansen 2301.03981]

HAL QCD results just above left-hand cut

Lattice results at $m_{\pi} \sim 350$ MeV find attractive interaction in $I{=}0$, only one energy near threshold [CLQCD 2206.06185]



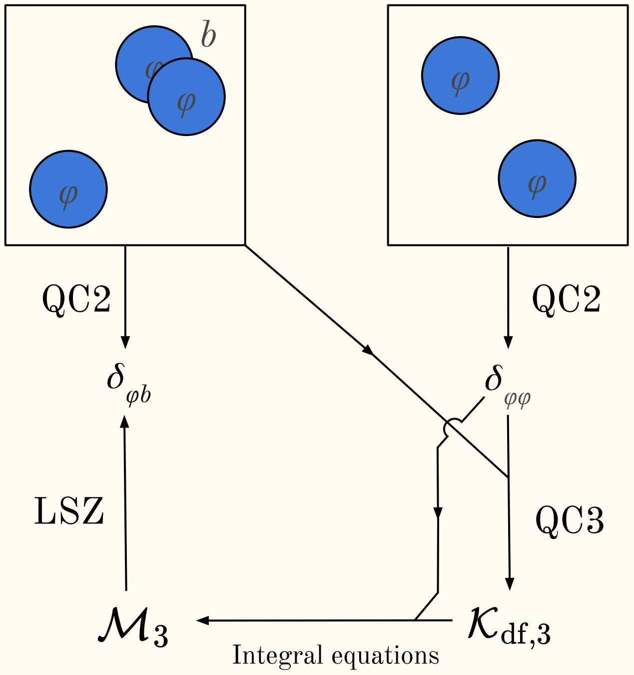
[HAL QCD 2302.04505]

See talks by:

- Jeremy Green (DD^* with distillation), Jul. 31 at 1:30 pm
- Emmanuel Ortiz Pacheco (diquark-antidiquark operators), Jul. 31 at 1:50 pm
- Sinya Aoki (T_{cc} with HAL QCD method), Jul. 31 at 2:10 pm
- Andre Baiao Raposo (Scattering on left-hand cut), Aug. 3 at 1:30 pm

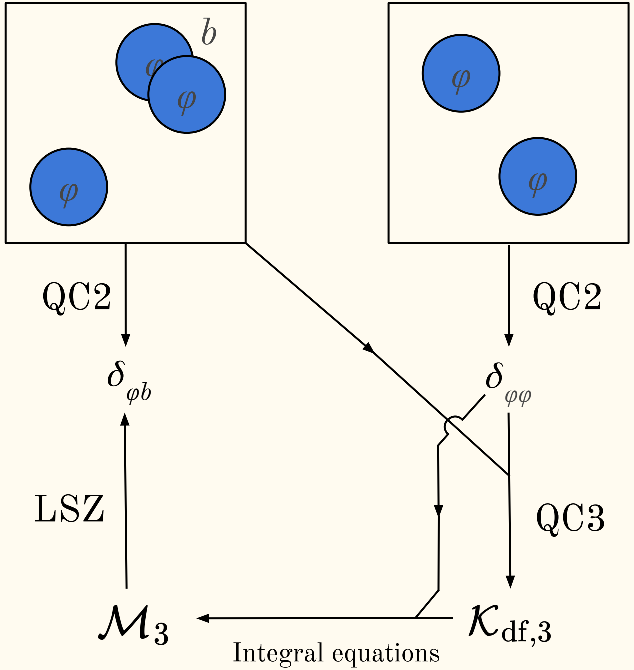
Three-particle formalism saves the day

Consider scattering of a bound state b (of two φ particles) and single particle φ



Three-particle formalism saves the day

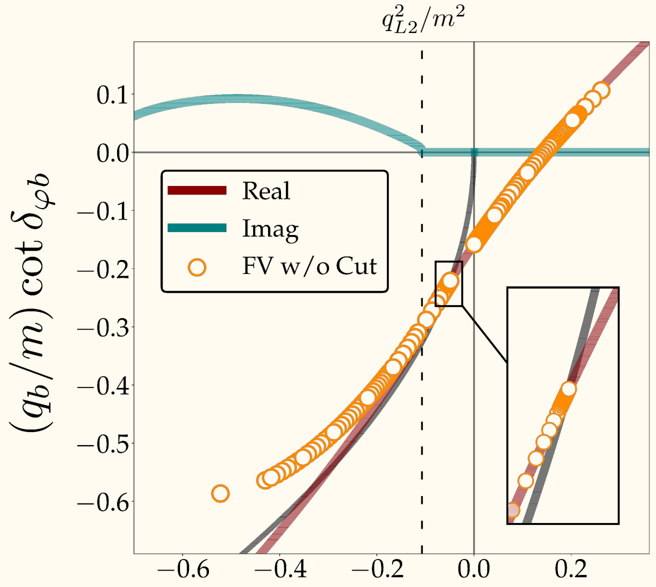
Consider scattering of a bound state b (of two φ particles) and single particle φ



See talks by:

- Md Habib E Islam (Breakdown from left-hand cut), Aug. 3 at 1:50 pm
- Steve Sharpe (Solving left-hand cut for the T_{cc}), Aug. 3 at 2:10 pm

Breakdown of equivalence between methods beyond left-hand cut demonstrated recently



[Dawid, Islam, Briceño 2303.04394]

[Orange data: F. Romero-López *et al.* 1908.02411]

Three-particle Quantization Conditions

- Most QCD resonance decays involve three or more particles
  $\omega(782) \to \pi\pi\pi$, $a_1(1260) \to \pi\pi\pi$, $N(1440) \to N\pi\pi$
- Many recent developments on the theoretical side (and their applications)
- Three competing formalisms to interpret three-particle finite-volume energies
  - Relativistic Field Theory (RFT) approach
    [Hansen & Sharpe 1408.5933, 1504.04248, ...]
  - Finite-volume unitarity (FVU) approach
    [Mai, Döring 1709.08222]
  - Non-relativistic effective field theory (NREFT)
    [Hammer, Pang, Rusetsky 1706.07700, 1707.02176]
    - Basis for recent formalism studying the Roper
      [Severt, Mai, Meißner 2212.02171]

<u>RFT QC</u>

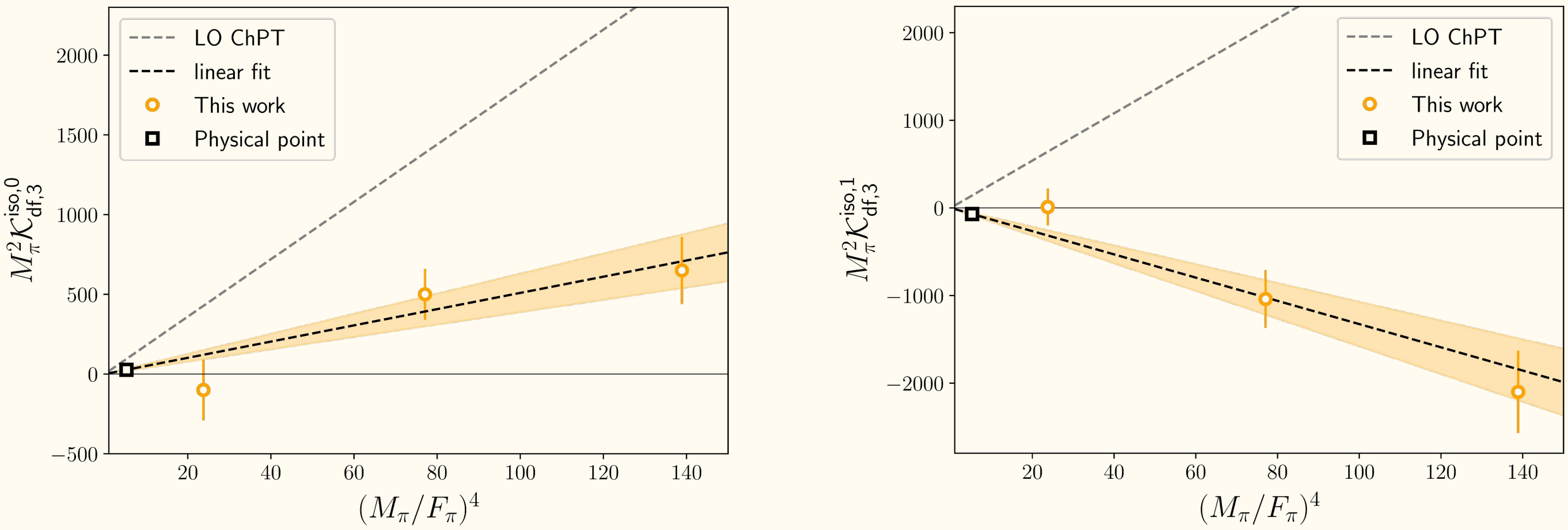
$$\det\left[F_3(E,\mathbf{P},L)^{-1} + \mathcal{K}_{\mathrm{df},3}(E^*)\right] = 0$$

- Equation in $k\ell m$ basis (spectator - dimer)
- F_3 contains both kinematic functions and the two-particle K-matrix
- $\mathcal{K}_{\mathrm{df},3}$ is a real, analytic, infinite-volume quantity but is scheme-dependent
- Must solve integral equation to obtain three-particle scattering amplitude

[Review: Hansen & Sharpe 1901.00483]

Three-pion K-matrix at NLO in ChPT

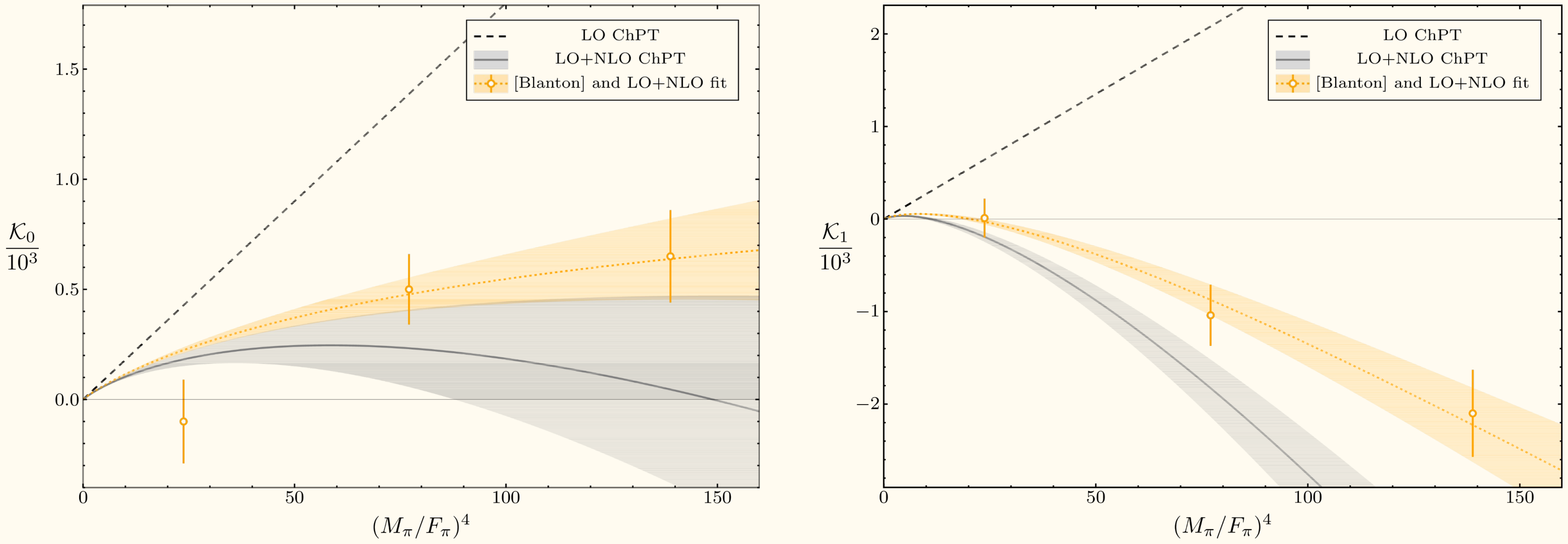
Initial worry regarding comparison to LO ChPT [T. Blanton, **ADH** *et al.* 2106.05590]



Three-pion K-matrix at NLO in ChPT

Initial worry regarding comparison to LO ChPT [T. Blanton, **ADH** *et al.* 2106.05590]

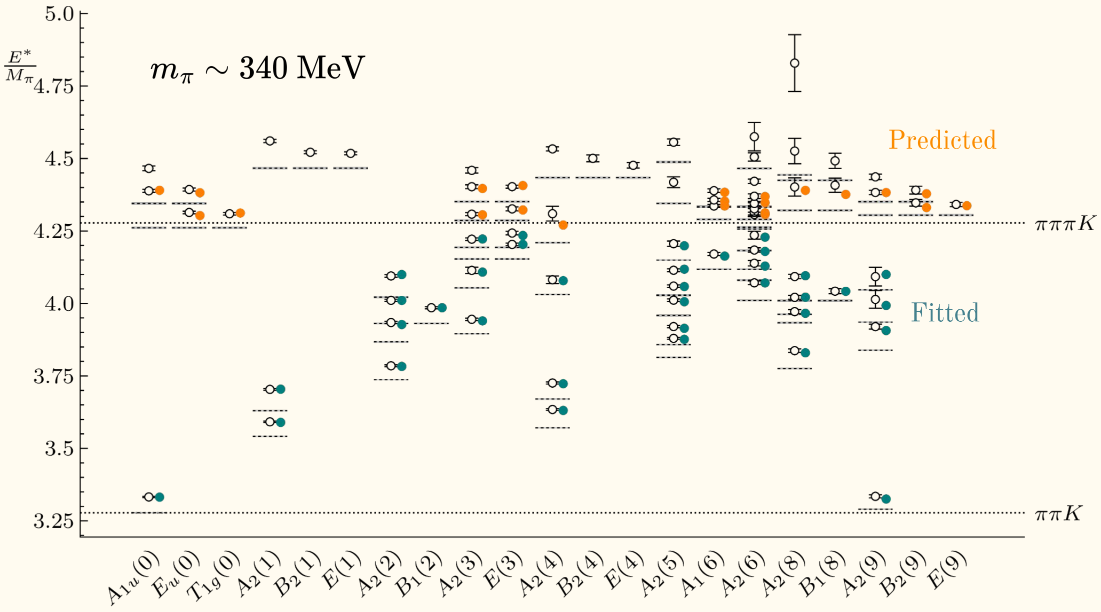
Significant NLO correction gives better agreement with lattice! [J. Baeza-Ballesteros *et al.* 2303.13206]

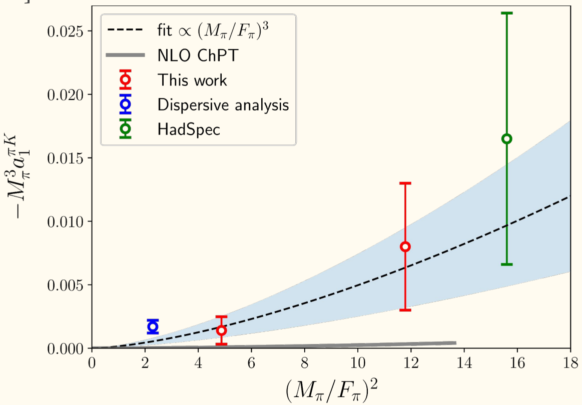


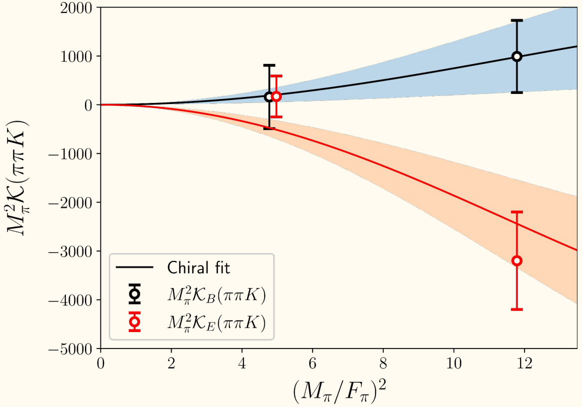
See talk by Mattias Sjö, Aug 3 at 2:30 pm

Mixed flavor three-hadron systems

- Formalism for "2+1" flavor systems recently developed [T. Blanton, *et al.* 2111.12734]
- Application to $\pi\pi K$ and $KK\pi$ systems [Z. Draper, **ADH** *et al.* 2302.13587]



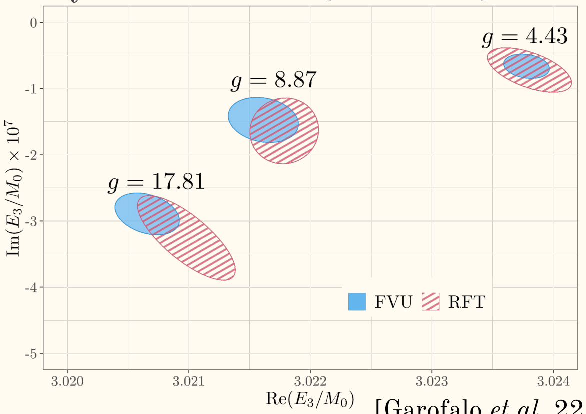




Present and future for three particles on a lattice

Recent progress

- RFT/FVU formalisms equivalent [1905.12007, 2007.16190, 2208.10587]
- Non-maximal isospin [2003.10974]
- Non-degenerate mesons [2106.05590]
- Integral equations with resonances [2211.05605]
- Non-zero spin [2303.10219, 2304.13635]
- Analytic continuations [2303.04394]



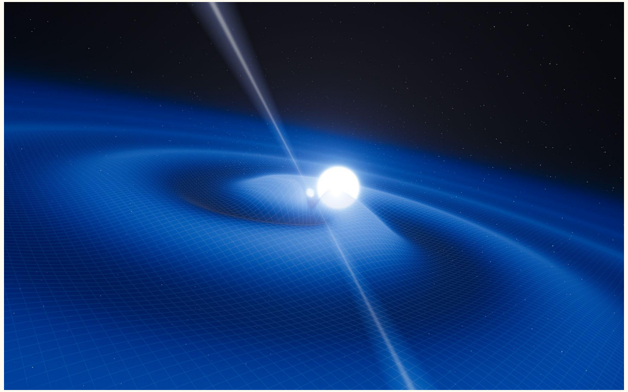
[Garofalo *et al.* 2211.05605]

See talks by:

- Sebastian Dawid (Analytic continuations), Aug. 3 at 2:50 pm
- Zachary Draper (3 spinning particles), Aug. 3 at 3:10 pm

On the horizon?

- Formalism for proper extraction of T_{cc}^+
- Formalism for Roper resonance
- Lattice data with three-neutron formalism

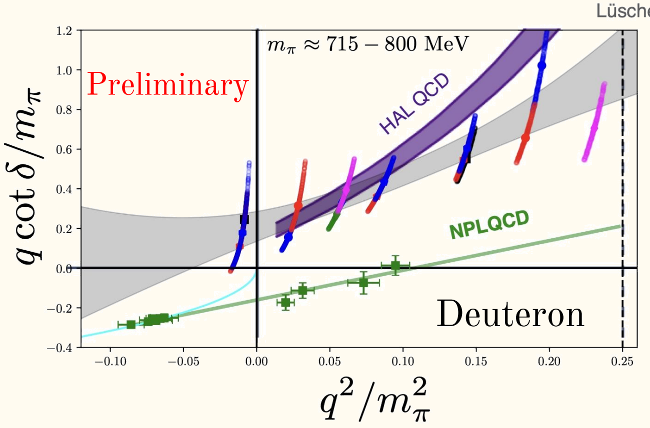


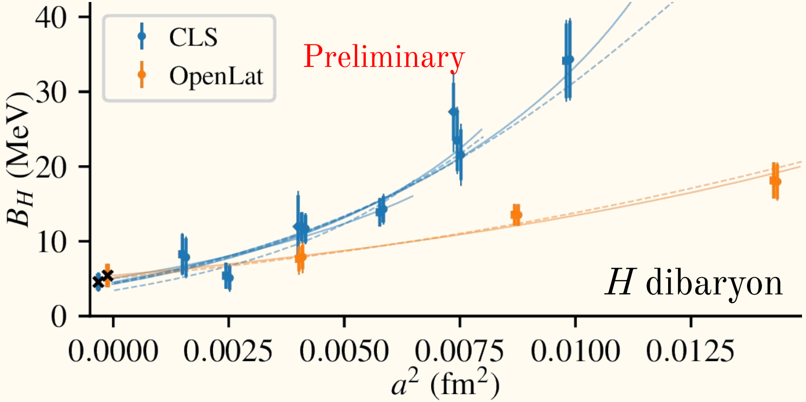
[Artist's impression of the pulsar PSR J0348+0432 and its white dwarf companion, **Credit:** ESO/L. Calçada, https://www.eso.org/public/images/eso1319c/]

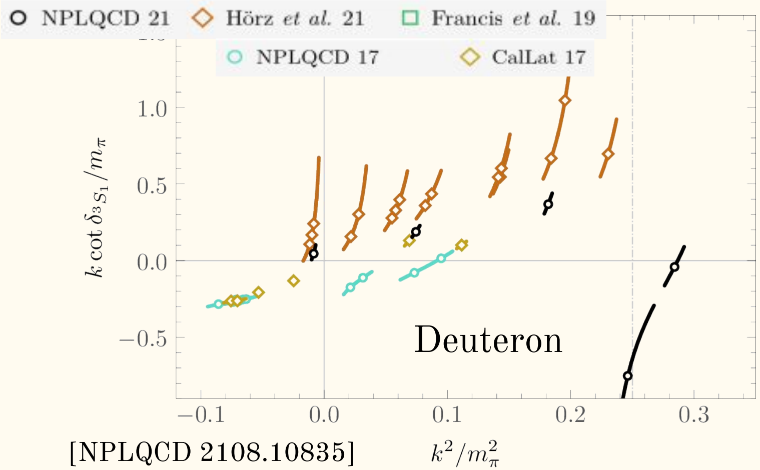
But we should not get too ahead of ourselves regarding three nucleons before...

Two-baryon calculations

- *NN* results in continuum see only virtual bound state [Mainz]
- GEVP results see no bound state, asymmetric correlators do [NPLQCD]
- Lüscher and HAL QCD method agree on same ensemble [CoSMoN]
- Universality of binding energy observed from two actions [BaSc: Mainz+CoSMoN]







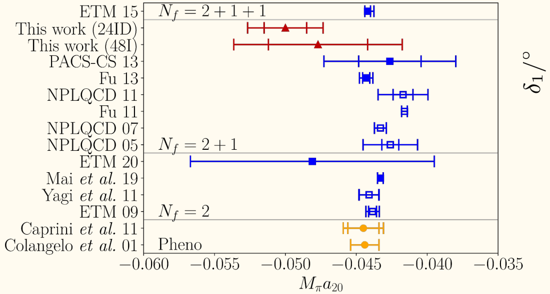
[NPLQCD 2108.10835]

See talks by:

- Takahiro Doi (Nucleon-hyperon, HAL QCD), Aug. 3 at 5:00 pm
- André Walker-Loud (NN, HAL QCD vs Lüscher), Aug. 3 at 5:20 pm
- Phiala Shanahan (NN, $m_\pi \sim 800, 170$ MeV), Aug. 3 at 5:40 pm
- Anthony Francis (SWF from OpenLat), Aug. 4 at 9:00 am

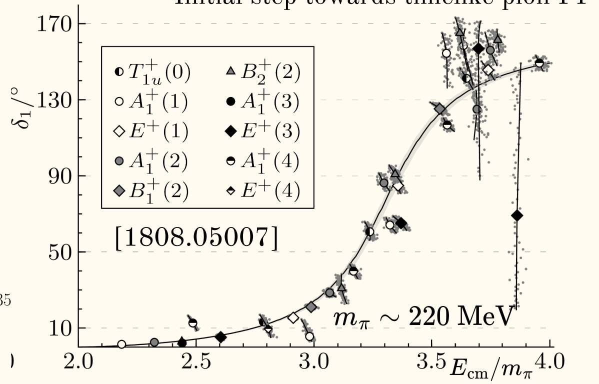
Incomplete selection of applications of formalism

π-π as a benchmark

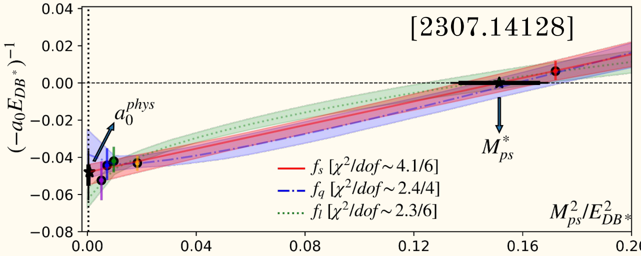


[2304.03313]

Initial step towards timelike pion FF

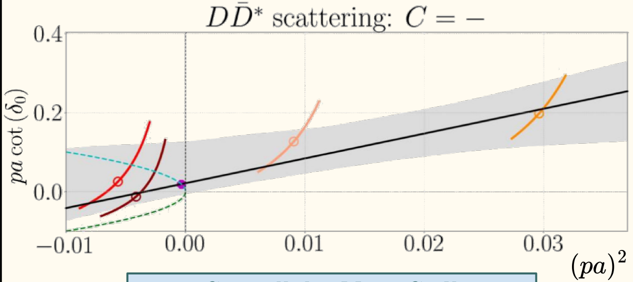


Tetraquark searches



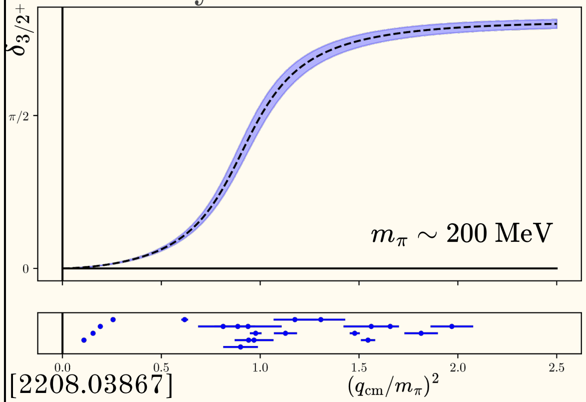
- Study of T_{bb} with HAL QCD method [2306.03565]
- Double $\bar{b}$ tetraquark from lattice NRQCD [2303.17295]

Charmonium



See talk by Mitja Sadl
Z_c, I=1, 1^{+-} & 1^{++}
Aug. 1 at 2:30 pm

Meson-Baryon



[2208.03867]

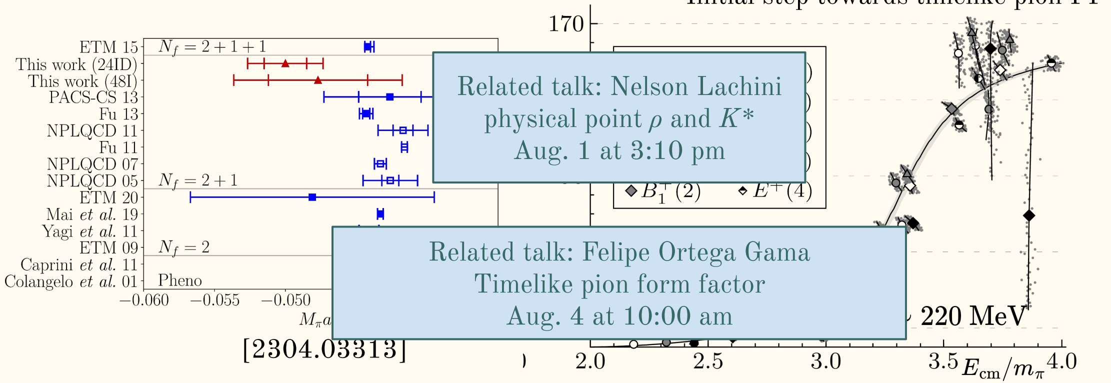
Studied with the HAL QCD method as well [2210.05395]

	m_π(MeV)	$M_\pi a_0^{3/2}$
Alexandrou *et al.* [2307.12846]	139	-0.13(4)
Bulava *et al.* [2208.03867]	200	-0.2735(81)
ChPT [1003.4444]	140	-0.0785(32)
Unitarized ChPT [1012.2233]	140	-0.0894(17)
Phenomenology	140	-0.101(4)

Initial step towards $N \to \Delta$ transition FF [2211.12278]

Incomplete selection of applications of formalism

π-π as a benchmark

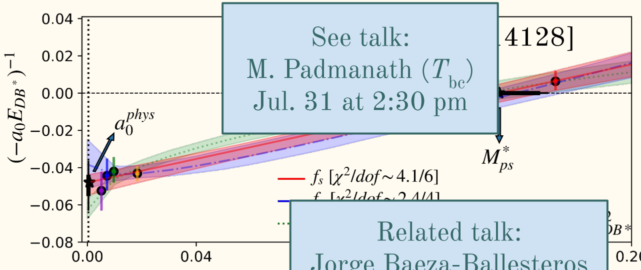


Related talk: Nelson Lachini
physical point ρ and K^*
Aug. 1 at 3:10 pm

Related talk: Felipe Ortega Gama
Timelike pion form factor
Aug. 4 at 10:00 am

[2304.03313]

Tetraquark searches

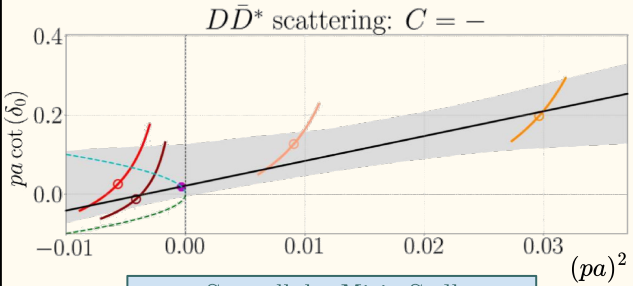


See talk:
M. Padmanath (T_{bc})
Jul. 31 at 2:30 pm

- Study of T_{bb} with HAL … 5]
- Double $\bar{b}$ tetraquark fr…
[2303.17295]

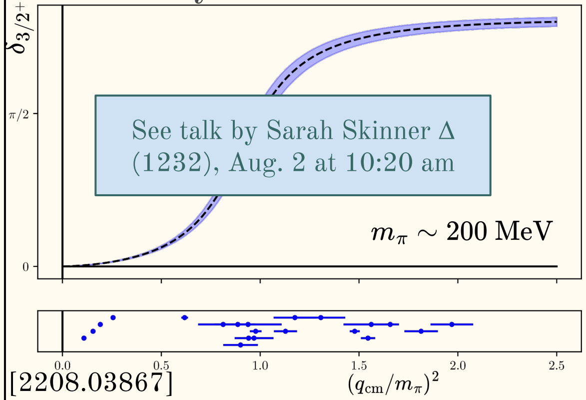
Related talk:
Jorge Baeza-Ballesteros
$T_{cs\ 0,1}(2900)$
Jul. 31 at 2:50 pm

Charmonium



See talk by Mitja Sadl
Z_c, I=1, 1^{+-} & 1^{++}
Aug. 1 at 2:30 pm

Meson-Baryon



See talk by Sarah Skinner Δ
(1232), Aug. 2 at 10:20 am

[2208.03867]

Studied with the HAL QCD method as well
[2210.05395]

Unitarized ChPT [1012.2233]	140	-0.0894(17)
Phenomenology	140	-0.101(4)

Related talk: Anthony Grebe
$N\pi$ scattering, Aug. 2 at 10:00 am

Initial step towards $N \rightarrow \Delta$ transition FF
[2211.12278]

Conclusions and Outlooks

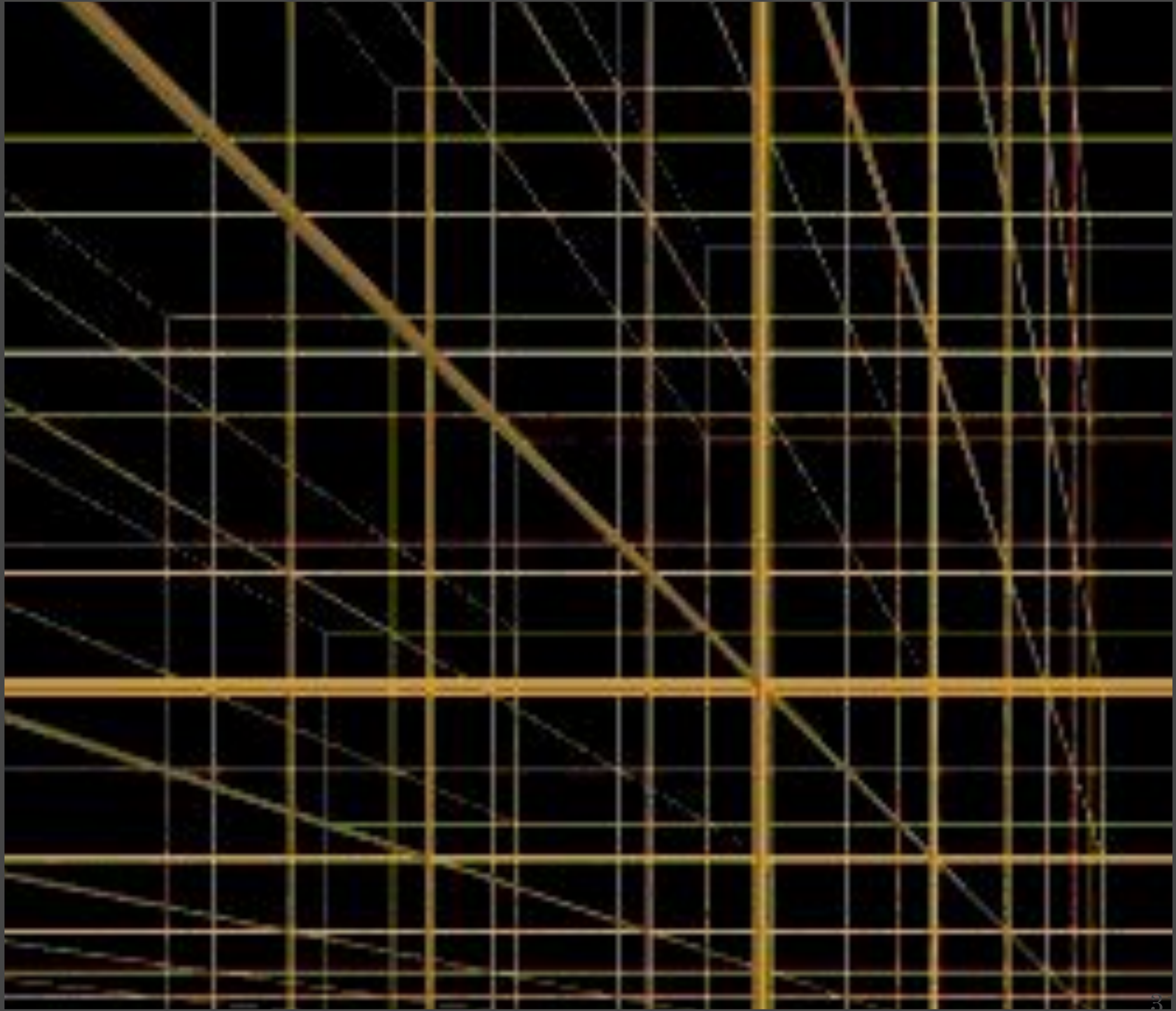
- Robust extractions of pole positions from both lattice and experimental data
- Applications of two-hadron interactions involving baryons becoming more prevalent
- Left-hand cuts beginning to be dealt with
- Progress for three particle systems continues

Exciting time for spectroscopy!

- Dispersion relations for other broad resonances deep in the complex plane
- Control over systematics becoming the norm
  - Excited-state contamination and discretization errors
- Three-particle formalism opening up possibilities to study new resonances
  - Roper, T_{cc}^{+}, and many others..

Stay tuned!

Thank you for your attention!



Math grid tessellation (https://gifer.com/)