Hadron spectroscopy and few-body dynamics

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Outline

Why study the spectrum of hadrons?

How to study spectroscopy from the lattice

Selected applications
Spectroscopy from the lattice

However:

- Bound states near threshold can have significant finite-volume effects
- Further, most hadrons are resonances, i.e. unstable states

How to proceed?:

- Robust extraction of resonance information from multi-hadron states on the lattice can be (and has been) achieved!

[A. Kronfeld 1203.1204]

Spectrum of QCD-‘stable’ states agrees well with experiment

[Lučher ‘86, ‘91; generalizations]
The limits of the quark model

The quark model was mostly successful at describing a wide range of the observed resonances, but is clearly incomplete

- It was puzzling that up until ~2000, all hadrons could be described by conventional baryons/mesons

- Exotic hadrons possible

- However, many questions
  - why is the Roper, \(N(1440)\) 1/2\(^+\), so light?
  - why is the \(\Lambda(1405)\) lighter than its nucleon counterpart, i.e. the \(N(1535)\) 1/2\(^-\)?
  - is the nonet of light scalar mesons better described as tetraquarks?
  - and lots more...

[Particle Data Group, *PTEP* 2022 083C01]

[R.L. Jaffe hep-ph/0409065]
Exotic candidates

Since the early 2000s, several resonances that do not fit in the quark model have appeared

- $X(3872)$ - could be the $\chi_{c1}(2P)$, but very different from expected behavior
- $Z_c(3900)^+$ - charged charmonium-like state, must be more than $c\bar{c}$
- $T_{cc}(3875)^+$ - two charm and overall integer spin, must be exotic to be a color singlet

[Belle hep-ex/0309032]

[BESIII 1303.5949]

[LHCb 2109.01056]
New hadrons at the LHC

[Image from https://www.nikhef.nl/~pkoppenb/particles.html]
New hadrons at the LHC

23 new exotic hadrons at the LHC
Extracting resonances from data

- Resonances appear as poles off the real-axis on the unphysical Riemann sheets of $T_{fi}$

$$S_{fi} = \delta_{fi} + i(2\pi)^4\delta^4(p^f - p^i)T_{fi}$$

- Each scattering threshold doubles the number of Riemann sheets in which the scattering amplitude lives

- Must infer poles from data on real axis

Goal: Reliably determine resonance properties by robustly extracting pole position and residue!
Lüscher two-particle formalism

Compact formula for quantization condition

\[
\det \left[ F(E_2, P, L)^{-1} + K_2(E_2^*) \right] = 0
\]

- \( E \) - finite-volume energies
- \( K_2 \) - 2-to-2 K-matrix
- \( F \) - known geometric function

Caveats:
- truncated at some max \( \ell \)
- only valid above left-hand cut and below 3 (or 4) particle threshold
- assumes continuum energies
- ignores exponentially small contributions

Alternatives:
- HAL QCD potential method [S. Aoki & T. Doi 2003.10730]
- Spectral functions from Euclidean correlators [J. Bulava & M. Hansen 1903.11735]
Finite-volume spectrum from correlator matrix

\[ C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle = \sum_{n=0}^\infty Z_i^{(n)} Z_j^{(n)*} e^{-E_n t}, \quad Z_j^{(n)} = \langle 0 | O_j | n \rangle \]

Toy Model:

\[ E_n = E_{n-1} + \frac{0.08}{\sqrt{n}}, \quad n = 1, 2, \ldots, 199, \quad E_0 = 0.20, \quad Z_j^{(n)} = \frac{(-1)^{j+n}}{1 + 0.05(j-n)^2} \]

Diagonal elements of \( C(t) \)

Eigenvalues of \( C(t) \)

Generalized eigenvalues of \( C(t) \)

[Plots courtesy of Colin Morningstar]
The $\Lambda(1405)$ - Two poles or one?

An $I=0$, $J^P=1/2^-$ resonance, with possible lower mass partner $\Lambda(1380)$ not predicted by quark model

Experiment tends to be mixed regarding this
- CLAS, BGOOD, ALICE, GlueX suggest two poles
- J-PARC consistent with one pole
- Global analysis says one
  [Anisovich et al, EPJA 2020]

Unitarized Chiral EFT predicts two poles

<table>
<thead>
<tr>
<th>Reference</th>
<th>high-mass pole [MeV]</th>
<th>low-mass pole [MeV]</th>
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<td>arXiv:1109.3005 &amp; 1201.6549</td>
<td>1424^{+7}<em>{-23} - i26^{+3}</em>{-14}</td>
<td>1381^{+18}<em>{-6} - i81^{+19}</em>{-8}</td>
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<td>1429^{+8}<em>{-7} - i12^{+2}</em>{-3}</td>
<td>1325^{+15}<em>{-15} - i90^{+12}</em>{-18}</td>
</tr>
</tbody>
</table>

[Particle Data Group, Chpt. 83, PTEP 2022 083C01]
What does QCD on the lattice say?

Recent determination of the $I=0$ coupled channel $\pi\Sigma-\bar{K}N$ scattering amplitudes with $m_\pi \sim 200$ MeV

Several parameterizations considered, with effective-range expansion as preferred fit

$$\frac{E_{cm}}{m_\pi} \tilde{K}_{ij} = A_{ij} + B_{ij} \Delta_{\pi\Sigma}$$

with $B_{00} = B_{11} = 0$, and where $\Delta_{\pi\Sigma} = \frac{E_{cm}^2 - (m_\pi + m_{\Sigma})^2}{(m_\pi + m_{\Sigma})^2}$

Final estimate for pole positions consistent with results from UChPT

$$E_1 = 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_a \text{ MeV},$$
$$E_2 = [1455(13)_{\text{stat}}(2)_{\text{model}}(17)_a$$
$$- i \times 11.5(4.4)_{\text{stat}}(4)_{\text{model}}(0.1)_a] \text{ MeV}.$$

See talks by:
- Fernando Romero-López (this work), Jul. 31 at 5:00 pm
- Kotaro Murakami (HAL QCD), Jul. 31 at 5:20 pm

[J. Bulava, ..., ADH et al. 2307.10413, 2307.13471]
Two-pole structure for other resonances?

Pole extractions can be unstable if located on sheets not directly connected to the physical one.

Lattice results for coupled $D\pi-D\eta-D_s\bar{K}$ showed strong dependence for second $D_0^*(2300)$ resonance pole on the scattering amplitude parameterization, thus was considered ‘unreliable’.

Poles with a small residue close to threshold look similar to poles with a large residue far from threshold.

[A. Asokan et al. 2212.07856]
[Data: HadSpec 1607.07093]
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Using constraints from ChPT for the parameterizations, stable pole positions could be extracted!

See related talk by Haobo Yan ($D^{(*)}\pi$ scattering), Aug. 1 at 2;50 pm.

[A. Asokan et al. 2212.07856]
[Data: HadSpec 1607.07093]
General structure of the scattering amplitude

In cases where the resonance pole lies deep in the complex plane, the full structure should be taken into account

- Unitarity ($SS^\dagger = 1$, $K$-matrix enforces this)
- Crossing (relation between scattering channels)
- Analyticity in Mandelstam variables (from causality)

Analytic properties can be imposed via dispersion relations

$$T(s, t, u) = \frac{1}{\pi} \int_{4m^2}^{\infty} ds' \frac{\text{Im} T(s', t, u)}{s' - (s + i\varepsilon)} + \frac{1}{\pi} \int_{-\infty}^{-t} ds' \frac{\text{Im} T(s', t, u)}{s' - (s + i\varepsilon)}$$

*May require subtractions in order to ignore contour at $\infty$

Leads to much more stable pole determination! (red points)
σ resonance from lattice with dispersion relations

Dispersion relation techniques can also be used with lattice data.

In general, the partial-wave projected amplitudes

\[ t_\ell(s) = \frac{1}{64\pi} \int d\cos \theta_s \, T(s, t, u) \, P_\ell(\cos \theta_s) \]

constrained from lattice data do not respect crossing symmetry. However, those that do will respect the dispersion relation

\[ \tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I', \ell} \int_0^\infty ds' \, K_{\ell I}^{I'I'}(s', s) \, \text{Im} \, t_{\ell'}(s'), \]

i.e. they will result in \( \tilde{t}_\ell^I(s) = t_\ell^I(s) \)

Again leads to more stable pole positions!
Speaking of left-hand cuts...

The $T_{cc}^+$ has been studied from the lattice using pion masses in which the $D^*$-meson is stable

Lüscher quantization condition not valid on the left-hand cut

- For $DD^*$ system, this cut can lie close to threshold, spoiling the conclusions drawn

Work towards extending the quantization condition is underway [Raposo and Hansen 2301.03981]

\[ m_\pi \sim 280 \text{ MeV} \]

\[ p \cot(\delta/E_{DD^*})^2 \]

[M. Padmanath & S. Prelovsek 2202.10110]
Speaking of left-hand cuts...

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HAL QCD results just above left-hand cut

Lattice results at $m_\pi \sim 350$ MeV find attractive interaction in $I=0$, only one energy near threshold [CLQCD 2206.06185]

See talks by:
- Jeremy Green ($DD^*$ with distillation), Jul. 31 at 1:30 pm
- Emmanuel Ortiz Pacheco (diquark-antidiquark operators), Jul. 31 at 1:50 pm
- Sinya Aoki ($T_{cc}$ with HAL QCD method), Jul. 31 at 2:10 pm
- Andre Baiao Raposo (Scattering on left-hand cut), Aug. 3 at 1:30 pm
Three-particle formalism saves the day

Consider scattering of a bound state \( b \) (of two \( \phi \) particles) and single particle \( \phi \)

\[
\begin{align*}
&\text{QC2} \\
&\delta_{\phi b} \\
&\text{LSZ} \\
&\mathcal{M}_3 \\
&\text{Integral equations} \\
&\mathcal{K}_{df,3} \\
&\text{QC3} \\
&\delta_{\phi\phi} \\
&\text{QC2}
\end{align*}
\]
Three-particle formalism saves the day

Consider scattering of a bound state $b$ (of two $\phi$ particles) and single particle $\phi$

Breakdown of equivalence between methods beyond left-hand cut demonstrated recently

See talks by:
- Md Habib E Islam (Breakdown from left-hand cut), Aug. 3 at 1:50 pm
- Steve Sharpe (Solving left-hand cut for the $T_{cc}$), Aug. 3 at 2:10 pm

[Dawid, Islam, Briceño 2303.04394]
[Orange data: F. Romero-López et al. 1908.02411]
Three-particle Quantization Conditions

- Most QCD resonance decays involve three or more particles
  \( \omega(782) \rightarrow \pi\pi\pi, \ a_1(1260) \rightarrow \pi\pi\pi, \ N(1440) \rightarrow N\pi\pi \)

- Many recent developments on the theoretical side (and their applications)

- Three competing formalisms to interpret three-particle finite-volume energies
  - Relativistic Field Theory (RFT) approach
    [Hansen & Sharpe 1408.5933, 1504.04248, ...]
  - Finite-volume unitarity (FVU) approach
    [Mai, Döring 1709.08222]
  - Non-relativistic effective field theory (NREFT)
    [Hammer, Pang, Rusetsky 1706.07700, 1707.02176]

  - Basis for recent formalism studying the Roper
    [Severt, Mai, Meißner 2212.02171]

\[ \text{RFT QC} \]

\[
\det \left[ F_3(E, P, L)^{-1} + \mathcal{K}_{df,3}(E^*) \right] = 0
\]

- Equation in \( k\ell m \) basis (spectator - dimer)

- \( F_3 \) contains both kinematic functions and the two-particle K-matrix

- \( \mathcal{K}_{df,3} \) is a real, analytic, infinite-volume quantity but is scheme-dependent

- Must solve integral equation to obtain three-particle scattering amplitude

[Review: Hansen & Sharpe 1901.00483]
Three-pion $K$-matrix at NLO in ChPT

Initial worry regarding comparison to LO ChPT  
[T. Blanton, ADH et al. 2106.05590]
Three-pion $K$-matrix at NLO in ChPT

Initial worry regarding comparison to LO ChPT  [T. Blanton, ADH et al. 2106.05590]

Significant NLO correction gives better agreement with lattice! [J. Baeza-Ballesteros et al. 2303.13206]

See talk by Mattias Sjö, Aug 3 at 2:30 pm
Mixed flavor three-hadron systems

- Formalism for “2+1” flavor systems recently developed [T. Blanton, et al. 2111.12734]
- Application to $\pi\pi K$ and $KK\pi$ systems [Z. Draper, ADH et al. 2302.13587]

$m_\pi \sim 340$ MeV

![Graph showing data points and fitted curves for $M_\pi^2(\pi\pi K)$ and $M_\pi^2(\pi K)$ against $(M_\pi/F_\pi)^2$.]
Present and future for three particles on a lattice

Recent progress

- RFT/FVU formalisms equivalent [1905.12007, 2007.16190, 2208.10587]
- Non-maximal isospin [2003.10974]
- Non-degenerate mesons [2106.05590]
- Integral equations with resonances [2211.05605]
- Non-zero spin [2303.10219, 2304.13635]
- Analytic continuations [2303.04394]

On the horizon?

- Formalism for proper extraction of $T^+_c$
- Formalism for Roper resonance
- Lattice data with three-neutron formalism

See talks by:

- Sebastian Dawid (Analytic continuations), Aug. 3 at 2:50 pm
- Zachary Draper (3 spinning particles), Aug. 3 at 3:10 pm

But we should not get too ahead of ourselves regarding three nucleons before...

[Artist’s impression of the pulsar PSR J0348+0432 and its white dwarf companion, Credit: ESO/L. Calçada, https://www.eso.org/public/images/eso1319c/]
Two-baryon calculations

- $NN$ results in continuum see only virtual bound state [Mainz]
- GEVP results see no bound state, asymmetric correlators do [NPLQCD]
- Lüscher and HAL QCD method agree on same ensemble [CoSMoN]
- Universality of binding energy observed from two actions [BaSc: Mainz+CoSMoN]

See talks by:
- Takahiro Doi (Nucleon-hyperon, HAL QCD), Aug. 3 at 5:00 pm
- André Walker-Loud (NN, HAL QCD vs Lüscher), Aug. 3 at 5:20 pm
- Phiala Shanahan (NN, $m_\pi \sim 800, 170$ MeV), Aug. 3 at 5:40 pm
- Anthony Francis (SWF from OpenLat), Aug. 4 at 9:00 am
Incomplete selection of applications of formalism

**π-π as a benchmark**

- Initial step towards timelike pion FF
  - Studied with the HAL QCD method as well
  - Double $b$ tetraquark from lattice NRQCD

**Charmonium**

- $D\bar{D}^*$ scattering: $C = -$ [2304.03313]

**Meson-Baryon**

- $m_\pi \sim 200$ MeV

**Tetraquark searches**

- Study of $T_{bb}$ with HAL QCD method [2306.03565]
- Double $b$ tetraquark from lattice NRQCD [2303.17295]

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See talk by Mitja Sadl
$Z_c, I=1, 1^+ & 1^{++}$
Aug. 1 at 2:30 pm

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[pdf]

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Incomplete selection of applications of formalism

**π-π as a benchmark**

**Initial step towards timelike pion FF**

- Related talk: Nelson Lachini
  - Physical point $\rho$ and $K^*$
  - Aug. 1 at 3:10 pm

- Related talk: Felipe Ortega Gama
  - Timelike pion form factor
  - Aug. 4 at 10:00 am

**Tetraquark searches**

**Study of $T_{bb}$ with HAL QCD method**
- Jul. 31 at 2:30 pm

**Double $\bar{b}$ tetraquark from lattice NRQCD**
- Jul. 31 at 2:50 pm

**Charmonium**

**Meson-Baryon**

- See talk by Sarah Skinner
  - $\Delta$ (1232), Aug. 2 at 10:20 am

- Related talk: Anthony Grebe
  - $N\pi$ scattering, Aug. 2 at 10:00 am

See talk by Mitja Sadl
- $Z_c$, $I=1, 1^+$ & $1^{++}$
- Aug. 1 at 2:30 pm
Conclusions and Outlooks

- Robust extractions of pole positions from both lattice and experimental data
- Applications of two-hadron interactions involving baryons becoming more prevalent
- Left-hand cuts beginning to be dealt with
- Progress for three particle systems continues

Exciting time for spectroscopy!
- Dispersion relations for other broad resonances deep in the complex plane
- Control over systematics becoming the norm
  - Excited-state contamination and discretization errors
- Three-particle formalism opening up possibilities to study new resonances
  - Roper, $T_{cc}^+$, and many others..

Stay tuned!
Thank you for your attention!