Transport and Connection to Heavy-ion Collisions
via heavy flavor probes

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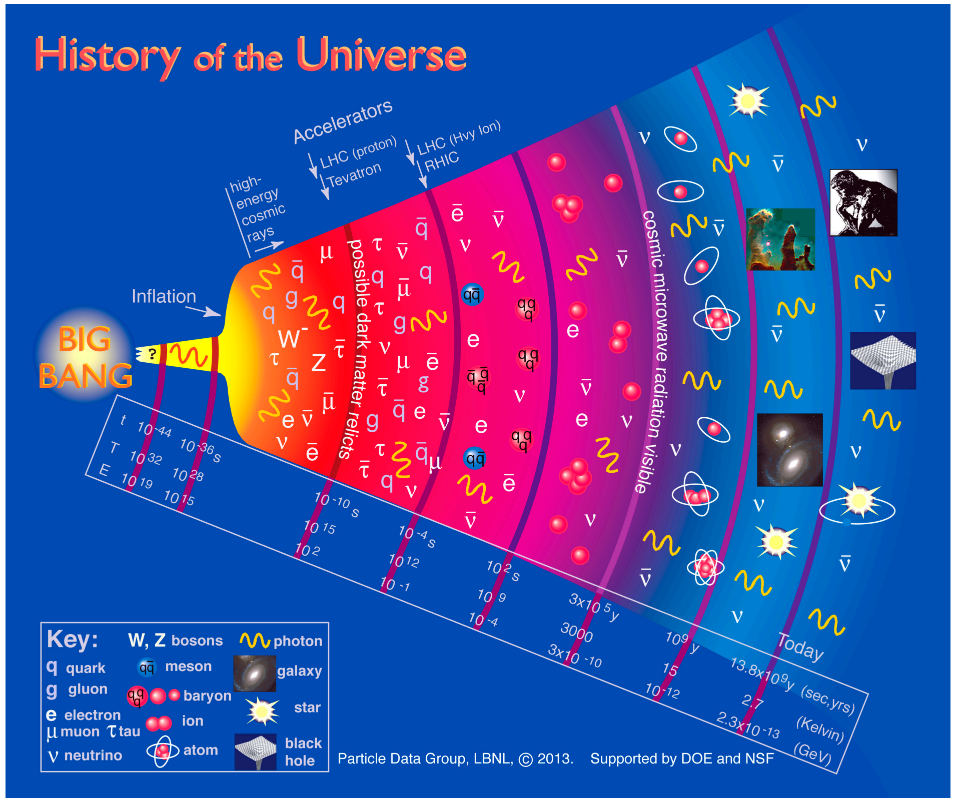
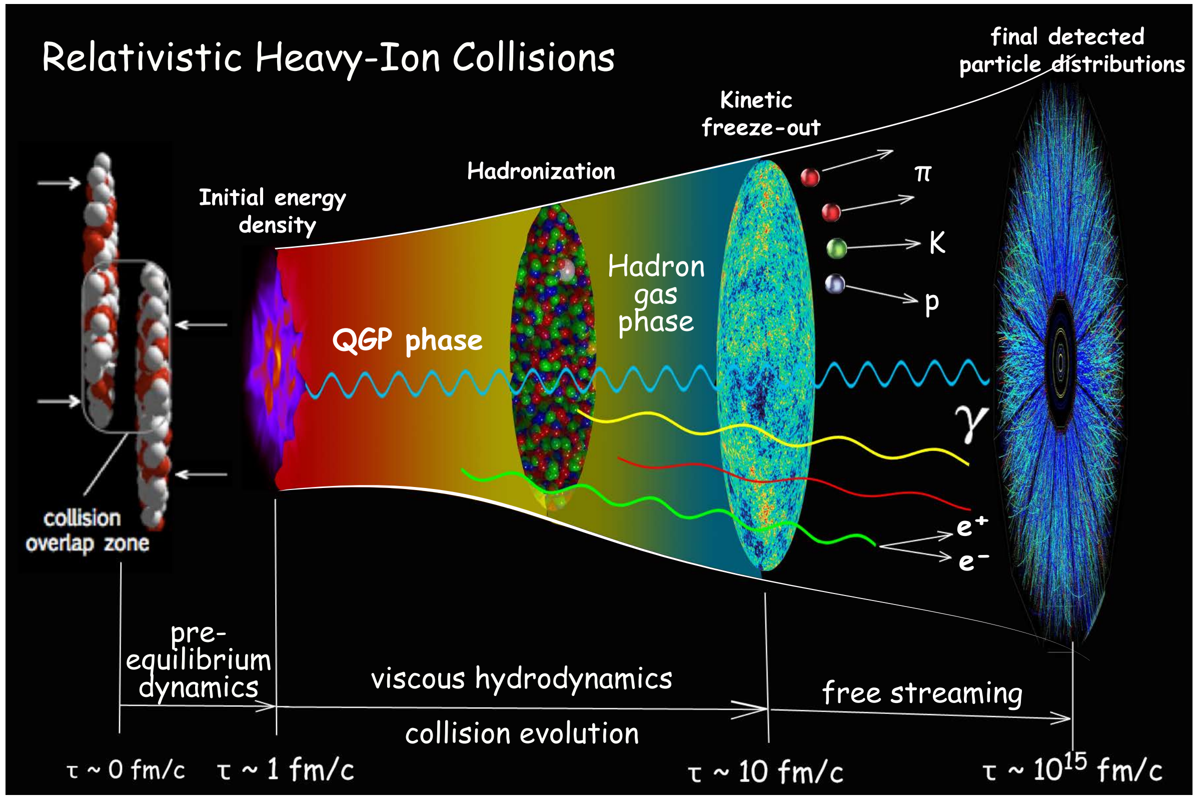
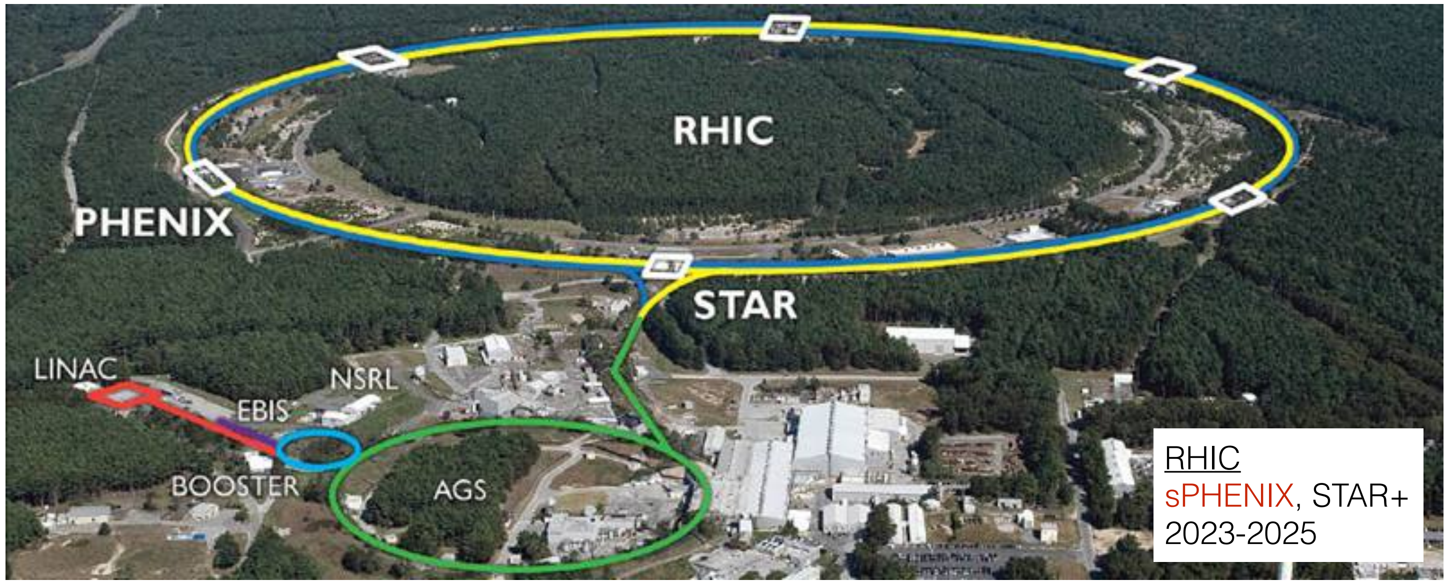
Understand the Big-Bang from the Little-Bang





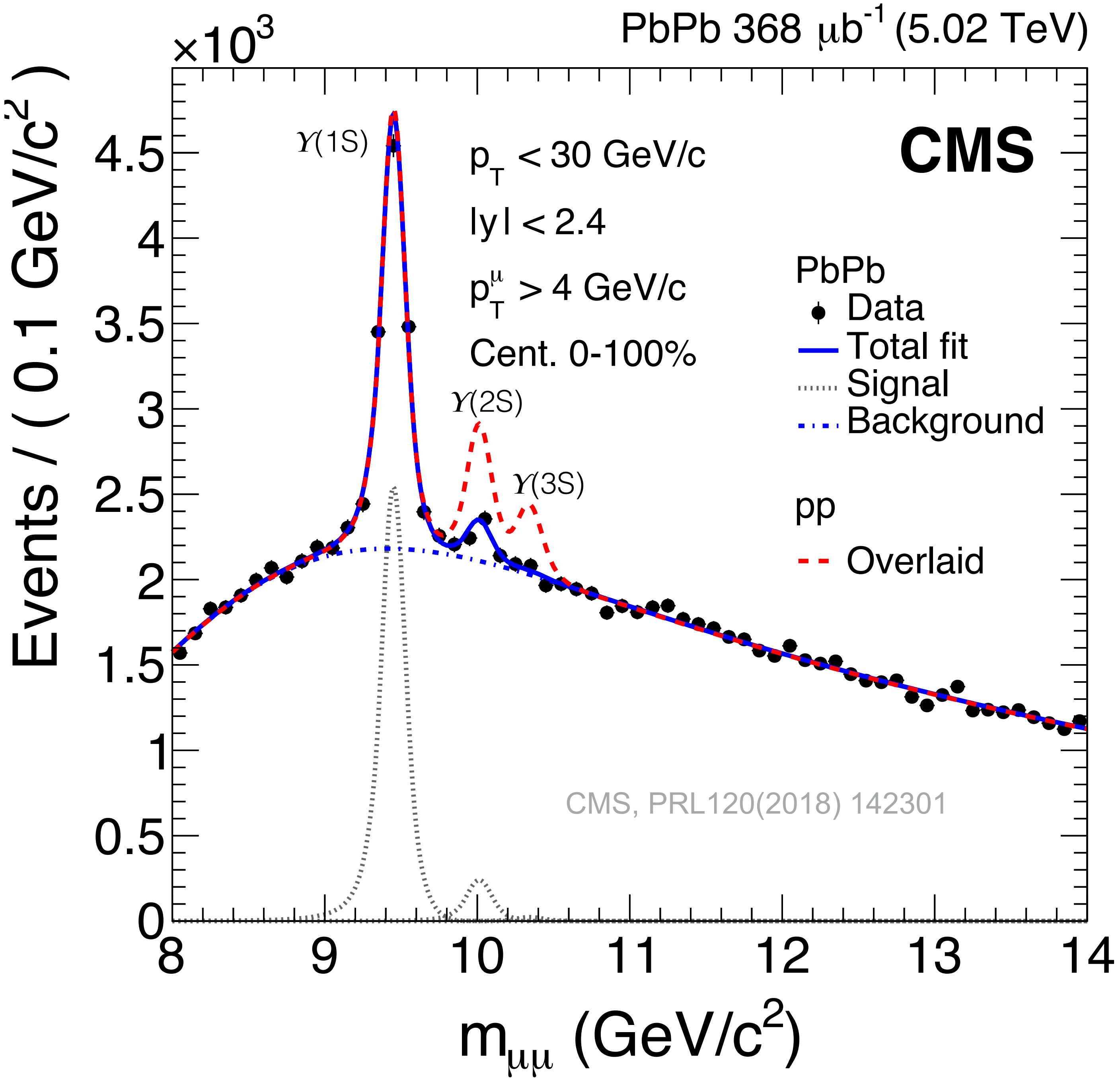
Mimic the early universe using HICs

LHC Runs 3-5
ALICE 2-3, CMS+, ATLAS+, LHC+
2022-2038



RHIC
sPHENIX, STAR+
2023-2025

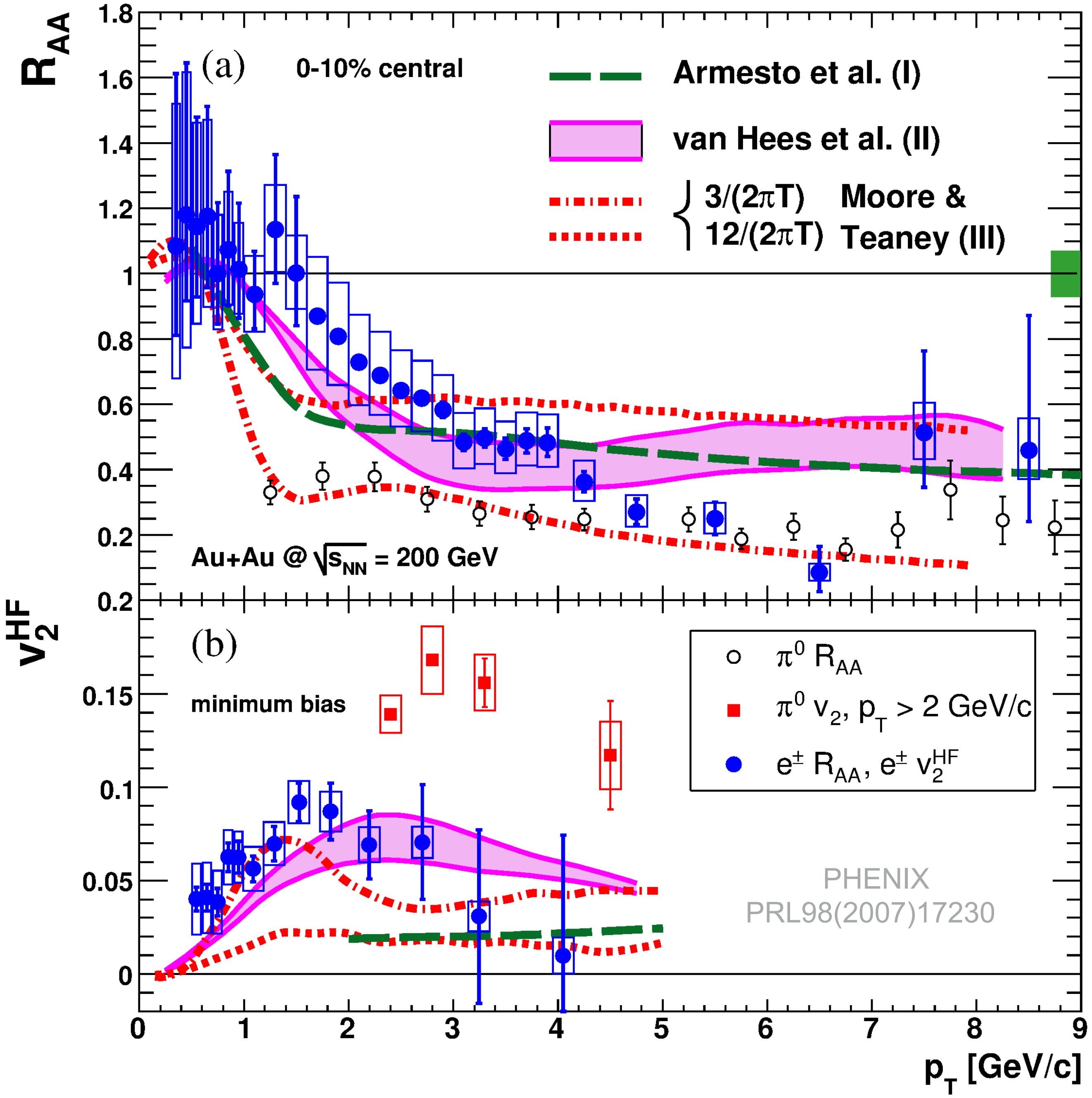
Sequential melting of bottomonium in HICs



CMS, PRL120(2018) 142301

Color screening of hot medium?

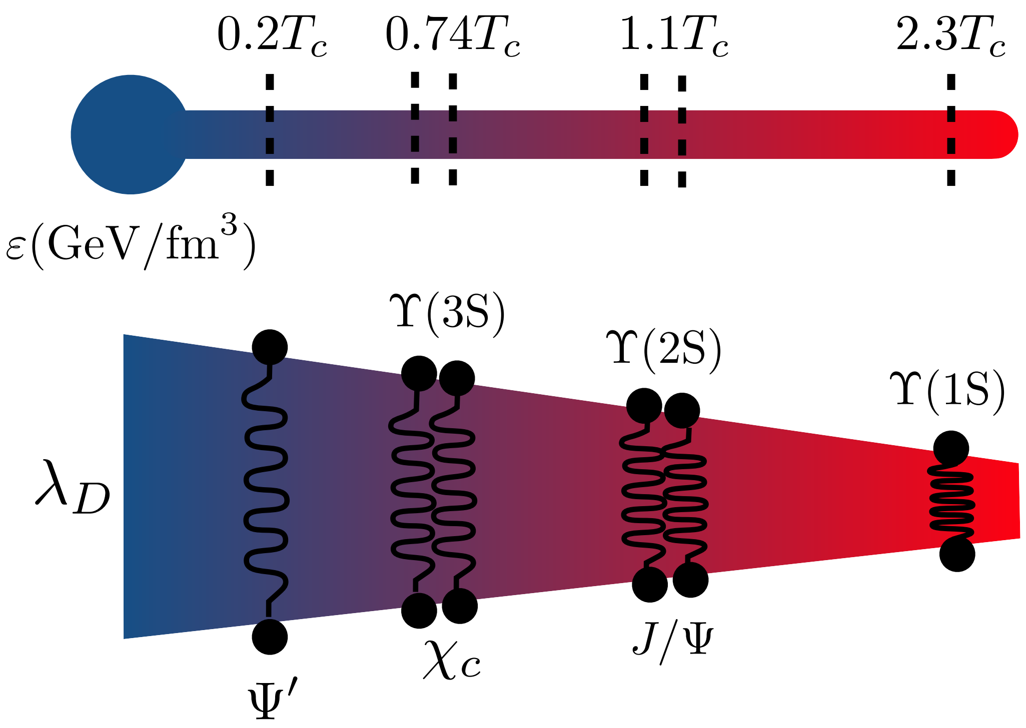
Heavy quark collective flow in HICs



Hot medium is a near-perfect fluid?

Heavy flavor probes

Probe the hot medium with different length scales…



2015 Long Run Plan Nuclear Science

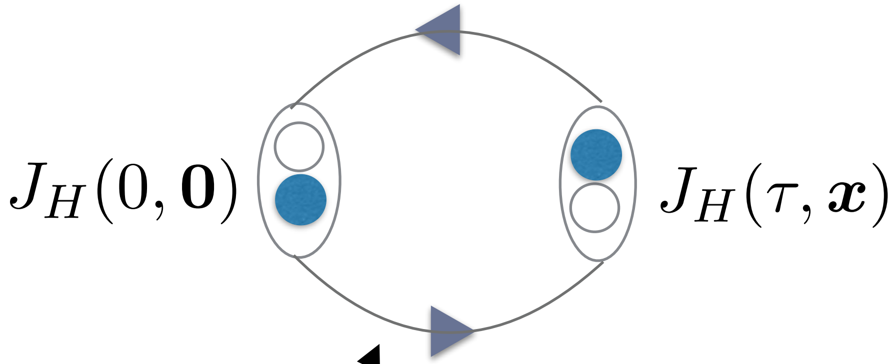
Where LQCD has been delivering value…

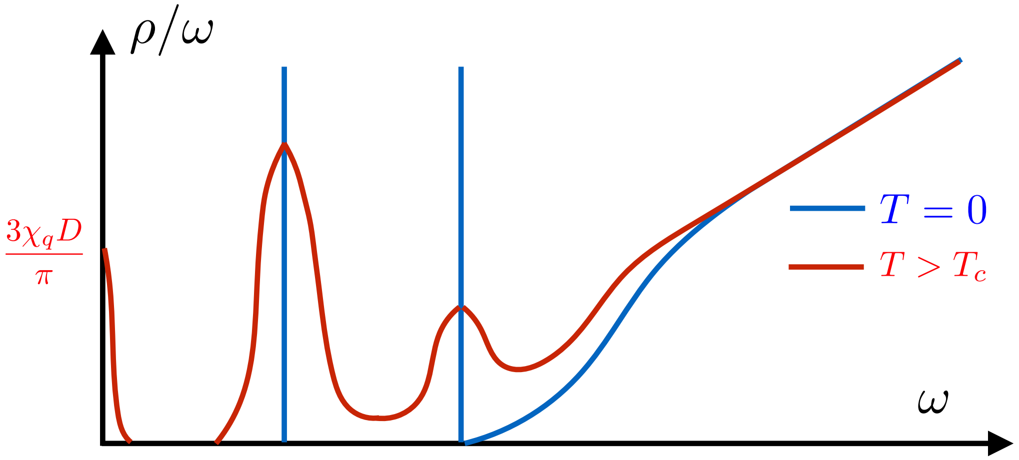
- In-medium quarkonium properties: masses, widths, melting T
- Complex quark-antiquark potential: Re[V], Im[V]
- Heavy quark (momentum) diffusion coefficient: D_s, kappa

Part I

In-medium quarkonium properties

Meson spectral function





$$G_H(\tau,\vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x}) = \int \frac{d\omega}{2\pi}\frac{\cosh(\omega(\tau T-1/2)/T)}{\sinh(\omega/2T)}\rho_H(\omega,\vec{p},T)$$

ill-posed

$K(\omega,\tau,T)$

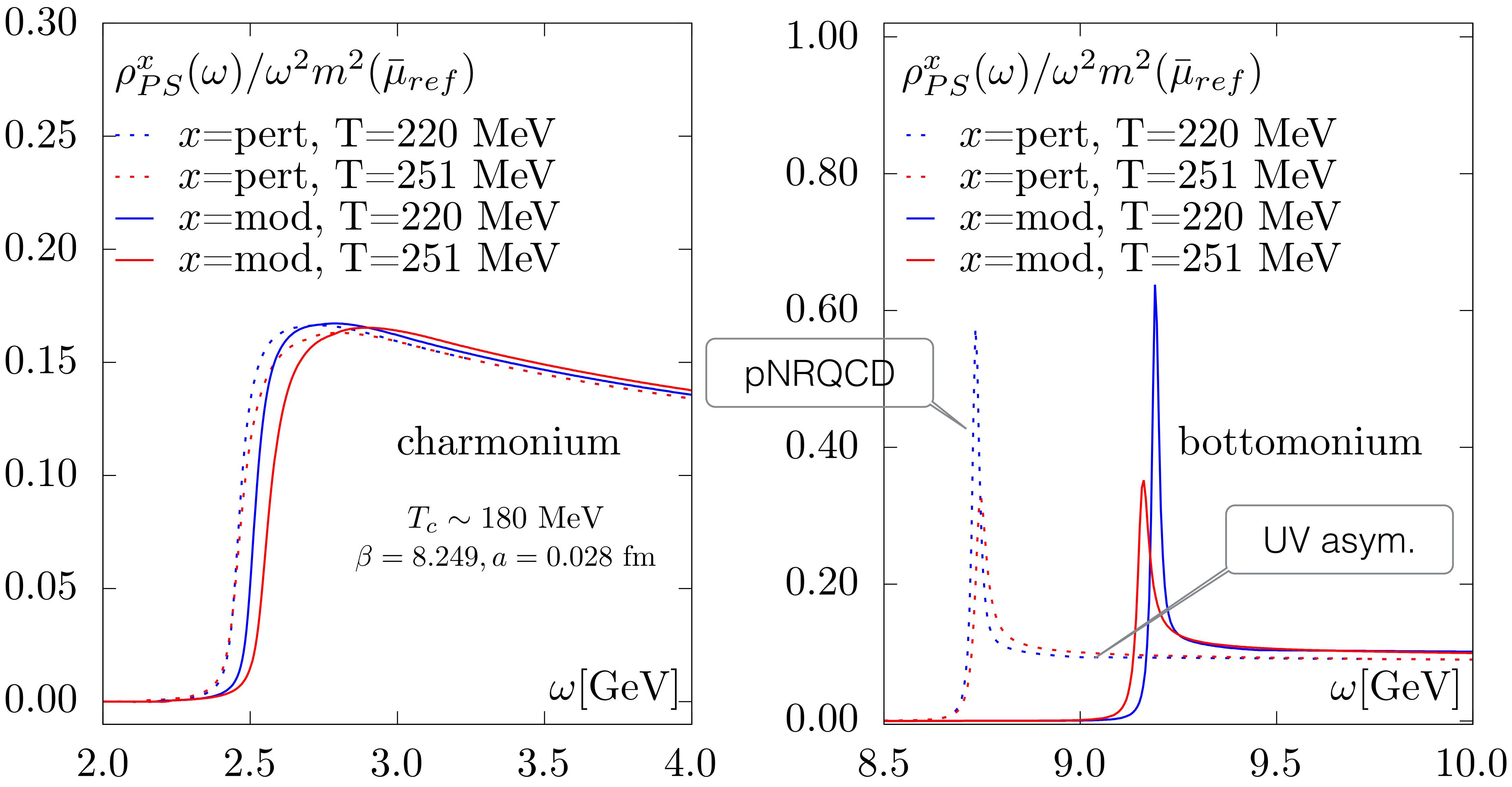
- Meson spectral function tells melting T and heavy quark diffusion
- Need prior info in the spectral reconstruction

Update: sparse modeling for charmonium 💬 *Junichi Takahashi, Tue 15:40 [232]*

Quarkonium spectral function (relativistic HQ)

Need very fine and large lattice for heavy quark $T = 1/(aN_\tau)$

First full QCD calculation with relativistic heavy quarks hotQCD, Few Body Syst. 64 (2023) 3, 52



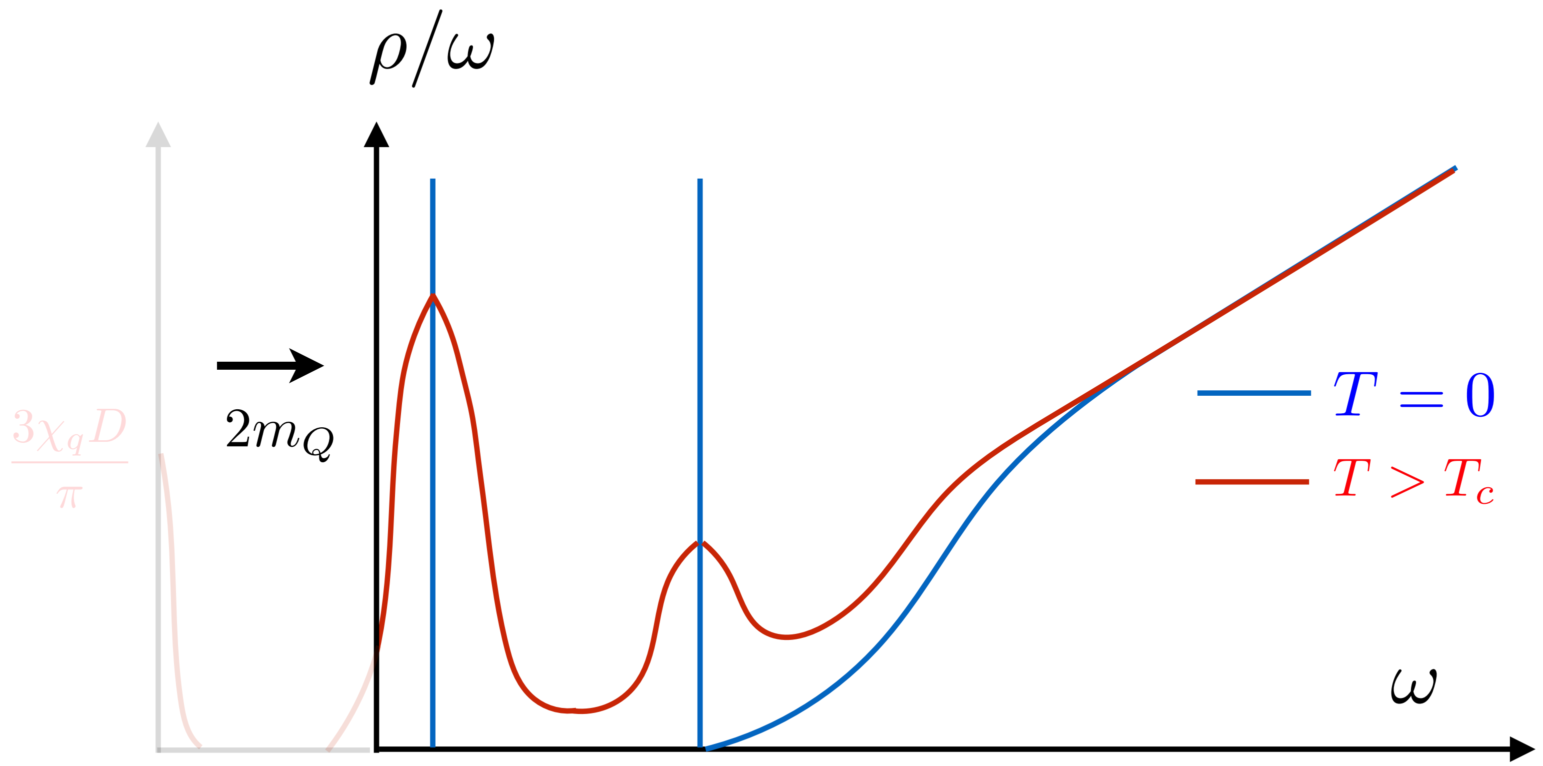
pNRQCD

UV asym.

- No resonance peaks are needed for charmonium eta_c down to 1.2 Tc
- Thermally broadened resonance peaks persist for eta_b up to 1.4 Tc

Non-relativistic heavy quark

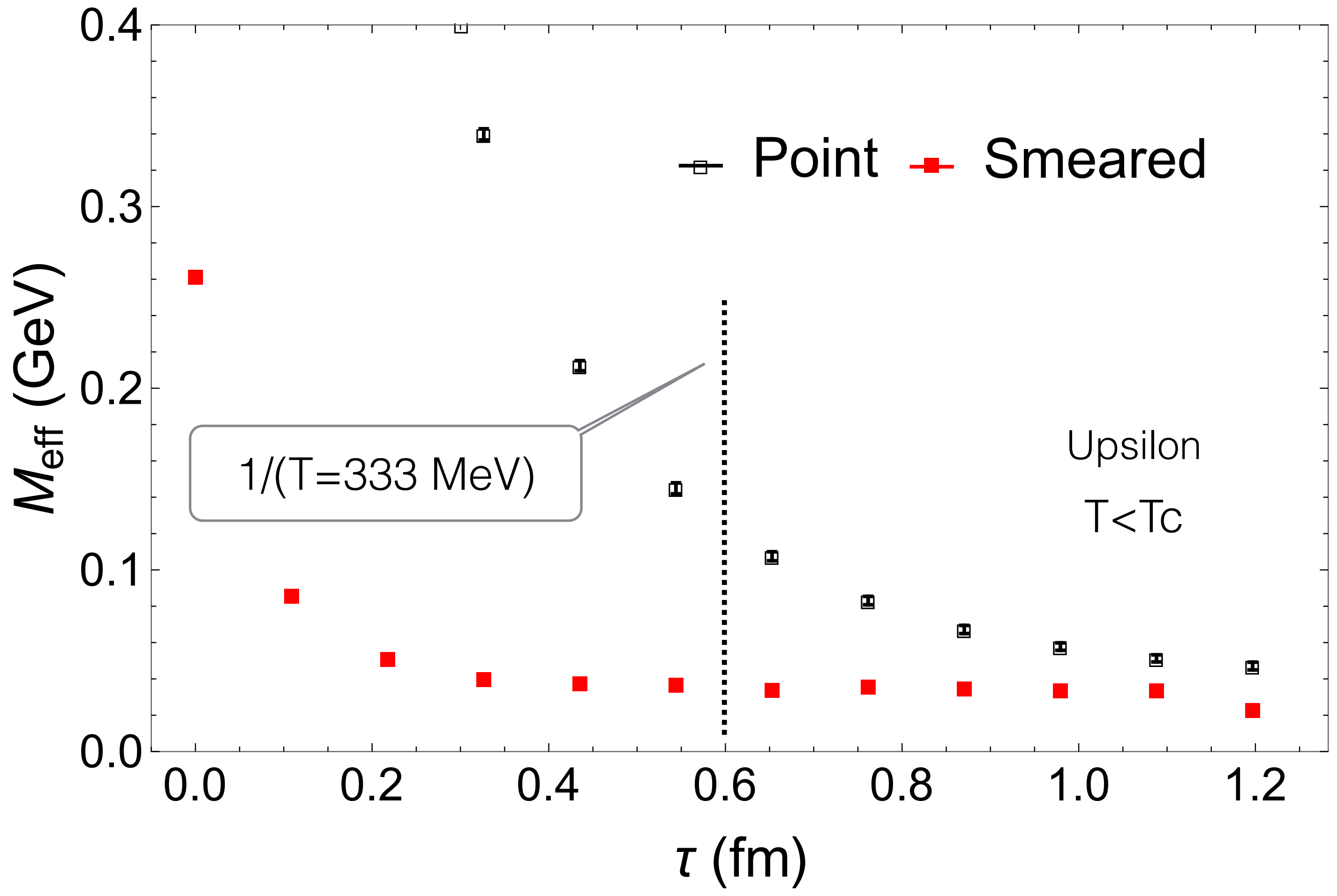
NRQCD becomes possible due to scale separation $\frac{\Lambda_{\text{QCD}}}{m_Q} \ll \frac{T_{\text{HIC}}}{m_Q} \ll 1$



G. Aarts, et al., JHEP 07 (2014) 097
S. Kim, et al., JHEP 11 (2018) 088
R. Larsen, et al., PRD 100 (2019) 7, 074506
R. Larsen, et al., PLB 800 (2020) 135119

Extended meson source

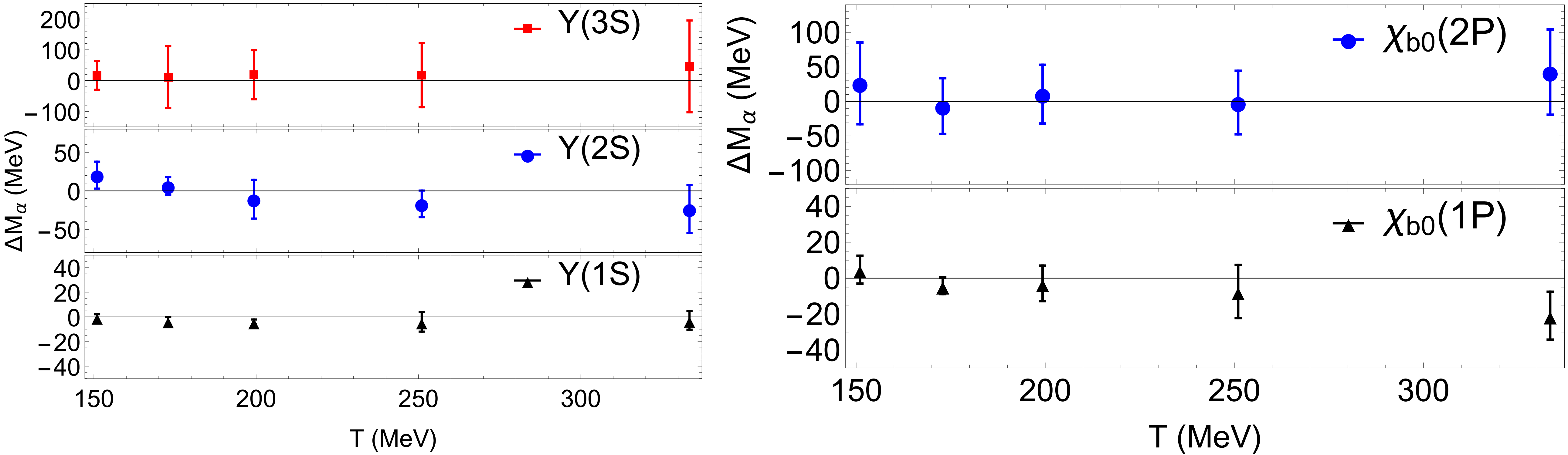
Excited states accessible from extended meson operator



R. Larsen, et al., PRD 100 (2019) 7, 074506

Update: anisotropic HISQ 💬 *Ioannis Trimis, Fri 9:20 [218]*

Thermal mass shift of bottomonium

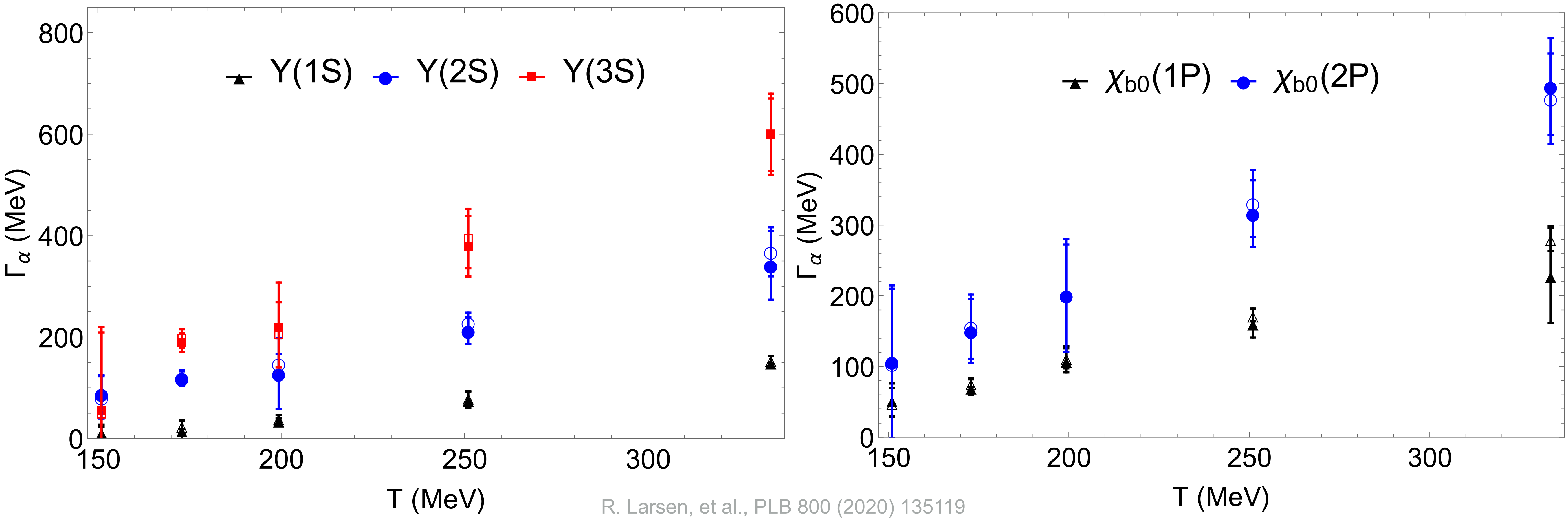


R. Larsen, et al., PLB 800 (2020) 135119

simple model: $\rho_\alpha^{\text{med}}(\omega,T) = A_\alpha^{\text{cut}}(T)\,\delta\left(\omega-\omega_\alpha^{\text{cut}}(T)\right) + A_\alpha(T)\exp\left(-\frac{[\omega-M_\alpha(T)]^2}{2{\Gamma_\alpha}^2(T)}\right)$

- No mass shift for all states

Thermal width of bottomonium



R. Larsen, et al., PLB 800 (2020) 135119

- Increasing thermal width with temperature for all states
- Thermal broadening follows the hierarchical increasing pattern

Part II
Complex quark-antiquark potential

Static quark potential in hot QCD

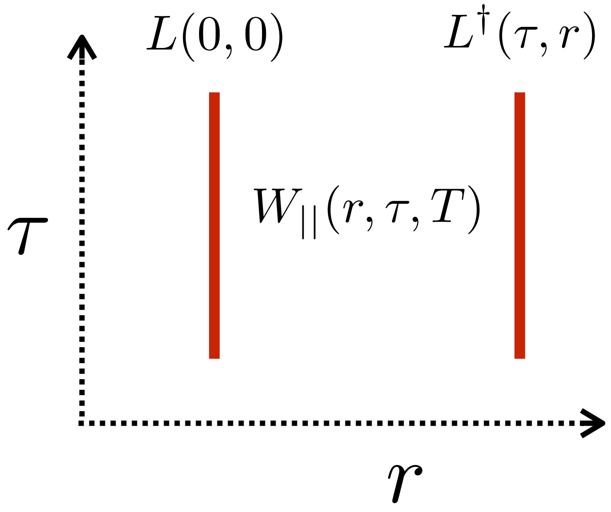
- Hard Thermal Loop resummed perturbation theory M. Laine, JHEP0703,054 (2007)

$$\lim_{t\to\infty} V^{(2)}_{>}(t,r) = -\frac{g^2C_F}{4\pi}\left[m_\mathrm{D} + \frac{\exp(-m_\mathrm{D}r)}{r}\right] - \frac{ig^2TC_F}{4\pi}\phi(m_\mathrm{D}r)$$

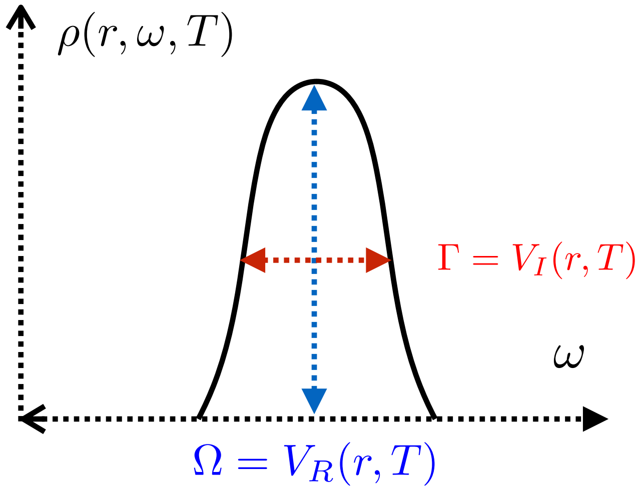
Imaginary part becomes important for physical bottomonium at T>250 MeV!

- Non-perturbative determination matters at around and above Tc

Wilson loop/thermal Wilson line correlators in Coulomb gauge A. Rothkopf et al., PRL. 108 (2012) 162001



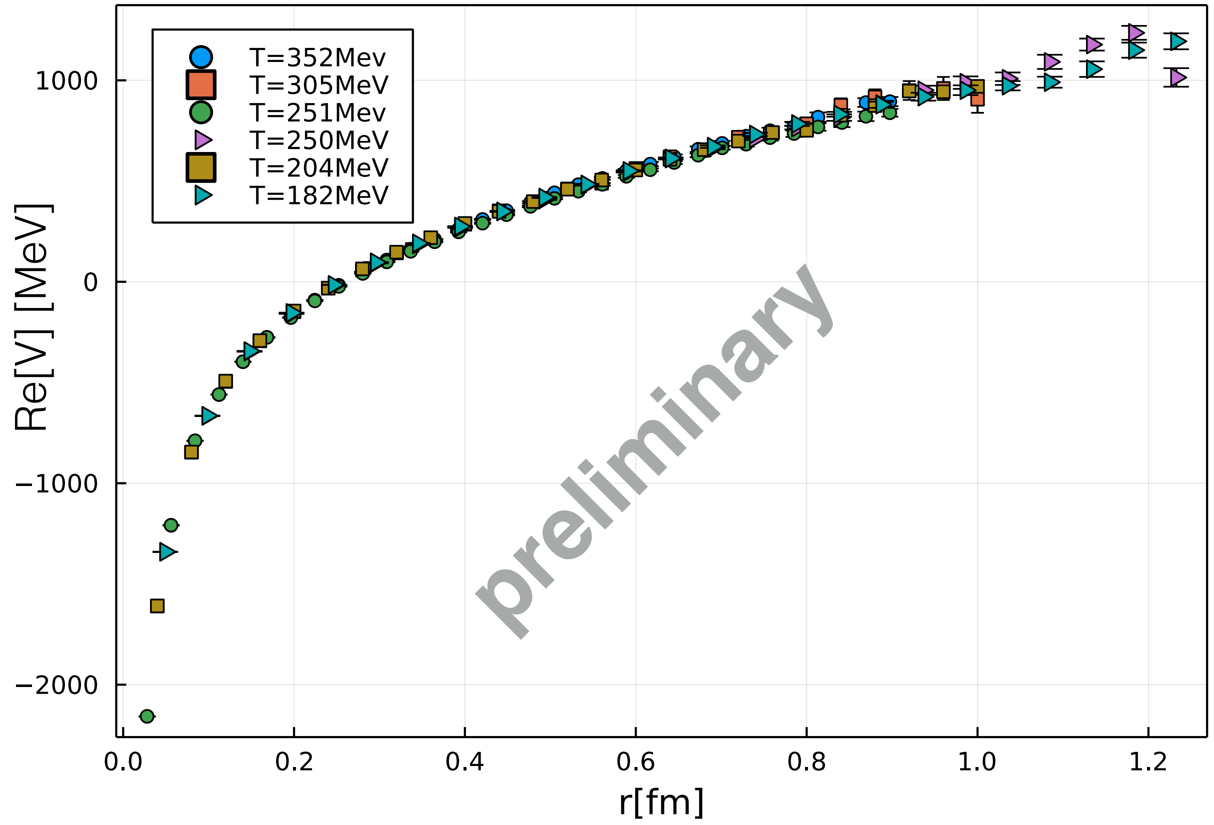
$$W_{||}(r,\tau,T) = \int d\omega\rho(r,\omega,T)e^{-\omega\tau}$$



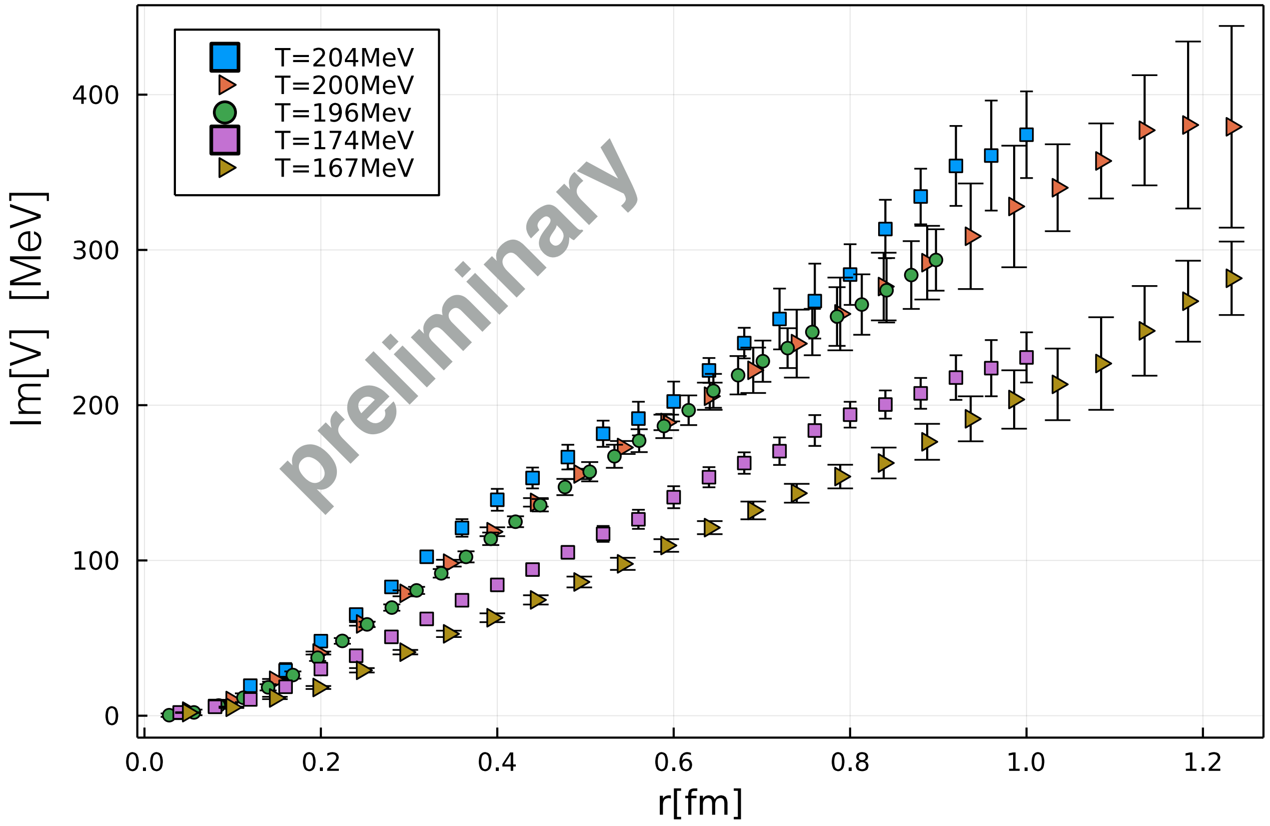
- Subtract continuum contribution from zero T correlators
- Model the potential as arguments of spectral function (modified Lorentzian)

Static quark potential in hot QCD

Real part: temperature insensitive



Imag. part: increasing with T & r



hotQCD, in preparation

- No color screening (no mass shift for bottomonium)

Update: hotQCD, complex potential 💬 *Peter Petreczky, Mon 14:10 [144]*

Part III

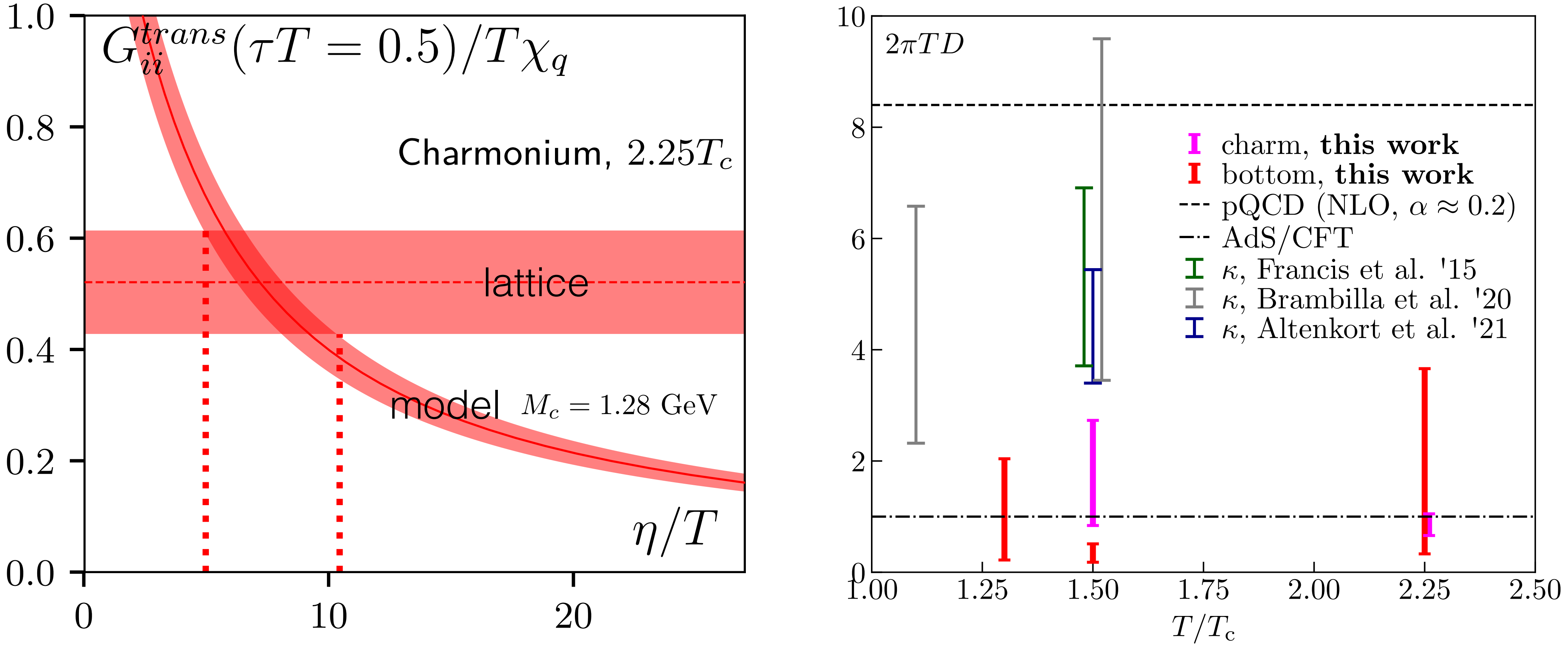
Heavy quark diffusion

Charm & bottom quark diffusion

Model the transport peak contribution of quarkonium correlators

$$G_{trans}(\tau) = G(\tau) - G_{mod}(\tau) = \int \frac{d\omega}{\pi} 3\chi_q D \frac{\omega\eta}{\omega^2+\eta^2} K(\omega,\tau,T)$$

H.-T. Ding, **HTS**, et al., PRD 104 (2021) 11, 114508



- Consistent with AdS/CFT

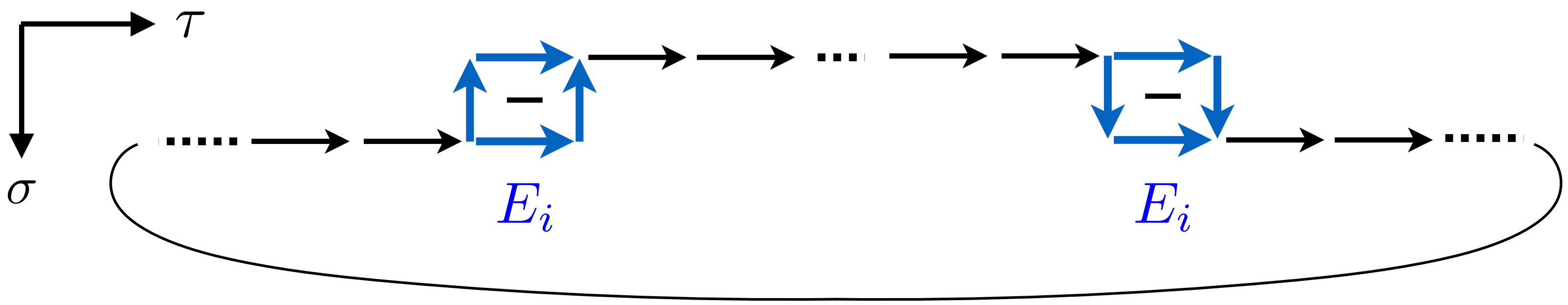
Static quark momentum diffusion

Heavy quark momentum diffusion coefficient from HQET

J. Casalderrey-Solana and D. Teaney, PRD 74, 085012
S. Caron-Huot et al., JHEP 0904 (2009) 053
A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

$$\frac{1}{2\pi T D} = \frac{\kappa}{4\pi T^3} = \frac{1}{2\pi T^2} \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$$

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2}\rangle \kappa_{\mathbf{B}} \qquad \langle \mathbf{v}^2\rangle = \frac{3T}{M}$$



Color-electric field correlation function

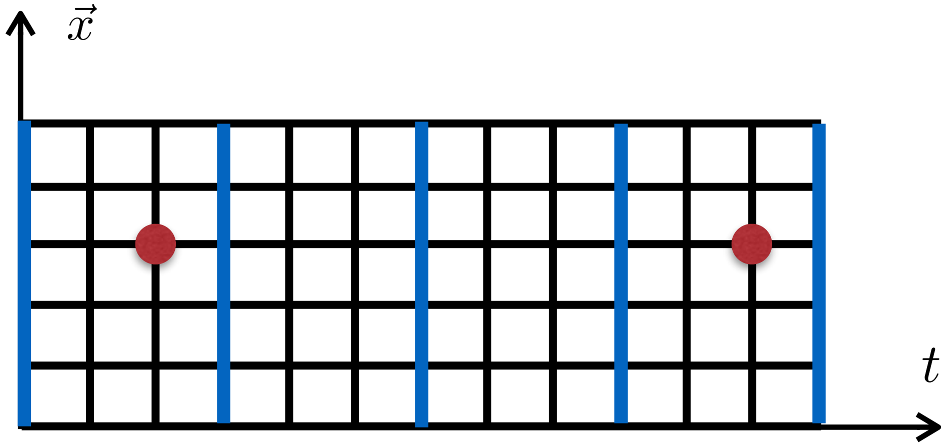
$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho(\omega, T)$$

- Cheaper to measure on the lattice
- No peak structures in spectral functions
- Absence of transport peak

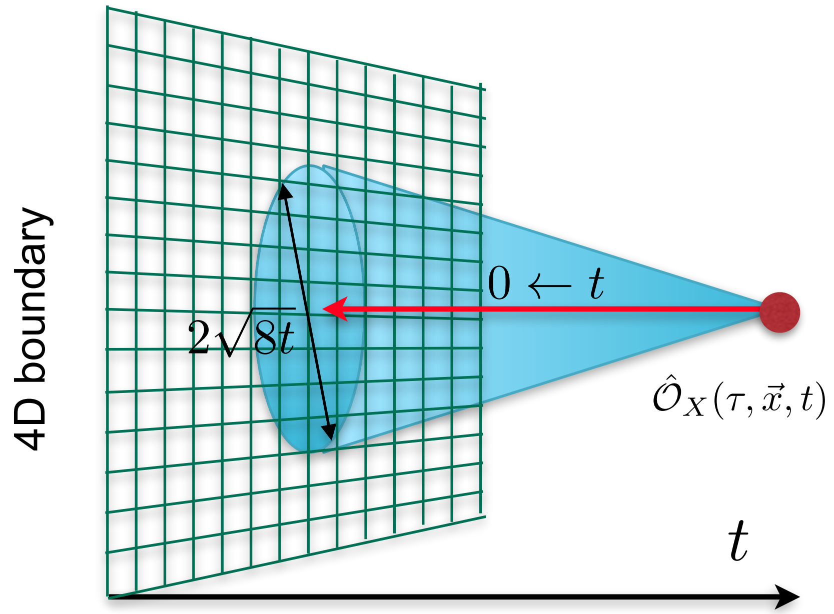
Update: quarkonium diffusion from adjoint chromo- electric/magnetic correlators 💬 *Viljami Leino, Tue 16:40 [357]*

Multi-level v.s. gradient flow

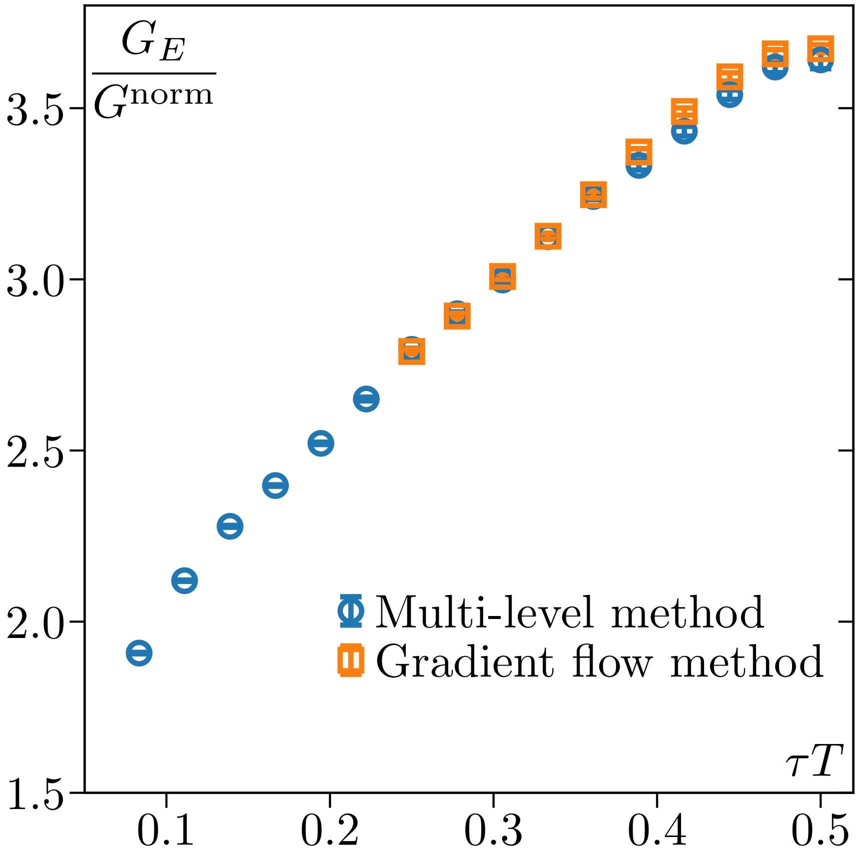
multi-level v.s. gradient flow



M. Luscher and P. Weisz, JHEP 09 (2001) 010



Luscher & Weisz, JHEP1102(2011)051
Narayanan & Neuberger, JHEP0603(2006)064

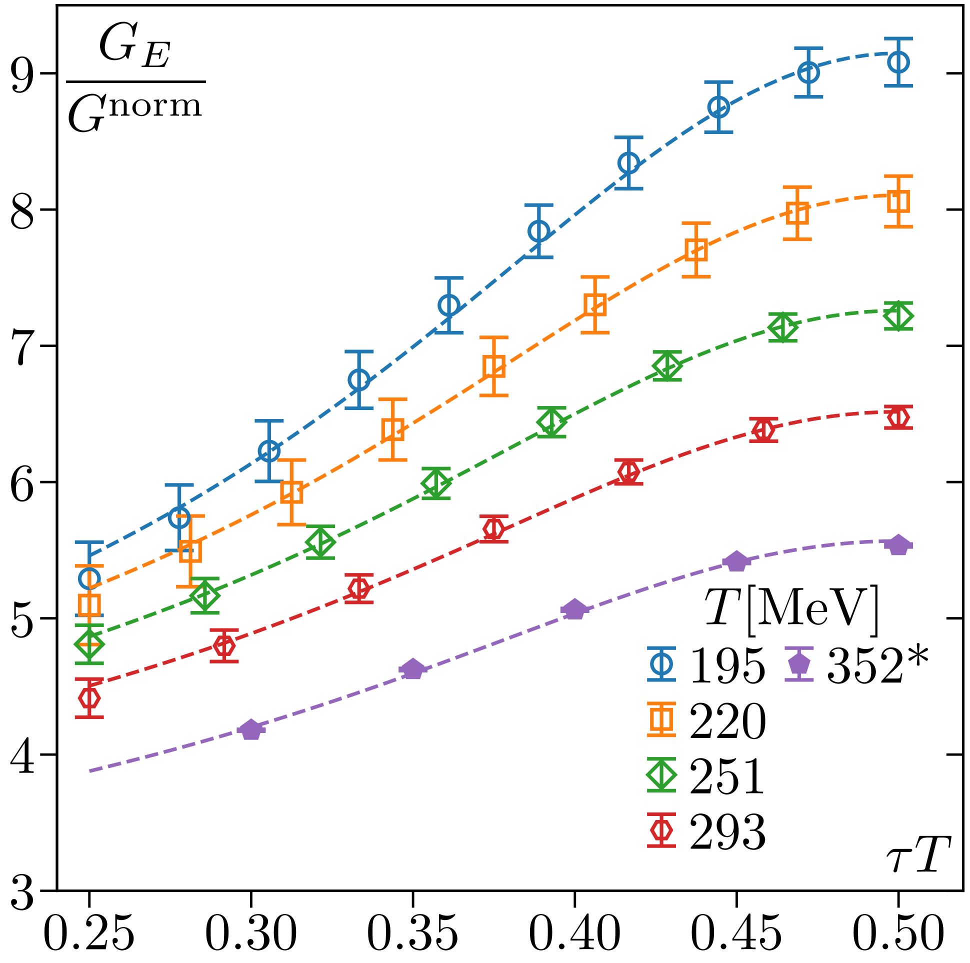


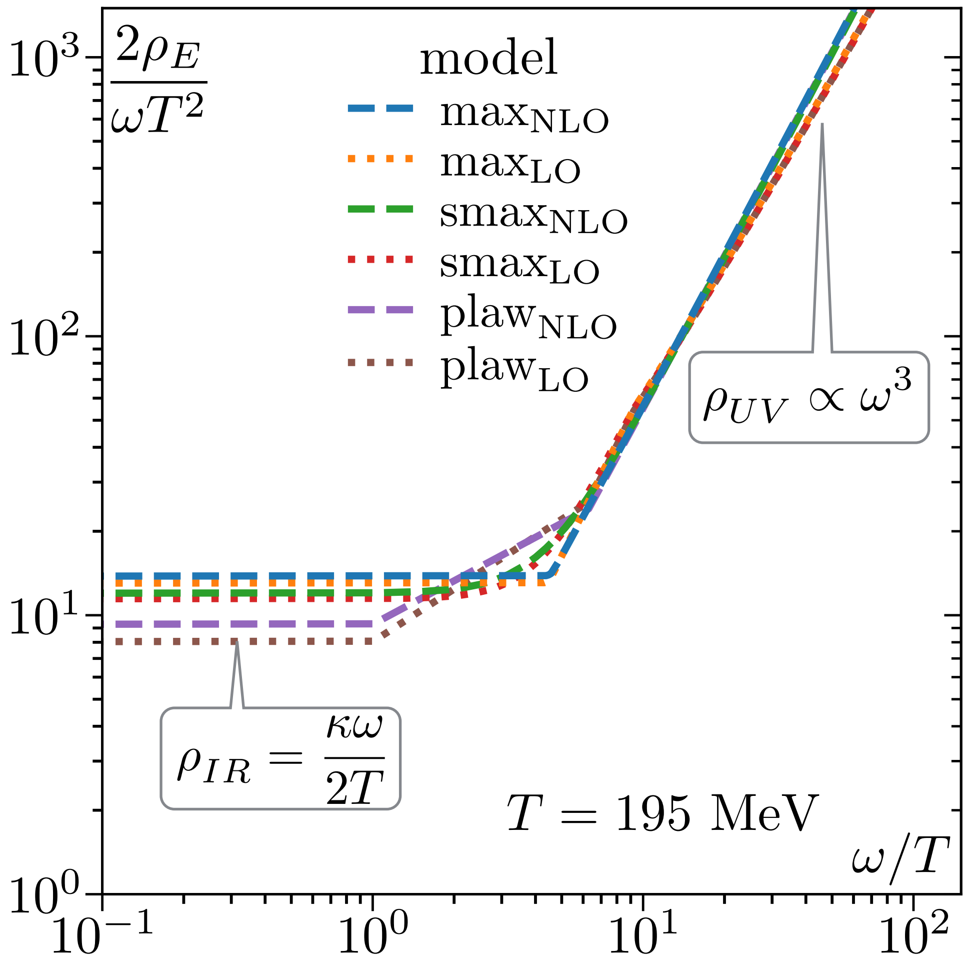
- Consistent quenched results from ML & GF
- Gradient flow paves the way to full QCD

L. Altenkort, **HTS**, et al., PRD 103 (2021) 1, 014511

Modeling the color-electric correlators

First full QCD calculation of kappa!

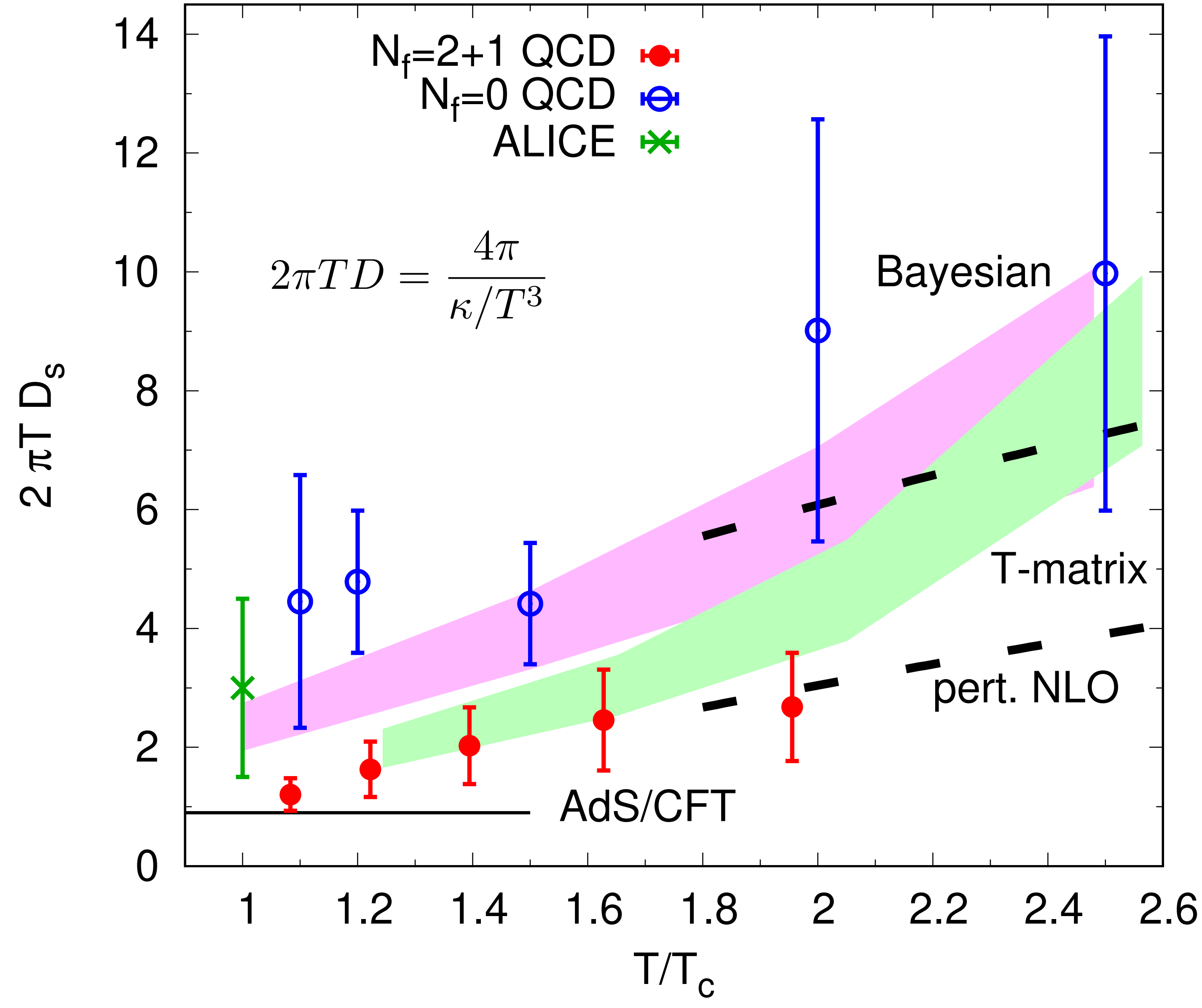




hotQCD, PRL 130 (2023) 23, 231902

- Wide temperature range with Mpi=320 MeV
- Much larger magnitudes in full QCD than in quenched QCD
- Good description of lattice data using different models

Summary of heavy quark diffusion coefficient



Quenched results (blue):

N. Branbilla, et al., PRD 102 (2020) 7, 074503

L. Altenkort, **HTS**, et al., PRD 103 (2021) 1, 014511

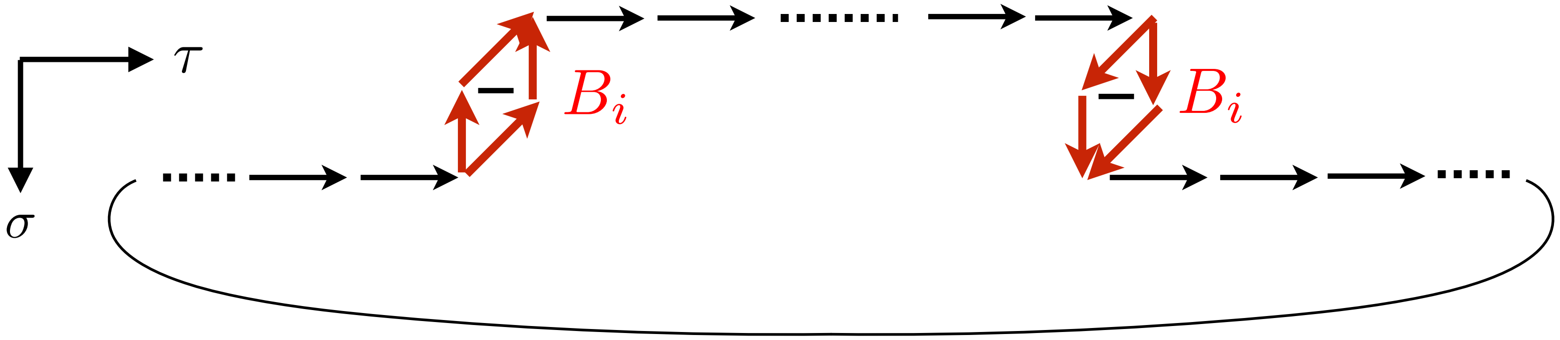
D. Banerjee, et al., arXiv: 2206.15471

Full LQCD results (red):

hotQCD, PRL 130 (2023) 23, 231902

- Agree with AdS/CFT at ~Tc (rapid equilibrium)
- Agree with T-matrix estimate at moderate T
- Agree with NLO perturbative estimate at large T
- Smaller than quenched estimates

Finite mass correction to HQ momentum diffusion



Color-magnetic field correlation function

A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

Charm & bottom quark not heavy enough

- Renormalization known in MSbar scheme via finite-volume scheme

D. Banerjee, et al., JHEP 08 (2022) 128

D. Guazzini, et al., JHEP 10 (2007) 081

$$Z_E = 1 + \delta Z_E \ = \ 1 + \mathcal{O}(g^4) \qquad Z_B = 1 + \delta Z_B \ = \ 1 + \frac{g^2 C_A}{(4\pi)^2}\left[\frac{1}{\epsilon} + 2\ln\left(\frac{\bar{\mu}e^{\gamma_E}}{4\pi T}\right) - 2\right] + \mathcal{O}(g^4)$$

- Matching under gradient flow solved

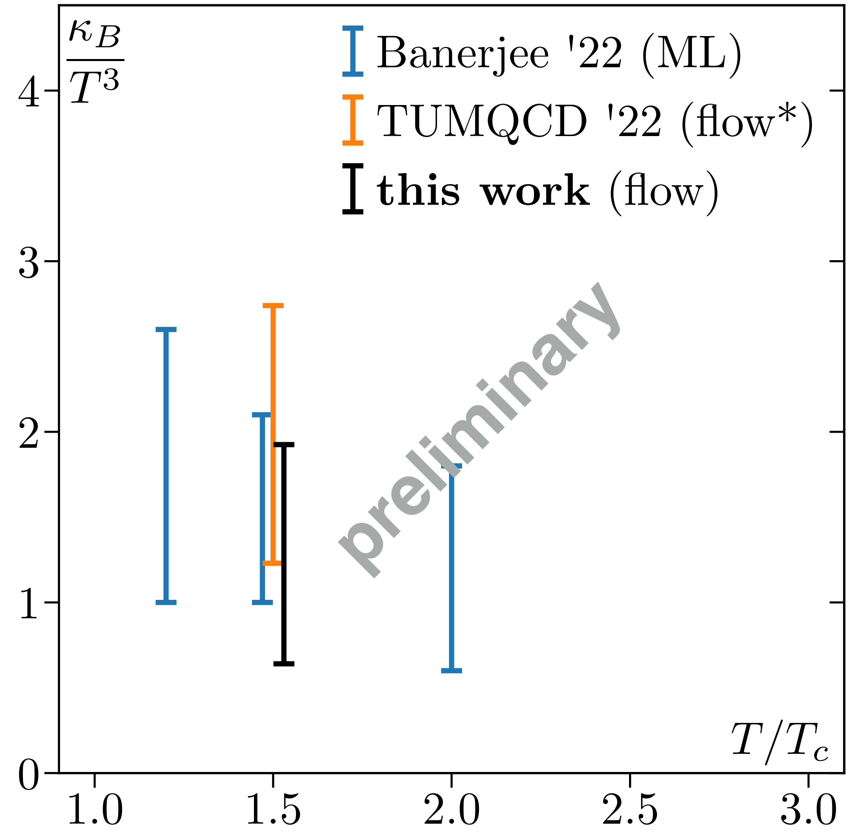
$$G_B = Z_B^2 Z^2 \langle G_B^{bare} \rangle_{\tau_F}$$

David Cruz, Guy Moore, in preparation

$$Z^2 = \left(1 - 2\frac{g^2 C_A}{16\pi^2}\ln(\mu^2\tau_F)\right)\left(1 + 2K\frac{g^2 C_A}{16\pi^2}\right) \equiv Z_f^2 Z_K^2$$

- Consistent quenched results among different calculations

L. Altenkort, **HTS**, et al. in preparation



Summary & outlook

Major achievements since Lattice 2022

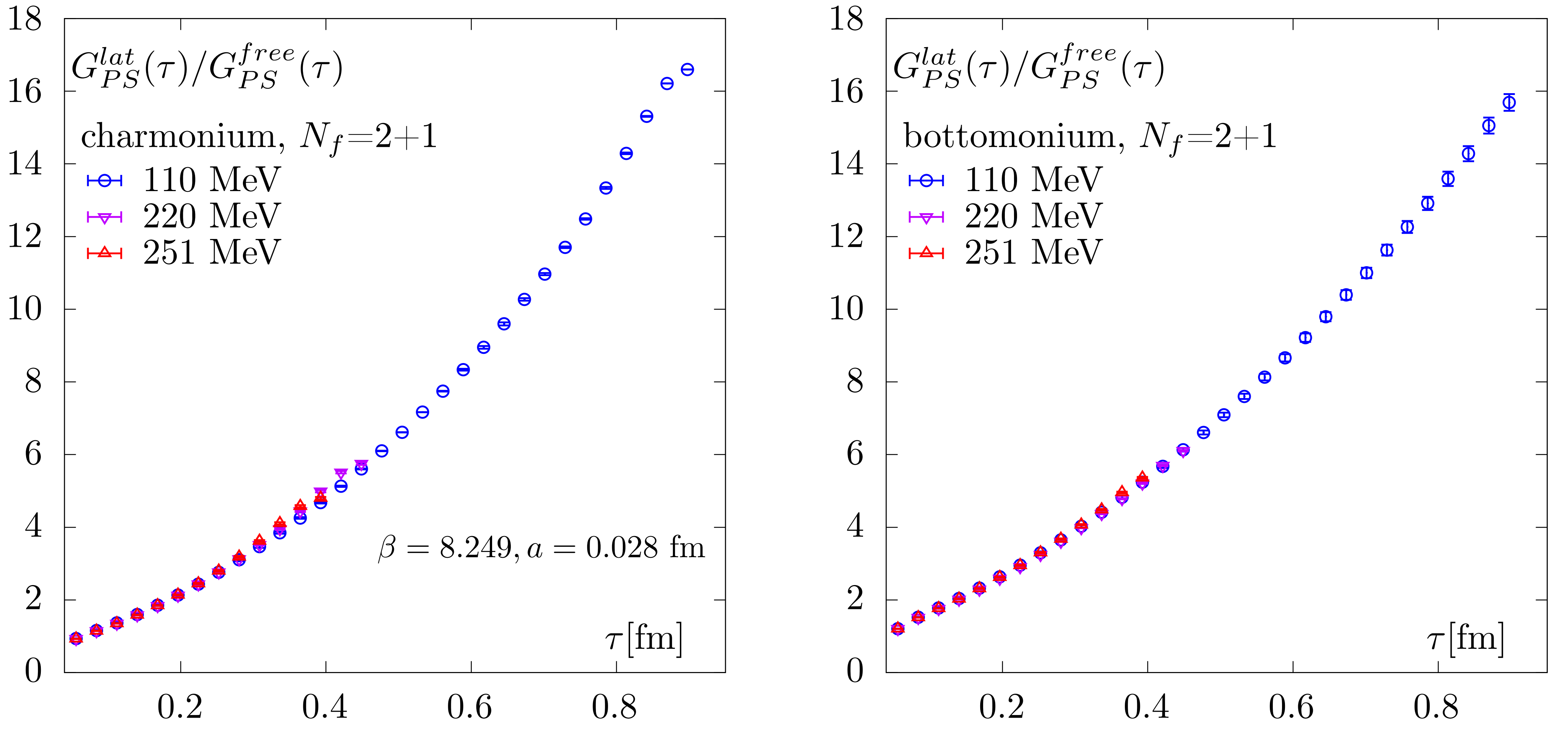
- First quarkonium study with relativistic heavy quarks in full QCD
  - ✓ Dissociation temperature of quarkonium
  - ◉ Heavy quark diffusion coefficient
- First full QCD calculation of static quark momentum diffusion
  - ✓ Much larger than in quenched case
  - ✓ Finite-mass correction is calculated in quenched case
  - ◉ Calculate finite-mass correction in full QCD
  - ◉ Calculate HQ momentum diffusion at physical pion mass
- No screening for Re[V] & increasing Im[V] with T & r
- No mass shift for bottomonium & increasing thermal width

Lattice QCD is in a phase to provide accurate and realistic inputs for HIC phenomenology

A special thank you to those who sent me materials and thoughts!
[Ali, Bala, Bazavov, Leino, Petreczky, Trimis, Weber]

Backup:Melting temperatures (relativistic)

✓ H.-T. Ding, et al., PRD 86, 014509 (2012), charmonium on quenched lattice
✓ Y. Burnier, et al., JHEP 1711 (2017) 206, eta_c & eta_b on quenched lattice in the continuum
✓ H.-T. Ding, et al., PRD 104 (2021) 11, 114508, J/Psi & Upsilon on quenched lattice in the continuum
✓ …
✓ **Update**: hotQCD, Few Body Syst. 64 (2023) 3, 52, quarkonium in Nf=2+1 HISQ sea



- potential NRQCD calculations applicable around the threshold M. Laine, JHEP 05 (2007) 028
- Ultraviolet asymptotics valid well above the threshold Y. Burnier, et al., EPJC72, 1902 (2012)
- Combine two parts by interpolation: Y. Burnier, et al., JHEP 11 (2017) 206

need higher temperatures and continuum limit

$$\rho_V^{pert}(\omega) = A^{match}\Phi(\omega)\rho_V^{\mathrm{pNRQCD}}(\omega)\theta(\omega^{match}-\omega) + \rho_V^{vac}(\omega)\theta(\omega-\omega^{match}) \Longrightarrow \rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$$

Backup:Melting temperatures (NRQCD)

Scale separation becomes an advantage in building NRQCD

- Absent of transport peak
- Unchanged physics for bound states
- Possibility of reaching HIC relevant temperatures

✓ G. Aarts, et al., JHEP 07 (2014) 097, bottomonium Nf=2+1 NRQCD, Mpi=400 MeV
✓ S. Kim, et al., JHEP 11 (2018) 088, quarkonium in Nf=2+1 NRQCD, Mpi=160 MeV

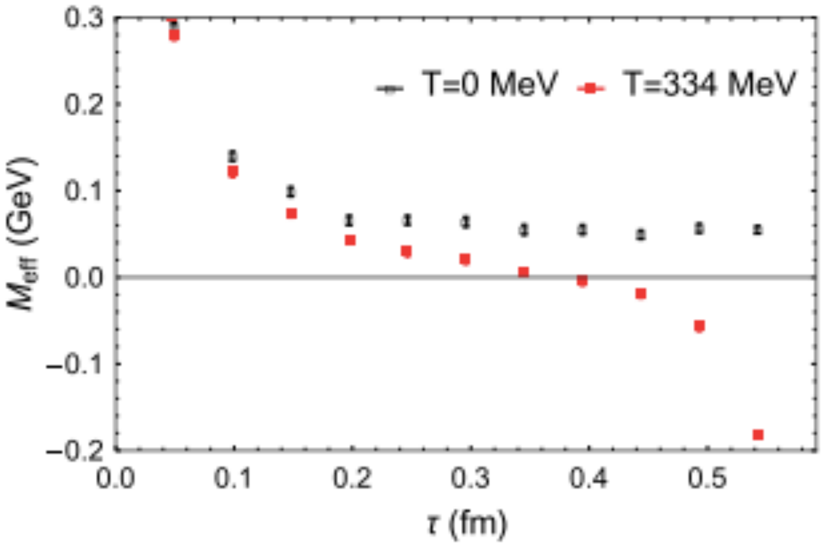
- Bottomonium S-wave survives up to T = 407MeV= 2.63Tc
- Bottomonium P-wave survives up to T = 185MeV= 1.19Tc
- Charmonium S-wave melts in 1.29 –1.35Tc
- Charmonium P-wave melts above 1.19Tc

✓ R. Larsen, et al., PRD 100 (2019) 7, 074506, bottomonium in Nf=2+1 NRQCD, Mpi=160 MeV, ground
✓ R. Larsen, et al., PLB 800 (2020) 135119, bottomonium in Nf=2+1 NRQCD, Mpi=160 MeV, excited

$$C_\alpha^{\mathrm{sub}}(\tau,T) = C_\alpha(\tau,T) - C_\alpha^{\mathrm{high}}(\tau)$$

kill low-energy tail

$$\rho_\alpha^{\mathrm{med}}(\omega,T) = A_\alpha^{\mathrm{cut}}(T)\,\delta\left(\omega-\omega_\alpha^{\mathrm{cut}}(T)\right) + A_\alpha(T)\exp\left(-\frac{\left[\omega-M_\alpha(T)\right]^2}{2\Gamma_\alpha^2(T)}\right)$$



continuum contribution subtracted from T=0 correlators
Long tail from the left of the peak cut by theta function

Backup:Melting temperatures (others)

Use anisotropic lattice for finer resolution in the temporal direction

✓ A. Jakovac, et al., PRD 75 (2007) 014506, quarknoium on quenched anisotropic lattice

Update: extension to HISQ! *Ioannis Trimis, Fri 9:20 [218]*

- Gauge anisotropy tuning performed with the Symanzik gradient flow
- Tuning of the strange quark mass and quark anisotropy using spectrum measurements on quenched ensembles
- Discussion on the impact of anisotropy on pion taste splittings for aHISQ

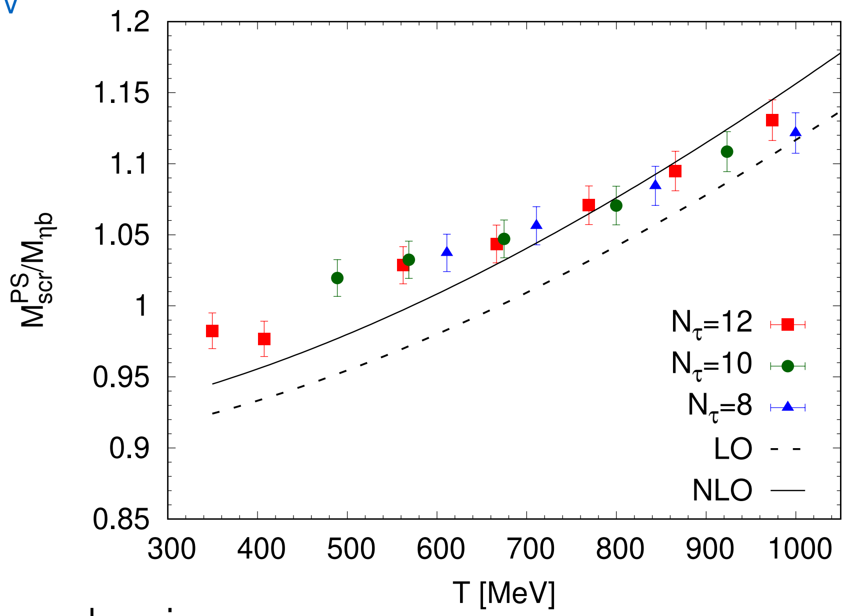
From the screening masses of quarkonium

✓ F. Karsch, et al., PRD 85 (2012) 114501, charmonium on Nf=2+1 lattice, Mpi=220 MeV, significant modification above 1.5Tc
✓ A. Bazavov, et al., PRD 91 (2015) 054503, charmonium on Nf=2+1 lattice, Mpi=160 MeV, significant modification above 1.3Tc
✓ P. Petrezcky, et al., PRD 104. 054511 (2021), bottomonium on Nf=2+1 lattice, Mpi=160 MeV

- In-medium screening masses from *spatial* bottomonium correlators
- Compare finite-T masses and zero-T masses

$$C_M(z) = \int dxdyd\tau \langle J_M(\mathbf{x}) J_M(0)\rangle = A_{NO}\cosh\left[M_{NO}\left(z-\frac{N_s}{2}\right)\right] - (-1)^z A_O \cosh\left[M_O\left(z-\frac{N_s}{2}\right)\right].$$

- the eta_b(1S) and Upsilon(1S) states melt at T > 500 MeV
- chi_b0(1P) and h_b(1P) melt at T > 350 MeV
- Need understand mechanism of dissociate at intermediate temperatures



- Different methods developed to investigate the dissociation of quarkonium
- Understanding dissociation from the binding energy and thermal width?
- (1S) charmonium melts at T≳ 1.2Tc
- (1S) bottomonium melts at T≳ 2.63Tc

Backup: Complex potential in hot QCD

- Hard Thermal Loop resummed perturbation theory M. Laine, JHEP0703,054 (2007)

$$\lim_{t\to\infty} V^{(2)}_{>}(t,r) = -\frac{g^2 C_F}{4\pi}\left[m_{\mathrm{D}} + \frac{\exp(-m_{\mathrm{D}} r)}{r}\right] - \frac{i g^2 T C_F}{4\pi}\phi(m_{\mathrm{D}} r)$$

Real part: binding energy ~ peak location
Imag part: thermal width ~ peak width

Imaginary part becomes important for physical bottomonium at T>250 MeV!

- Non-perturbative determination matters at around and above Tc

Lattice formulation via Wilson loop/thermal Wilson line correlators in Coulomb gauge

A. Rothkopf et al., PRL. 108 (2012) 162001

1. Wilson loop as static quarkonium correlator

$$\langle(\bar{Q}Q)(\bar{Q}Q)^\dagger\rangle \overset{m_Q\to\infty}{=} W_\square(r,t)$$

2. Real-time evolution ignoring non-potential effects

$$i\partial_t W_\square(r,t) \simeq V(r) W_\square(r,t)$$

3. Solution of potential in spectra representation

$$V(r) = \lim_{t\to\infty}\frac{i\partial_t W_\square(r,t)}{W_\square(r,t)} = \lim_{t\to\infty}\frac{\int d\omega\,\omega\rho(r,\omega)e^{-i\omega t}}{\int d\omega\rho(r,\omega)e^{-i\omega t}}$$

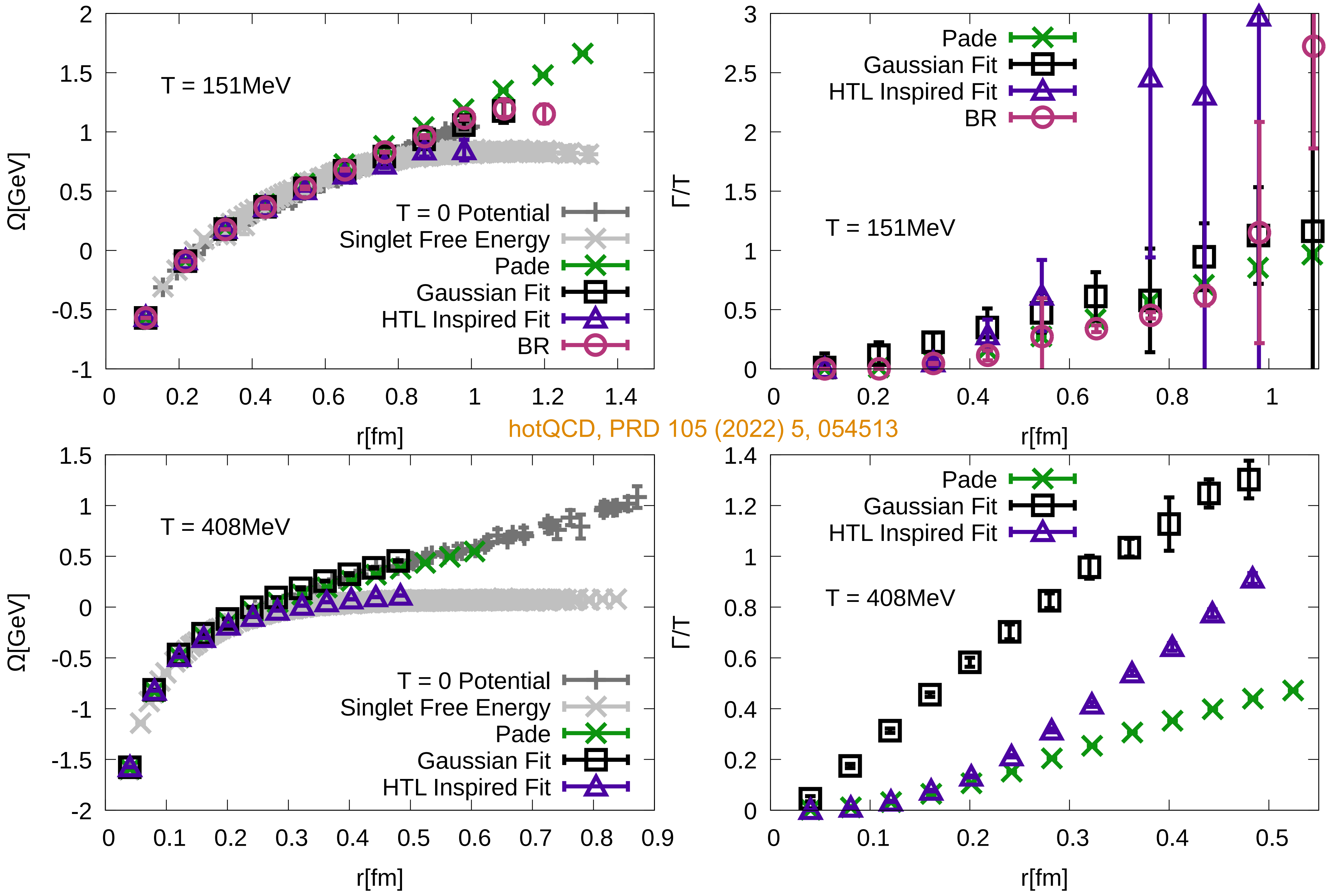
4. Re[V] and Im[V] as arguments of spectra

$$W_\square(r,t) = \int d\omega\rho(r,\omega)e^{-i\omega t} \quad \text{<—>} \quad W_\square(r,\tau) = \int d\omega\rho(r,\omega)e^{-\omega\tau}$$

- Model of the spectral function: Gaussian, Lorentzian, Pade, HTL, Bayesian analysis…

✓ A. Rothkopf, et al., PRL108 (2012) 162001, quenched via WLoop, first lattice study, MEM
✓ Y. Burnier, A. Rothkopf, PRD 86, 051503 (2012), quenched via WLoop, Modified Breit-Wigner
✓ Y. Burnier, A. Rothkopf, PRD 87, 114019 (2013), quenched via WLine, HTL fits
✓ Y. Burnier, A. Rothkopf, PRL.111, 182003 (2013), quenched via WLoop, BR method
✓ Y. Burnier, et al., JHEP 12 (2015) 101, Nf=2+1, Mpi=160 MeV via WLoop, Gaussian
✓ hotQCD, PRD 105 (2022) 5, 054513, Nf=2+1, Mpi=160 MeV via WLine, Gaussian, Pade, HTL, BR

Backup: Complex potential at T=0 and T>0

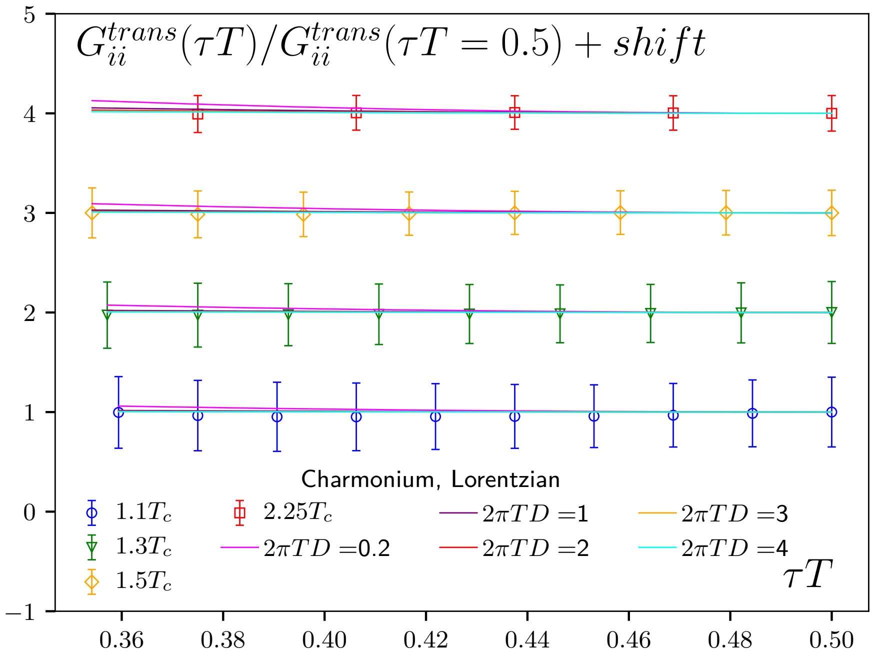


hotQCD, PRD 105 (2022) 5, 054513

HISQ, Nt=12

Real part determines the binding energy.
Imginary part gives the thermal width, characterizing the thermal effects on the dilepton production from bottomonium.
The imaginary part originating from the Landau-damping of low-frequency gauge fields.

Backup: HQ diffusion at physical quark masses



- Reconstruct the transport contribution:

$$G_{ii}^{trans}(\tau T)=G_{ii}(\tau T)-G_{ii}^{mod}(\tau T)$$

- Modeling the transport peak

$$G_{trans}(\tau)=\int\frac{d\omega}{\pi}K(\omega,\tau)\rho_{ii}^{trans}(\omega)$$

$$\rho_{ii}^{trans}(\omega)=3\chi_q D\frac{\omega\eta^2}{\omega^2+\eta^2}$$

Backup: kappa$_E$ from lattice QCD

- Leading term of heavy quark diffusion using ML and GF

  ✓ A. Francis, et al., PRD92 (2015)116003, quenched using ML
  ✓ L. Altenkort, et al., PRD 103 (2021) 1, 014511, quenched using GF
  ✓ TUMQCD, PRD 107 (2023) 5, 054508, quenched using GF
  ✓ D. Banerjee, et al., arXiv: 2206.15471, quenched using ML
  ✓ **Update**: hotQCD, PRL 130 (2023) 23, 231902, Nf=2+1 HISQ, Mpi=320 MeV, using GF

- Construct a kinetic mass dependent momentum diffusion coefficient

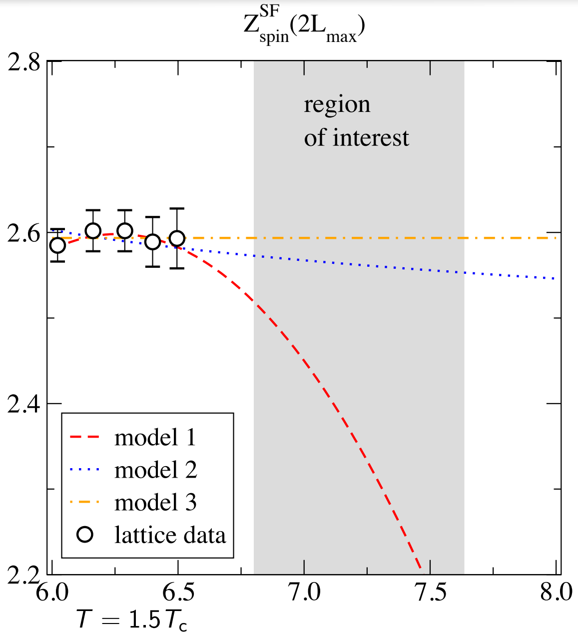
$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_q}\sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Big|^{\eta\ll|\omega|\lesssim\omega_{\rm UV}} \quad + \quad D = T/(\eta M) \quad \Longrightarrow \quad D = \frac{2T^2}{\kappa^{(M)}}$$

S. Caron-Huot et al., JHEP 0904 (2009) 053

Backup:Finite mass correction kappa$_B$

charm & bottom
not heavy enough

✓ D. Banerjee, et al., JHEP 08 (2022) 128, quenched using ML
✓ TUMQCD, PRD 107 (2023) 5, 054508, quenched using GF
✓ L. Altenkort, et al. in preparation, quenched&full QCD using GF



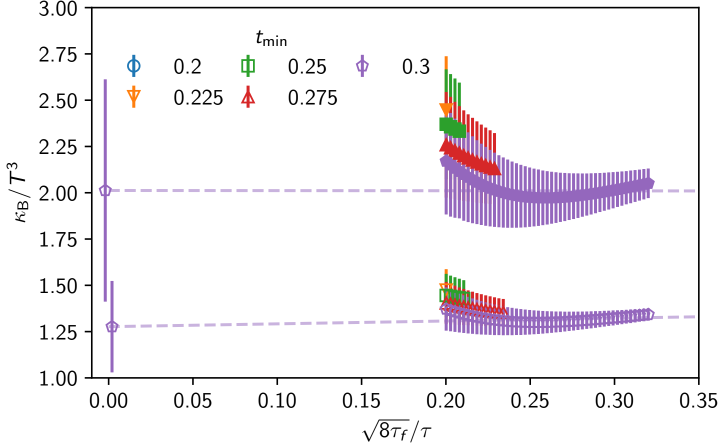
- Renormalization known in MSbar scheme via finite-volume scheme

$$Z_E = 1 + \delta Z_E \;=\; 1 + \mathcal{O}(g^4) \qquad Z_B = 1 + \delta Z_B \;=\; 1 + \frac{g^2 C_A}{(4\pi)^2}\left[\frac{1}{\epsilon} + 2\ln\left(\frac{\bar\mu e^{\gamma_E}}{4\pi T}\right) - 2\right] + \mathcal{O}(g^4)$$

$$\frac{[G_B(\tau)]_{\rm physical}}{[G_B(\tau)]_{\rm bare,L}} = \left\{\frac{\Phi_{\overline{\rm ms}}(\bar\mu = 19.179T)}{\Phi_{\rm RGI}} \times \frac{\Phi_{\rm RGI}}{\Phi_{\rm SF}(\frac{1}{2L_{\rm max}})} \times Z^{\rm SF}_{\rm spin}(2L_{\rm max})\right\}^2 \qquad Z^{\rm SF}_{\rm spin}(2L_{\rm max}) \overset{\text{model 1}}{\approx} 2.58 + 0.14\,(\beta - 6) - 0.27(\beta - 6)^2$$

D. Guazzini, et al., JHEP 10 (2007) 081 D. Banerjee, et al., JHEP 08 (2022) 128

- Renormalization under gradient flow TUMQCD, PRD 107 (2023) 5, 054508



$$G_B^{\rm flow,UV}(\tau,\tau_F) = (1 + \gamma_0 g^2 \ln(\mu\sqrt{8\tau_F}))^2 Z_{\rm flow} G_B^{\overline{\rm MS},\rm UV}(\tau,\mu) + h_0 \cdot (\tau_F/\tau)$$

$$\rho_B^{\rm UV}(\omega,\tau_F) = Z_{\rm flow}\frac{g^2(\mu)\omega^3}{6\pi}(1 + g^2(\mu)(\beta_0 - \gamma_0)\ln(\mu^2/(A\omega^2)) + g^2(\mu)\gamma_0\ln(8\tau_F\mu^2)$$

workaround by spf reconstruction at finite flow time treating renormalization a fit parameter

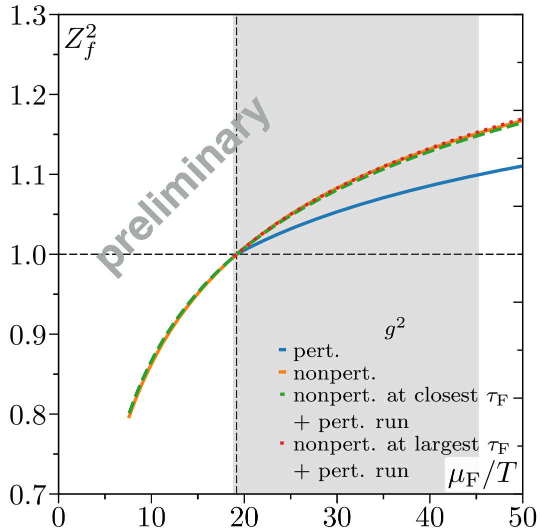
Direct solve of the renormalization under flow !

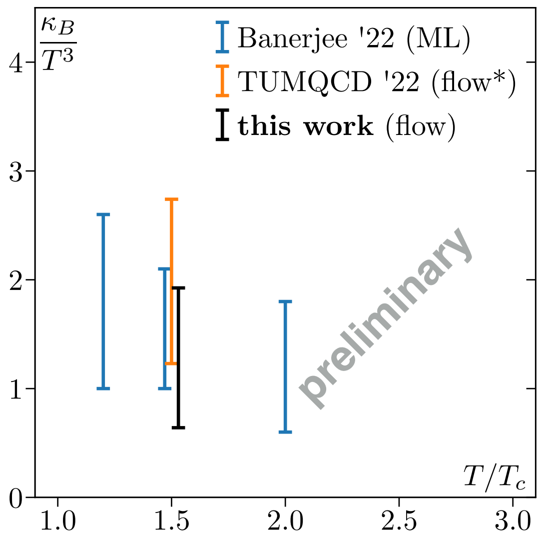
$$G_B = Z_B^2 Z^2 \langle G_B^{bare}\rangle_{\tau_F}$$

$$Z^2 = \left(1 - 2\frac{g^2 C_A}{16\pi^2}\ln(\mu^2\tau_F)\right)\left(1 + 2K\frac{g^2 C_A}{16\pi^2}\right) \equiv Z_f^2 Z_K^2$$

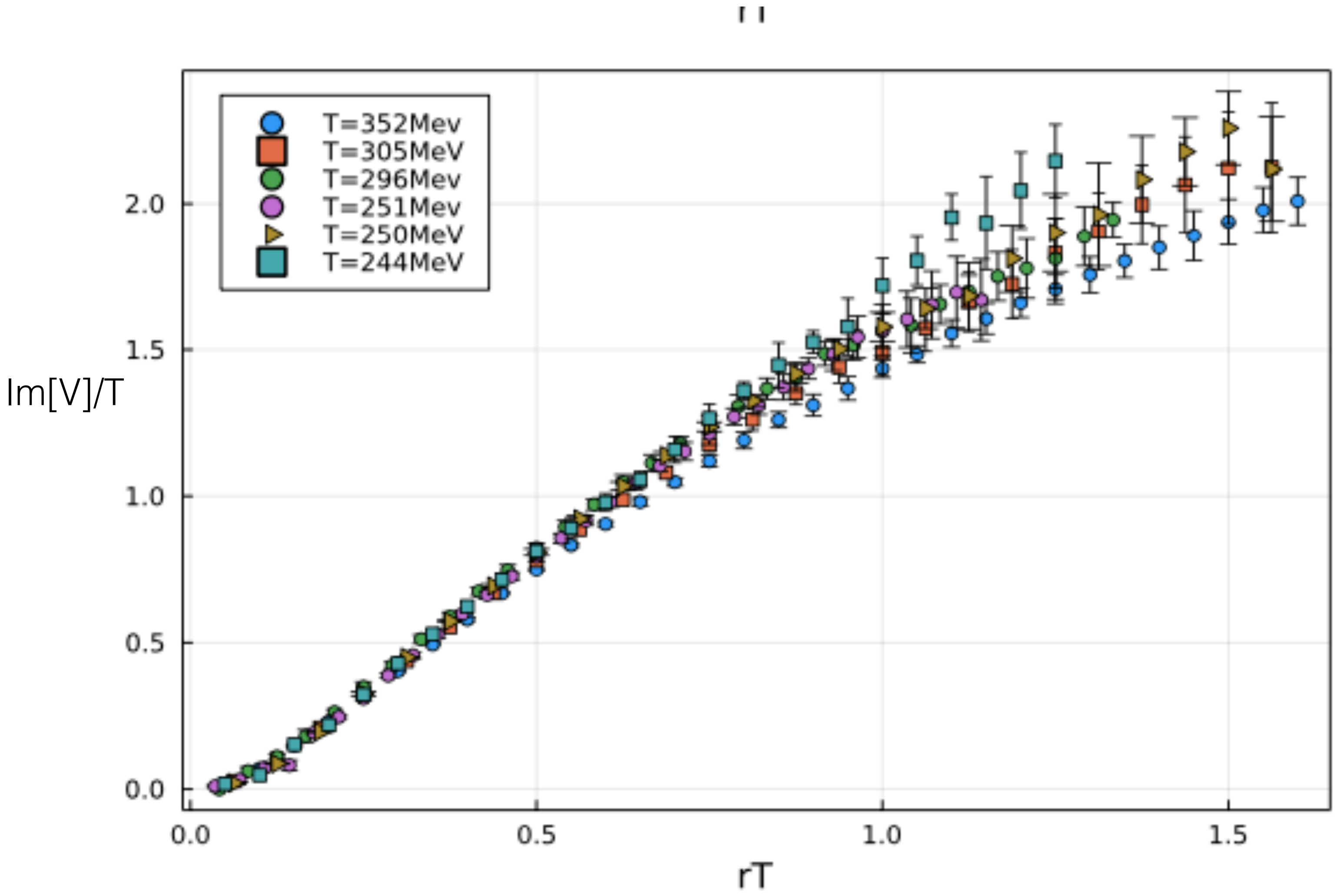
$$K = 8/3 + \gamma_E - \ln(2)$$

L. Altenkort, et al. in preparation





Backup: why no screening



t>>1/T for free energy, dissociation & recombination
1/Im[V] < 1/T decays within time scale