B-meson semileptonic decays from highly improved staggered quarks

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Motivation

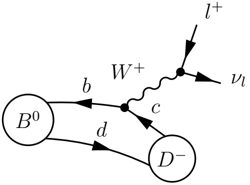
- Semileptonic decays are a rich source of information for determining CKM matrix elements.
- Relatively simple decay processes – measured in accelerator experiments, require theoretical input from lattice QCD to extract fundamental parameters.
- Desire precise measurements of $|V_{xb}|$ from multiple decay processes to test the consistency of the Standard Model.

Semileptonic decays

SL Decay processes critical inputs for heavy flavor studies.

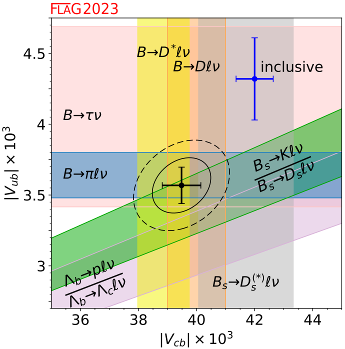
Lattice predictions needed for:

- Extracting CKM matrix elements from expt'l measurements
- Pure SM predictions of R-ratios
- SM predictions $\frac{d\Gamma}{dq^2}$, etc.



Lattice calculations based on 2- & 3-point correlators give matrix elements $\rightarrow f_i(q^2)$

Stress-testing the CKM paradigm



- Inclusive/exclusive discrepancies for $|V_{ub}|$ and $|V_{cb}|$
- Also discrepancies from SM expectations in $R(D, D^*, J/\psi, ...)$ see e.g. Snowmass 2205.15373
- $\rightarrow$ Want high accuracy SM predictions for sl decays

Outline

1. Intro & Motivation.
2. Computational framework.
3. Status and preliminary results.
   - Two-point and three-point correlators.
   - Form factor results.
   - Renormalization.
4. Summary & Outlook.

Heavy quarks

Treatment of c and especially b quarks challenging in lattice simulation due to lattice artifacts which grow as $(am_h)^n$

- May use an effective theory framework to handle the b quark.
  - Fermilab method, RHQ, OK, NRQCD
  - Pros: Solves problem w/ am_h artifacts.
  - Cons: Requires matching, can still have ap artifacts.

- Also possible to use relativistic fermion provided a is sufficiently small $am_c \ll 1$, $am_b < 1$.
  - Use improved actions e.g. $\mathcal{O}(a^2) \to \mathcal{O}(\alpha_s a^2)$
  - Pros: Absolutely normalised current, straightforward continuum extrap.
  - Cons: Numerically expensive, extrapolate $m_h \to m_b$.

allhisq simulations

- Here we simulate *all* quarks with the HISQ action.
- Unified treatment for wide range of $B_{(s)}$ (and $D_{(s)}$) to pseudoscalar tr
  - $B_{(s)} \to D_{(s)}$
  - $B_{(s)} \to K$
  - $B \to \pi$
- Ensembles with (HISQ) sea quarks down to physical at each lattice spacing.

MILC ensembles [1004.0342, 1212.4768]

- HISQ fermion action.
  - Discretization errors begin at $\mathcal{O}(\alpha_s a^2)$.
  - Designed for simulating heavy quarks (m_c and higher at current lattice spacings).

- Symanzik-improved gauge action, takes into account $\mathcal{O}(N_f \alpha_s a^2)$ effects of HISQ quarks in sea. [0812.0503]

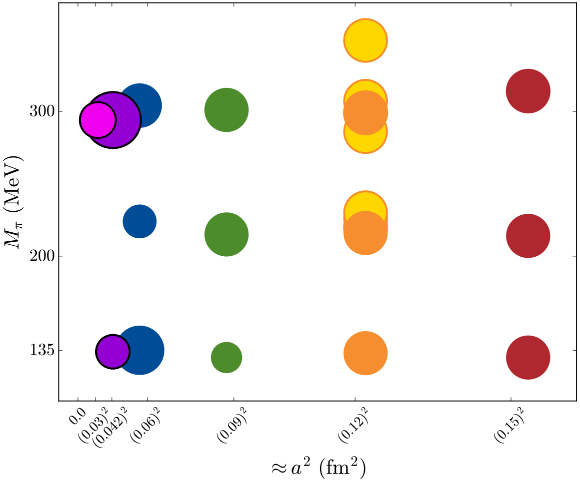
- Multiple lattice spacings down to ~ 0.042 (now 0.03) fm.

- Effects of u/d, s, and c quarks in the sea.

- Multiple light-quark input parameters down to physical pion mass.
  - Chiral fits.
  - Reduce statistical errors.

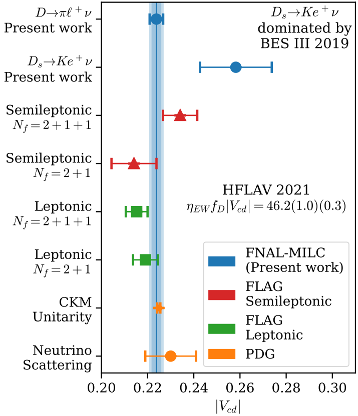
MILC ensemble parameters

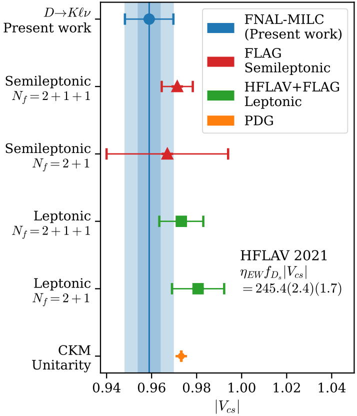
1712.09262



Results for D decays

2212.12648





all hisq b

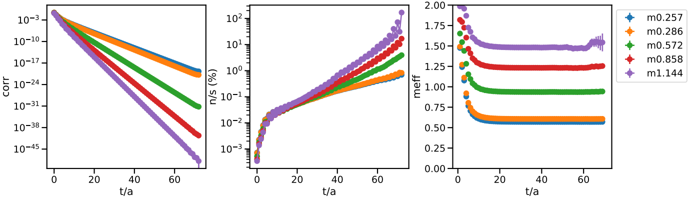
- Use a heavy valence mass h as a proxy for the b quark.
- Work at a range of m_h, with $am_c < am_h \lesssim 1$ on each ensemble. On sufficiently fine ensembles, m_h is near to m_b (e.g. m_b at $am_h \approx 0.65$ on $a = 0.03$ fm).
- Map out physical dependence on m_h, remove discretisation effects $\sim (am_h)^{2n}$ using information from several ensembles. Extrapolate results $a^2 \to 0, m_h \to m_b$.

Preliminary results

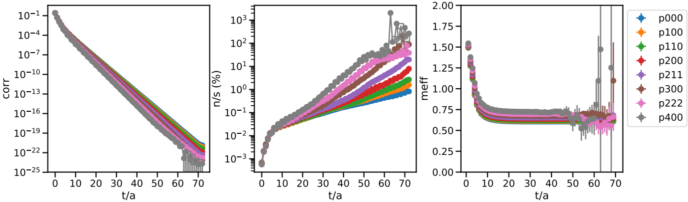
Two point functions

Consider $B_{(s)} \to D_{(s)}$ decays for $a = 0.06$ fm, $m_l/m_s = 0.1$.

- Compute $H_{(s)}$ mesons at a range of am_h values:

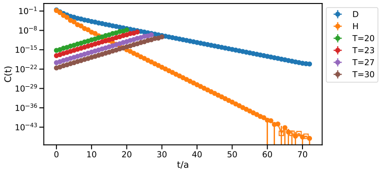


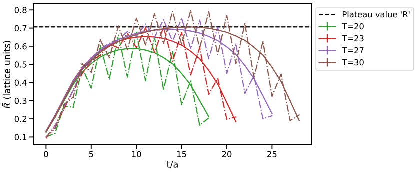
- $D_{(s)}$ mesons for a range of momenta:



Three point functions

- Generate three-point functions for scalar, vector, and tensor current insertions, $\langle D_{(s)}(T)\, J(t)\, H^\dagger_{(s)}(0)\rangle$.
- Fit simultaneously with two-point functions to extract the matrix elements of interest $\rightarrow \langle D_{(s)}|J|H_{(s)}\rangle$





- We use scalar (S), and vector (V^0, V^i) current insertions to extract the form factors f_0 and f_+.

Extracting form factors

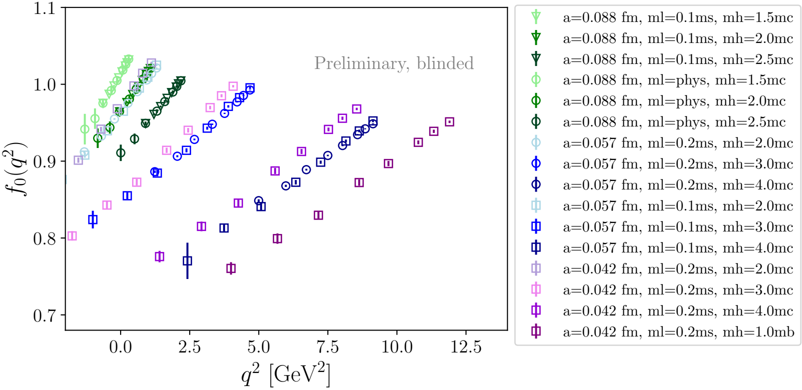
$$f_0(q^2) = \frac{m_h - m_\ell}{M_H^2 - M_L^2} \langle L|S|H\rangle$$

$$f_\parallel(q^2) = Z_{V^0} \frac{\langle L|V^0|H\rangle}{\sqrt{2M_H}}$$

$$f_\perp(q^2) = Z_{V^i} \frac{\langle L|V^i|H\rangle}{\sqrt{2M_H}} \frac{1}{p_L^i} ,$$

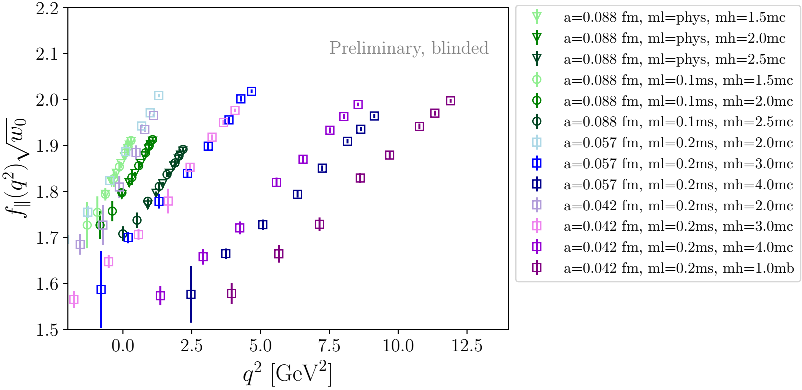
$$f_+ = \frac{1}{\sqrt{2M_H}} \left(f_\parallel + (M_H - E_L) f_\perp \right) .$$

$B_s \to D_s$: $f_0(q^2)$



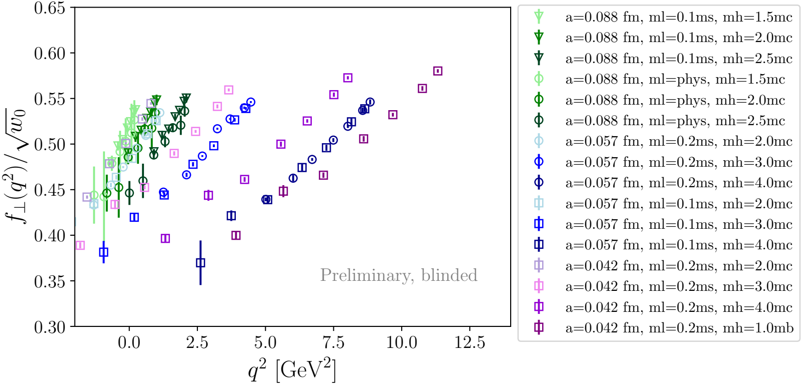
- Good precision out to $\mathtt{p} = \mathtt{400}$
- Rightmost points on figure have $m_h = m_b$

$B_s \to D_s$: $f_\parallel(q^2)$



- Good precision out to $\mathtt{p} = 400$
- Rightmost points on figure have $m_h = m_b$

$B_s \to D_s$: $f_\perp(q^2)$

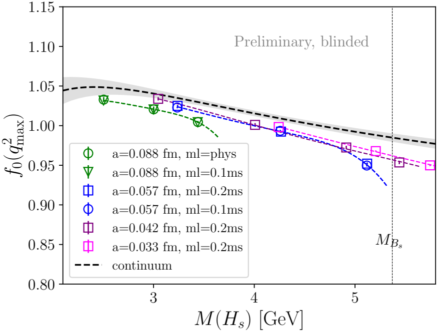


- Good precision out to $\mathtt{p} = 400$
- Rightmost points on figure have $m_h = m_b$

$B_s \to D_s$ - a simple $f_0(q^2_{\text{max}})$ fit

Basic fit parameterizing M_H dependence and heavy quark discretization.

$$f_0(q^2_{\text{max}})[M_H, am_h] = \sum_{ij} c_{ij} \left(\frac{1}{M_H}\right)^i \left(am_h\right)^{2j}$$



Good precision obtained ($\sim 0.5\%$) at M_{B_s}.

Chiral/cont. extrapolations

Build from chiral forms used in D analysis.

$$f_{0,\parallel,\perp}(E) = \frac{c_0}{E+\Delta}(1 + \cdots + c_H \chi_{H_s} + \cdots)$$

$$\Delta = \frac{M_{D^*}^2 - M_{D_s}^2 - M_K^2}{2M_{D_s}}, \qquad \chi_{H_s} = \frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} - \frac{\Lambda_{\mathrm{HQET}}}{M_{D_s}^{\mathrm{PDG}}}$$

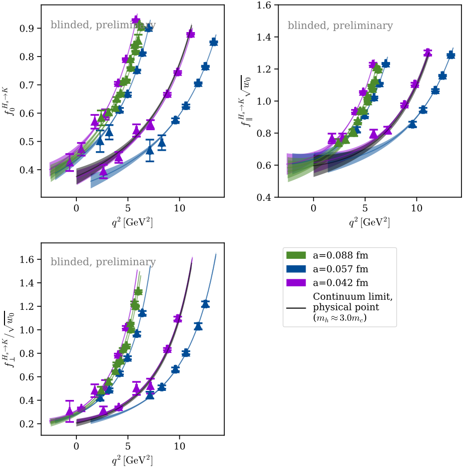
Generalize to incorparate HQET expansion:

$$c_0 \to c_0 + c_1 \frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} + \cdots, \quad \Delta \to \frac{M_{D^*}^2 - M_{D_s}^2 - M_K^2}{2M_{H_s}} \text{ (1st order)}$$

$$\chi_{H_s} = \frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} - \frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}^{\text{``phys''}}}$$

$H_s \to K$

Here building off D_s chiral analysis, working out towards B_s.



- Data at 2–$3m_c$, 3 lattice spacings, 3 $m_{l,\mathrm{sea}}$ values
- Note 0.057 fm has $m_h \approx 2.2, 3.3 m_c$
- Reasonable $\chi^2/\mathrm{dof} = 0.92, 1.79, 0.75$ for $f_0, f_\parallel, f_\perp$

Current normalization

Normalization of vector currents

We renormalize the vector current by applying the partially conserved vector current (PCVC) relation directly to extracted matrix elements:

$$\partial_\mu V_\mu^{\mathrm{cons}} = (m_h - m_l) S$$

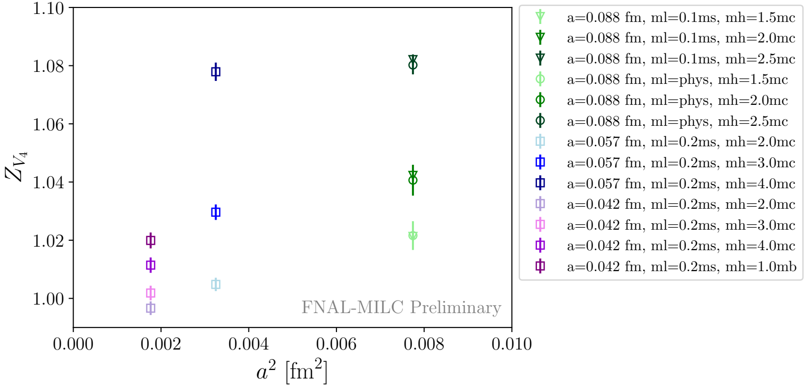
Applied to our lattice matrix elements,

$$Z_{V^0}(M_H - E_L)\langle L|V^0|H\rangle + Z_{V^i}\mathbf{q}\cdot\langle L|\mathbf{V}|H\rangle = (m_h - m_l)\langle L|S|H\rangle \,,$$

where V^0 is local and V^i is a one-link current.

Renormalization - Z_{V_4}

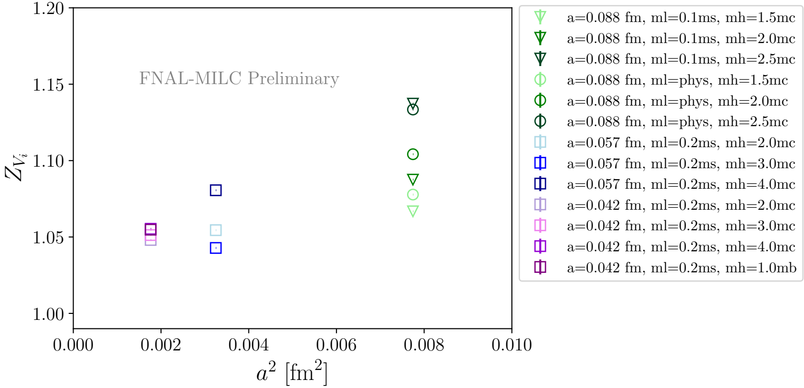
- Z_{V_4} determined from zero-momentum vector and scalar correlators.
- $Z_{V^0}(M_H - E_L)\langle L|V^0|H\rangle = (m_h - m_l)\langle L|S|H\rangle$



- Z-factors tend towards 1 as $a \to 0$, $am \to 0$.

Renormalization - Z_{V_i}

- Z_{V_i} determined from non-zero momentum correlators.
- Here use $\mathbf{p} = (3, 0, 0)$ data (need to fit/optimize).



- Z-factors tend towards 1 as $a \to 0$, $am \to 0$.

Summary & Outlook

- Unified treatment for range of semileptonic decays.
- HISQ action used for *all* quarks.
- Good statistical precision (percent-level) achieved.
- Small discretization effects.
- Will permit *interpolation* in both m_l and m_h.
- Extending production to vector final states ($B_{(s)} \to D_{(s)^*}$). Stay tuned!

Thank you!