B-meson semileptonic decays from highly improved staggered quarks

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Motivation

- Semileptonic decays are a rich source of information for determining CKM matrix elements.

- Relatively simple decay processes – measured in accelerator experiments, require theoretical input from lattice QCD to extract fundamental parameters.

- Desire precise measurements of $|V_{xb}|$ from multiple decay processes to test the consistency of the Standard Model.
Semileptonic decays

SL Decay processes critical inputs for heavy flavor studies.

Lattice predictions needed for:

- Extracting CKM matrix elements from expt’l measurements
- Pure SM predictions of R-ratios
- SM predictions $\frac{d\Gamma}{dq^2}$, etc.

Lattice calculations based on 2- & 3-point correlators give matrix elements $\rightarrow f_i(q^2)$
Stress-testing the CKM paradigm

- Inclusive/exclusive discrepancies for $|V_{ub}|$ and $|V_{cb}|$
- Also discrepancies from SM expectations in $R(D, D^*, J/\psi, \ldots)$ see e.g. Snowmass 2205.15373
- → Want high accuracy SM predictions for sl decays
Outline

1. Intro & Motivation.

2. Computational framework.

   ▶ Two-point and three-point correlators.
   ▶ Form factor results.
   ▶ Renormalization.

Heavy quarks

Treatment of $c$ and especially $b$ quarks challenging in lattice simulation due to lattice artifacts which grow as $(am_h)^n$

- May use an effective theory framework to handle the $b$ quark.
  - Fermilab method, RHQ, OK, NRQCD
  - Pros: Solves problem w/ $am_h$ artifacts.
  - Cons: Requires matching, can still have $ap$ artifacts.

- Also possible to use relativistic fermion provided $a$ is sufficiently small $am_c \ll 1$, $am_b < 1$.
  - Use improved actions e.g. $O(a^2) \rightarrow O(\alpha_s a^2)$
  - Pros: Absolutely normalised current, straightforward continuum extrap.
  - Cons: Numerically expensive, extrapolate $m_h \rightarrow m_b$. 
allhisq simulations

- Here we simulate *all* quarks with the HISQ action.

- Unified treatment for wide range of $B_s$ (and $D_s$) to pseudoscalar tr
  - $B_s \rightarrow D_s$
  - $B_s \rightarrow K$
  - $B \rightarrow \pi$

- Ensembles with (HISQ) sea quarks down to physical at each lattice spacing.
MILC ensembles

- **HISQ fermion action.**
  - Discretization errors begin at $O(\alpha_s a^2)$.
  - Designed for simulating heavy quarks ($m_c$ and higher at current lattice spacings).

- **Symanzik-improved gauge action,** takes into account $O(N_f \alpha_s a^2)$ effects of HISQ quarks in sea. [0812.0503]

- **Multiple lattice spacings down to $\sim 0.042$ (now 0.03) fm.**

- **Effects of $u/d$, $s$, and $c$ quarks in the sea.**

- **Multiple light-quark input parameters down to physical pion mass.**
  - Chiral fits.
  - Reduce statistical errors.
MILC ensemble parameters

\[ a^2 \approx (\text{fm}^2) \]

\[ M_\pi (\text{MeV}) \]

\[ \approx a^2 (\text{fm}^2) \]

\[ \begin{array}{c}
0.0 \\
(0.03)^2 \\
(0.06)^2 \\
(0.09)^2 \\
(0.12)^2 \\
(0.15)^2 \\
\end{array} \]
Results for $D$ decays

$|V_{cd}|$

$|V_{cs}|$

$D \rightarrow \pi \ell^+ \nu$
Present work

$D_s \rightarrow K e^+ \nu$
Present work

Semileptonic $N_f = 2 + 1 + 1$

Semileptonic $N_f = 2 + 1$

Leptonic $N_f = 2 + 1 + 1$

CKM Unitarity

Neutrino Scattering

$D_s \rightarrow K e^+ \nu$
dominated by BES III 2019

$\eta_{EW, f D} |V_{cd}| = 46.2(1.0)(0.3)$

$F_{\text{NBF-MILC}}$
(Present work)

$HFLAV$

$\eta_{EW, f D} |V_{cs}| = 245.4(2.4)(1.7)$

$F_{\text{NBF-MILC}}$
(Present work)

$HFLAV$+FLAG

Leptonic

PDG

$|V_{cd}|$

$|V_{cs}|$
all hisq $b$

- Use a heavy valence mass $h$ as a proxy for the $b$ quark.

- Work at a range of $m_h$, with $am_c < am_h \lesssim 1$ on each ensemble. On sufficiently fine ensembles, $m_h$ is near to $m_b$ (e.g. $m_b$ at $am_h \approx 0.65$ on $a = 0.03$ fm).

- Map out physical dependence on $m_h$, remove discretisation effects $\sim (am_h)^{2n}$ using information from several ensembles. Extrapolate results $a^2 \to 0, m_h \to m_b$. 
Preliminary results
Two point functions

Consider $B(s) \rightarrow D(s)$ decays for $a = 0.06 \; \text{fm}$, $m_l/m_s = 0.1$.

- Compute $H(s)$ mesons at a range of $am_h$ values:

- $D(s)$ mesons for a range of momenta:
Three point functions

- Generate three-point functions for scalar, vector, and tensor current insertions, $\langle D(s)(T) J(t) H_{(s)}^\dagger(0) \rangle$.
- Fit simultaneously with two-point functions to extract the matrix elements of interest $\rightarrow \langle D(s)|J|H_{(s)} \rangle$

- We use scalar ($S$), and vector ($V^0, V^i$) current insertions to extract the form factors $f_0$ and $f_+$. 
Extracting form factors

\[ f_0(q^2) = \frac{m_h - m_\ell}{M_H^2 - M_L^2} \langle L|S|H \rangle \]

\[ f_\parallel(q^2) = Z_{V^0} \frac{\langle L|V^0|H \rangle}{\sqrt{2M_H}} \]

\[ f_\perp(q^2) = Z_{V^i} \frac{\langle L|V^i|H \rangle}{\sqrt{2M_H}} \frac{1}{p_L^i} , \]

\[ f_+ = \frac{1}{\sqrt{2M_H}} \left( f_\parallel + (M_H - E_L)f_\perp \right) . \]
$B_s \rightarrow D_s: f_0(q^2)$

- Good precision out to $p = 400$
- Rightmost points on figure have $m_h = m_b$
$B_s \rightarrow D_s: f_{\parallel}(q^2)$

- Good precision out to $p = 400$
- Rightmost points on figure have $m_h = m_b$
\( B_s \rightarrow D_s: f_\perp(q^2) \)

- Good precision out to \( p = 400 \)
- Rightmost points on figure have \( m_h = m_b \)
$B_s \rightarrow D_s$ - a simple $f_0(q^2_{\text{max}})$ fit

Basic fit parameterizing $M_H$ dependence and heavy quark discretization.

$$f_0(q^2_{\text{max}})[M_H, am_h] = \sum_{i,j} c_{ij} \left( \frac{1}{M_H} \right)^i (am_h)^{2j}$$

Good precision obtained ($\sim 0.5\%$) at $M_{B_s}$.
Chiral/cont. extrapolations

Build from chiral forms used in $D$ analysis.

$$f_{0,||,\perp}(E) = \frac{c_0}{E + \Delta} (1 + \cdots + c_H \chi_{Hs} + \cdots)$$

$$\Delta = \frac{M_{D^*}^2 - M_{Ds}^2 - M_K^2}{2M_{Ds}}, \quad \chi_{Hs} = \frac{\Lambda_{HQET}}{M_{Hs}} - \frac{\Lambda_{HQET}}{M_{PDG}^{D_s}}$$

Generalize to incorporate HQET expansion:

$$c_0 \rightarrow c_0 + c_1 \frac{\Lambda_{HQET}}{M_{Hs}} + \cdots, \quad \Delta \rightarrow \frac{M_{D^*}^2 - M_{Ds}^2 - M_K^2}{2M_{Hs}} \quad (1st \ order)$$

$$\chi_{Hs} = \frac{\Lambda_{HQET}}{M_{Hs}} - \frac{\Lambda_{HQET}}{M_{Hs}^{"phys"}}$$
Here building off $D_s$ chiral analysis, working out towards $B_s$. 

- Data at $2-3m_c$, 3 lattice spacings, 3 $m_{l,\text{sea}}$ values
- Note $0.057$ fm has $m_h \approx 2.2, 3.3m_c$
- Reasonable $\chi^2/\text{dof} = 0.92, 1.79, 0.75$ for $f_0, f_\parallel, f_\perp$
Current normalization
Normalization of vector currents

We renormalize the vector current by applying the partially conserved vector current (PCVC) relation directly to extracted matrix elements:

$$\partial_{\mu} V_{\mu}^{\text{cons}} = (m_h - m_l) S$$

Applied to our lattice matrix elements,

$$Z_{V^0}(M_H - E_L) \langle L|V^0|H \rangle + Z_{V^i} \mathbf{q} \cdot \langle L|V|H \rangle = (m_h - m_l) \langle L|S|H \rangle,$$

where $V^0$ is local and $V^i$ is a one-link current.
Renormalization - $Z_{V_4}$

- $Z_{V_4}$ determined from zero-momentum vector and scalar correlators.
- $Z_{V^0}(M_H - E_L)\langle L|V^0|H \rangle = (m_h - m_l)\langle L|S|H \rangle$

- $Z$-factors tend towards 1 as $a \to 0$, $am \to 0$. 

![Graph showing $Z_{V_4}$ vs. $a^2$ with data points and labels indicating different a, ml, mh combinations]
Renormalization - $Z_{V_i}$

- $Z_{V_i}$ determined from non-zero momentum correlators.
- Here use $p = (3, 0, 0)$ data (need to fit/optimize).

- $Z$-factors tend towards 1 as $a \to 0$, $am \to 0$. 
Summary & Outlook

- Unified treatment for range of semileptonic decays.
- HISQ action used for all quarks.
- Good statistical precision (percent-level) achieved.
- Small discretization effects.
- Will permit interpolation in both $m_l$ and $m_h$.
- Extending production to vector final states ($B_s \rightarrow D_{(s)^*}$). Stay tuned!
Thank you!