2023 LATTICE

Quark flavor physics with lattice QCD

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The Standard Model of particle physics has the fascinating property that each of the five different fermion representations of the gauge group $SU(3) \times SU(2) \times U(1)$ comes in three copies – the *generations*:

$$
\begin{array}{cclccc}
 & & & 1 & 2 & 3 \\
Q'_L & = & \Bigg(& \begin{pmatrix} u'_L \\ d'_L \end{pmatrix}, & \begin{pmatrix} c'_L \\ s'_L \end{pmatrix}, & \begin{pmatrix} t'_L \\ b'_L \end{pmatrix} \Bigg), \\
U'_R & = & (& u'_R, & c'_R, & t'_R \quad), \\
D'_R & = & (& d'_R, & s'_R, & b'_R \quad), \\
L'_L & = & \Bigg(& \begin{pmatrix} \nu_e{}'_L \\ e'_L \end{pmatrix}, & \begin{pmatrix} \nu_\mu{}'_L \\ \mu'_L \end{pmatrix}, & \begin{pmatrix} \nu_\tau{}'_L \\ \tau'_L \end{pmatrix} \Bigg), \\
E'_R & = & (& e'_R, & \mu'_R, & \tau'_R \quad).
\end{array}
$$

The species labels u, c, t, d, s, b, e, μ, τ are called *flavors*. The $'$ indicates the use of the gauge basis.

In the absence of flavor-violating interactions, we would have a $U(3)^5$ global flavor symmetry.

In the Standard Model, the only origin of flavor symmetry violation (and CP violation) is the Yukawa interaction of the fermions with the Higgs field ϕ:

$$\mathcal{L}_{\text{Yukawa}} = -\overline{Q'_{Li}} Y^U_{ij} U'_{Rj} \tilde{\phi} - \overline{Q'_{Li}} Y^D_{ij} D'_{Rj} \phi - \overline{L'_{Li}} Y^E_{ij} E'_{Rj} \phi + \text{h.c.}$$

When ϕ acquires its vacuum expectation value $\langle\phi\rangle = (0, v/\sqrt{2})$, these couplings produce the fermion mass terms. In the quark sector, the unitary field transformations that diagonalize the mass matrices,

$$\begin{aligned} U'_L &= V^U_L U_L, \quad & U'_R &= V^U_R U_R, \\ D'_L &= V^D_L D_L, \quad & D'_R &= V^D_R D_R, \end{aligned}$$

do not cancel in the charged current coupling to the W field,

$$\mathcal{L}_{\text{c.c.}} \;=\; -\frac{g}{\sqrt{2}} \overline{U'_{Li}} \gamma^\mu D'_{Li}\, W^+_\mu + \text{h.c.} \;=\; -\frac{g}{\sqrt{2}} \overline{U_{Li}} \underbrace{(V^{U\dagger}_L V^D_L)_{ij}}_{=V_{ij}} \gamma^\mu D_{Lj}\, W^+_\mu + \text{h.c.},$$

giving rise to the unitary Cabibbo-Kobayashi-Maskawa quark mixing matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

After eliminating unobservable phase factors, V can be written in terms of four parameters.

Some of the fundamental questions in flavor physics are:

- What is the origin of the three generations?
- What is the origin of the hierarchies in the fermion masses and mixing matrices?
- Are there other sources of flavor-violating interactions and CP violation beyond the Standard Model?

In most of the more fundamental theories that have been proposed to address the deficiencies of the Standard Model, the answer to the third question is "yes". The precision study of flavor-changing processes is therefore a powerful tool for discovering new physics.

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Parametrization of the Kobayashi-Maskawa Matrix

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The quark mixing of the weak-interaction current in the standard model is described by the 3×3 Kobayashi-Maskawa (KM) matrix[1]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{1}$$

The element V_{us} is quite well determined to be equal to 0.22. This and other information suggest that V differs from unity by a small quantity. Here we set

$$0.22 = V_{us} = \lambda \tag{2}$$

and consider an expansion of V in powers of λ.

A recent measurement of the lifetime τ_B of B particles yields the result[2]

$$V_{cb} \approx 0.06 . \tag{3}$$

This suggests to us that V_{cb} is of order λ^2 rather than λ so that we set

$$V_{cb} = A\lambda^2$$

with $A \approx \frac{5}{4}$. To order λ^2 the KM matrix can then be written

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & 0 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ 0 & -A\lambda^2 & 1 \end{pmatrix} .$$

We now want to go to order λ^3. Unitarity then prescribes the following form:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1 - \rho - i\eta) & -\lambda^2 A & 1 \end{pmatrix}, \quad (4)$$

where two new parameters ρ and η must be introduced.

Given the values of λ and A we look for empirical constraints on ρ and η. If we neglect CP nonconservation for the moment, terms of the order λ^4 (which enter along the diagonal and in the $\lambda^2 A$ terms) are too small to be of importance given experimental and theoretical uncertainties. Therefore the simple form (4) is adequate for present analyses. The only significant constraint now comes from the limit on the ratio of $b \to u$ to $b \to c$ transitions which yields[4]

$$|V_{ub}/V_{cb}| < 0.2 \tag{6}$$

or

$$\rho^2 + \eta^2 < 1 . \tag{7}$$

For the K^0 system CP-nonconserving effects depend on $V_{td} \times V_{ts}$; because $V_{ts} \sim \lambda^2$ whereas $V_{us} \sim \lambda$ the characteristic CP-nonconserving parameter is

$$\lambda^4 A^2 \eta = s_2 s_3 \sin\delta \leqslant 4 \times 10^{-3} . \tag{9}$$

Determination of $\lambda = \dfrac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}$

Assuming the CKM unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + \underbrace{|V_{ub}|^2}_{\approx 0} = 1$, a determination of $|V_{ud}|$ alone already gives us $|V_{us}|$ as well, and hence λ.

The most precise direct result for $|V_{ud}|$ comes from the study of superallowed $0^+ \to 0^+$ nuclear β decays, which are pure vector transitions and therefore fairly insensitive to nuclear/nucleon structure [E. Blucher and W. Marciano, 2023 Review of Particle Physics, Sec. 67]:

$$|V_{ud}| = 0.97373(11)_{\text{exp.}}(9)_{\text{RC}}(27)_{\text{NS}}.$$

This result alone would give $\lambda = 0.2277(13)$.

But is the unitarity relation actually satisfied?

We also have the following experimental results:

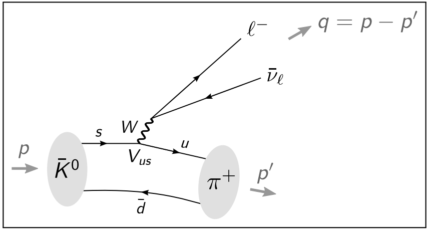
$$\frac{\Gamma(K^{\pm} \to \mu^{\pm}\nu[\gamma])}{\Gamma(\pi^{\pm} \to \mu^{\pm}\nu[\gamma])} = 1.3367(28),$$

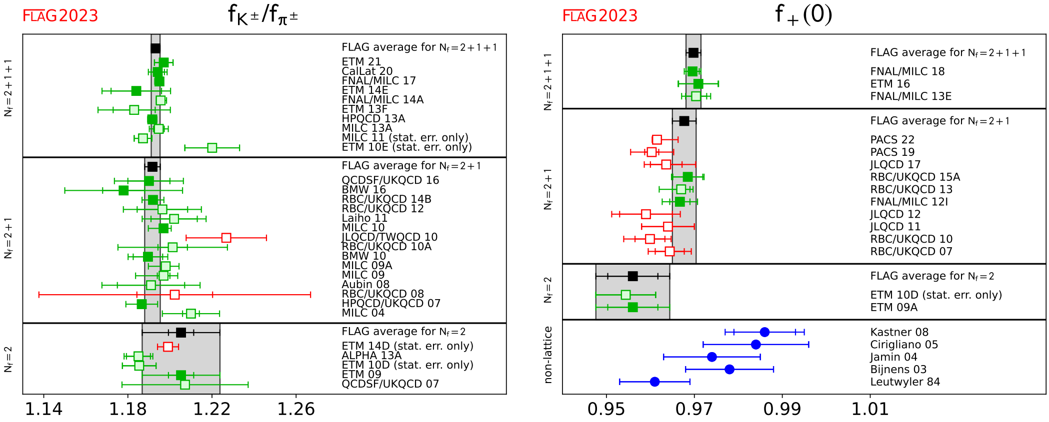
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(K \to \pi\ell\nu[\gamma]) \underset{\text{non-lattice theory}}{\Rightarrow} f_+(K \to \pi, q^2 = 0)|V_{us}| = 0.21635(38)(3).$$

[E. Blucher and W. Marciano, 2023 Review of Particle Physics, Sec. 67]

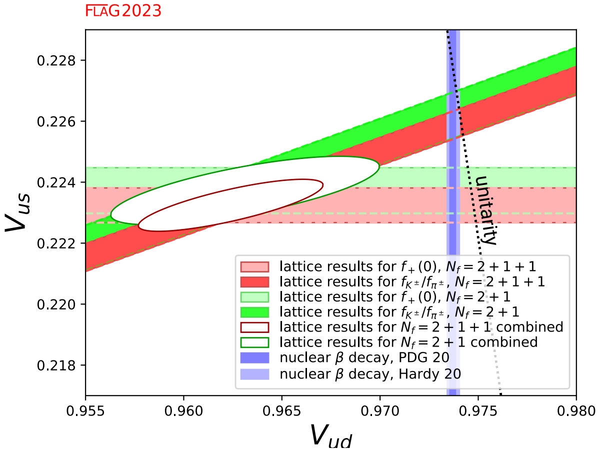
To get $|V_{us}/V_{ud}|$ and $|V_{us}|$, we need lattice-QCD calculations of the ratio of decay constants $f_{K^\pm}/f_{\pi^\pm}$ and of the form factor $f_+(K \to \pi, q^2 = 0)$:

$$\begin{aligned}
\langle 0|\, \bar{u}\gamma^\mu\gamma_5 d\, |\pi^-(p)\rangle &= ip^\mu f_{\pi^-}\,, \\
\langle 0|\, \bar{u}\gamma^\mu\gamma_5 s\, |K^-(p)\rangle &= ip^\mu f_{K^-}\,, \\
\langle \pi^+(p')|\, \bar{u}\gamma^\mu s\, |\bar{K}^0(p)\rangle &= \left[(p+p')^\mu - \frac{m_K^2 - m_\pi^2}{q^2} q^\mu\right] f_+(K \to \pi, q^2) \\
&\quad + \frac{m_K^2 - m_\pi^2}{q^2} q^\mu f_0(K \to \pi, q^2).
\end{aligned}$$




FLAG2023
$f_{K^\pm}/f_{\pi^\pm}$
$N_f=2+1+1$
$N_f=2+1$
$N_f=2$
FLAG average for $N_f=2+1+1$
ETM 21
CalLat 20
FNAL/MILC 17
ETM 14E
FNAL/MILC 14A
ETM 13F
HPQCD 13A
MILC 13A
MILC 11 (stat. err. only)
ETM 10E (stat. err. only)
FLAG average for $N_f=2+1$
QCDSF/UKQCD 16
BMW 16
RBC/UKQCD 14B
RBC/UKQCD 12
Laiho 11
MILC 10
JLQCD/TWQCD 10
RBC/UKQCD 10A
BMW 10
MILC 09A
MILC 09
Aubin 08
RBC/UKQCD 08
HPQCD/UKQCD 07
MILC 04
FLAG average for $N_f=2$
ETM 14D (stat. err. only)
ALPHA 13A
ETM 10D (stat. err. only)
ETM 09
QCDSF/UKQCD 07
1.14
1.18
1.22
1.26
FLAG2023
$f_+(0)$
$N_f=2+1+1$
$N_f=2+1$
$N_f=2$
non-lattice
FLAG average for $N_f=2+1+1$
FNAL/MILC 18
ETM 16
FNAL/MILC 13E
FLAG average for $N_f=2+1$
PACS 22
PACS 19
JLQCD 17
RBC/UKQCD 15A
RBC/UKQCD 13
FNAL/MILC 12I
JLQCD 12
JLQCD 11
RBC/UKQCD 10
RBC/UKQCD 07
FLAG average for $N_f=2$
ETM 10D (stat. err. only)
ETM 09A
Kastner 08
Cirigliano 05
Jamin 04
Bijnens 03
Leutwyler 84
0.95
0.97
0.99
1.01

Here are the results for the CKM matrix elements compared to those from nuclear beta decays:



The results from $\dfrac{\Gamma(K^\pm \to \mu^\pm \nu[\gamma])}{\Gamma(\pi^\pm \to \mu^\pm \nu[\gamma])}$ shown here also use QED corrections calculated on the lattice [M. Di Carlo *et al.*, `arXiv:1904.08731`/PRD 2019] — see Matteo's plenary talk.

Can these tensions be explained with new physics?

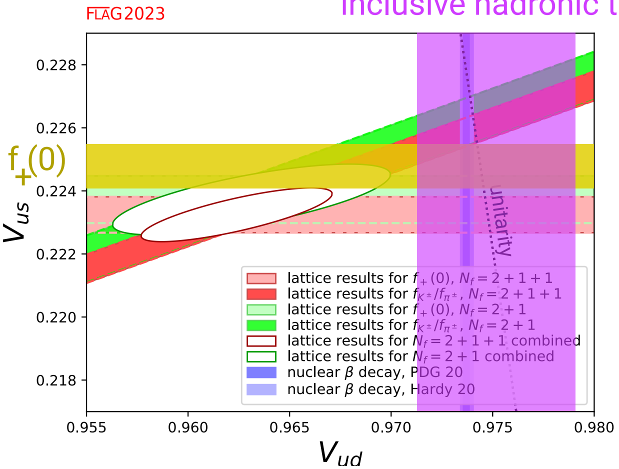
Yes! For example, TeV-scale vector-like quarks can introduce small right-handed couplings that will do the job, and can also explain the W-boson-mass anomaly [B. Belfatto, S. Trifinopoulos, 2302.14097].

Contributions relevant to $|V_{ud}|$ and $|V_{us}|$ determinations (these are all links to Indico!):

- "Light meson decay constants from Möbius domain-wall fermions on gradient flowed HISQ ensembles," Zack Hall Results: $f_\pi/\sqrt{2} = 92.6(1.0)$ MeV, $f_K/\sqrt{2} = 110.3(1.3)$ MeV
- "$|V_{us}|$ from kaon semileptonic form factor in $N_f = 2+1$ QCD at the physical point on $(10\text{ fm})^4$," Takeshi Yamazaki Result: $f_+(0) = 0.9634(24)_{\text{stat.}}$, $|V_{us}| = 0.22477(70)_{\text{stat.}}$
- "Inclusive hadronic decay rate of the τ lepton from lattice QCD," Antonio Evangelista
  First fully nonperturbative lattice calculation using spectral reconstruction! Result ($\bar{u}d$-flavor channel): $|V_{ud}| = 0.9752(39)$
- "Lattice Calculation of Electromagnetic Corrections to $K_{\ell 3}$ decay," Norman Christ
- "Finite-volume collinear divergences in radiative corrections to meson leptonic decays," Antonin Portelli
- "Structure-dependent electromagnetic finite-volume effects through order $1/L^3$," Nils Hermansson Truedsson
- "Radiative Electroweak box correction to pion, kaon and Nucleon β decay," Jun-sik Yoo
- "Isospin-breaking and electromagnetic corrections to weak decays" (plenary), Matteo Di Carlo

A. Evangelista et al.
inclusive hadronic tau decay rate

T. Yamazaki et al. $f_+(0)$



Now that we have λ (up to some tensions), let's move to the next Wolfenstein parameter, A. Its definition is

$$A\lambda^2 = \frac{\lambda}{|V_{us}|}|V_{cb}|.$$

The next task is therefore to determine $|V_{cb}|$.

The most important processes currently used to determine $|V_{cb}|$ are

- Inclusive $B \to X_c \ell \nu$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)
- Exclusive $B \to D \ell \nu$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)
- Exclusive $B \to D^* \ell \nu$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)
- Exclusive $B_s \to D_s \mu \nu$ (LCHb)
- Exclusive $B_s \to D_s^* \mu \nu$ (LCHb)

The exclusive determinations use form factors from lattice QCD.

The most precise inclusive determinations use the heavy-quark/operator-product expansion in powers of $1/m_b$ and α_s, where hadronic matrix elements of $\Delta B = 0$ matrix elements are fitted to experimental data; these calculations use lattice input for m_b, m_c, α_s.

There is also substantial progress with lattice calculations of inclusive processes. This was covered thoroughly in the Lattice 2022 plenary talks by Takeshi Kaneko and John Bulava. Given the limited time, I will omit this important topic here.

$B \to D^*$ form factors

This year, two new lattice calculations of the $B \to D^*$ form factors were published. Below is a comparison of their parameters to the 2021 Fermilab/MILC calculation.

	Fermilab/MILC 2105.14019/EPJC 2022	HPQCD 2304.03137/PRD 2023	JLQCD 2306.05657
$u, d, s, (c)$-quark action	AsqTad (2+1)	HISQ (2+1+1)	domain wall (2+1)
b-quark action	Fermilab clover	HISQ	domain wall
B-meson mass	$m_{\text{kin}} \approx m_{\text{phys}}$	$m \lesssim 0.93\ m_{\text{phys}}$	$m \lesssim 0.74\ m_{\text{phys}}$
m_π (MeV)	180 - 560*	135 - 329*	230 - 500
a (fm)	0.045 - 0.15	0.044 - 0.090	0.044 - 0.080
#(source-sink separations)	2 (T, $T+1$)	3	4

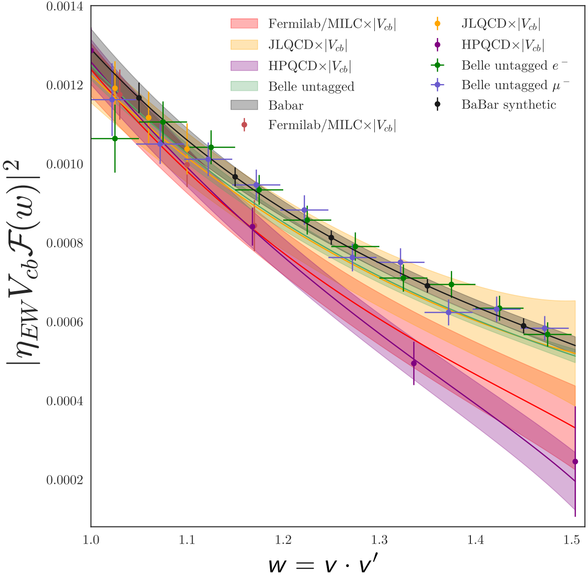
*These are the masses of the lightest pion (taste γ_5)

This figure shows the combination of form factors that appears in the $B \to D^*\ell\nu$ differential decay rate.

The black and green curves are from BGL fits to the experimental data.

The Fermilab/MILC and HPQCD lattice results have a steeper slope than the experimental data.

[Figure by A. Vaquero]

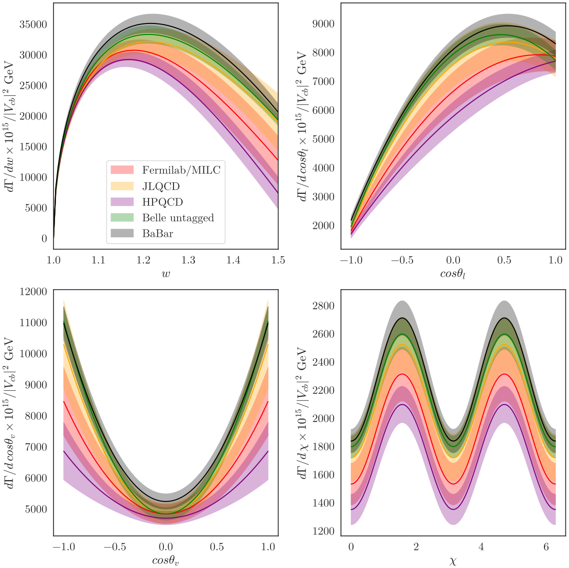


Shown here are the $B \to D^{*}\ell\nu$ differential decay rate (top left) and three angular observables.

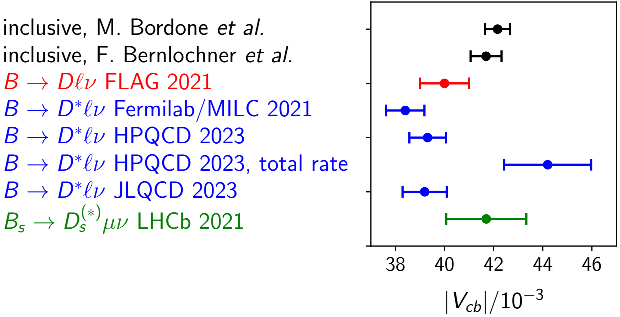
The black and green curves are from BGL fits to the experimental data.

There is a significant tension between the HPQCD predictions and the experimental data.

[Figures by A. Vaquero]



$|V_{cb}|$ summary



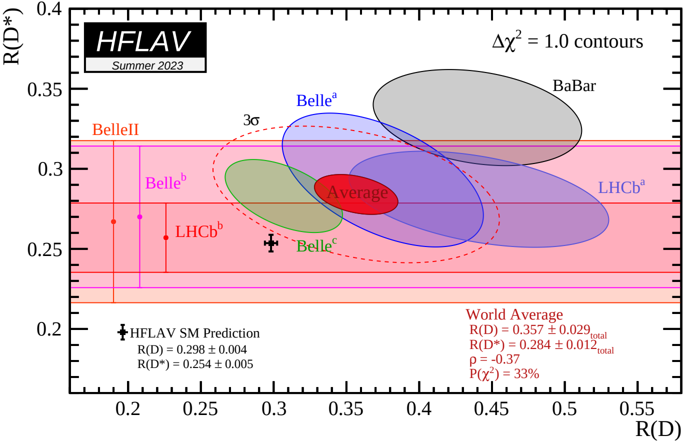
Inclusive, M. Bordone *et al.*: 2107.00604/PLB 2021
Inclusive, F. Bernlochner *et al.* (first extraction using q^2 moments): 2205.10274/JHEP 2022
$B \to D$ form factors: Fermilab/MILC 1503.07237/PRD 2015 and HPQCD 1505.03925/PRD 2015
$B_s \to D_s^{(*)}$ form factors: HPQCD 1904.02046/PRD 2019; 1906.00701/PRD 2020

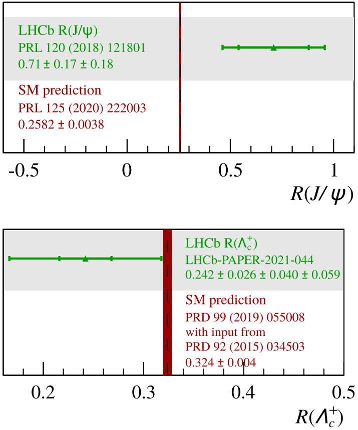
Belle II also has early $|V_{cb}|$ results: see Chunhui Chen's talk at Lepton Photon 2023

Contributions relevant to $|V_{cb}|$ determinations:

- "B-meson semileptonic decays from highly improved staggered quarks," Andrew Lytle
- "Semileptonic Form Factors for $B \to D^* \ell \nu$ Decays using the Oktay-Kronfeld Action," Benjamin Jaedon Choi
- "Progress report on data analysis of 2 point correlation functions for semileptonic decay $B_{(s)} \to D_{(s)}^{(*)} \ell \nu$ form factors," Seungyeob Jwa
- "Hadronic susceptibilities for b to c transitions from two point correlation functions," Aurora Melis
- "Chebyshev and Backus-Gilbert reconstruction for inclusive semileptonic $B_{(s)}$-meson decays from Lattice QCD," Alessandro Barone

$$R(H_c) = \frac{\Gamma(H_b \to H_c \tau \nu)}{\Gamma(H_b \to H_c \ell \nu)}$$



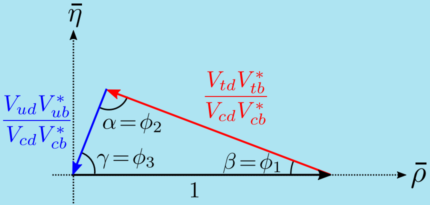


Can the excesses in the mesonic decays be explained with new physics? Yes! [Many papers on hep-ph.]
Explaining simultaneously $R(\Lambda_c) \leq R(\Lambda_c)_{\mathrm{SM}}$ (with heavy NP) is not possible [M. Fedele *et al.*, `2211.14172`/PRD 2023]

The remaining two Wolfenstein parameters are ρ and η, or, to ensure exact unitarity, $\bar{\rho}$ and $\bar{\eta}$:

$$V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]} A\lambda^3(\bar{\rho} + i\bar{\eta}).$$

Also note that $\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$, and the orthogonality of the first and third columns of the CKM matrix, $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, can be represented as a triangle in the complex plane with apex $\bar{\rho} + i\bar{\eta}$:



The magnitude $|V_{ub}| = A\lambda^3\sqrt{\rho^2 + \eta^2}$ can be determined from b-hadron semileptonic decays.

The most important processes currently used to determine $|V_{ub}|$ are

- Inclusive $B \to X_u \ell \nu$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)
- Exclusive $B \to \pi \ell \nu$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)
- Exclusive $B \to \rho \ell \nu$ and $B \to \omega \ell \nu$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments, still using light-cone sum rules)
- Exclusive $B_s \to K_s \mu \nu$ (LCHb)
- Exclusive $\Lambda_b \to p \mu \nu$ (LCHb)
- Exclusive $B \to \tau \nu$ (BaBar, Belle, Belle II, and older experiments)

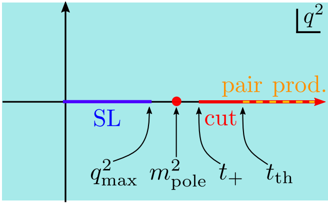
The inclusive determination of $|V_{ub}|$ is more difficult compared to $|V_{cb}|$ due to the large $b \to c \ell \bar{\nu}$ background. Cutting away this contribution with a requirement on the lepton energy leaves only the endpoint region with $2E_\ell / m_b \sim 1$, where the local HQE breaks down. In this region, one needs to use a light-cone OPE, such that the HQE parameters are replaced by nonlocal matrix elements, the so-called shape functions.

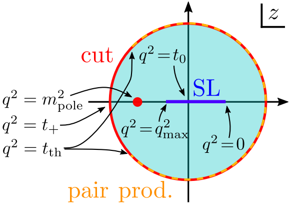
The exclusive determinations using $B \to \pi \ell \nu$, $B_s \to K_s \mu \nu$, $\Lambda_b \to p \mu \nu$, $B \to \tau \nu$ use form factors and the B decay constant from lattice QCD.

Work is underway to calculate the $B \to \rho(\to \pi\pi) \ell \nu$ form factors in lattice QCD using the Lellouch-Lüscher method, as discussed in Luka's plenary talk.

The plots on the next few slides show form factors as a function of the variable z, which is defined as

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \, .$$





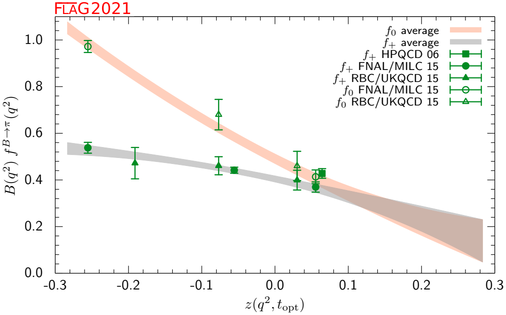
Furthermore, some of the plots show $B(q^2)f(q^2)$ instead of $f(q^2)$, where $B(q^2) = (1 - m^2_{\mathrm{pole}}/q^2)$.

$B \to \pi$: 2023 FLAG web update

New calculation by JLQCD [2203.04938/PRD 2022].
We recently included it in the FLAG average:

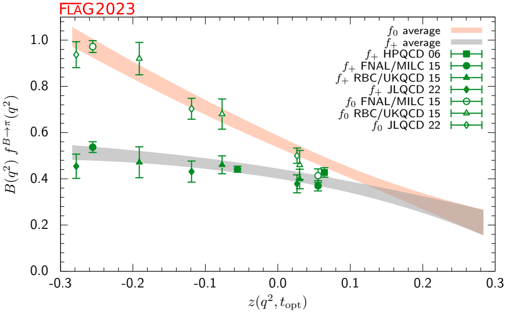
HPQCD 06	2+1 Asqtad, NRQCD *b* (not included in fit)
FNAL/MILC 15	2+1 Asqtad, Fermilab *b*
RBC/UKQCD 15	2+1 DWF, RHQ *b*
JLQCD 22	2+1 DWF, DWF *b*

Old



$\chi^2/\text{dof} = 0.82$

New, including JLQCD 22



$\chi^2/\text{dof} = 3.63$

All uncertainties rescaled by $\sqrt{\chi^2/\text{dof}}$

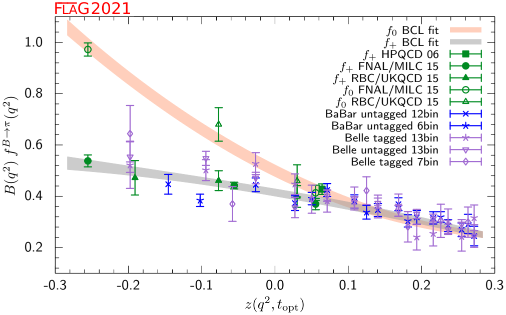
Uncertainty of $a_0^{0,+}$ increased
Uncertainty of a_1^0, $a_{1,2}^+$ decreased

$B \to \pi$: 2023 FLAG web update

New calculation by JLQCD reported at FPCP 2022.
We recently included it in the FLAG average:

HPQCD 06	2+1 Asqtad, NRQCD *b* (not included in fit)
FNAL/MILC 15	2+1 Asqtad, Fermilab *b*
RBC/UKQCD 15	2+1 DWF, RHQ *b*
JLQCD 22	2+1 DWF, DWF *b*

Old

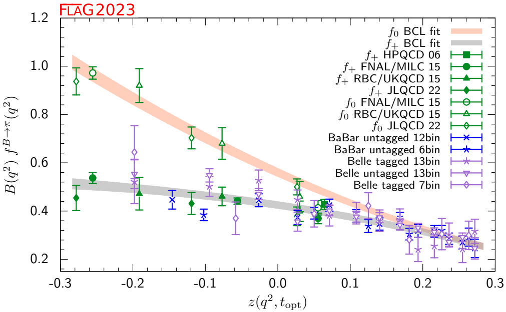


$\chi^2/\mathrm{dof} = 1.41$

All uncertainties rescaled by $\sqrt{\chi^2/\mathrm{dof}}$

$|V_{ub}| = 3.74(17)$

New, including JLQCD 22



$\chi^2/\mathrm{dof} = 1.88$

All uncertainties rescaled by $\sqrt{\chi^2/\mathrm{dof}}$

$|V_{ub}| = 3.64(16)$

$B_s \to K$: new 2023 calculation by RBC/UKQCD J. Flynn *et al.*, 2303.11280/PRD 2023

The calculation uses $N_f = 2+1$ domain-wall fermions, RHQ b, and "mostly nonperturbative" renormalization.

The main changes compared to the 2015 RBC/UKQCD calculation are

- 1 new ensemble

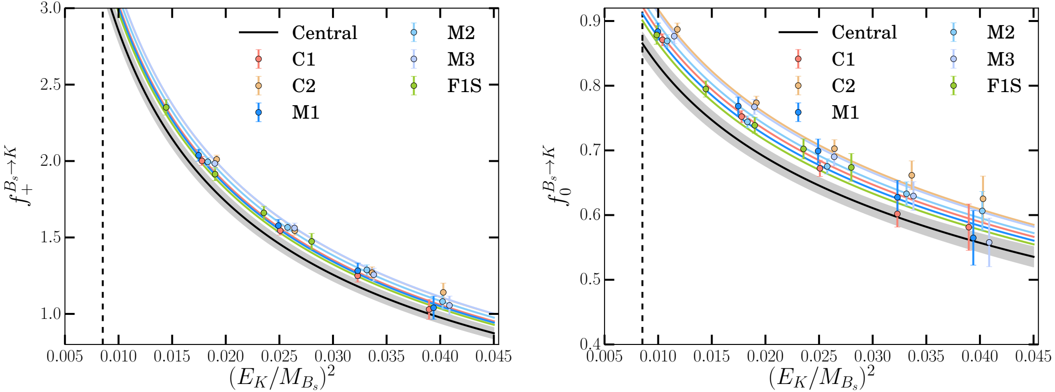
	L/a	T/a	L_s	a^{-1}/GeV	am_l	am_s^{sea}	am_s^{phys}	M_π/MeV	# cfgs	# sources	
C1	24	64	16	1.7848(50)	0.005	0.040	0.03224(18)	340	1636	1	
C2	24	64	16	1.7848(50)	0.010	0.040	0.03224(18)	434	1419	1	
M1	32	64	16	2.3833(86)	0.004	0.030	0.02477(18)	301	628	2	
M2	32	64	16	2.3833(86)	0.006	0.030	0.02477(18)	363	889	2	
M3	32	64	16	2.3833(86)	0.008	0.030	0.02477(18)	411	544	2	
F1S	48	96	12	2.785(11)	0.002144	0.02144	0.02167(20)	268	98	24	New

- New determination of lattice spacings, new tuning of valence m_s and of b-quark RHQ paramaters
- Chiral-continuum extrapolation performed directly for f_+, f_0, instead of $f_\parallel$, $f_\perp$
- Extrapolation to $q^2 = 0$ using new approach for dispersive bounds [J. Flynn, A. Jüttner, J. Tsang, 2303.11285]

$B_s \to K$: new 2023 calculation by RBC/UKQCD

J. Flynn *et al.*, 2303.11280/PRD 2023

$$f_X^{B_s\to K}(M_\pi, E_K, a^2) = \frac{\Lambda}{E_K+\Delta_X}\left[c_{X,0}\left(1+\frac{\delta f(M_\pi^s)-\delta f(M_\pi^p)}{(4\pi f_\pi)^2}\right) + c_{X,1}\frac{\Delta M_\pi^2}{\Lambda^2} + c_{X,2}\frac{E_K}{\Lambda} + c_{X,3}\frac{E_K^2}{\Lambda^2} + c_{X,4}(a\Lambda)^2\right]$$

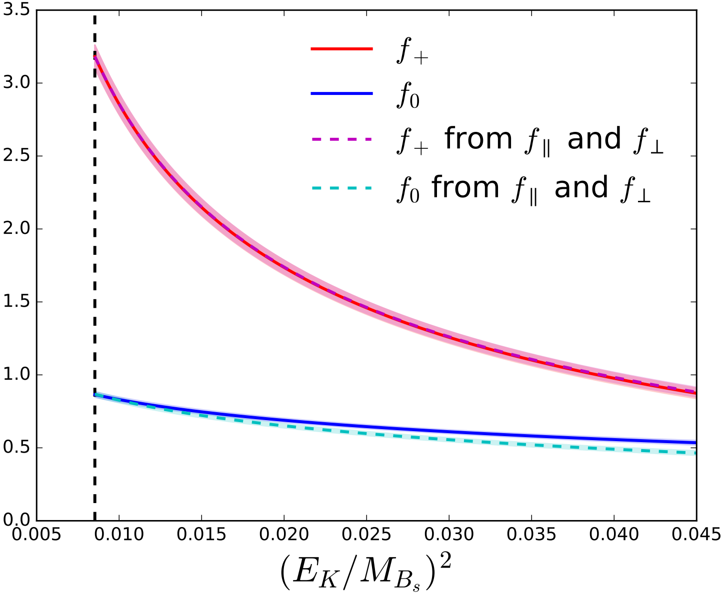


The pole mass differences $\Delta_+ = -42.1$ MeV and $\Delta_0 = 263$ MeV are specific for f_+ and f_0.

RBC/UKQCD 15 and FNAL/MILC 19 used fit models with the same poles for $f_\perp$ and $f_\parallel$.

$B_s \to K$: new 2023 calculation by RBC/UKQCD

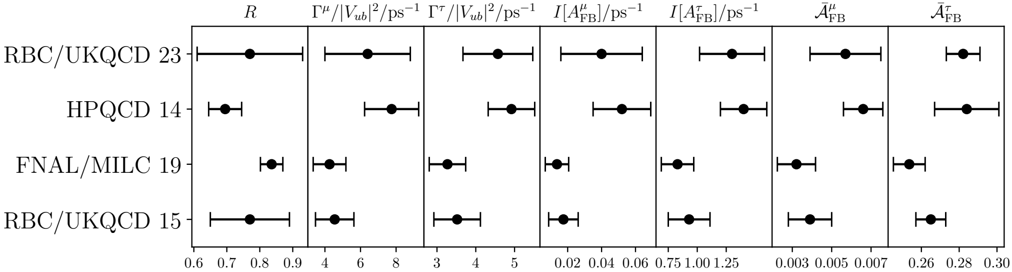
J. Flynn *et al.*, 2303.11280/PRD 2023



$B_s \to K$: new 2023 calculation by RBC/UKQCD

J. Flynn *et al.*, 2303.11280/PRD 2023

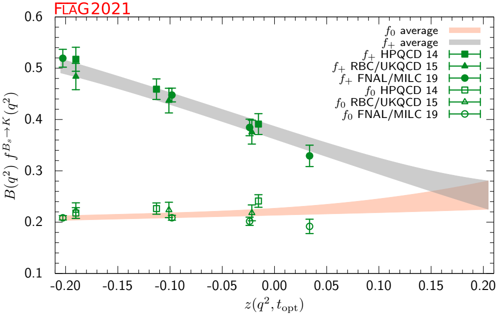
This figure compares predictions for decay rates an angular observables from different lattice calculations.



$B_s \to K$: my unofficial update of the FLAG average

Replacing RBC/UKQCD 15 by RBC/UKQCD 23

Old

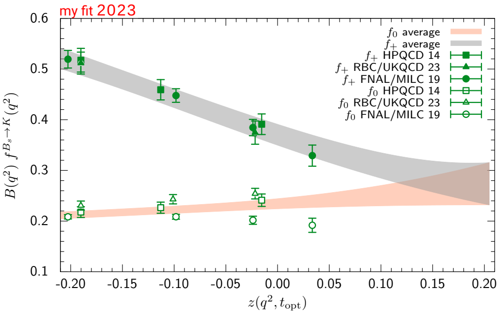


$\chi^2/\text{dof} = 1.54$

All uncertainties rescaled by $\sqrt{\chi^2/\text{dof}}$

$$\frac{\Gamma(B_s \to K\mu\nu)}{|V_{ub}|^2} = 6.28(0.67)\ \text{ps}^{-1}$$

New

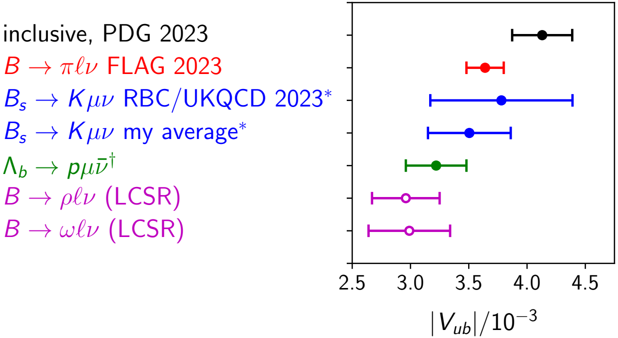


$\chi^2/\text{dof} = 3.82$

All uncertainties rescaled by $\sqrt{\chi^2/\text{dof}}$

$$\frac{\Gamma(B_s \to K\mu\nu)}{|V_{ub}|^2} = 6.5(1.1)\ \text{ps}^{-1}$$

$|V_{ub}|$ summary



*This actually uses $\mathcal{B}(B_s \to K\mu\nu)/\mathcal{B}(B_s \to D_s\mu\nu)$ and $\mathcal{B}(B_s \to D_s\mu\nu)/\mathcal{B}(B \to D\mu\nu)$ from LHCb [2012.05143, 2001.03225] and $\mathcal{B}(B \to D\mu\nu)$ from PDG

†This actually uses $\mathcal{B}(\Lambda_b \to p\mu\bar{\nu})/\mathcal{B}(\Lambda_b \to \Lambda_c\mu\bar{\nu})$ from LHCb [1504.01568/Nat.Phys. 2015] and $|V_{cb}| = 40.8(1.4) \times 10^{-3}$ from PDG. Form factors from W. Detmold, C. Lehner, S. Meinel, 1503.01421/PRD 2015

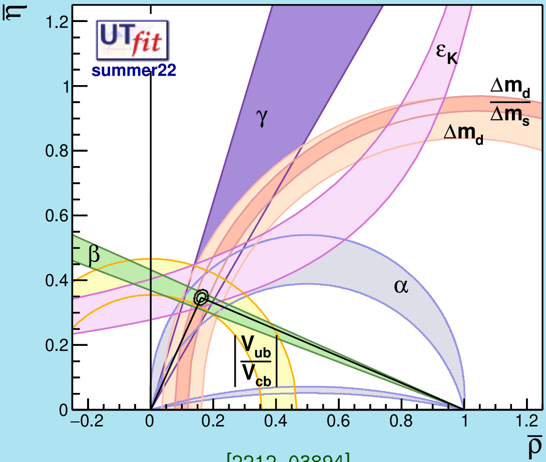
$B \to \rho\ell\nu$, $B \to \omega\ell\nu$ using LCSR: form factors from A. Bharucha, D. Straub, R. Zwicky, 1503.05534/JHEP 2016; fit from F. Bernlochner, M. Prim, D. Robinson, 2104.05739/PRD 2021

Belle II also has early $|V_{ub}|$ results: see Chunhui Chen's talk at Lepton Photon 2023

Contributions relevant to $|V_{ub}|$ determinations:

- “$B_s \to K\ell\nu$ form factors from lattice QCD with domain-wall heavy quarks,” Protick Mohanta
- “Bayesian inference for form-factor fits regulated by unitarity and analyticity,” Andreas Jüttner
- “Semileptonic form factors for exclusive $B_s \to K\ell\nu$ decays,” Ryan Hill
- “Form factors for semileptonic B-decays with HISQ light quarks and clover b-quarks in Fermilab interpretation,” Hwancheol Jeong
- “B-meson semileptonic decays from highly improved staggered quarks,” Andrew Lytle
- “Status of next-generation $\Lambda_b \to p, \Lambda, \Lambda_c$ form-factor calculations,” Stefan Meinel
- “A strategy for B-physics observables in the continuum limit,” Rainer Sommer
- “m_b and $f_{B^{(*)}}$ of 2+1 flavor QCD from a combination of continuum limit static and relativistic results,” Alessandro Conigli
- “Electroweak transitions involving resonances” (plenary), Luka Leskovec

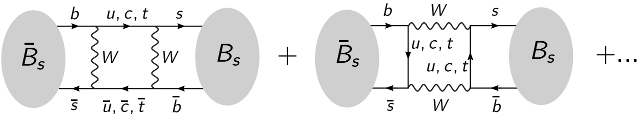
Other constraints on the Wolfenstein parameters $\bar{\rho}$, $\bar{\eta}$



[2212.03894]

- α from *CP* violation in e.g. $B^0(\overline{B^0}) \to \pi\pi, \pi\rho, \rho\rho$
- β from *CP* violation in e.g. $B^0(\overline{B^0}) \to J/\psi K_S$
- γ from *CP* violation in e.g. $B^- \to D^0(\overline{D^0})(\to f)K^-$
- Δm_d, $\dfrac{\Delta m_d}{\Delta m_s}$: $B^0/\overline{B^0}$, $B^0_s/\overline{B^0_s}$ mixing mass differences – uses hadronic matrix elements from lattice QCD
- ϵ_K: indirect *CP* violation in the neutral kaon system – uses hadronic matrix elements from lattice QCD
- ϵ'_K (not shown): direct *CP* violation in the neutral kaon system – uses hadronic matrix elements from lattice QCD

NB: much of the uncertainty in ϵ_K and Δm_d comes from $|V_{cb}|$.



The measured values of the $B^0_{(s)} - \overline{B^0_{(s)}}$ oscillation frequencies are [HFLAV 2023]

$$\Delta m_d = 0.5065(19)\ \text{ps}^{-1},$$
$$\Delta m_s = 17.765(6)\ \text{ps}^{-1}.$$

The hadronic matrix elements currently taken from lattice QCD for the Standard-Model calculation of Δm_d and Δm_s are

$$\langle \overline{B^0_q} | O_q^{\Delta B=2} | B^0_q \rangle = \tfrac{8}{3} f^2_{B_q} m^2_{B_q} B_{B_q} \quad \text{where} \quad O_q^{\Delta B=2} = [\bar{b}\gamma_\mu(1-\gamma_5)q][\bar{b}\gamma^\mu(1-\gamma_5)q].$$

The kaon CP violation parameters ϵ_K and ϵ'_K are defined through

$$\frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} \approx \epsilon_K + \epsilon'_K, \qquad \frac{A(K_L^0 \to \pi^0\pi^0)}{A(K_S^0 \to \pi^0\pi^0)} \approx \epsilon_K - 2\epsilon'_K.$$

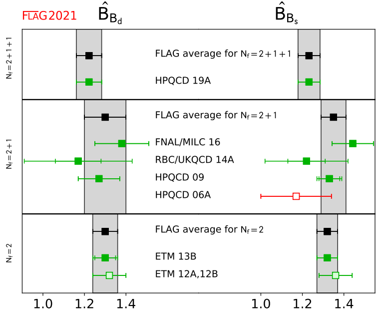
The measured values are [PDG 2023]

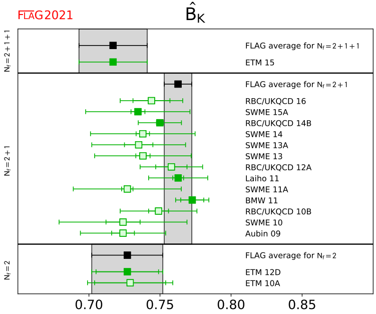
$$\epsilon_K = 2.228(11) \times 10^{-3},$$
$$\mathrm{Re}(\epsilon'_K/\epsilon_K) = 1.66(23) \times 10^{-3}.$$

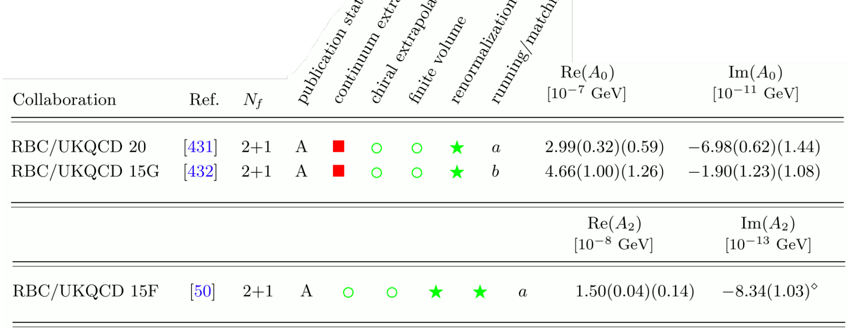
The hadronic matrix elements currently taken from lattice QCD for the Standard-Model calculation of ϵ_K and ϵ'_K are

- $\langle \pi\pi | O_i^{\Delta S=1} | K^0 \rangle$ for seven different four-quark operators $O_i^{\Delta S=1}$
- $\langle \overline{K^0} | O^{\Delta S=2} | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K$ where $O^{\Delta S=2} = [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma^\mu(1-\gamma_5)d]$

UTfit currently takes the contributions from nonlocal two-current matrix elements from a chiral-perturbation-theory calculation [A. Buras, D. Guadagnoli, G. Isidori, `arXiv:1002.3612`/PLB 2010], but they can also be calculated in lattice QCD [N. Christ, `1201.2065`; Z. Bai *el al.*, `1406.0916`/PRL 2014; B. Wang, `2301.01387`; A. Jackura, R. Briceńo, M. Hansen, `2212.09951`].







Collaboration	Ref.	N_f	publication status	continuum extrapo.	chiral extrapolatio	finite volume	renormalization	running/matching	$\mathrm{Re}(A_0)$ [10^{-7} GeV]	$\mathrm{Im}(A_0)$ [10^{-11} GeV]
RBC/UKQCD 20	[431]	2+1	A	🟥	○	○	★	a	2.99(0.32)(0.59)	−6.98(0.62)(1.44)
RBC/UKQCD 15G	[432]	2+1	A	🟥	○	○	★	b	4.66(1.00)(1.26)	−1.90(1.23)(1.08)
									$\mathrm{Re}(A_2)$ [10^{-8} GeV]	$\mathrm{Im}(A_2)$ [10^{-13} GeV]
RBC/UKQCD 15F	[50]	2+1	A	○	○	★	★	a	1.50(0.04)(0.14)	$-8.34(1.03)^{\diamond}$

$$\mathrm{Re}(\epsilon'_K/\epsilon_K)_{\mathrm{SM}} = 2.17(26)(62)(50) \times 10^{-3}$$

[R. Abbott *et al.*, 2004.09440/PRD 2020]

Contributions discussing ϵ_K, ϵ'_K, Δm_d, Δm_s:

- "$B_{(s)}$-mixing parameters from all-domain-wall-fermion simulations," Justus Tobias Tsang
- "Using Gradient Flow to Renormalise Matrix Elements for Meson Mixing and Lifetimes," Matthew Black
- "Nonperturbative renormalization of HQET operators in position space," Joshua Lin
- "Operator mixing and non-perturbative running of $\Delta F = 2$ four-fermion operators," Riccardo Marinelli
- "2023 update of ϵ_K with lattice QCD inputs," Weonjong Lee

  Significant deviation between experimental value and SM prediction when using exclusive $|V_{cb}|$
- "New result for ϵ' in $K \to \pi\pi$ decay using periodic boundary conditions," Masaaki Tomii

$$\mathrm{Re}(\epsilon'_K/\epsilon_K)_{\mathrm{SM}} = 2.94(0.52)_{\mathrm{stat}}(1.11)_{\mathrm{syst}}(0.50)_{\mathrm{EM/IB}} \times 10^{-3}$$

The 2022 Standard-Model global fit of Wolfenstein parameters by UTfit gives

$$\begin{aligned} \lambda &= 0.22519(83), \\ A &= 0.828(11), \\ \bar{\rho} &= 0.161(10), \\ \bar{\eta} &= 0.347(10), \end{aligned}$$

which corresponds to

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97431(19) & 0.22517(81) & 0.003715(93)\, e^{-i(65.1(1.3))^o} \\ -0.22503(83)\, e^{+i(0.0351(1))^o} & 0.97345(20)\, e^{-i(0.00187(5))^o} & 0.0420(5) \\ 0.00859(11)\, e^{-i(22.4(7))^o} & -0.04128(46)\, e^{+i(1.05(3))^o} & 0.999111(20) \end{pmatrix}.$$

[UTfit Collaboration, 2212.03894]

Selected further processes

Direct determinations of $|V_{cd}|$ and $|V_{cs}|$

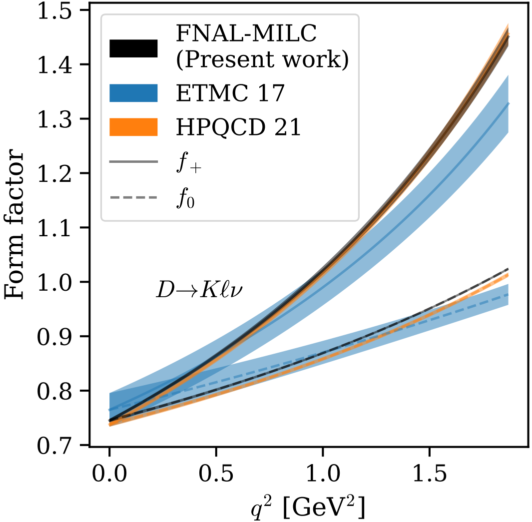
With $|V_{cd}| = 0.22503(83)$ and $|V_{cs}| = 0.97345(20)$ predicted precisely by the global fit (without charm decays), it is interesting to check whether direct determinations are compatible with these values.

Experimental data are more precise for semileptonic $D_{(s)}$ decays compared to leptonic $D_{(s)}$ decays.

In the past, leptonic decays nevertheless gave the most precise $|V_{cs}|$ and $|V_{cd}|$ because lattice results for decay constants are more precise than for form factors.

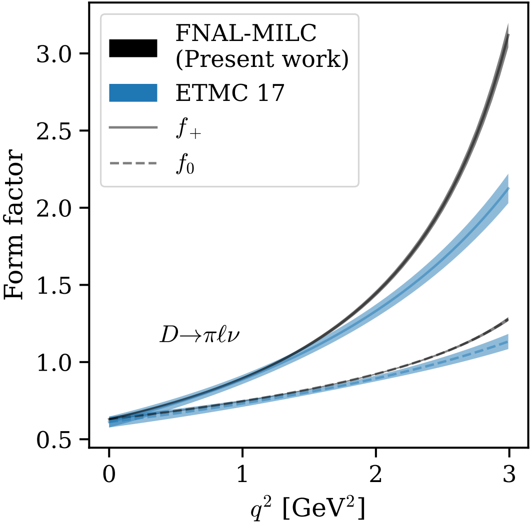
Now, lattice results for semileptonic decays have reached high precision and semileptonic decays give the most precise $|V_{cs}|$ and $|V_{cd}|$.

New Fermilab/MILC calculation of $D \to \pi$, $D_{(s)} \to K$ form factors ($N_f = 2+1+1$ HISQ)



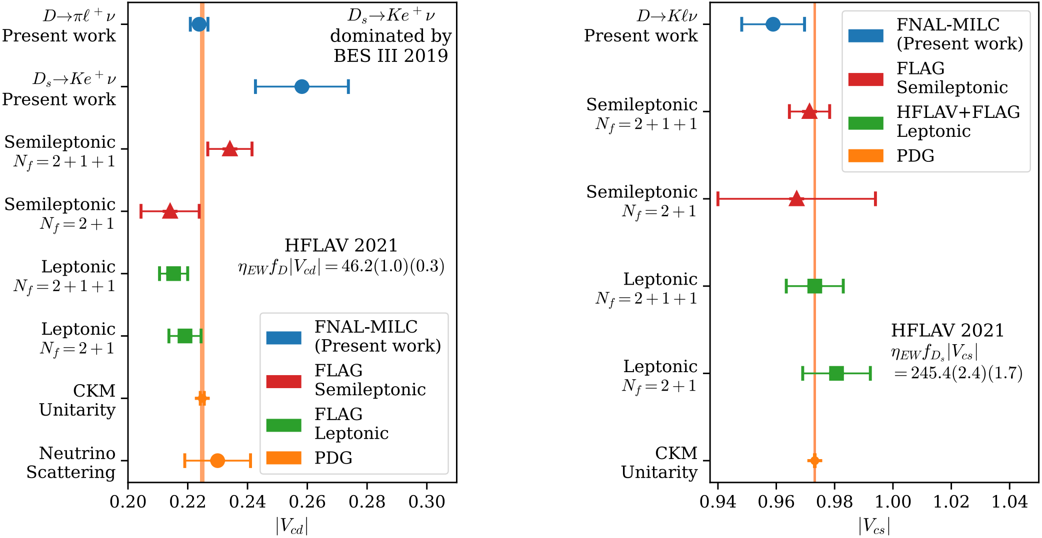
[A. Bazavov *et al.*, 2212.12648/PRD 2023]

New Fermilab/MILC calculation of $D \to \pi$, $D_{(s)} \to K$ form factors ($N_f = 2+1+1$ HISQ)



[A. Bazavov *et al.*, 2212.12648/PRD 2023]

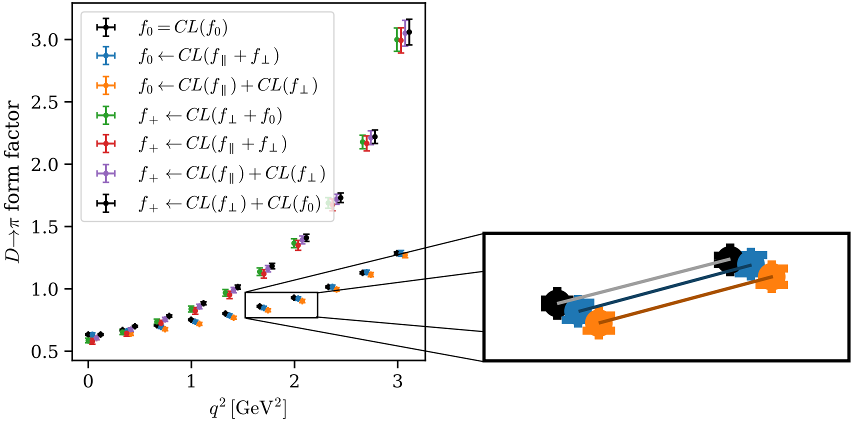
New Fermilab/MILC calculation of $D \to \pi$, $D_{(s)} \to K$ form factors ($N_f = 2+1+1$ HISQ)



[A. Bazavov *et al.*, 2212.12648/PRD 2023] (I removed the blue bands and added unitarity bands)

New Fermilab/MILC calculation of $D \to \pi$, $D_{(s)} \to K$ form factors ($N_f = 2+1+1$ HISQ)

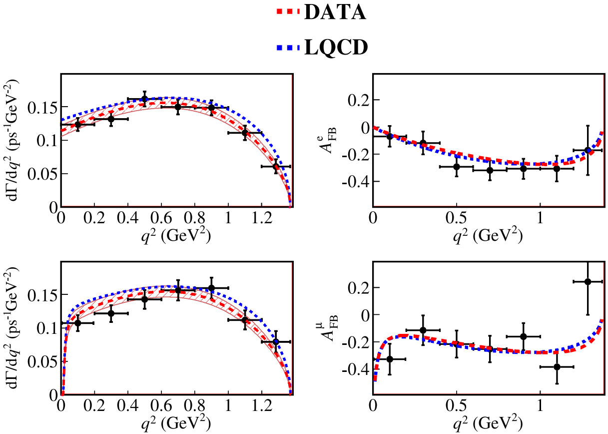
Dependence on the form-factor basis used for the continuum extrapolation:

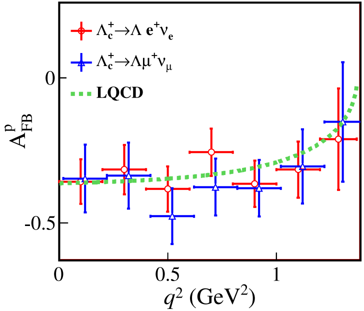


[A. Bazavov *et al.*, 2212.12648/PRD 2023] (I added the magnification box)

Thanks to Andreas Jüttner for pointing out this figure.

Measurements of the $\Lambda_c \to \Lambda(\to p\pi)\ell\nu$ decay distributions by BESIII



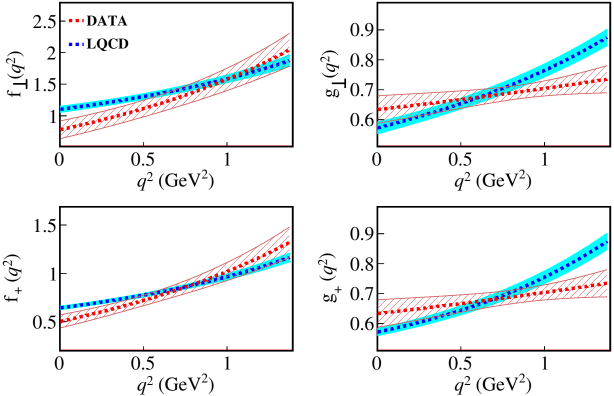


From total rates: $|V_{cs}| = 0.937 \pm 0.014_{\mathcal{B}} \pm 0.024_{\text{LQCD}} \pm 0.007_{\tau_{\Lambda_c}}$

[Data: BESIII, `2306.02624`; LQCD: S. Meinel, `arXiv:1611.09696`/PRL 2017]

Measurements of the $\Lambda_c \to \Lambda(\to p\pi)\ell\nu$ decay distributions by BESIII

The form-factor model fitted by BESIII to their data is in some tension with the lattice-QCD predictions.



[Data: BESIII, `2306.02624`; LQCD: S. Meinel, `arXiv:1611.09696`/PRL 2017]

There is an independent LQCD calculation of the $\Lambda_c \to \Lambda$ form factors by H. Bahtiyar [2107.13909/Turk.J.Phys. 2021], but it used only a single $N_f = 2$ ensemble on a $16^3 \times 32$ lattice with $a \approx 0.16$ fm, $m_\pi \approx 550$ MeV.

Contributions relevant to $|V_{cs}|$ or $|V_{cd}|$ determinations:

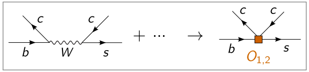
- “Studies on finite-volume effects in the inclusive semi-leptonic decays of charmed mesons,” Ryan Kellermann
- “Structure-dependent form factors in radiative leptonic decays of the D_s meson with Domain Wall fermions,” Davide Giusti
- “Finite-volume collinear divergences in radiative corrections to meson leptonic decays,” Antonin Portelli
- “Form factors for the charm-baryon semileptonic decay $\Xi_c \to \Xi \ell \nu$ from domain-wall lattice QCD,” Callum Farrell
- “Towards charm physics with stabilised Wilson Fermions,” Justus Kuhlmann

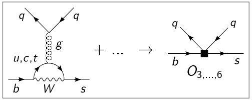
Weak effective Hamiltonian for $b \to s\ell^+\ell^-$ decays

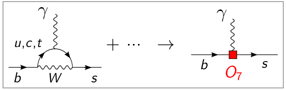
$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

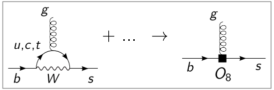
with

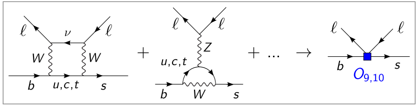
$$\begin{aligned}
O_1 &= \bar{c}^b \gamma^\mu b_L^a \;\; \bar{s}^a \gamma_\mu c_L^b, \\
O_2 &= \bar{c}^a \gamma^\mu b_L^a \;\; \bar{s}^b \gamma_\mu c_L^b, \\
O_7 &= (e\, m_b)/(16\pi^2)\;\; \bar{s}\sigma^{\mu\nu} b_R \;\; F_{\mu\nu}^{(\text{e.m.})}, \\
O_9 &= e^2/(16\pi^2)\;\; \bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \ell, \\
O_{10} &= e^2/(16\pi^2)\;\; \bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \gamma_5 \ell, \\
&\dots
\end{aligned}$$











In the Standard Model, $\overline{\text{MS}}$ scheme, at $\mu = 4.2$ GeV,

C_1	C_2	C_7	C_9	C_{10}	…
−0.288	1.010	−0.336	4.275	−4.160	…

[Computed using EOS, `https://eos.github.io/`]

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays

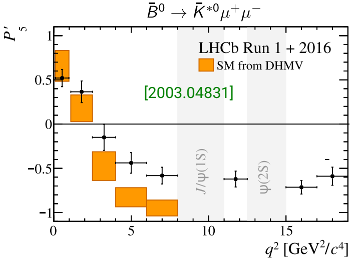
For a generic decay $H_b \to H_s\ell^+\ell^-$:

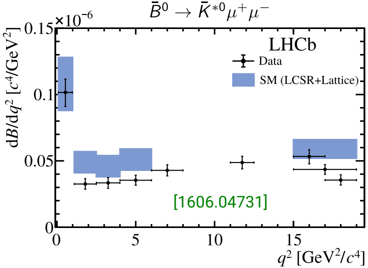
Contributions from O_7, O_9, O_{10}: $\langle H_s(p')|\, \bar{s}\Gamma b\, |H_b(p)\rangle$ → local form factors, can be calculated using lattice QCD.
[C. Bouchard *et al.*, 1306.2384/PRD 2013; R. Horgan, Z. Liu, S. Meinel, M. Wingate, 1310.3722/PRD 2014; J. Bailey *et al.*, 1509.06235/PRD 2016; W. Detmold, S. Meinel, 1602.01399/PRD 2016; S. Meinel, G. Rendon, 2107.13140/PRD 2022; W. Parrott, C. Bouchard, C. Davies, 2207.12468/PRD 2023]

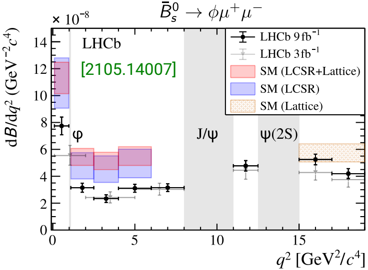
Contributions from $O_{1,\ldots,6}$, O_8: $\int d^4x\ e^{iq\cdot x}\,\langle H_s(p')|\,\mathrm{T}\ O_i(0)\ J^{\mu}_{\mathrm{e.m.}}(x)\,|H_b(p)\rangle$ → nonlocal form factors, very challenging for lattice QCD (see [K. Nakayama, T. Ishikawa, S. Hashimoto, 2001.10911] for first steps).

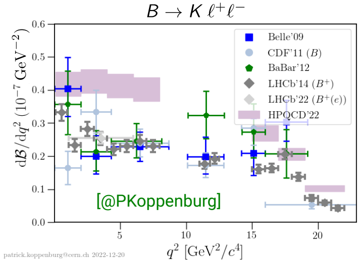
Continuum treatment using local OPE at high q^2 and QCDF/light-cone OPE at low q^2.
Recently, also combined with dispersive bounds and $\mathcal{B}(H_b \to H_s\,J/\psi)$ [N. Gubernari, D. van Dyk, J. Virto, 2011.09813/JHEP 2021; N. Gubernari, M. Reboud, D. van Dyk, J. Virto 2206.03797/JHEP 2022].

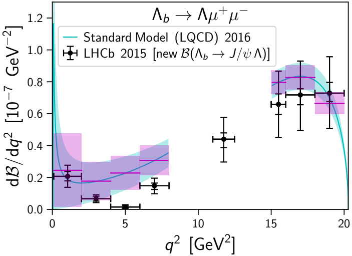
Deviations from SM predictions in $b \to s\ell^+\ell^-$ angular observables and differential branching fractions

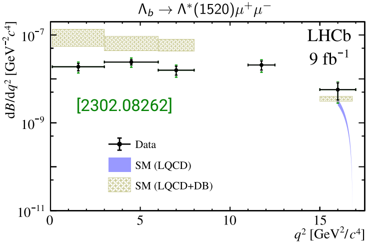

$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$


$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$


$\bar{B}_s^0 \to \phi\mu^+\mu^-$

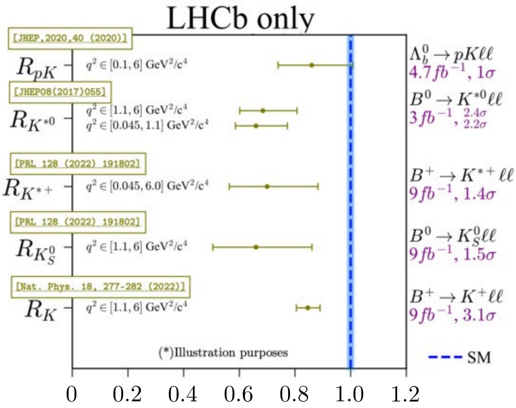

$B \to K\,\ell^+\ell^-$


$\Lambda_b \to \Lambda\mu^+\mu^-$


$\Lambda_b \to \Lambda^*(1520)\mu^+\mu^-$

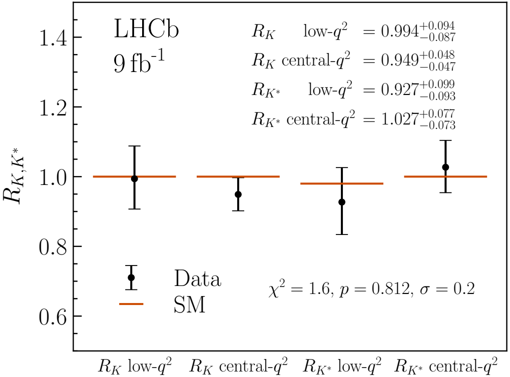
For many years, prior to December 2022, it appeared that the deviations violate lepton-flavor universality, based on measurements of, for example

$$R_K \equiv \frac{\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} \frac{\mathrm{d}\mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)}{\mathrm{d}q^2} \mathrm{d}q^2}{\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} \frac{\mathrm{d}\mathcal{B}(B^+ \to K^+ e^+ e^-)}{\mathrm{d}q^2} \mathrm{d}q^2},$$



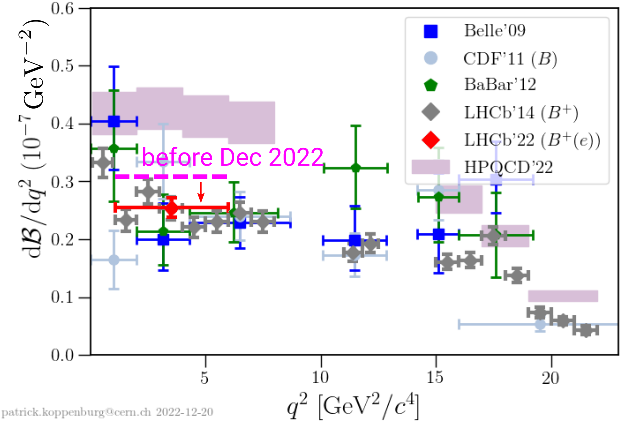
[P. de Simone, Talk at ALPS 2023]

But the LHCb results had an error (hadrons misidentified as electrons). In December 2022, LHCb published a new analysis:



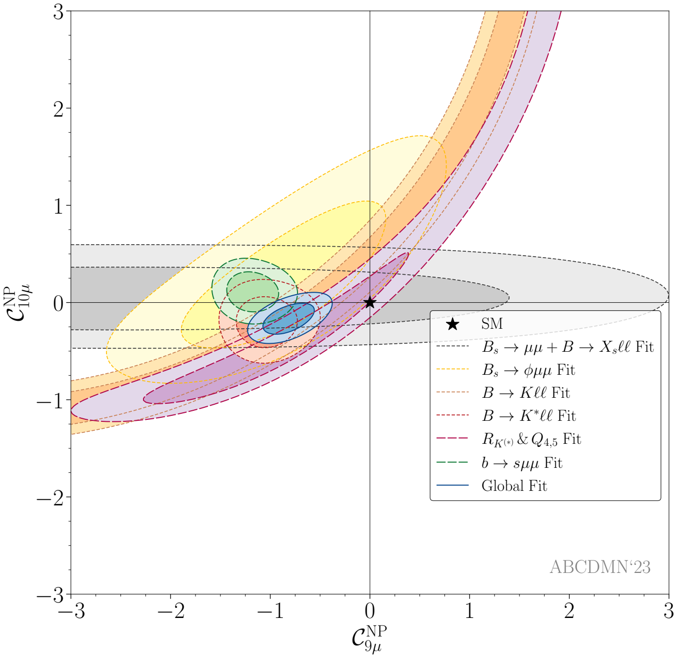
[LHCb Collaboration, `arXiv:2212.09153`/PRD 2023]

Note that only the $B \to K^{(*)}e^+e^-$ decay rate measurements have changed, and are now lower:



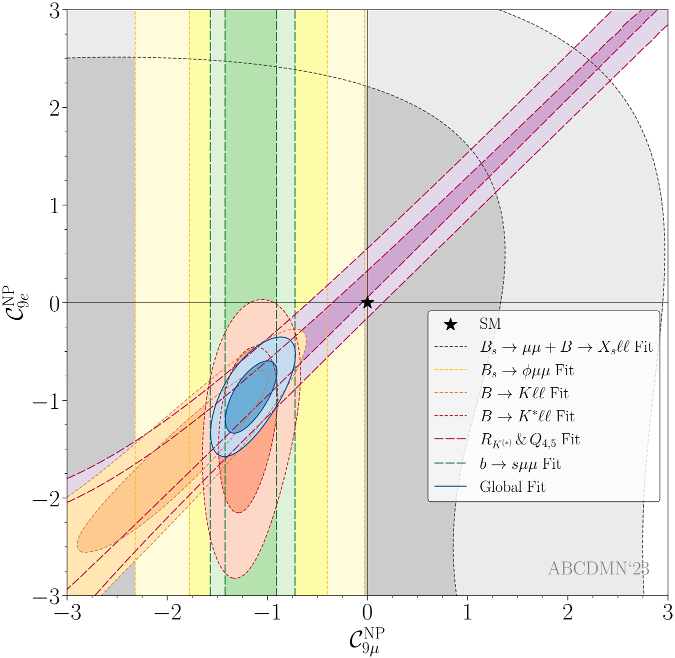
Fits of new-physics contributions to the muonic Wilson coefficients only show a tension between the $b \to s\mu^+\mu^-$ observables and LFUV observables.

[M. Algueró *et al.*, `2304.07330`/EPJC 2023]



Good fits are obtained by allowing new-physics contributions to both the electronic and muonic Wilson coefficients. [M. Algueró *et al.*, `2304.07330`/EPJC 2023]

Possible new-physics models are discussed, for example, in [A. Greljo, J. Salko, A. Smolkovič, P. Stangl, `2212.10497`/JHEP 2023]



Contributions discussing local form factors relevant for rare b decays:

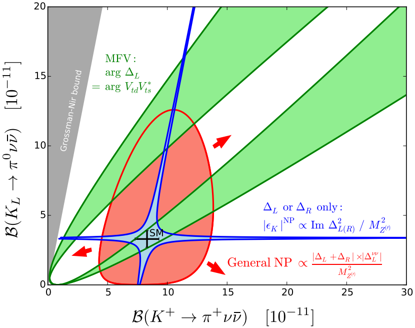
- "B-meson semileptonic decays from highly improved staggered quarks," Andrew Lytle
- "Form factors for semileptonic B-decays with HISQ light quarks and clover b-quarks in Fermilab interpretation," Hwancheol Jeong
- "Status of next-generation $\Lambda_b \to p, \Lambda, \Lambda_c$ form-factor calculations," Stefan Meinel

Calculations of the $B \to K^*(\to K\pi)$ local form factors with the proper Lellouch-Lüscher approach (see Luka's talk) are needed.

For the $B_s \to \phi$ form factors, I think it is worth doing new calculations even in the narrow-width approximation.

Semileptonic rare kaon decays with neutrinos

Shown on the right are SM predictions and possible BSM modifications of the $K_L \to \pi^0 \bar{\nu}\nu$ and $K^+ \to \pi^+ \bar{\nu}\nu$ branching fractions [A. Buras, D. Buttazzo, R. Knegjens, `arXiv:1507.08672`/JHEP 2015].



The current experimental results are

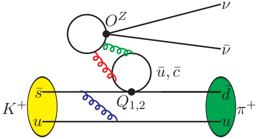
$$\mathcal{B}(K_L \to \pi^0 \bar{\nu}\nu) < 3.0 \times 10^{-9} \text{ (90\% CL)},$$

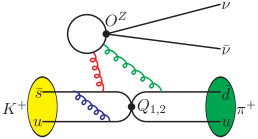
[KOTO, `1810.09655`/PRL 2019]

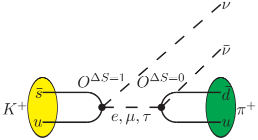
$$\mathcal{B}(K^+ \to \pi^+ \bar{\nu}\nu) = (10.6^{+4.0}_{-3.4}|_{\mathrm{stat}} \pm 0.9|_{\mathrm{syst}}) \times 10^{-11}$$

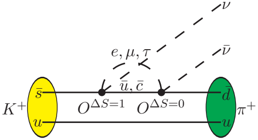
[NA62, `2103.15389`/JHEP 2021].

The SM prediction for $\mathcal{B}(K^+ \to \pi^+ \bar{\nu}\nu)$ receives an $\mathcal{O}(5\%)$ contribution from nonlocal matrix elements, which can be calculated on the lattice [N. Christ *et al.*, `1910.10644`/PRD 2019] and will become more relevant as the experimental precision improves in the future.







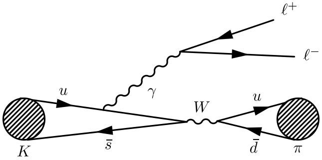


Semileptonic rare kaon or hyperon decays with charged leptons

Some experimental results for charged-lepton modes are [PDG 2023]

$$\mathcal{B}(K^+ \to \pi^+ e^+ e^-) = 3.00(9) \times 10^{-7},$$
$$\mathcal{B}(K^+ \to \pi^+ \mu^+ \mu^-) = 9.17(14) \times 10^{-8},$$
$$\mathcal{B}(\Sigma^+ \to p^+ \mu^+ \mu^-) = 2.4^{+1.7}_{-1.3} \times 10^{-8}.$$

The SM predictions for these processes are dominated by nonlocal matrix elements, which can be calculated on the lattice [P. Boyle *et al.*, `2202.08795`/PRD 2023; F. Erben, `2212.09595`/Lattice 2022].



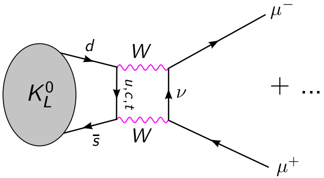
$K_L \to \mu^+\mu^-$

The branching fraction of this rare decay is measured precisely [PDG 2023, dominated by BNL Experiment 871]:

$$\mathcal{B}(K_L \to \mu^+\mu^-) = 6.84(11) \times 10^{-9}.$$

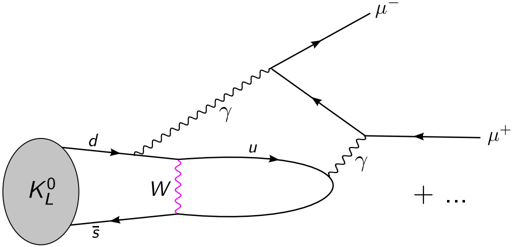
There are two types of contributions to the decay amplitude:

Local



Just need the kaon decay constant.

Nonlocal



Two quark electromagnetic currents and $O^{\Delta S=1}$, all at different spacetime points. Very challenging for lattice QCD. [N. Christ *et al.*, PoS LATTICE2019 128]

Contributions on rare kaon or hyperon decays:

- “$K_L \to \mu^+\mu^-$ from lattice QCD,” En-Hung Chao
- “Comparing phenomenological estimates of dilepton decays of pseudoscalar mesons with lattice QCD,” Bai-Long Hoid
- “Status of the exploratory calculation of the rare hyperon decay,” Raoul Hodgson
- “Rare K decays off and on the lattice,” Amarjit Soni

CP violation in charm decays

CP violation in charm decays was discovered in 2019 by LHCb, with the time-averaged result [1903.08726/PRL 2019]

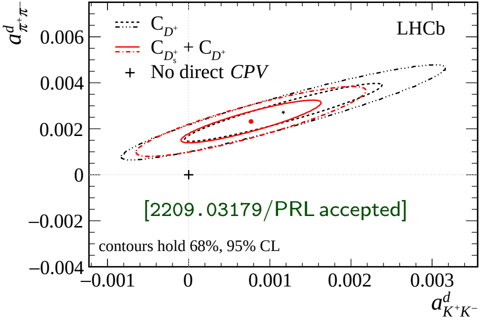
$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-15.4 \pm 2.9) \times 10^{-4},$$

where

$$A_{CP}(f;t) = \frac{\Gamma(D^0(t) \to f) - \Gamma(\overline{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\overline{D}^0(t) \to f)}.$$

The time-dependent analysis shows that ΔA_{CP} is dominated by direct CPV.

More recently, LHCb also determined the individual asymmetries:



[2209.03179/PRL accepted]

CP violation in charm decays

$$\Delta A_{CP}^{\mathrm{LHCb}} = (-15.4 \pm 2.9) \times 10^{-4}$$

Standard-Model predictions for ΔA_{CP} vary substantially depending on the methods used to estimate the nonperturbative QCD contributions. For example

$\Delta A_{CP}^{\mathrm{SM}} \approx 2 \times 10^{-4}$ [A. Khodjamirian, A. Petrov, `arXiv:1706.07780`/PLB 2017]

$\Delta A_{CP}^{\mathrm{SM}} \approx -4 \times 10^{-4}$ [A. Pich, E. Solomonidi, L. Silva, `arXiv:2305.11951`]

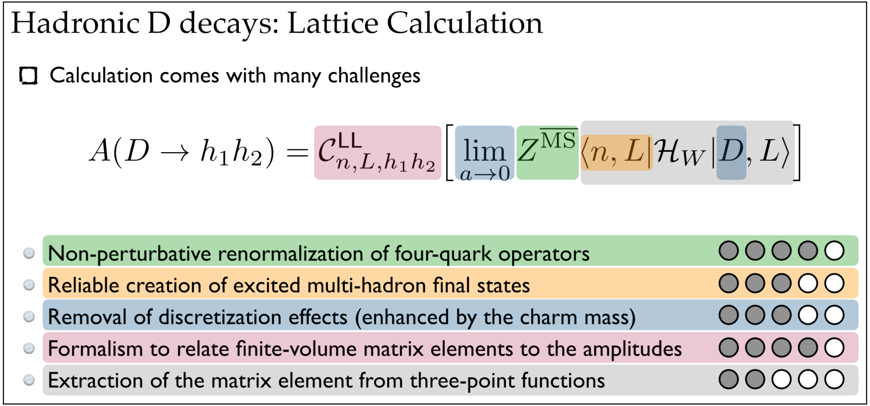
$\Delta A_{CP}^{\mathrm{SM}} \approx -16 \times 10^{-4}$ [S. Schacht, A. Soni, `2110.07619`/PLB 2021]

It is currently unclear whether the LHCb observation is a signal of new physics or consistent with the SM.

Progress toward a lattice-QCD calculation of $\Delta A_{CP}^{\mathrm{SM}}$

"Towards hadronic D decays at the $SU(3)$ flavour symmetric point, " Maxwell Hansen

Ongoing lattice calculation of $D \to K\pi$ matrix elements at $m_\pi = m_K \approx 420$ MeV.



Hadronic D decays: Lattice Calculation

☐ Calculation comes with many challenges

$$A(D \to h_1 h_2) = \mathcal{C}^{\mathsf{LL}}_{n,L,h_1h_2}\left[\lim_{a\to 0} Z^{\overline{\mathrm{MS}}} \langle n, L|\mathcal{H}_W|D, L\rangle\right]$$

Non-perturbative renormalization of four-quark operators	●●●●○
Reliable creation of excited multi-hadron final states	●●●○○
Removal of discretization effects (enhanced by the charm mass)	●●●○○
Formalism to relate finite-volume matrix elements to the amplitudes	●●●●○
Extraction of the matrix element from three-point functions	●●○○○

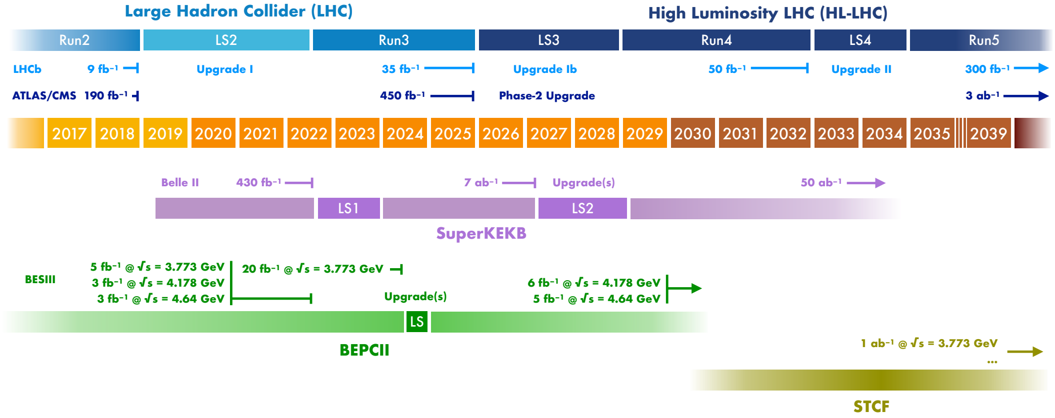
Conclusions

Quark flavor physics is exciting and may lead to the discovery of physics beyond the Standard Model. We already see interesting deviations between measurements and SM predictions that have inspired substantial model-building work and demonstrate possible routes to discovery.

Lattice-QCD calculations are essential for quark flavor physics. There has been excellent progress, and we need to continue and expand this work to make the best use of existing precise measurements and to keep up with the expected experimental progress in the coming years.

It is very valuable to have multiple calculations from different groups with different methods. Tensions between some of the lattice results for semileptonic form factors have emerged, indicating that uncertainties were underestimated in some cases.

The future prospects in quark flavor physics are discussed, for example, in the reports of the Snowmass 2021 topical groups RF1 (Weak decays of b and c quarks) [`2208.05403`] and RF2 (Weak Decays of Strange and Light Quarks) [`2209.07156`].



+ KOTO, NA62, HIKE, PIONEER, REDTOP, JEF, ...