2023 LATTICE

Kenneth G. Wilson Award

for excellence in Lattice Field Theory

To recognize outstanding contributions in Lattice Field Theory

Previous awardees

2013 - André Walker-Loud

2014 - Gergely Endrödi

2015 - Stefan Meinel

2016 - Antonin Portelli

2017 - Raúl Briceño

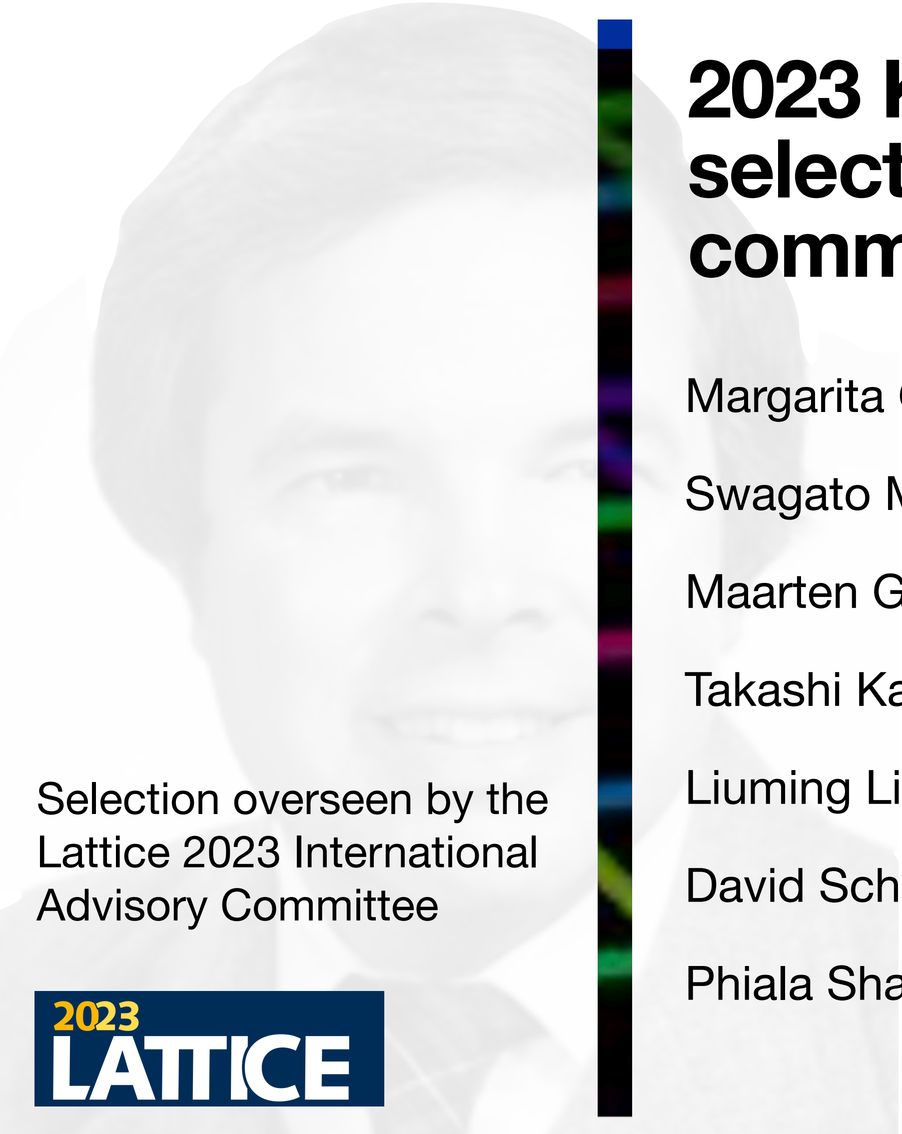
2018 - Zohreh Davoudi

2019 - Luchang Jin

2020 - Phiala Shanahan

2021 - Maxwell Hansen

2022 - Yong Zhao



2023 KWLA selection committee

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Takashi Kaneko

Liuming Liu

David Schaich

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Selection overseen by the Lattice 2023 International Advisory Committee

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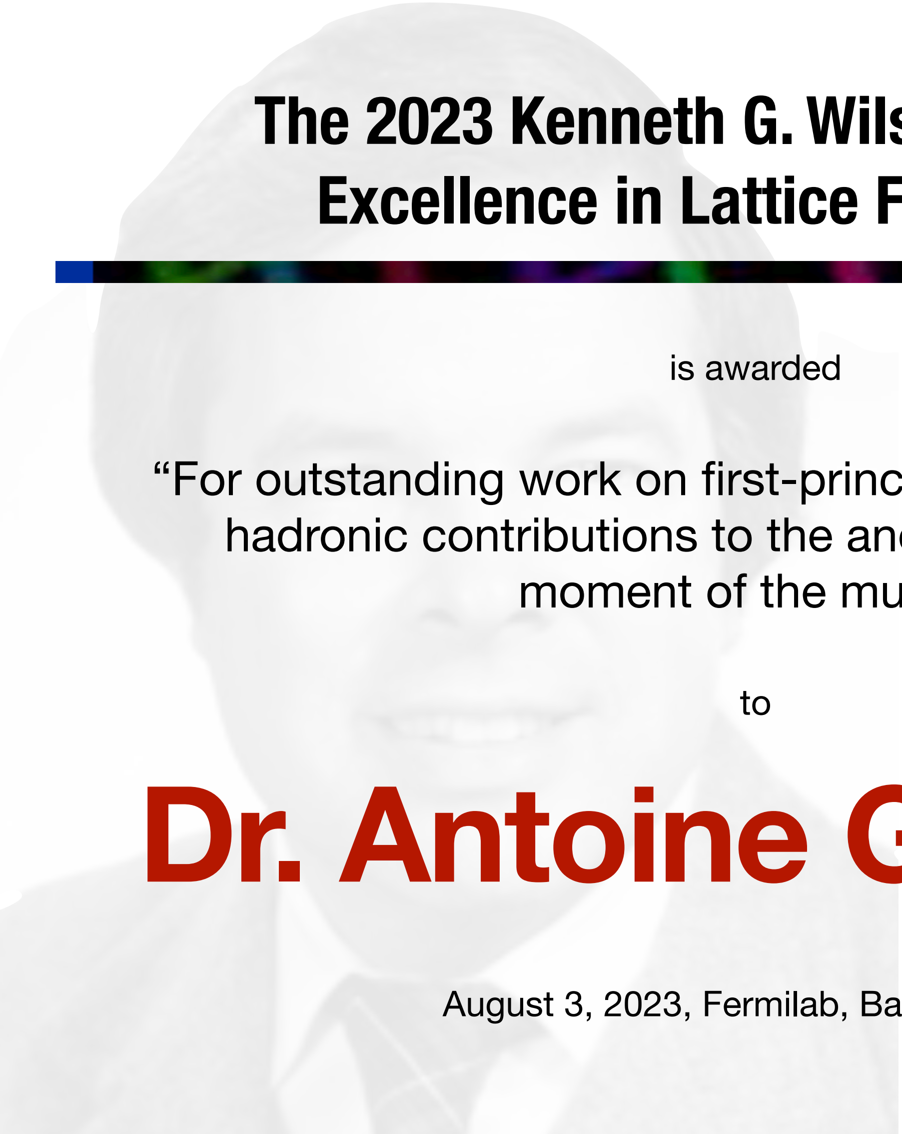
The 2023 Kenneth G. Wilson Award for Excellence in Lattice Field Theory

is awarded

“For outstanding work on first-principles computations of hadronic contributions to the anomalous magnetic moment of the muon".

to



The 2023 Kenneth G. Wilson Award for Excellence in Lattice Field Theory

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“For outstanding work on first-principles computations of hadronic contributions to the anomalous magnetic moment of the muon".

to

Dr. Antoine Gérardin

August 3, 2023, Fermilab, Batavia, USA

Hadronic contributions to the anomalous magnetic moment of the muon

Antoine Gérardin

40th International Symposium on Lattice Field Theory
Fermilab - August 3, 2023

Aix*Marseille université
anr agence nationale de la recherche
CPT Centre de Physique Théorique

"For outstanding work on first-principles computations of hadronic contributions to the anomalous magnetic moment of the muon"

Special thanks to the KWLA Selection Committee and the Conference Organizers

to Benoit Blossier (my PhD supervisor in Orsay, France)

to Harvey Meyer, Hartmut Wittig and Andreas Nyffeler from Mainz

to Laurent Lellouch and my new colleagues from BMW

> *"For outstanding work on first-principles computations of hadronic contributions to the anomalous magnetic moment of the muon"*

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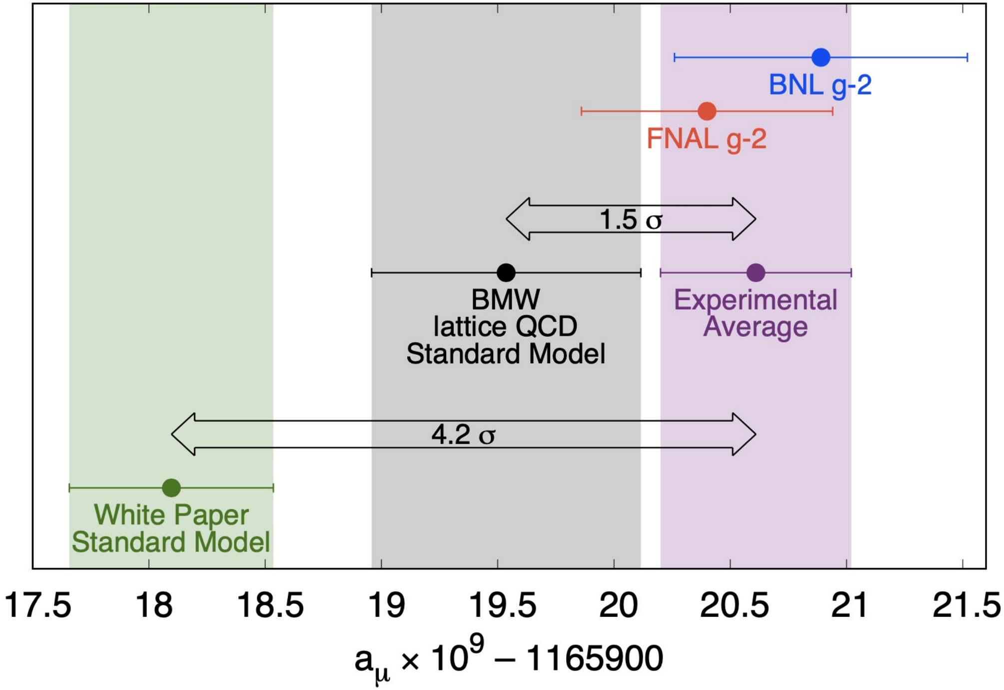
The work presented in this talk is the result of a team effort

I would like to thank all my collaborators and colleagues who contributed to this work

$$\vec{\mu} = g \left(\frac{e}{2m_\mu} \right) \vec{S}$$

g : Landé g-factor ($g = 2$ at the classical level)

$$a_\mu = \frac{g-2}{2}$$



"*The anomalous magnetic moment of the muon in the Standard Model*" [Phys.Rept. 887 (2020) 1-166]

Contribution	$a_\mu \times 10^{11}$
- **QED** (10^{th} order)	$116\ 584\ 718.931 \pm 0.104$
- **Electroweak**	153.6 ± 1.0
- **Strong interaction**	
HVP (LO)	$6\ 931 \pm 40$
HVP (NLO)	-98.3 ± 0.7
HVP (NNLO)	12.4 ± 0.1
HLbL	92 ± 18
Total (Standard Model)	$116\ 591\ 810 \pm 43$
Experiment	$116\ 592\ 061 \pm 41$

- Theory and experimental precisions are comparable (2020 White Paper status)
- Error budget **dominated by hadronic contributions : LO-HVP and HLbL**

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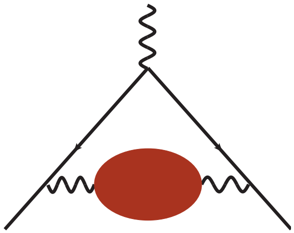
HLbL	92 ± 18
Total (Standard Model)	$116\,591\,810 \pm 43$
Experiment	$116\,592\,061 \pm 41$

$\rightarrow$ **2.4 reduction (expected)**

- Theory and experimental precisions are comparable (2020 White Paper status)
- Error budget **dominated by hadronic contributions : LO-HVP and HLbL**

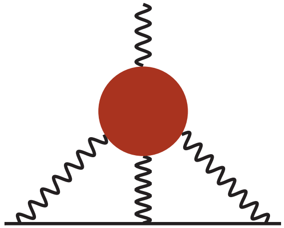
- **Hadronic Vacuum Polarisation** (HVP, α^2)



- Blobs : all intermediate hadronic states $(\pi^+\pi^-, \pi^+\pi^-\pi^0, K^+K^-, \cdots)$
- Precision physics (target precision $\sim 0.2\%$)
- $\Pi_{\mu\nu}(Q) =$  $= \int \mathrm{d}^4x\, e^{iQ\cdot x} \, \langle V_\mu(x) V_\nu(0) \rangle$

$$a_\mu^{\mathrm{LO-HVP}} = (6\ 931 \pm 40) \times 10^{-11}$$

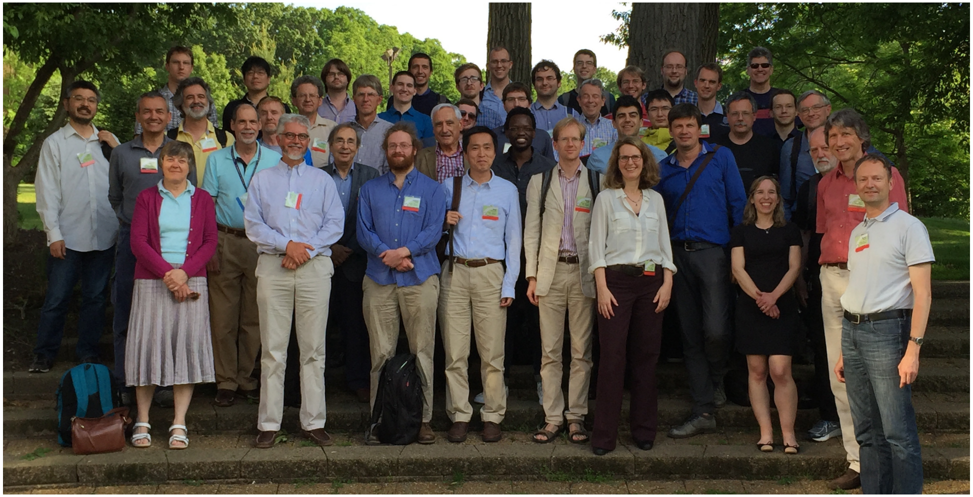
- **Hadronic Light-by-Light scattering** (HLbL, α^3)



- Hadronic light-by-light tensor $\Pi_{\mu\nu\lambda\sigma}(p_1, p_2, p_3)$
- More difficult, but 10% precision would be enough

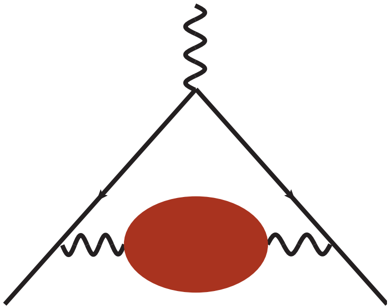
$$a_\mu^{\mathrm{HLbL}} = (92 \pm 18) \times 10^{-11}$$

- First workshop organized by the muon $g-2$ theory initiative : (almost) at Fermilab



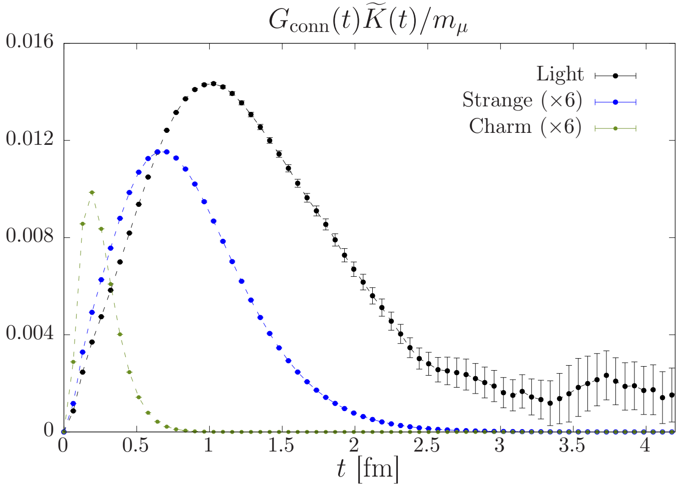
- Lattice calculations were not yet competitive with data-driven approaches (HVP and HLbL)
  - $\rightarrow$ 10 lattice talks on lattice HVP
  - $\rightarrow$ 3 lattice talks on lattice HLbL
  - $\rightarrow$ 2021 White Paper average ($g-2$ *theory initiative*) dominated by non-lattice inputs
- Situation is drastically different today
  - $\rightarrow$ exciting time to work on this topic
  - $\rightarrow$ lattice now plays a decisive role in our understanding of had. contributions

Hadronic vacuum polarization



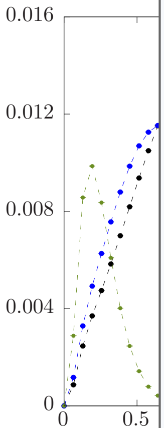
- Time-momentum representation [Blum '02] [Bernecker, Meyer '11]

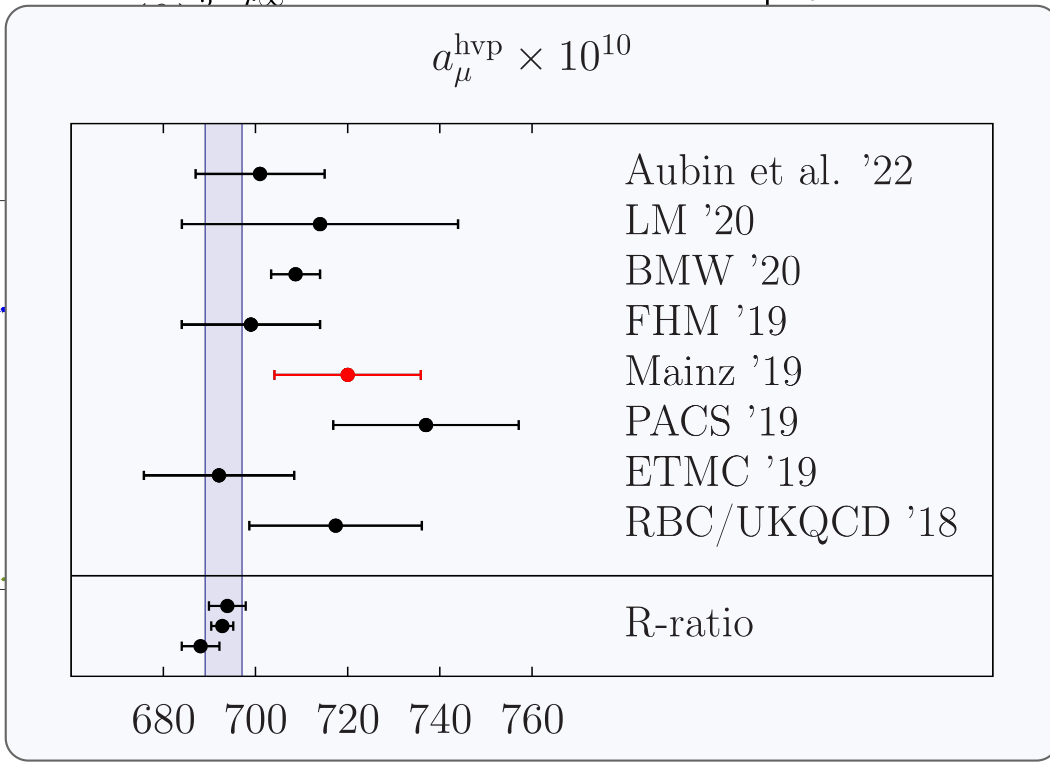
$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\ K(t)\ G(t)\ , \qquad G(t) = -\frac{1}{3}\sum_{k=1}^{3}\sum_{\vec{x}} \langle J_k(x) J_k(0)\rangle$$



- Noise problem (light-quark contribution)
- Finite-volume effects : $\mathcal{O}(3\%)$ for $L \approx 6$ fm
- Continuum extrapolation
- QED / strong isospin breaking corrections

- Time-momentum representation [Blum '02] [Bernecker, Meyer '11]





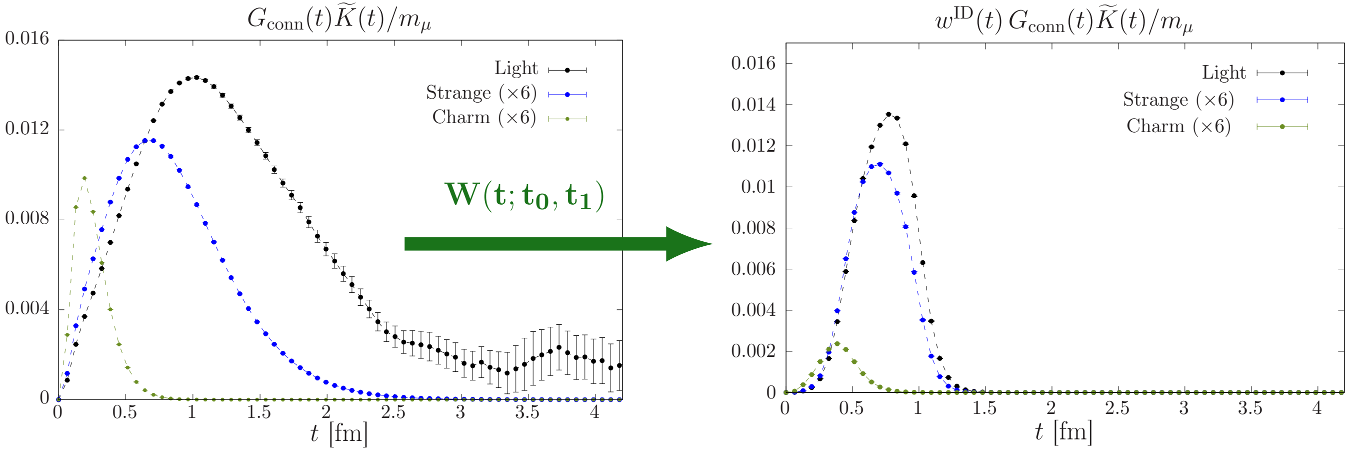
contribution)

) for $L \approx 6$ fm

ng corrections

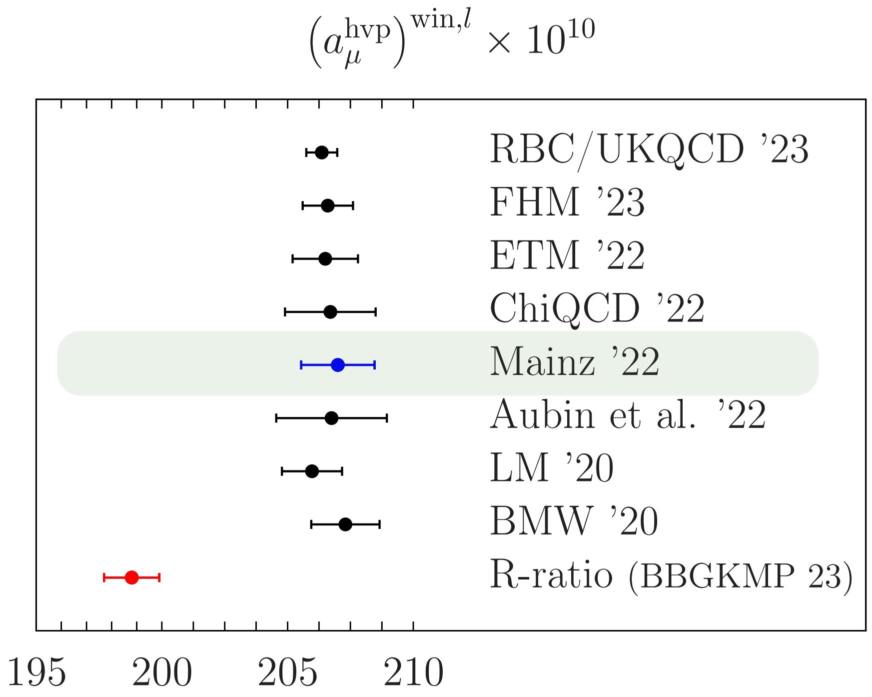
[RBC/UKQCD 2018]

$$a_\mu^{\rm win} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t G(t)\,\widetilde{K}(t)\,\mathbf{W(t;t_0,t_1)}$$



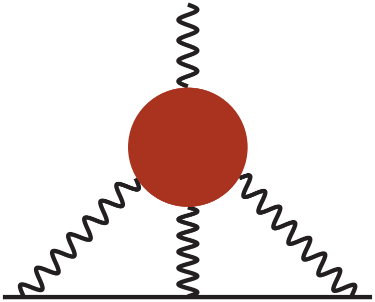
- **Intermediate window** : $\sim 30\%$ of the total contribution
- Easier to compute on the lattice (and accessible from R-ratio data !)
  - → 1-2 permille statistical precision can be reached on the integrand
  - → small finite-volume effects, small electromagnetic correction
- Data-driven : 2π contribution in the region $600\ \mathrm{MeV} \leq \sqrt{s} \leq 900\ \mathrm{MeV}$ (around the rho peak) :
  - → relative contribution of 55%-60% on both $a_\mu^{\rm LO-HVP}$ and $a_\mu^{\rm win}$!
  - → $\sqrt{s} \leq 600\ \mathrm{MeV}$ slightly suppressed, $\sqrt{s} \geq 900\ \mathrm{MeV}$ slightly enhanced.

[talk by L. Lellouch on Tuesday]

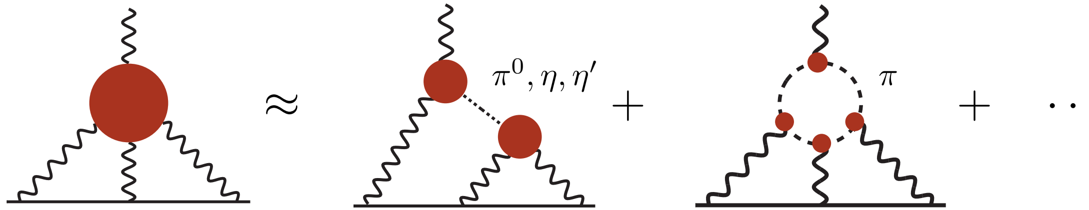


- complete calculation with the Mainz group in '22
- significant tension between (all !) lattice calculations vs data-driven approach (here shown for the light-quark connected contribution in the isospin limit)
- we will learn more about it tomorrow afternoon !

Hadronic light-by-light scattering



$a_\mu^{\mathrm{HLbL}} =$



Dispersive framework ('21)	$a_\mu \times 10^{11}$
π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy q	3 ± 1
sum	92 ± 19
Mainz '22	109.6 ± 15.9
RBC/UKQCD '23	124.7 ± 15.2

Two approaches on the lattice :

π^0, η, η' : accessible on the lattice

direct lattice calculation

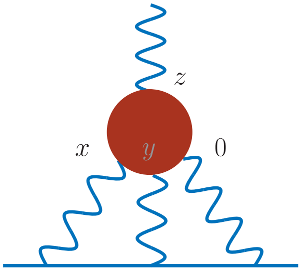
Strong synergy between the two approaches !

Collaborators : N. Asmussen, E. Chao, J. Green, R.J. Hudspith, H.B Meyer, A. Nyffeler

- Our setup : QED part in position space (continuum + infinite volume)

$$a_\mu^{\rm HLbL} = \frac{me^6}{3} \int \mathrm{d}^4y \int \mathrm{d}^4x \, \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \, i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$$

$$i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y) = -\int \mathrm{d}^4z \, z_\rho \, \langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0) \rangle$$



- ▶ $\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$: four-point correlation function computed on the lattice
- ▶ $\mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$: QED part, computed semi-analytically in position-space
  - → avoid $1/L^2$ finite-volume effects from the massless photons ⇒ $\sim e^{-m_\pi L/2}$
  - → pre-computed : efficient numerical implementation ($\ll$ to solver time)
  - → not unique : different choices strongly affect the signal/noise ratio [RBC/UKQCD]
- ▶ At given y : weight function factorizes as a function of x and a function of z (linear)
  - → allows very efficient numerical implementation

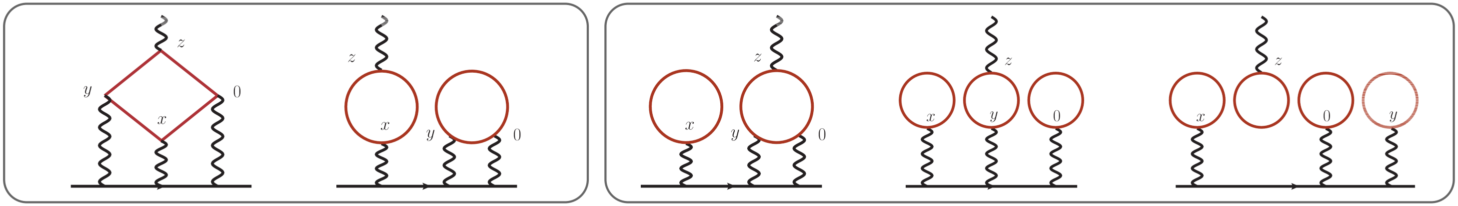
$$a_\mu^{\text{HLbL}} = \frac{me^6}{3} \int \mathrm{d}^4y \int \mathrm{d}^4x \, \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y)$$

$$i\widehat{\Pi}_{\rho,\mu\nu\lambda\sigma}(x,y) = -\int \mathrm{d}^4z \, z_\rho \, \langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0) \rangle$$

- 8 of the 12 integrals (over x and z) are done exactly $\to f(y)$
- Once all indices have been contracted : $f(y) = f(|y|)$ (rotational invariant)

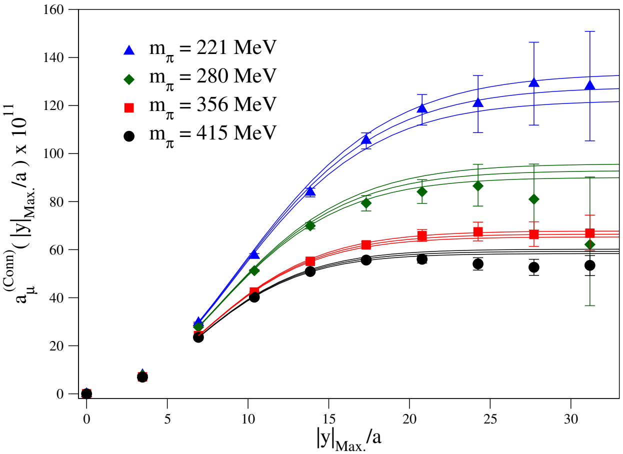
  $\to$ sample the integrand by selecting for a few values of $|y|$
- Partially integrated sum :

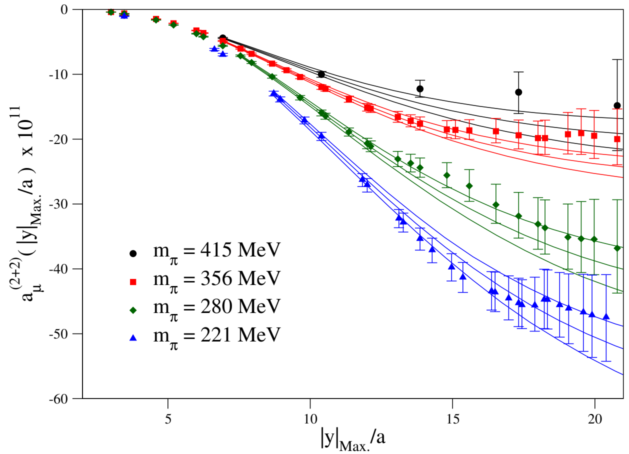
$$a_\mu^{\text{hlbl}}(|y|_{\max}) = \int_0^{|y|_{\max}} d|y| \; f(|y|)$$



[Eur.Phys.J.C 81 (2021) 7, 651]

- Connected and disconnected contributions as $m_\pi \to m_\pi^{\text{phys}}$

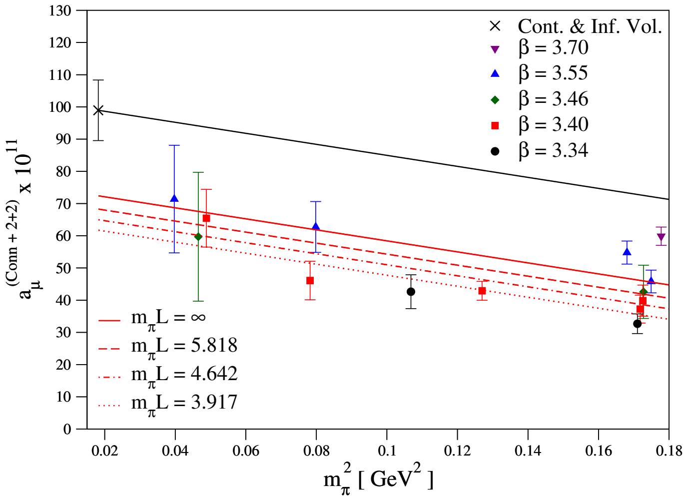




- Statistical noise increases rapidly as $m_\pi \to m_\pi^{\text{phys}}$

  $\to$ extend data using the model $f(|y|) = A|y|^3 \exp(-M|y|)$ (works well for the π^0-pole)

- Strong pion mass dependence observed, but suppressed in the sum of the two contributions



Dominant sources of uncertainty :

- Statistical error increases as $m_\pi \to m_\pi^{\mathrm{phys}}$
- Finite-volume effects $\propto \exp(-m_\pi L/2)$
- Continuum extrapolation

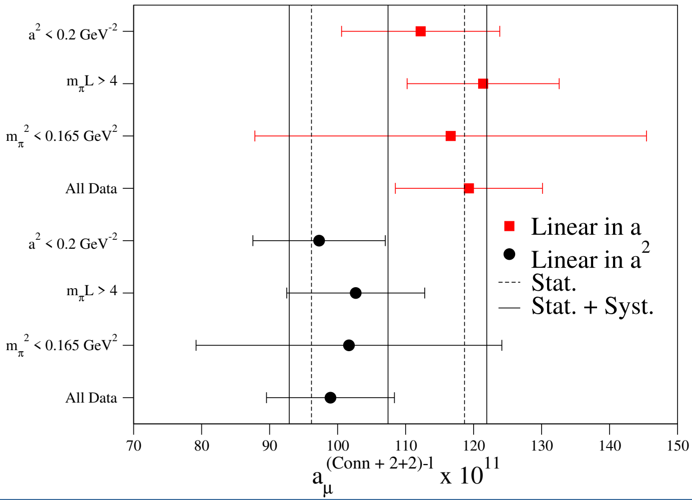
- Light-quark contribution :

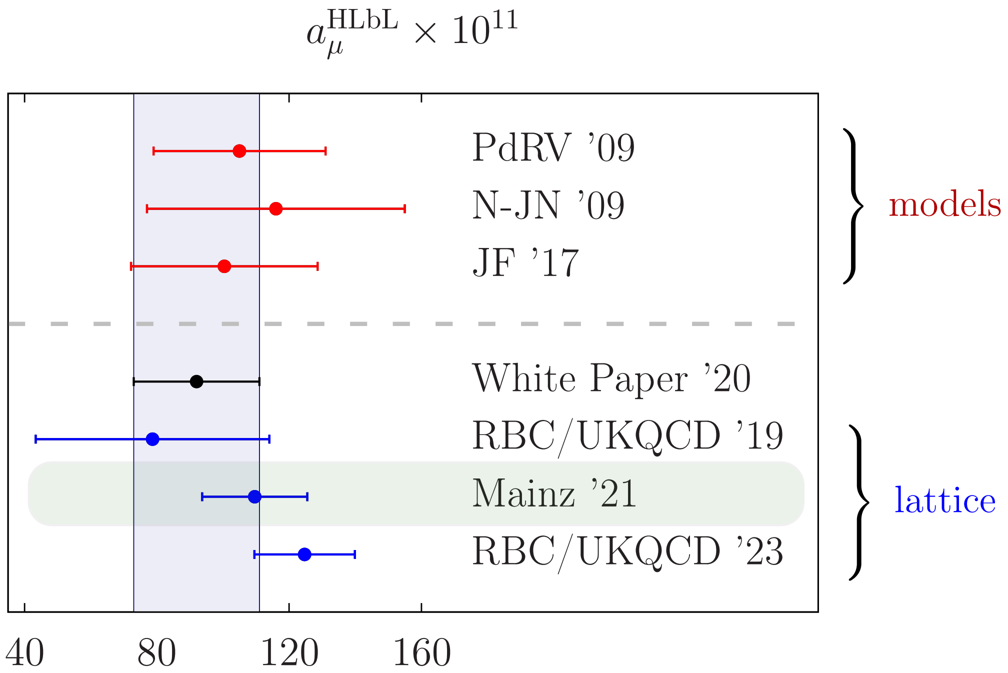
$$a_\mu^{\mathrm{hlbl,light}} = (107.4 \pm 14.6) \times 10^{-11}$$

- Total contribution $(l + s + c)$

$$a_\mu^{\mathrm{hlbl}} = (109.6 \pm 15.9) \times 10^{-11}$$

[Eur.Phys.J.C 80 (2020) 9, 869]
[Eur.Phys.J.C 81 (2021) 7, 651]
[Eur.Phys.J.C 82 (2022) 8, 664]
[JHEP 04 (2023) 040]



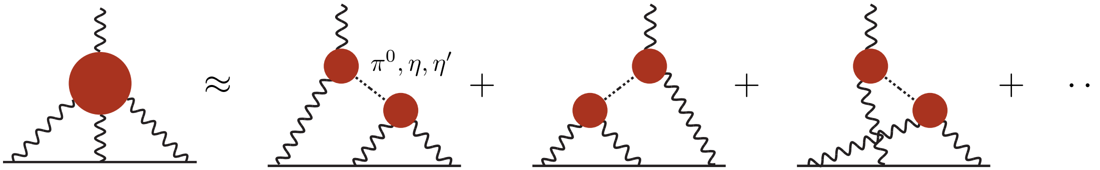


- First lattice QCD results are now published

  $\rightarrow$ good agreement with the dispersive framework (precision $\sim$ 15%)

- Treatment of systematic errors : finite-volume, chiral extrapolation, ...

  $\rightarrow$ strongly relies on the knowledge of the pseudoscalar transition form factors !



[Jegerlehner & Nyffeler '09]

$$a_\mu^{\mathrm{HLbL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \; w_1(Q_1,Q_2,\tau)\, \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2)\, \mathcal{F}_{P\gamma^*\gamma^*}(-Q_2^2, 0) + w_2(Q_1,Q_2,\tau)\, \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\, \mathcal{F}_{P\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0)$$

Integrand concentrated at small momenta below 2 GeV

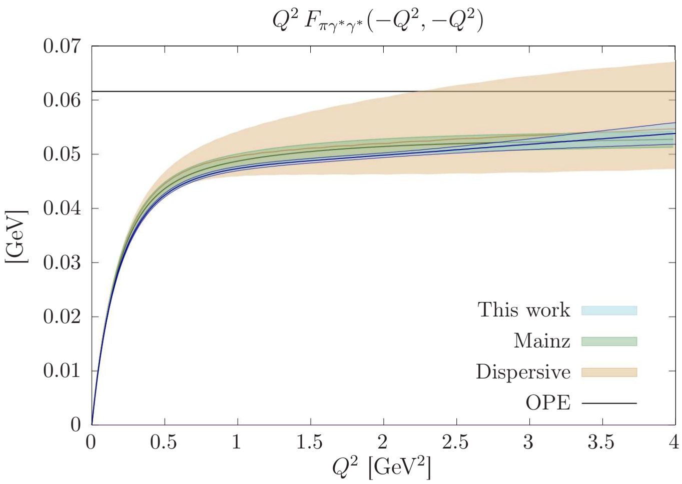
Transition form factors

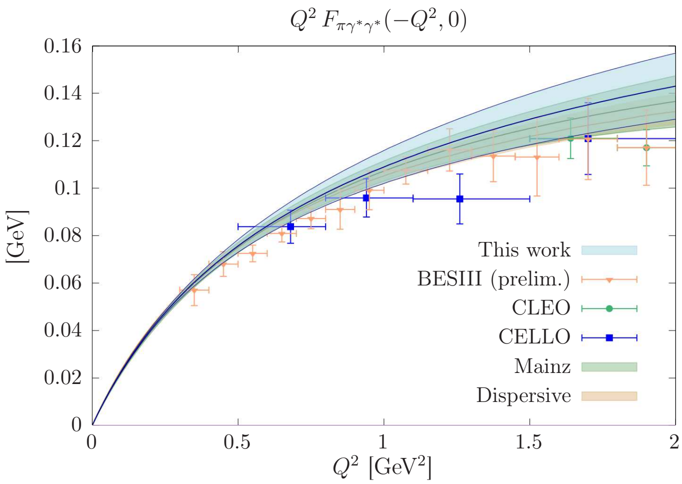
$$\epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha\, q_2^\beta\; \mathcal{F}_{P\gamma\gamma}(q_1^2, q_2^2) = i \int \mathrm{d}^4x\, e^{iq_1x}\, \langle 0|T\{J_\mu(x)J_\nu(0)\}|P(\vec{p})\rangle = M_{\mu\nu}(q_1^2, q_2^2)$$

In Euclidean space-time : [Ji & Jung '01] [Cohen et al. '08] [Feng et al. '12]

$$M_{\mu\nu}^E(-Q_1^2, -Q_2^2) = -\int \mathrm{d}\tau\, e^{\omega_1\tau} \int \mathrm{d}^3x\, e^{-i\vec{q}_1\vec{x}}\, \langle 0|T\left\{J_\mu(\vec{x},\tau)J_\nu(\vec{0},0)\right\}|P(\vec{p})\rangle = \int_{-\infty}^{\infty} \mathrm{d}\tau\, \widetilde{A}_{\mu\nu}^{(\pi^0)}(\tau)\, e^{\omega_1\tau}$$

Collaborators : H.B Meyer and A. Nyffeler (Phys.Rev.D 94 (2016) and Phys.Rev.D 100 (2019))





$$\Gamma(\pi^0 \to \gamma\gamma) = 7.17(0.49)\ \text{eV}$$

$$a_\mu^{\text{HLbL};\pi^0} = 59.7(3.6) \times 10^{-11}$$

→ PrimEx-II : $7.802(52)_{\text{stat}}(105)_{\text{syst}}$ eV

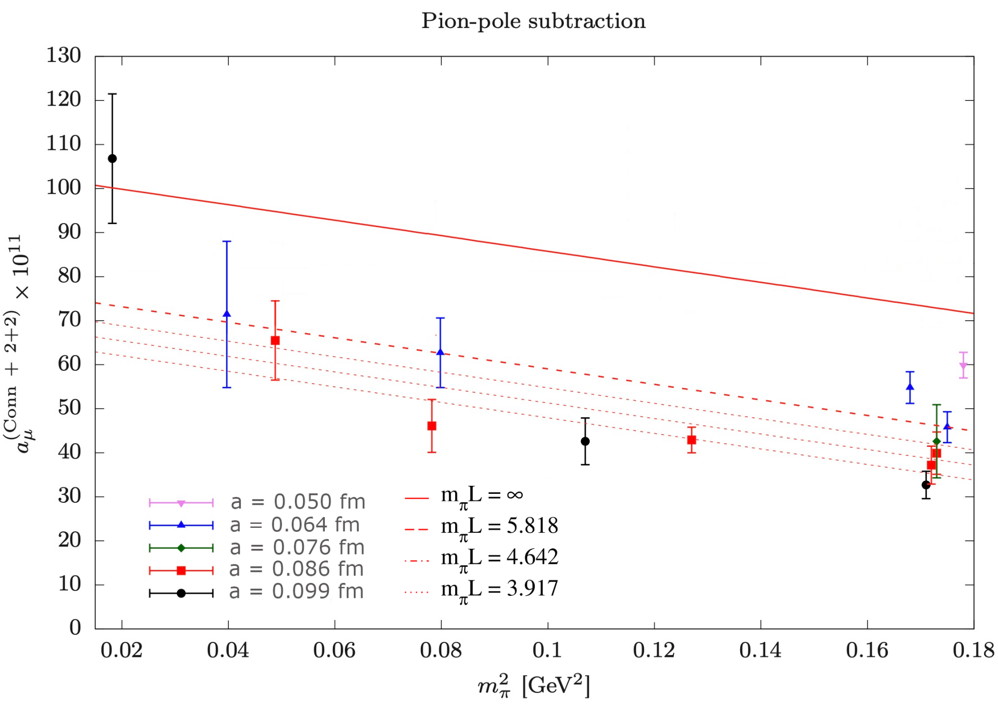
→ ≈ 60% of pseudoscalar-pole

Other lattice estimates :

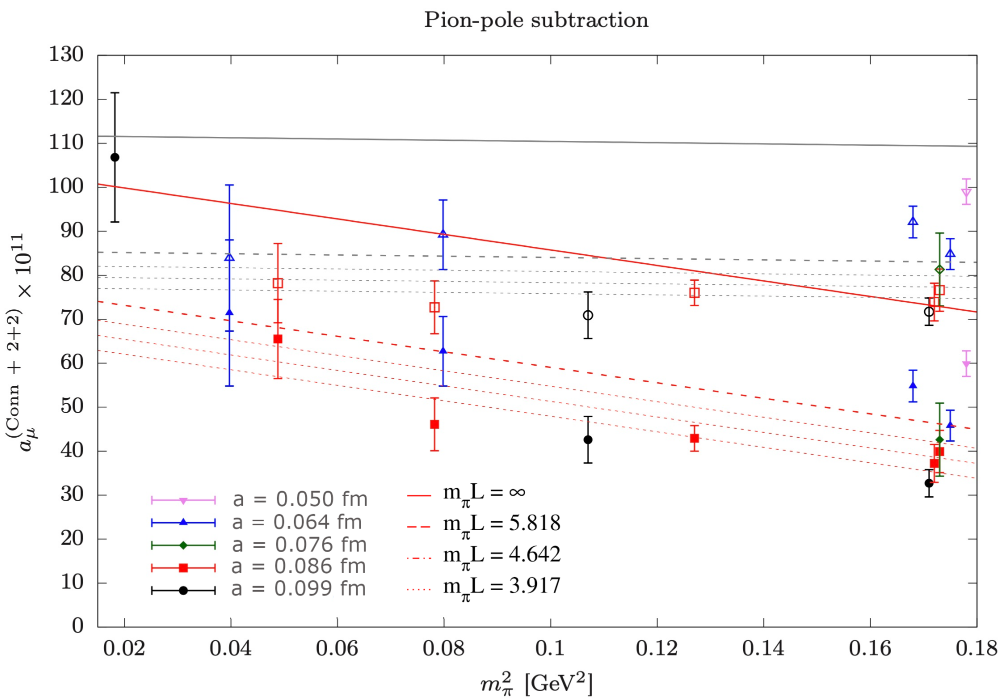
→ BMW '23 : $a_\mu^{\text{HLbL};\pi^0} = 57.8(2.0) \times 10^{-11}$ → (talk by W. Verplanke on Tuesday)

→ ETM (prelim) : $a_\mu^{\text{HLbL};\pi^0} = 56.7(3.2) \times 10^{-11}$ → (talk by S. Burri on Tuesday)

Dispersive framework : $a_\mu^{\text{HLbL};\pi^0} = 63.6(2.7) \times 10^{-11}$ [Hoferichter et. al '18] → 1.7 σ larger



- Results obtained with the Mainz group [Eur.Phys.J.C 81 (2021) 7, 651]
- Statistical precision deteriorates rapidly at low pion masses
- Dashed lines : finite-volume correction → based on pion TFF !



- Open symbols : $a_\mu^{\mathrm{hlbl,cor}}(a, m_\pi) = a_\mu^{\mathrm{hlbl,data}}(a, m_\pi) + \left(a_\mu^{\pi^0,\mathrm{phys}(a,m_\pi)} - a_\mu^{\pi^0}(a, m_\pi) \right)$
- Correction term : from our dedicated lattice QCD calculation [Phys.Rev.D 100 (2019) 3]

Significantly improve the chiral extrapolation ! Allow to improve on the continuum extrapolation at heavier pion masses.

Pion-pole contribution $\approx$ 65-70% of the pseudoscalar-pole contribution

- Next step : extend the calculation to include both the η and η'

  In principle, the same strategy works for the η ...
  ... but not for the η' (excited state)

  $\rightarrow$ need to study the mixing between the η and η'

  $\rightarrow$ noisy disconnected diagrams need to be evaluated

- Work done within the BMW collaboration using staggered quarks

  $\rightarrow$ talk by W. Verplanke on Tuesday

  $\rightarrow$ Recent preprint 2305.04570 [hep-lat]

Collaborators : W. Verplanke, G. Wang, Z. Fodor, J. Guenther, L. Lellouch, K. Szabo, L. Varnhorst

Study the mixing of the η-η' in the isospin basis

$$\mathcal{O}_8(x) = \frac{1}{\sqrt{6}}\left(\overline{u}\gamma_5 u(x) + \overline{d}\gamma_5 d(x) - 2\overline{s}\gamma_5 s(x)\right),$$
$$\mathcal{O}_0(x) = \frac{1}{\sqrt{3}}\left(\overline{u}\gamma_5 u(x) + \overline{d}\gamma_5 d(x) + \overline{s}\gamma_5 s(x)\right).$$

Spectral decomposition of the 2×2 matrix of 3-point correlation functions :

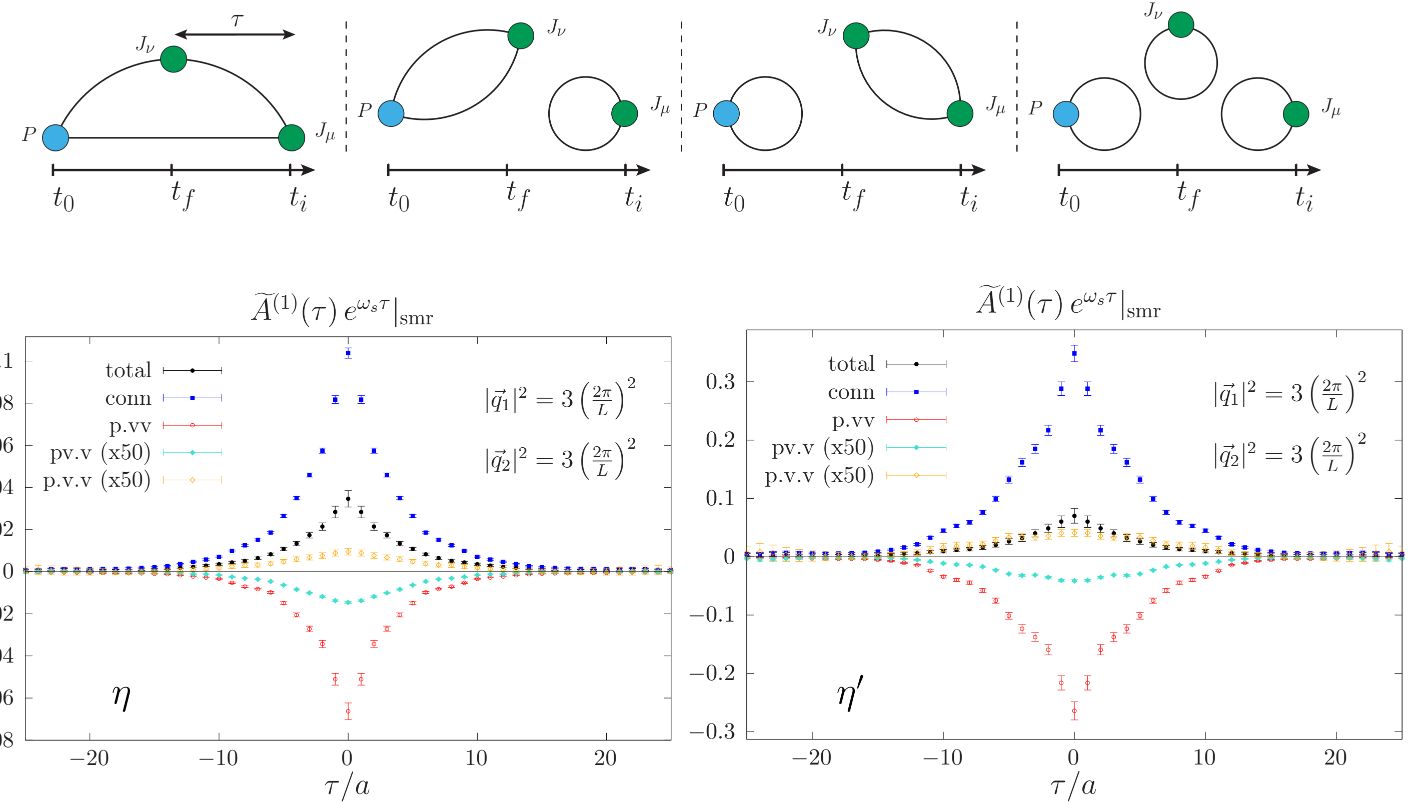
$$C^{(i)}_{\mu\nu}(\tau, t_P) = \sum_{\vec{z}} \langle 0|J_\mu(\vec{z},\tau)J_\nu(\vec{0},0)|\eta(\vec{p})\rangle e^{-i\vec{q}_1\cdot\vec{z}} \times \frac{1}{2E_\eta}\langle \eta(\vec{p})|\mathcal{O}_i|0\rangle e^{E_\eta(t_0-t_f)}$$
$$+ \sum_{\vec{z}} \langle 0|J_\mu(\vec{z},\tau)J_\nu(\vec{0},0)|\eta'(\vec{p})\rangle e^{-i\vec{q}_1\cdot\vec{z}} \times \frac{1}{2E_{\eta'}}\langle \eta'(\vec{p})|\mathcal{O}_i|0\rangle e^{E'_\eta(t_0-t_f)} + \cdots$$

Matrix notation :

$$\begin{pmatrix} C^{(8)}_{\mu\nu} \\ C^{(0)}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} T^{(8)}_\eta & T^{(8)}_{\eta'} \\ T^{(0)}_\eta & T^{(0)}_{\eta'} \end{pmatrix} \begin{pmatrix} \widetilde{A}^{(\eta)}_{\mu\nu} \\ \widetilde{A}^{(\eta')}_{\mu\nu} \end{pmatrix},$$

$$\mathcal{F}_{P\gamma\gamma}(q_1^2, q_2^2) \propto \int_{-\infty}^{\infty} \mathrm{d}\tau\, \widetilde{A}_{\mu\nu}(\tau)\, e^{\omega_1 \tau}$$

2305.04570 [hep-lat]



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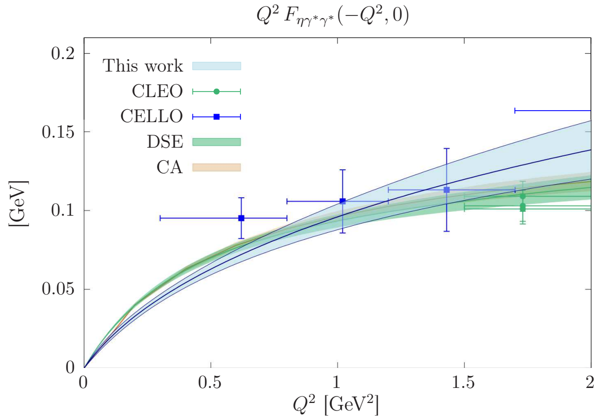
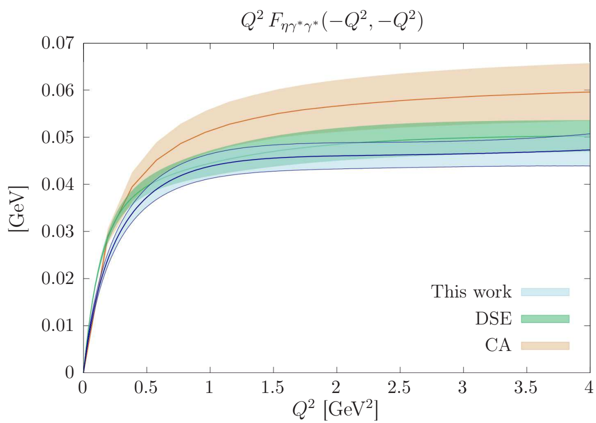
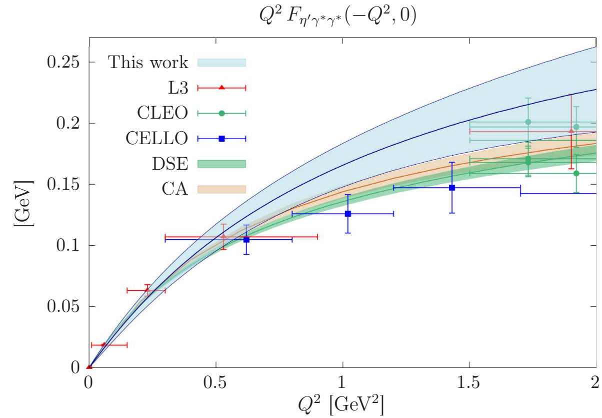
2305.04570 [hep-lat]

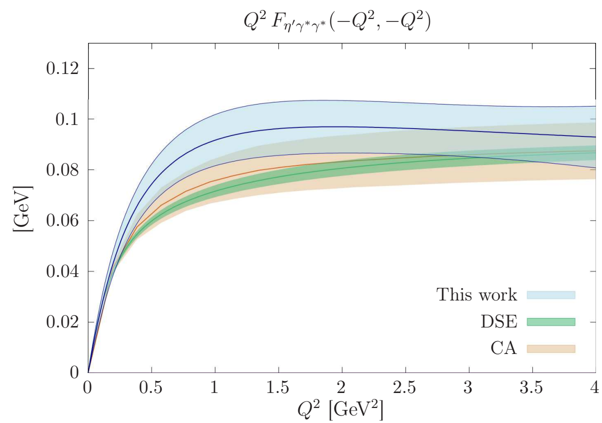
η-meson





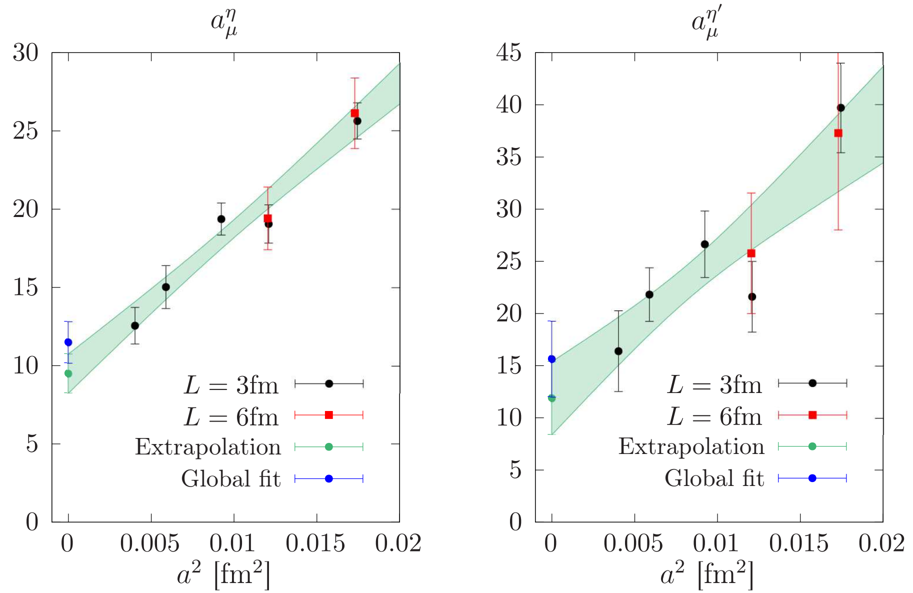
η'—meson





2305.04570 [hep-lat]

$$a_\mu^{\mathrm{HLbL};P} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \; w_1(Q_1,Q_2,\tau)\; \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2)\; \mathcal{F}_{P\gamma^*\gamma^*}(-Q_2^2, 0) + w_2(Q_1,Q_2,\tau)\; \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\; \mathcal{F}_{P\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0)$$



$$a_\mu^{\mathrm{HLbL};\eta} = 11.6(1.6)_{\mathrm{stat}}(0.5)_{\mathrm{syst}}(1.1)_{\mathrm{FSE}} \times 10^{-11}$$
$$a_\mu^{\mathrm{HLbL};\eta'} = 15.7(3.9)_{\mathrm{stat}}(1.1)_{\mathrm{syst}}(1.3)_{\mathrm{FSE}} \times 10^{-11}$$

Canterbury approximants [PRD 95, 054026 (2017)]

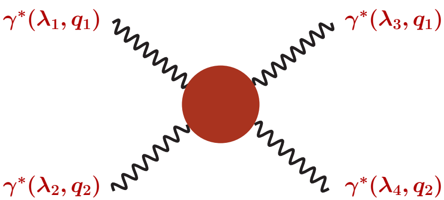
$\rightarrow a_\mu^{\mathrm{HLbL};\eta} = 16.3(1.4) \times 10^{-11}$

$\rightarrow a_\mu^{\mathrm{HLbL};\eta'} = 14.5(1.9) \times 10^{-11}$

2305.04570 [hep-lat]

Collaborators : J. Green, O. Gryniuk, G. von Hippel, H.B Meyer, V. Pascalutsa and H. Wittig , Phys.Rev.D 98 (2018) 7, 074501

- Forward scattering amplitudes $\mathcal{M}_{\lambda_3\lambda_4\lambda_1\lambda_2}$

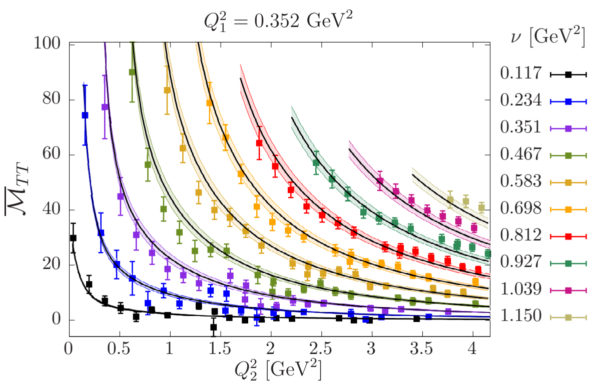


- 8 independent helicity amplitudes

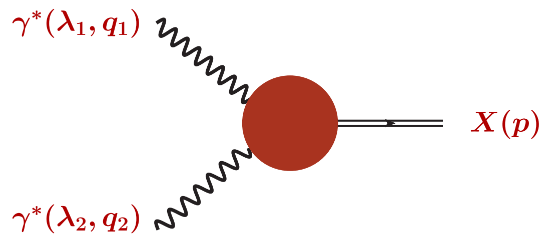
$$\mathcal{M}_{\lambda_1'\lambda_2'\lambda_1\lambda_2} = \mathcal{M}_{\mu\nu\rho\sigma}\ \epsilon^{*\mu}(\lambda_1')\ \epsilon^{*\nu}(\lambda_2')\ \epsilon^{\rho}(\lambda_1)\ \epsilon^{\sigma}(\lambda_2)$$

$$\overline{\mathcal{M}}_{\alpha}(\nu) = \frac{4\nu^2}{\pi}\int_{\nu_0}^{\infty} d\nu'\ \frac{\sqrt{X'}\ \sigma_\alpha/\tau_\alpha(\nu')}{\nu'(\nu'^2 - \nu^2 - i\epsilon)}$$

Lattice calculation 4pt correlator



$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \to \mathrm{X}(p_\mathrm{X})$ fusion cross sections



↪ Input : transition form factors

↪ Assume monopole/dipole masses (fit parameters)

- **HLbL contribution** : already close to the target precision of 10%.
  $\rightarrow$ good agreement between lattice calculations (at the level of 15%)
  $\rightarrow$ dominant contribution to the dispersive framework (π^0, η, η'-pole) available from lattice

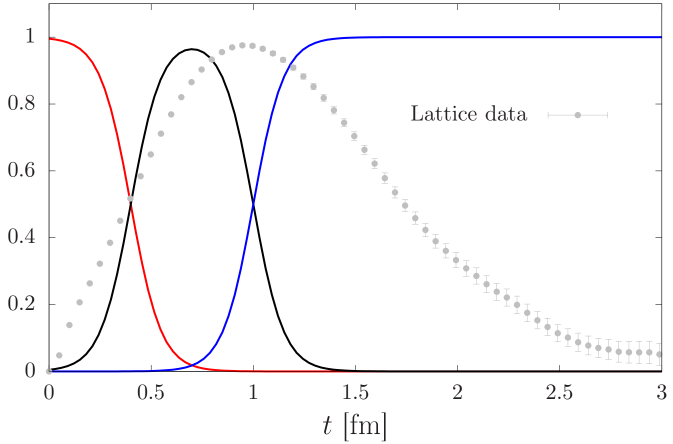
- **HVP contribution**
  $\rightarrow$ target precision : a few-permille
  $\rightarrow$ significant progress on the lattice : now competitive in terms of error
  $\rightarrow$ ... but this increase of precision comes with a new puzzle : tension with the R-ratio
  $\rightarrow$ need to be understood if one wants to agree on *one* Standard Model estimate

Lattice QCD plays a major role in this search for new physics !

- Experimental update from Fermilab next week (August 10, 2023) !
  $\rightarrow$ 21 Brookhaven raw statistics accumulated in 2023

Thanks !

Backup slides



$$a_\mu^{\mathrm{win}} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t G(t)\, K(t)\, W(t; t_0, t_1)$$

→ Short distances (SD)

→ Intermediate distances (ID)

→ Long distances (LD)

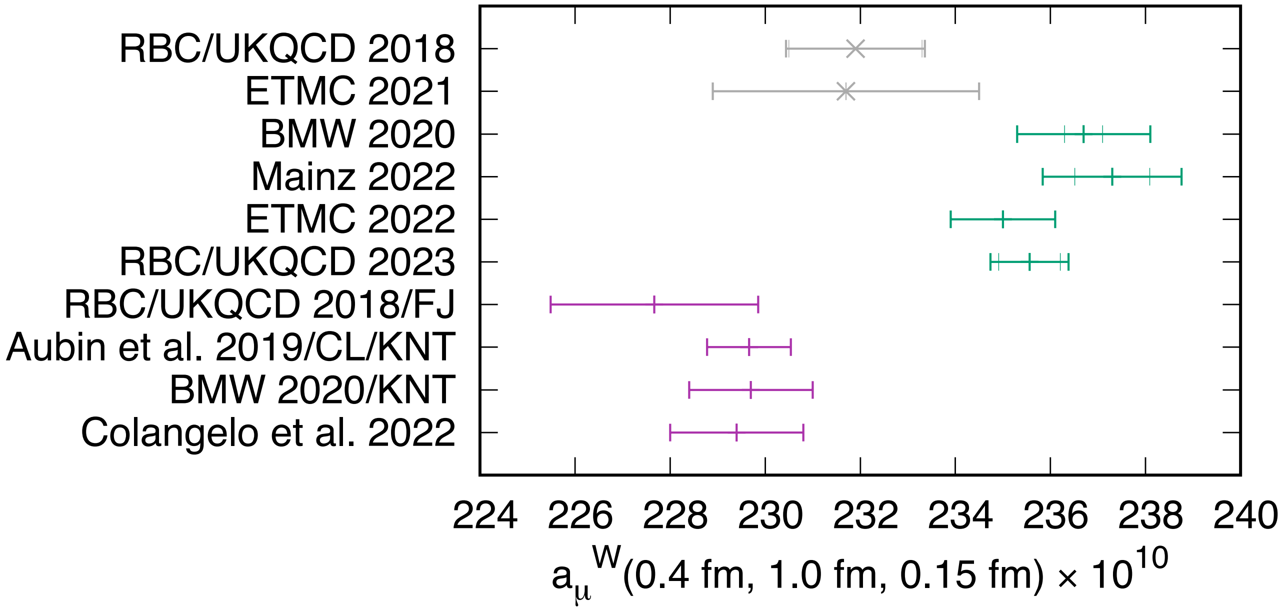
- By construction, the sum over the 3 windows gives the full contribution

$$a_\mu^{\mathrm{LO-HVP}} = a_\mu^{\mathrm{win,SD}} + a_\mu^{\mathrm{win,ID}} + a_\mu^{\mathrm{win,LD}}$$

- Each window observable is subject to very different systematic errors

Short-distance	Intermediate-distance	Long-distance
stat. precise	stat. precise	noise problem
discretization effects	small finite volume effect	finite volume corrections
		large taste breaking (staggered)

figure taken from RBC/UKQCD [2301.08696 [hep-lat]]



- R-ratio dominated by the 2π channel ($\sim 80\%$) [hep-lat : 2306.16808]

Slide taken from the talk given by Maarten Golterman on Tuesday :

Potential impact of new CMD3 2pi data

Replace 2-pion data between 0.33 and 1.2 GeV by CMD3 data, keeping KNT19 data elsewhere
(preliminary)