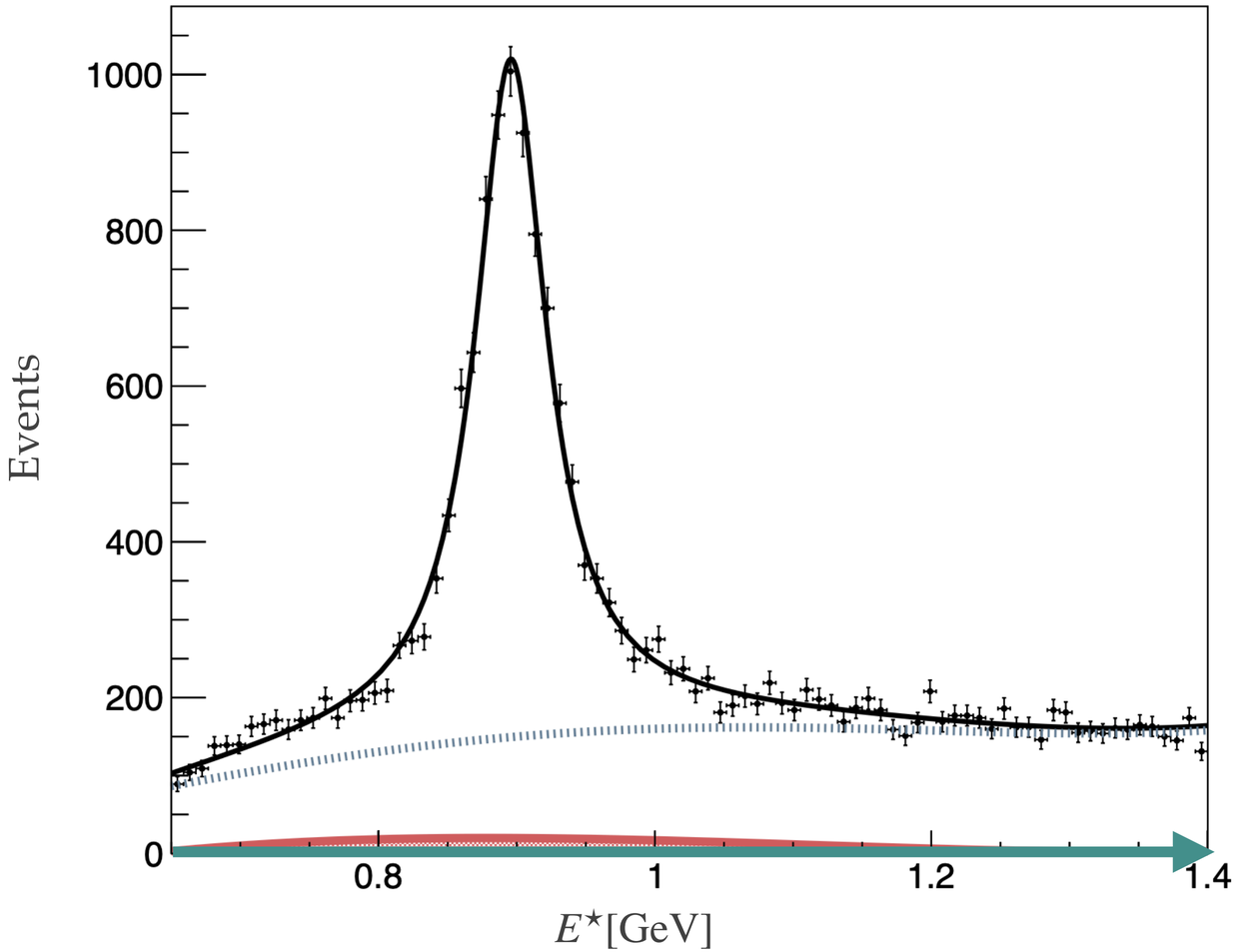
Luka Leskovec

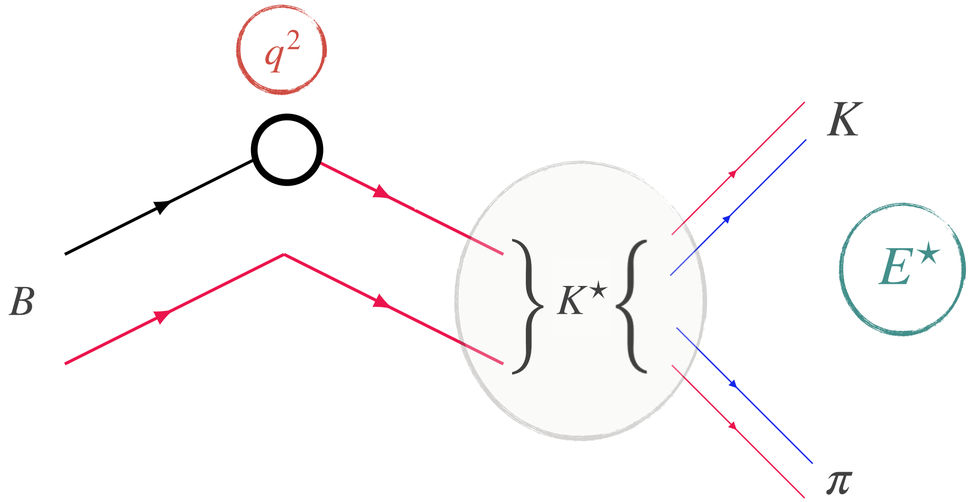
Electroweak Transitions Involving Resonances

The International Symposium on Lattice Field Theory
Fermilab, August 4, 2023

University *of Ljubljana*
Faculty *of Mathematics and Physics*

Institut
"Jožef Stefan"
Ljubljana, Slovenija



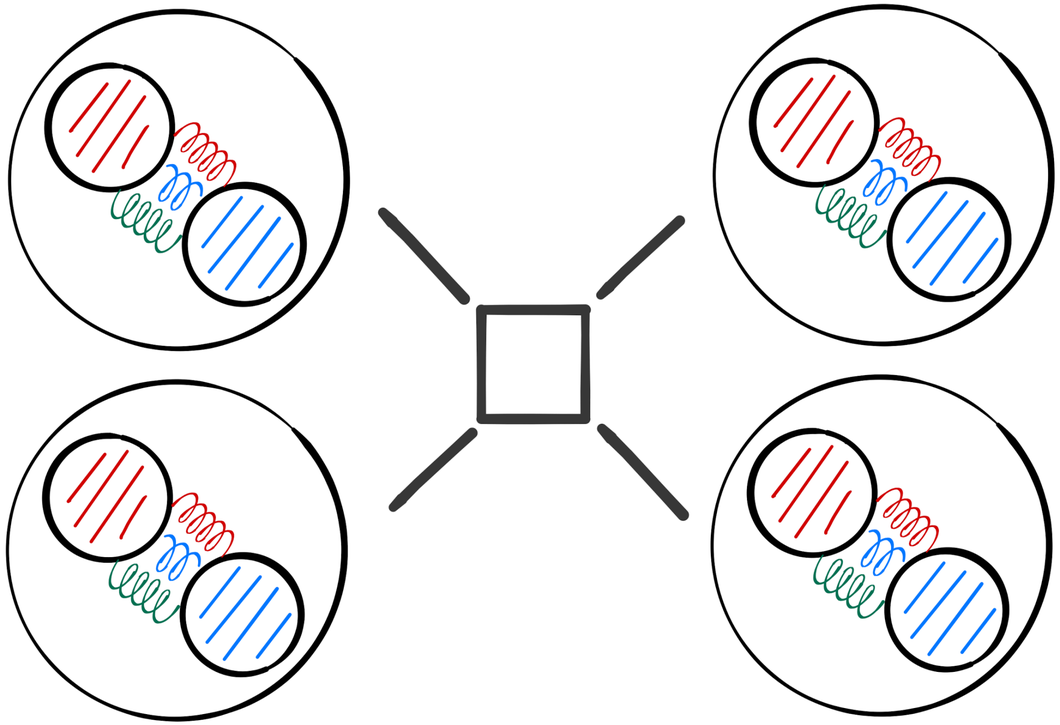


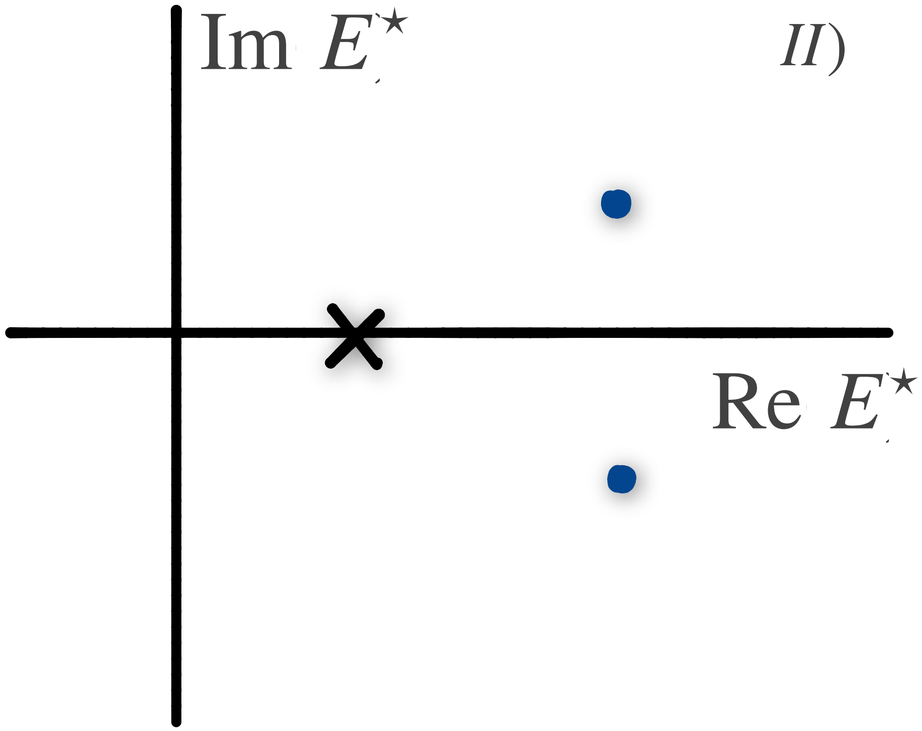
resonances

pretty “bumps” in expt. data

S. Wehle thesis [Belle]
10.3204/PUBDB-2016-03770

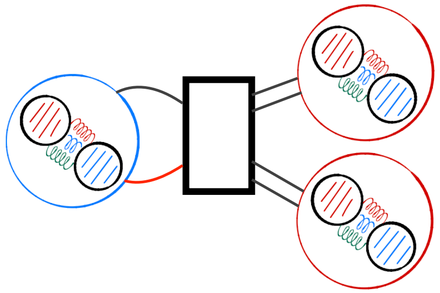
poles, not bumps





ElectroWeak Transitions with Resonances

purely hadronic EW decays



e.g. $K \to \pi\pi$

- CP violation

KTeV

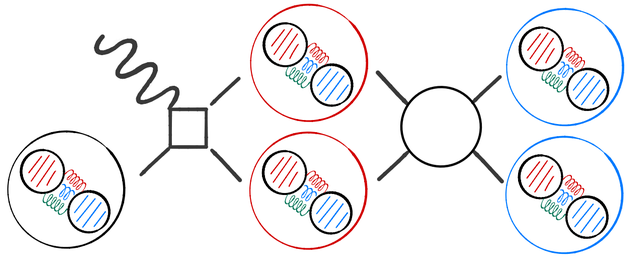
C. Kelly, Mon@13:50

M. Tomii, Thu@14:30

M. Hansen, Thu@14:50

caveat: plenty more processes

photoproduction of resonances

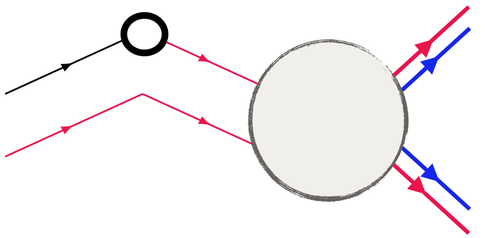


e.g. $\pi\gamma \to \pi\pi$

- photocouplings
- dispersive approaches to $g_{\mu-2}$

F. Ortega Gama, Fri@10:00

semileptonic processes



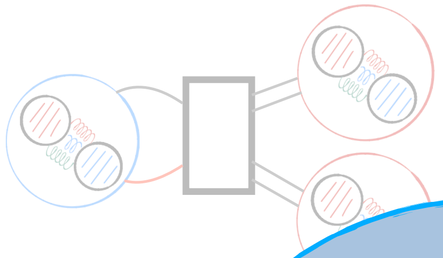
e.g. $B \to \pi\,\pi\,\ell\,\bar{\nu}$

- understanding SM pheno
- enable BSM searches

A. Grebe, Wed@10:20

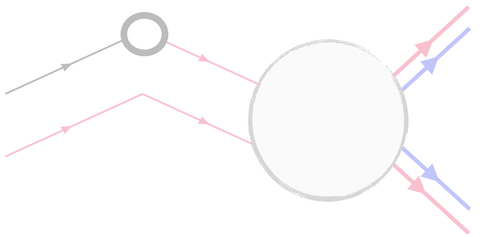
ElectroWeak Transitions with Resonances

purely hadronic EW decays



e.g. K

semileptonic processes



$B \to \pi\,\pi\,\ell\,\bar{\nu}$

- matter
- CP vio

ng SM pheno

earches

KTeV

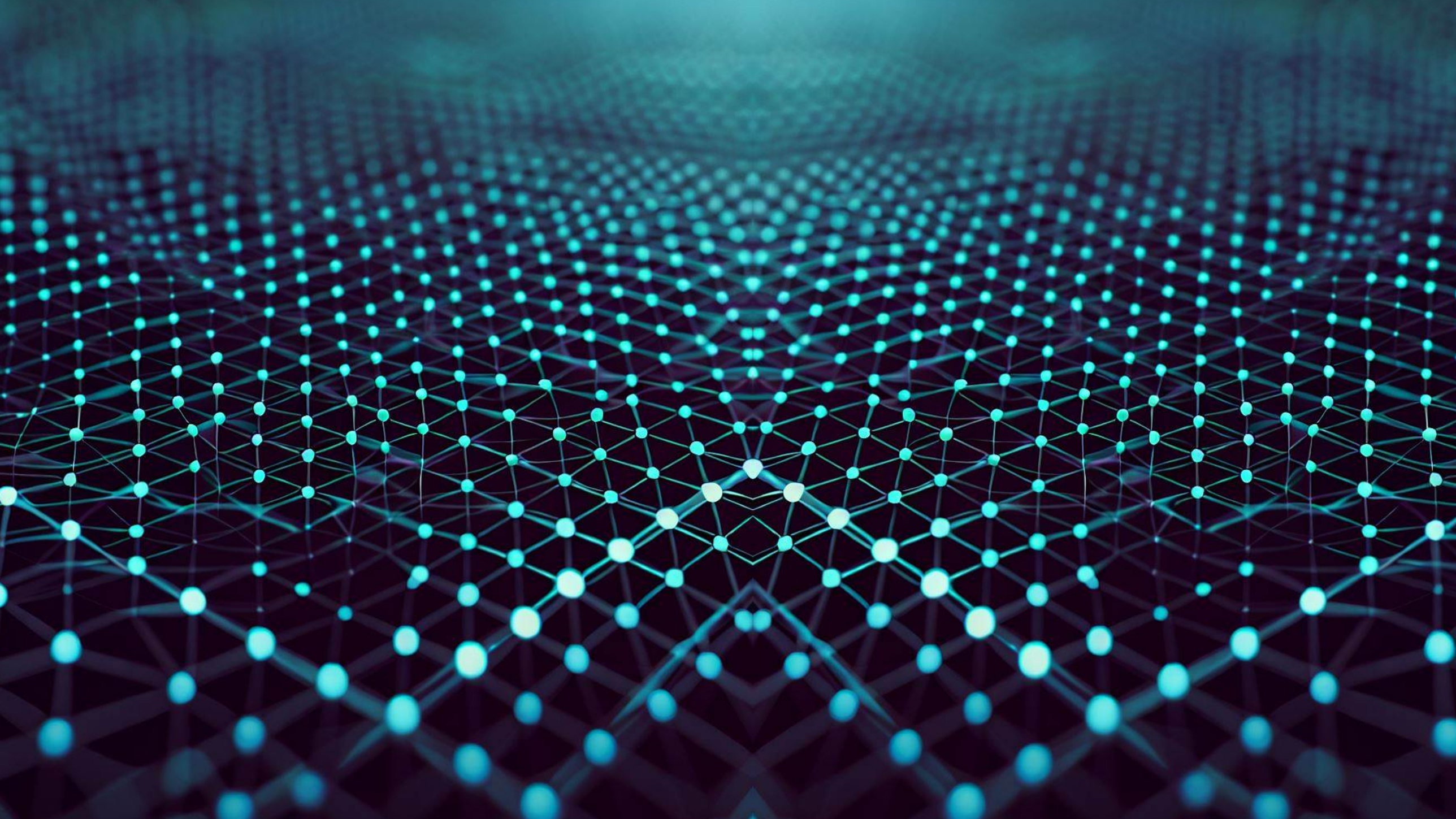
Raul Briceño, Fri@10:00

$\gamma \to \pi\pi\pi$ & $K \to \pi\pi\pi$

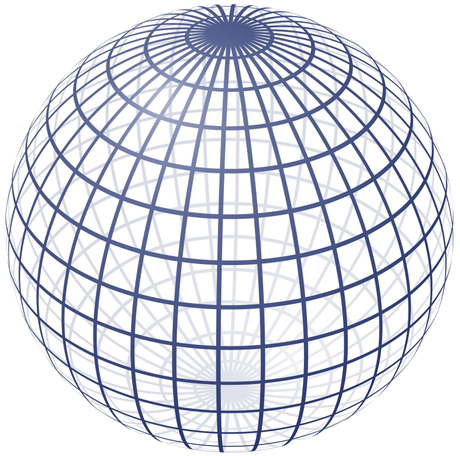
Jefferson Lab

Belle II

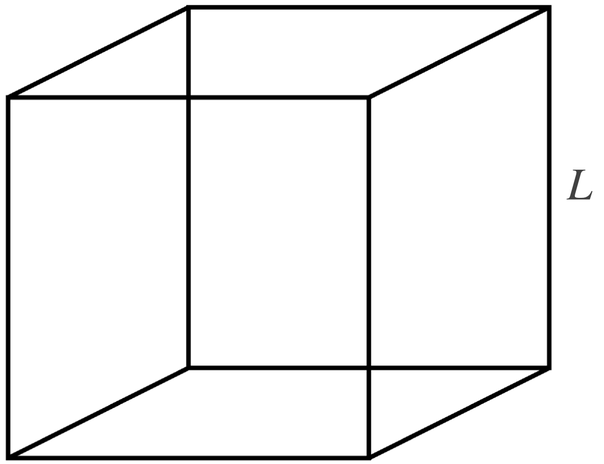
LHCb



life on the lattice



many-to-one

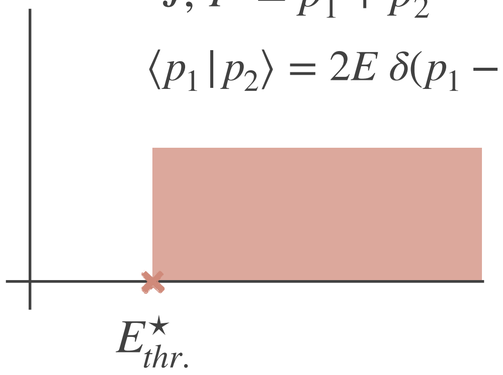


infinite volume:

- $O(3)$ symmetry
- infinite irreps (J^P)

$$J, \vec{P} = \vec{p}_1 + \vec{p}_2$$

$$\langle p_1 | p_2 \rangle = 2E\ \delta(p_1 - p_2)$$



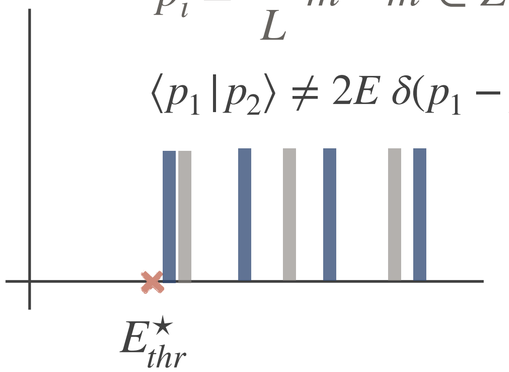
finite volume:

- discrete symmetries, Λ

$$L, \Lambda, \vec{P} = \vec{p}_1 + \vec{p}_2$$

$$\vec{p}_i = \frac{2\pi}{L}\vec{m} \quad \vec{m} \in Z^3$$

$$\langle p_1 | p_2 \rangle \neq 2E\ \delta(p_1 - p_2)$$



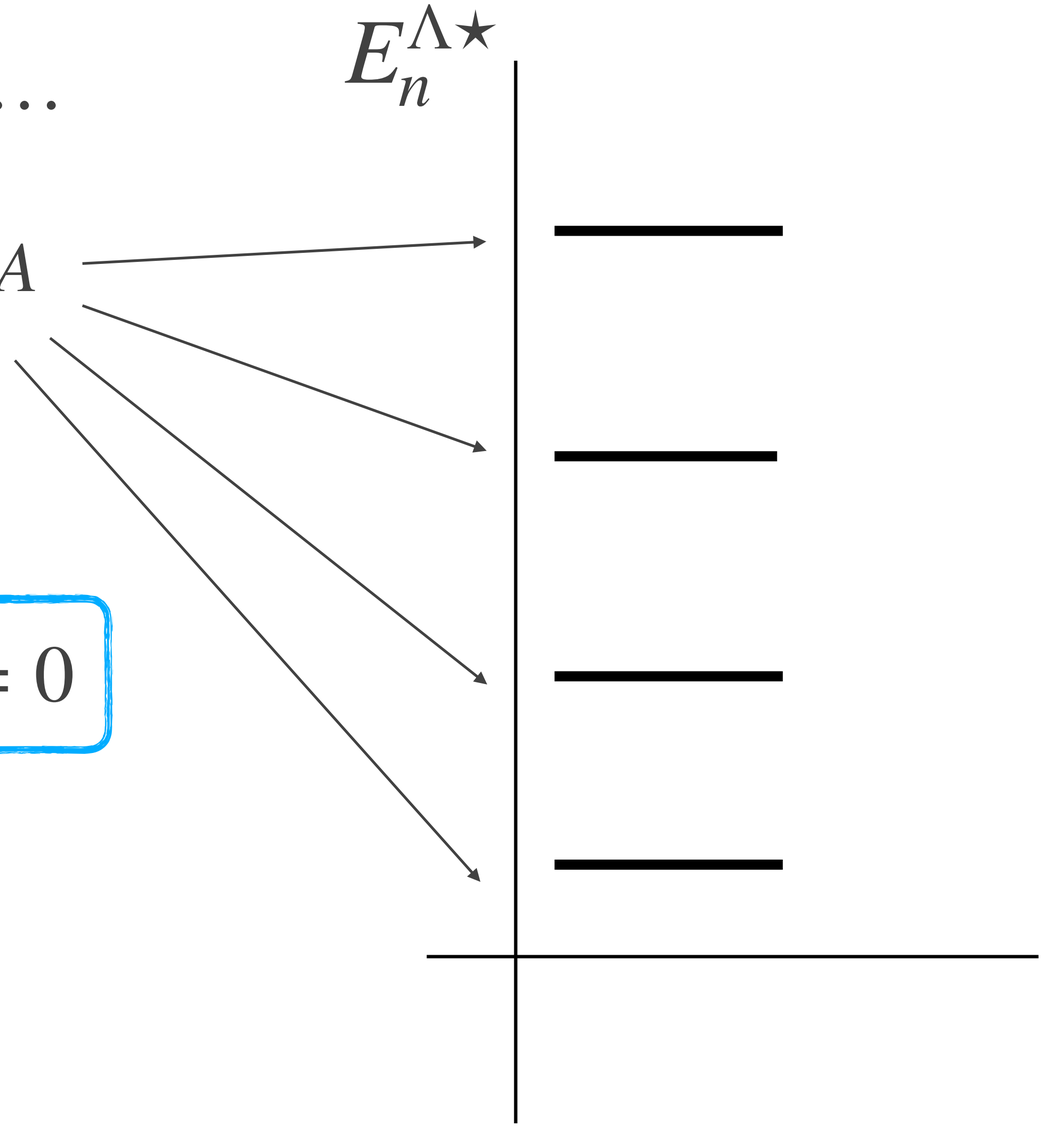
lattice states: energies

$$C_L^{(2)} = \bigcirc\!\!=\!\!\bigcirc + \bigcirc\!\!=\!\!\bullet\!\!=\!\!\bigcirc + \ldots$$

$$C_L^{(2)} = C_\infty^{(2)} - A' \frac{1}{F^{-1}(E^\star) + T(E^\star)} A$$

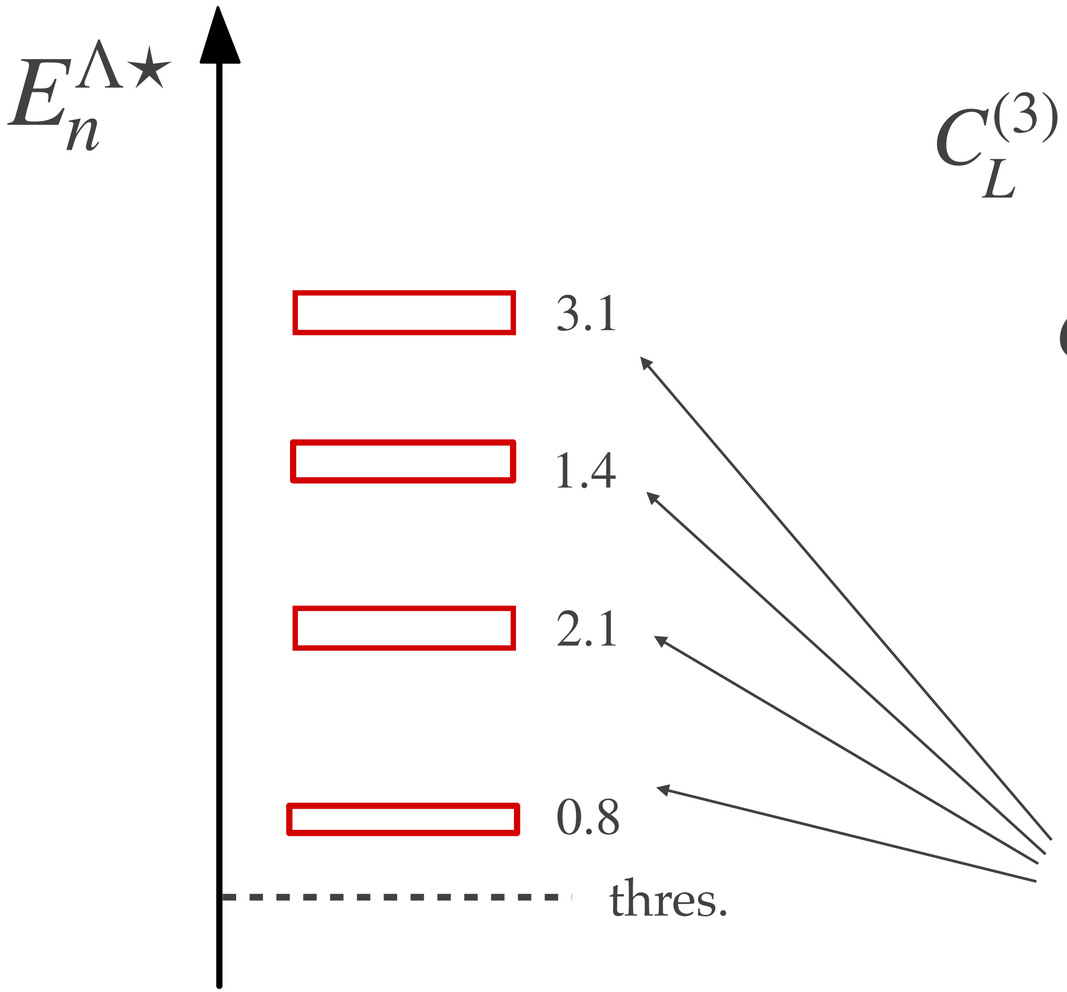
discrete spectrum where:

$$\det\left[F^{-1}(E^\star) + T(E^\star)\right] = 0$$



Luscher NPB354
Rummukainen, Gottlieb hep-lat/9503028
Kim, Sharpe, Sachrajda hep-lat/0507006
Leskovec, Prelovsek hep-lat/1202.2145
Briceno 1401.3312
Briceno, Dudek, Young 1706.06223
Woss, Wilson, Dudek 2001.08474
[and many more]

lattice states: normalization



$$C_L^{(3)} = \quad + \quad + \ldots$$

$$C_L^{(3)} = C_\infty^{(3)} - A'RA$$

$$R_n = \lim_{E \to E_n} \frac{E - E_n}{F^{-1} + T}$$

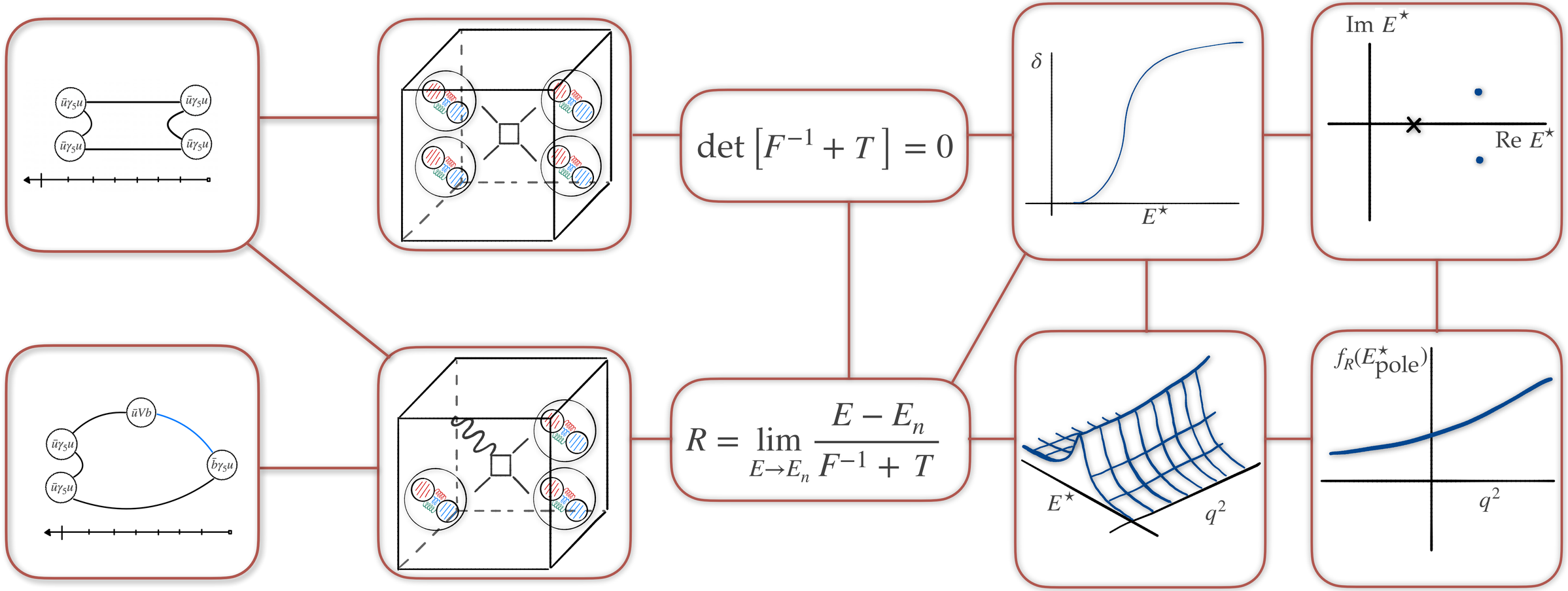
normalization of finite-volume states

$$|E_n^{\Lambda\star}\rangle_L \sim \sqrt{R_n}\,|p_1 p_2(E^\star = E_n^{\Lambda\star})\rangle_\infty$$

the "Lellouch-Lüscher" factor

Lellouch, Luscher hep-lat/0003023
Lin, Sachrajda, Testa hep-lat/0104006
…
Briceno, Hansen, Walker-Loud 1406.5965
Briceno, Hansen 1502.04314
Briceno, Dudek, LL 2105.02017

a cartoon "how-to" with $B \to \pi\pi\ell\bar{\nu}$

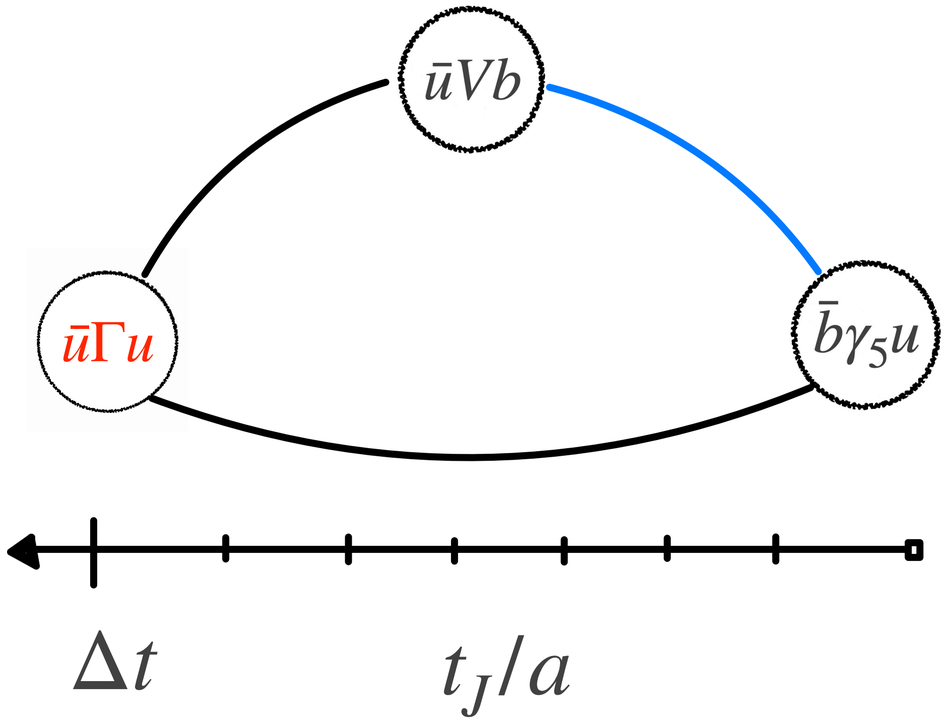


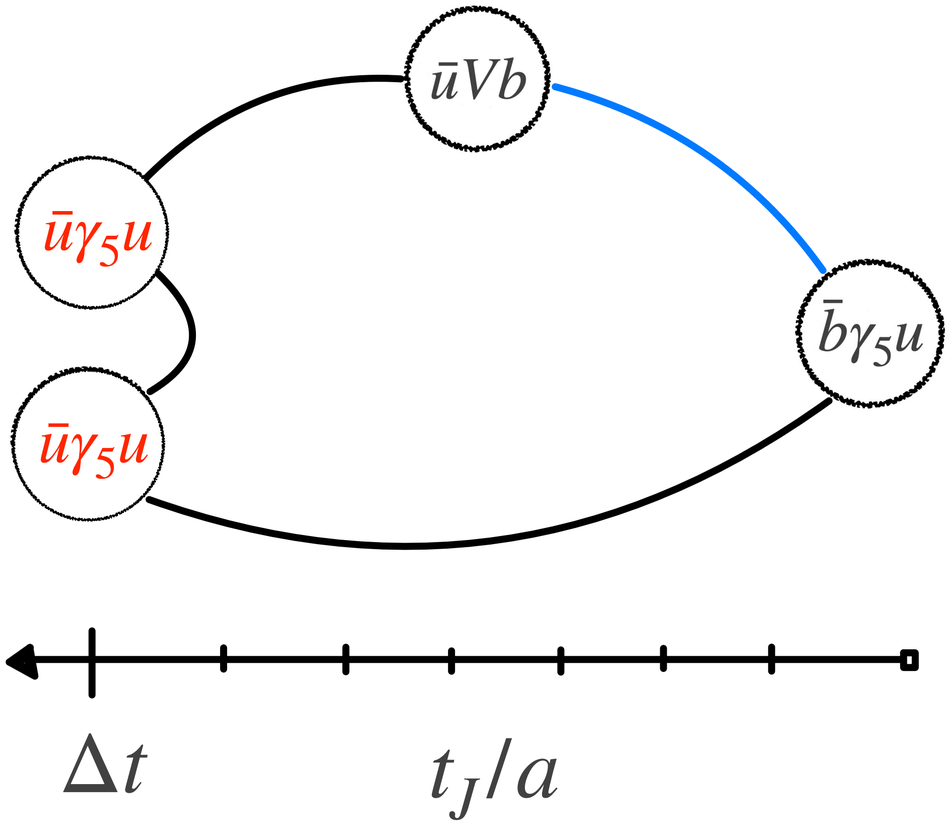
in collaboration with:
S. Meinel, M. Petschlies, C. Alexandrou,
J.W. Negele, S. Paul, A. Pochinsky

Briceño-Hansen-Walker-Loud approach

3-point functions

$$C_{3,i} = \langle\ O_i(\vec{p}, \Lambda)\ \ V^\mu\ \ O_B(\vec{p}_B)\ \rangle$$





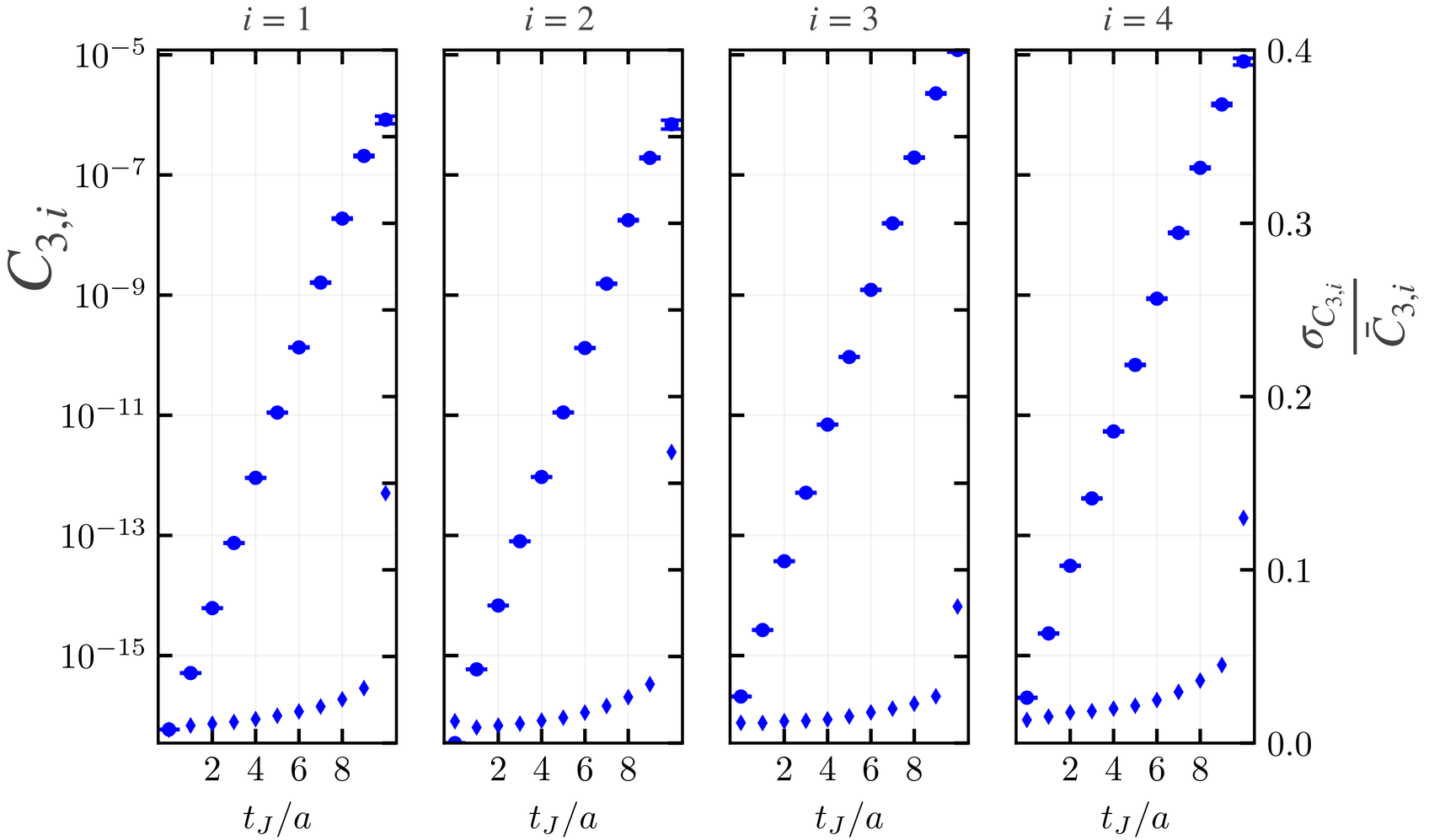
3-point function lattice data

$\vec{p} = \frac{2\pi}{L}[1,0,1]$
$\Lambda = B_2$

$\mu = z$

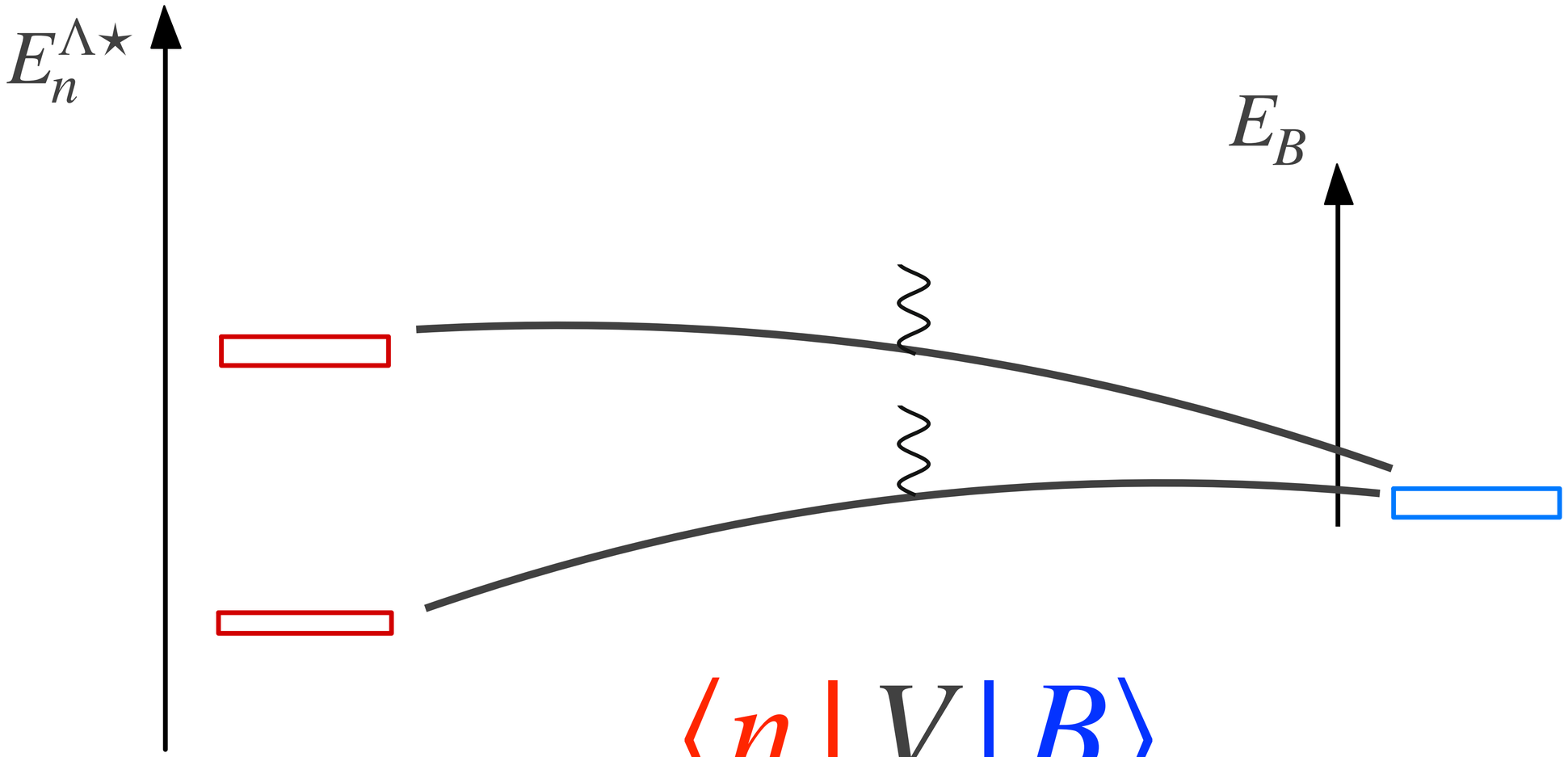
$\vec{p}_B = \frac{2\pi}{L}[0,0,1]$

$$C_{3,i} = < O_i(\vec{p}, \Lambda)\ V^\mu\ O_B(\vec{p}_B)\rangle$$



$O_1 = \bar{u}\Gamma_{B_1}u$
$O_2 = \bar{u}\gamma_t\Gamma_{B_1}u$
$O_3 = \pi(\vec{p}_1)\pi(\vec{p}_2)$
$O_4 = \pi(\vec{p}_1)\pi(\vec{p}_2)$

matrix elements



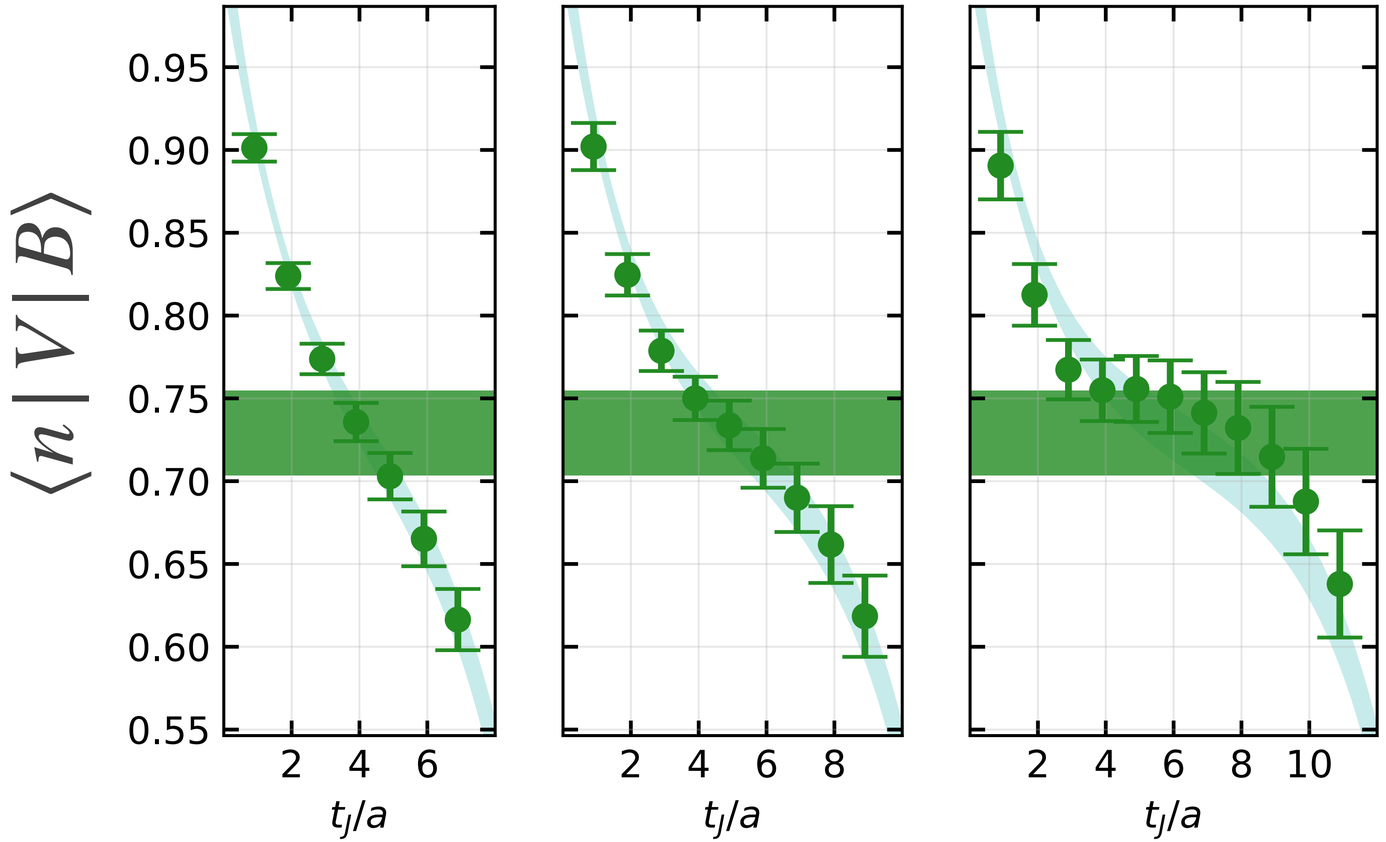
$$\langle n | V | B \rangle$$

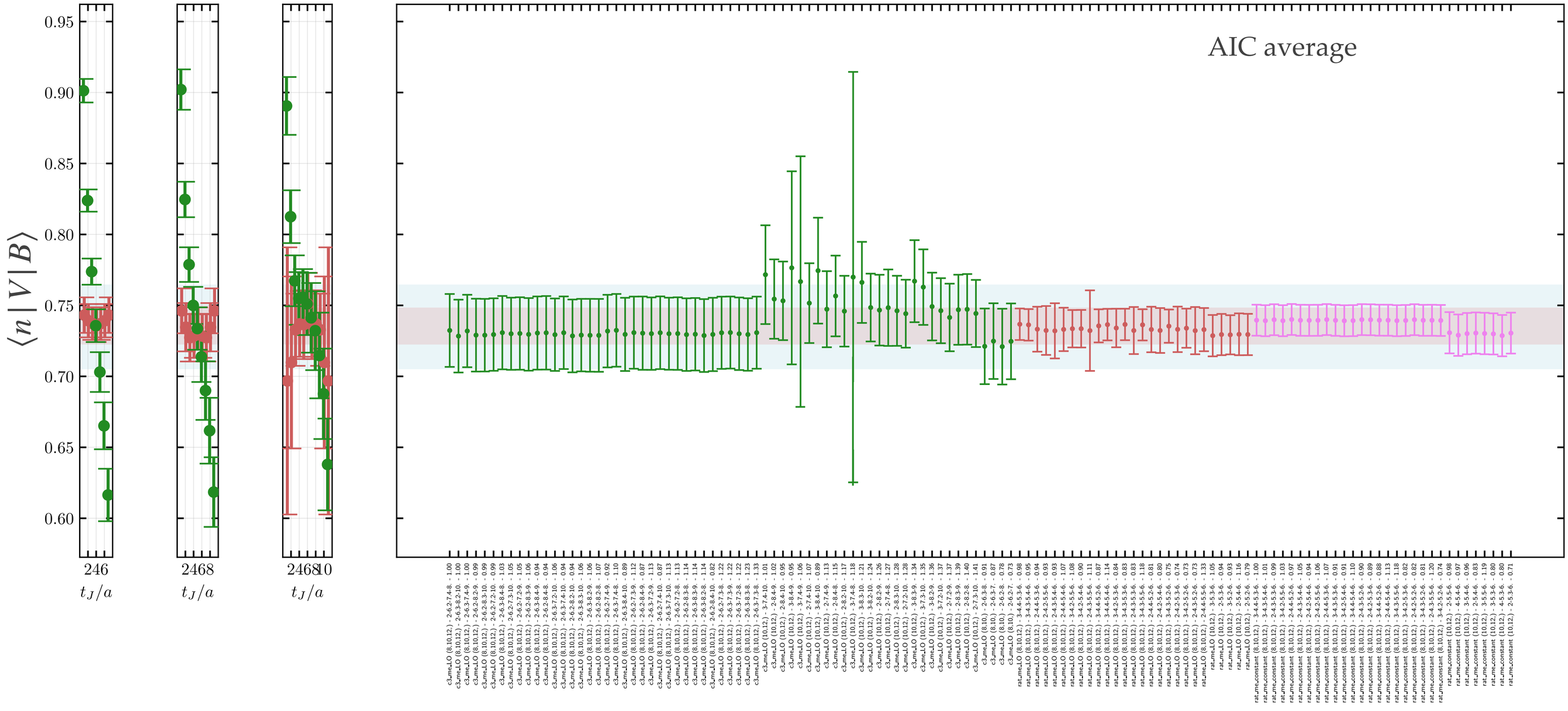
state projection

$$C_{3,i} = \sum_{n \in [\pi\pi]} Z_i^n \langle n \,|\, V \,|\, B \rangle Z_B \frac{e^{-E_n(\Delta t - t)} e^{-E_B t}}{2E_n 2E_B}$$

$$C_3^n = u_i^n C_{3,i}$$

weights from GEVP





a multitude of models…

Jay & Neil, 2008.01069

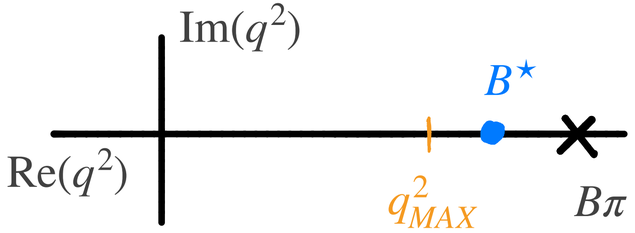
transition amplitude

Boyd, Grinstein, Lebed hep-ph/9412324
Bourrely, Caprini, Lellouch 0807.2722
Alexandrou, LL, Meinel et al. 1807.08357

$$\langle \pi\pi, E^\star \,|\, V \,|\, B, p_B \rangle_\infty = \frac{2iV(E^\star, q^2)}{m_B + 2m_\pi} \epsilon_{\mu\nu\alpha\beta}\, \varepsilon^{\nu *}\, p^\alpha\, p_B^\beta$$

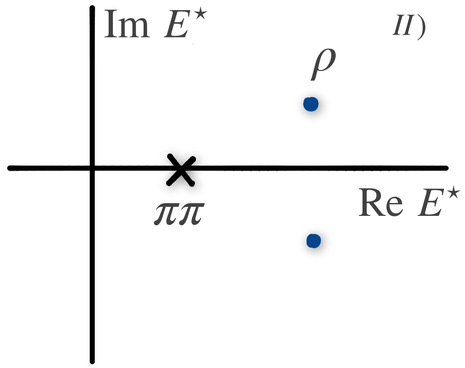
$$q = p_f - p_i$$
$$E^\star = 2\sqrt{m_\pi^2 + k^2}$$

$$V(E^\star, q^2) = F(E^\star, q^2) \frac{T(E^\star)}{k}$$



$$F(E^\star, q^2) = \frac{1}{1 - \frac{q^2}{m_P^2}} \sum_{n,m} A_{n,m}\, z^n(q^2)\, (E^{\star 2} - E_{thr}^2)^m$$

$$T = \frac{E^\star \Gamma_i}{m_R^2 - E^{\star 2} - iE^\star \Gamma_i}$$



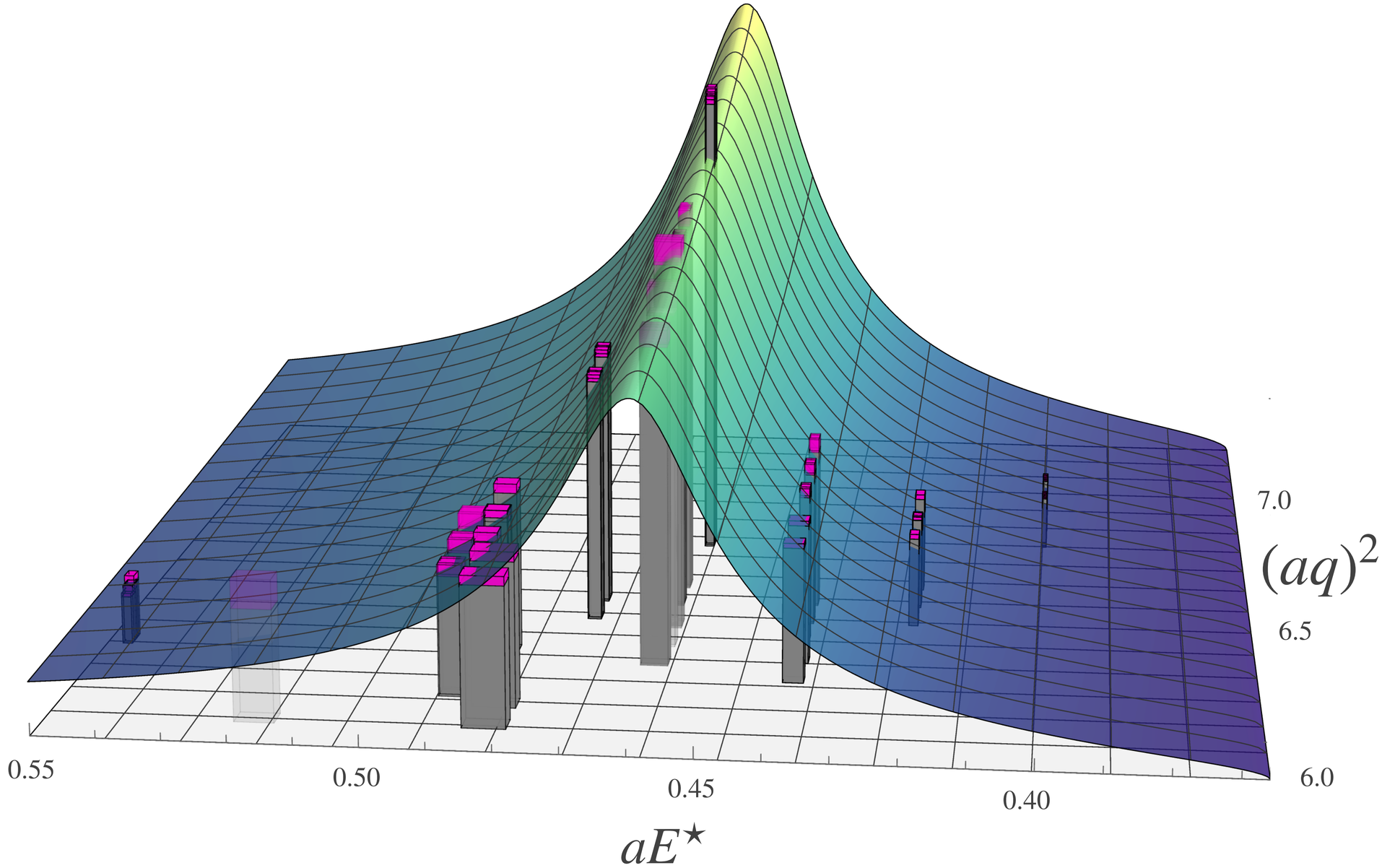
$$\Gamma_I = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{p^3}{E^{\star 2}}$$

$$\Gamma_{II} = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{p^3}{E^{\star 2}} \frac{1 + (k_R r_0)^2}{1 + (k r_0)^2}$$

$$\langle n \,|\, V \,|\, B \rangle_L = \sqrt{R_n}\ \langle \pi\pi, E^\star \,|\, V \,|\, B, p_B \rangle_\infty$$

"Lellouch-Lüscher"
factor

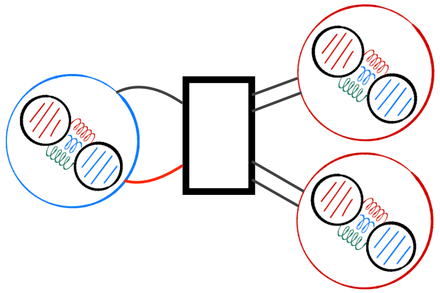
$$\chi^2 = \left[\langle n|V|B\rangle_L^{data} - \langle n|V|B\rangle_L^{model})\right]_i C_{ij}^{-1} \left[\langle n|V|B\rangle_L^{data} - \langle n|V|B\rangle_L^{model})\right]_j$$



SOME APPLICATIONS

ElectroWeak Transitions with Resonances

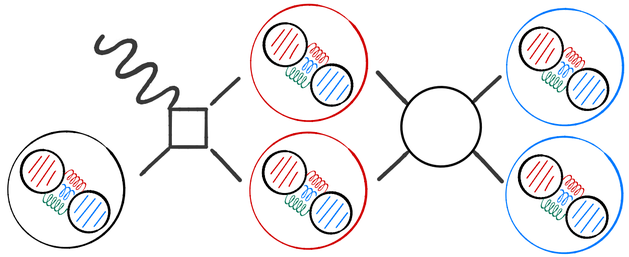
purely hadronic EW decays



e.g. $K \to \pi\pi$

- matter / antimatter content
- CP violation

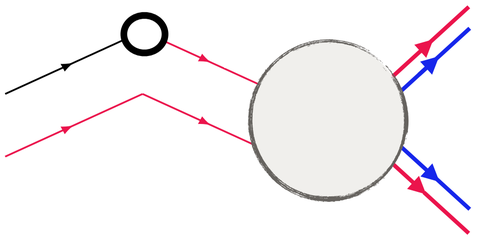
photoproduction of resonances



e.g. $\pi\gamma \to \pi\pi$

- photocouplings
- dispersive approaches to $g_{\mu-2}$

semileptonic processes



e.g. $B \to \pi\,\pi\,\ell\,\bar{\nu}$

- understanding SM pheno
- enable BSM searches

KTeV

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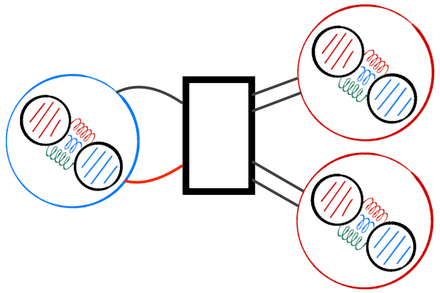

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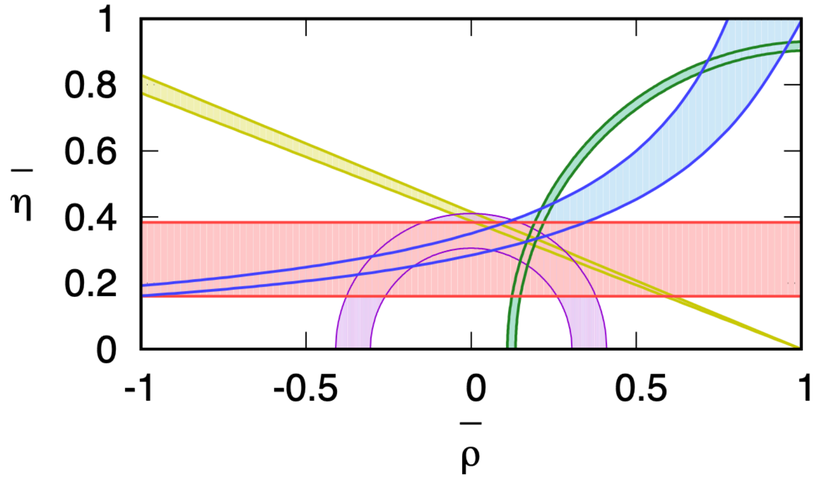
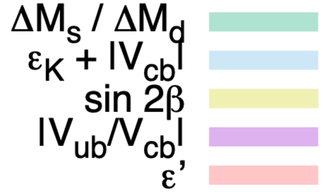
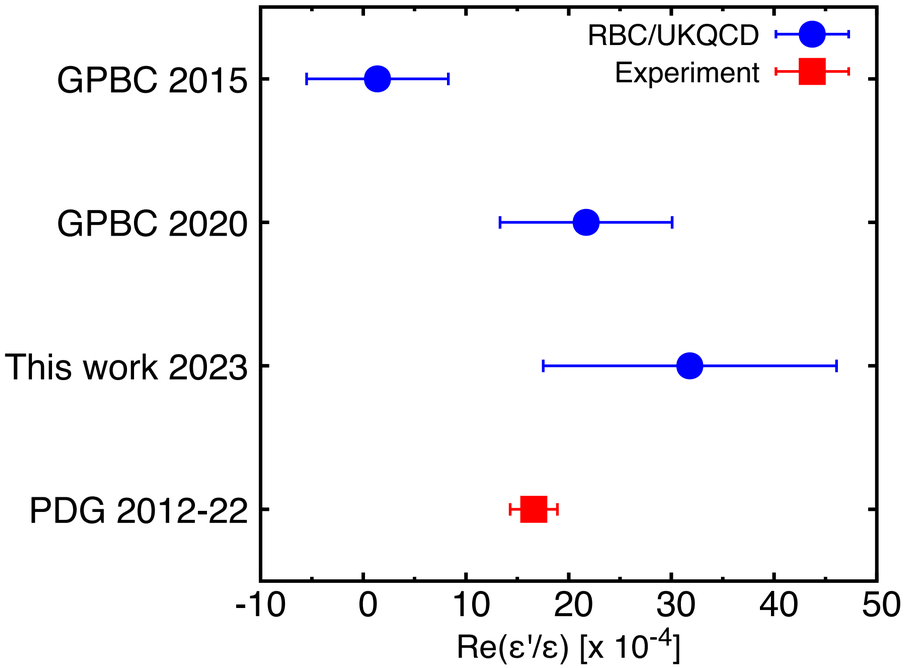
$K \to \pi\pi$ I: RBC/UKQCD - G-parity and periodic





RBC/UKQCD, hep-lat/2004.09440

- G-twisted BC
- m_π^{phys}



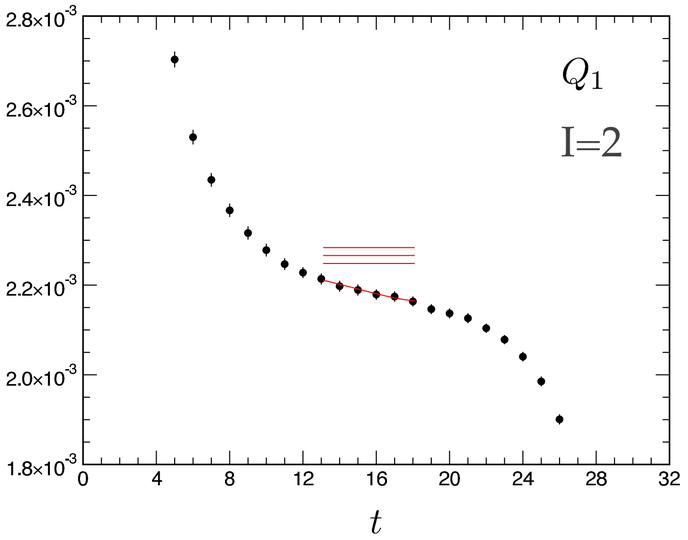
RBC/UKQCD, 2306.06781

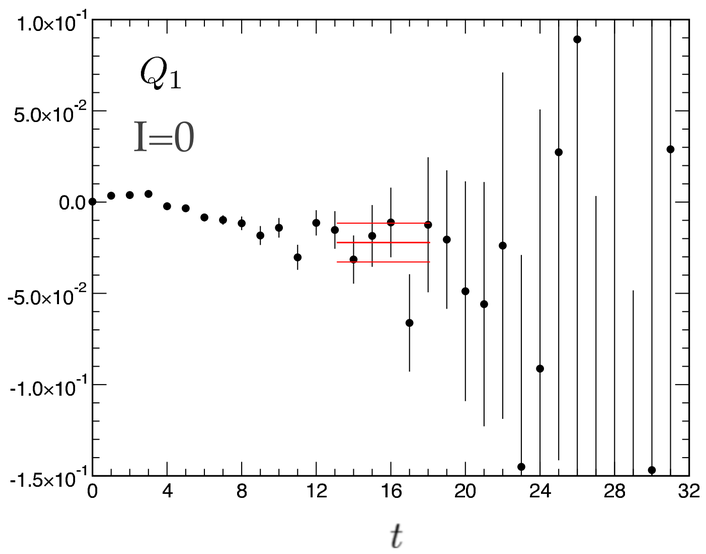
- periodic BC
- m_π^{phys}
- optimized ops from GEVP

$K \to \pi\pi$ II: Ishizuka et al.

- Wilson-Clover
- $m_\pi = 260$ MeV
- don't give up

	$q^* = 1/a$	$q^* = \pi/a$	Exp.
$\mathrm{Re}A_2\ (\times 10^{-8}\ \mathrm{GeV})$	2.431 ± 0.019	2.463 ± 0.019	1.479 ± 0.004
$\mathrm{Re}A_0\ (\times 10^{-8}\ \mathrm{GeV})$	51.3 ± 27.5	45.9 ± 24.1	33.2 ± 0.2
$\mathrm{Re}A_0/\mathrm{Re}A_2$	21.1 ± 11.3	18.6 ± 9.8	22.45 ± 0.06
$\mathrm{Im}A_2\ (\times 10^{-12}\ \mathrm{GeV})$	-1.0754 ± 0.0064	-0.6611 ± 0.0042	
$\mathrm{Im}A_0\ (\times 10^{-12}\ \mathrm{GeV})$	-89.1 ± 135.5	-64.9 ± 112.4	
$\mathrm{Re}(\epsilon'/\epsilon)(\times 10^{-3})$	1.94 ± 5.72	1.95 ± 5.77	1.66 ± 0.23





Ishizuka et al., hep-lat/1809.03893

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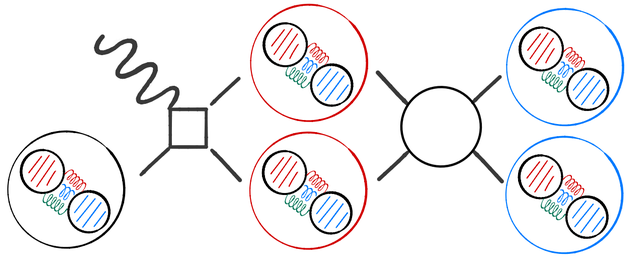

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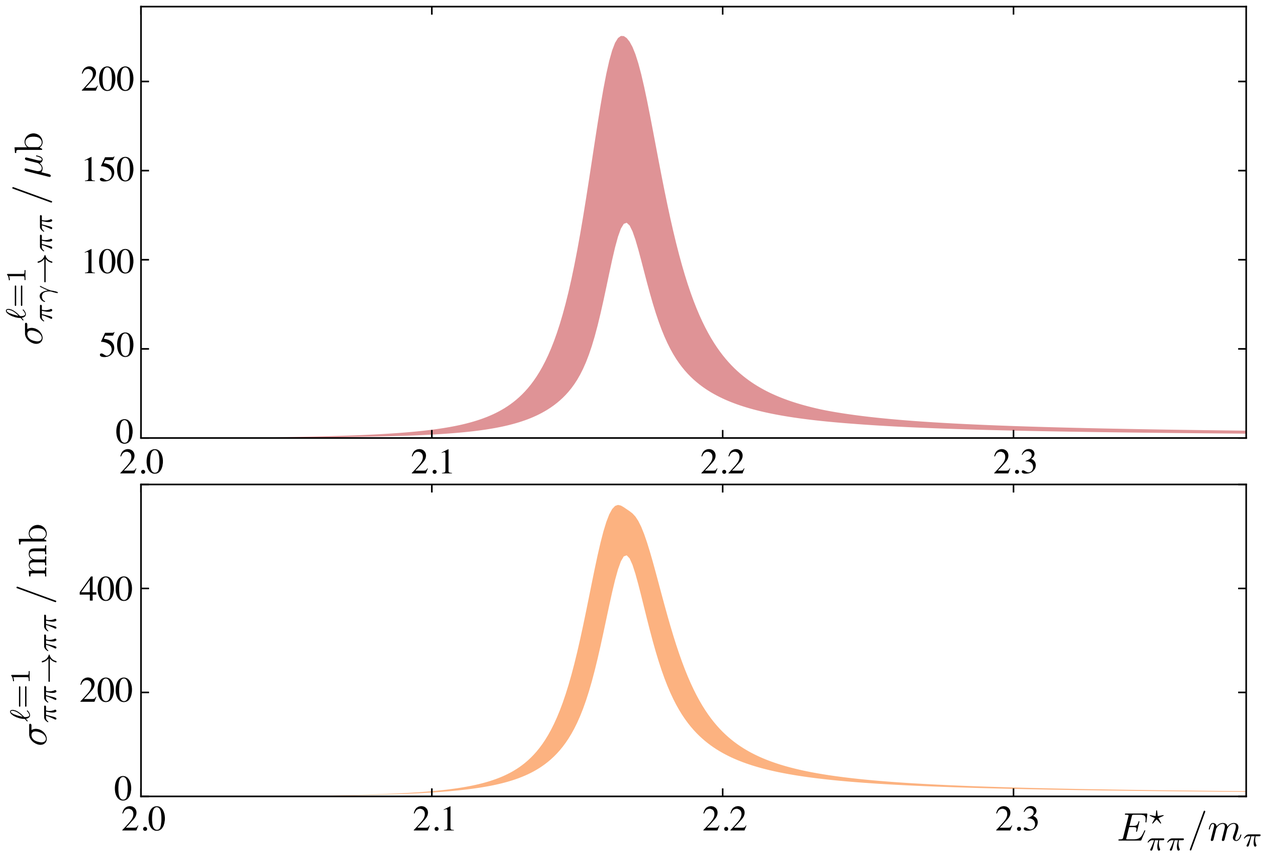
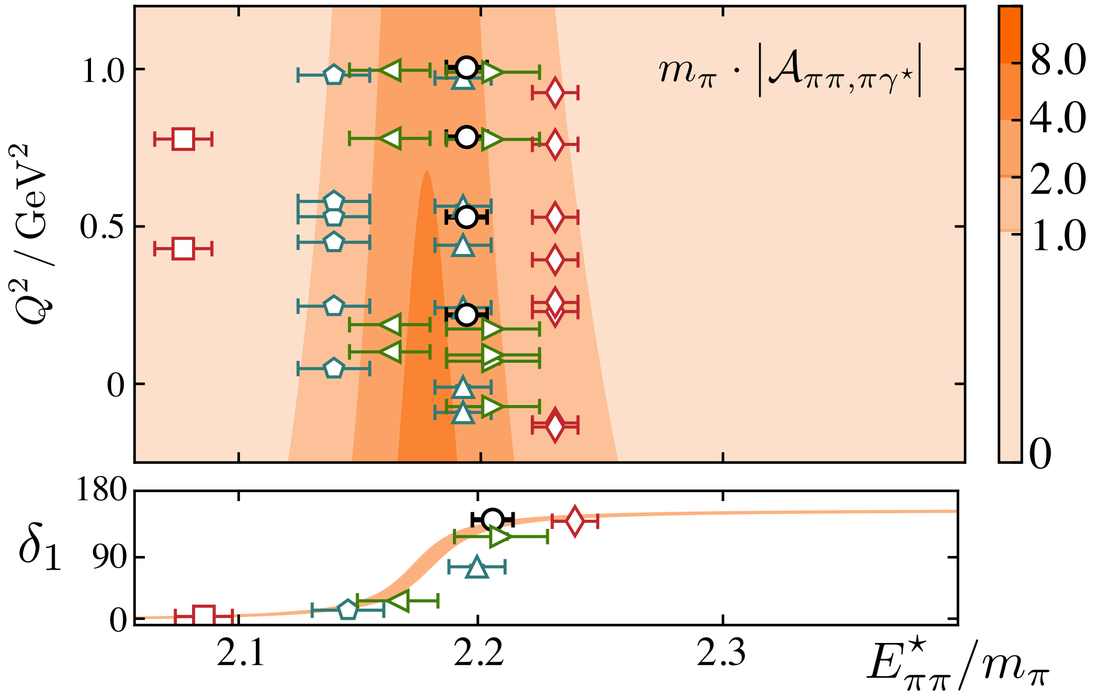
$\pi\gamma \to \pi\pi$ I: HadSpec





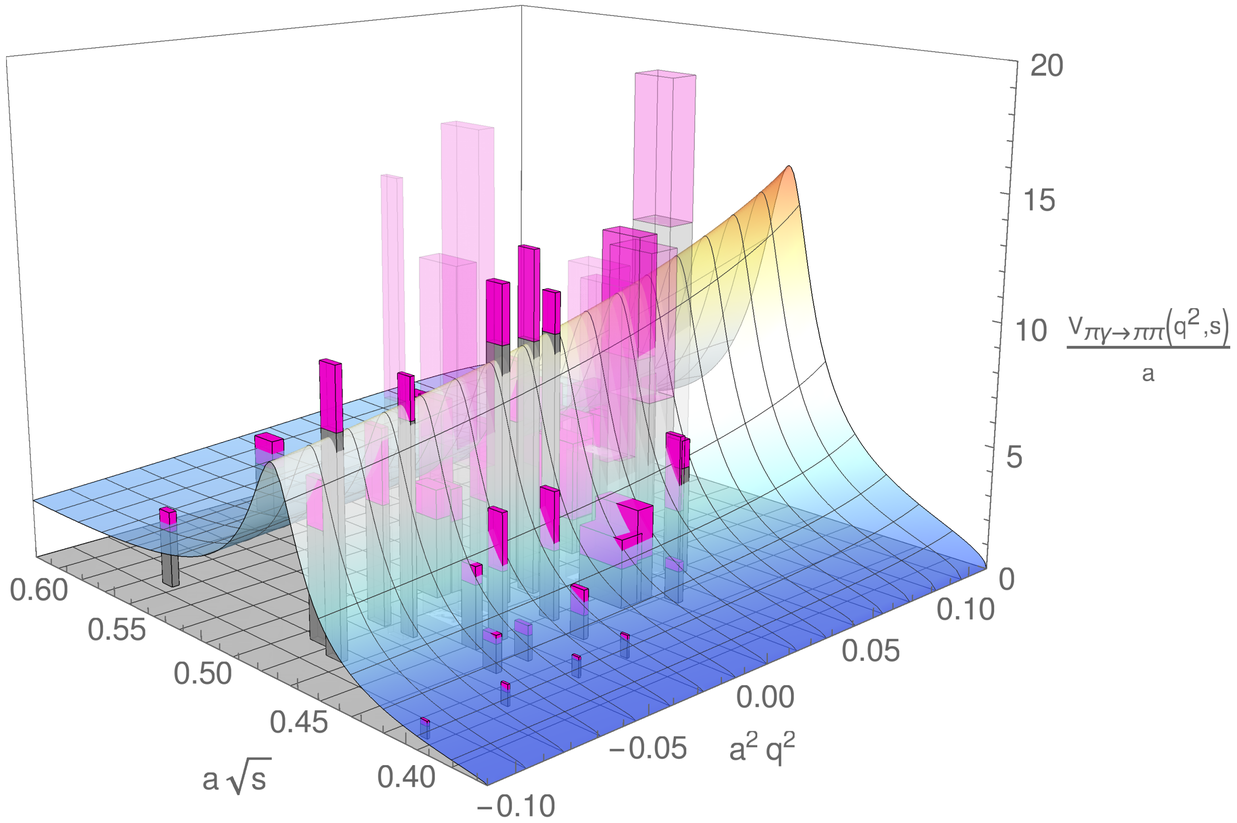
- $m_\pi = 400\,\mathrm{MeV}$
- anisotropic $N_f = 2 + 1$
- Clover-Wilson

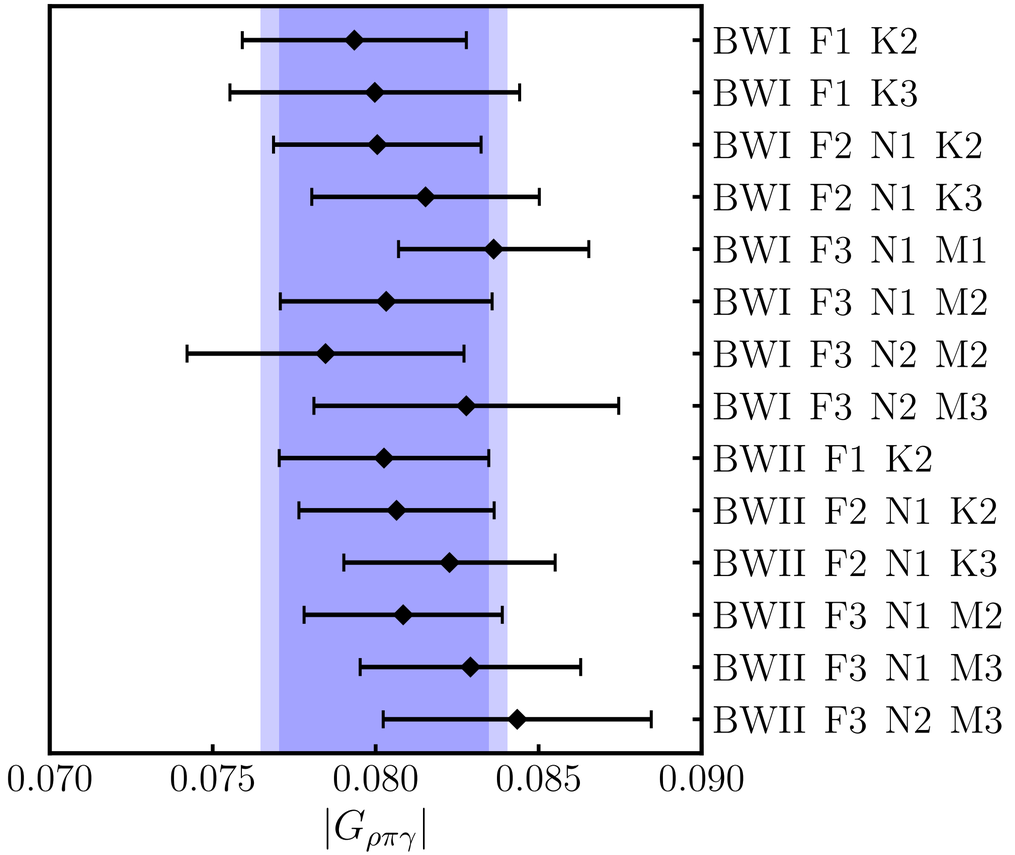
HadSpec, 1507.06622
HadSpec, 1604.03530

$\pi\gamma \to \pi\pi$ II: LHPC

C. Alexandrou, LL, S. Meinel, M. Petschlies et al. 1807.08357

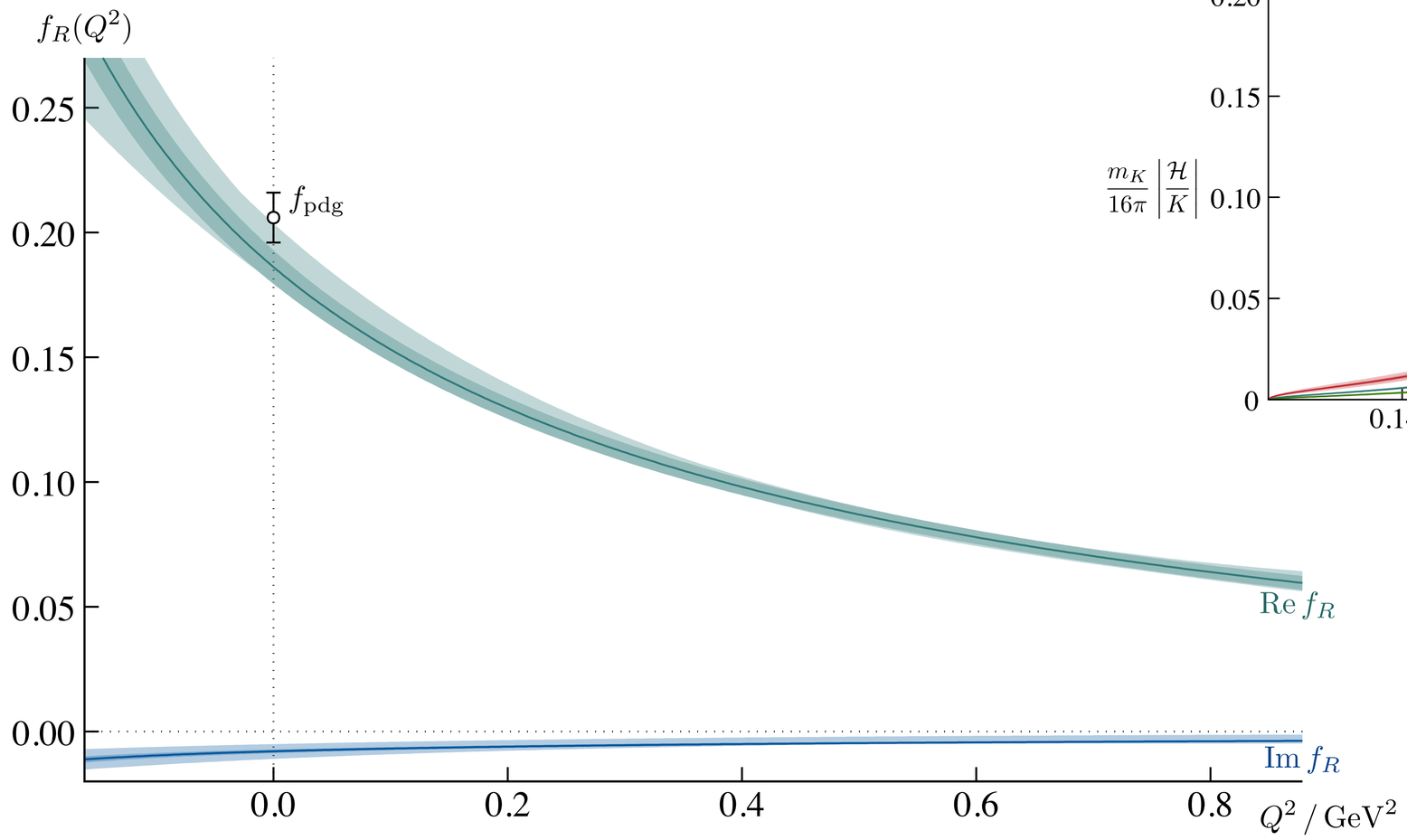
- $m_\pi = 317$ Mev
- Wilson-Clover
- isotropic $N_f = 2 + 1$

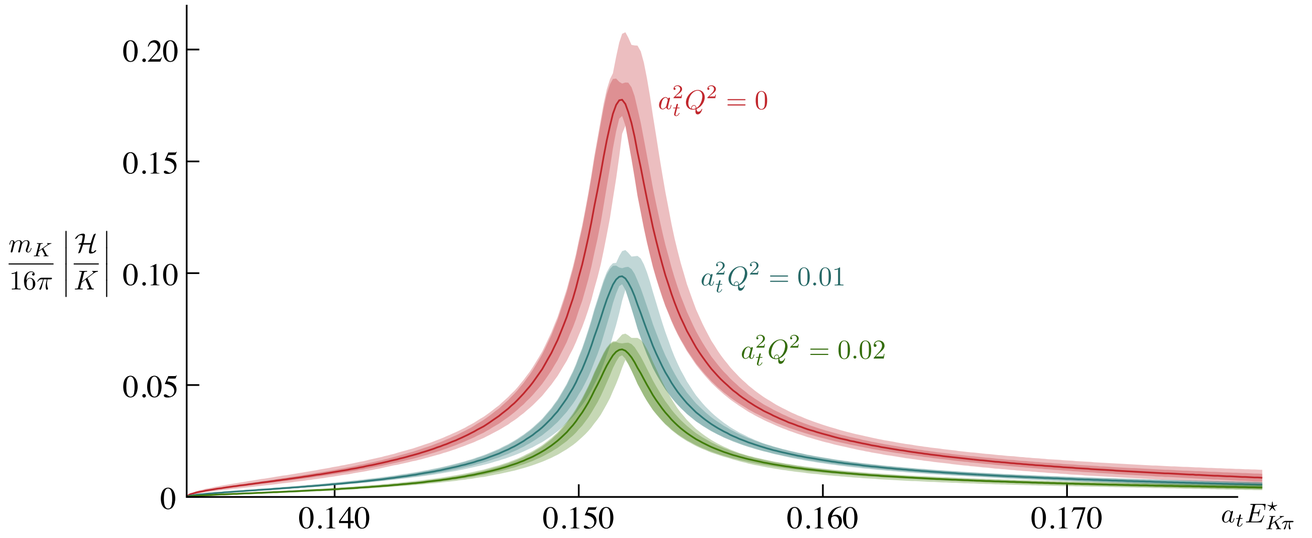




$K\gamma \to K\pi$: HadSpec

HadSpec, 2208.13755





- $m_\pi = 280$ MeV
- Clover-Wilson
- anisotropic $N_f = 2 + 1$

ElectroWeak Transitions with Resonances

purely hadronic EW decays

e.g. $K \to \pi\pi$

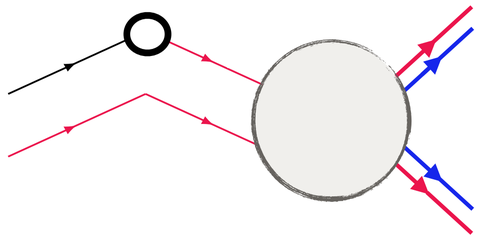
- matter/antimatter content
- CP violation

photoproduction of resonances

e.g. $\pi\gamma \to \pi\pi$

- photocouplings
- dispersive approaches to $g_{\mu-2}$

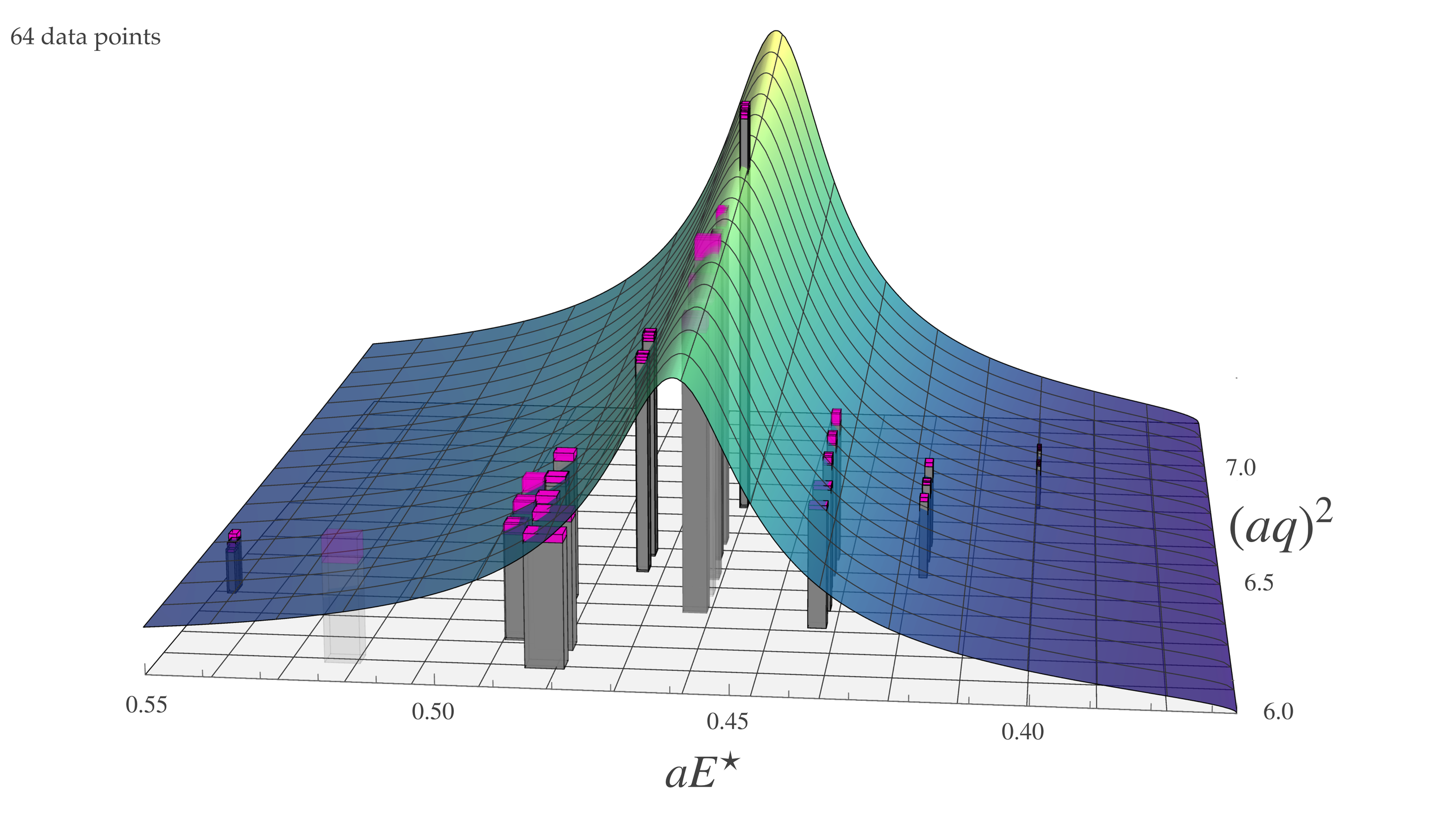
semileptonic processes



e.g. $B \to \pi\,\pi\,\ell\,\bar{\nu}$

- understanding SM pheno
- enable BSM searches

J-PARC KLeV NA62 KTeV GlueX Jefferson Lab g-2 BESIII DUNE Belle II LHCb


64 data points
7.0
$(aq)^2$
6.5
6.0
0.55
0.50
0.45
0.40
$aE^\star$

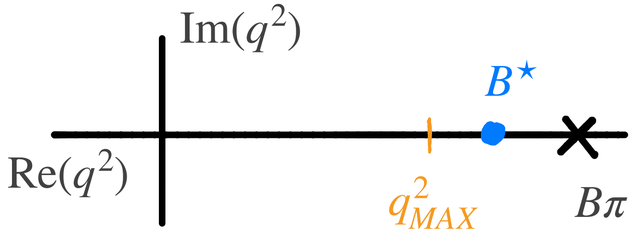
transition amplitude

Boyd, Grinstein, Lebed hep-ph/9412324
Bourrely, Caprini, Lellouch 0807.2722
Alexandrou, LL, Meinel et al. 1807.08357

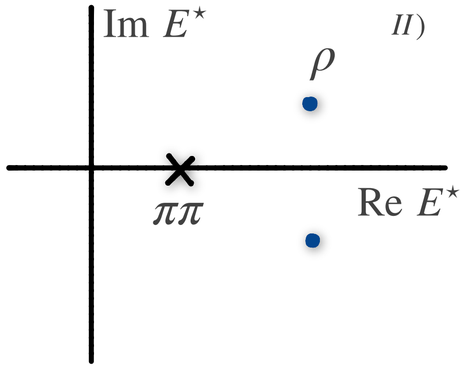
$$\langle \pi\pi, E^\star \,|\, V \,|\, B, p_B \rangle_\infty = \frac{2iV(E^\star, q^2)}{m_B + 2m_\pi} \epsilon_{\mu\nu\alpha\beta}\, \varepsilon^{\nu *}\, p^\alpha\, p_B^\beta$$

$$q = p_f - p_i$$
$$E^\star = 2\sqrt{m_\pi^2 + k^2}$$

$$V(E^\star, q^2) = F(E^\star, q^2) \frac{T(E^\star)}{k}$$



$$F(E^\star, q^2) = \frac{1}{1 - \frac{q^2}{m_P^2}} \sum_{n,m} A_{n,m}\, z^n(q^2)\, (E^{\star 2} - E_{thr}^2)^m$$



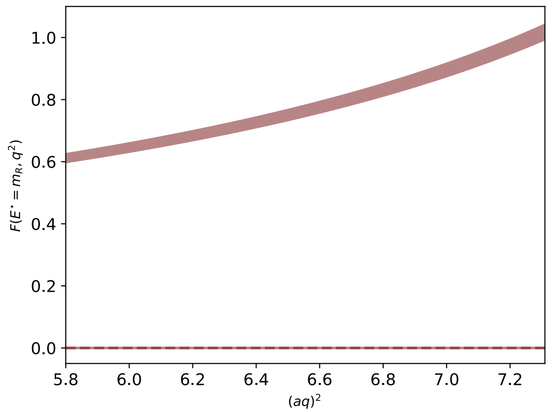
$$T = \frac{E^\star \Gamma_i}{m_R^2 - E^{\star 2} - iE^\star \Gamma_i}$$

$$\Gamma_I = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{p^3}{E^{\star 2}}$$

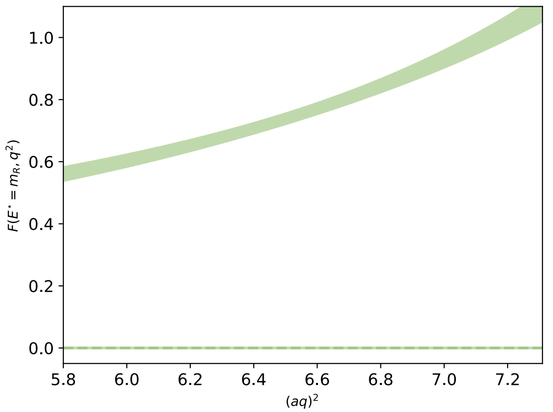
$$\Gamma_{II} = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{p^3}{E^{\star 2}} \frac{1 + (k_R r_0)^2}{1 + (k r_0)^2}$$

details I

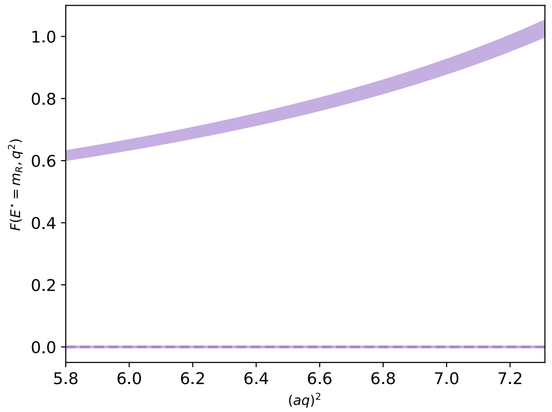
- FF3N0M0_TBWI
  - $\chi^2/\mathrm{dof} = 74.3/63 = 1.179$
  - $A_{0,0} = 0.2413(66)$



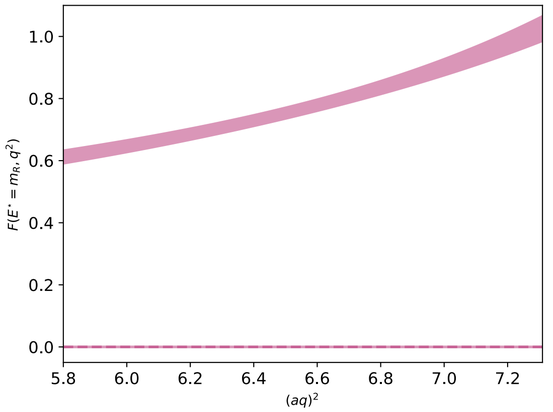
- FF3N1M0_TBWI
  - $\chi^2/\mathrm{dof} = 66.8/62 = 1.07$
  - $A_{0,0} = 0.2256(87)$
  - $A_{1,0} = -0.41(18)$



- FF3N0M1_TBWI
  - $\chi^2/\mathrm{dof} = 39.1/62 = 0.630$
  - $A_{0,0} = 0.279(12)$
  - $A_{0,1} = -0.061(13)$

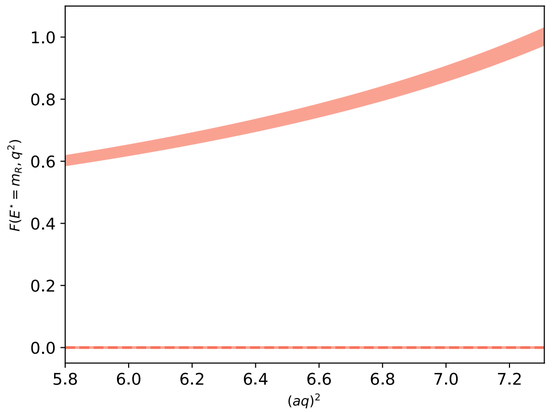


- FF3N1M1_TBWI
  - $\chi^2/\mathrm{dof} = 29.9/60 = 0.499$
  - $A_{0,0} = 0.260(16)$
  - $A_{1,0} = -0.88(50)$
  - $A_{0,1} = -0.031(21)$
  - $A_{1,1} = 1.46(82)$

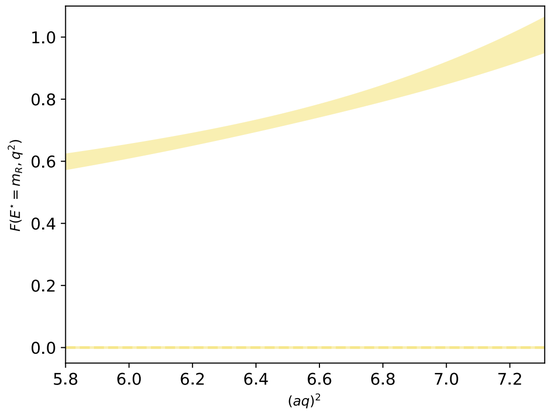


details II

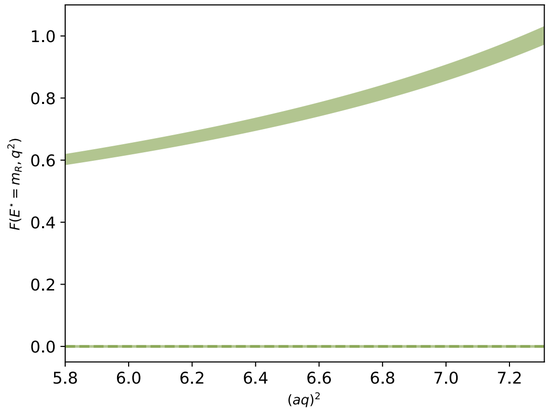
- FF3N0M0_TBWII
  - $\chi^2/\mathrm{dof} = 34.3/63 = 0.545$
  - $A_{0,0} = 0.2376(71)$



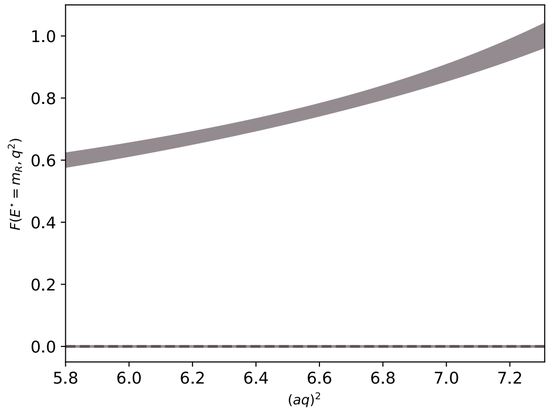
- FF3N1M0_TBWII
  - $\chi^2/\mathrm{dof} = 34.0/62 = 0.549$
  - $A_{0,0} = 0.2365(89)$
  - $A_{1,0} = -0.03(22)$



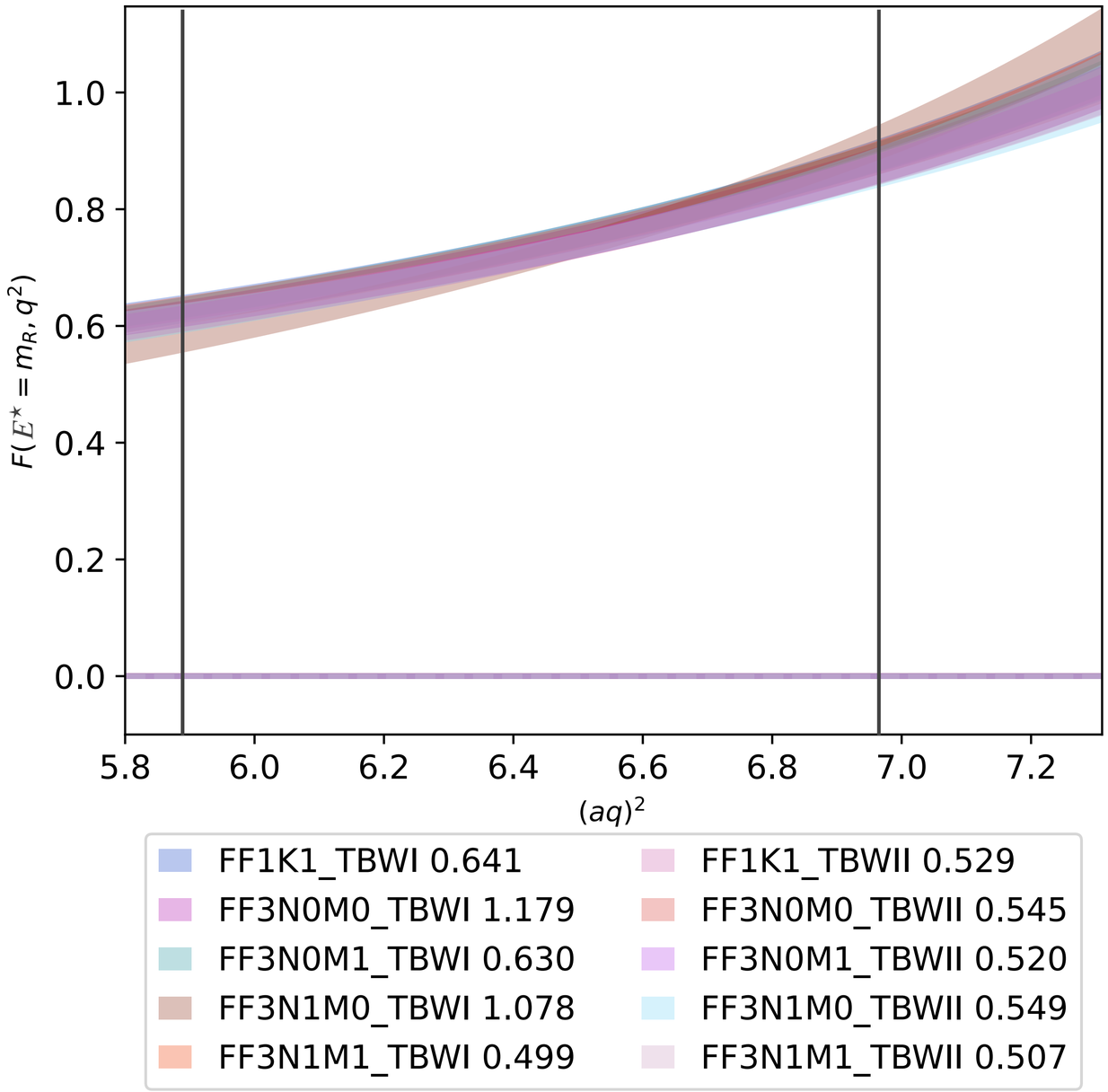
- FF3N0M1_TBWII
  - $\chi^2/\mathrm{dof} = 32.2/62 = 0.520$
  - $A_{0,0} = 0.240(23)$
  - $A_{0,0} = -0.003(33)$



- FF3N1M1_TBWII
  - $\chi^2/\mathrm{dof} = 30.4/60 = 0.507$
  - $A_{0,0} = 0.232(20)$
  - $A_{1,0} = -0.27(60)$
  - $A_{0,1} = 0.008(28)$
  - $A_{1,1} = 0.45(1.02)$



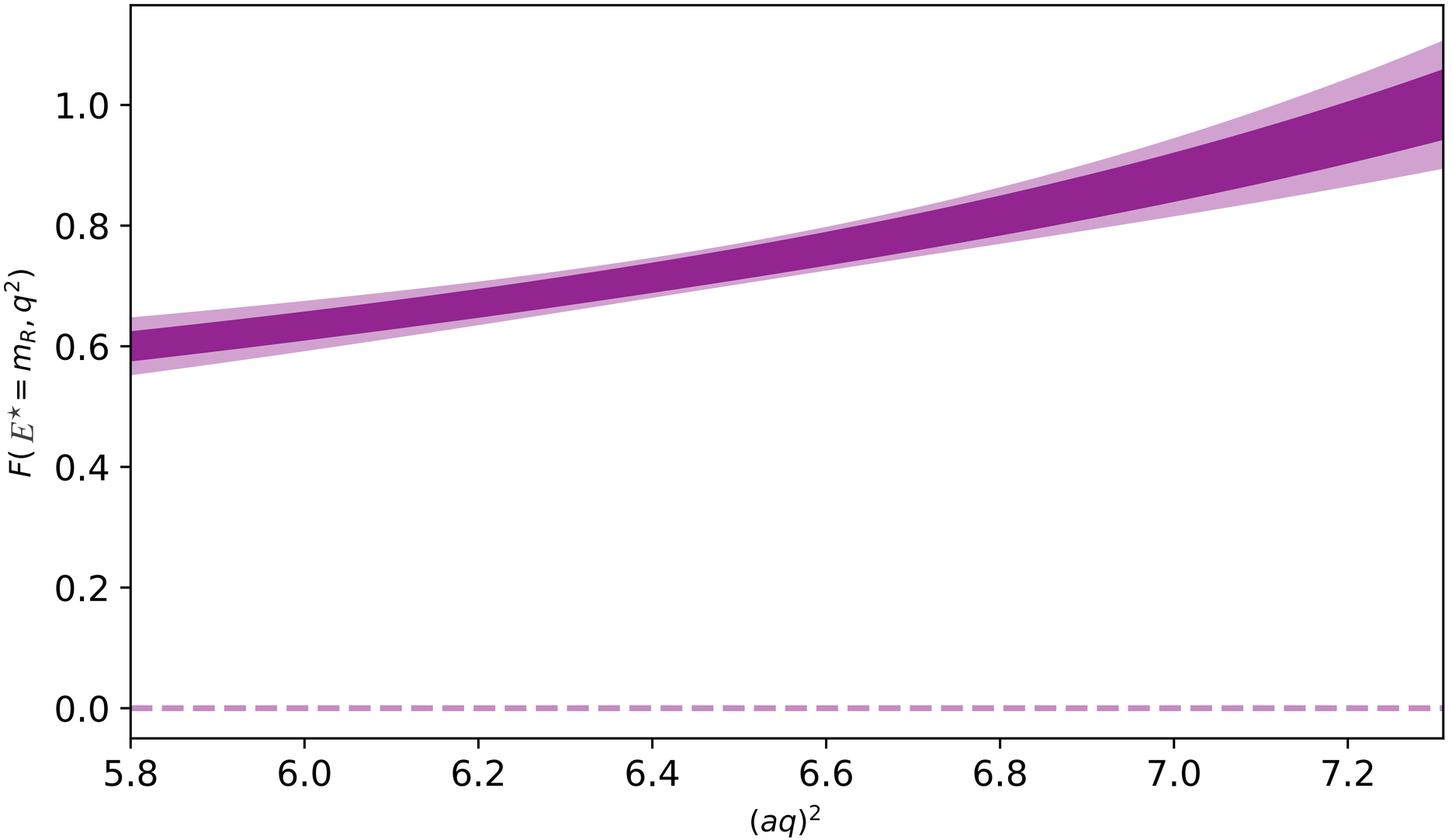
different parameterizations

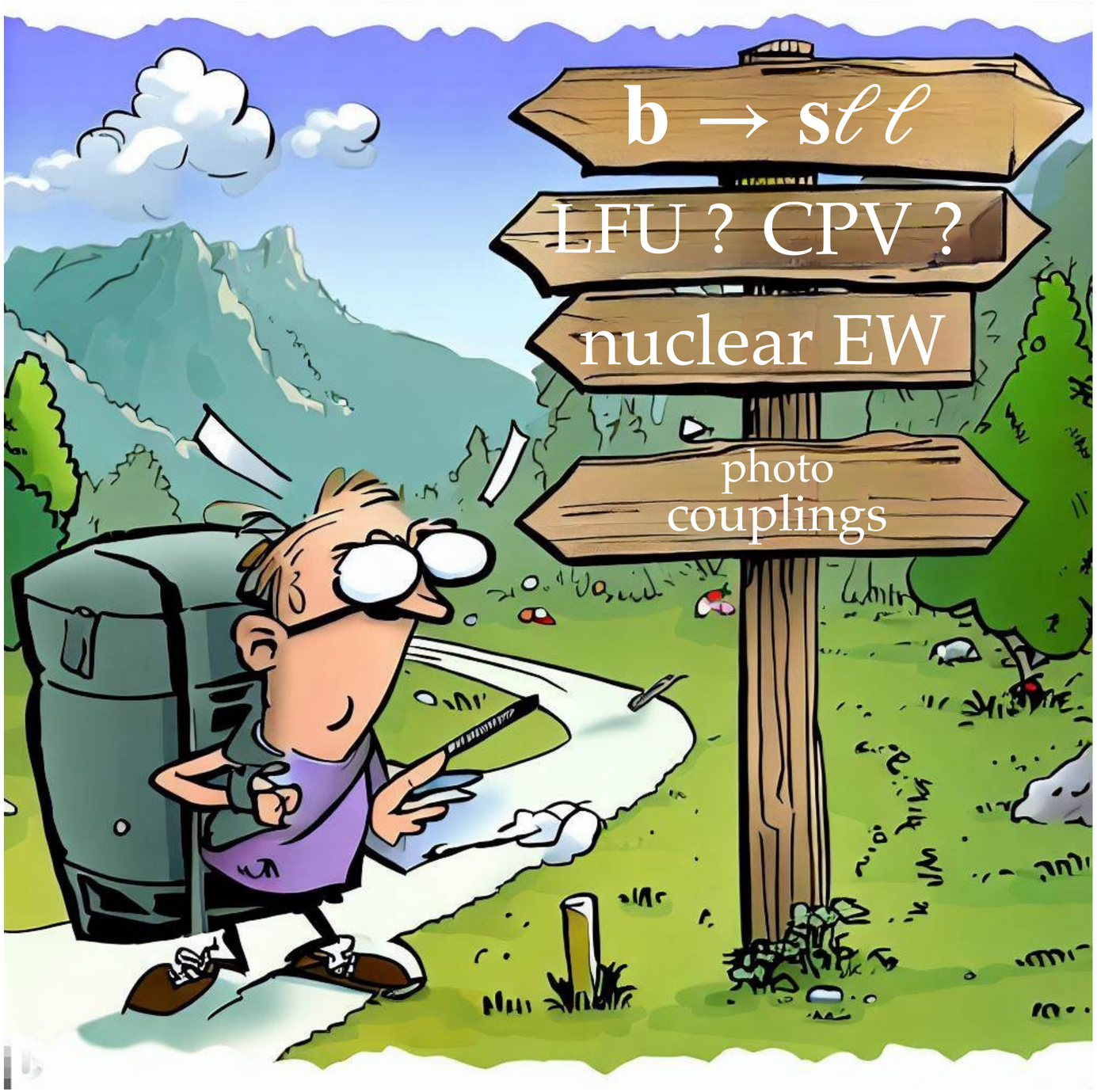


- $E^\star$ dependence statistically significant, albeit small
- different $E^\star$ dependence in BWI and BWII
- FF3N1M1_TBWII - chosen as central

different parameterizations

$$F(q^2_{MAX}) = 1.001 \pm 0.059 \pm 0.106$$





- a great start to a (more) complete understanding of SM

“One might think that having a finite volume (…) makes it even more difficult to extract the transition amplitudes. In the present paper we wish to show that this is actually not so.“

—*Lellouch & Lüscher, hep-lat/0003023*