Adjoint chromoelectric (-magnetic) correlators with gradient flow

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Outline

We are interested in the following correlation function between chromoelectric E (-magnetic B) fields connected with an adjoint Wilson line Φ:

$$\mathcal{E}(t) = \langle 0| gE^{i,a}(t,0)\Phi_{ab}(t,0) gE^{i,b}(0,0)|0\rangle$$

- Theoretical background
- Lattice implementation
- Gradient flow
- Current results

Zero T: Gluelumps

- The two lowest lying gluelump states 1^{+-} and 1^{--} given by BB and EE correlators respctively
- Extraction as the ground state mass of the correlator
- There are more gluelumps given by more complicated operators (out of scope of this talk)
- Simples test case and allows setting everything in units of the lowest gluelump
- Recent full spectrum extraction available by Herr *et.al.*2306.09902

See e.g. Bali and Pinedda PRDD69 (2004)

Zero T moments in pNRQCD

- Moments of EE-correlator appear in pNRQCD
  Brambilla *et.al.*PRL88 2002, Brambilla *et.al.*PRD67 2003,

  Brambilla *et.al.*JHEP04 2020

$$\mathcal{E}_n = \frac{T_{\mathrm{F}}}{N_{\mathrm{c}}} \int_0^\infty \mathrm{d}t \, t^n \mathcal{E}(t)$$

- $\mathcal{E}_3$ simplest case to start with
  - Needs to be nonperturbatively calculated
  - Describes the inclusive annihilation rate of a P-wave spin-triplet into light hadrons

$$\Gamma_{\chi_{QJ}} = \frac{3N_c}{2\pi}|R'(0)|^2 \frac{32}{M^4}\left[\mathrm{Im} f_1(^3P_J)(\Lambda) + \mathrm{Im} f_8(^3S_1)\frac{2T_F}{9N_c}\mathcal{E}_3(\Lambda)\right]$$

  - Where Q is charm or bottom, M the mass of χ_{QJ} and f_1 and f_2 matching coefficients known Petrelli *et.al.*Nucl.Phys.B514 1998

Finite T: heavy quarkonium diffusion

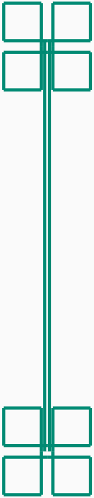
- The relaxation time of a heavy quarkonium in a quark gluon plasma is defined trough a diffusion process
- Compare to heavy quark diffusion coefficient κ defined with fundamental Wilson lines W:

$$\langle W(\beta,t)gE(t,0)W(t,0)gE(0,0)\rangle/\langle W(\beta,0)\rangle$$

- Quarkonium differs from single quarks by adjoint representation and the diffusion coefficient κ is given by $\mathcal{E}$
- Perturbatively κ same up to NLO to be same between quarks and quarkonium
- For quarkonium the mass shift is also related to the diffusion process via $\gamma \sim \mathcal{E}_0$
- In theory one can also definite a symmetric correlator

See e.g. Brambilla *et.al.*PRDD 97 (2018), Brambilla *et.al.*PRDD 97 (2018)

Discretization



- Use Clover discretizaton

$$E_i = \frac{1}{2iga^2}\left(\Pi_{i0} - \Pi_{i0}^\dagger\right)$$

$$\Pi_{\mu\nu} = \frac{1}{4}\left(P_{\mu,\nu} + P_{\nu,-\mu} + P_{-\mu,-\nu} + P_{-\nu,\mu}\right)$$

- Adjoint operators are related to their fundamental counterparts

$$E^a = \mathrm{Tr}(E\lambda^a)\,, \quad \Phi_{ab} = \mathrm{Tr}(U^\dagger\lambda^a U\lambda^b)$$

$$E^{i,a}(t,0)\Phi_{ab}(t,0)gE^{i,b}(0,0) =$$

$$2\mathrm{Tr}(E(0)U^\dagger E(t)U) - \frac{2}{3}\mathrm{Tr}(E(0))\mathrm{Tr}(E(t))$$

- Similarly for BB-correlator, the symmetric operator and adjoint Polyakov loops

See e.g. Bali *et.al.*PRD56 (1997)

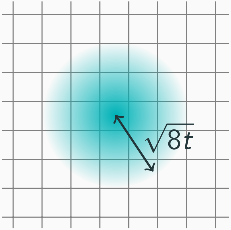
Divergences

- Any Wilson line comes with $\sim 1/a$ linear divergence
  - Related to renormalon ambiguity in dimreg
  - Needs to be fixed to correct scheme for proper physical results
- Discretization of E-fields comes with lattice only multiplicative renormalization Z_E. Similarly for Z_B.
- EE and BB correlators have an anomalous dimensions. Starting at $(O)(\alpha)$ for B and $\mathcal{O}(\alpha^2)$for E.
  - Has been relevant in earlier projects

    Brambilla *et.al.*PRD 107 (2023); Banerjee *et.al.*JHEP 08 (2022)
  - Ignored at current analysis, will be included later
- $\mathcal{E}_n$ for $n < 4$ need to be regularized for $t \to 0$
  - match to perturbation theory
- The renormalization constants that don't depend on t will divide out for gluelump masses, the linear divergence gives a mass shift to the gluelumps

Gradient flow

$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu}\,,$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}]\,.$$
$$B_{0,\mu} = A_\mu \;\leftarrow \text{ the original gauge field}$$



- Evolve gauge along fictitious time t towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t}$
- Automatically renormalizes gauge invariant observables
- Zero flow time limit needed to connect to real physics
- Need to flow enough but not too much to avoid overlap, restrict to:

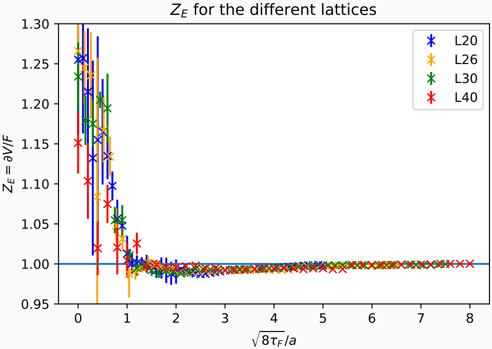
$$1 < \sqrt{8\tau_f}/a < \frac{t-2}{2}$$

Lüscher JHEP 1008, 071 (2010)

Gradient flow and divergences/renormalization

- The discretization effect $Z_{E,B}$ becomes one at $\sqrt{8\tau_f} \gtrsim 1$

  see Julian Mayer-Steudte's talk 15:10
- Linear divergence changes: $\frac{1}{a} \to \frac{1}{\sqrt{8\tau_f}}$
- Less divergence at large flow time, but care needed for zero flow time limit



- Anomalous dimensions become log-divergences at zero flow time limit
  - Currently ignored, future should take into account

Simulation Details

V_S	N_T	β	a [fm]	T/T_c	$N_{\rm conf}$
20^3	6	6.284	0.060	1.848	764
20^3	8	6.284	0.060	1.386	620
40^3	6	6.816	0.030	3.765	226

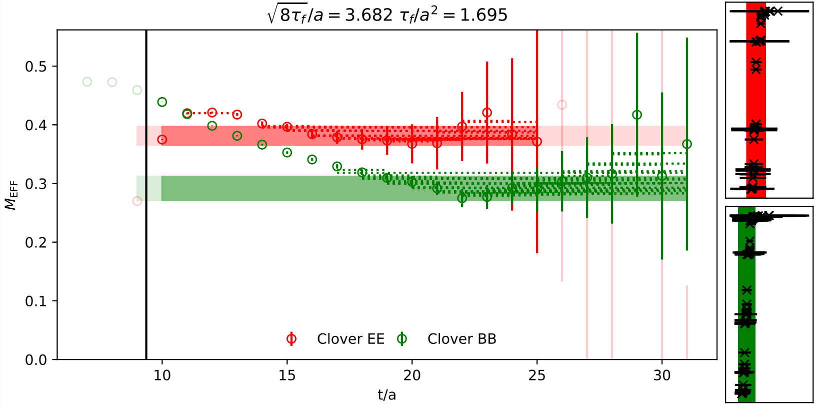
(a) finite T ensembles

V_S	N_T	β	a [fm]	T/T_c	$N_{\rm conf}$
20^3	40	6.284	0.060	0.277	6000
26^3	56	6.481	0.046	0.261	6000
30^3	60	6.594	0.040	0.283	6000
40^3	80	6.816	0.030	0.282	3300

(b) Zero T ensembles

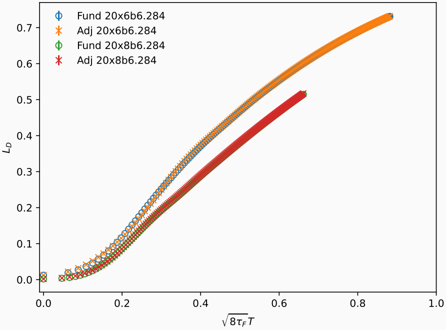
- Wilson gauge action, Pure gauge simulations at zero and finite T
- Flow GF use Lüscher-Weisz action and adaptive solver
- MILC code for simulations
- Scale setting either with S. Necco *et.al.*Nucl. Phys. B622 (2002) or with t_0

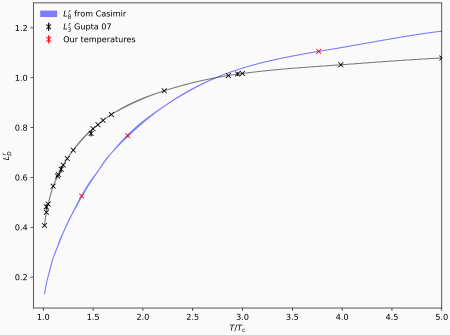
Test case: gluelump masses



- Lowest gluelump masses a good candidate to test different divergence subtractions due to existing lattice results
- Fit effective masses within allowed τ_f range
- Vary over multiple fits with Akaike information criterion

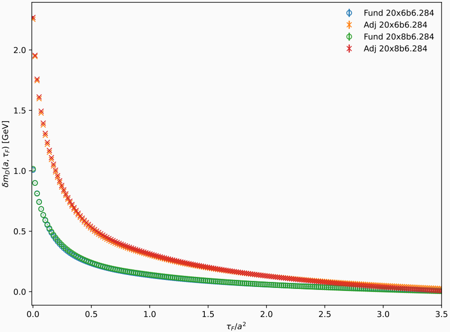
Method 1: Polyakov loop renormalization

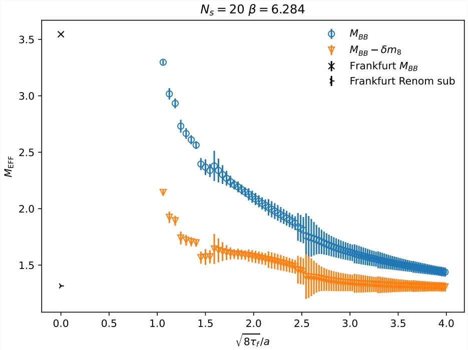




- Linear divergence of fundamental Polyakov loops have been renormalized before Gupta *et.al.*PRD77 2008
- Observe Casimir scaling in finite T and get adjoint divergence trough that

Method 1: results

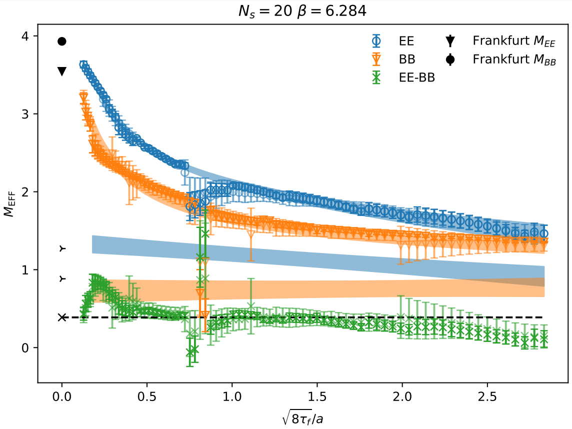




- Seems to work reasonably well at large flow times
- Residual curvature at small flow times still need to be understood

Frankfurt points from: Herr *et.al.*2306.09902

Method 2: Direct fit



- We know the divergence is $\sim 1/\sqrt{8\tau_f}$, we can fit it
- Currently at finite lattice spacing, continuum limit under progress
- Seems to work good up to a scheme contributions starting at $\mathcal{O}(\alpha^2)$

Frankfurt points from: Herr *et.al.*2306.09902

Conclusions

- The correlation function between two chromoelectric fields can be used in many applications
- Gradient flow can solve or ease many of the associated divergences
- Tested a two different ways to remove the linear divergence
- Future:
  - Continuum limits to properly attain flow time scaling
  - Zero flow time limits
  - Measure the actual observables mentioned in motivation

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Thank you!