Adjoint chromoelectric (-magnetic) correlators with gradient flow

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Lattice 2023
Fermilab
August 1st 2023
We are interested in the following correlation function between chromoelectric $E$ (-magnetic $B$) fields connected with an adjoint Wilson line $\Phi$:

$$\mathcal{E}(t) = \langle 0 | gE^{i,a}(t, 0)\Phi_{ab}(t, 0)gE^{i,b}(0, 0) | 0 \rangle$$

- Theoretical background
- Lattice implementation
- Gradient flow
- Current results
- The two lowest lying gluelump states $1^{+-}$ and $1^{--}$ given by $BB$ and $EE$ correlators respectively
- Extraction as the ground state mass of the correlator
- There are more gluelumps given by more complicated operators (out of scope of this talk)
- Simples test case and allows setting everything in units of the lowest gluelump
- Recent full spectrum extraction available by Herr et.al.2306.09902

See e.g. Bali and Pinedda PRDD69 (2004)
Zero $T$ moments in pNRQCD

- Moments of EE-correlator appear in pNRQCD
  Brambilla et.al. PRL88 2002, Brambilla et.al. PRD67 2003,
  Brambilla et.al. JHEP04 2020

\[ \mathcal{E}_n = \frac{T_F}{N_c} \int_0^\infty dt t^n \mathcal{E}(t) \]

- $\mathcal{E}_3$ simplest case to start with
  - Needs to be nonperturbatively calculated
  - Describes the inclusive annihilation rate of a P-wave spin-triplet into light hadrons

\[ \Gamma_{\chi_{QJ}} = \frac{3N_c}{2\pi} \left|R'(0)\right|^2 \frac{32}{M^4} \left[ \text{Im} f_1(3P_J)(\Lambda) + \text{Im} f_8(3S_1) \frac{2T_F}{9N_c} \mathcal{E}_3(\Lambda) \right] \]

- Where $Q$ is charm or bottom, $M$ the mass of $\chi_{QJ}$ and $f_1$ and $f_2$ matching coefficients known Petrelli et.al. Nucl.Phys.B514 1998
The relaxation time of a heavy quarkonium in a quark gluon plasma is defined through a diffusion process. Compare to heavy quark diffusion coefficient $\kappa$ defined with fundamental Wilson lines $W$:

$$\langle W(\beta, t)gE(t, 0)W(t, 0)gE(0, 0)\rangle/\langle W(\beta, 0) \rangle$$

Quarkonium differs from single quarks by adjoint representation and the diffusion coefficient $\kappa$ is given by $E$.

Perturbatively $\kappa$ same up to NLO to be same between quarks and quarkonium.

For quarkonium the mass shift is also related to the diffusion process via $\gamma \sim \mathcal{E}_0$.

In theory one can also definite a symmetric correlator.

See e.g. Brambilla et. al. PRDD 97 (2018), Brambilla et. al. PRDD 97 (2018)
Discretization

- Use Clover discretization

\[ E_i = \frac{1}{2i\alpha^2} \left( \Pi_{i0} - \Pi_{i0}^\dagger \right) \]

\[ \Pi_{\mu\nu} = \frac{1}{4} \left( P_{\mu,\nu} + P_{\nu,-\mu} + P_{-\mu,-\nu} + P_{-\nu,\mu} \right) \]

- Adjoint operators are related to their fundamental counterparts

\[ E^a = \text{Tr}(E\lambda^a), \quad \Phi_{ab} = \text{Tr}(U^\dagger \lambda^a U \lambda^b) \]

\[ E^{i,a}(t,0)\Phi_{ab}(t,0)gE^{i,b}(0,0) = \]

\[ 2\text{Tr}(E(0)U^\dagger E(t)U) - \frac{2}{3}\text{Tr}(E(0))\text{Tr}(E(t)) \]

- Similarly for BB-correlator, the symmetric operator and adjoint Polyakov loops

See e.g. Bali et al. PRD56 (1997)
Divergences

- Any Wilson line comes with $\sim 1/a$ linear divergence
  - Related to renormalon ambiguity in dimreg
  - Needs to be fixed to correct scheme for proper physical results
- Discretization of E-fields comes with lattice only multiplicative renormalization $Z_E$. Similarly for $Z_B$.
- EE and BB correlators have an anomalous dimensions. Starting at $\mathcal{O}(\alpha)$ for B and $\mathcal{O}(\alpha^2)$ for E.
  - Has been relevant in earlier projects
    Brambilla *et al.* PRD 107 (2023); Banerjee *et al.* JHEP 08 (2022)
  - Ignored at current analysis, will be included later
- $E_n$ for $n < 4$ need to be regularized for $t \to 0$
  - match to perturbation theory
- The renormalization constants that don’t depend on $t$ will divide out for gluelump masses, the linear divergence gives a mass shift to the gluelumps.
Gradient flow

\[
\partial_t B_{t,\mu} = - \frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu},
\]

\[
G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}].
\]

\[
B_{0,\mu} = A_\mu \leftarrow \text{the original gauge field}
\]

- Evolve gauge along fictitious time \( t \) towards minima of \( S_{YM} \)
- Diffuses the initial gauge field with radius \( \sqrt{8t} \)
- Automatically renormalizes gauge invariant observables
- Zero flow time limit needed to connect to real physics
- Need to flow enough but not too much to avoid overlap, restrict to:

\[
1 < \sqrt{8\tau_f/a} < \frac{t - 2}{2}
\]

Lüscher JHEP 1008, 071 (2010)
Gradient flow and divergences/renormalization

- The discretization effect $Z_{E,B}$ becomes one at $\sqrt{8\tau_f} \gtrsim 1$
  see Julian Mayer-Steudte’s talk 15:10
- Linear divergence changes:
  \[
  \frac{1}{a} \rightarrow \frac{1}{\sqrt{8\tau_f}}
  \]
- Less divergence at large flow time, but care needed for zero flow time limit
- Anomalous dimensions become log-divergences at zero flow time limit
  - Currently ignored, future should take into account
Simulation Details

(a) finite T ensembles

(b) Zero T ensembles

- Wilson gauge action, Pure gauge simulations at zero and finite T
- Flow GF use Lüscher-Weisz action and adaptive solver
- MILC code for simulations
- Scale setting either with S. Necco et al. Nucl. Phys. B622 (2002) or with $t_0$
Test case: gluelump masses

- Lowest gluelump masses a good candidate to test different divergence subtractions due to existing lattice results
- Fit effective masses within allowed $\tau_f$ range
- Vary over multiple fits with Akaike information criterion
Method 1: Polyakov loop renormalization

- Linear divergence of fundamental Polyakov loops have been renormalized before Gupta et al. PRD77 2008
- Observe Casimir scaling in finite $T$ and get adjoint divergence through that
- Seems to work reasonably well at large flow times
- Residual curvature at small flow times still need to be understood

Frankfurt points from: Herr et.al.2306.09902
Method 2: Direct fit

- We know the divergence is $\sim 1/\sqrt{8T_f}$, we can fit it
- Currently at finite lattice spacing, continuum limit under progress
- Seems to work good up to a scheme contributions starting at $\mathcal{O}(\alpha^2)$

Frankfurt points from: Herr et.al.2306.09902
• The correlation function between two chromoelectric fields can be used in many applications
• Gradient flow can solve or ease many of the associated divergences
• Tested a two different ways to remove the linear divergence
• Future:
  • Continuum limits to properly attain flow time scaling
  • Zero flow time limits
  • Measure the actual observables mentioned in motivation
Conclusions

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Thank you!