

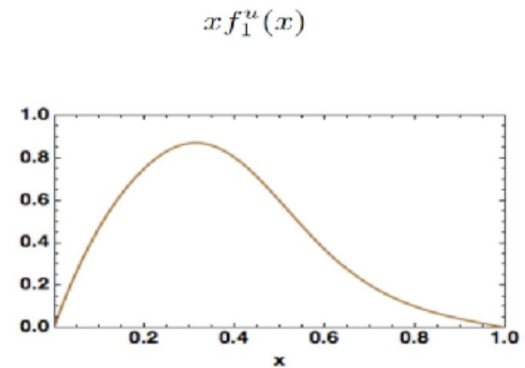
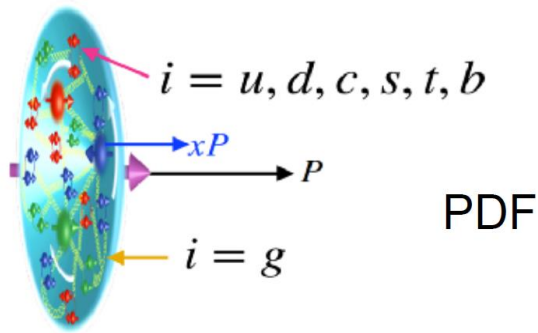
Renormalization of quasi transverse-momentum-dependent parton distribution function (quasi TMD PDF) on the lattice

Speaker: Kuan Zhang

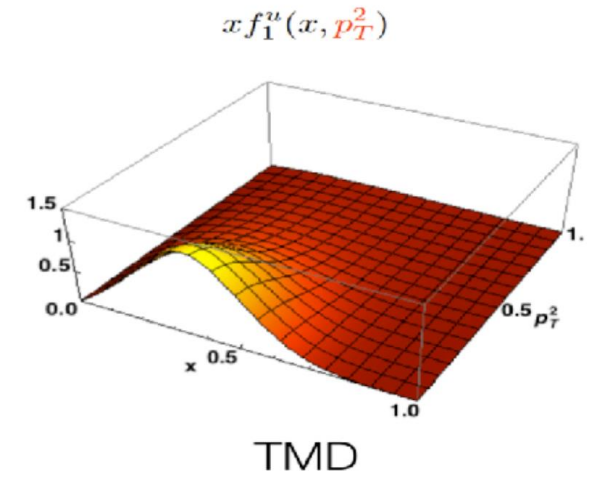
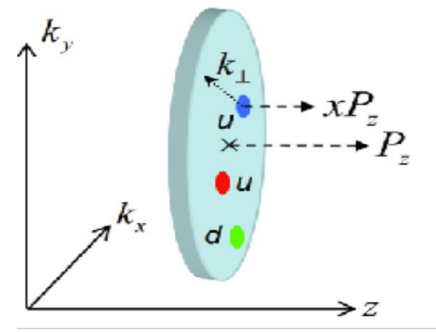
Xiangdong Ji, Yi-Bo Yang, Fei Yao, Jian-Hui Zhang

Lattice 2023 2023.07.31-2023.08.04

Motivation

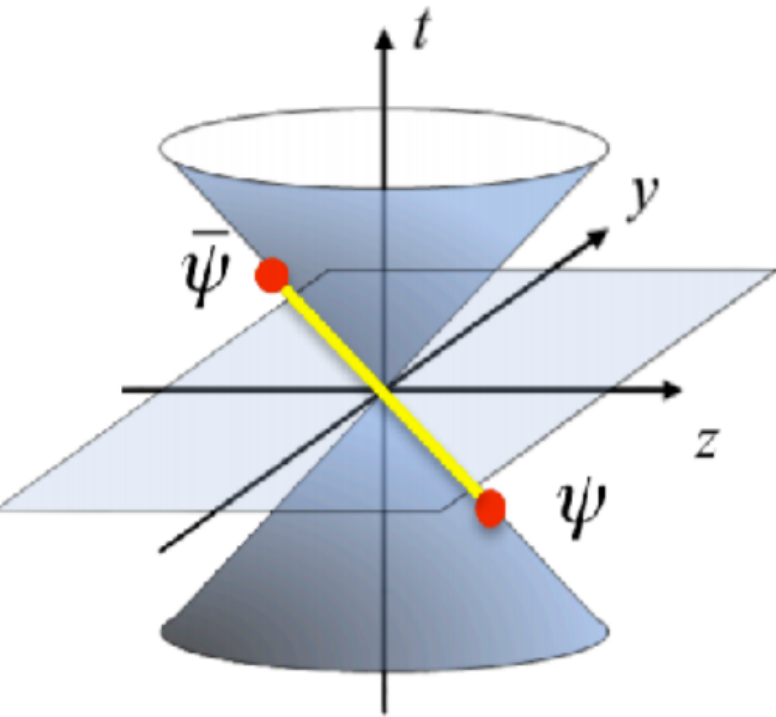


- TMDPDF describes the hadron structure in the transverse direction, and is much less known.



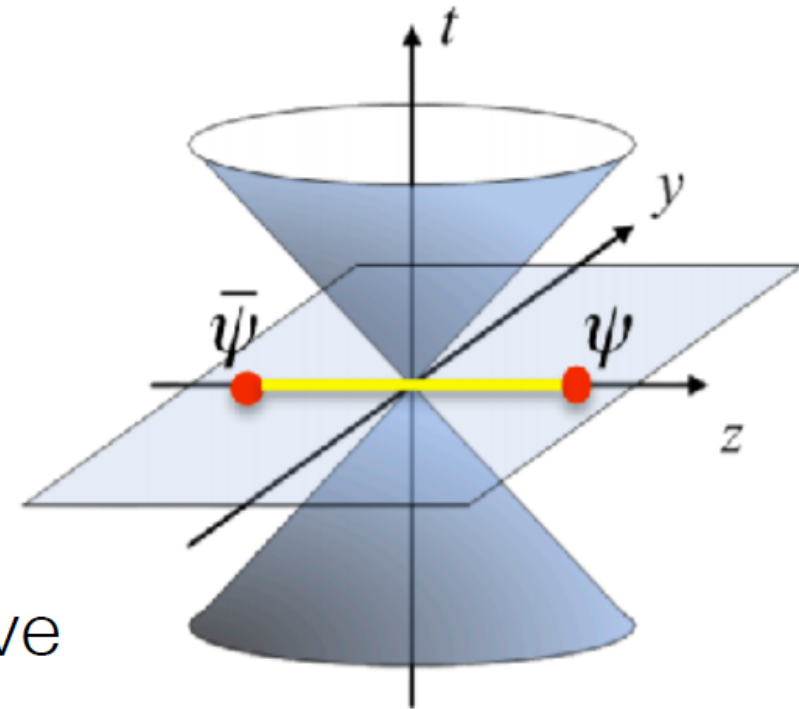
- How to determine it from first principle?

Large Momentum Effect Theory (LaMET)



Light-cone PDF : Cannot be calculated on the lattice

← Lorentz boost and perturbative matching



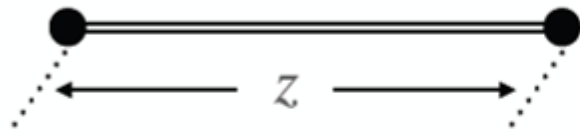
Quasi-PDF : Directly calculable on the lattice

Outline

- 1、 Renormalization of quasi PDF operator in RI/MOM scheme
- 2、 Renormalization of quasi TMD-PDF operator
 - 2.1 RI/MOM scheme
 - 2.2 Wilson loop + Short distance hadron matrix element
- 3、 Numerical details
- 4、 Summary

1、Quasi-PDF operator

$$O_{\gamma_t}(z) = \bar{q}(0)\gamma_t U_z(0,z)q(z)$$

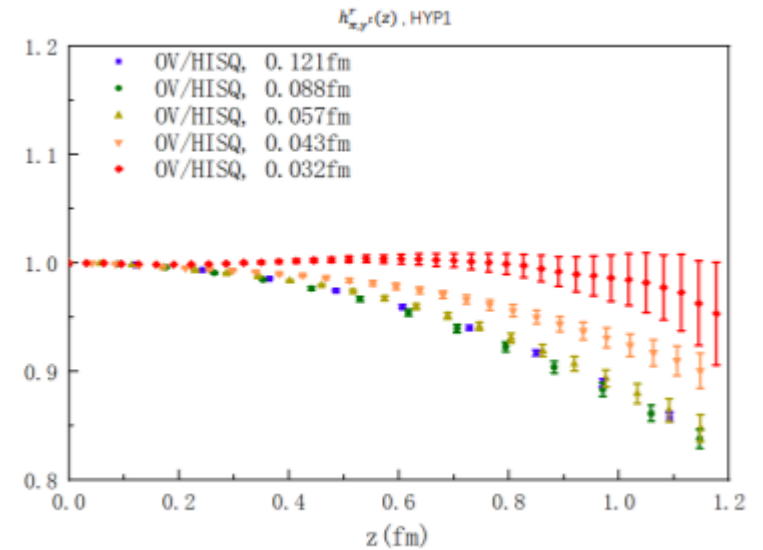
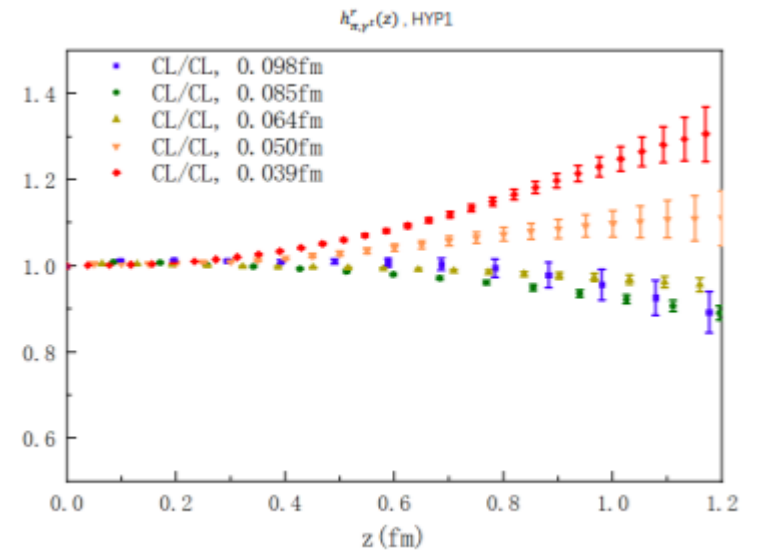
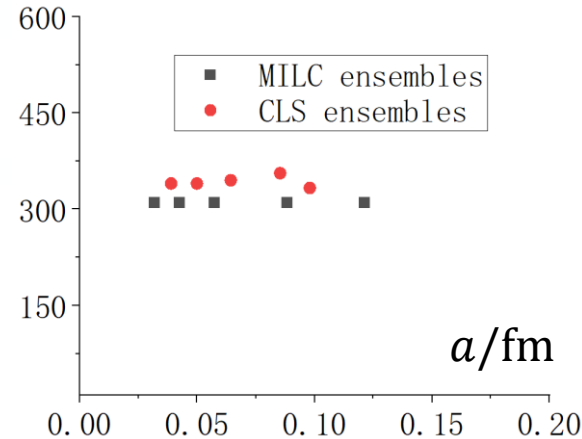


quasi-PDF operator

RI/MOM
renormalization

$$\langle \pi | O_{\gamma_t}(z) | \pi \rangle^{\text{RI/MOM}} \equiv \frac{\langle \pi | O_{\gamma_t}(z) | \pi \rangle}{\langle q | O_{\gamma_t}(z) | q \rangle}$$

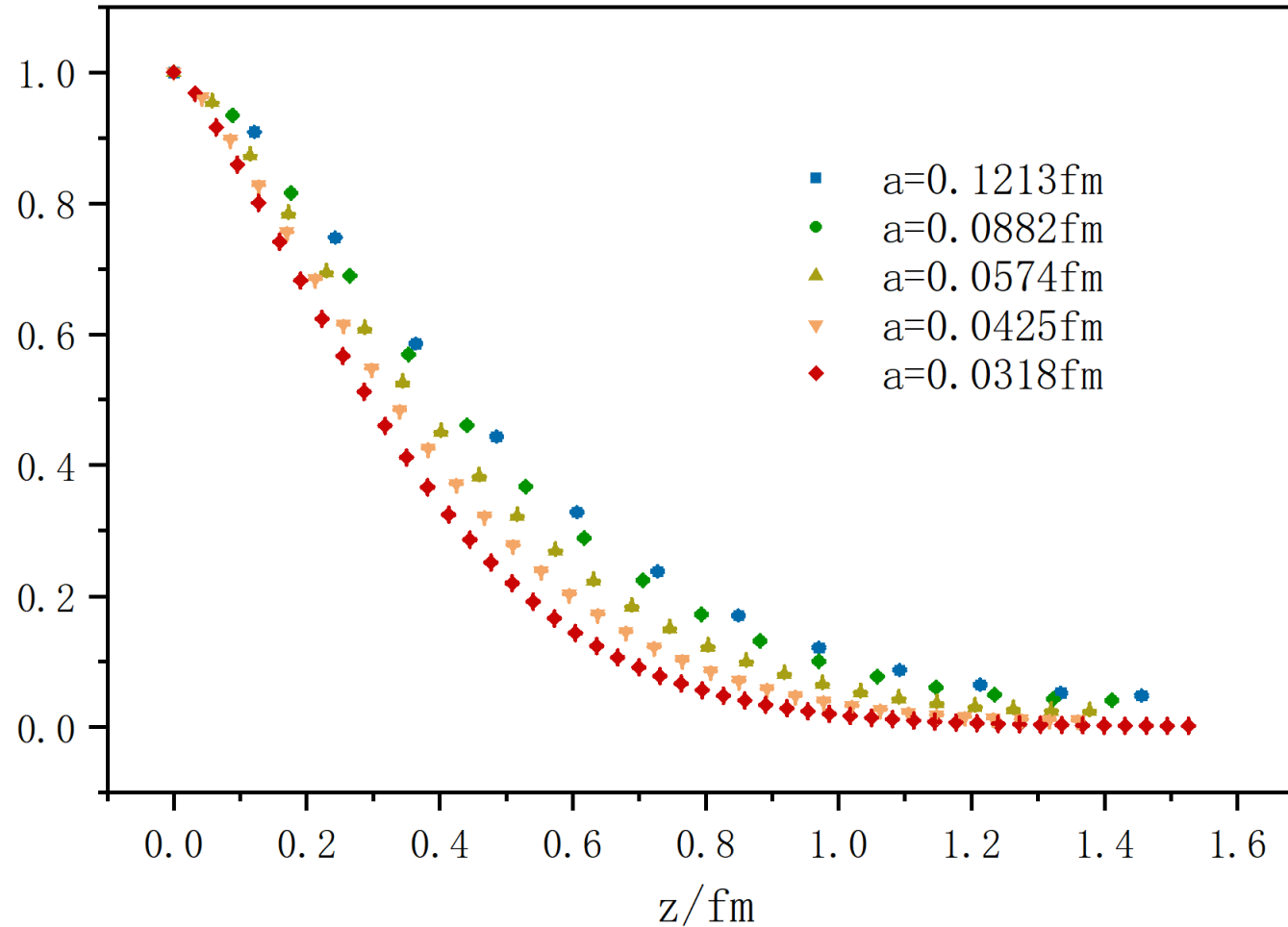
m_π/MeV



Coulomb gauge quasi PDF (preliminary)

Xiang Gao, Wei-Yang Liu, Yong Zhao, arXiv: 2306.14960

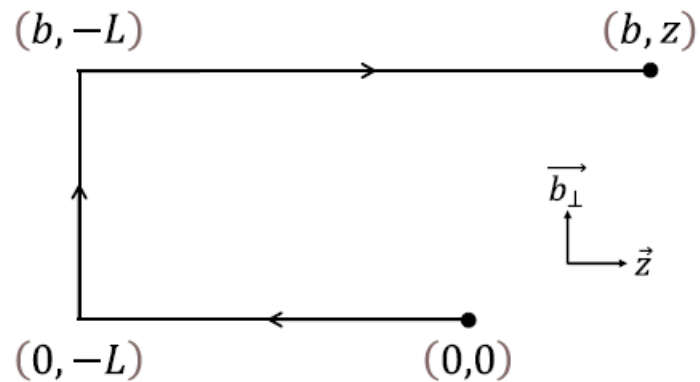
$$O_c(z) = \bar{\psi}_c(0)\Gamma\psi_c(z\hat{n}_z)$$



2、 Quasi TMD-PDF on lattice

- Matrix element
$$\tilde{h}_{\chi,\gamma_t}(b, z, L, P_z; 1/a) = \langle \chi(P_z) | O_{\gamma_t}(b, z, L) | \chi(P_z) \rangle,$$
$$O_{\Gamma}(b, z, L) \equiv \bar{\psi}(\vec{0}_{\perp}, 0) \Gamma \mathcal{W}(b, z, L) \psi(\vec{b}_{\perp}, z)$$
$$h_{\chi,\gamma_t}(b, z, P_z; 1/a) = \lim_{L \rightarrow \infty} \frac{\tilde{h}_{\chi,\gamma_t}(b, z, L, P_z; 1/a)}{\sqrt{Z_E(b, 2L + z; 1/a)}}$$

- Staple-shaped link



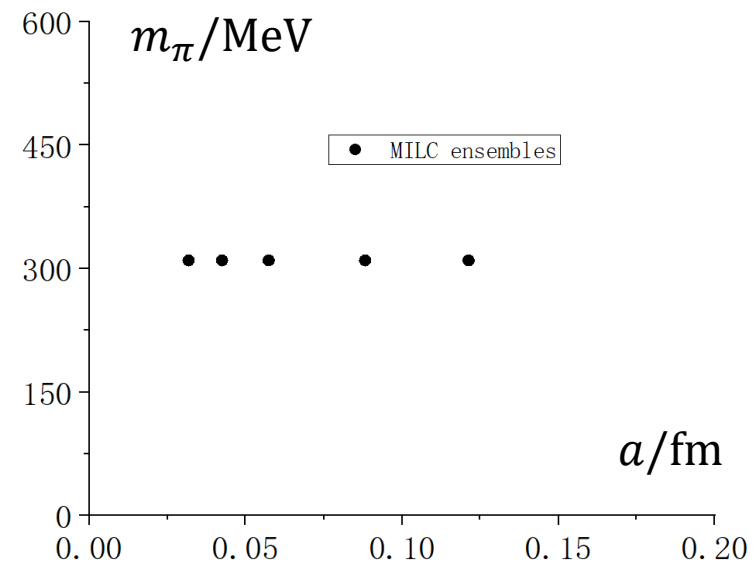
X. Ji, Y. Liu, Y. S. Liu, J.-H. Zhang, and Y. Zhao, Rev. Mod.Phys. 93, 035005 (2021).

X. Ji, Y. Liu, and Y.-S. Liu, Nucl. Phys. B955, 115054 (2020).

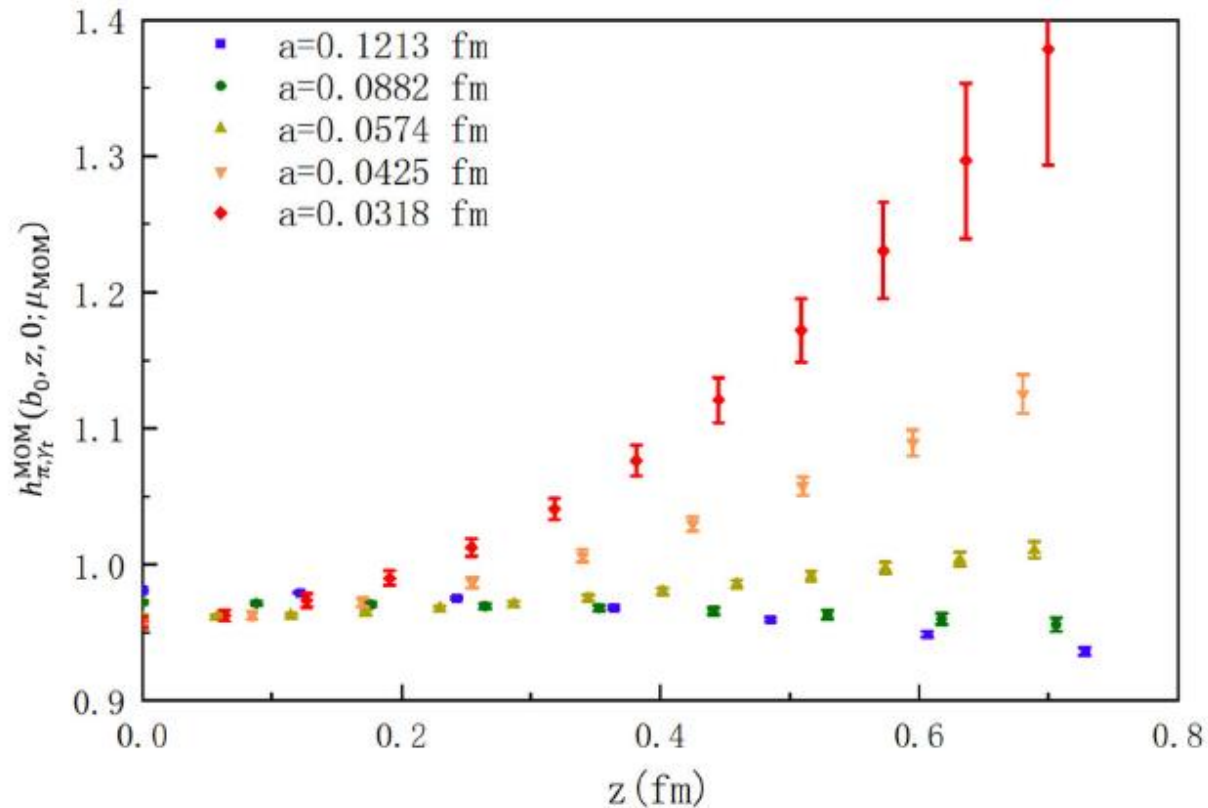
M. A. Ebert, I.W. Stewart, and Y. Zhao, J. High Energy Phys. 09 (2019) 037.

Lattice setup

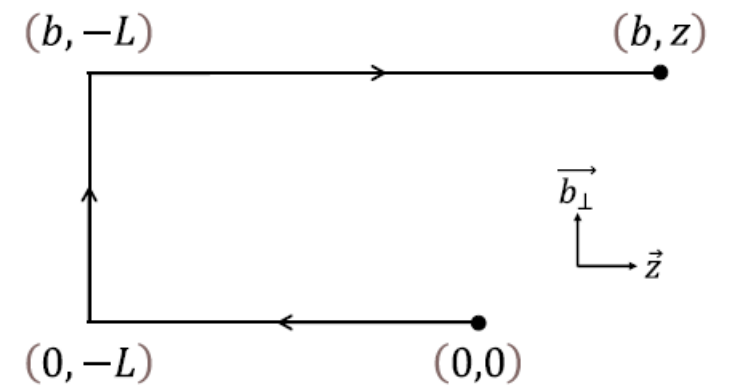
- $\langle \pi(0) | O_{\gamma_t}(b, z) | \pi(0) \rangle$ pion ME (matrix element) in rest frame wall source
- $\langle q(p) | O_{\gamma_t}(b, z) | q(p) \rangle$ $(p_x, p_y, p_z, p_t) = \frac{2\pi}{La} (0, 5, 0, 5)$ $|p| \sim 3 \text{ GeV}$
- 1-step HYP smearing on staple link
- Clover & overlap valence quark



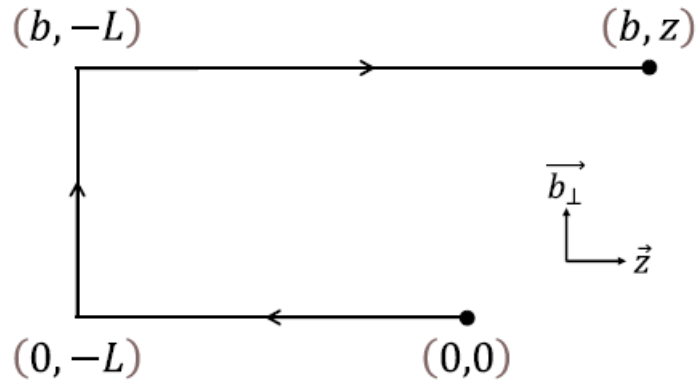
2.1 Staple link ME renormalized in RI/MOM scheme



$$\begin{aligned}
 h_{\chi, \Gamma}^{\text{MOM}}(b, z, P_z; p) &= \sum_{\Gamma'} [Z^{\text{MOM}}(b, z, p; 1/a)]_{\Gamma \Gamma'}^{-1} h_{\chi, \Gamma'}(b, z, P_z; 1/a) \\
 &= \lim_{L \rightarrow \infty} \sum_{\Gamma'} [\tilde{Z}^{\text{MOM}}(b, z, L, p; 1/a)]_{\Gamma \Gamma'}^{-1} \tilde{h}_{\chi, \Gamma'}(b, z, L, P_z; 1/a),
 \end{aligned}$$



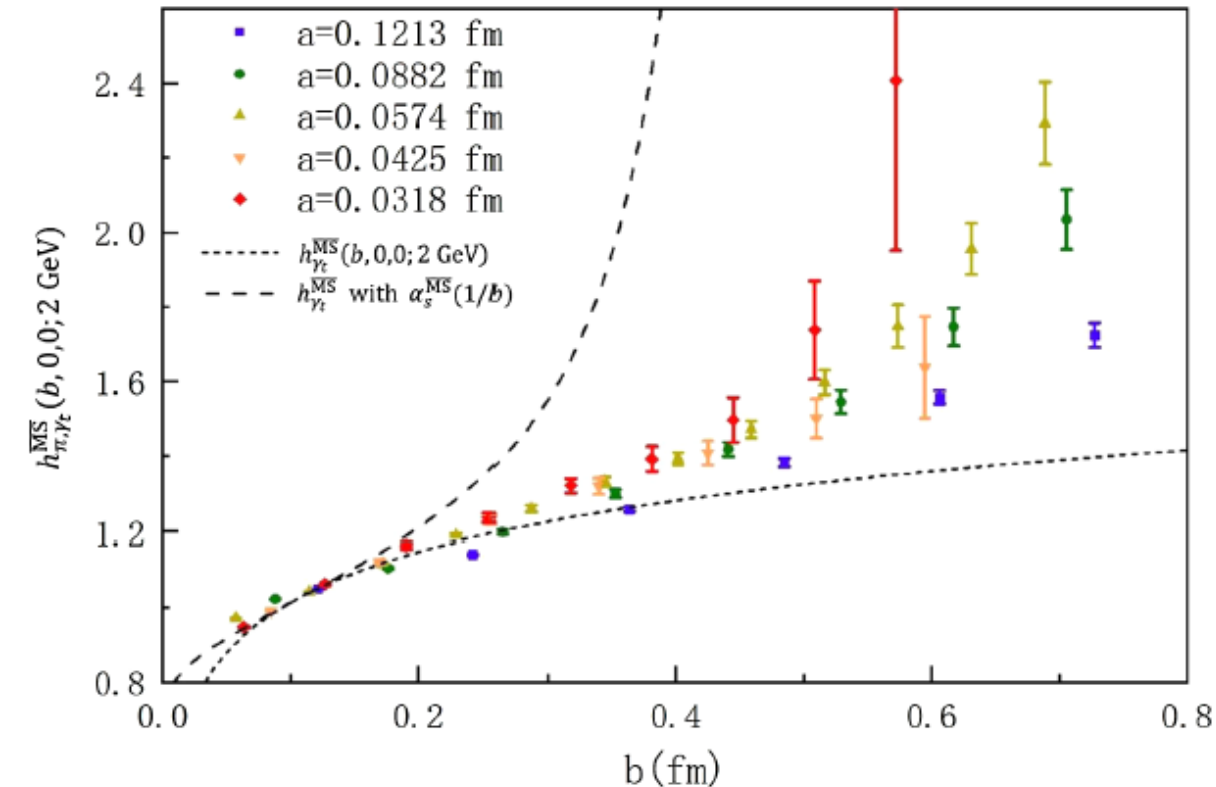
2.2 Staple link ME renormalized in SDR scheme



$$h_{\chi, \gamma_t}^{\text{SDR}} \left(b, z, P_z; \frac{1}{b_0} \right) = \frac{h_{\chi, \gamma_t}(b, z, P_z; 1/a)}{h_{\pi, \gamma_t}(b_0, z_0 = 0, 0, 1/a)}$$

$$h_{\chi, \gamma_t}^{\overline{\text{MS}}}(b_0, z_0, 0; \mu) = 1 + \frac{\alpha_s C_F}{2\pi} \left\{ \frac{1}{2} + 3\gamma_E - 3 \ln 2 + \frac{3}{2} \ln[\mu^2(b_0^2 + z_0^2)] - 2 \frac{z_0}{b_0} \arctan \frac{z_0}{b_0} \right\} + \mathcal{O}(\alpha_s^2)$$

$$h_{\chi, \gamma_t}^{\overline{\text{MS}}}(b, z, P_z; \mu) = h_{\gamma_t}^{\overline{\text{MS}}}(b_0, 0, 0; \mu) h_{\chi, \gamma_t}^{\text{SDR}} \left(b, z, P_z; \frac{1}{b_0} \right)$$



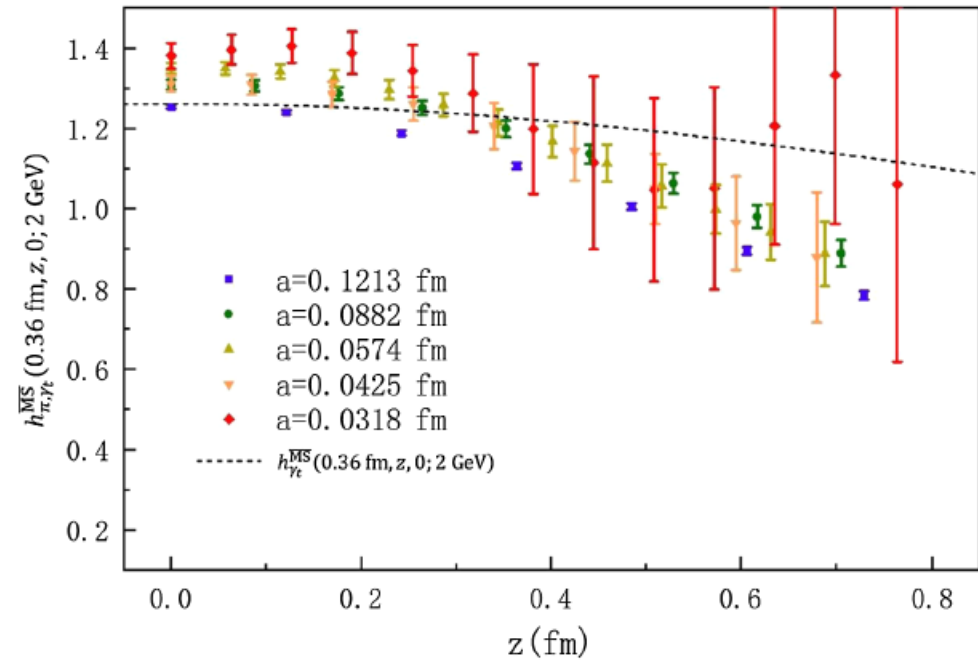
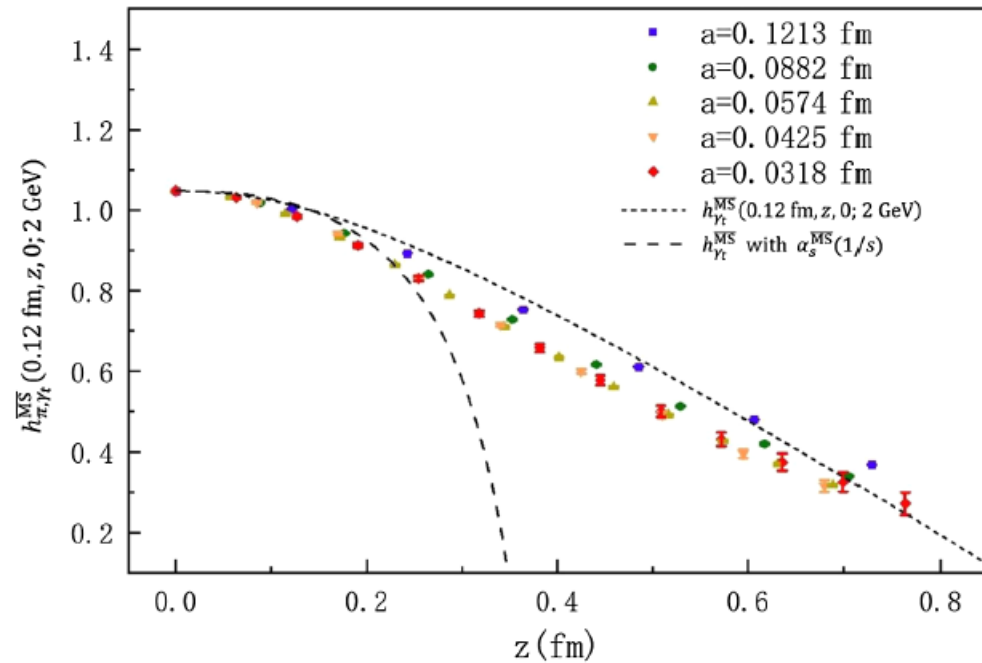
The UV divergence is of a multiplicative structure

M. A. Ebert, I.W. Stewart, and Y. Zhao, *J. High Energy Phys.* 03 (2020) 099
 J. R. Green, K. Jansen, and F. Steffens, *Phys. Rev. D* 101, 074509 (2020).

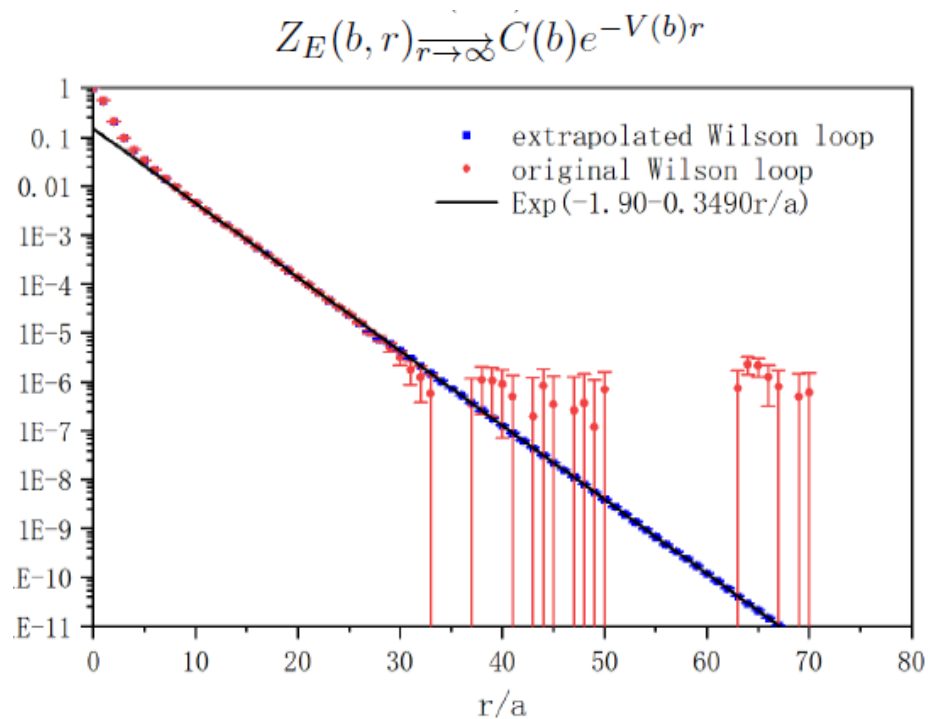
The mixing for nonchiral lattice fermion actions appears to be negligibly small

Y. Ji, J.-H. Zhang, S. Zhao, and R. Zhu, *Phys. Rev. D* 104, 094510 (2021).
 Q.-A. Zhang, et.al, LPC, *Phys. Rev. Lett.* 125, 192001(2020)
 Constantia Alexandrou, et.al, arXiv:2305.11824

2.2 Staple link ME renormalized in SDR scheme



3.1 Extrapolation of Wilson loop

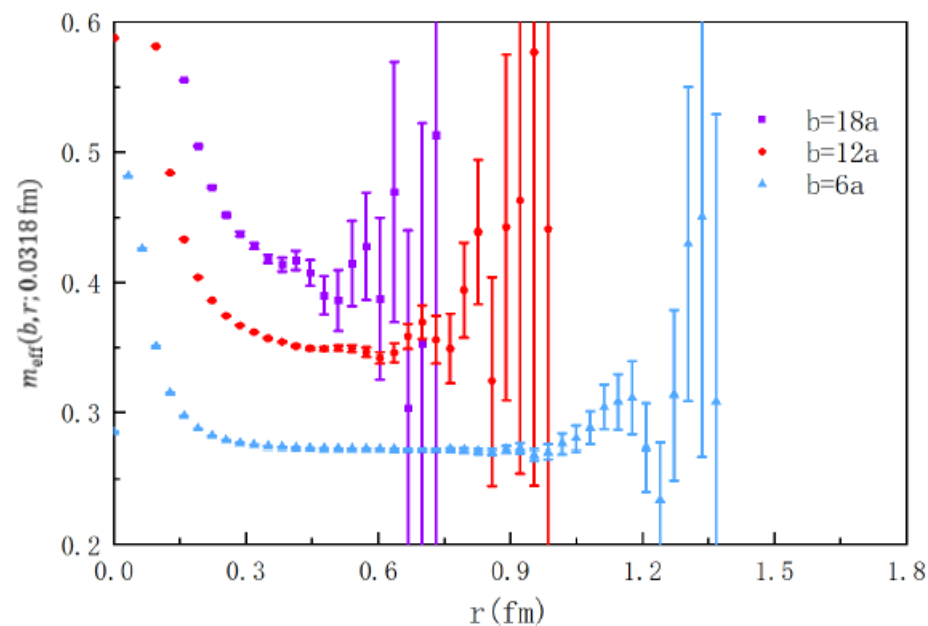
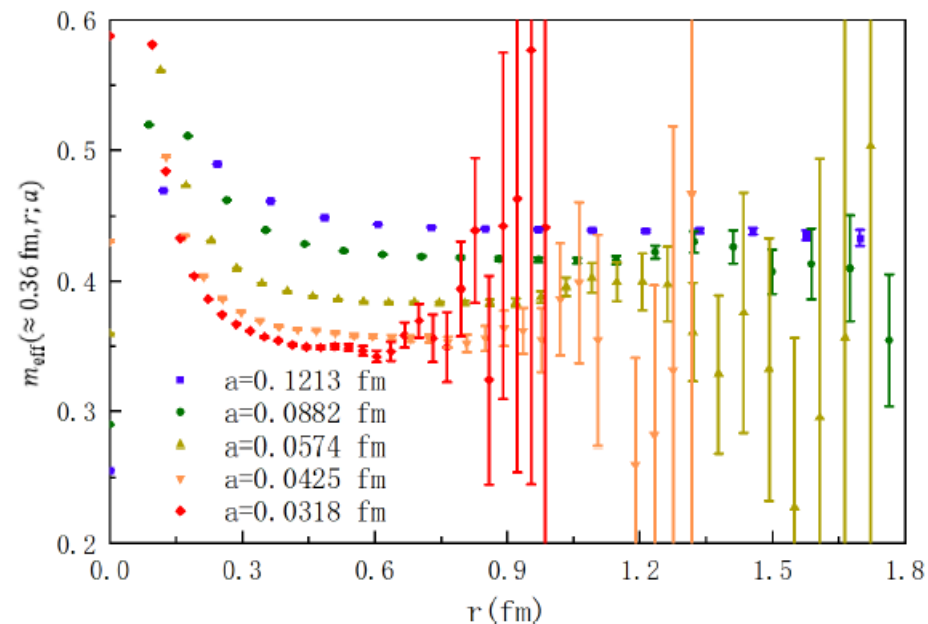


$$e^{c(b;a) - V(b;a)r}$$

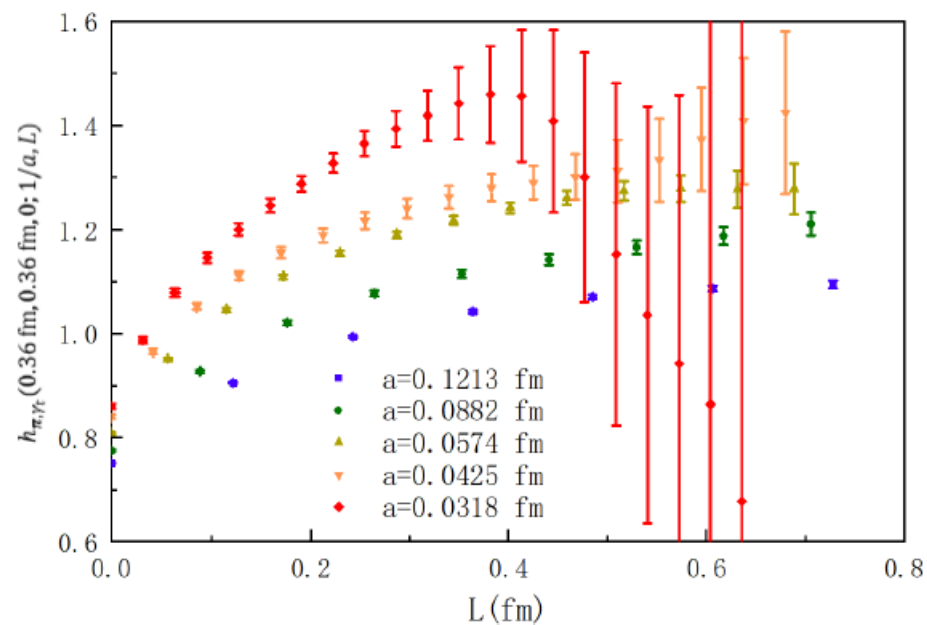
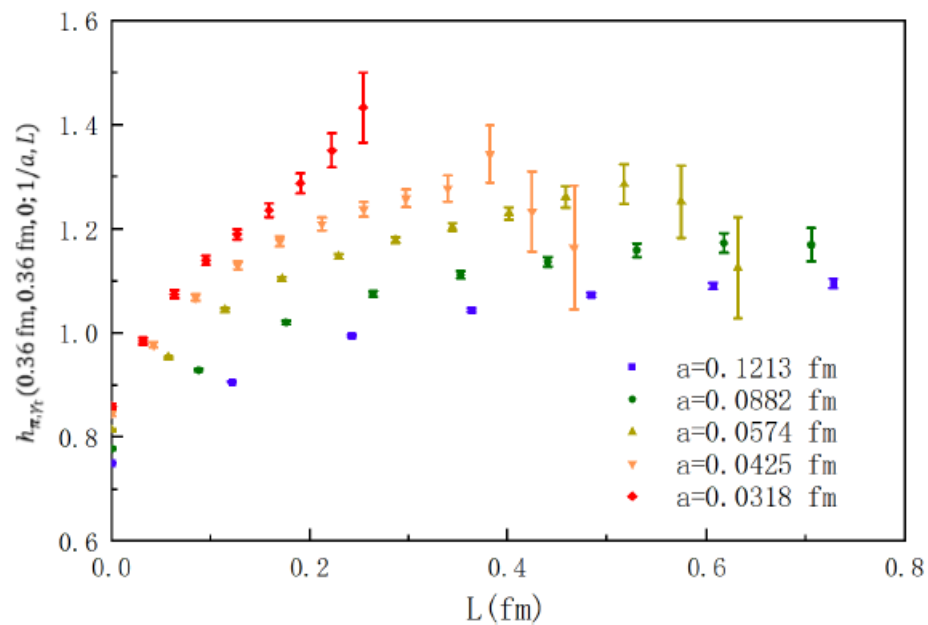
$$c(12a; 0.0318\text{fm}) = -1.90(0.02) \quad V(12a; 0.0318\text{fm}) = 0.3490(0.0015)$$

$$\chi^2/\text{d.o.f.} = 0.63$$

$$m_{\text{eff}}(b, r; a) = \ln \frac{Z_E(b, r; a)}{Z_E(b, r+1; a)}$$

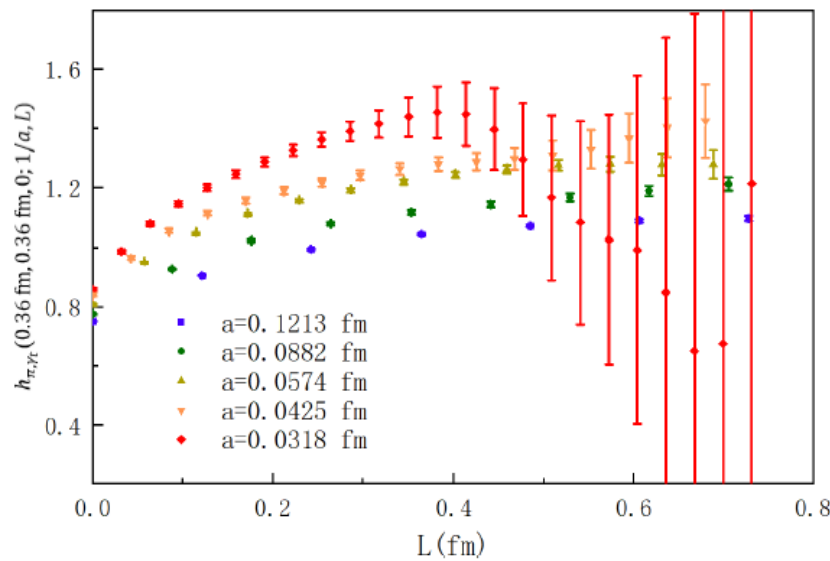
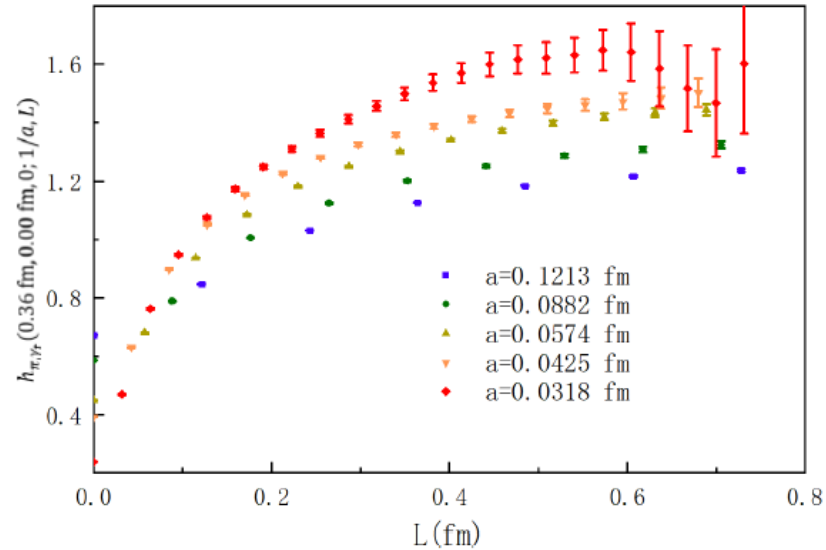
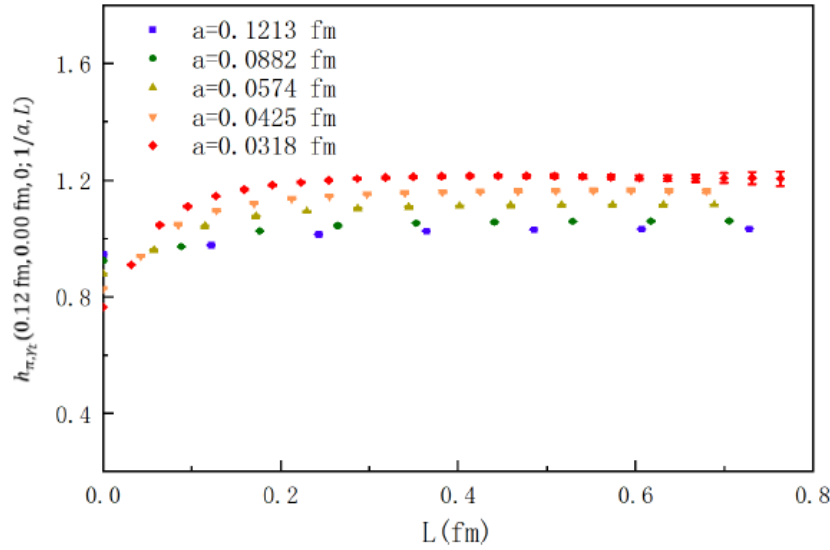


3.1 Extrapolation of Wilson loop



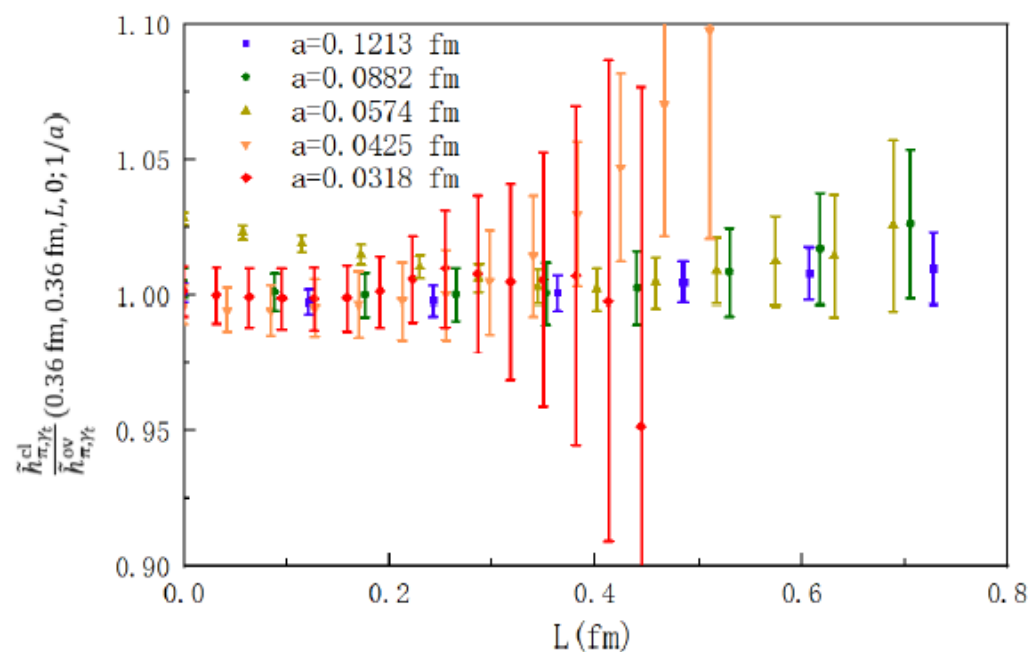
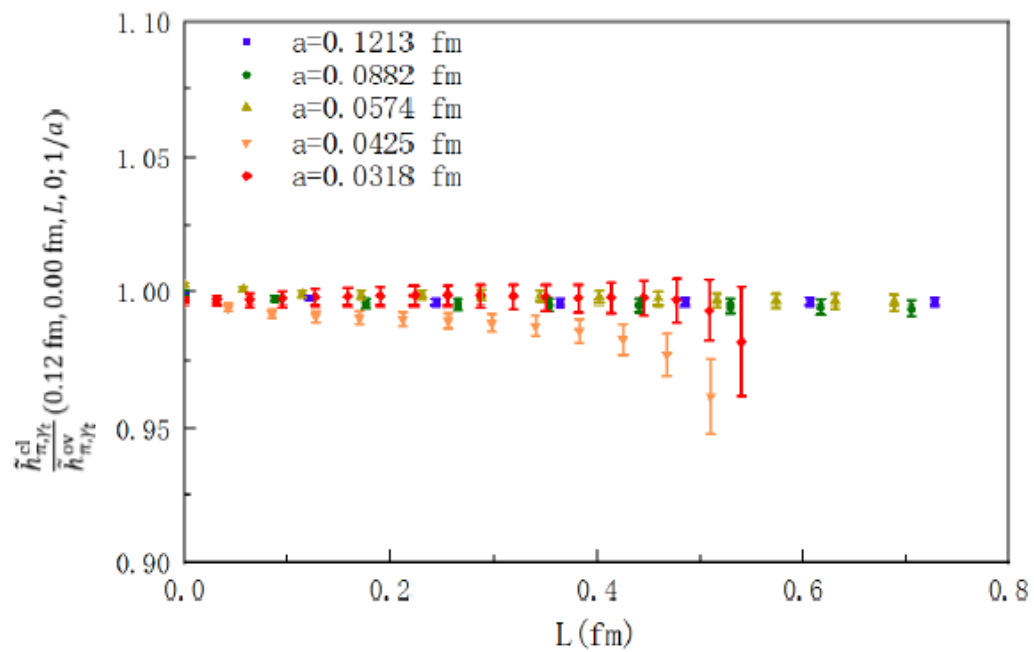
$$h_{\chi,\gamma_t}(b, z, P_z; 1/a) = \lim_{L \rightarrow \infty} \frac{\tilde{h}_{\chi,\gamma_t}(b, z, L, P_z; 1/a)}{\sqrt{Z_E(b, 2L + z; 1/a)}}$$

3.2 L dependence of the matrix elements



To balance the statistical uncertainty and the systematic uncertainty, we take $L = 6a$ for MILC12, $L = 8a$ for MILC09, $L = 11a$ for MILC06, $L = 13a$ for MILC04, and $L = 14a$ for MILC03 in most cases. For the cases with larger b on the MILC03 ensemble, we set $L = b$ (more specifically, we set $L = 16a$ for $b = 16a$, and $L = 18a$ for $b = 18a$).

3.3 Action dependence



3.4 Operator mixing

$$\mathcal{M}_{\mathcal{P}\Gamma}(b, z; 1/a) = \frac{\text{Abs} \left[\text{Tr} \left[\mathcal{P} \langle q(p) | \bar{q} \Gamma \mathcal{W}(b, z, L) q | q(p) \rangle \right] \right]}{\text{Abs} \left[\text{Tr} \left[\Gamma \langle q(p) | \bar{q} \Gamma \mathcal{W}(b, z, L) q | q(p) \rangle \right] \right]}$$

$M_{\mathcal{P}\Gamma}^{\text{cl}}(4a, 4a; a = 0.0882 \text{ fm})$

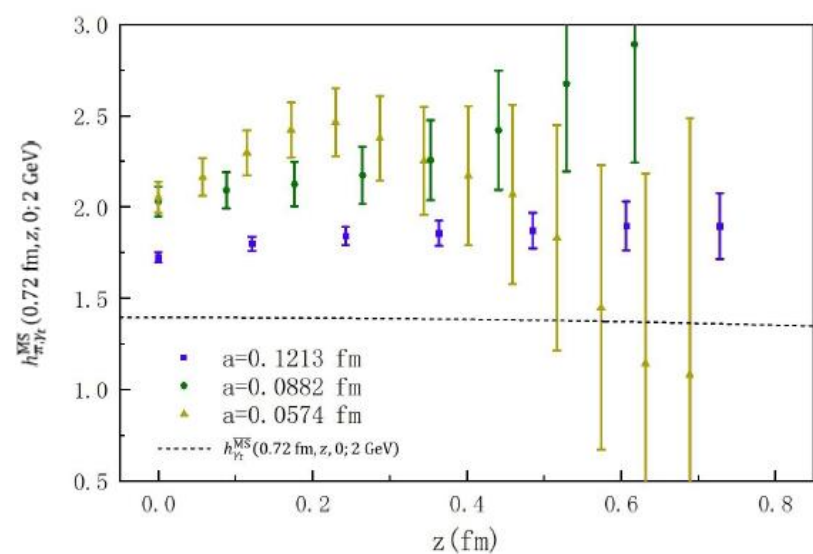
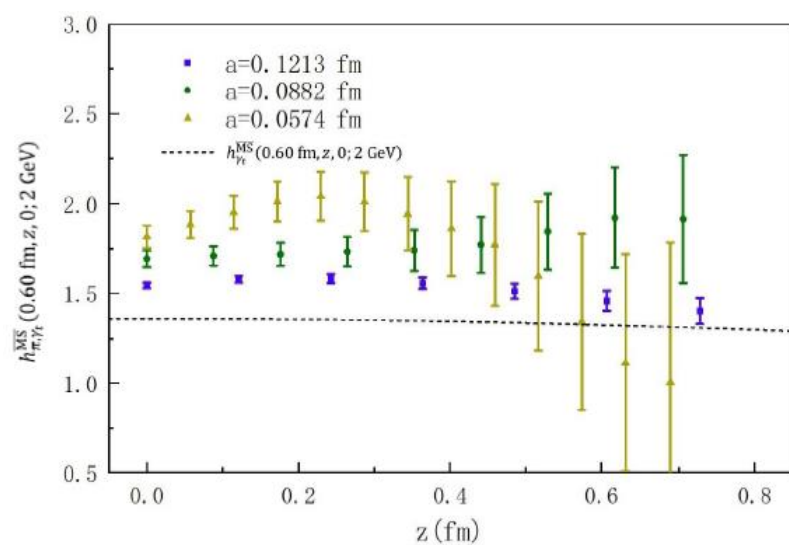
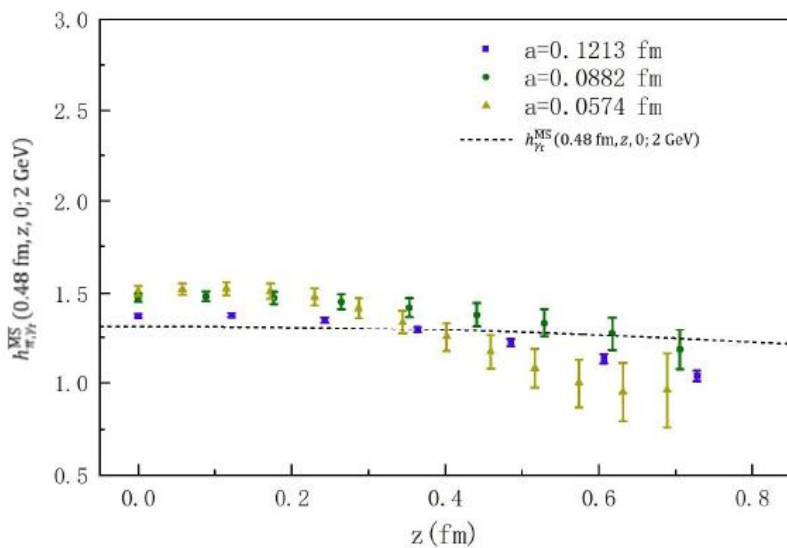
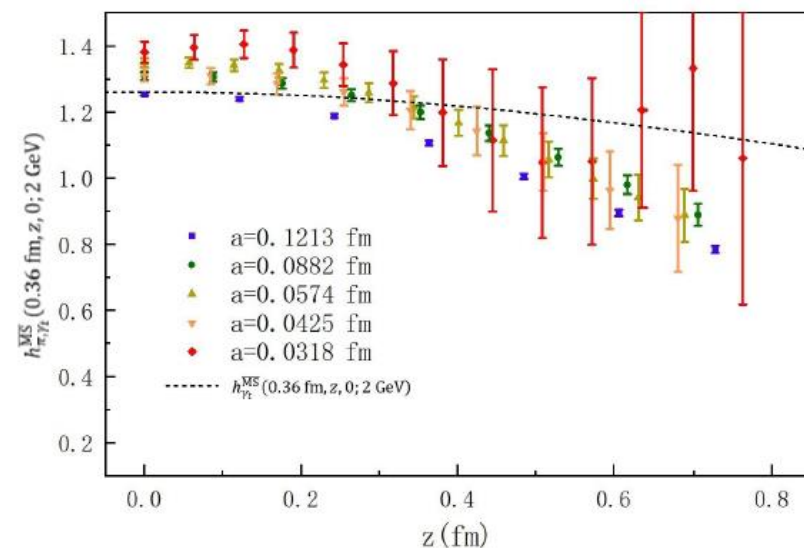
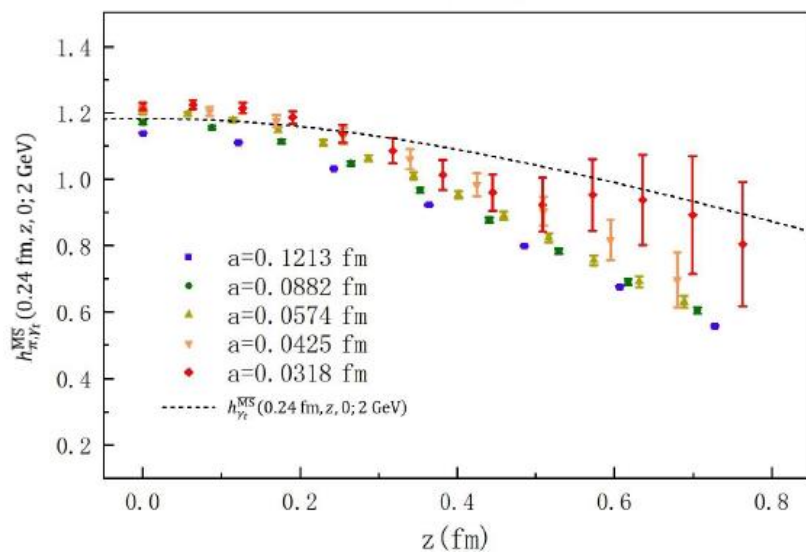
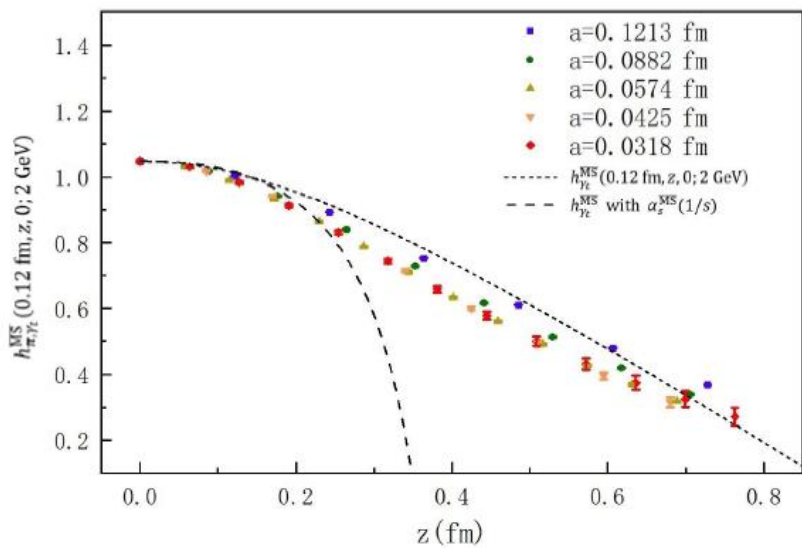
| $\Gamma \backslash \mathcal{P}$ | 1 | γ^t | γ^x | γ^y | γ^z | σ^{tx} | σ^{ty} | σ^{tz} | σ^{xy} | σ^{xz} | σ^{yz} | $\gamma^5 \gamma^t$ | $\gamma^5 \gamma^x$ | $\gamma^5 \gamma^y$ | $\gamma^5 \gamma^z$ | γ^5 |
|---------------------------------|------|------------|------------|------------|------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------------|---------------------|---------------------|---------------------|------------|
| 1 | 1.00 | 0.05 | 0.06 | 0.05 | 0.07 | 0.03 | 0.00 | 0.11 | 0.03 | 0.06 | 0.11 | 0.02 | 0.00 | 0.02 | 0.00 | 0.00 |
| γ^t | 0.01 | 1.00 | 0.04 | 0.01 | 0.06 | 0.04 | 0.00 | 0.15 | 0.00 | 0.02 | 0.00 | 0.00 | 0.10 | 0.07 | 0.03 | 0.01 |
| γ^x | 0.06 | 0.05 | 1.00 | 0.05 | 0.05 | 0.00 | 0.00 | 0.02 | 0.00 | 0.15 | 0.02 | 0.10 | 0.00 | 0.10 | 0.00 | 0.00 |
| γ^y | 0.01 | 0.01 | 0.04 | 1.00 | 0.05 | 0.00 | 0.00 | 0.00 | 0.04 | 0.02 | 0.15 | 0.07 | 0.10 | 0.00 | 0.03 | 0.01 |
| γ^z | 0.06 | 0.05 | 0.03 | 0.05 | 1.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.03 | 0.00 | 0.02 | 0.00 | 0.02 | 0.00 | 0.00 |
| σ^{tx} | 0.02 | 0.03 | 0.00 | 0.00 | 0.02 | 1.00 | 0.05 | 0.04 | 0.01 | 0.05 | 0.00 | 0.00 | 0.02 | 0.07 | 0.04 | 0.11 |
| σ^{ty} | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 1.00 | 0.05 | 0.04 | 0.00 | 0.05 | 0.01 | 0.07 | 0.01 | 0.06 | 0.15 |
| σ^{tz} | 0.10 | 0.15 | 0.02 | 0.00 | 0.00 | 0.04 | 0.05 | 1.00 | 0.00 | 0.05 | 0.01 | 0.00 | 0.02 | 0.07 | 0.01 | 0.02 |
| σ^{xy} | 0.02 | 0.00 | 0.00 | 0.03 | 0.02 | 0.01 | 0.05 | 0.00 | 1.00 | 0.05 | 0.04 | 0.07 | 0.02 | 0.00 | 0.04 | 0.11 |
| σ^{xz} | 0.07 | 0.02 | 0.15 | 0.02 | 0.04 | 0.05 | 0.00 | 0.04 | 0.05 | 1.00 | 0.04 | 0.02 | 0.00 | 0.01 | 0.00 | 0.00 |
| σ^{yz} | 0.10 | 0.00 | 0.02 | 0.15 | 0.00 | 0.00 | 0.05 | 0.01 | 0.04 | 0.05 | 1.00 | 0.07 | 0.02 | 0.00 | 0.01 | 0.02 |
| $\gamma^5 \gamma^t$ | 0.02 | 0.00 | 0.11 | 0.07 | 0.03 | 0.00 | 0.01 | 0.00 | 0.07 | 0.04 | 0.06 | 1.00 | 0.04 | 0.01 | 0.05 | 0.00 |
| $\gamma^5 \gamma^x$ | 0.00 | 0.11 | 0.00 | 0.10 | 0.00 | 0.02 | 0.06 | 0.04 | 0.02 | 0.00 | 0.04 | 0.05 | 1.00 | 0.05 | 0.04 | 0.03 |
| $\gamma^5 \gamma^y$ | 0.02 | 0.07 | 0.11 | 0.00 | 0.03 | 0.07 | 0.01 | 0.06 | 0.00 | 0.04 | 0.00 | 0.01 | 0.04 | 1.00 | 0.05 | 0.00 |
| $\gamma^5 \gamma^z$ | 0.00 | 0.02 | 0.00 | 0.02 | 0.00 | 0.02 | 0.06 | 0.01 | 0.02 | 0.00 | 0.01 | 0.05 | 0.04 | 0.05 | 1.00 | 0.16 |
| γ^5 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.09 | 0.06 | 0.02 | 0.09 | 0.00 | 0.02 | 0.00 | 0.04 | 0.00 | 0.14 | 1.00 |

$M_{\mathcal{P}\Gamma}^{\text{ov}}(4a, 4a; a = 0.0882 \text{ fm})$

| $\Gamma \backslash \mathcal{P}$ | 1 | γ^t | γ^x | γ^y | γ^z | σ^{tx} | σ^{ty} | σ^{tz} | σ^{xy} | σ^{xz} | σ^{yz} | $\gamma^5 \gamma^t$ | $\gamma^5 \gamma^x$ | $\gamma^5 \gamma^y$ | $\gamma^5 \gamma^z$ | γ^5 |
|---------------------------------|------|------------|------------|------------|------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------------|---------------------|---------------------|---------------------|------------|
| 1 | 1.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.05 | 0.00 | 0.19 | 0.05 | 0.06 | 0.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| γ^t | 0.01 | 1.00 | 0.07 | 0.04 | 0.08 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.20 | 0.06 | 0.05 | 0.00 |
| γ^x | 0.01 | 0.07 | 1.00 | 0.07 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.19 | 0.00 | 0.19 | 0.00 | 0.00 |
| γ^y | 0.01 | 0.04 | 0.07 | 1.00 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.20 | 0.00 | 0.05 | 0.00 |
| γ^z | 0.01 | 0.07 | 0.03 | 0.07 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 | 0.04 | 0.00 | 0.00 |
| σ^{tx} | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.07 | 0.04 | 0.02 | 0.07 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 |
| σ^{ty} | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 1.00 | 0.08 | 0.06 | 0.00 | 0.08 | 0.00 | 0.01 | 0.00 | 0.01 | 0.15 |
| σ^{tz} | 0.20 | 0.01 | 0.00 | 0.00 | 0.00 | 0.04 | 0.07 | 1.00 | 0.01 | 0.07 | 0.02 | 0.00 | 0.01 | 0.01 | 0.00 | 0.04 |
| σ^{xy} | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.07 | 0.01 | 1.00 | 0.07 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 |
| σ^{xz} | 0.06 | 0.00 | 0.01 | 0.00 | 0.00 | 0.08 | 0.00 | 0.06 | 0.08 | 1.00 | 0.06 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 |
| σ^{yz} | 0.20 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.07 | 0.02 | 0.04 | 0.07 | 1.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.04 |
| $\gamma^5 \gamma^t$ | 0.00 | 0.01 | 0.20 | 0.06 | 0.05 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 1.00 | 0.07 | 0.04 | 0.08 | 0.00 |
| $\gamma^5 \gamma^x$ | 0.00 | 0.19 | 0.00 | 0.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 1.00 | 0.07 | 0.05 | 0.00 |
| $\gamma^5 \gamma^y$ | 0.00 | 0.06 | 0.20 | 0.01 | 0.05 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.04 | 0.07 | 1.00 | 0.08 | 0.00 |
| $\gamma^5 \gamma^z$ | 0.00 | 0.04 | 0.00 | 0.04 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.03 | 0.07 | 1.00 | 0.01 |
| γ^5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.06 | 0.04 | 0.17 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 0.01 | 1.00 |

3.5 Matrix element at larger b

$$h_{\chi,\gamma_t}^{\overline{\text{MS}}}(b, z, P_z; \mu) = h_{\gamma_t}^{\overline{\text{MS}}}(b_0, 0, 0; \mu) h_{\chi,\gamma_t}^{\text{SDR}}\left(b, z, P_z; \frac{1}{b_0}\right)$$



3.6 TMD wave function matrix element

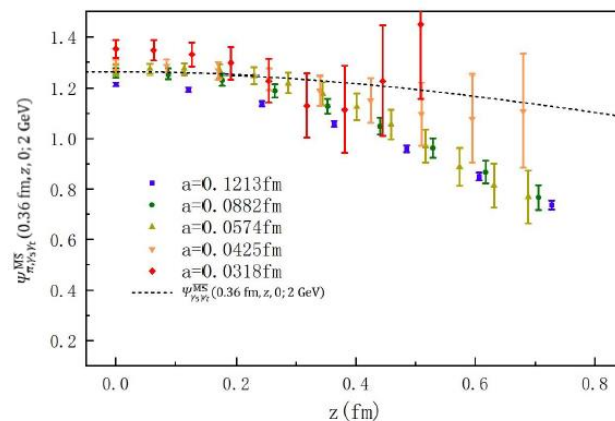
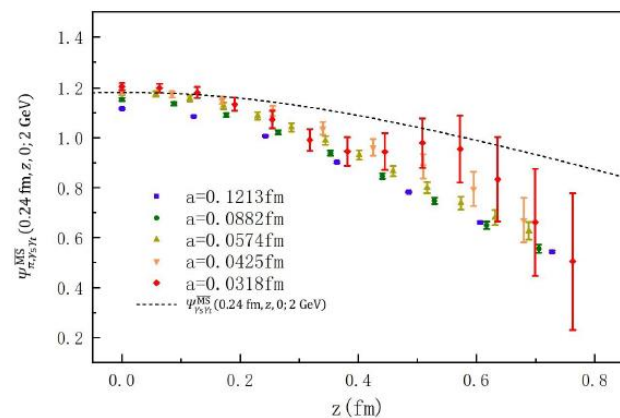
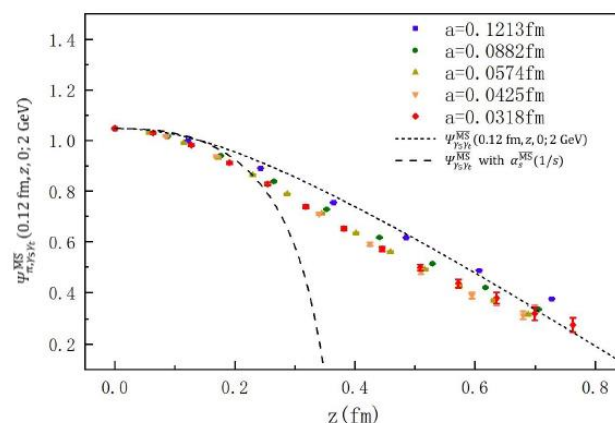
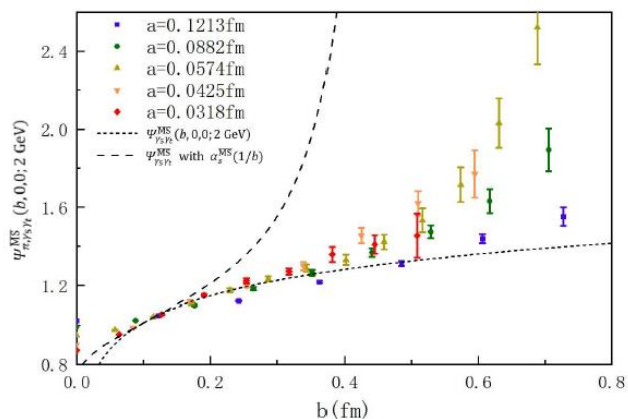
$$\Psi_{\pi,\gamma_5\gamma_t}(b, z, P_z; 1/a) = \lim_{L \rightarrow \infty} \frac{\langle O_{\gamma_5\gamma_t}(b, z, L) | \pi(P_z) \rangle}{\langle O_{\gamma_5\gamma_t}(0, 0, 0) | \pi(P_z) \rangle \sqrt{Z_E(b, 2L + z; 1/a)}},$$

$$\Psi_{\pi,\gamma_5\gamma_t}^{\text{SDR}}(b, z, P_z; \frac{1}{b_0}) = \frac{\Psi_{\pi,\gamma_5\gamma_t}(b, z, P_z; 1/a)}{\Psi_{\pi,\gamma_5\gamma_t}(b_0, z_0 = 0, 0, 1/a)}.$$

$$\Psi_{\gamma_5\gamma_t}^{\overline{\text{MS}}}(b_0, z_0, 0; \mu) = h_{\chi,\gamma_t}^{\overline{\text{MS}}}(b_0, z_0, 0; \mu)$$

$$= 1 + \frac{\alpha_s C_F}{2\pi} \left\{ \frac{1}{2} + 3\gamma_E - 3\ln 2 + \frac{3}{2} \ln[\mu^2(b_0^2 + z_0^2)] - 2 \frac{z_0}{b_0} \arctan \frac{z_0}{b_0} \right\} + \mathcal{O}(\alpha_s^2),$$

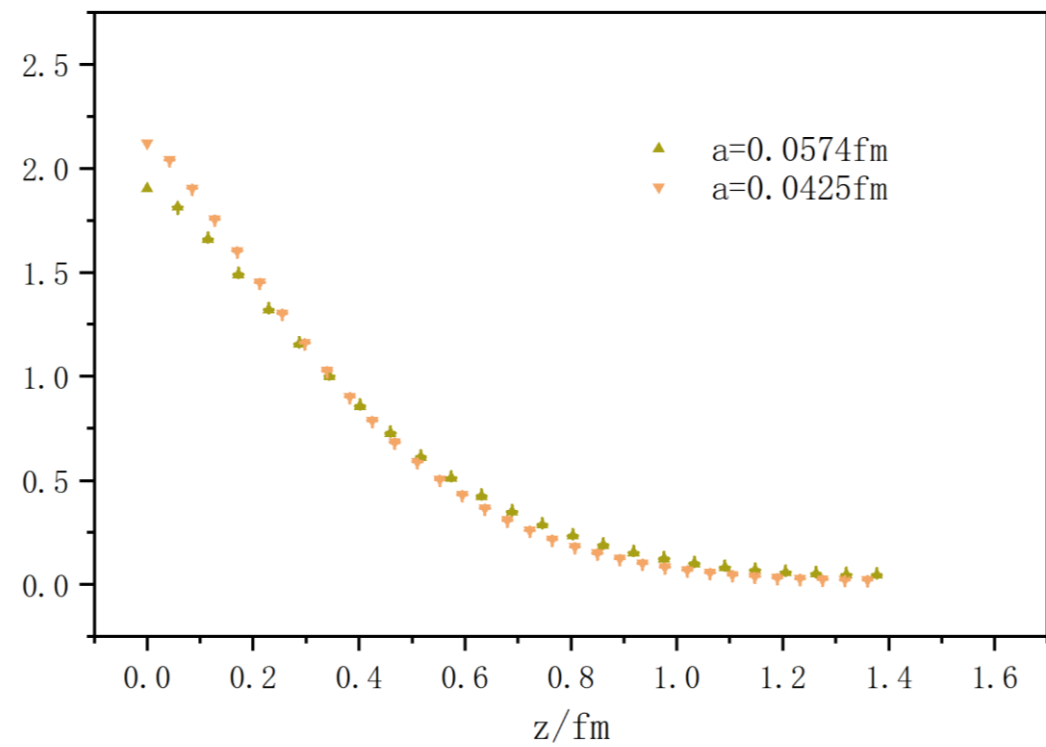
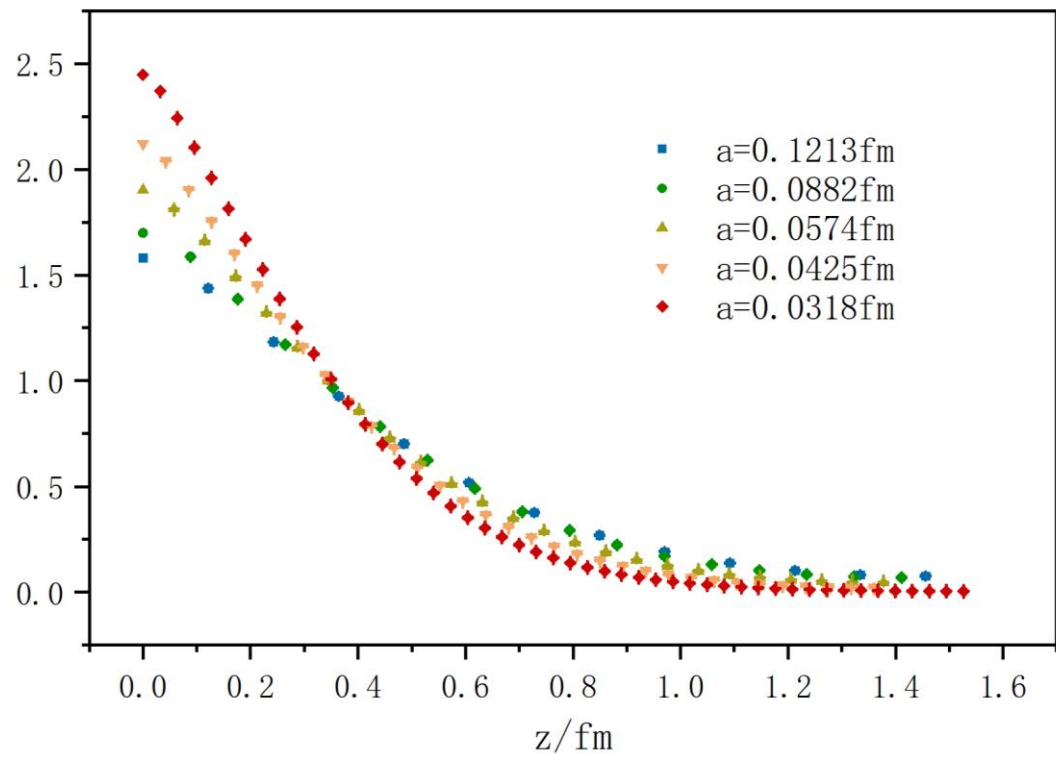
$$\Psi_{\pi,\gamma_5\gamma_t}^{\overline{\text{MS}}}(b, z, P_z; \mu) = \psi_{\gamma_5\gamma_t}^{\overline{\text{MS}}}(b_0, 0, 0; \mu) \Psi_{\pi,\gamma_t}^{\text{SDR}}(b, z, P_z; \frac{1}{b_0}).$$



4、 Summary

- The square root of the **Wilson loop** combined with the **short distance hadron matrix element** provides a successful method to **remove all ultraviolet divergences** of the quasi TMD-PDF operator
- Quasi TMD-PDF renormalized in **RI/MOM** scheme suffers from **residual linear divergence**.

backup



Hybrid-renormalization

X. Ji. et.al. , NPB964(2021)115311

Self-renormalization

Y.-K. Huo. et.al. ,LPC, NPB969(2021)115443

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