Proton Helicity GPDs from lattice QCD

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In collaboration with:
S. Bhattacharya, K. Cichy, M. Constantinou, J. Dodson, X. Gao, A. Metz, S. Mukherjee, P. Petreczky, A. Scapellato, F. Steffens, Y. Zhao

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Outline

❖ Work relies on approach proposed for the unpolarized

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

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- Theory component

Shohini Bhattacharya's talk

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- ❖ Work relies on approach proposed for the unpolarized
- ❖ Theory component — Shohini Bhattacharya's talk
- ❖ Lattice Component (this talk)

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  - ❖ Background
  - ❖ Lattice Methodology
  - ❖ Results
    - Matrix Elements
    - Lorentz invariant amplitudes
    - Quasi-GPDs
    - Light-Cone GPDs

Generalized Parton Distributions

❖ **GPDs are rich in information:**

- Reflect spatial distribution of partons in transverse plane
- Hadron mechanical properties are stored in GPDs
- Information on spin

❖ **… but not well studied:**

- extracted from off-forward kinematic (unlike PDFs)
- Multi-variable quantities; dependence upon x, t and ξ (unlike PDFs)
- Inferred from Compton form factors from experimental data (e.g., DVCS)

❖ **Helicity proton GPDs:**

- Two GPDs: $\widetilde{H}, \widetilde{E}$

$$F^{[\gamma^+\gamma_5]}(z,\Delta,P) = \bar{u}(p_f,\lambda')\left[\gamma^+\gamma_5\widetilde{H}(z,\xi,t) + \frac{\Delta^+\gamma_5}{2m}\widetilde{E}(z,\xi,t)\right]u(p_i,\lambda)$$

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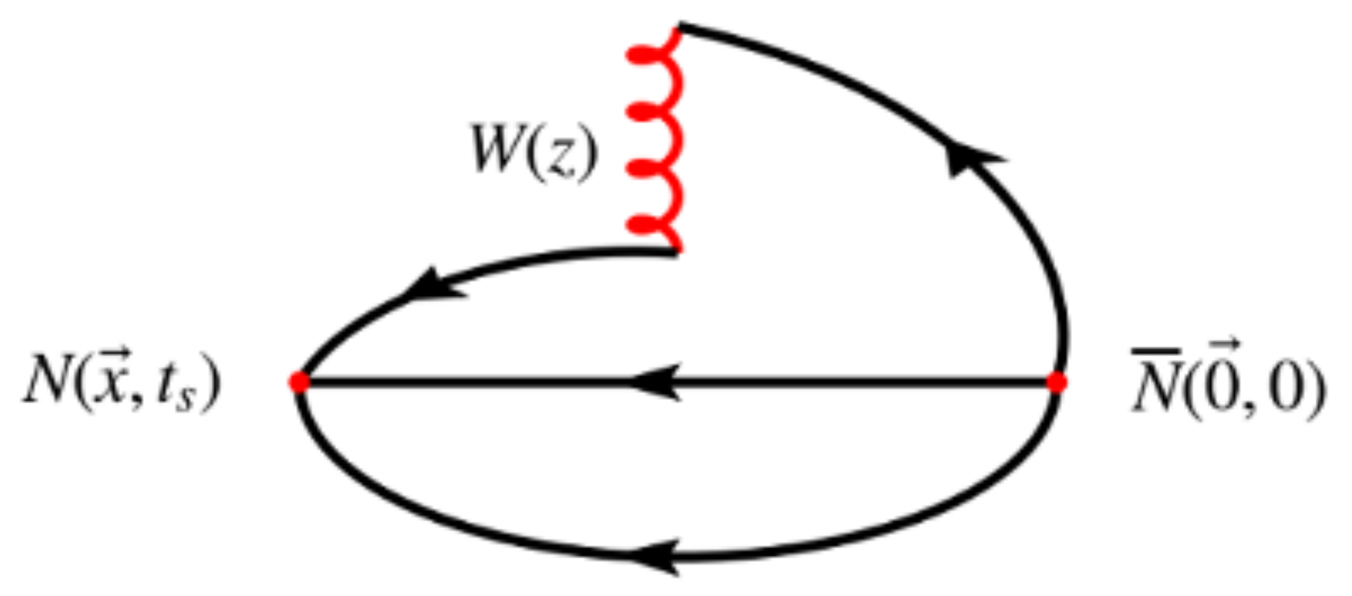
How can we complement information if access is difficult?

Methodology on the Lattice

- Extraction of matrix elements (helicity): $\left\langle N(P_f)\,|\,\bar{\Psi}(z)\gamma^{\mu}\gamma_5\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\right\rangle$



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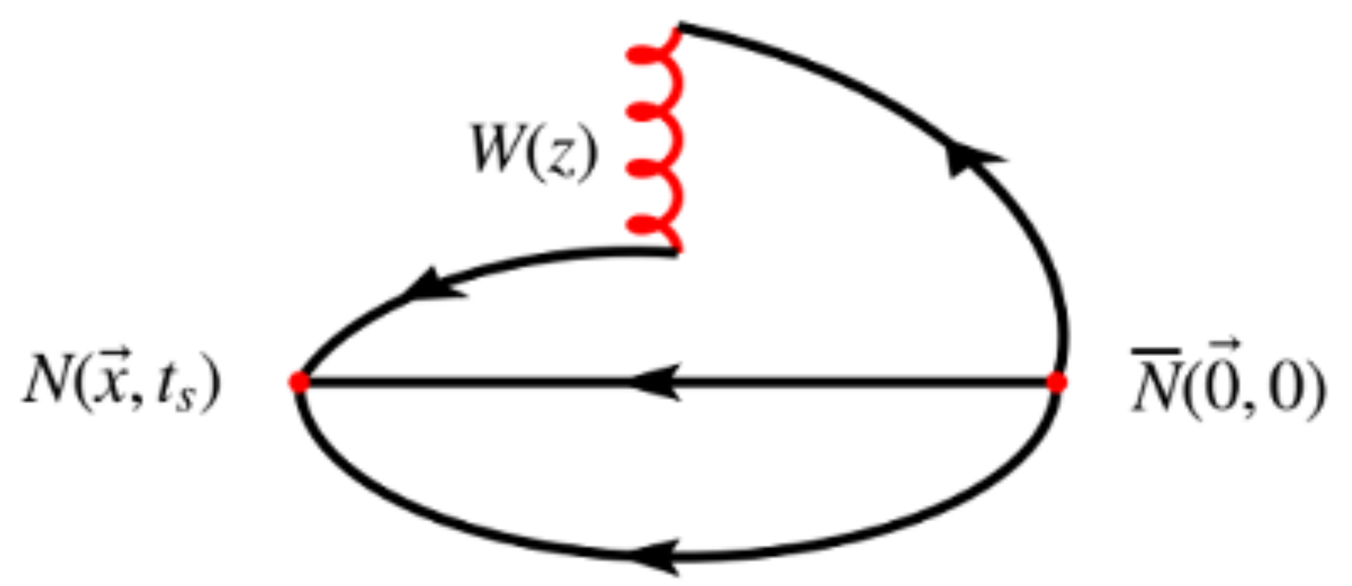

- Choice of frame:
  - Symmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p}_f = P_3\hat{z} + \vec{\Delta}/2$
  - Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \; \vec{p}_f = P_3\hat{z}$

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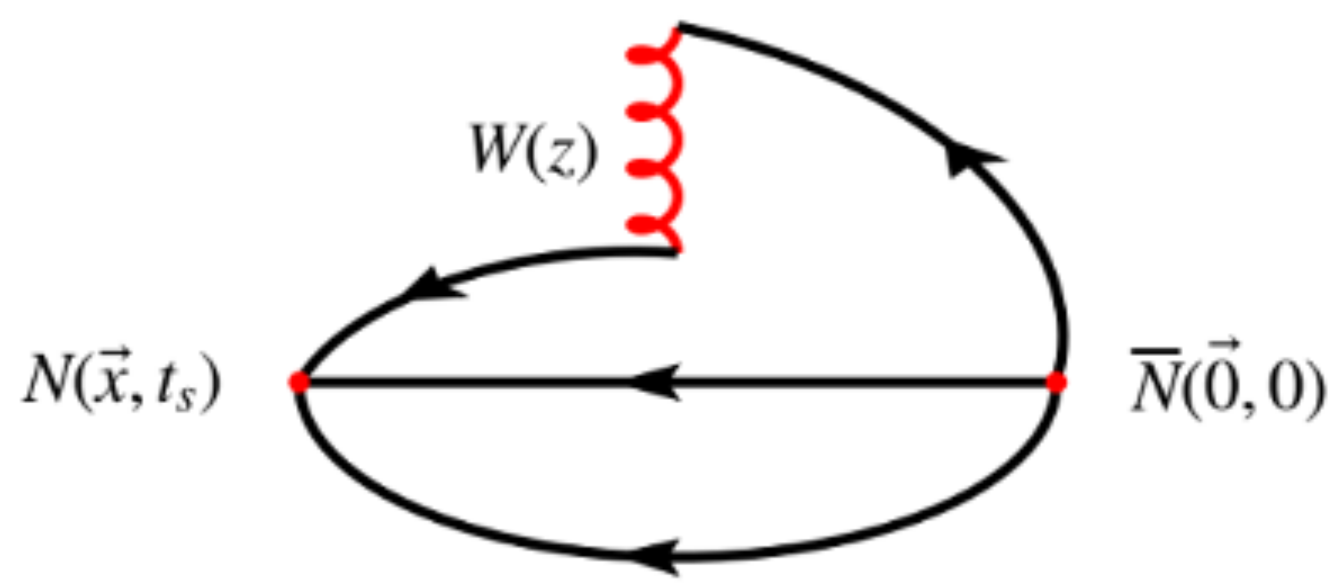

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❖ Isolation of ground state: single-state fit (plateau fit)

$$R_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau) = \frac{C_\mu^{3\text{pt}}(\Gamma_\kappa, z, p_f, p_i; t_s, \tau)}{C^{2\text{pt}}(\Gamma_0, p_f; t_s)}\sqrt{\frac{C^{2\text{pt}}(\Gamma_0, p_i, t_s-\tau)C^{2\text{pt}}(\Gamma_0, p_f, \tau)C^{2\text{pt}}(\Gamma_0, p_f, t_s)}{C^{2\text{pt}}(\Gamma_0, p_f, t_s-\tau)C^{2\text{pt}}(\Gamma_0, p_i, \tau)C^{2\text{pt}}(\Gamma_0, p_i, t_s)}} \xrightarrow[\tau\gg a]{t_s-\tau\gg a} \Pi_\mu(\Gamma_\kappa, z, p_f, p_i; t_s)$$

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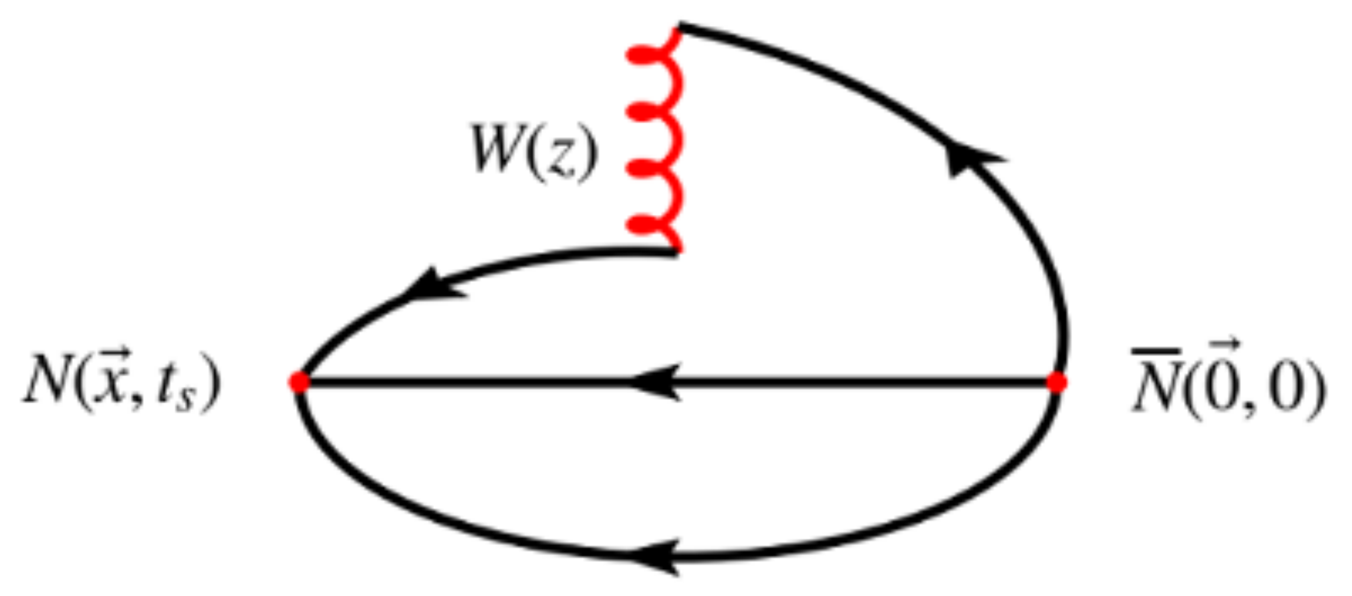

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❖ Parameterization of matrix elements (Lorentz Invariant)

$$\widetilde{F}^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z \Delta}}{m}\tilde{A}_1 + \gamma^\mu\gamma_5\tilde{A}_2 + \gamma_5\left(\frac{P^\mu}{m}\tilde{A}_3 + mz^\mu\tilde{A}_4 + \frac{\Delta^\mu}{m}\tilde{A}_5\right) + m\gamma_\nu z^\nu\gamma_5\left(\frac{P^\mu}{m}\tilde{A}_6 + mz^\mu\tilde{A}_7 + \frac{\Delta^\mu}{m}\tilde{A}_8\right)\right]u(p_i,\lambda)$$

The matrix elements depend upon 8 linearly-independent Lorentz invariant amplitudes → $\tilde{A}_i\left(z\cdot P, z\cdot\Delta, \Delta^2, z^2\right)$

Methodology on the Lattice

❖ Extraction of quasi-GPDs using the amplitudes

Standard $\gamma^3\gamma_5$ definition:

$$\tilde{\mathcal{H}}_3(z \cdot P, z \cdot \Delta, \Delta^2) = \tilde{A}_2 + zP_3\tilde{A}_6 - m^2z^2\tilde{A}_7 - z\Delta_3\tilde{A}_8$$

$$\tilde{\mathcal{E}}_3(z, P, \Delta) = 2\frac{P_3}{\Delta_3}\tilde{A}_3 + 2m^2\frac{z}{\Delta_3}\tilde{A}_4 + 2\tilde{A}_5$$

$$F^{[\gamma^3\gamma_5]} = \frac{1}{2P_0}\bar{u}(p_f, \lambda')[\gamma^3\gamma_5\tilde{\mathcal{H}}(x, \xi, t; P_3) + \frac{\Delta_3\gamma_5}{2m}\tilde{\mathcal{E}}(x, \xi, t; P_3)]u(p_i, \lambda)$$

Lorentz invariant definition

$$\tilde{\mathcal{H}}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) = \tilde{A}_2 + (P \cdot z)\tilde{A}_6 + (\Delta \cdot z)\tilde{A}_8$$

$$\tilde{\mathcal{E}}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) = 2\frac{P \cdot z}{\Delta \cdot z}\tilde{A}_3 + 2\tilde{A}_5$$

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❖ Renormalization functions: RI-MOM.

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❖ Fourier-like transform to x-space (Backus-Gilbert)

[Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

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❖ Extract light cone-GPDs using matching formalism [Liu, et al., Phys. Rev. D 100, 034006 (2019)]

Decomposition (selected)

Working with zero-skewness, we cannot extract $\widetilde{\mathscr{E}}$ due to the $\gamma^3\gamma_5$ decomposition

$$F^{[\gamma^3\gamma_5]}\left(x,\Delta;P^3\right)=\frac{1}{2P^0}\bar{u}(p_f,\lambda')\left[\gamma^3\gamma_5\widetilde{\mathscr{H}}\left(x,\xi,t;P^3\right)+\frac{\Delta^3\gamma_5}{2m}\widetilde{\mathscr{E}}\left(x,\xi,t;P^3\right)\right]u(p_i,\lambda)$$

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Symmetric frame ($\xi=0$)

$$\Pi_0^s(\Gamma_1)=K\left(\frac{E\Delta_1(E+m)}{4m^3}\tilde{A}_3\right)$$

$$\Pi_1^s(\Gamma_0)=K\left(\frac{-2E\Delta_2 z(E(E+m)-P_3^2)}{m^3}\tilde{A}_1-\frac{P_3\Delta_2}{4m^2}\tilde{A}_2\right)$$

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$$\Pi_0^a(\Gamma_1)=K\Delta_1\left(\frac{(E_f+m)}{4m^2}\tilde{A}_2+\frac{(E_f+E_i)(E_f+m)}{8m^3}\tilde{A}_3+\frac{(E_f-E_i)(E_f+m)}{4m^3}\tilde{A}_5+\frac{(E_f+E_i)P_3z}{8m^2}\tilde{A}_6+\frac{(E_f-E_i)P_3z}{4m^2}\tilde{A}_8\right)$$

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Lorentz Invariance

$\tilde{A}_i$ ✓

$\Pi_i^{s/a}$ ✗

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★ **Frame dependence of matrix elements due to kinematic coefficients of $\tilde{A}_i$** ★

Lattice Setup

❖ $N_f = 2 + 1 + 1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

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❖ Calculation of symmetric and asymmetric frame

- Symmetric frame:
  Each $\vec{\Delta}$ requires new calculation

- Asymmetric frame:
  Several $\vec{\Delta}$ values grouped in the same production run (e.g. $\{\vec{\Delta}$ = (100), (200), (300), …})

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	329	16	10528
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	$(\pm2, \pm2, 0)$	1.38	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.77	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	269	8	17216
asymm	±1.25	$(\pm1, \pm1, 0)$	0.34	0	16	195	8	24960
asymm	±1.25	(±2,0,0), (0,±2,0)	0.65	0	8	269	8	17216
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.81	0	16	195	8	24960
asymm	±1.25	(±2,±2,0)	1.24	0	16	195	8	24960
asymm	±1.25	(±3,0,0), (0,±3,0)	1.38	0	8	269	8	17216
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.52	0	16	195	8	24960
asymm	±1.25	(±4,0,0), (0,±4,0)	2.29	0	8	269	8	17216

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frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	329	16	10528
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
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Computationally efficient setup

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❖ $N_f = 2+1+1$ Twisted mass fermions with a clover term

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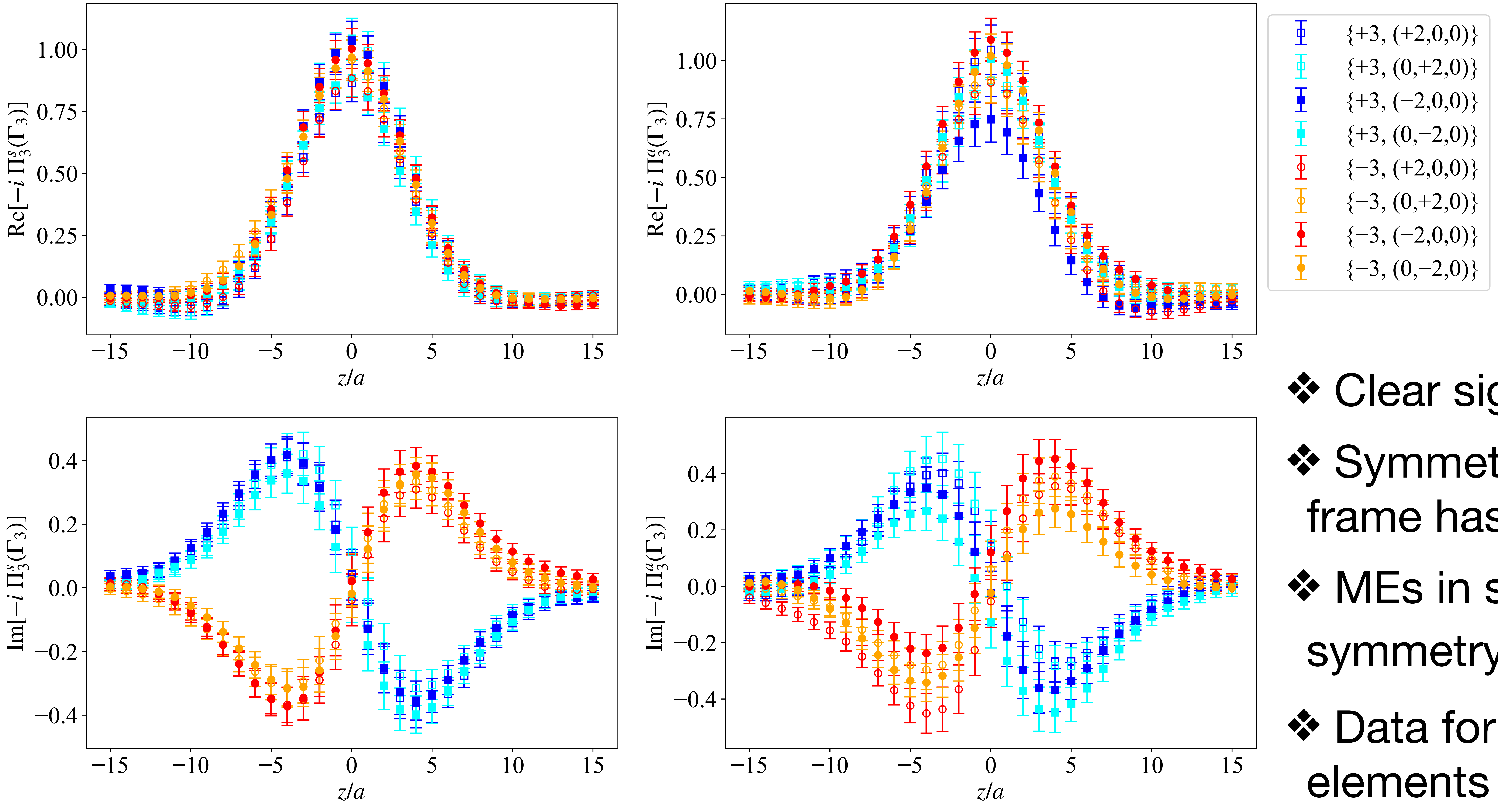
❖ Strategy: decomposition of amplitudes for each kinematic setup $(\pm P_3, \pm\vec{\Delta}, \pm z)$

❖ Exploitation of $\tilde{A}_i$ symmetry properties with respect to $(\pm P_3, \pm\vec{\Delta}, \pm z)$

Matrix Elements: $\Pi_3^{s/a}(\Gamma_3)$



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- ❖ Clear signal in both frames
- ❖ Symmetric frame and asymmetric frame has similar magnitude
- ❖ MEs in symmetric frame have definite symmetry properties in $\pm z, \pm P_3$
- ❖ Data for asymmetric frame matrix elements show small asymmetries

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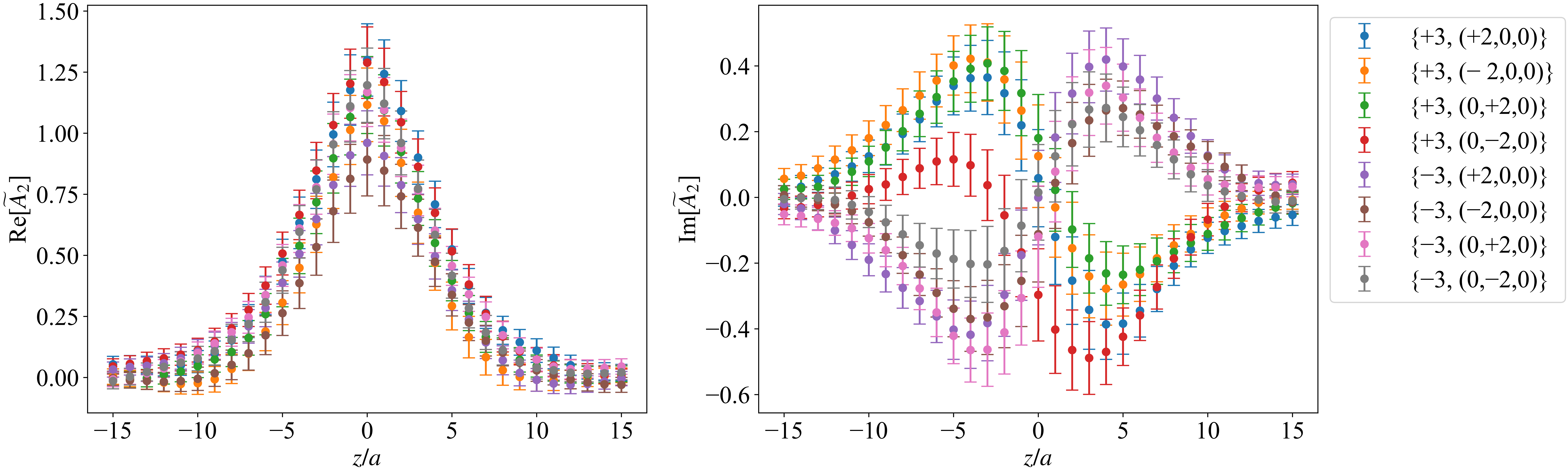

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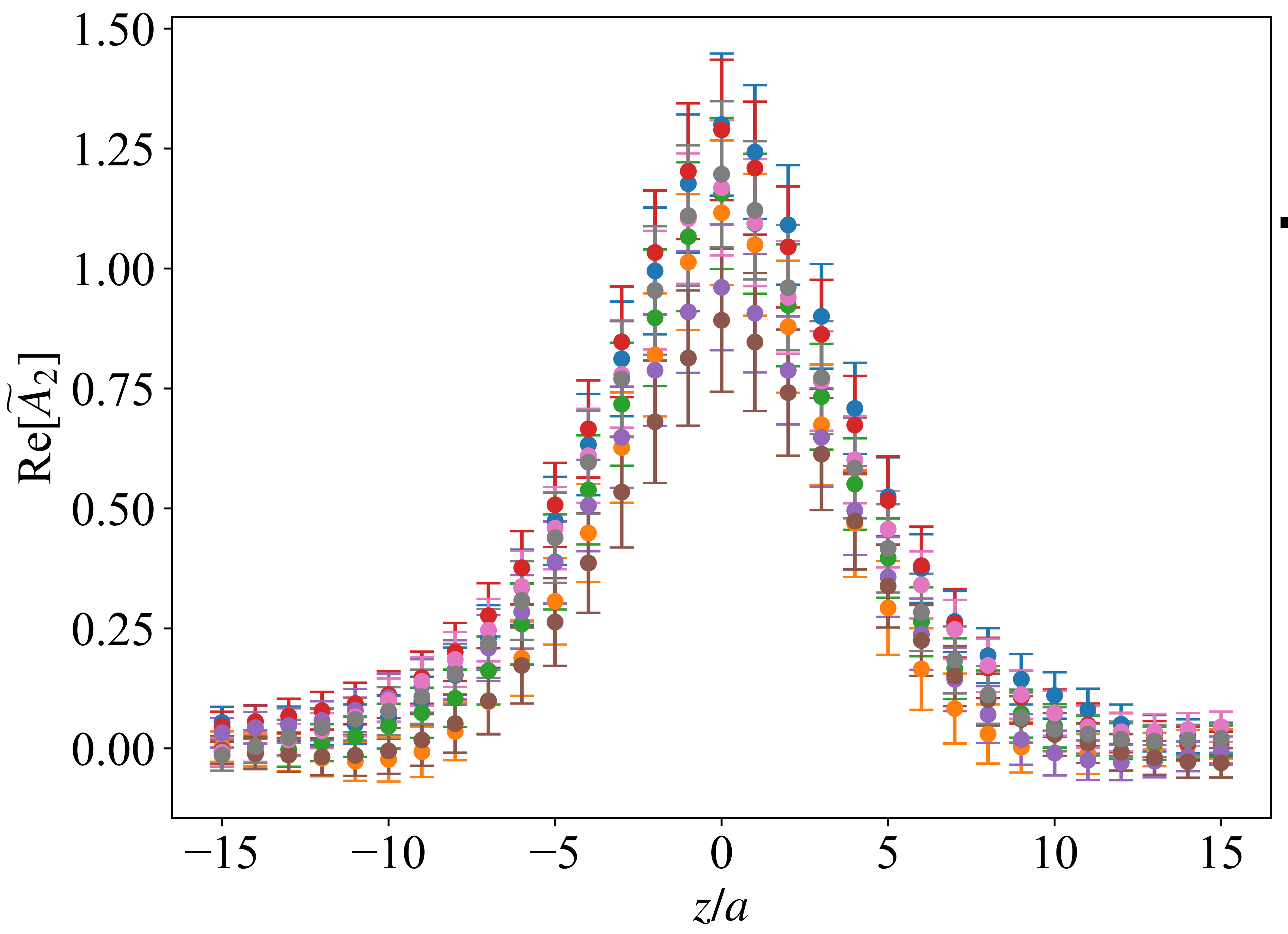

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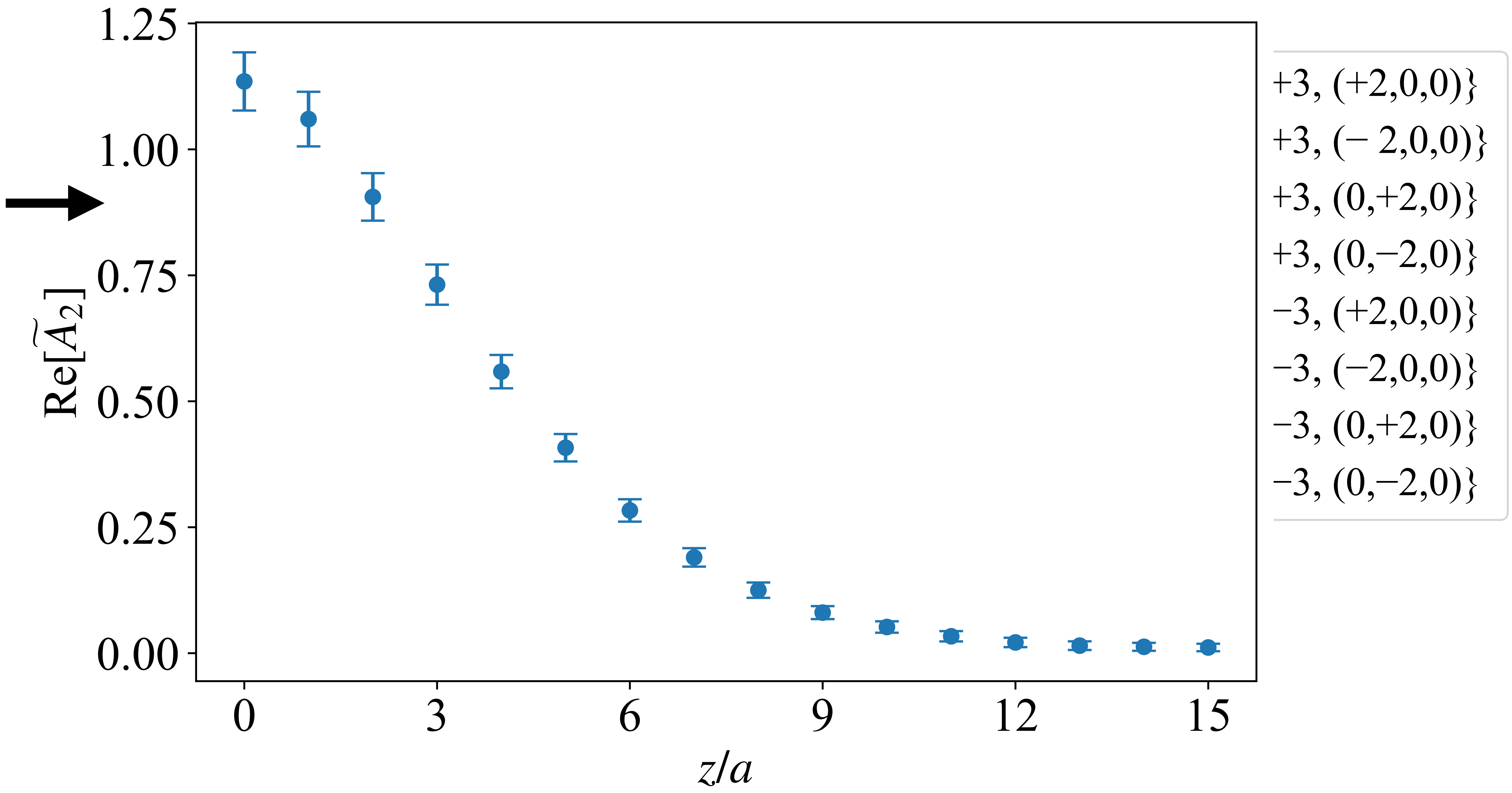

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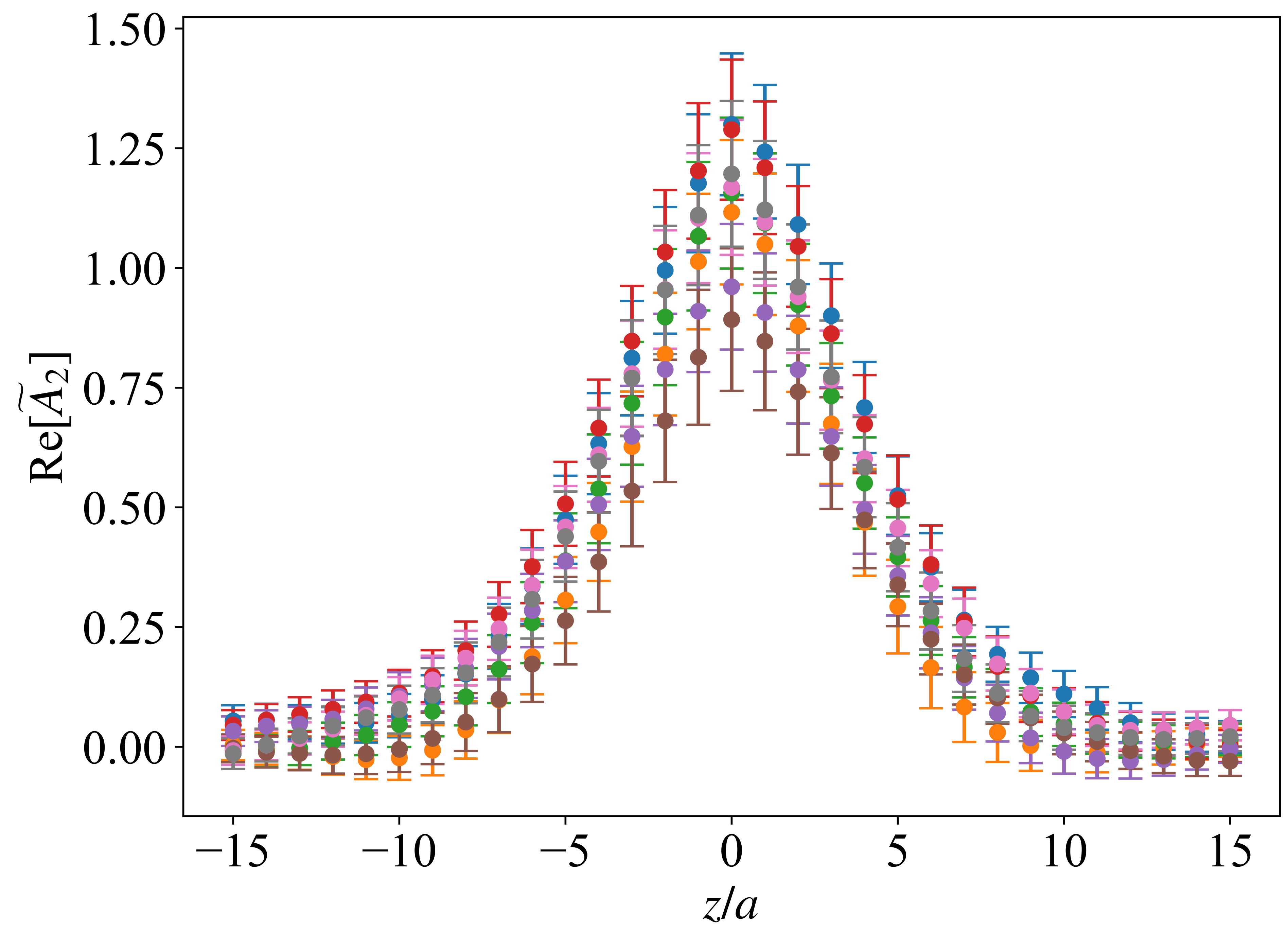

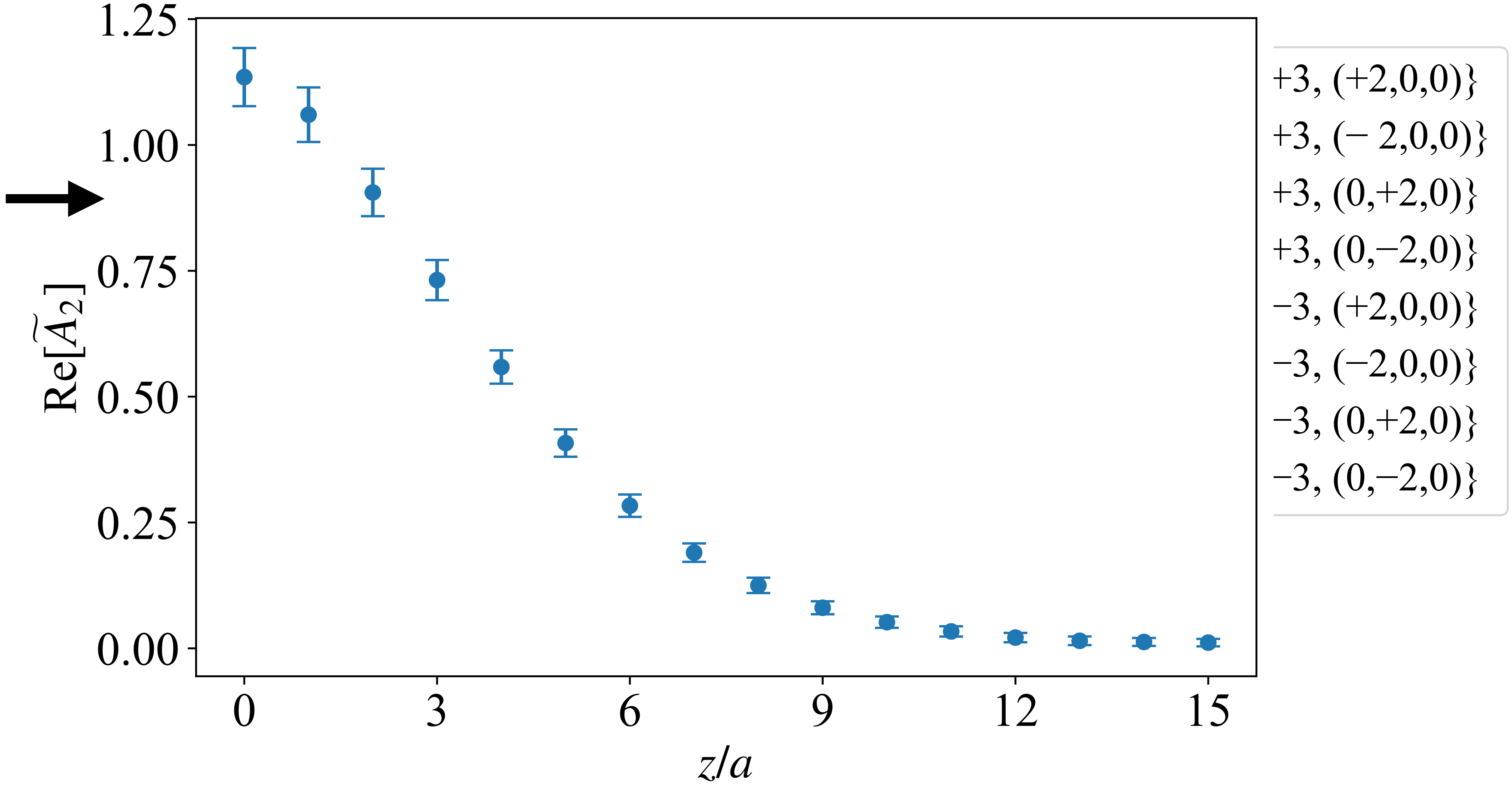

❖ Data can be combined according to symmetry properties

$$-\tilde{A}_i^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = \tilde{A}_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad i = 1,3,6$$

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Amplitudes

$|P_3| = 1.25\,\text{GeV}$
$-t_s = 0.69\,\text{GeV}^2$
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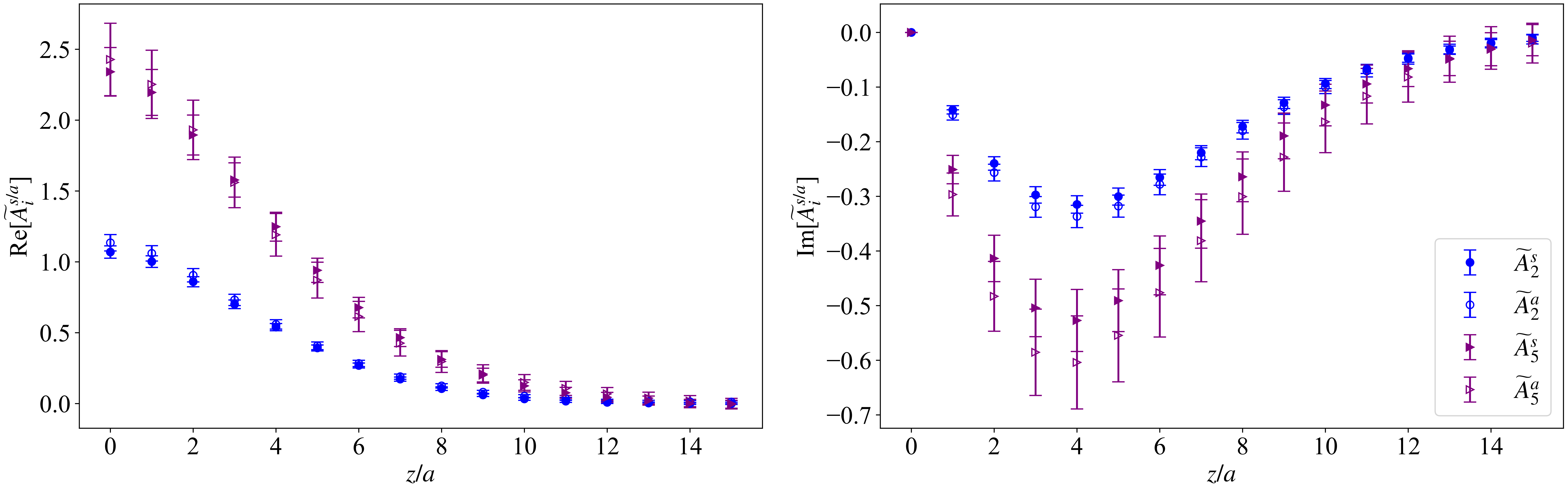
❖ Frame comparison for A_2 and A_5



❖ Theoretical expectation: amplitudes are Lorentz invariant for same $-t$ value

❖ We keep $P_3, \overrightarrow{\Delta}$ fixed in both frames $\Rightarrow \; -t_s = 0.69\,\text{GeV}^2, -t_a = 0.65\,\text{GeV}^2$

❖ Slight deviance due to $-t_s \approx -t_a$, ($\sim 5\,\%$) but close enough for a comparison

❖ Remaining amplitudes are either:

- very small in magnitude ($\tilde{A}_1, \tilde{A}_6, \tilde{A}_7$)
- theoretically zero at zero skewness ($\tilde{A}_3, \tilde{A}_4, \tilde{A}_8$)

Quasi-GPDs

❖ Recall that at zero skewness

$$F^{[\gamma^3\gamma_5]}\left(x,\Delta;P^3\right)=\frac{1}{2P^0}\bar{u}(p_f,\lambda')\left[\gamma^3\gamma_5\widetilde{\mathscr{H}}\left(x,\xi,t;P^3\right)+\frac{\Delta^3\gamma_5}{2m}\widetilde{\mathscr{E}}\left(x,\xi,t;P^3\right)\right]u(p_i,\lambda)$$

❖ Our quasi-GPDs can be related to the LI amplitudes

$$(\xi=0)\qquad \begin{aligned}\tilde{\mathscr{H}}_3(\tilde{A}_i;z)&=\tilde{A}_2+P_3z\tilde{A}_6-m^2z^2\tilde{A}_7 &&\textbf{Standard}\\ \tilde{\mathscr{H}}(\tilde{A}_i;z)&=\tilde{A}_2+P_3z\tilde{A}_6 &&\textbf{Lorentz Invariant}\end{aligned}$$

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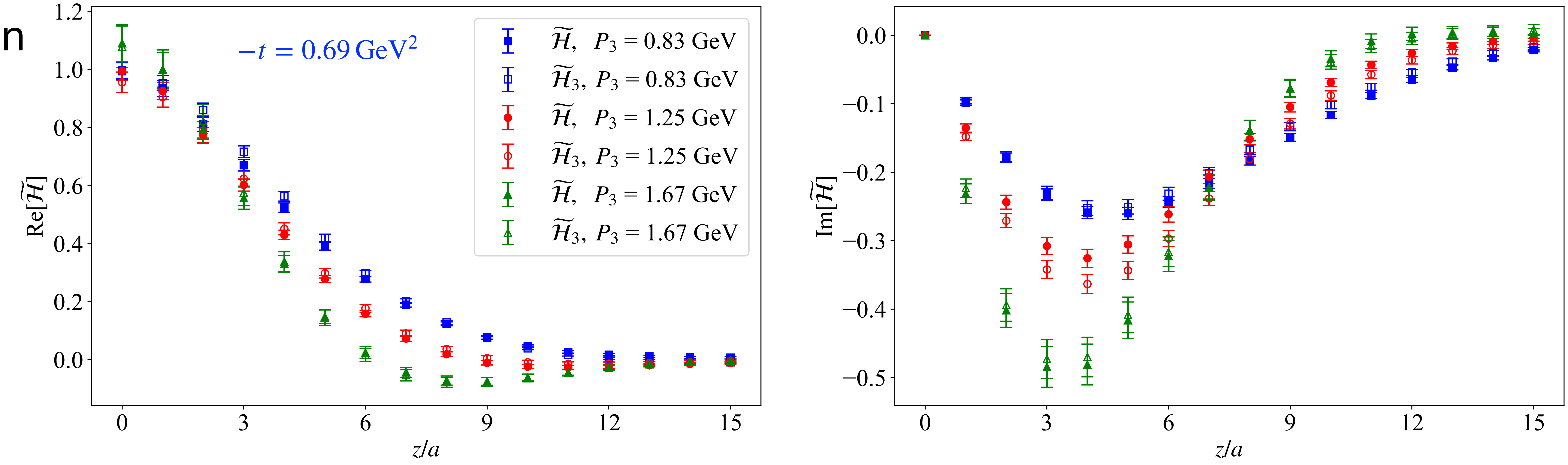
❖ Definition comparison

❖ P_3 dependence



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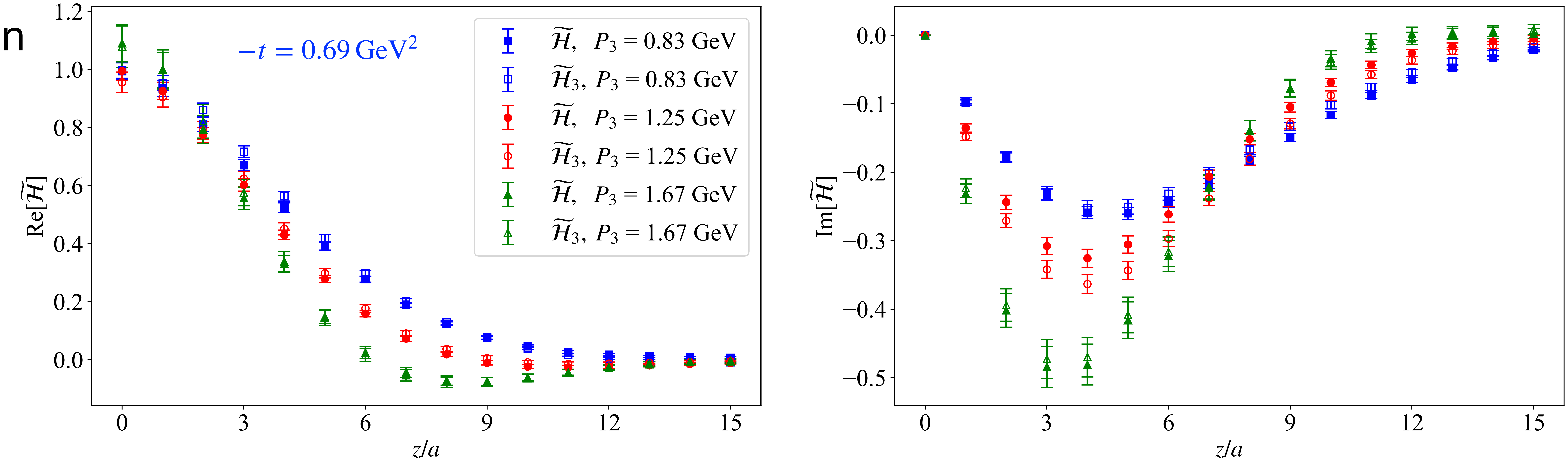


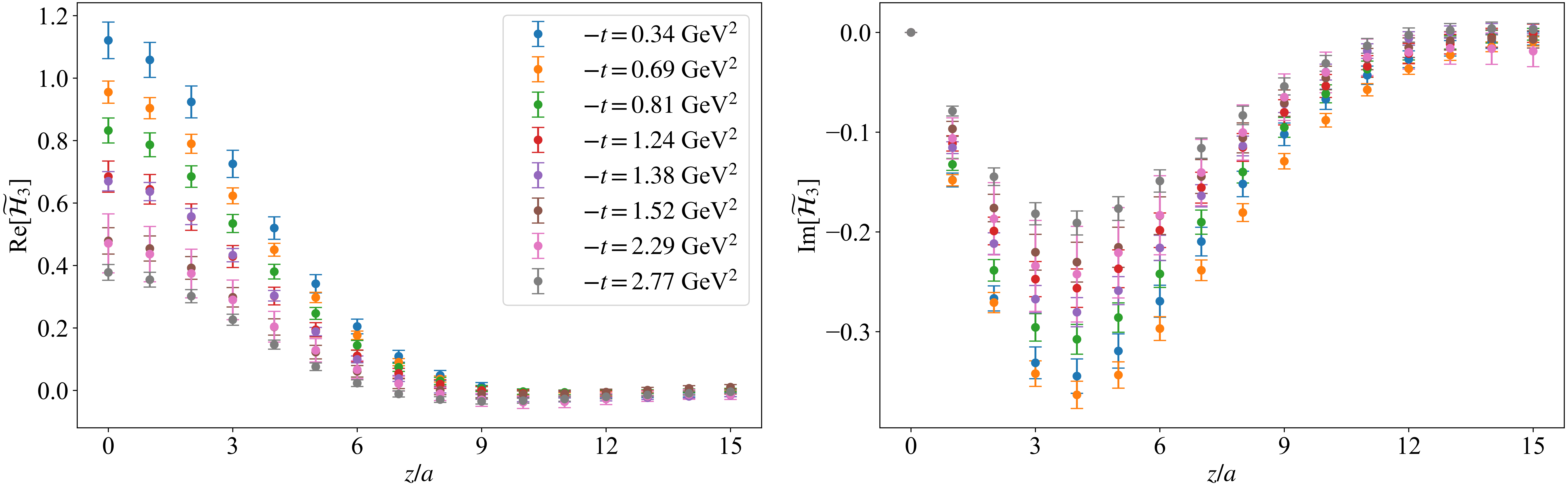
❖ Two definitions for quasi-H lead to compatible results (small difference in Im part at $P_3 = 1.25\,\mathrm{GeV}$)

❖ Imaginary part enhances with P_3 increase

❖ Real part decays faster to zero for the highest P_3 value

Quasi-GPDs

❖ Momentum transfer dependence at fixed $|P_3| = 1.25\,\mathrm{GeV}$



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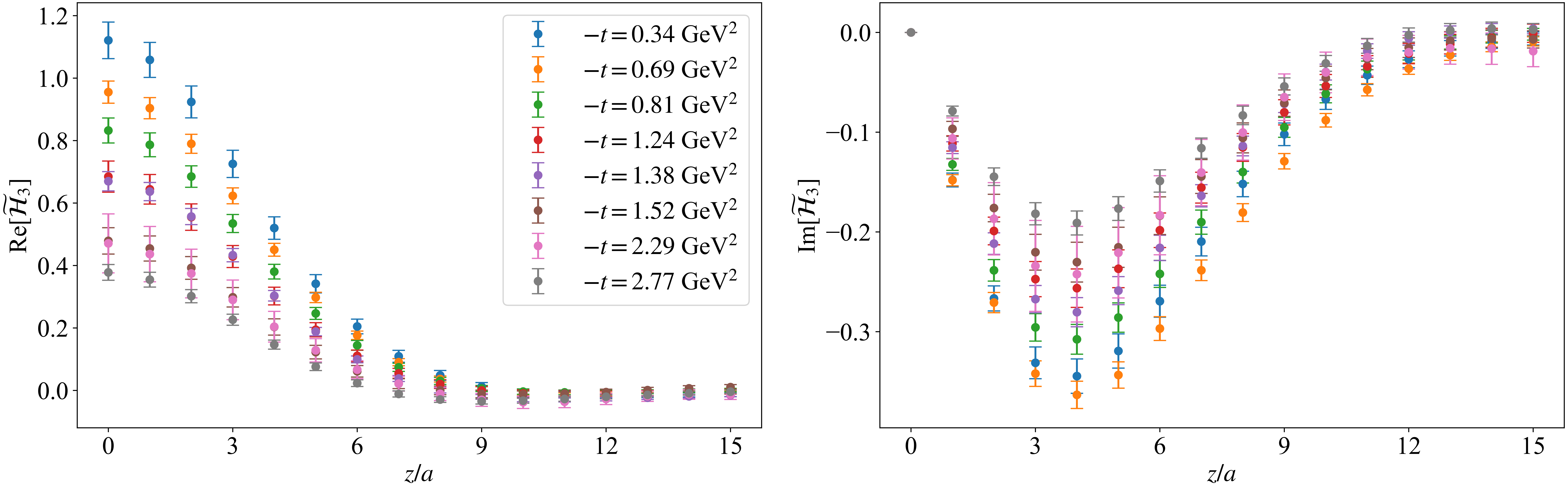

❖ Decreased magnitude as $-t$ increases

❖ Difference in magnitude between $-t$ points due to $\tilde{\mathscr{H}}_3$ depending on $\tilde{A}_7$

From Position to Momentum

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❖ Use Backus-Gilbert approach: [Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

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  - Model-independent
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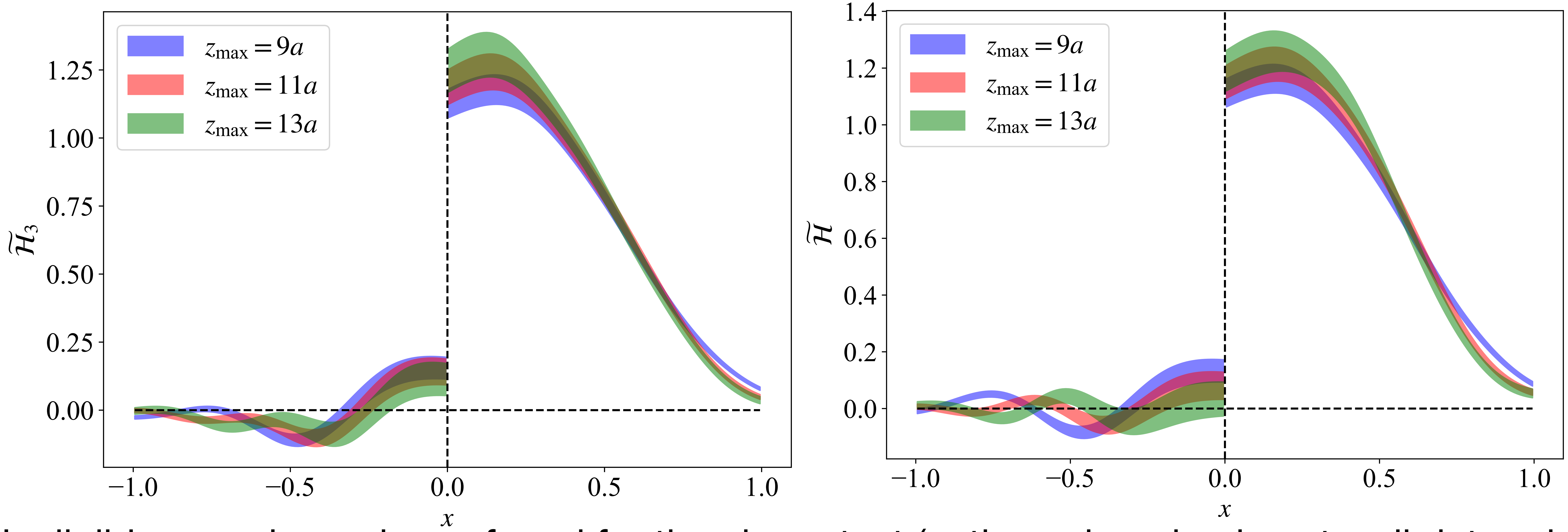
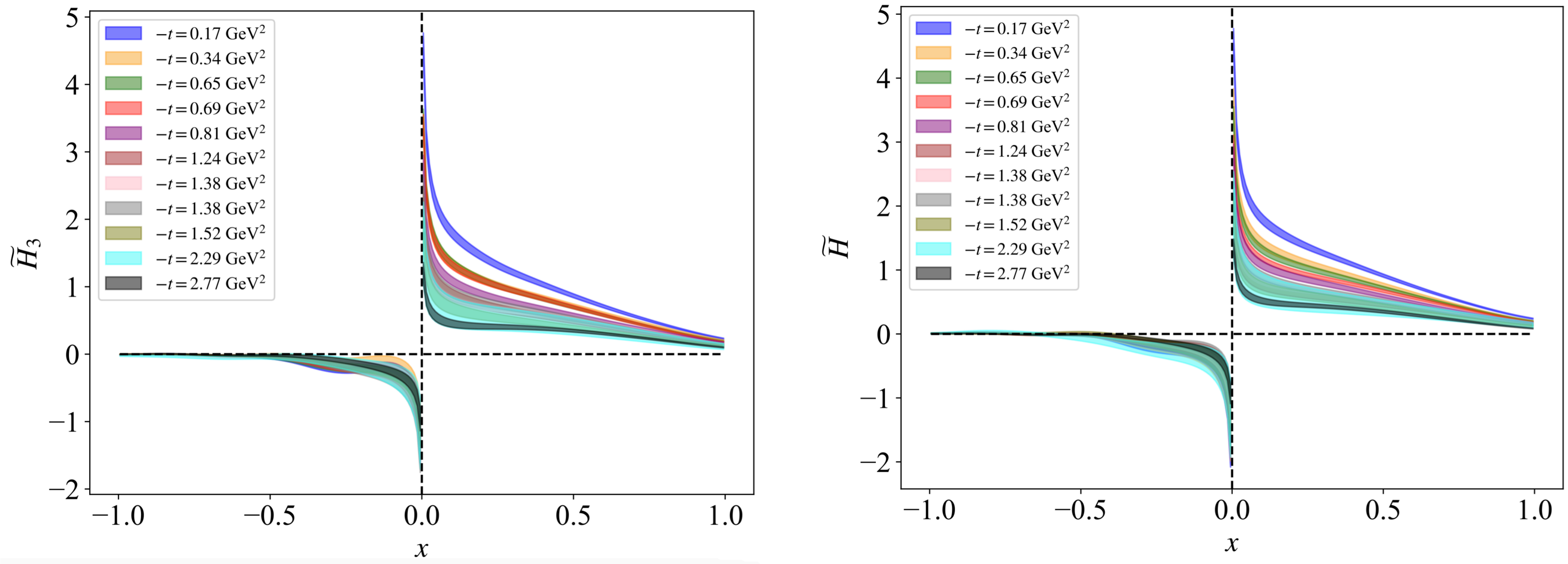
- Test of $z_{\rm max}$ dependence in BG reconstruction for $|P_3| = 1.25$ GeV, $-t = 0.65$ GeV^2:



- Negligible $z_{\rm max}$ dependence found for the above test (anti-quark region is not well determined)
- Statistical errors increase for larger $z_{\rm max}$
- Chosen value: $z_{\rm max} = 11a$

Light-Cone GPDs



❖ Similar statistical accuracy for both definitions

❖ As $-t$ increases, the magnitude of $H-$GPD becomes smaller

❖ Smooth dependence in $-t$

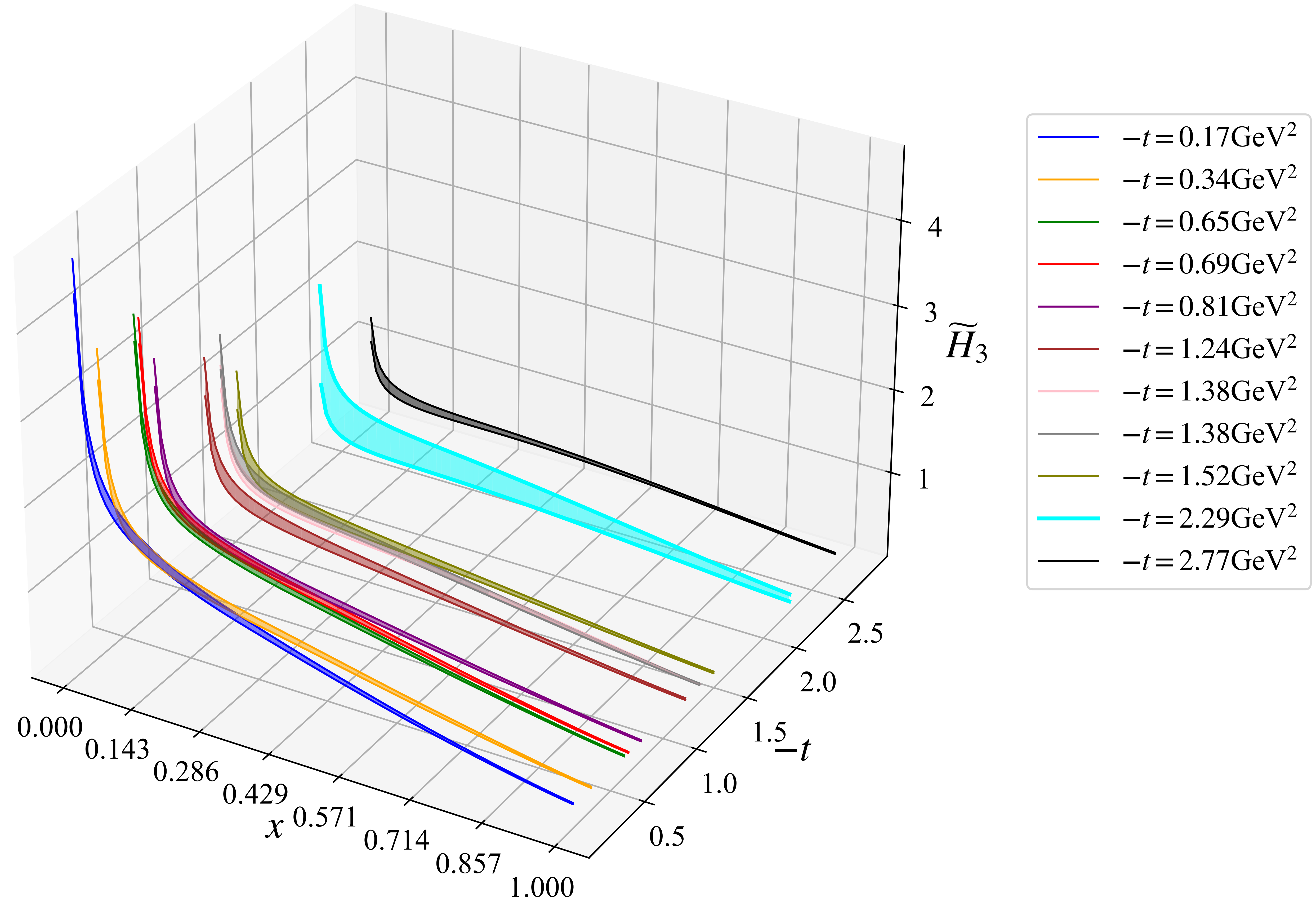
❖At $-t > 1.5\ \mathrm{GeV}^2$, the $H-$GPD are compatible within errors

Light-Cone GPDs

❖ $H-$GPD: $-t$ and x dependence

❖ Good signal for all values of $-t$

❖ Large values of $-t$ not reliably extracted due to higher-twist effects; obtained at no extra computational cost.



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❖ Implementation of asymmetric frame allows us to obtain results in a computationally less expensive way

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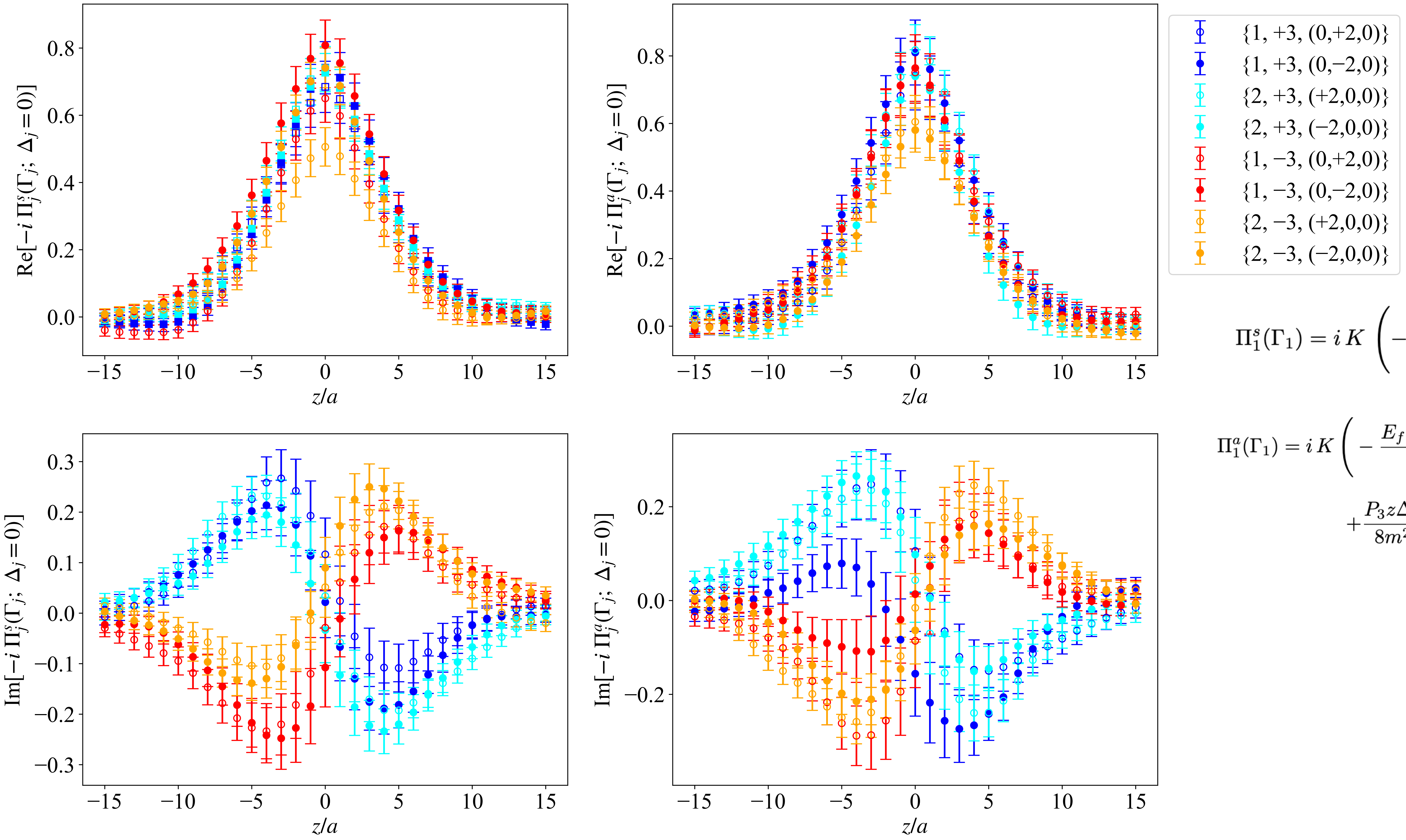
Thank You!!!

Acknowledgements

- ❖ U.S. Department of Energy, Office of Nuclear Physics, Early Career Award under Grant No. DE-SC0020405
- ❖ PLGrid Infrastructure by Prometheus in Cracow
- ❖ Poznan Supercomputing and Networking Center by Eagle
- ❖ Interdisciplinary Centre for Mathematical and Computational Modeling of the Warsaw University by Okeanos
- ❖ Academic Computer Center in Gdańsk by Tryton

Backup slides

Matrix Elements: $\Pi_j^{s/a}(\Gamma_j)$



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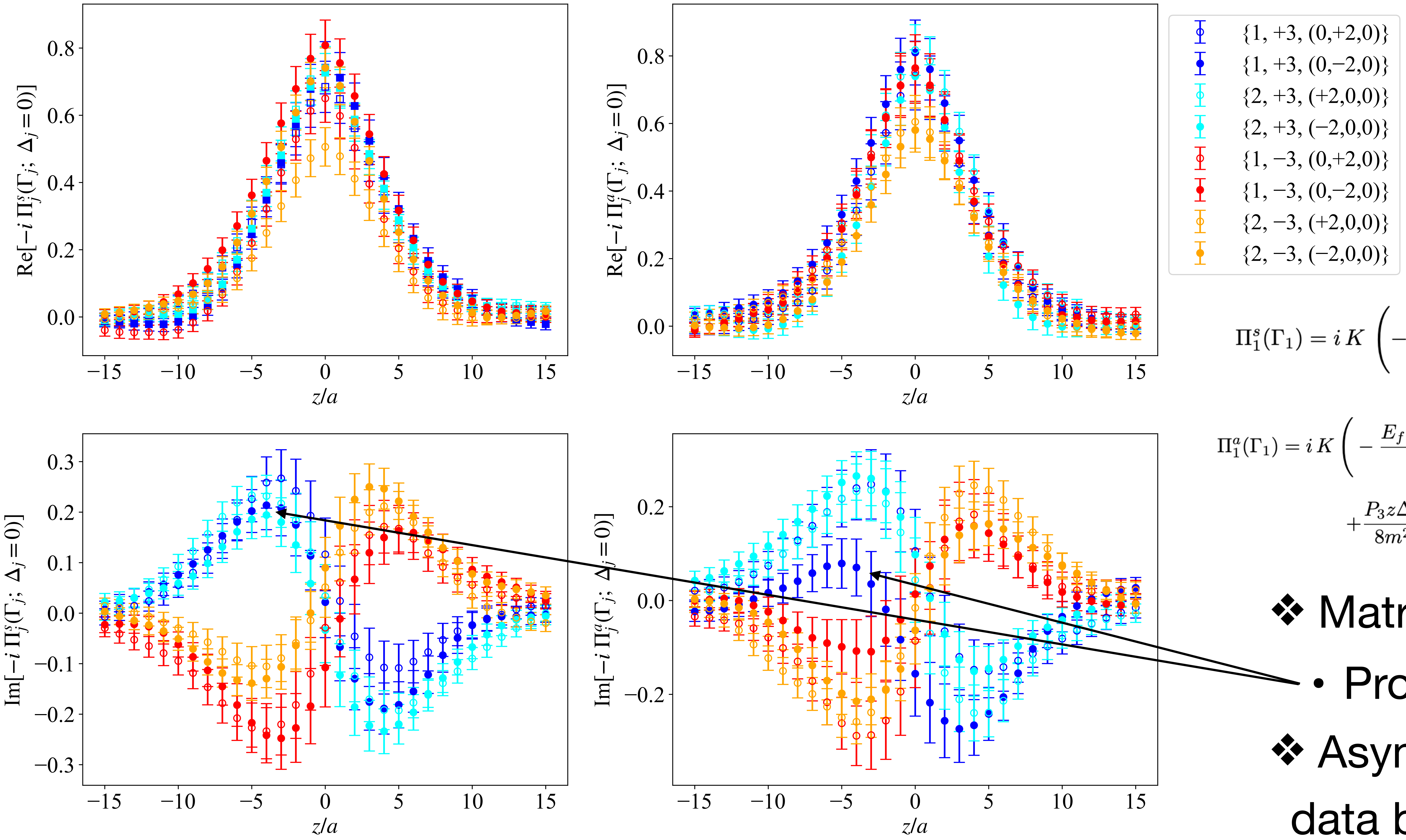

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- ❖ Matrix elements are frame dependent
  - Prominent in imaginary part
- ❖ Asymmetric frame: larger deviation of data between $\pm z$, $\pm P_3$, $\pm \vec{\Delta}$ cases
- ❖ $\Pi_j(\Gamma_j)$ more noisy than $\Pi_3(\Gamma_3)$

Amplitude Decomposition

- ❖ Matrix elements disentangle in 8 LI amplitudes $\widetilde{A}_i$
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$$\widetilde{A}_5 = -\frac{2\,i\,Em^2\left(E^2+Em-P_3^2\right)}{\Delta^2(E+m)\,(E^2-P_3^2)}\Pi_2^s(\Gamma_2) + \frac{Em^2P_3}{\Delta(E+m)\,(E^2-P_3^2)}\Pi_2^s(\Gamma_0) + \frac{2\,i\,Em}{\Delta^2}\Pi_1^s(\Gamma_1)$$

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Symmetric Frame Decomposition

$$\widetilde{A}_2 = \frac{E P_3 \Delta}{2(E+m)\left(E^2-P_3^2\right)} \Pi_2^s(\Gamma_0) + \frac{i\,E\left(P_3^2 - E(E+m)\right)}{(E+m)(E-P_3)(E+P_3)} \Pi_2^s(\Gamma_2)\,,$$

$$\widetilde{A}_5 = -\frac{2\,i\,Em^2\left(E^2+Em-P_3^2\right)}{\Delta^2(E+m)\left(E^2-P_3^2\right)} \Pi_2^s(\Gamma_2) + \frac{Em^2 P_3}{\Delta(E+m)\left(E^2-P_3^2\right)} \Pi_2^s(\Gamma_0) + \frac{2\,i\,Em}{\Delta^2} \Pi_1^s(\Gamma_1)$$

Asymmetric Frame Decomposition

$$\widetilde{A}_2 = \frac{2P_3\Delta m^2}{(E_f+m)(E_i+m)\left(2m^2+E_f(E_i-E_f)\right)} \frac{\Pi_2^a(\Gamma_0)}{K} + \frac{2\,i\,(E_f-E_i-2m)m^2}{(E_i+m)\left(2m^2+E_f(E_i-E_f)\right)} \frac{\Pi_2^a(\Gamma_2)}{K}$$

$$\begin{aligned}\widetilde{A}_5 = &-\frac{2(E_f+E_i)P_3 m^4}{E_f(E_f+m)(E_i+m)\left(E_f^2 - E_iE_f - 2m^2\right)\Delta} \frac{\Pi_2^a(\Gamma_0)}{K} + \frac{(E_f+E_i)m^3}{E_f^2(E_i+m)\Delta} \frac{\Pi_0^a(\Gamma_1)}{K} \\ &+ \frac{2\,i\,(E_f-E_i-2m)m^4}{E_f(E_f-E_i)(E_i+m)\left(E_f^2-E_iE_f-2m^2\right)} \frac{\Pi_2^a(\Gamma_2)}{K} + \frac{P_3 m^3}{E_f^2(E_f+m)(E_i+m)} \frac{\Pi_0^a(\Gamma_3)}{K} \\ &- \frac{i\,(E_f+E_i)m^3}{E_f^2(E_f-E_i)(E_i+m)} \frac{\Pi_1^a(\Gamma_1)}{K} + \frac{i\,(E_f+E_i)P_3 m^3}{E_f^2(E_f+m)(E_i+m)\Delta} \frac{\Pi_1^a(\Gamma_3)}{K}\,,\end{aligned}$$

Amplitude Decomposition

- ❖ Matrix elements disentangle in 8 LI amplitudes $\widetilde{A}_i$
- ❖ For each setup of $\pm z, \pm P_3, \pm \vec{\Delta}$, we disentangle the amplitudes
  - For example, at $\vec{\Delta} = (\Delta, 0, 0)$

Symmetric Frame Decomposition

$$\widetilde{A}_2 = \frac{E P_3 \Delta}{2(E+m)\,(E^2 - P_3^2)} \Pi_2^s(\Gamma_0) + \frac{i\,E\left(P_3^2 - E(E+m)\right)}{(E+m)(E-P_3)(E+P_3)} \Pi_2^s(\Gamma_2)\,,$$

$$\widetilde{A}_5 = -\frac{2\,i\,E m^2\left(E^2 + Em - P_3^2\right)}{\Delta^2(E+m)\,(E^2-P_3^2)} \Pi_2^s(\Gamma_2) + \frac{E m^2 P_3}{\Delta(E+m)\,(E^2 - P_3^2)} \Pi_2^s(\Gamma_0) + \frac{2\,i\,E m}{\Delta^2} \Pi_1^s(\Gamma_1)$$

Asymmetric Frame Decomposition

$$\widetilde{A}_2 = \frac{2P_3\Delta m^2}{(E_f+m)(E_i+m)\,(2m^2 + E_f(E_i - E_f))} \frac{\Pi_2^a(\Gamma_0)}{K} + \frac{2\,i\,(E_f - E_i - 2m)m^2}{(E_i+m)\,(2m^2 + E_f(E_i - E_f))} \frac{\Pi_2^a(\Gamma_2)}{K}$$

$$\begin{aligned}\widetilde{A}_5 = &-\frac{2(E_f+E_i)P_3 m^4}{E_f(E_f+m)(E_i+m)\left(E_f^2 - E_i E_f - 2m^2\right)\Delta} \frac{\Pi_2^a(\Gamma_0)}{K} + \frac{(E_f+E_i)m^3}{E_f^2(E_i+m)\Delta} \frac{\Pi_0^a(\Gamma_1)}{K} \\ &+ \frac{2\,i\,(E_f - E_i - 2m)m^4}{E_f(E_f-E_i)(E_i+m)\left(E_f^2 - E_i E_f - 2m^2\right)} \frac{\Pi_2^a(\Gamma_2)}{K} + \frac{P_3 m^3}{E_f^2(E_f+m)(E_i+m)} \frac{\Pi_0^a(\Gamma_3)}{K} \\ &- \frac{i\,(E_f+E_i)m^3}{E_f^2(E_f-E_i)(E_i+m)} \frac{\Pi_1^a(\Gamma_1)}{K} + \frac{i\,(E_f+E_i)P_3 m^3}{E_f^2(E_f+m)(E_i+m)\Delta} \frac{\Pi_1^a(\Gamma_3)}{K}\,,\end{aligned}$$

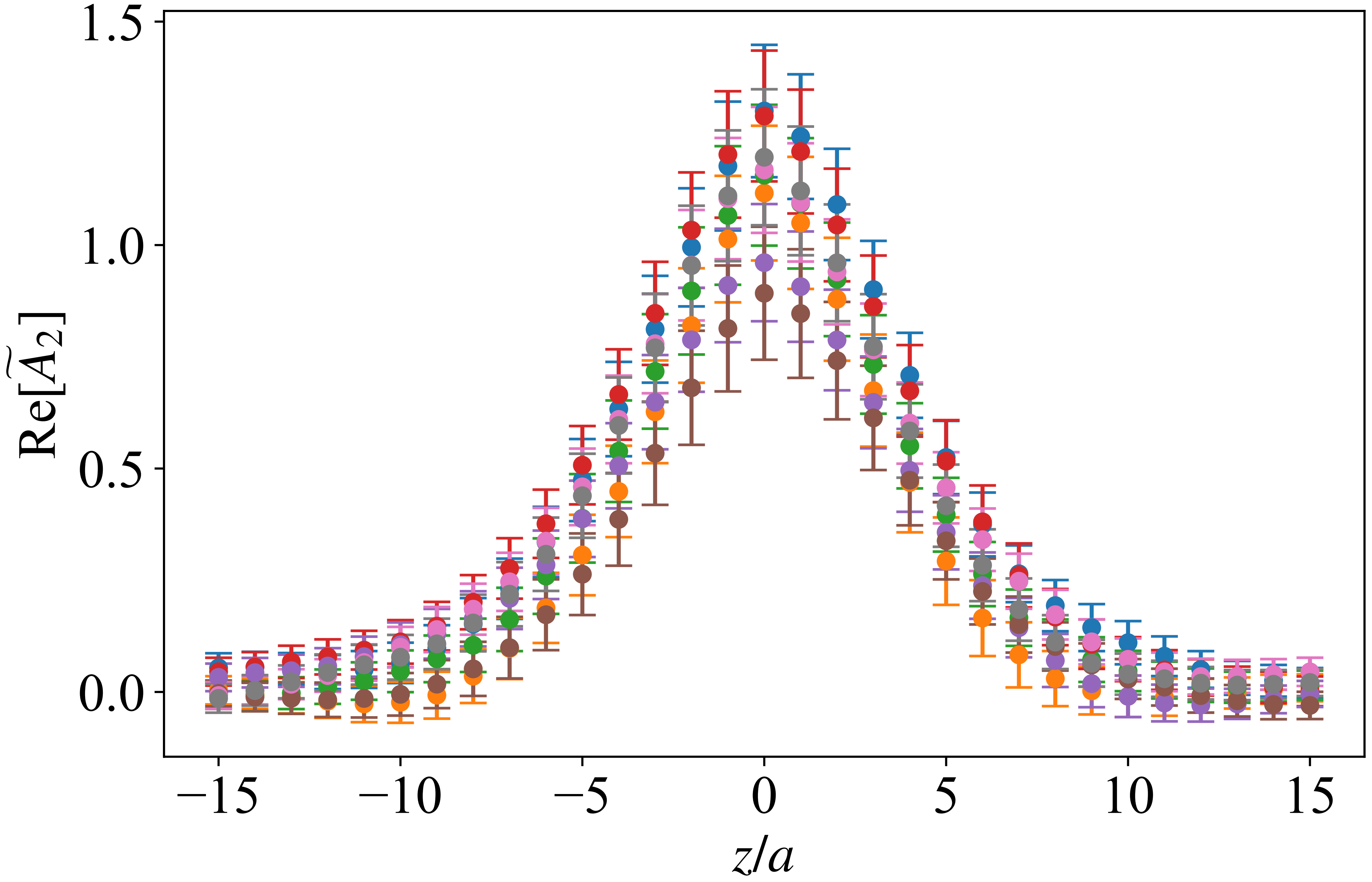
- ❖ Asymmetric frame: more matrix elements in each $\widetilde{A}_i$

Amplitudes

❖ Symmetry Properties

$$-\tilde{A}_i^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = \tilde{A}_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad i = 1,3,6$$

$$\tilde{A}_i^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = \tilde{A}_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad i = 2,4,5,7,8$$



Amplitudes

❖ Symmetry Properties

$$-\tilde{A}_i^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = \tilde{A}_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad i = 1,3,6$$

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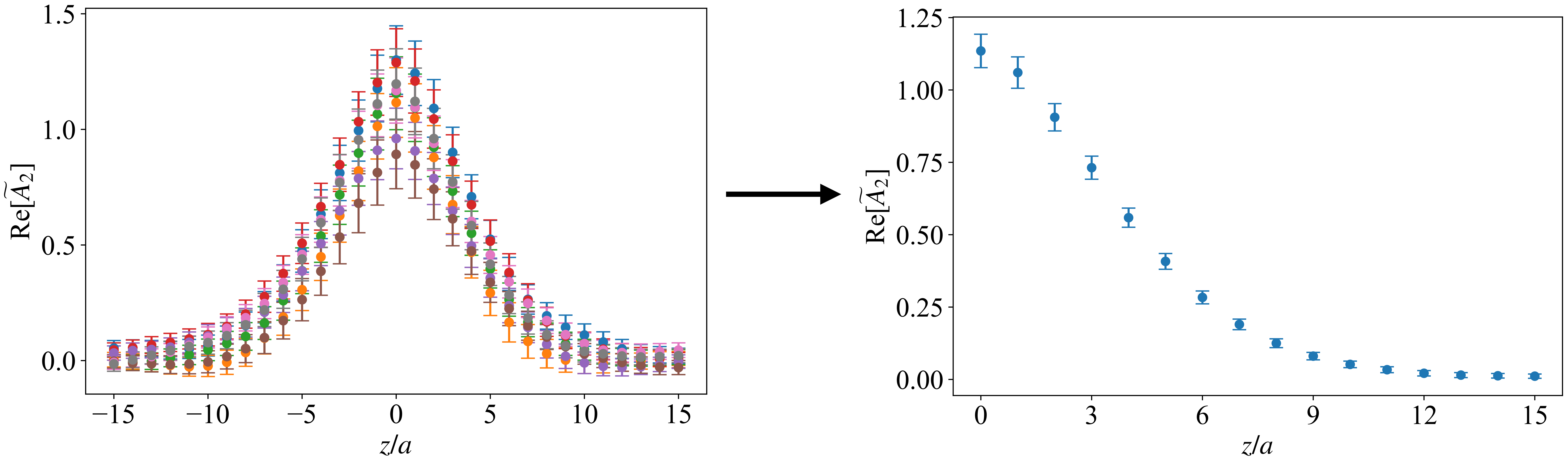
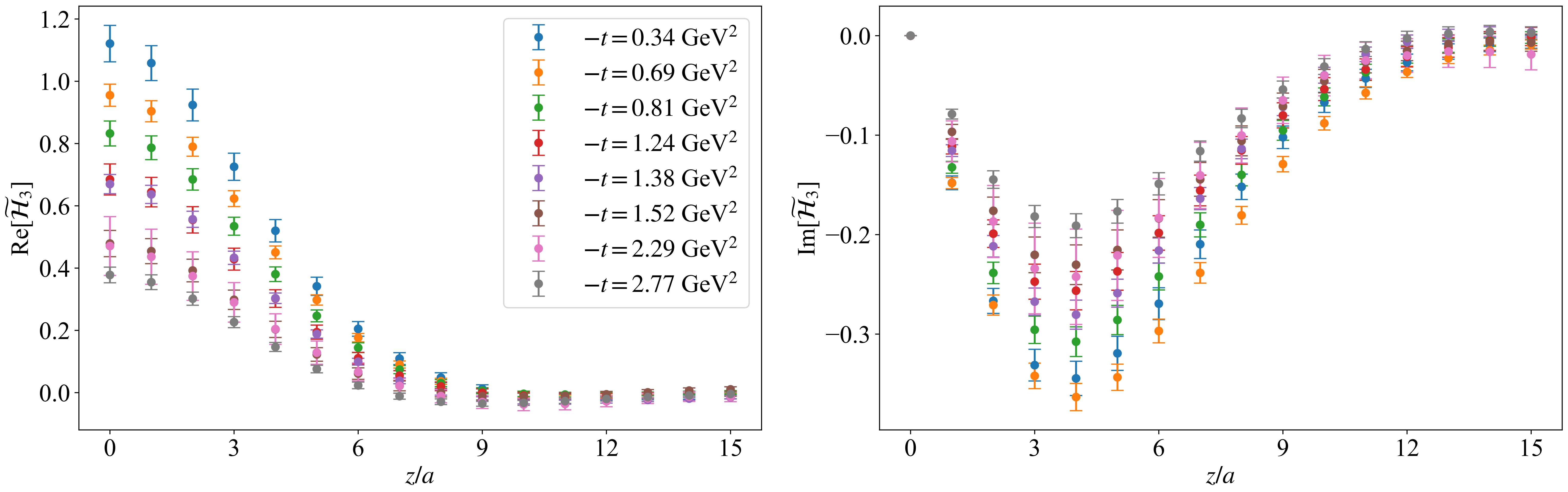


❖ We find that statistical errors reduce by $\sim 1/\sqrt{8}$ when the 8 kinematic cases are combined

Quasi-GPDs

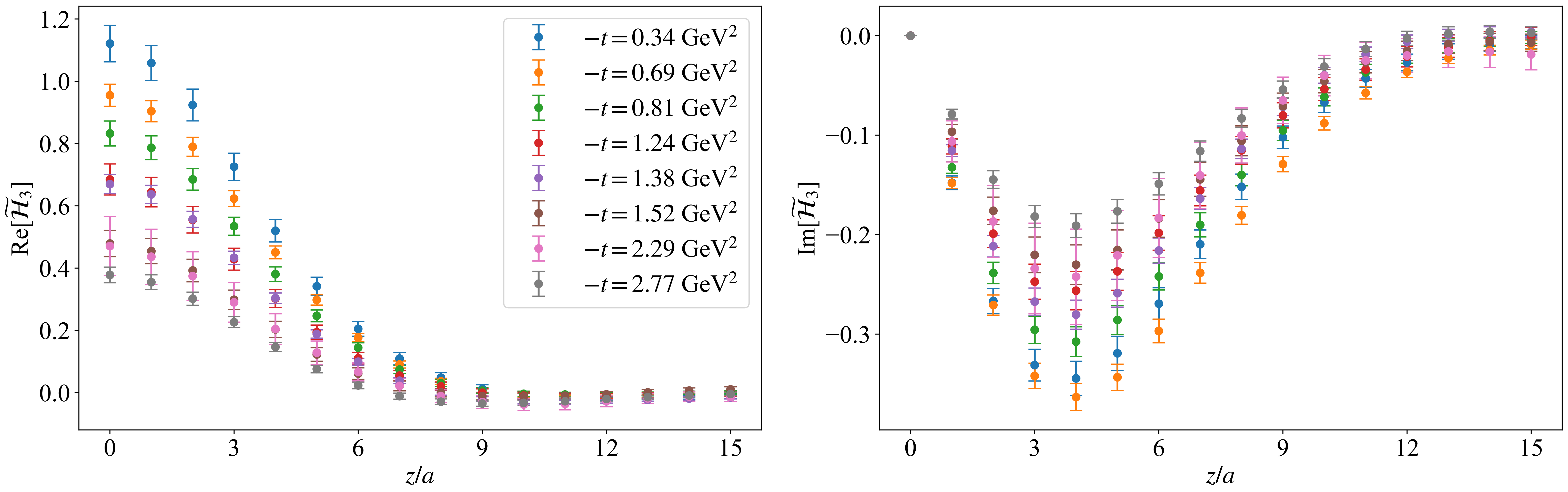
❖ Momentum transfer dependence at fixed $|P_3| = 1.25\,\mathrm{GeV}$



← $-t$ dependence for $\tilde{\mathscr{H}}_3$

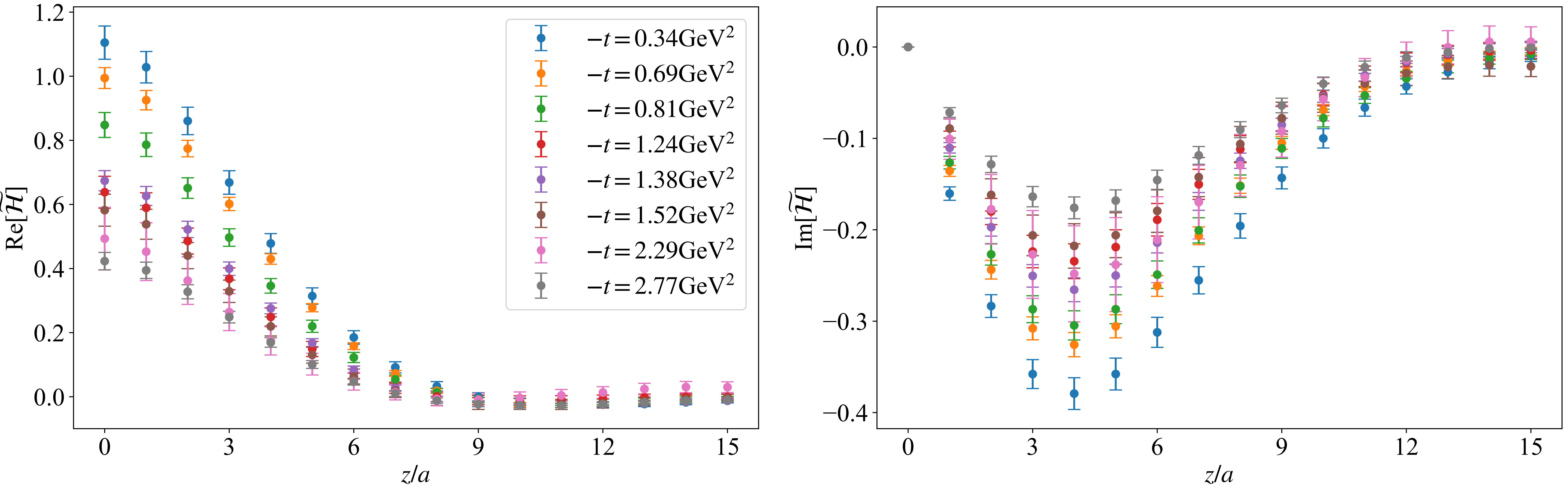
Quasi-GPDs

❖ Momentum transfer dependence at fixed $|P_3| = 1.25\,\mathrm{GeV}$



← $-t$ dependence for $\tilde{\mathscr{H}}_3$

$-t$ dependence for $\tilde{\mathscr{H}}$ →



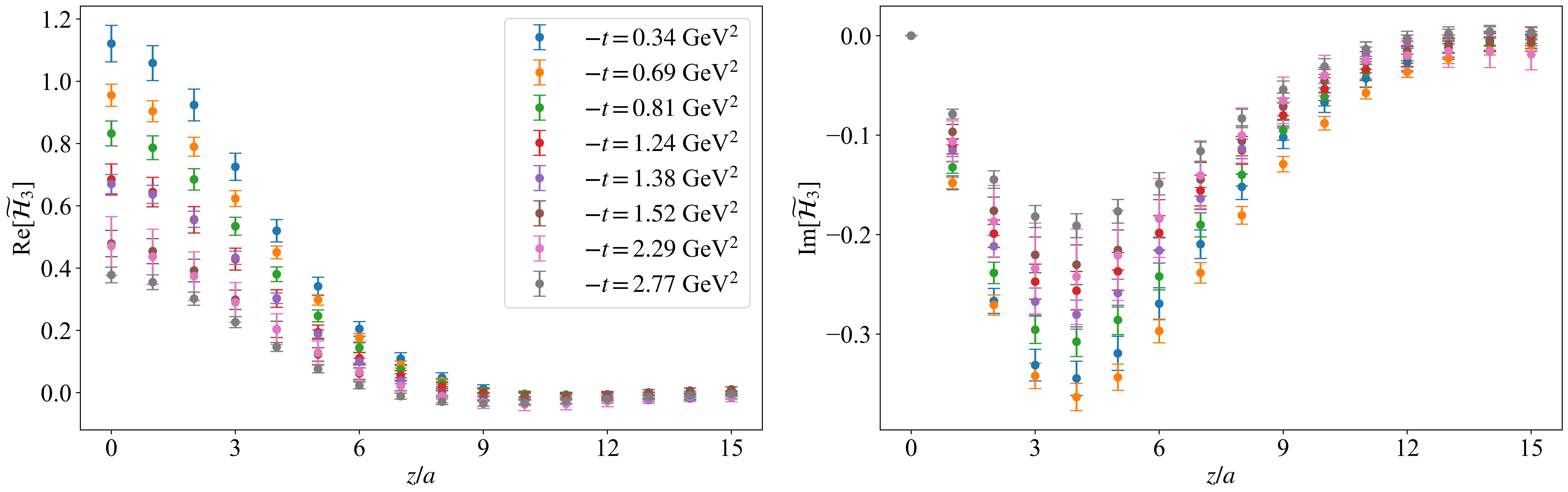
$(\xi = 0)$

$$\tilde{\mathscr{H}}_3(\tilde{A}_i; z) = \tilde{A}_2 + P_3 z \tilde{A}_6 - m^2 z^2 \tilde{A}_7$$

$$\tilde{\mathscr{H}}(\tilde{A}_i; z) = \tilde{A}_2 + P_3 z \tilde{A}_6$$

Quasi-GPDs

❖ Momentum transfer dependence at fixed $|P_3| = 1.25\,\mathrm{GeV}$

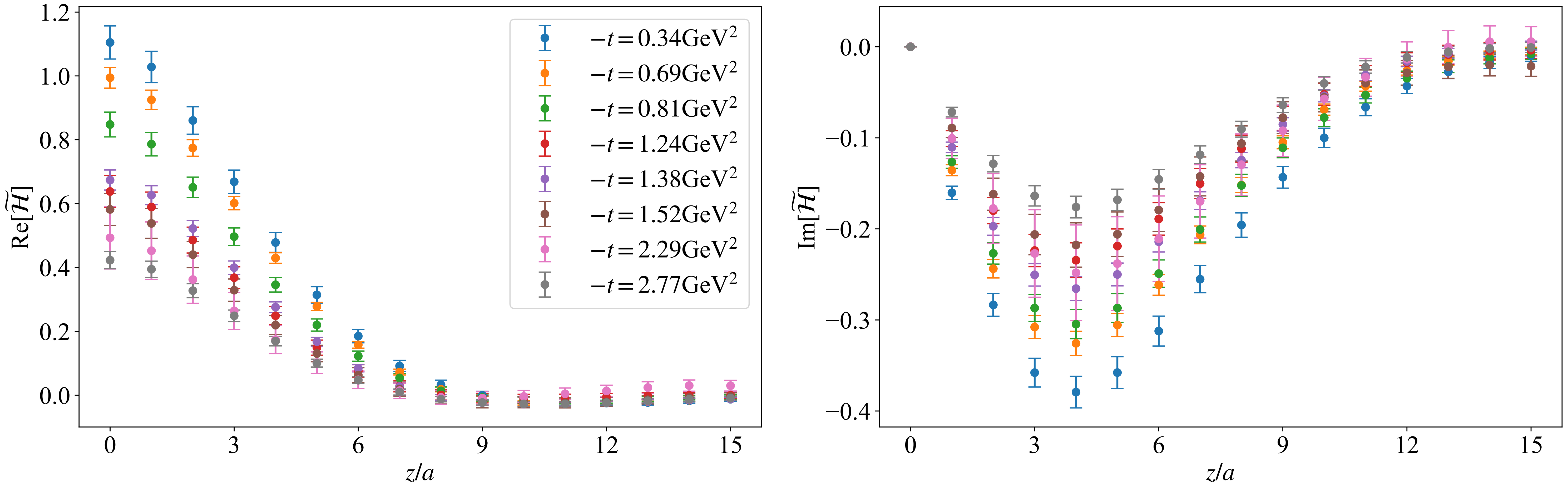


← $-t$ dependence for $\tilde{\mathscr{H}}_3$

$-t$ dependence for $\tilde{\mathscr{H}}$

$$(\xi = 0) \quad \tilde{\mathscr{H}}_3(\tilde{A}_i; z) = \tilde{A}_2 + P_3 z \tilde{A}_6 - m^2 z^2 \tilde{A}_7$$

$$\tilde{\mathscr{H}}(\tilde{A}_i; z) = \tilde{A}_2 + P_3 z \tilde{A}_6$$

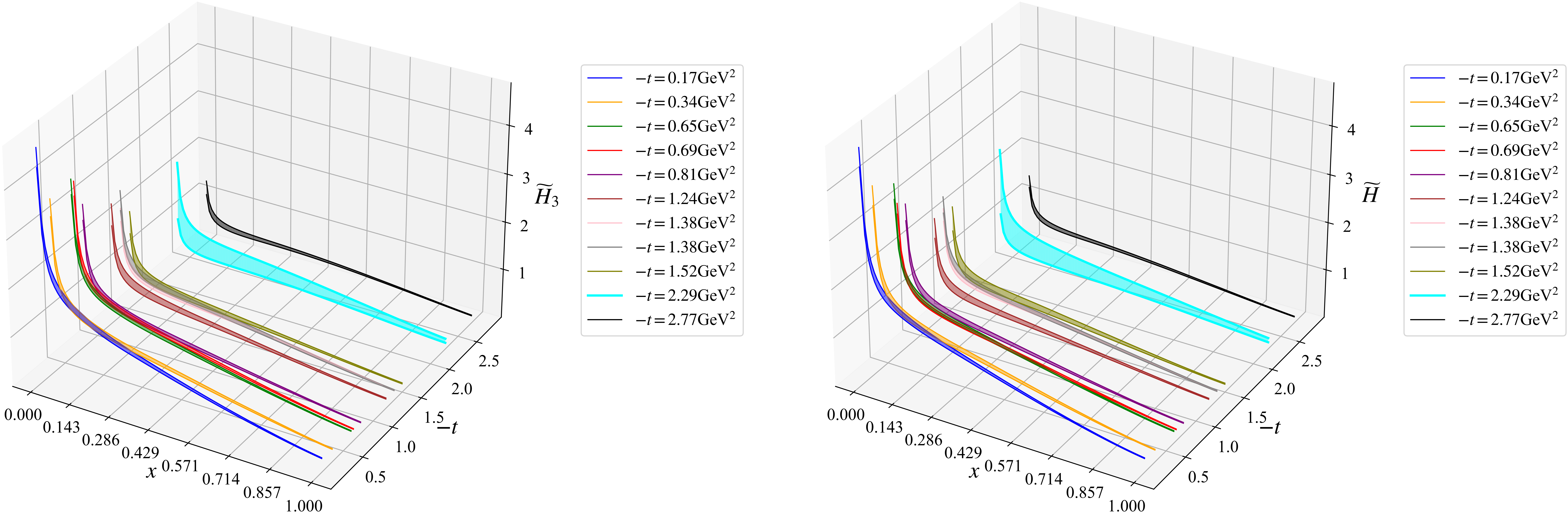


❖ Decreased magnitude as $-t$ increases

❖ Difference in magnitude between $-t$ points due to $\tilde{\mathscr{H}}_3$ depending on $\tilde{A}_7$

Light-Cone GPDs

❖ $H-$GPD: $-t$ and x dependence



❖ Good signal for all values of $-t$

❖ Large values of $-t$ not reliably extracted due to higher-twist effects; obtained at no extra computational cost.