Universität Bielefeld
BERGISCHE UNIVERSITÄT WUPPERTAL
HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

QCD EQUATION OF STATE IN THE PRESENCE OF MAGNETIC FIELDS AT LOW DENSITY

LATTICE 2023

FERMILAB, ILLINOIS, US

DEAN VALOIS

IN COLLABORATION WITH

S. BORSÁNYI, B. BRANDT, G. ENDRŐDI, J. GÜNTHER & R. KARA

OUTLINE

1. WHY $\vec{B}$ & μ ?

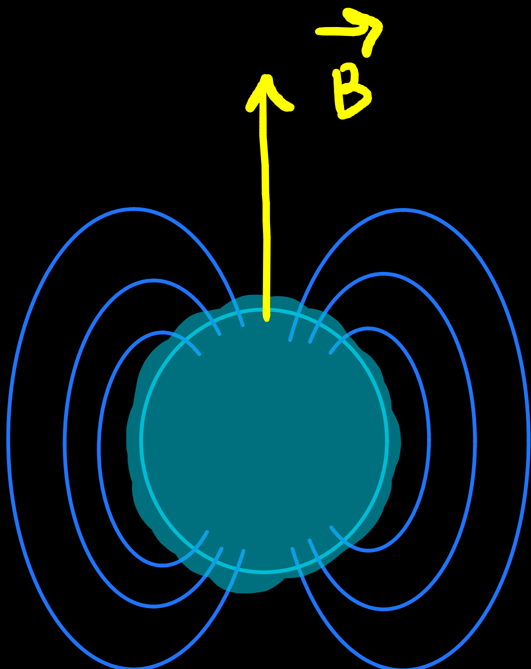
2. THEORETICAL SETUP

3. PRELIMINARY RESULTS

4. CONCLUSIONS

Motivation

MOTIVATION



NEUTRON STARS

$eB \sim 1\,\mathrm{MeV}^2$

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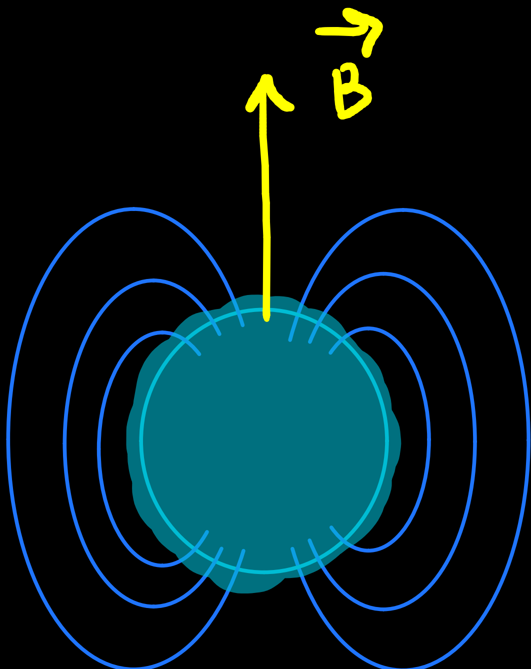

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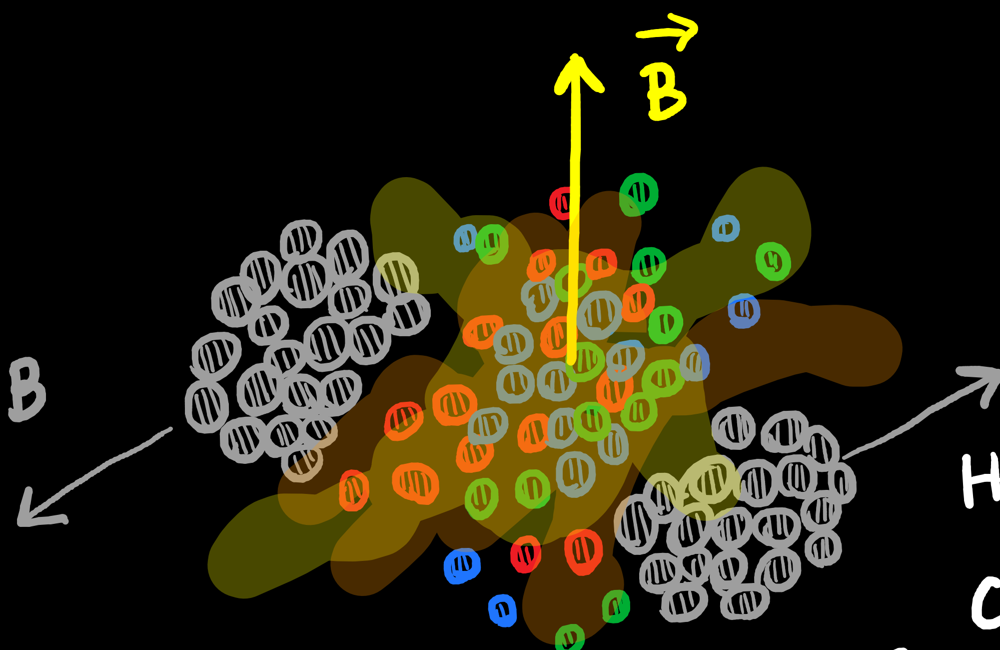

HEAVY-ION COLLISIONS

$0.01\,\mathrm{GeV}^2 \lesssim eB \lesssim 0.25\,\mathrm{GeV}^2$

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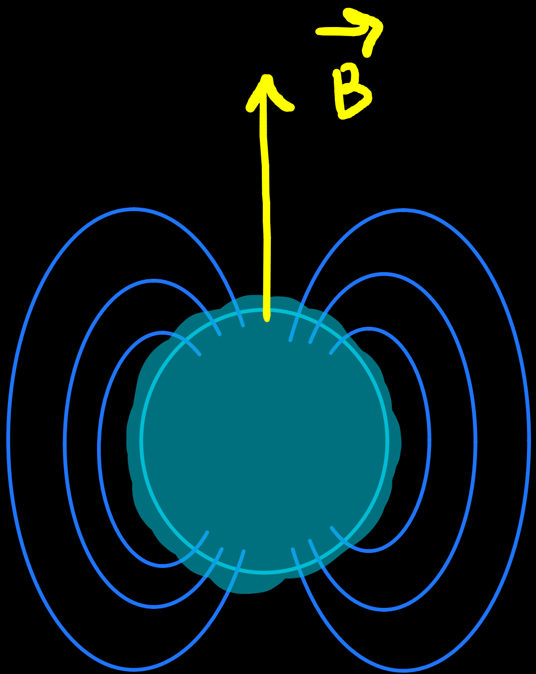

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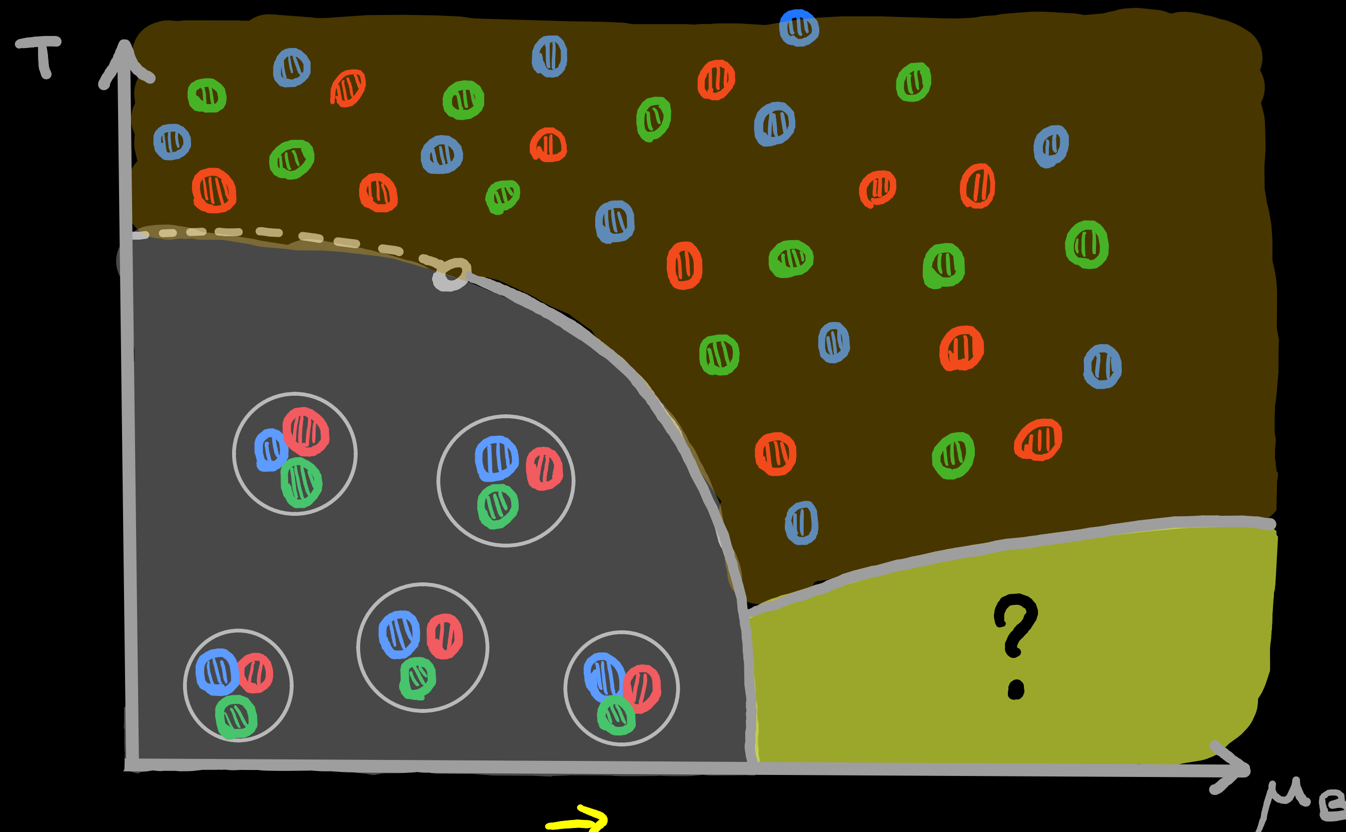

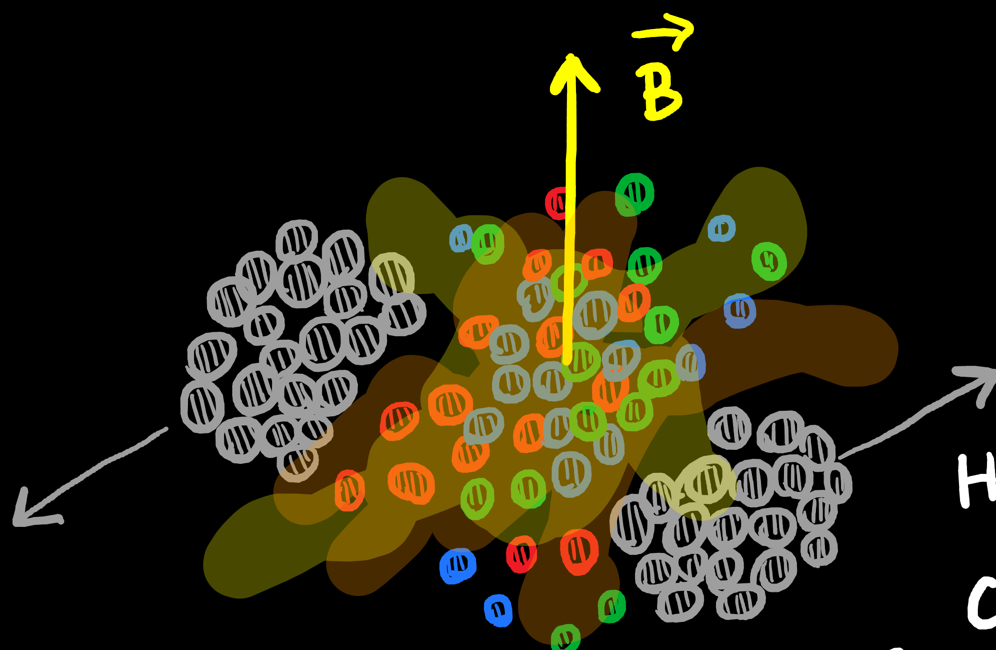

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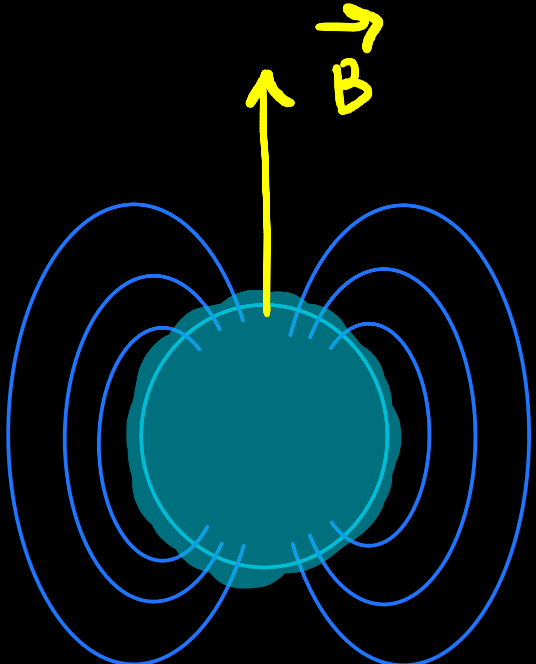

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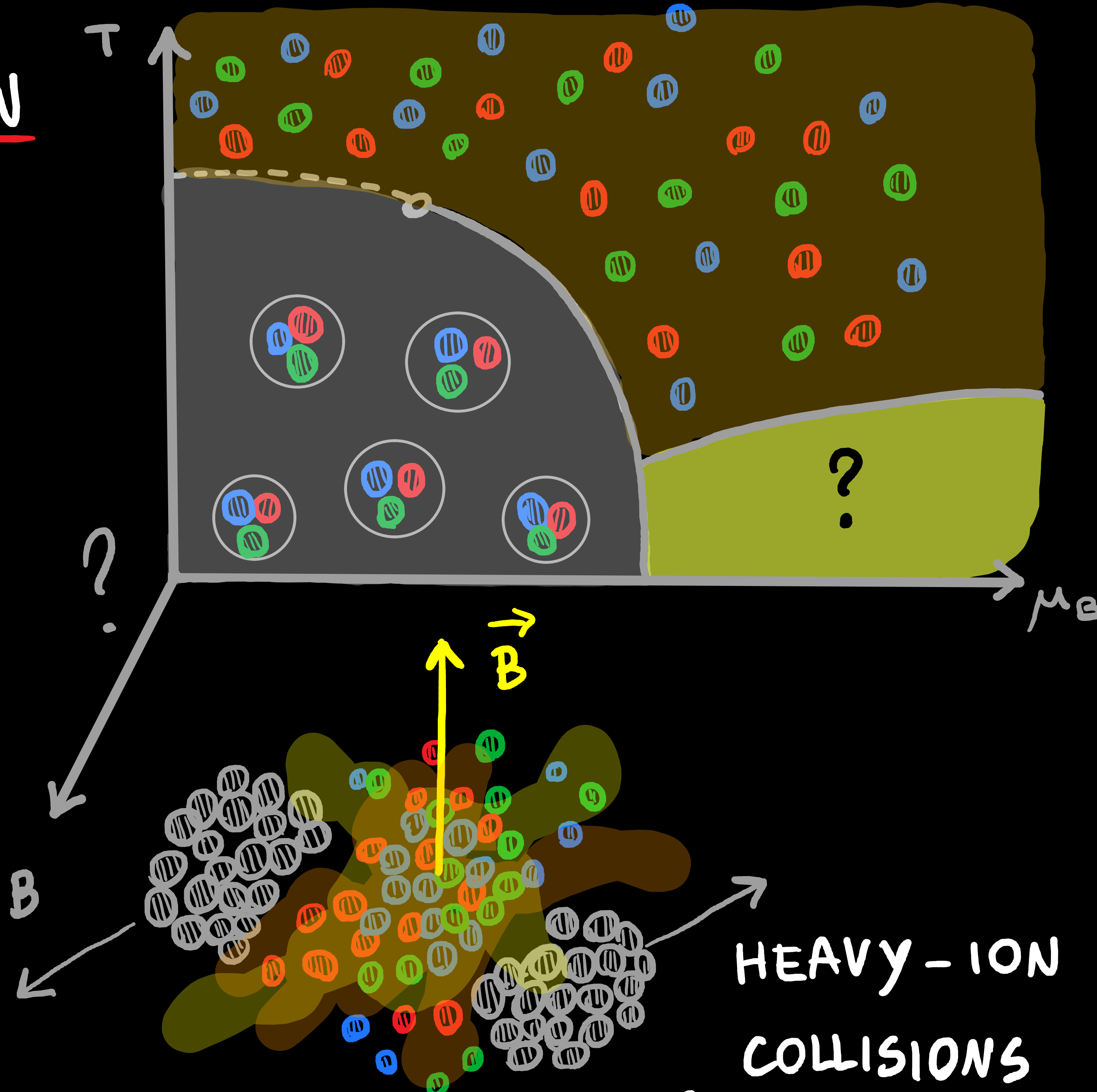

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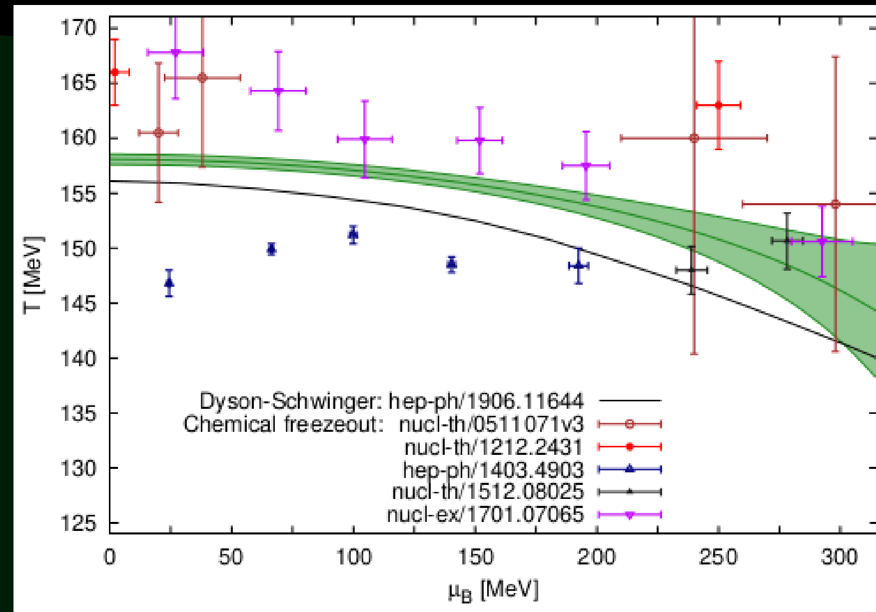

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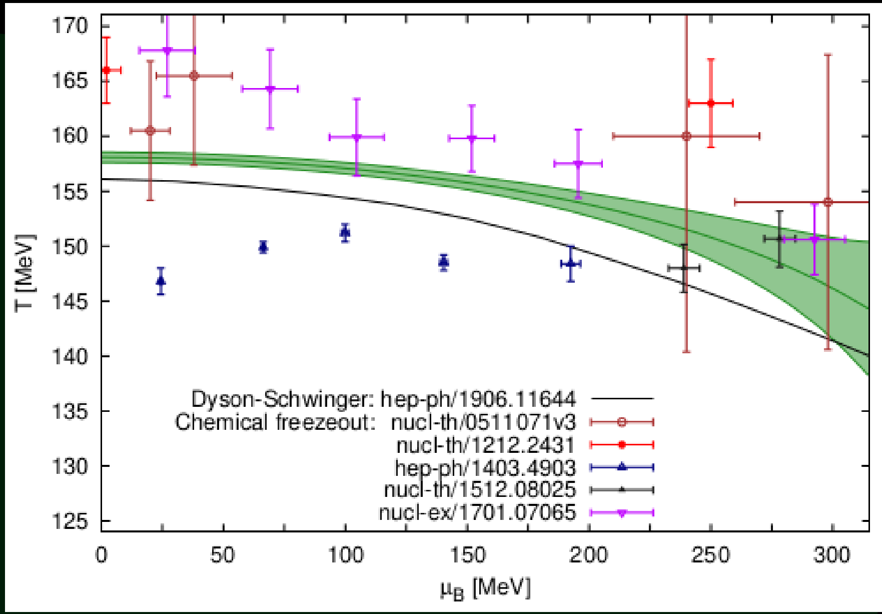

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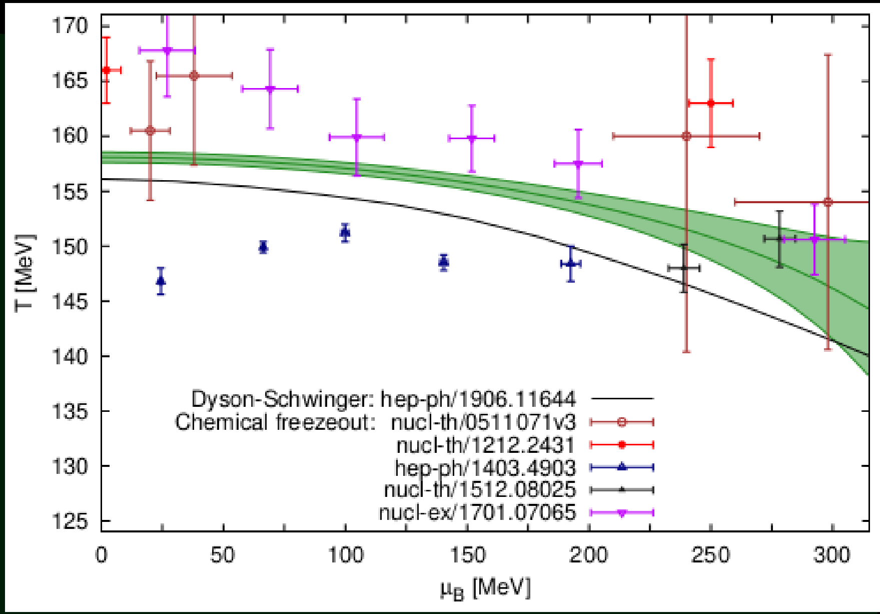
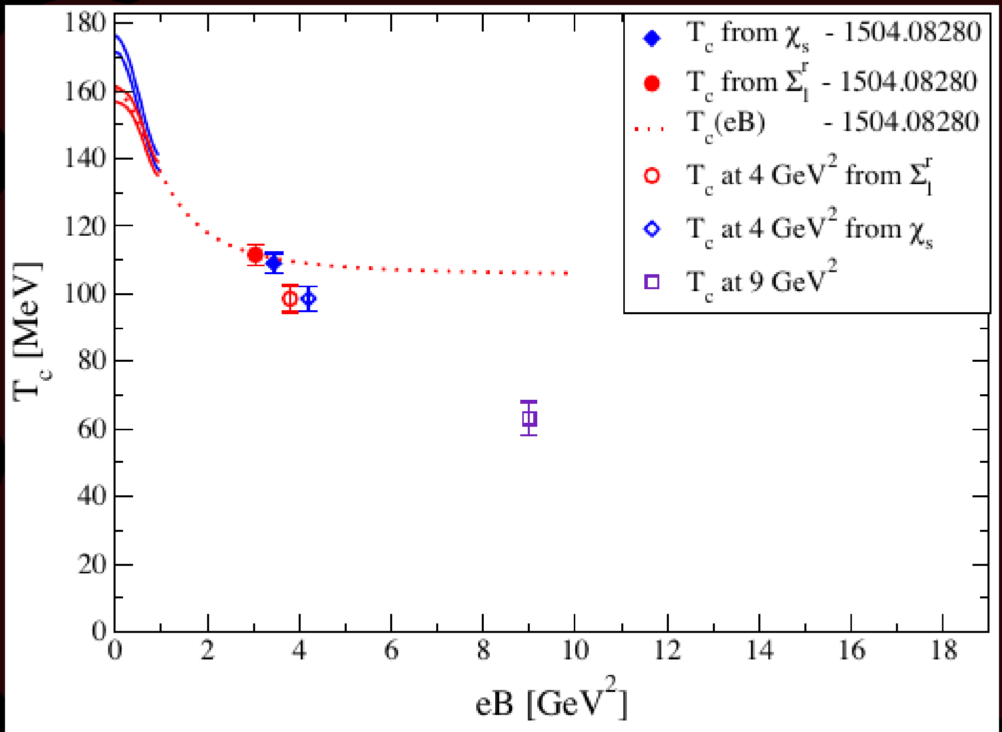
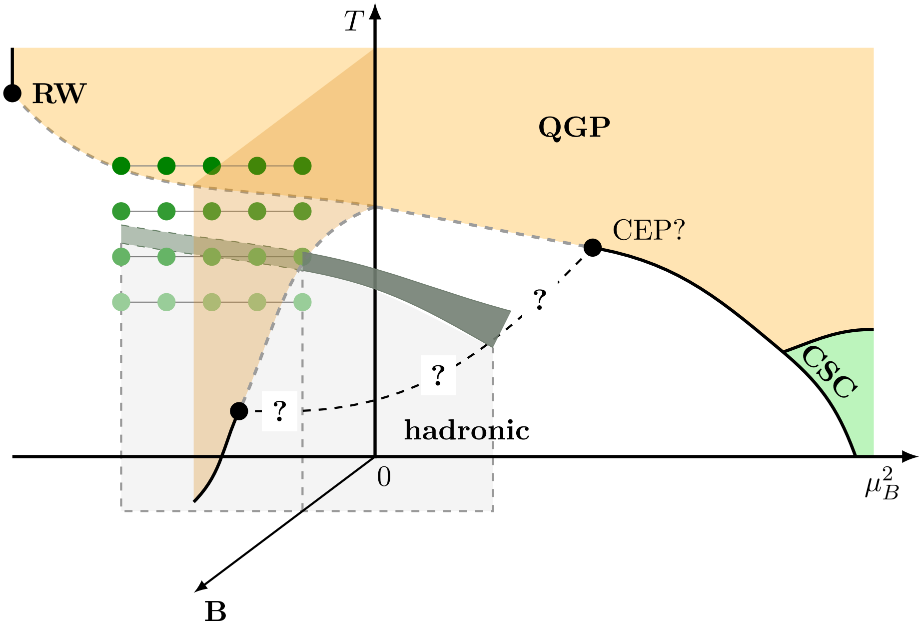




- AT $B \neq 0$, $\mu_B = 0$?

  - NO SIGN PROBLEM
  - CRITICAL ENDPOINT AT $4\,\mathrm{GeV}^2 < eB < 9\,\mathrm{GeV}^2$

[1]


T
RW
QGP
CEP?
?
?
?
hadronic
CSC
0
μ_B^2
B

UNIFORM $\vec{B}$ ON THE LATTICE $(A_y = Bx)$

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$$u_x = \begin{cases} e^{-iqyBL_x}, & \text{IF } x = L_x - a \\ 1, & \text{OTHERWISE} \end{cases}$$

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- QUANTIZATION $\quad qB = \dfrac{2\pi N_b}{L_x L_y}, \quad N_b \in \mathbb{Z}$

OUR SETUP

2+1+1

STAGGERED FERMIONS

PHYSICAL MASSES

$32^3 \times 8$ LATTICE /

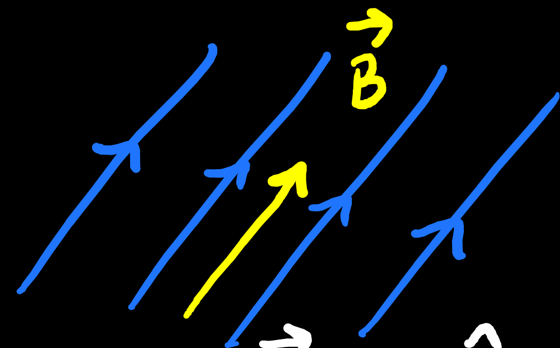
4-STOUT SYMANZIK

ACTION

TAYLOR

EXPANSION

IN μ_B



UNIFORM $\vec{B} \parallel \hat{z}$

$0.5\ \mathrm{GeV}^2$ $0.8\ \mathrm{GeV}^2$

THE EoS

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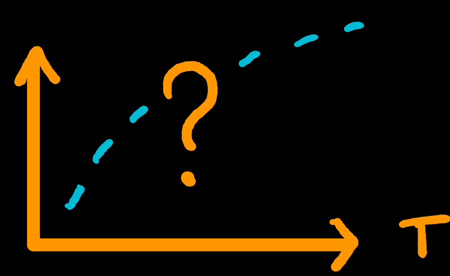

$$\frac{P}{T^4} = \sum_{ijk} \frac{\chi_{ijk}}{i!\,j!\,k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

THE EoS

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$$\chi_{ijk} \equiv \left(\frac{\partial}{\partial \hat{\mu}_B}\right)^i \left(\frac{\partial}{\partial \hat{\mu}_Q}\right)^j \left(\frac{\partial}{\partial \hat{\mu}_S}\right)^k \ln Z \Big|_{\vec{\mu}=\vec{0}} \quad , \quad \vec{\mu} \equiv (\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)$$

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$$\frac{P}{T^4} = c_0 + \frac{\chi_{200}}{2} \hat{\mu}_B^2 + \frac{\chi_{020}}{2} \hat{\mu}_Q^2 + \frac{\chi_{002}}{2} \hat{\mu}_S^2 +$$

$$+ \chi_{110} \hat{\mu}_B \hat{\mu}_Q + \chi_{101} \hat{\mu}_B \hat{\mu}_S + \chi_{011} \hat{\mu}_Q \hat{\mu}_S + \mathcal{O}(4)$$

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$\hat{\mu}_B$, $\hat{\mu}_Q$ & $\hat{\mu}_S$ ARE NOT INDEPENDENT!

MATCHING EXPERIMENTAL CONDITIONS

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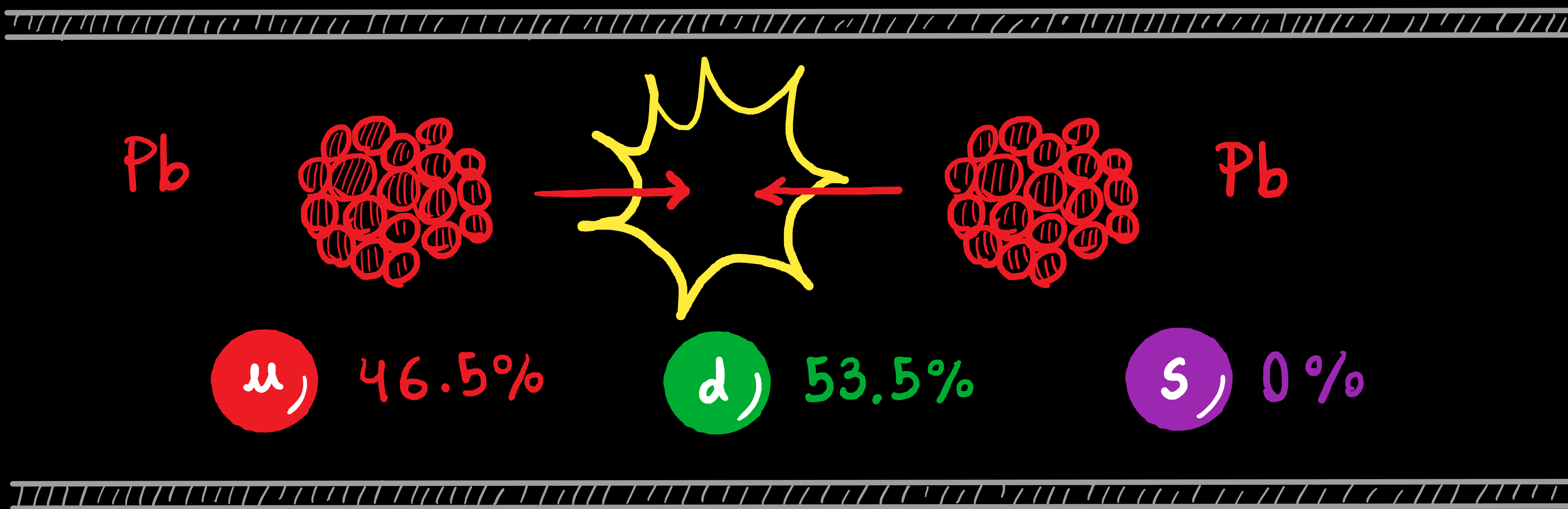

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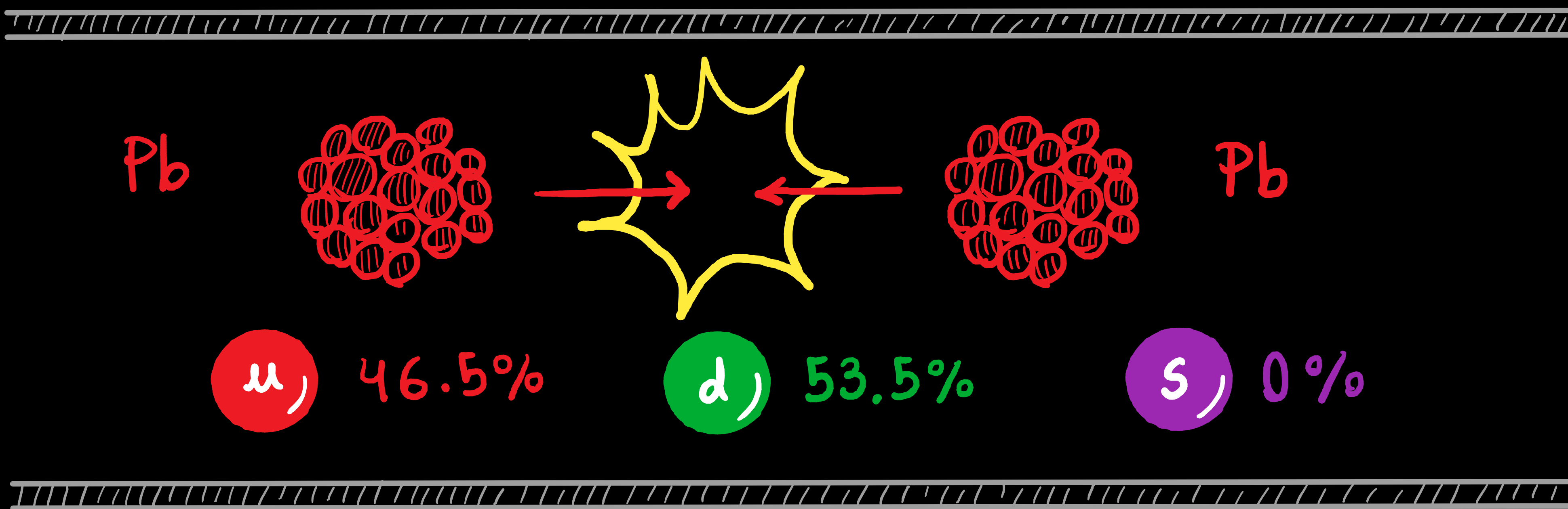
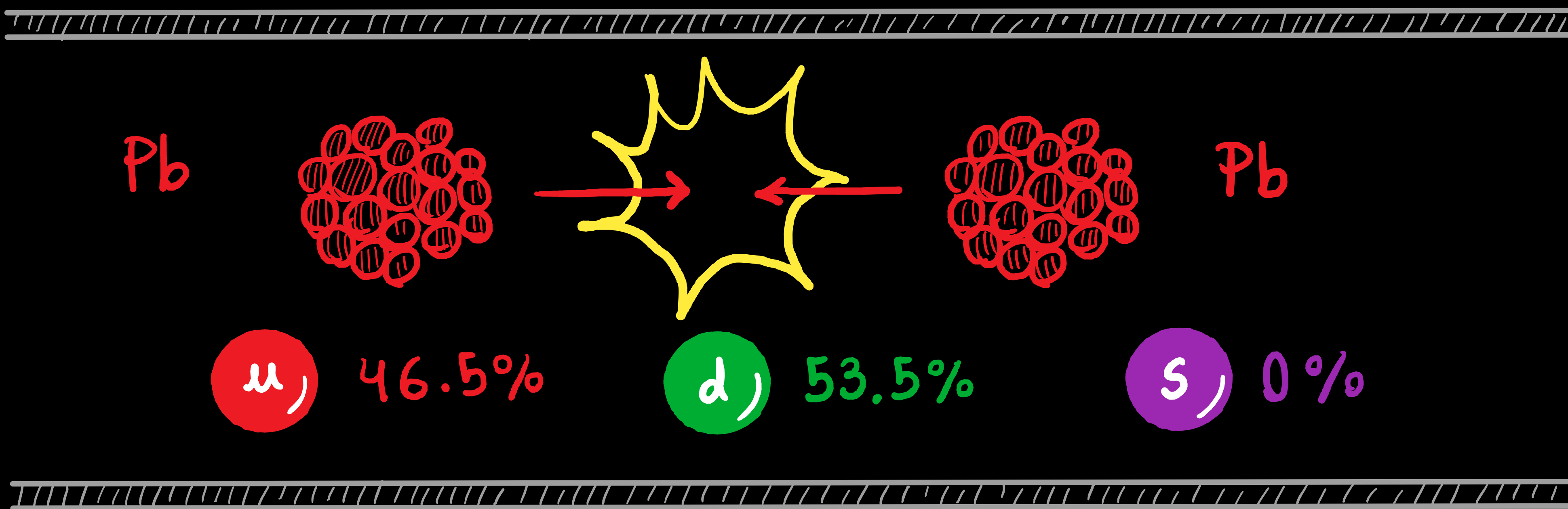


1. $\langle m_s \rangle = 0$ (STRANGENESS NEUTRALITY)

MATCHING EXPERIMENTAL CONDITIONS



1. $\langle m_s \rangle = 0$ (STRANGENESS NEUTRALITY)

2. $$\frac{\langle m_Q \rangle}{\langle m_B \rangle} = \frac{\frac{2}{3} \times 0.465 - \frac{1}{3} \times 0.535}{\frac{1}{3} \times 0.465 + \frac{1}{3} \times 0.535} \simeq 0.4$$

$$\hat{\mu}_Q = q_1 \hat{\mu}_B + q_3 \hat{\mu}_B^3 + \dots$$

$$\hat{\mu}_S = s_1 \hat{\mu}_B + s_3 \hat{\mu}_B^3 + \dots$$

FOR q_1 AND s_1:

$$q_1 = \frac{0.4(\chi_{200}\chi_{002} - \chi_{101}^2) - (\chi_{110}\chi_{002} - \chi_{110}\chi_{011})}{(\chi_{020}\chi_{002} - \chi_{011}^2) - 0.4(\chi_{110}\chi_{002} - \chi_{101}\chi_{011})}$$

$$s_1 = -\frac{\chi_{101}}{(\chi_{002})^2} - \frac{\chi_{011}}{(\chi_{002})^2} \cdot q_1$$ [6]

IMPOSING EXPERIMENTAL CONSTRAINTS...

$$P/T^4 = c_0 + \left(\frac{\chi_{200}}{2} + \frac{\chi_{020}}{2} q_1^2 + \frac{\chi_{002}}{2} s_1^2 + \chi_{110} q_1 + \chi_{101} s_1 + \chi_{011} q_1 s_1 \right) \hat{\mu}_B^2 + \mathcal{O}(4)$$

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C_2

L.O. CONTRIBUTION

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IN THE PRESENCE OF $\vec{B}$: $\chi_{ijk} = \chi_{ijk}(T, \vec{B})$

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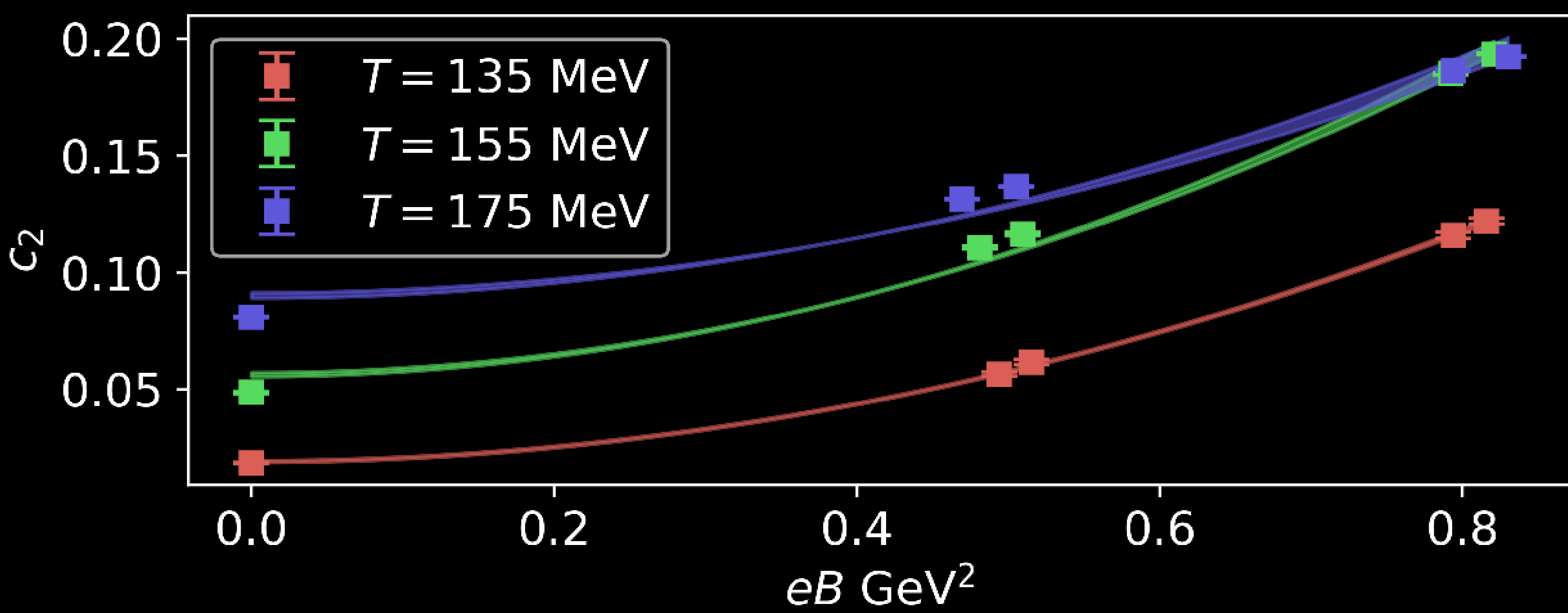

STRANGENESS NEUTRALITY AT FINITE $\vec{B}$

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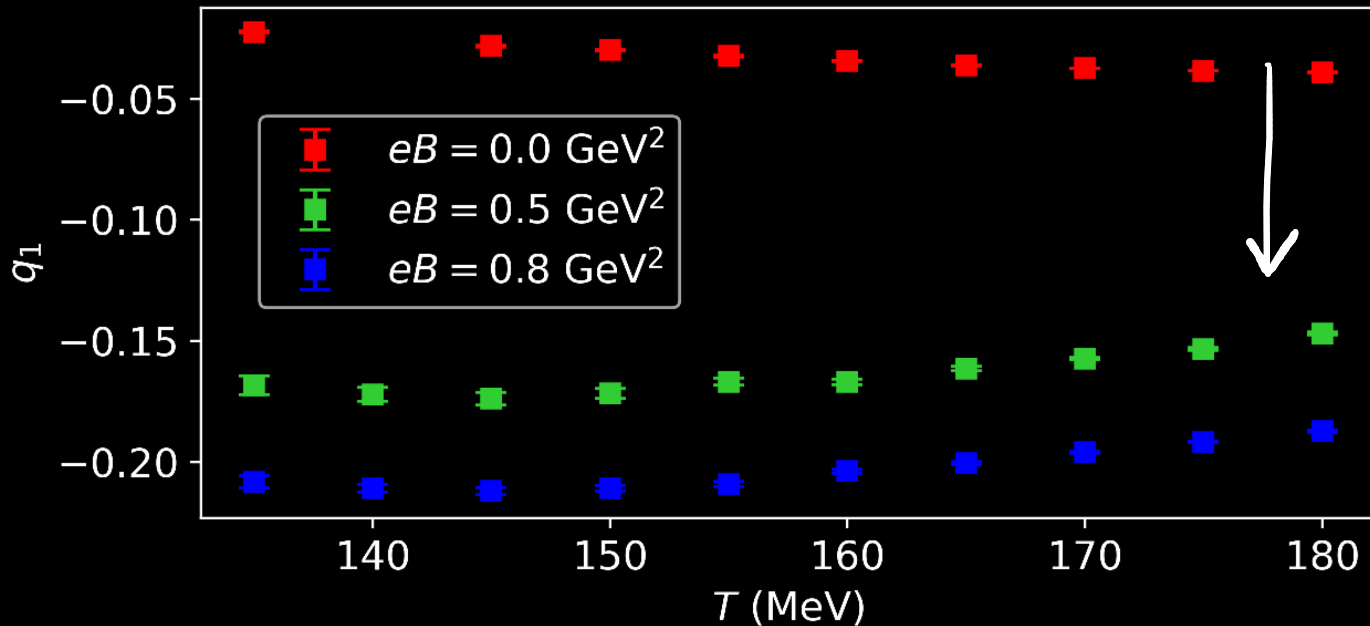
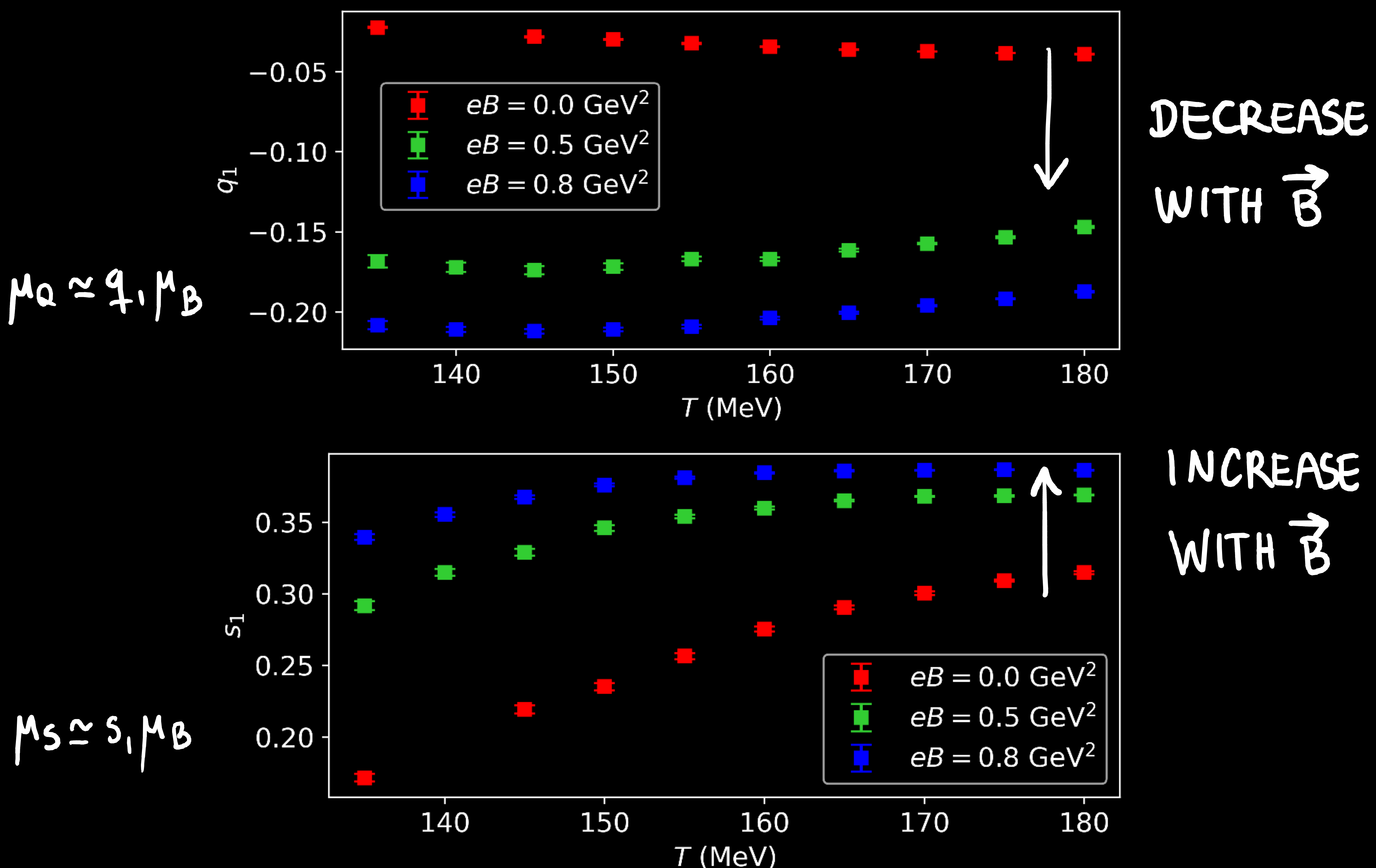
$\mu_Q \simeq q_1 \mu_B$



DECREASE WITH $\vec{B}$

STRANGENESS NEUTRALITY AT FINITE $\vec{B}$



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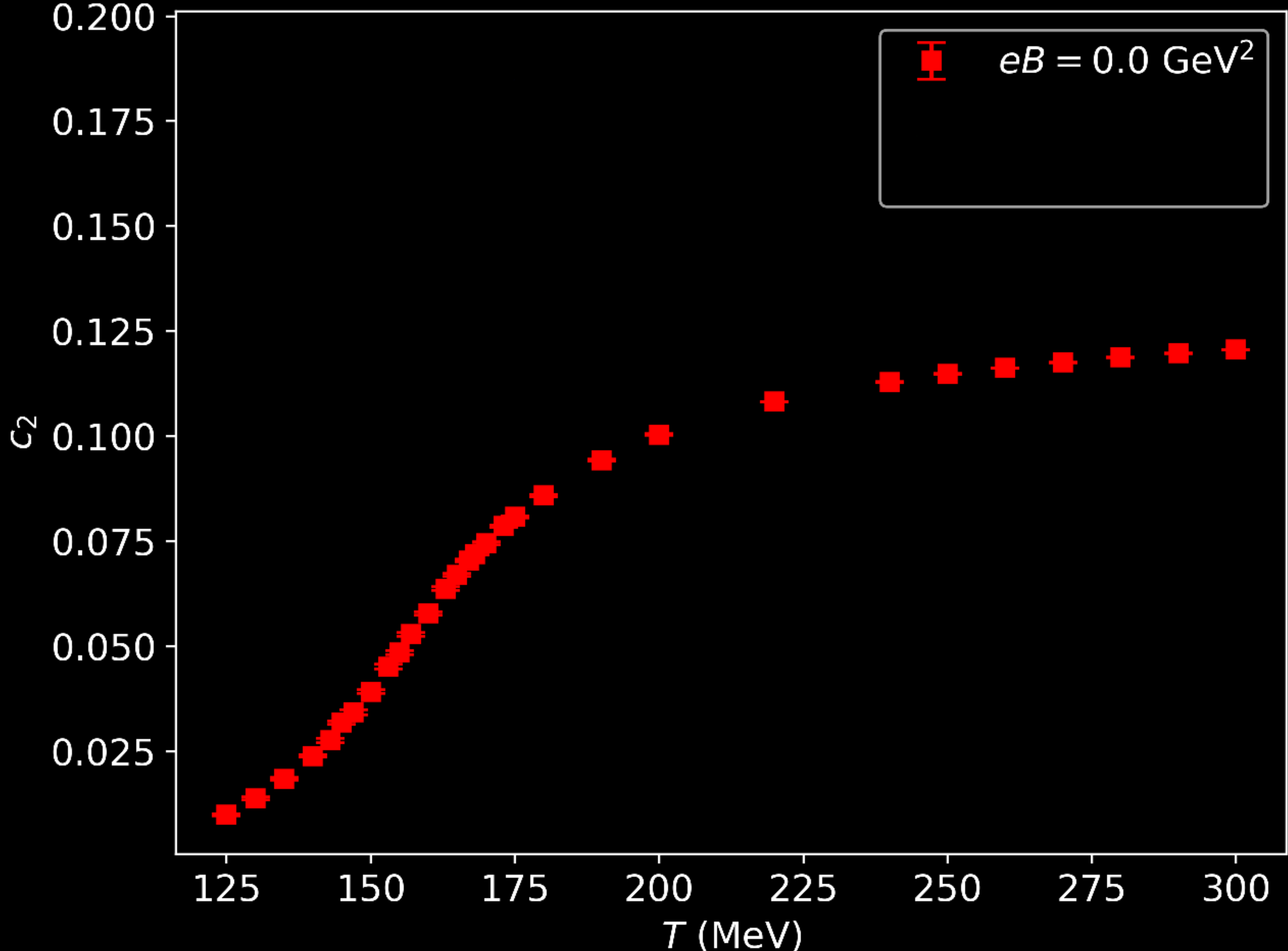

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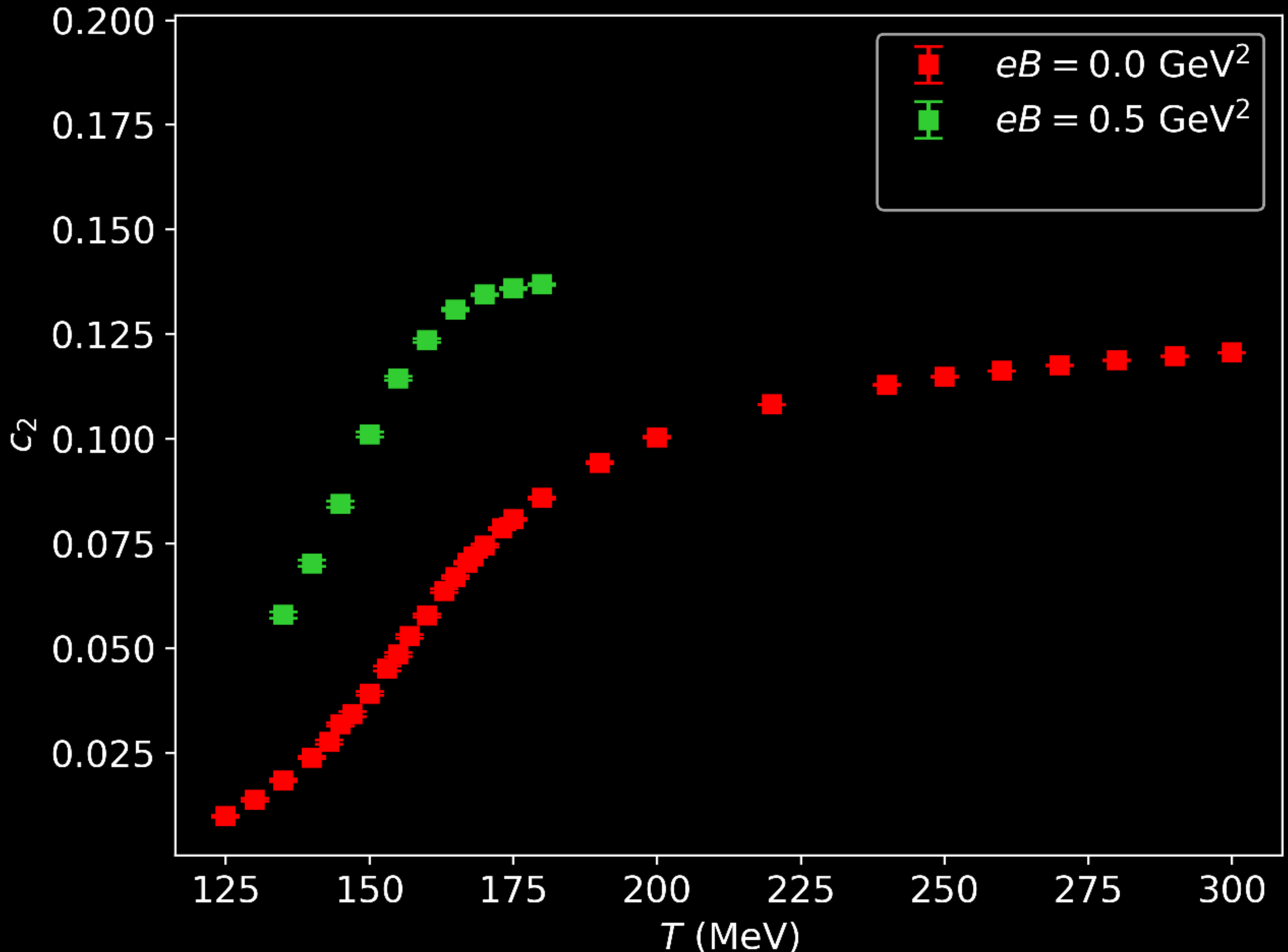

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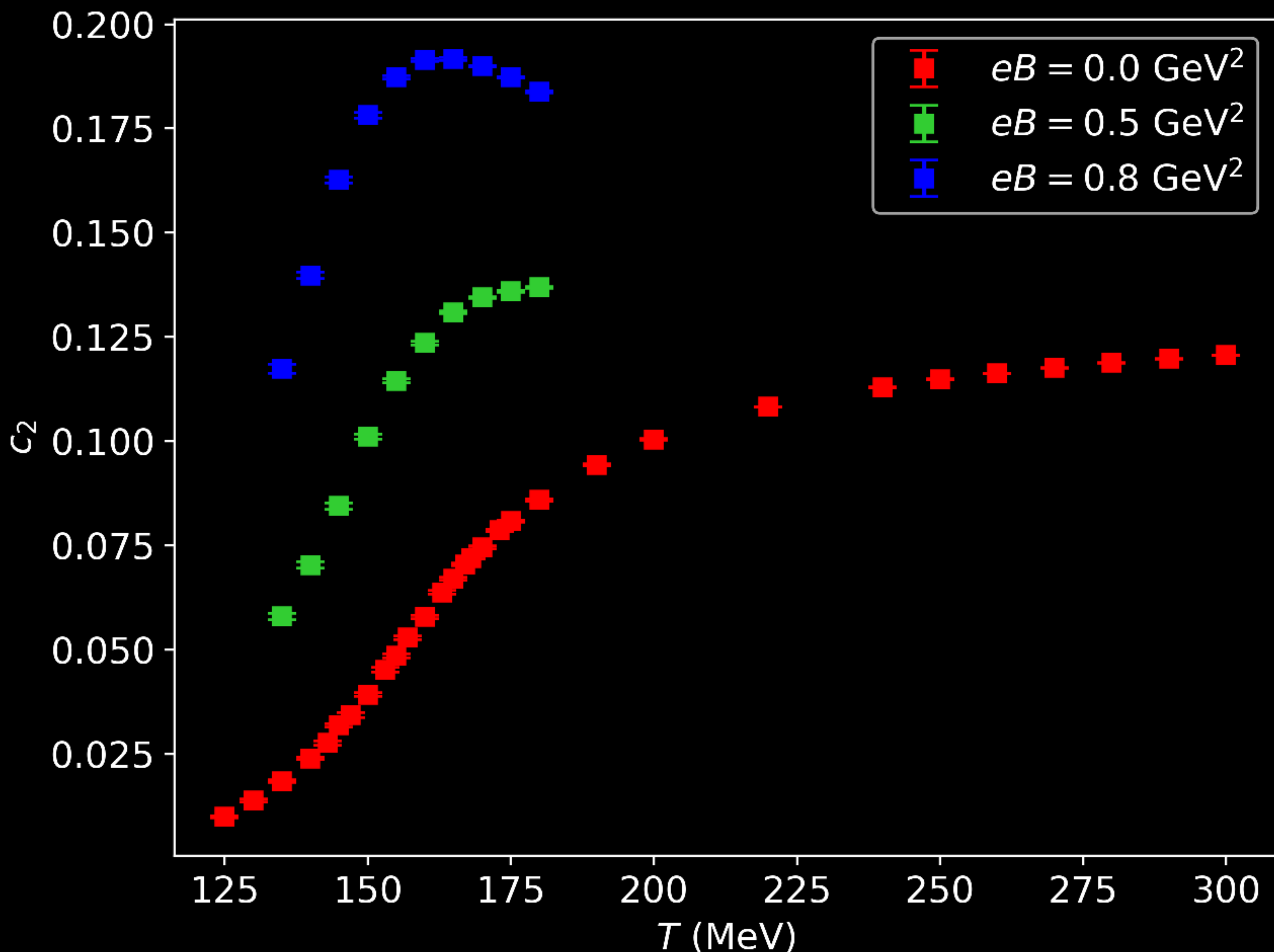

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THANK YOU !

REFERENCES

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