Lattice QCD input for neutrino-Z scattering

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• USQCD Community white paper:

• Snowmass 2021 White Paper
  Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics,
  phenomenology, and neutrino event generators. e-Print: 2203.09030 [hep-ph]
Outline:

- What LQCD can provide for ν-nucleus oscillation experiments
  - Axial and vector form factors of nucleons [nuclei are much more challenging]
  - Nuclear corrections \((p, n) \rightarrow (C^{12}, O^{16}, Ar^{40})\)
- Challenges to the calculations of nucleon matrix elements
  - Signal-to-noise falls as \(e^{-(M_N-1.5M_\pi)\tau}\)
  - Excited states in nucleon correlation functions
  - Extrapolation in \(\{a, M_\pi, M_\pi L\}\)
- FF must satisfy PCAC
  - What we learned from \(\langle N(p_f) | A_4(q) | N(p_i) \rangle\)
  - Towers of \(N\pi, N\pi\pi\), states contribute to axial and PS correlators
- Comparison of published results for \(g_A, G_A\)
- Comparison with MINERvA and ν-\(D\) analyses
- Summary of unpublished results for \(g_A, G_A\)
- Transition matrix elements; Results for \(G_E, G_M\)
- Future
ν energy-range covers complex physics

- Neutrino energy and flux not known precisely
- Dynamics of struck Argon nucleus is too complex to simulate directly and connect to final states seen in the detectors
Ultimate Goal: Inputs for DUNE

Two matrix elements for $\nu - ^{40}Ar$ scattering.

$$\langle X | A_\mu (q) | ^{40}Ar \rangle$$

$$\langle X | V_\mu (q) | ^{40}Ar \rangle$$

Building blocks for different energy regions: starting with nucleons

$$\langle p | J_\mu^w (q) | n \rangle$$  \hspace{2cm} Quasi-elastic

$$\langle n\pi | J_\mu^w (q) | n \rangle, \langle \Delta | J_\mu^w (q) | n \rangle$$  \hspace{2cm} Resonant

$$\langle X | J_\mu^w (q) | n \rangle$$  \hspace{2cm} DIS
Including nuclear effects in complex nuclear targets

Nuclear many body Hamiltonian takes as input matrix elements involving successively more multi-particles

- One nucleon \[ \langle p | J^+_{\mu}(q) | n \rangle \]

- Transition \[ \langle n\pi | J^{w}_{\mu}(q) | n \rangle, \langle \Delta | J^{w}_{\mu}(q) | n \rangle \]

- Two nucleon \[ \langle n p | J^{w^+}_{\mu}(q) | n n \rangle \]

Discussed by Noemi Rocco

See also Snowmass 2021 white paper:
The ν-n differential cross-section:

\[
\frac{d\sigma}{dQ^2} \left( \begin{array}{c}
\nu_l + n \rightarrow l^- + p \\
\bar{\nu}_l + p \rightarrow l^+ + n
\end{array} \right) = \frac{M^2G_F^2\cos^2\theta_c}{8\pi E_\nu^2} \left\{ A(Q^2) \pm B(Q^2) \frac{(s-u)}{M^2} + C(Q^2) \frac{(s-u)^2}{M^4} \right\},
\]

\[
A(Q^2) = \frac{(m^2 + Q^2)}{M^2} \left[ (1 + \tau)F_A^2 - (1 - \tau)F_1^2 + \tau(1 - \tau)F_2^2 + 4\tau F_1 F_2 \\
- \frac{m^2}{4M^2} \left( (F_1 + F_2)^2 + (F_A + 2F_P)^2 - 4 \left( 1 + \frac{Q^2}{4M^2} \right) F_P^2 \right) \right],
\]

\[
B(Q^2) = \frac{Q^2}{M^2} F_A(F_1 + F_2),
\]

\[
C(Q^2) = \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2).
\]

\[
\langle NA_\mu N \rangle \rightarrow \text{linear combination of } F_A, \tilde{F}_P
\]

\[
\langle NV_\mu N \rangle \rightarrow G_E, G_M
\]

\[F_A = \text{axial form factor}\]
\[G_E = F_1 - \tau F_2 \quad \text{Electric}\]
\[G_M = F_1 + F_2 \quad \text{Magnetic}\]
\[\tau = \frac{Q^2}{4M^2}\]
\[M = M_p = 939 \text{ MeV}\]
\[m = \text{mass of the lepton}\]
Analysis of $(e, \mu, \nu)-n$ scattering involves
5 Form Factors & 3 charges $g_A, \mu, g_P^*$

- $G_E(Q^2)$ Electric
- $G_M(Q^2)$ Magnetic
- $G_A(Q^2)$ Axial
- $\tilde{G}_P(Q^2)$ Induced pseudoscalar
- $G_{P}(Q^2)$ Pseudoscalar (extracted from $\langle NPN \rangle \rightarrow G_P$)
- Lattice methodology is common: all calculated at the same time
- Precise experimental data exist for $G_E(Q^2)$ and $G_M(Q^2)$
- Axial ward identity (PCAC) relates $G_A(Q^2)$, $\tilde{G}_P(Q^2)$, $G_P(Q^2)$

- $G_E(Q^2 = 0) = 1$ Conserved vector charge
- $G_M(Q^2 = 0) = \mu = 4.7058$ Magnetic moment
- $G_A(Q^2 = 0) = g_A = 1.276(2)$ Axial charge
- $\tilde{G}_P(Q^2 = 0.88m_\mu^2) = g_P^* = 8.06(55)$ Induced pseudoscalar charge
Lattice QCD gives us

2-point function

\[ \langle \Omega | \hat{N}_\tau^\dagger \hat{N}_0 | \Omega \rangle \]
\[ \Gamma^{2pt}(\tau) = \sum_i |A_i|^2 e^{-E_i \tau} \]

3-point functions

\[ \langle \Omega | \hat{N}_\tau^\dagger O(t) \hat{N}_0 | \Omega \rangle \]
\[ \Gamma^{3pt}(t, \tau) = \sum_{i,j} A_i^* A_j \langle i | O | j \rangle e^{-E_i \tau - E_j (\tau - t)} \]
Calculations of nucleon 2,3-point functions using LQCD are mature

Spectrum (energies $E_i$, amplitudes $A_i$) and ME are extracted from fits to the spectral decomposition of 2- and 3-point functions

$$\Gamma^{2pt}(\tau) = \sum_{i} |A_i|^2 e^{-E_i \tau}$$

$$\Gamma^{3pt}_O(t, \tau) = \sum_{i,j} A_i^* A_j \langle i | O | j \rangle e^{-E_i t - E_j (\tau - t)}$$

Extract $\langle 0 | O | 0 \rangle$

All states $|i\rangle$ with the same quantum numbers as $N$ contribute unless $A_i$ (or $A_j$) is zero

Radial excited States:
- $N(1440), N(1710)$
- Towers of multihadrons states
  - $N(\bar{k})\pi(\bar{k}) > 1200 \text{ MeV}$
  - $N(0)\pi(\bar{k})\pi(\bar{k}) > 1200 \text{ MeV}$

but removing ESC from multihadron states remains a challenge
Challenges

• Need large \(\tau\) to “kill” states with small mass gap (\(\Delta M \sim 300\))

• Cannot go to large enough \(\tau\) because the signal/noise degrades as \(e^{-(M_N - 1.5M_\pi)\tau}\)
  – Signal: 2-pt: \(\tau \sim 2\text{fm}\) ; 3-pt: \(\tau \sim 1.5\text{fm}\)

• Typical interpolating operator \(\hat{N}\) couples to the nucleon, its excitations and multi-hadron states with the same quantum numbers

• As \(\vec{q} \to 0\), the towers of physical \(N\pi, N\pi\pi\) states become arbitrarily dense above \(\sim 1230\text{ MeV}\) (the \(\Delta\) region)

• Quantities impacted by \(N\pi, N\pi\pi\), states should be analyzed on \(M_\pi \lesssim 200\text{ MeV}\) ensembles

• Excited states that give significant contribution to a particular ME are not known a priori. \(\chi PT\) is a very useful guide

• The potential of variational methods for isolating the ground state is just starting to be realized!
• Corrections from pion loops are in all matrix elements
• Loops that originate or end at sources are ESC. These can be removed by a perfect source.
• Loops that originate on the nucleon line give rise to both: corrections to the physical result and excited state contributions (from pion going on-shell in Minkowski)
• The latter are suppressed exponentially by the mass gap
• Unless there are large cancellations, both should be considered in (i) removing excited state contamination in getting the data, and in (ii) the final chiral fits to the data
Extract Axial-vector Form Factors, $G_A, \tilde{G}_P, G_P$

3-point functions $\rightarrow$ ground state matrix elements $\rightarrow$ Form factors

$$\langle N(p_f) | A^\mu(q) | N(p_i) \rangle = \bar{u}(p_f) \left[ \gamma^\mu G_A(q^2) + q_\mu \frac{\tilde{G}_P(q^2)}{2M} \right] \gamma_5 u(p_i)$$

$$\langle N(p_f) | P(q) | N(p_i) \rangle = \bar{u}(p_f) G_P(q^2) \gamma_5 u(p_i)$$

PCAC [$\partial_\mu A_\mu = 2mP$] relates $G_A, \tilde{G}_P, G_P$
Constraints once FF are extracted from ground state matrix elements

1) PCAC \((\partial_{\mu} A_{\mu} = 2\hat{m}P)\) requires
\[
2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N} \tilde{G}_P(Q^2)
\]

2) In any [nucleon] ground state
\[
\partial_4 A_4 = (E_q - M_0) A_4
\]

3) \(G_A, \tilde{G}_P\) extracted from
\[
\langle N(p_f)|A_i(q)|N(p_i)\rangle
\]
must be consistent with
\[
\langle N(p_f)|A_4(q)|N(p_i)\rangle
\]
Decomposition of ground state matrix elements: 
\[ \langle N_\tau A_\mu(t)N_0 \rangle \] provides an over-determined set

Choosing “3” the direction of spin projection

\[
\langle N(p_f)\mid A_{1,2}(q)\mid N(p_i) \rangle \rightarrow - \frac{q_{1,2}q_3}{2M} \tilde{G}_P
\]

\[
\langle N(p_f)\mid A_3(q)\mid N(p_i) \rangle \rightarrow - \left[ \frac{q_3^2}{2M} \tilde{G}_P - (M + E)G_A \right]
\]

\[
\langle N(p_f)\mid A_4(q)\mid N(p_i) \rangle \rightarrow -q_3 \left[ \frac{E-M}{2M} \tilde{G}_P - G_A \right]
\]

Gives both \( G_A, \tilde{G}_P \)

Redundant. Dominated by excited states
\( \chi PT: \ N\pi \) state coupling large in the axial channel

Enhanced coupling to \( N\pi \) state: Since the pion is light, the vertex \( \bullet \) can be anywhere in the lattice 3-volume

\[ A_i^* \langle i | A_4 | j \rangle \sim V^{-1} \]

\[ \sim V \]

\[ \langle N_\tau A_4(t)N_0 \rangle \] has large ESC

Fits with \( N\pi \) state preferred

\[
\tilde{R}_{54}(n^2 = 1) = 0.91(27) \\
\chi^2/28 = 5.80, p = 0.00 \\
\Delta E_1 = 0.20(4), \Delta M_1 = 0.18(5) \\
\tau : \infty \sim 19 \quad 17 \quad 15 \quad 13 \\
\{4, 3^*\}
\]

\[
\tilde{R}_{54}(n^2 = 1) = 0.67(26) \\
\chi^2/126 = 1.48, p = 0.00 \\
\Delta E_1 = 0.09(1), \Delta M_1 = 0.10(1) \\
\tau : \infty \sim 19 \quad 17 \quad 15 \quad 13 \\
\{4^{N\pi}, 2^{\text{sim}}\}
\]

Data driven evidence for \( N\pi \) state.
Including \( N\pi \) state also addressed PCAC

2017→2019: Resolution of PCAC and PPD


On including low mass $N_{p=0}\pi_p$ and $N_p\pi_-$ excited states neglected in previous works, FF satisfy PCAC and PPD at $\sim5%$

$$\frac{\hat{m}G_P}{M_N G_A} + \frac{Q^2 \tilde{G}_P}{4M_N^2 G_A} = 1$$

$$\frac{Q^2 + M^2_\pi \tilde{G}_P(Q^2)}{4M_N^2 G_A(Q^2)} = 1$$

Also see RQCD Collaboration: JHEP 05 (2020) 126, 1911.13150
\[ N\pi \text{ state in the axial channel} \]

\[ A_\mu(\tau, \vec{q}) \]

Ground State

\[ \hat{p} = 0 \]

Excited State

Mass gaps extracted from fits match the above picture

\[ \Delta M : N(n) + \pi(-n) \quad \Delta E : N(0) + \pi(n) \]

\[ \Delta M_{1A4} \text{ and } \Delta E_{1A4} \text{ are outputs of 2-state fits and not driven by priors} \]
How large is the “$N\pi$” effect?

Output of a simultaneous fit to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ (called $\{4^{N\pi}, 2^{sim}\}$ fit) increases the form factors by:

- $G_A \sim 5\%$
- $\tilde{G}_P \sim 35\%$
- $G_P \sim 35\%$

Standard 3-state fit to $\langle P \rangle$

Simultaneous 2-state to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ correlators
Essential steps in the analysis

• Remove ESC from correlation functions
• Decompose into form factors to get $G(Q^2)$ on each ensemble
• Parameterize this $G(Q^2)$
• Perform CCFV extrapolation to get $G(Q^2)$ in the continuum
• Parameterize this $G(Q^2)|_{cont}$

Model averaging should include model choices at each step that have significant effect on result
If ESC is the largest systematic and fits do not select between \{A_i, E_i\}

- 2-state fit: Model average different $E_1$
- 3-state fit: Model average over \{$E_1, E_2$\}

$\chi^2 / \text{dof}$ roughly constant
Consistency in the extraction of $g_A$

- $g_A$ from forward ME versus $g_A = G_A(Q^2 \rightarrow 0)$
- With / without including $N\pi$ state in the analysis
- PCAC

Spectrum from $\Gamma^2$

$g_A$ (Forward ME)

$g_A = G_A(Q^2 \rightarrow 0)$

$G_A, \tilde{G}_P, G_P$ do not satisfy PCAC

$G_A, \tilde{G}_P, G_P$ with $N\pi$ satisfy PCAC
Calculations reviewed in 2305.11330

<table>
<thead>
<tr>
<th>Collab.</th>
<th>Ens.</th>
<th>Excited State</th>
<th>$M_\pi$</th>
<th>$Q^2$</th>
<th>Continuum-chiral-finite-volume extrap</th>
<th>$g_A$</th>
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<tr>
<td>PNDME 23</td>
<td>13</td>
<td>With $N\pi$</td>
<td>2 physical</td>
<td>$z^2 + z^2$</td>
<td>CCFV</td>
<td>1.292(53)(24)</td>
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<tr>
<td>Mainz 22</td>
<td>14</td>
<td>Simultaneous ESC, $Q^2$</td>
<td>2 physical</td>
<td>$z^2$</td>
<td>CCFV</td>
<td>1.225(39)(25)</td>
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<td>NME 21*</td>
<td>7 (12)</td>
<td>With $N\pi$</td>
<td>2 physical</td>
<td>$z^2$</td>
<td>Ignore ${a, M_\pi^2 M_\pi^2 L}$ dependence</td>
<td>1.32(6)(5)</td>
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<td>ETMC 20</td>
<td>3</td>
<td>Without $N\pi$</td>
<td>3 physical</td>
<td>data</td>
<td>${a}$</td>
<td>1.283(22)</td>
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<tr>
<td>RQCD 19/23</td>
<td>36 (47)</td>
<td>With $N\pi$ only for $\tilde{G}_P, G_P$</td>
<td>data</td>
<td></td>
<td>[1.284^{28}_{27}]</td>
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</table>

Mainz: PRD 106, 074503 (2022)  
ETMC: PRD 103, 034509 (2021)
Comparing axial form factor from LQCD

A consensus is emerging

\[ g_A = 1.281(53) \]

\[ \langle r_A^2 \rangle = 0.498(56)\text{fm}^2 \]
Comparing prediction of x-section using AFF from $\nu - D$ and PNDME with MINERvA data

$\theta_{\mu} \leq 20^0$, $1.5 \text{ GeV} \leq p_{\mu} \leq 20 \text{ GeV}$

$\bar{\nu}_\mu p \rightarrow \mu^+ n$

deuterium fit for $F_A$
PNDME $F_A$
hydrogen data

$10^{38} \frac{d\sigma}{dQ^2} \left[ \text{cm}^2/\text{GeV}^2 \right]$

$Q^2 [\text{GeV}^2]$

$Q^2 < 0.2 \text{ GeV}^2$  $0.2 < Q^2 < 1$  $Q^2 > 1 \text{ GeV}^2$


Oleksandr Tomalak, Rajan Gupta, Tanmoy Bhattacharyya, arXiv:2307.14920
Mapping the AFF

• $0 < Q^2 < 0.2 \text{ GeV}^2$
  – This region will get populated by simulations with $M_\pi \approx 135 \text{ MeV}$, $a \to 0$, $M_\pi L > 4$
  – MINER$\nu$A data has large errors
  – Characterized by $g_A$ and $\langle r_A^2 \rangle$ and $G_A(Q^2)$ parameterized by a $z$-expansion with a few terms

• $0.2 < Q^2 < 1 \text{ GeV}^2$
  – Lattice data mostly from $M_\pi > 200 \text{ MeV}$ simulations
  – Competitive with MINER$\nu$A data. Cross check of each other

• $Q^2 > 1 \text{ GeV}^2$
  – Lattice needs new ideas
  – MINER$\nu$A and future experiments
Update from ETMC ($3 M_\pi \approx 135$ MeV ensembles)

2+1+1-flavor twisted mass ensembles

<table>
<thead>
<tr>
<th>Ens. ID</th>
<th>latt. Vol.</th>
<th>$\alpha$ [fm]</th>
<th>$Lm_\pi$</th>
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<td>cB211.072.64 (cB64)</td>
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<td>$96^3 \times 192$</td>
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Excited state fits
- 2-state checked against 3-state
- $N\pi$ state not included
- 1st excited state mass $\approx xx$ MeV

PCAC test of form factors

$$r_{PCAC} = \frac{m_q G_5(Q^2) + \frac{Q^2}{4m_N^2} G_P(Q^2)}{G_A(Q^2)}$$

Large cut-off effects in twisted mass involving pions
Update from CalLAT Collaboration
(A. Meyer, A. Walker-Loud)

domain-wall on HISQ calculation using sequential prop through sink
$48^3 \times 64$ ensemble ($a12m130$): $a^{-1} = 1.66$ GeV; $M_\pi = 132$ MeV
Gaussian sources for quark propagators
1000 X 32 (configurations X measurements)
Updates from PACS

Talk by Ryutaro Tsuji

Stout smeared O(a) improved Wilson quark and Iwasaki gauge actions. 2+1 flavors

<table>
<thead>
<tr>
<th>Lattice size</th>
<th>128$^4$</th>
<th>160$^4$</th>
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<tr>
<td>Spatial volume</td>
<td>(10.9 fm)$^3$</td>
<td>(10.1 fm)$^3$</td>
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<tr>
<td>Pion mass</td>
<td>135 MeV</td>
<td>135 MeV</td>
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<tr>
<td>Nucleon mass</td>
<td>0.935(11) GeV</td>
<td>0.946(3) GeV</td>
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<td>Lattice spacing</td>
<td>0.086 fm</td>
<td>0.063 fm</td>
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<tr>
<td>$</td>
<td>t_{\text{sink}} - t_{\text{src}}</td>
<td>/a$</td>
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<tr>
<td>Renormalization</td>
<td>SF, RI-MOM/SMOM</td>
<td>SF</td>
</tr>
</tbody>
</table>

Preliminary

Tuned exponential sources show very little excited-state effects in axial
2+1-flavor domain-wall-fermions
48^3 \times 96 ensemble: a^{-1} = 1.730(4) GeV
Gaussian sources for quark propagators
120 configurations, ## measurements

Data at \( \tau = 8, 9, 10 \) do not show significant change indicating small excited state effect

Slower fall-off than PNDME 23 data
Comparison with unpublished data

- **Figures**: Various datasets and models are compared, including:
  - **LHP/RBC/UKQCD**
  - **PACS**
  - **CalLAT**
  - **ETM**

- **Dipole Model**: $M_A = 1.285(26)$ GeV
- **$z$ Expansion**
  - $k_{\text{max}} = 4$: LQCD (no sum rules)
  - $k_{\text{max}} = 8$: LQCD
  - $k_{\text{max}} = 8$: D
- **LQCD fit matrix elements**: $\langle Q^2 \rangle \sim \langle \langle Q^2 \rangle \rangle$

- **PACS**
  - $g_A = 1.268(24)$
  - $r_A^2 = 0.359(23)$ [fm$^2$]
Roper transition helicity amplitude using \( \langle NV_\mu V_\nu N \rangle \) hadronic tensor

Extracted using a simple spectral decomposition of \( H_{\mu\nu} \) that gives reasonable estimates

Talk by Raza Sufian for \( \chi QCD \)
The extraction of electric and magnetic form factors is insensitive to the details of the excited states.

Vector meson dominance $\rightarrow N\pi\pi$ state should contribute (some evidence).

The form factors do not show significant dependence on the lattice spacing or the quark mass.

Good agreement with the Kelly curve. Validates the lattice methodology.

Improve precision and get data over larger range of parameter values.
Multi-hadron states

\[ \langle n \pi^+ | J^+_{\mu}(q) | n \rangle \]

\[ \langle n p | J^+_{\mu}(q) | n n \rangle \]

See
- Barca et al, 2211.12278, 2110.11908
- Nuclear matrix elements from lattice QCD for electroweak and beyond-Standard-Model processes, 2008.11160 [hep-lat]
Looking ahead

• Challenges in lattice calculations of nucleon matrix elements:
  – Signal to noise degrades as $e^{-(M_N - 1.5M_\pi)t}$
  – removing multi-hadrons excited states to get ground state ME
  – including multi-hadrons in initial and/or final state for transition ME

• Continue to develop a robust analysis strategy for removing dominant excited states in various nucleon matrix elements

• Improve chiral and continuum extrapolation. Simulate at more $\{a, M_\pi\}$

• Current $0.04 < Q^2 < 1 \text{ GeV}^2$ Extend to larger $Q^2$ for DUNE

• Transition matrix elements

• Goal: Perform a comprehensive analysis of scattering data with input of lattice results for $g_A, G_E(Q^2), G_M(Q^2), G_A(Q^2), G_P(Q^2)$

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Improvements in algorithms and computing power are needed to reach few percent precision