Lattice QCD input for neutrino-Z scattering

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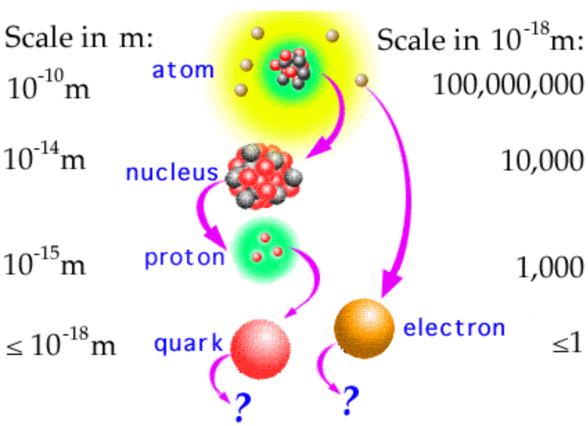

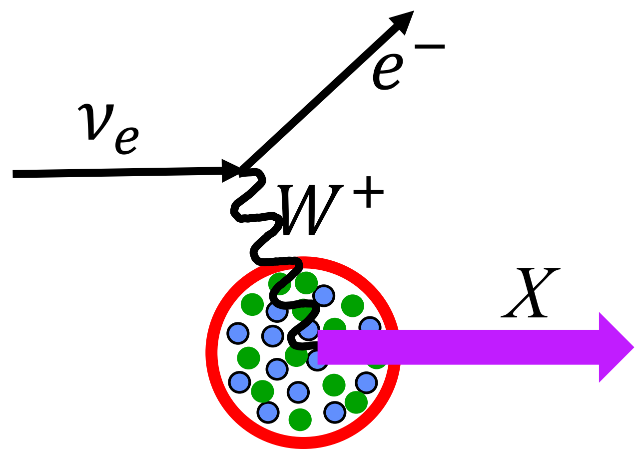

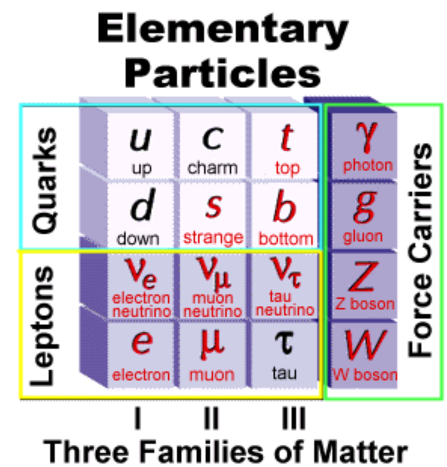

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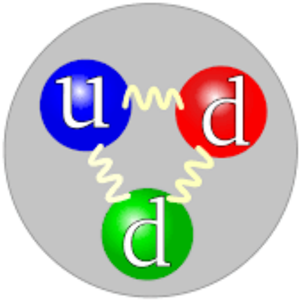
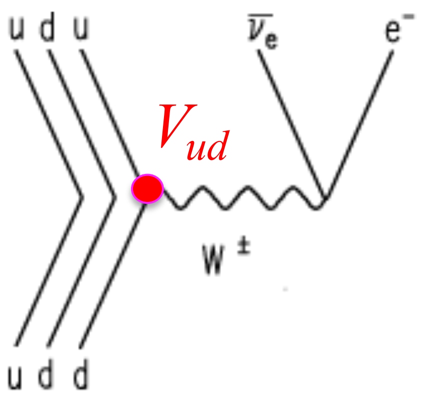
- USQCD Community white paper:
  Lattice QCD and Neutrino-Nucleus Scattering, *Eur.Phys.J.A* 55 (2019) 11, 196
- Snowmass 2021 White Paper
  Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators. e-Print: 2203.09030 [hep-ph]

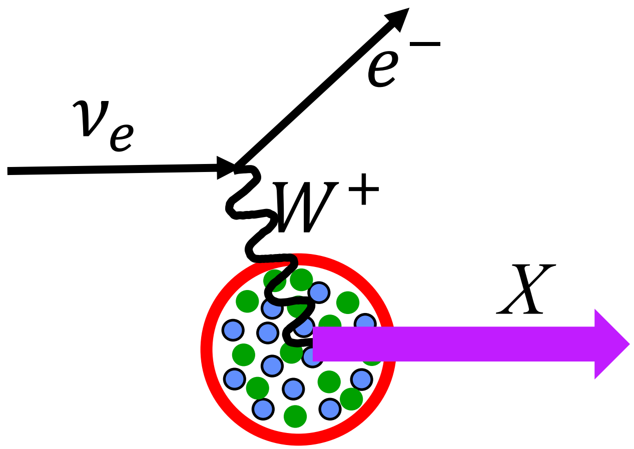
Outline:

- What LQCD can provide for ν-nucleus oscillation experiments
  - Axial and vector form factors of nucleons [nuclei are much more challenging]
  - Nuclear corrections $(p, n) \rightarrow (C^{12}, O^{16}, Ar^{40})$
- Challenges to the calculations of nucleon matrix elements
  - Signal-to-noise falls as $e^{-(M_N - 1.5 M_\pi)\tau}$
  - Excited states in nucleon correlation functions
  - Extrapolation in $\{a, M_\pi, M_\pi L\}$
- FF must satisfy PCAC
  - What we learned from $\langle N(p_f) | A_4(q) | N(p_i) \rangle$
  - Towers of $N\pi$, $N\pi\pi$, states contribute to axial and PS correlators
- Comparison of published results for g_A, G_A
- Comparison with MINERvA and ν-D analyses
- Summary of unpublished results for g_A, G_A
- Transition matrix elements; Results for G_E , G_M
- Future

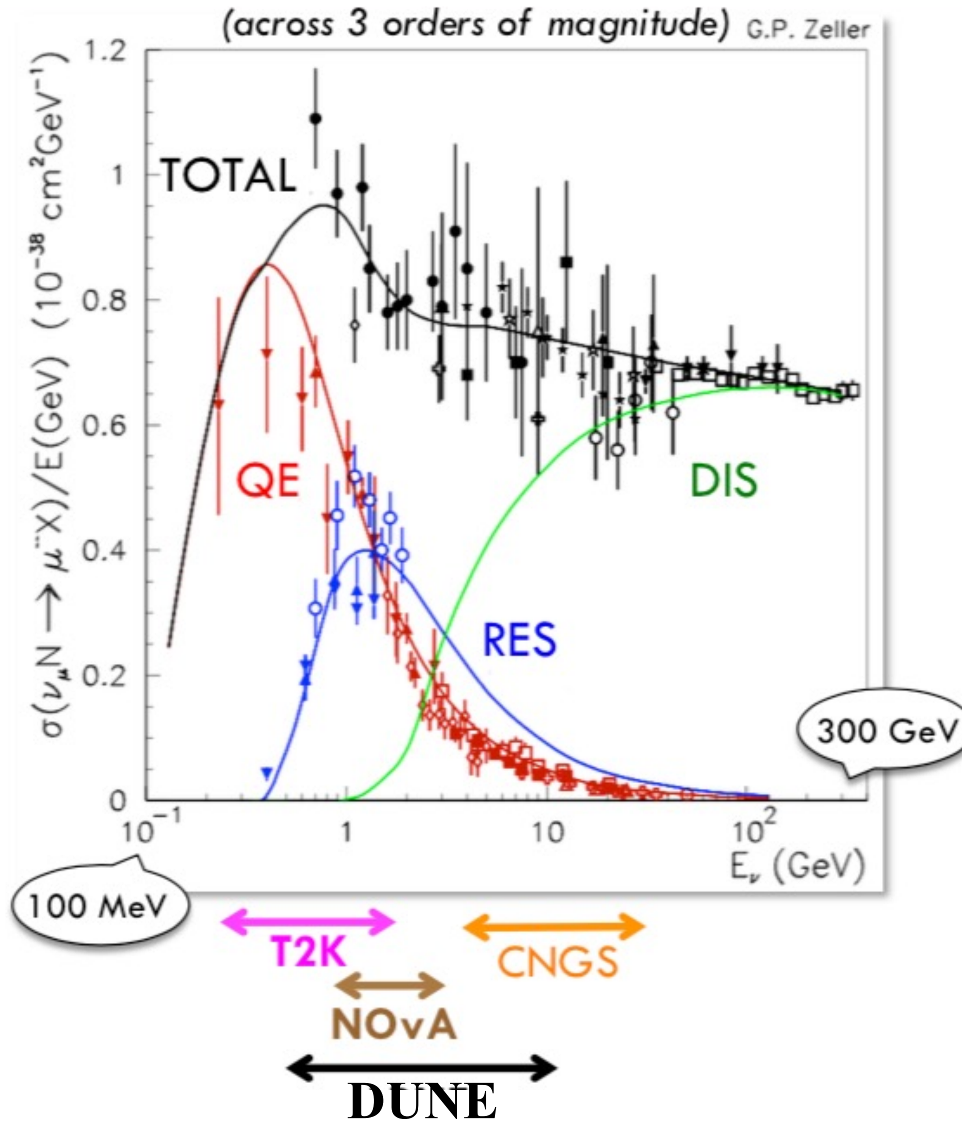




ν energy-range covers complex physics



- Neutrino energy and flux not known precisely
- Dynamics of struck Argon nucleus is too complex to simulate directly and connect to final states seen in the detectors



Ultimate Goal: Inputs for DUNE

Two matrix elements for $\nu - {}^{40}Ar$ scattering.

$$\langle X \,|\, A_\mu(q) \,|\, {}^{40}Ar \rangle$$

$$\langle X \,|\, V_\mu(q) \,|\, {}^{40}Ar \rangle$$

Building blocks for different energy regions: starting with nucleons

$\langle p \,|\, J_\mu^W(q) \,|\, n \rangle$ Quasi-elastic

$\langle n\pi \,|\, J_\mu^W(q) \,|\, n \rangle$, $\langle \Delta \,|\, J_\mu^W(q) \,|\, n \rangle$ Resonant

$\langle \mathrm{X} \,|\, J_\mu^W(q) \,|\, n \rangle$ DIS

Including nuclear effects in complex nuclear targets

Nuclear many body Hamiltonian takes as input matrix elements involving successively more multi-particles

- One nucleon $\langle\, p \,|\, J_\mu^+(q) \,|\, n \,\rangle$
- Transition $\langle\, n\pi \,|\, J_\mu^w(q) \,|\, n \,\rangle$, $\langle\, \Delta | \, J_\mu^w(q) \,|\, n \,\rangle$
- Two nucleon $\langle\, n\, p \,|\, J_\mu^{w+}(q) \,|\, n\, n \,\rangle$

Discussed by Noemi Rocco

See also Snowmass 2021 white paper:

The ν-n differential cross-section:

$$\frac{d\sigma}{dQ^2}\begin{pmatrix} \nu_l + n \to l^- + p \\ \bar{\nu}_l + p \to l^+ + n \end{pmatrix}$$
$$= \frac{M^2 {G_F}^2 cos^2\theta_c}{8\pi {E_\nu}^2}\left\{A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right\},$$

$$\begin{aligned} A(Q^2) &= \frac{(m^2+Q^2)}{M^2}\Big[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\ &\quad -\frac{m^2}{4M^2}\left((F_1+F_2)^2 + (F_A+2F_P)^2 - 4\left(1+\frac{Q^2}{4M^2}\right)F_P^2\right)\Big], \\ B(Q^2) &= \frac{Q^2}{M^2}F_A(F_1+F_2), \\ C(Q^2) &= \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2). \end{aligned}$$

$\langle N A_\mu N \rangle \to$ linear combination of F_A , $\tilde{F}_P$

$\langle N V_\mu N \rangle \to G_E$, G_M

F_A = axial form factor
$G_E = F_1 - \tau F_2$ Electric
$G_M = F_1 + F_2$ Magnetic
$\tau = Q^2/4M^2$
$M = M_p = 939$ MeV
m=mass of the lepton

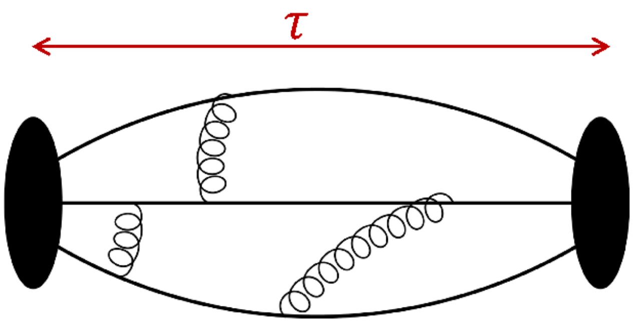
Analysis of (e, μ, ν)-n scattering involves 5 Form Factors & 3 charges g_A, μ, g_p^*

- $G_E(Q^2)$ Electric
- $G_M(Q^2)$ Magnetic
- $G_A(Q^2)$ Axial
- $\tilde{G}_P(Q^2)$ Induced pseudoscalar
- $G_P(Q^2)$ Pseudoscalar (extracted from $\langle NPN \rangle \rightarrow G_P$)
- Lattice methodology is common: all calculated at the same time
- Precise experimental data exist for $G_E(Q^2)$ and $G_M(Q^2)$
- Axial ward identity (PCAC) relates $G_A(Q^2)$, $\tilde{G}_P(Q^2)$, $G_P(Q^2)$

- $G_E(Q^2 = 0)$ $= 1$ Conserved vector charge
- $G_M(Q^2 = 0)$ $= \mu = 4.7058$ Magnetic moment
- $G_A(Q^2 = 0)$ $= g_A = 1.276(2)$ Axial charge
- $\tilde{G}_P(Q^2 = 0.88 m_\mu^2) = g_p^* = 8.06(55)$ Induced pseudoscalar charge

Lattice QCD gives us

2-point function



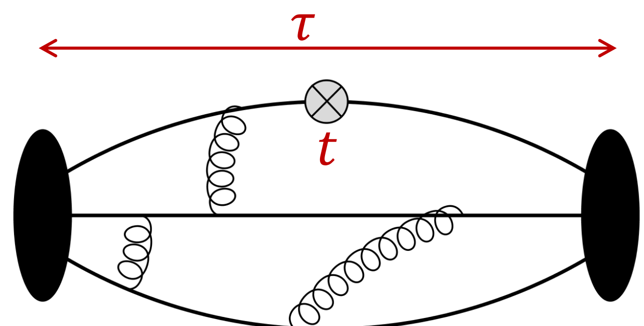
$$\langle \Omega \,|\, \widehat{N_\tau}^{\dagger} \; \widehat{N}_0 \,|\, \Omega \rangle$$

$$\Gamma^{2pt}(\tau) = \sum_i |A_i|^2 e^{-E_i \tau}$$

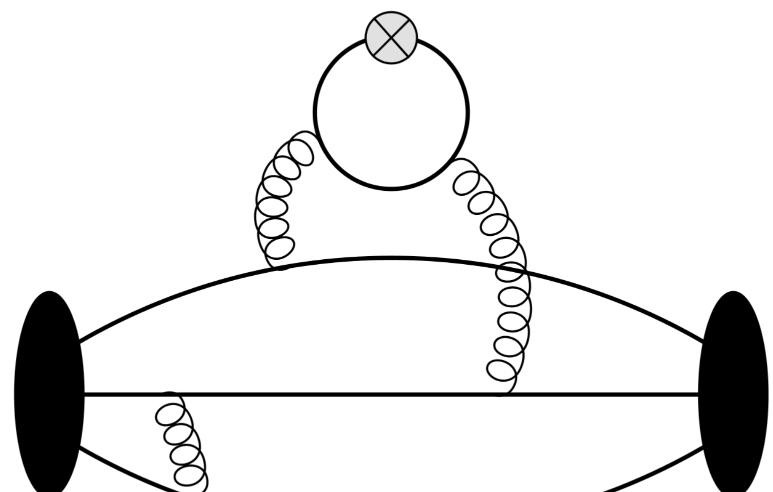
$$\langle \Omega \,|\, \widehat{N_\tau}^{\dagger} \, O(t) \, \widehat{N}_0 \,|\, \Omega \rangle$$

$$\Gamma_O^{3pt}(t,\tau) = \sum_{i,j} A_i^* A_j \langle i|O|j\rangle e^{-E_i t - E_j(\tau - t)}$$

3-point functions



Connected



Disconnected

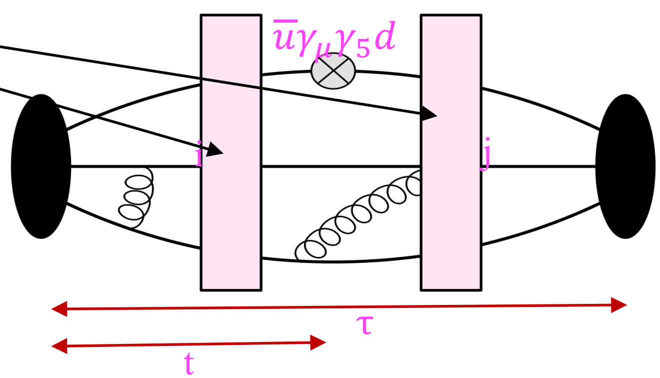
Calculations of nucleon 2,3-point functions using LQCD are mature

Spectrum (energies E_i, amplitudes A_i) and ME are extracted from fits to the spectral decomposition of 2- and 3-point functions

$$\Gamma^{2pt}(\tau) = \sum_i |A_i|^2 e^{-E_i \tau}$$

$$\Gamma_O^{3pt}(t,\tau) = \sum_{i,j} A_i^* A_j \underbrace{\langle i|O|j\rangle}_{\text{Extract } \langle 0|\,O\,|\,0\,\rangle} e^{-E_i t - E_j(\tau - t)}$$

All states $|i\rangle$ with the same quantum numbers as N contribute unless A_i (or A_j) is zero



Radial excited States:
N(1440), N(1710)
Towers of multihadrons states

$N(\vec{k})\pi(-\vec{k})$ > 1200 MeV

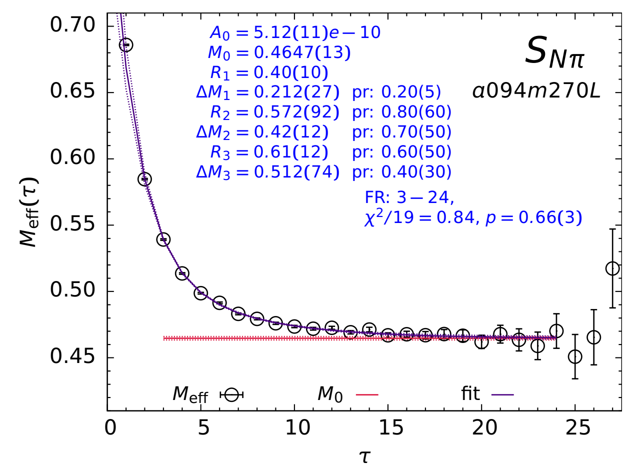
$N(0)\pi(\vec{k})\pi(-\vec{k})$ > 1200 MeV

but removing ESC from multihadron states remains a challenge

Challenges

- Need large τ to "kill" states with <u>*small*</u> mass gap ($\Delta M \sim 300$)

- Cannot go to large enough τ because the signal/noise degrades as $e^{-(M_N - 1.5 M_\pi)\tau}$
  - Signal: 2-pt: $\tau \sim 2$fm ; 3-pt: $\tau \sim 1.5$fm



- Typical interpolating operator $\hat{N}$ couples to the nucleon, its excitations and multi-hadron states with the same quantum numbers

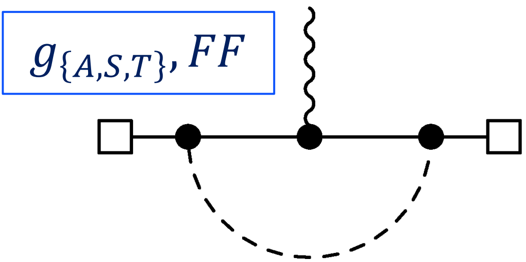
- As $\vec{q} \rightarrow 0$, the towers of physical $N\pi$, $N\pi\pi$, states become arbitrarily dense above ~1230 MeV (the **Δ** region)

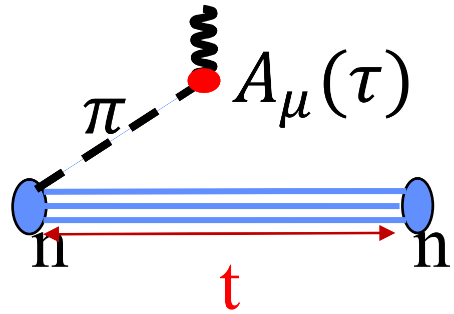
- Quantities impacted by Nπ, Nππ, states should be analyzed on $M_\pi \lesssim 200\ MeV$ ensembles

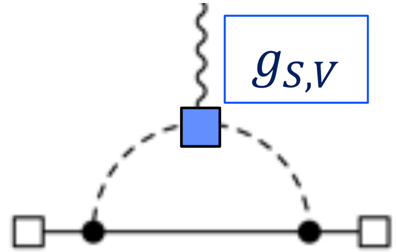
- Excited states that give significant contribution to a particular ME are not known *a priori*. *χPT is a very useful guide*

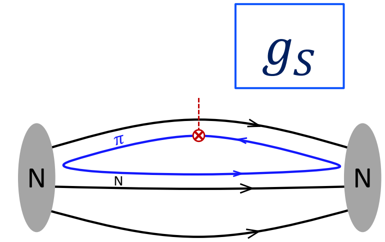
- *The potential of variational methods for isolating the ground state is just starting to be realized!*

χPT and excited states



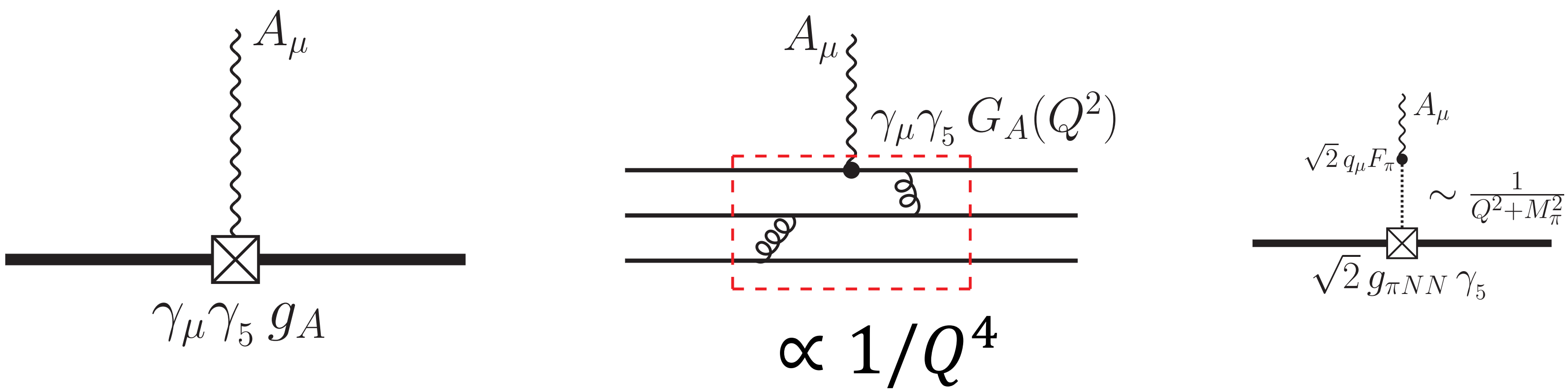






- Corrections from pion loops are in all matrix elements
- Loops that originate or end at sources are ESC. These can be removed by a perfect source.
- Loops that originate on the nucleon line give rise to both: corrections to the physical result and excited state contributions (from pion going on-shell in Minkowski)
- The latter are suppressed exponentially by the mass gap
- Unless there are large cancellations, both should be considered in (i) removing excited state contamination in getting the data, and in (ii) the final chiral fits to the data

Extract Axial-vector Form Factors, $G_A, \widetilde{G}_P, G_P$



3-point functions → ground state matrix elements → Form factors

$$\langle N(p_f) | A^\mu(q) | N(p_i) \rangle = \bar{u}(p_f) \left[\gamma^\mu G_A(q^2) + q_\mu \frac{\tilde{G}_P(q^2)}{2M} \right] \gamma_5 u(p_i)$$

$$\langle N(p_f) | P(q) | N(p_i) \rangle = \bar{u}(p_f) G_P(q^2) \gamma_5\, u(p_i)$$

PCAC [$\partial_\mu A_\mu = 2mP$] relates $G_A,\ \tilde{G}_P,\ G_P$

Constraints once FF are extracted from ground state matrix elements

1) PCAC ($\partial_u A_u = 2\hat{m}$P) requires

$$2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

2) In any [nucleon] ground state

$$\partial_4 A_4 = \left(E_q - M_0\right)A_4$$

3) G_A, $\tilde{G}_P$ extracted from $\langle N(p_f)|A_i(q)|N(p_i)\rangle$

must be consistent with $\langle N(p_f)|A_4(q)|N(p_i)\rangle$

Decomposition of ground state matrix elements: $\langle N_\tau A_\mu(t) N_0 \rangle$ provides an over-determined set

Choosing "3" the direction of spin projection

$$\langle N(p_f) | A_{1,2}(q) | N(p_i) \rangle \to -\frac{q_{1,2} q_3}{2M} \tilde{G}_P$$

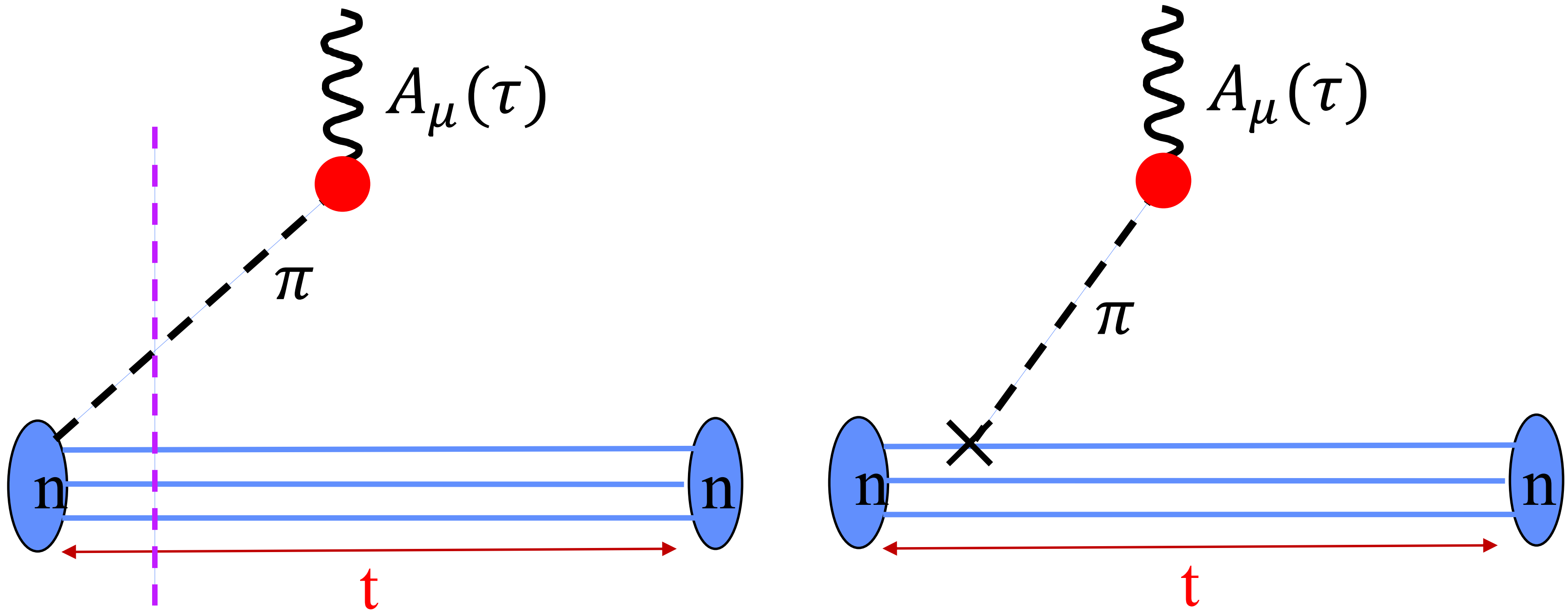
$$\langle N(p_f) | \, A_3(q) \, | N(p_i) \rangle \to - [\frac{q_3^2}{2M} \tilde{G}_P - (M + E) G_A]$$

Gives both $G_A, \ \tilde{G}_P$

$$\langle N(p_f) | \, A_4(q) \, | N(p_i) \rangle \to -q_3 \, [\frac{E-M}{2M} \tilde{G}_P - G_A]$$

Redundant. Dominated by excited states

χPT: $N\pi$ state coupling large in the axial channel



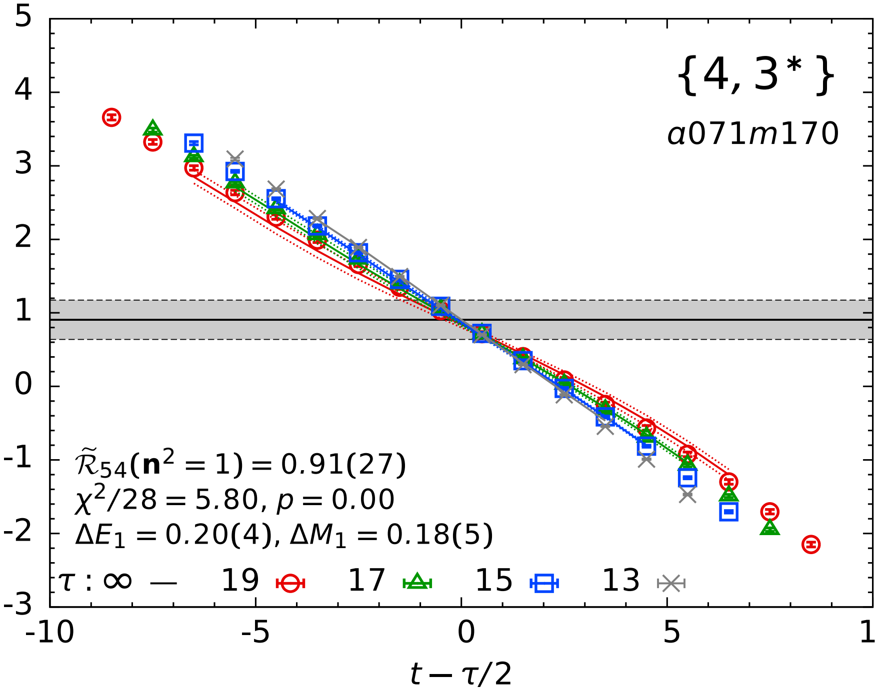
Enhanced coupling to $N\pi$ state: Since the pion is light, the vertex can be anywhere in the lattice 3-volume

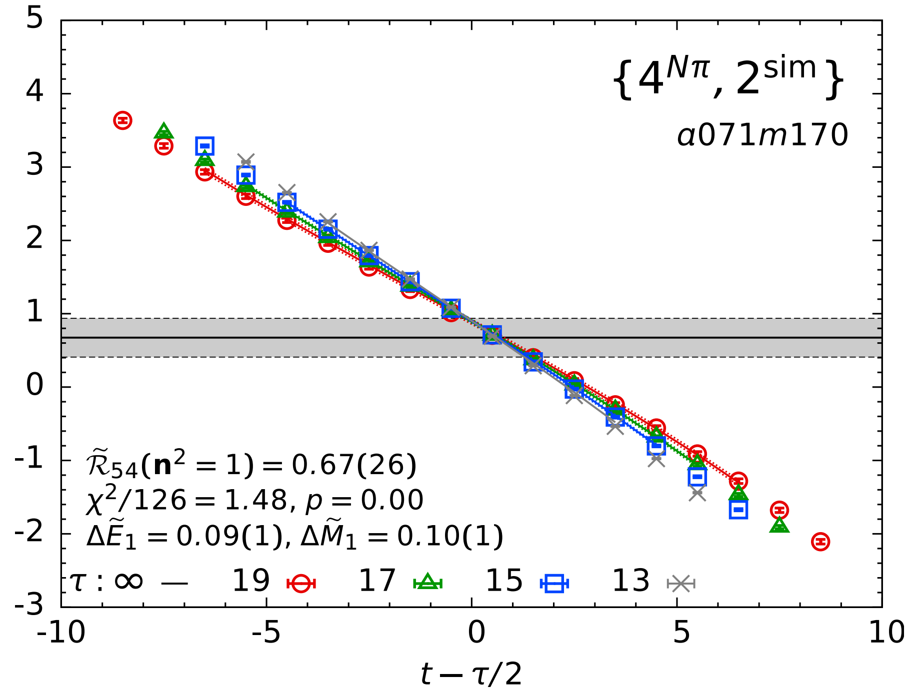
$$A_i^* \quad \langle i|A_4|j\rangle$$

$$\sim V^{-1} \qquad\qquad \sim V$$

Oliver Bär: Phys. Rev. D 99, 054506 (2019), Phys. Rev. D 100, 054507 (2019)

$\langle N_\tau A_4(t) N_0 \rangle$ has large ESC
Fits with $N\pi$ state preferred





Data driven evidence for $N\pi$ state.
Including $N\pi$ state also addressed PCAC

Gupta et al, PhysRevD.96.114503 (2017) → Jang et al, PRL 124 (2020) 072002

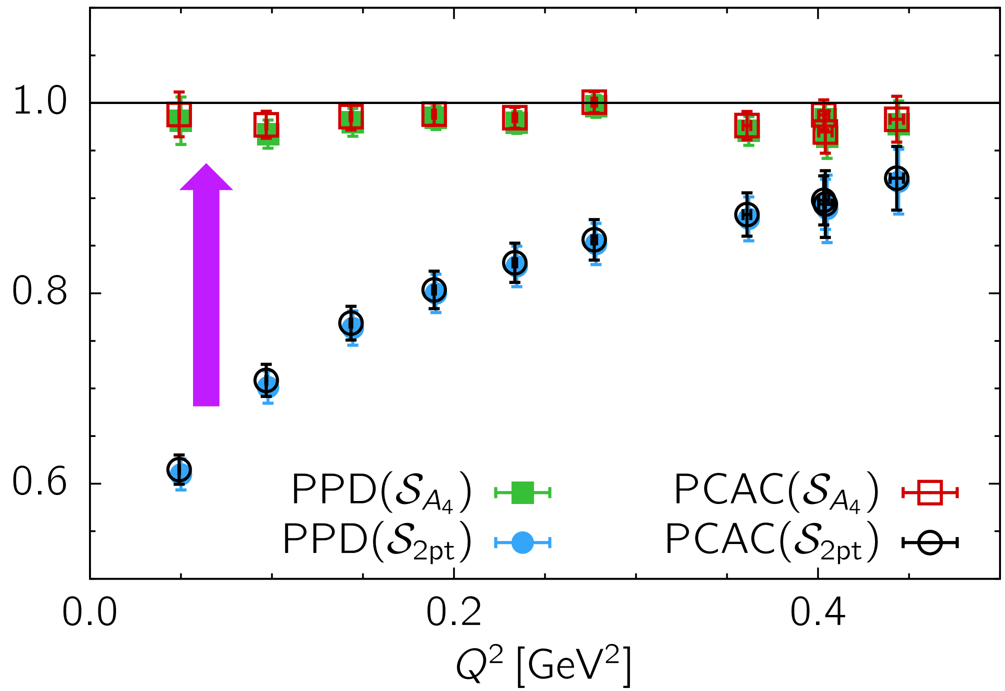
2017→2019: Resolution of PCAC and PPD

Gupta et al, PhysRevD.96.114503 → Jang et al, PRL 124 (2020) 072002

On including low mass $N_{p=0}\pi_p$ and $N_p\pi_{-p}$ excited states neglected in previous works, FF satisfy PCAC and PPD at ~5%

$$\frac{\hat{m}G_P}{M_N G_A} + \frac{Q^2 \tilde{G}_P}{4M_N^2 G_A} = 1$$

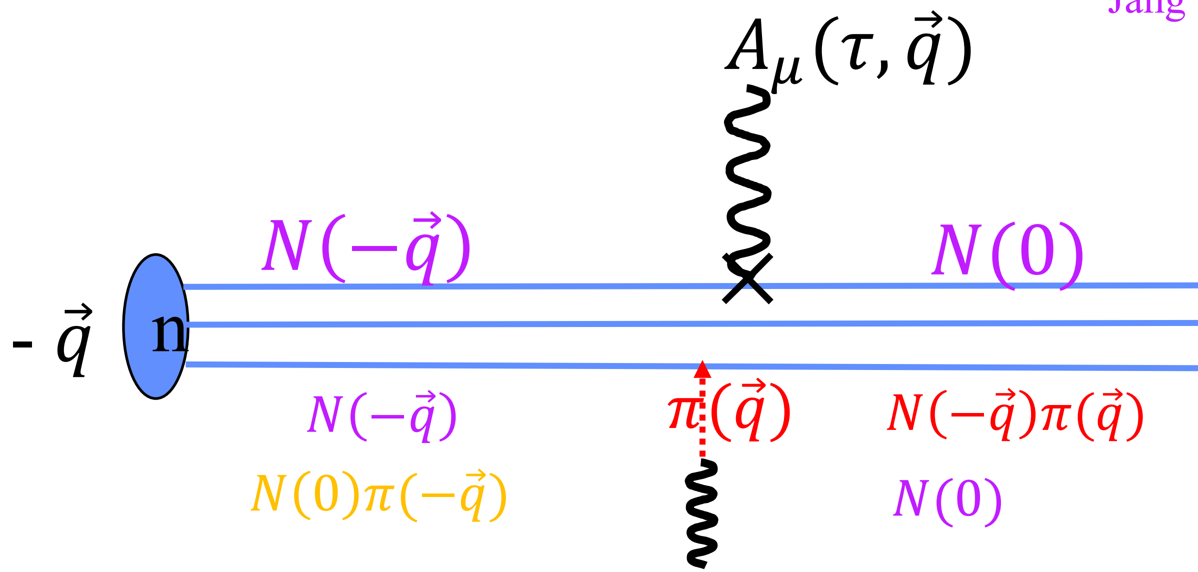
$$\frac{Q^2 + M_\pi^2}{4M_N^2} \frac{\tilde{G}_P(Q^2)}{G_A(Q^2)} = 1$$



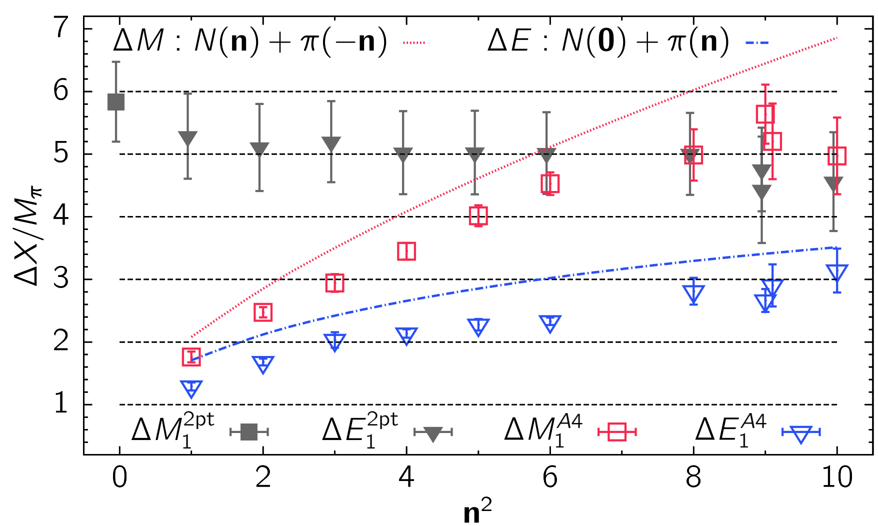
•Also see RQCD Collaboration: *JHEP* 05 (2020) 126, 1911.13150

$N\pi$ state in the axial channel

Jang et al, PRL 124 (2020) 072002



Mass gaps extracted from fits match the above picture

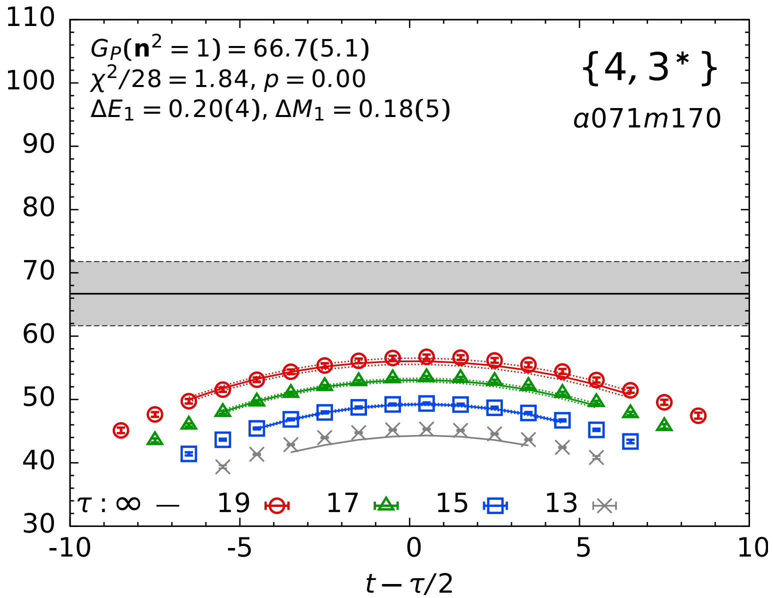


ΔM_1^{A4} and ΔE_1^{A4} are outputs of 2-state fits and not driven by priors

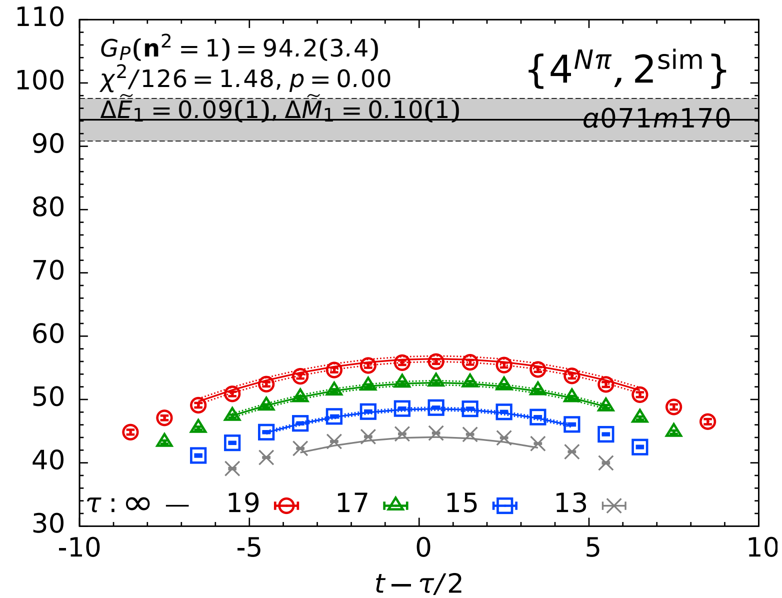
How large is the “$N\pi$” effect?

Output of a simultaneous fit to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ (called $\{4^{N\pi}, 2^{sim}\}$ fit) increases the form factors by:

$G_A \sim 5\ \%$
$\tilde{G}_P \sim 35\ \%$
$G_P \sim 35\ \%$



Standard 3-state fit to $\langle P \rangle$



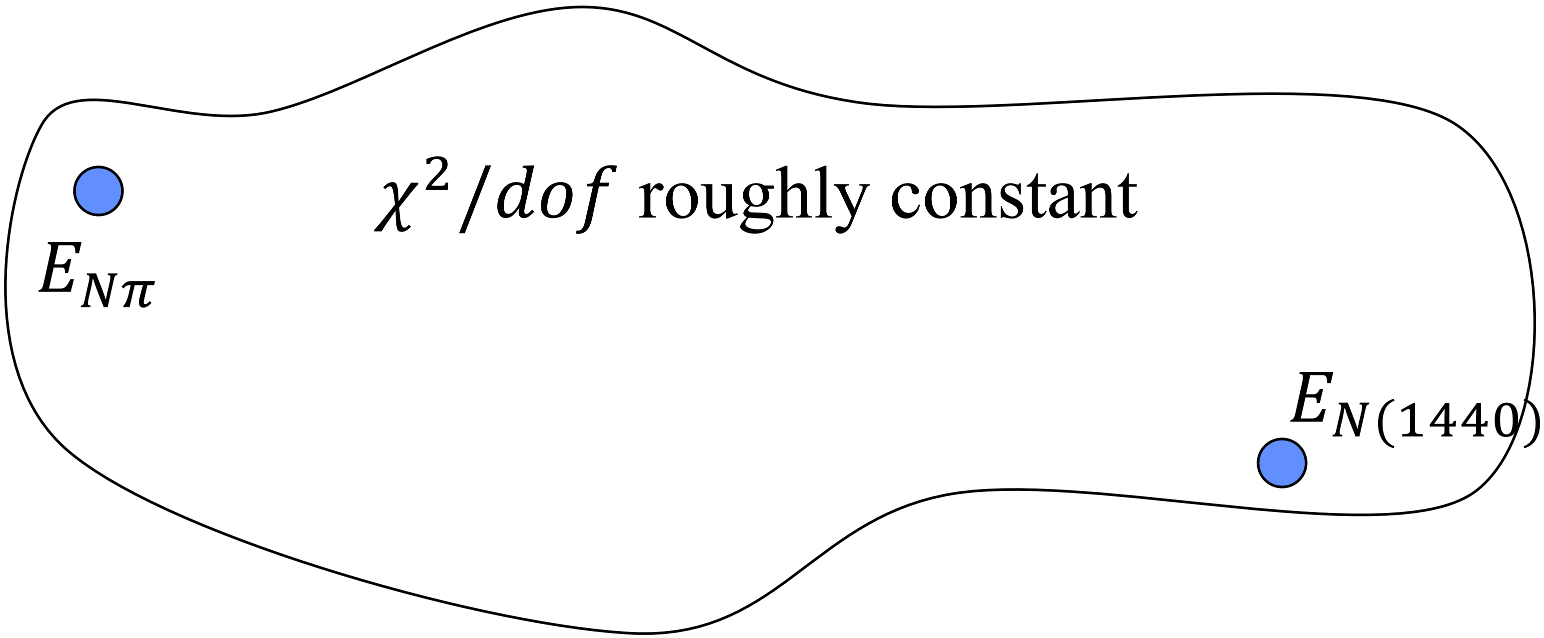
Simultaneous 2-state to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ correlators

Essential steps in the analysis

- Remove ESC from correlation functions
- Decompose into form factors to get $G(Q^2)$ on each ensemble
- Parameterize this $G(Q^2)$
- Perform CCFV extrapolation to get $G(Q^2)$ in the continuum
- Parameterize this $G(Q^2)|_{cont}$

Model averaging should include model choices at each step that have significant effect on result

If ESC is the largest systematic and fits do not select between $\{A_i, E_i\}$



- 2-state fit: Model average different E_1
- 3-state fit: Model average over $\{E_1, E_2\}$

Consistency in the extraction of g_A

- g_A from forward ME versus $g_A = G_A(Q^2 \to 0)$
- With / without including $N\pi$ state in the analysis
- PCAC

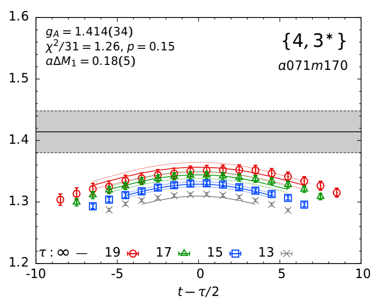
Spectrum from Γ^2

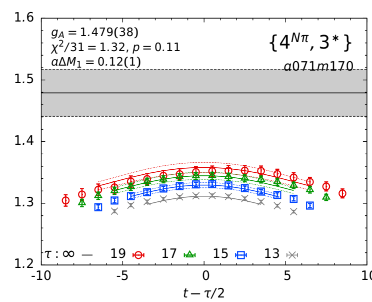
$N\pi$ included in fits (via A_4 or priors)

g_A (Forward ME)

1.22(5)

1.28(5)





$g_A = G_A(Q^2 \to 0)$

1.19(5)

1.32(6)

$G_A, \tilde{G}_P, G_P$ do not satisfy PCAC

$G_A, \tilde{G}_P, G_P$ with $N\pi$ satisfy PCAC

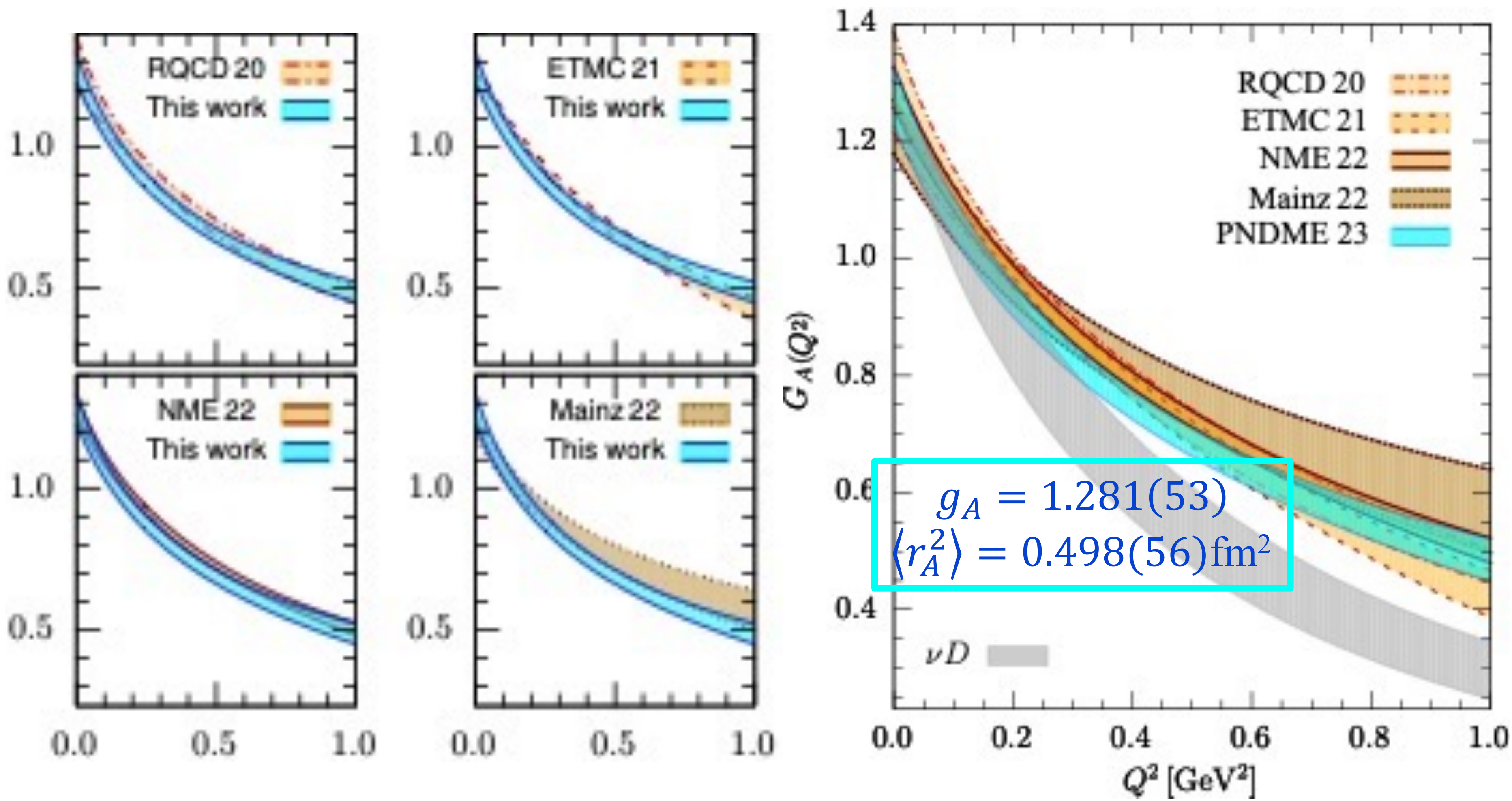
Calculations reviewed in 2305.11330

Collab.	**Ens.**	**Excited State**	M_π	Q^2	**Continuum-chiral-finite-volume extrap**	g_A
PNDME 23	13	With $N\pi$	2 physical	$z^2 + z^2$	CCFV	1.292(53)(24)
Mainz 22	14	Simultaneous ESC, Q^2	2 physical	z^2	CCFV	1.225(39)(25)
NME 21*	7 (12)	With $N\pi$	2 170MeV	z^2	Ignore $\{a, M_\pi^2\ M_\pi^2\mathrm{L}\}$ dependence	1.32(6)(5)
ETMC 20	3	Without $N\pi$	3 physical	data	$\{a\}$	1.283(22)
RQCD 19/23	36 (47)	With $N\pi$ only for $\tilde{G}_P, G_P$		data		$[1.284^{28}_{27}]$

PNDME: arXiv:2305.11330,
NME: PRD 105, 054505 (2022),
RQCD: JHEP 05, 126 (2020), PRD 107, L051505 (2023)

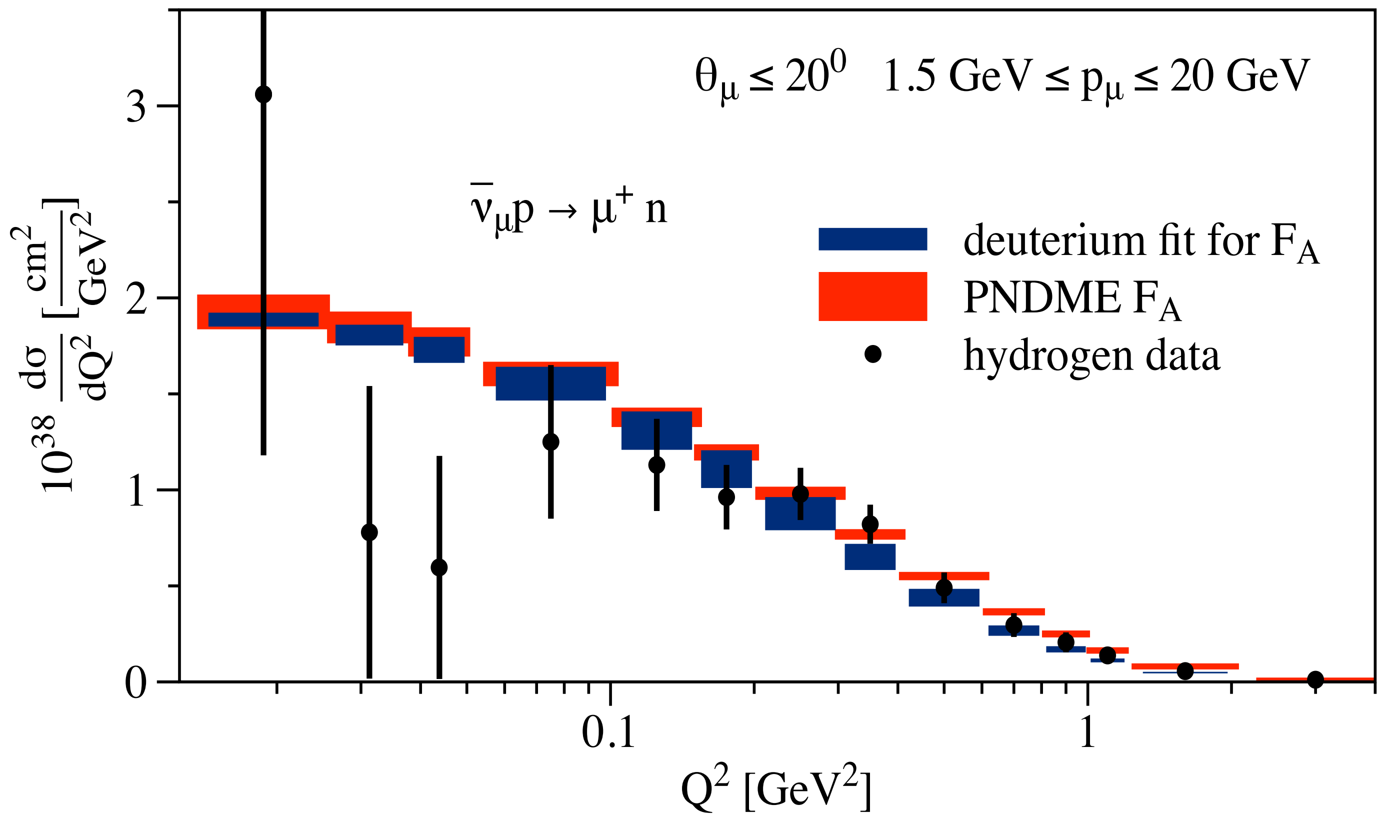
Mainz: PRD 106, 074503 (2022)
ETMC: PRD 103, 034509 (2021)

Comparing axial form factor from LQCD



A consensus is emerging

Comparing prediction of x-section using AFF from $\nu - D$ and PNDME with MINERvA data



$\underbrace{\hphantom{xxxxxxxxxx}}_{Q^2 < 0.2\ \mathrm{GeV}^2}$ $\underbrace{\hphantom{xxxxxx}}_{0.2 < Q^2 < 1}$ $\underbrace{\hphantom{xxxxxxxxxx}}_{Q^2 > 1\ \mathrm{GeV}^2}$

T. Cai, et al., (MINERvA) Nature **volume 614**, pages 48–53 (2023); Phys. Rev. Lett. 130, 161801 (2023)

Oleksandr Tomalak, Rajan Gupta, Tanmoy Bhattacharya, arXiv:2307.14920

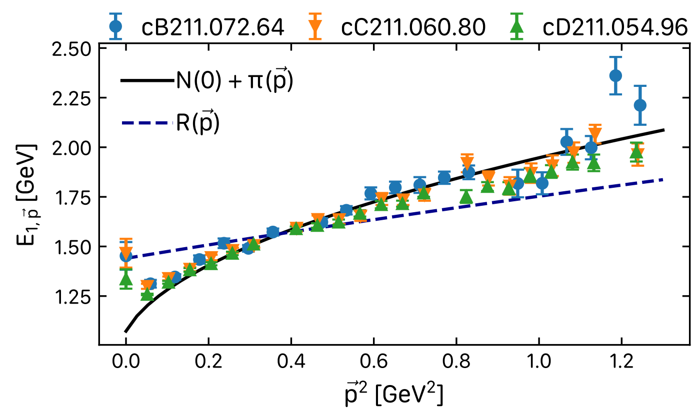
Mapping the AFF

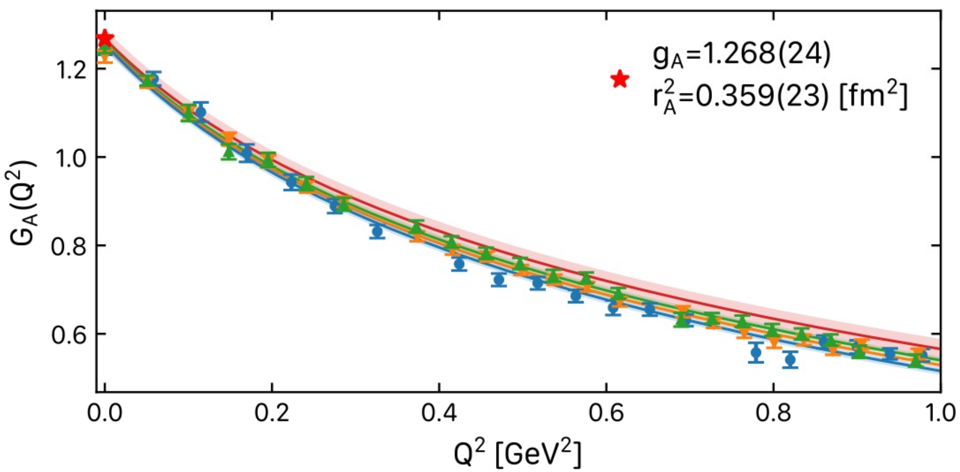
- $0 < Q^2 < 0.2\ \mathrm{GeV}^2$
  - This region will get populated by simulations with $M_\pi \approx 135$ MeV , a → 0, $M_\pi L > 4$
  - MINERνA data has large errors
  - Characterized by g_A and $\langle r_A^2 \rangle$ and $G_A(Q^2)$ parameterized by a z-expansion with a few terms
- $0.2 < Q^2 < 1\ \mathrm{GeV}^2$
  - Lattice data mostly from $M_\pi > 200$ MeV simulations
  - Competitive with MINERνA data. Cross check of each other
- $Q^2 > 1\ \mathrm{GeV}^2$
  - Lattice needs new ideas
  - MINERνA and future experiments

Update from ETMC (3 $M_\pi \approx 135$ MeV ensembles)

2+1+1-flavor twisted mass ensembles

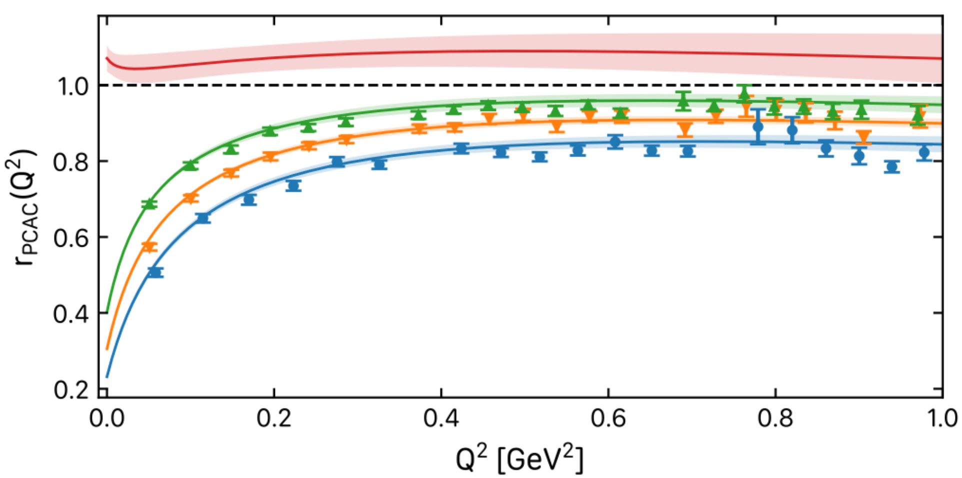
Ens. ID	latt. Vol.	a [fm]	Lm_π
cB211.072.64 (cB64)	$64^3\times128$	0.080	3.62
cC211.060.80 (cC80)	$80^3\times160$	0.069	3.78
cD211.054.96 (cD96)	$96^3\times192$	0.057	3.90





Excited state fits

- 2-state checked against 3-state
- $N\pi$ state not included
- 1st excited state mass ≈ *xx* MeV



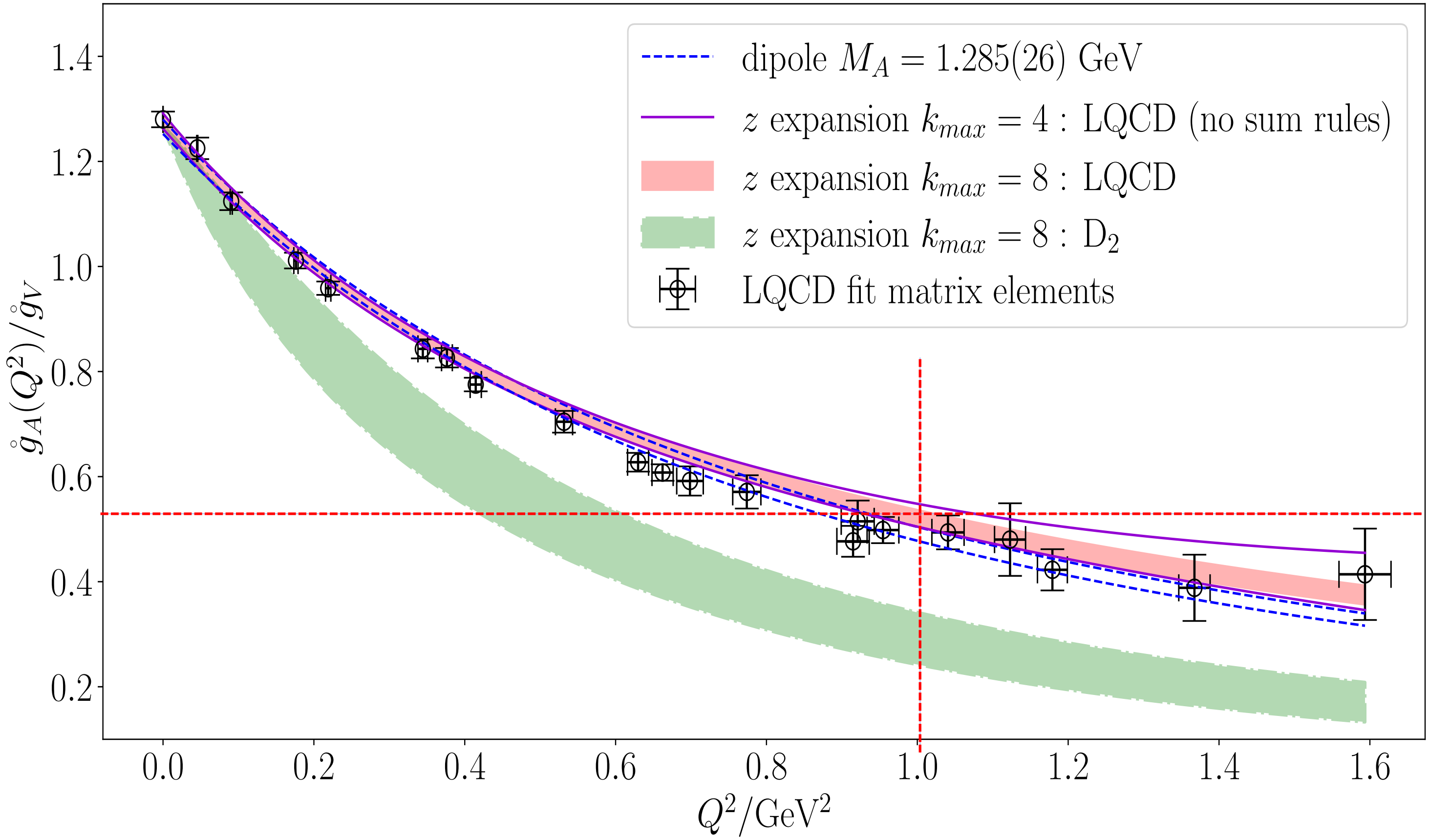
PCAC test of form factors

$$r_{\text{PCAC}} = \frac{\frac{m_q}{m_N} G_5(Q^2) + \frac{Q^2}{4m_N^2} G_P(Q^2)}{G_A(Q^2)}$$

Large cut-off effects in twisted mass involving pions

Update from CalLAT Collaboration
(A. Meyer, A. Walker-Loud)

domain-wall on HISQ calculation using sequential prop through sink
$48^3 \times 64$ ensemble (a12m130): $a^{-1} = 1.66$ GeV; $M_\pi = 132$ MeV
Gaussian sources for quark propagators
1000 X 32 (configurations X measurements)

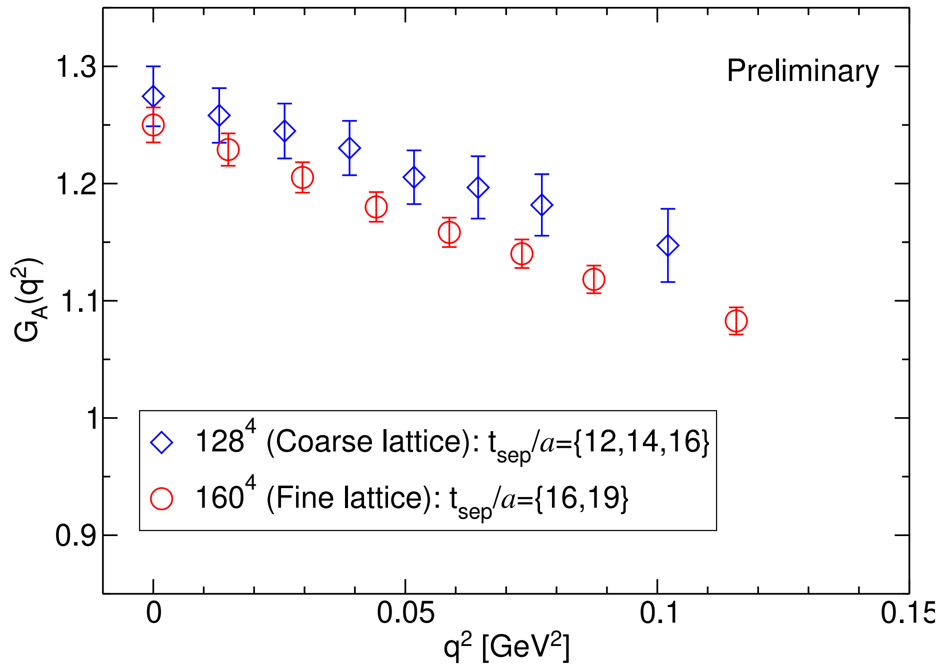


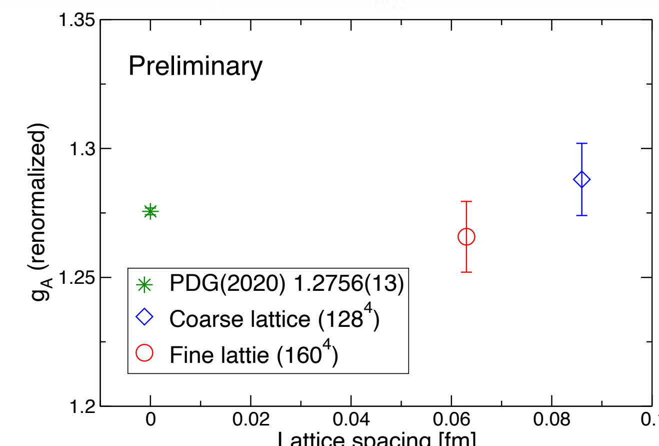
Updates from PACS

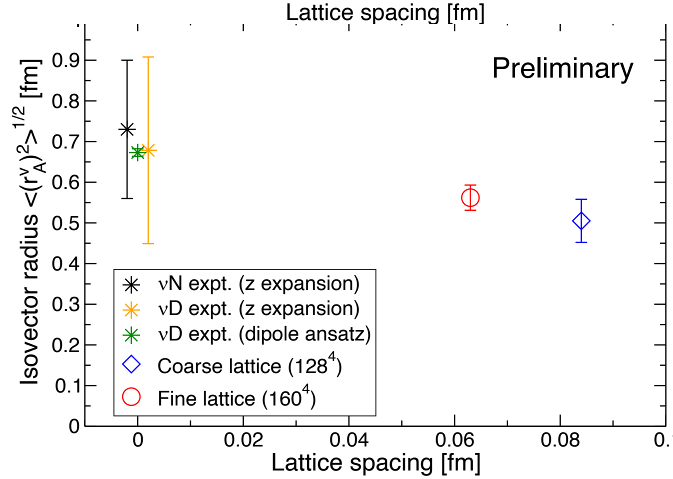
Talk by Ryutaro Tsuji

Stout smeared O(a) improved Wilson quark and Iwasaki gauge actions. 2+1 flavors

Lattice size	128^4	160^4
Spatial volume	$(10.9\ \text{fm})^3$	$(10.1\ \text{fm})^3$
Pion mass	135 MeV	135 MeV
Nucleon mass	0.935(11) GeV	0.946(3) GeV
Lattice spacing	0.086 fm	0.063 fm
$\lvert t_{\text{sink}} - t_{\text{src}}\rvert/a$	10, 12, 14, 16	13, 16, 19
Renormalization	SF, RI-MOM/SMOM	SF







Tuned exponential sources show very little excited-state effects in axial

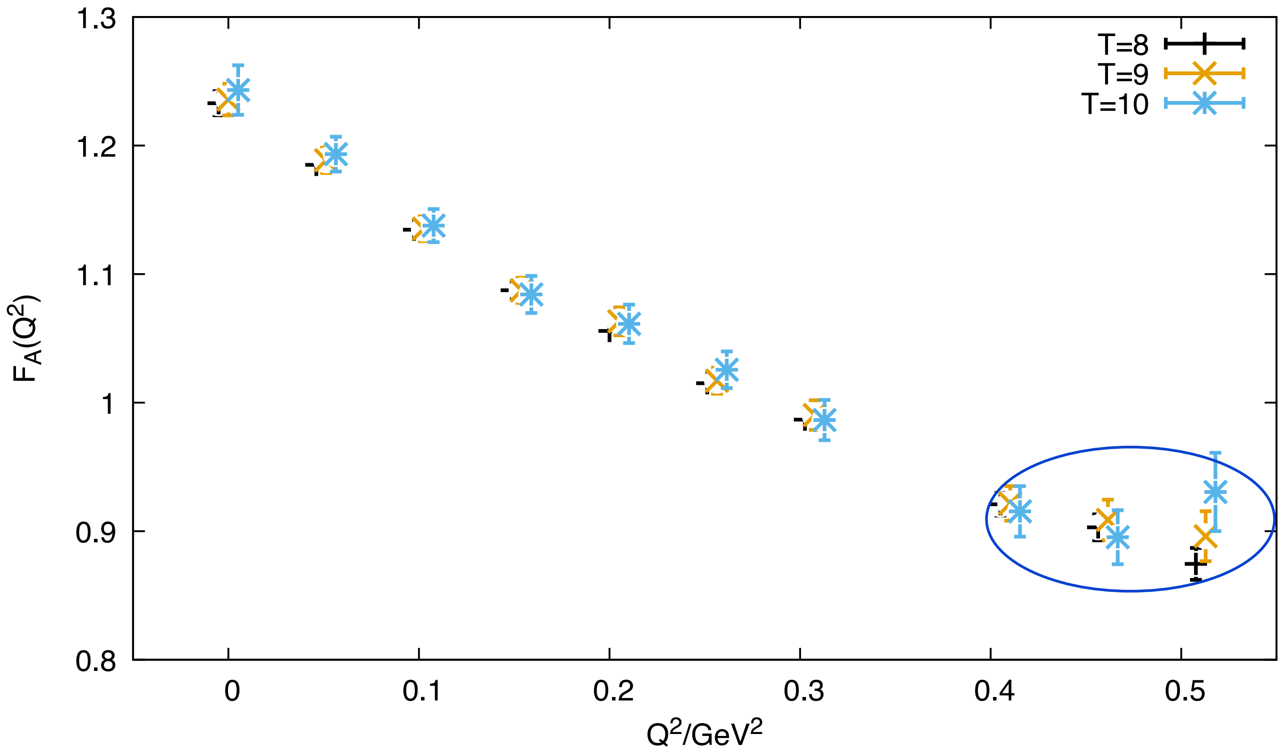
Update from LHP/RBC/UKQCD Collaboration

(S. Ohta arXiv:2211.16018)

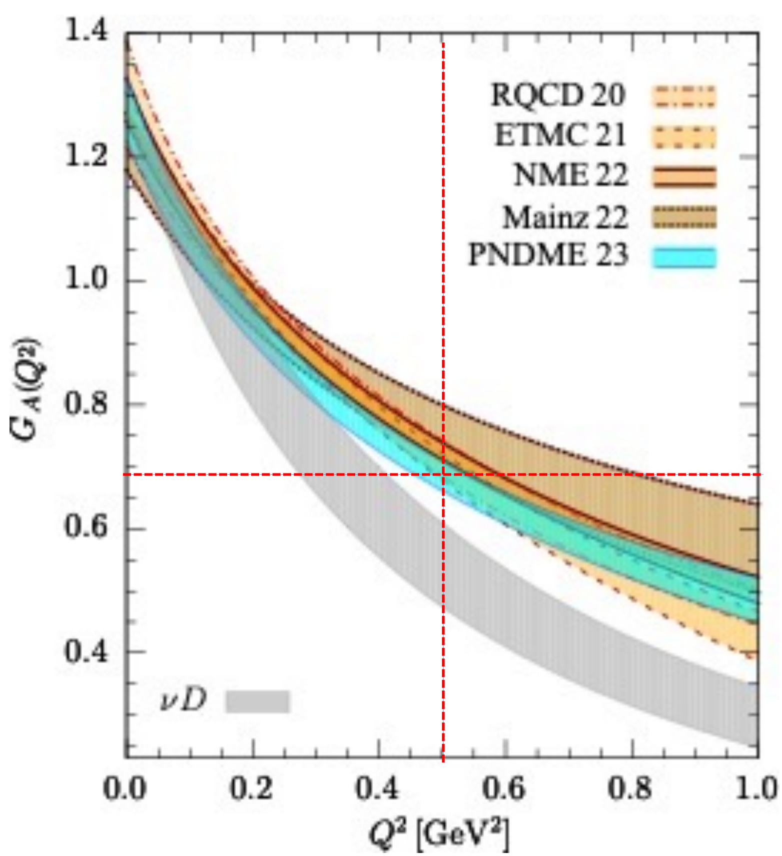
2+1-flavor domain-wall-fermions
$48^3 \times 96$ ensemble: $a^{-1} = 1.730(4)$ GeV
Gaussian sources for quark propagators
120 configurations, ## measurements

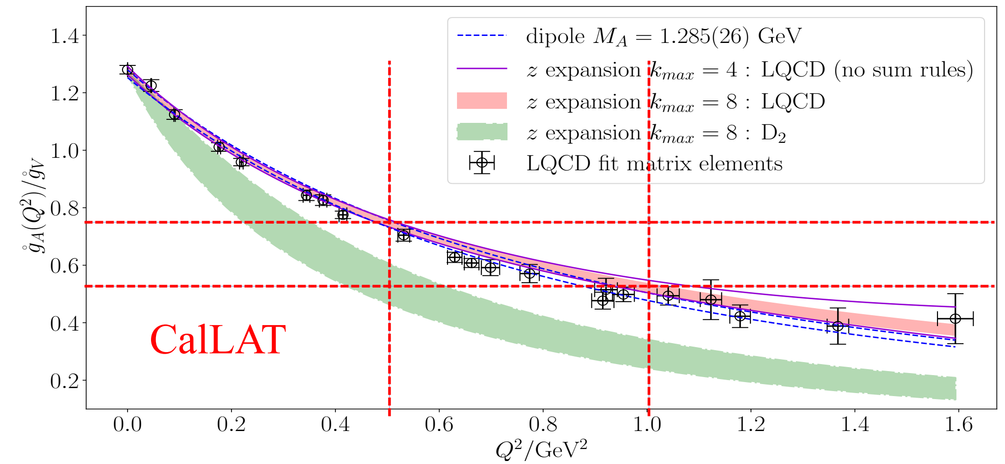
Data at $\tau = 8{,}9{,}10$ do not show significant change indicating small excited state effect

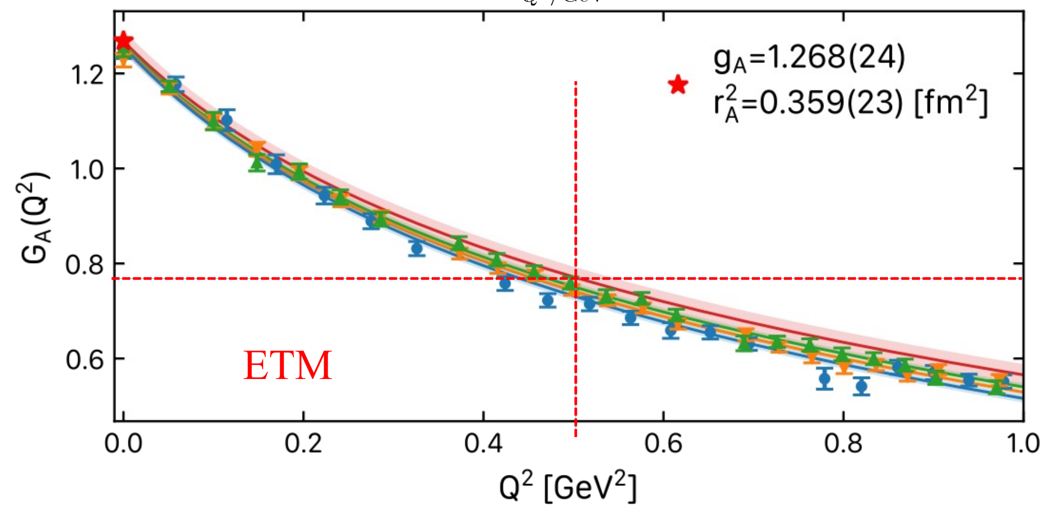
Slower fall-off than PNDME 23 data

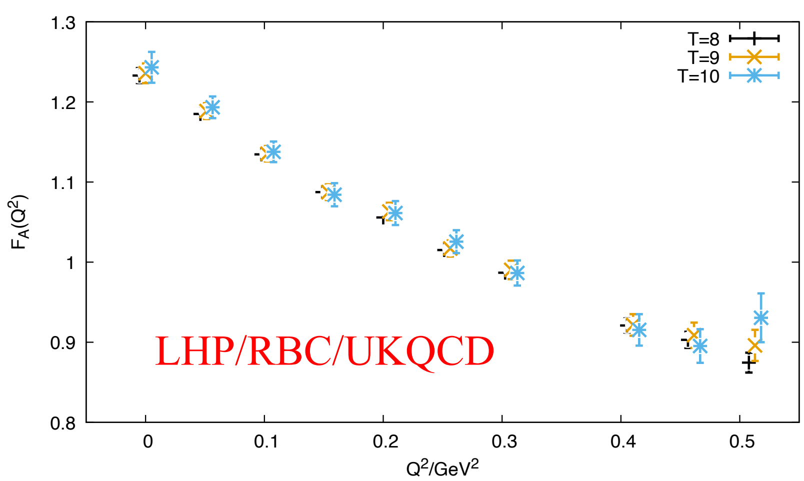


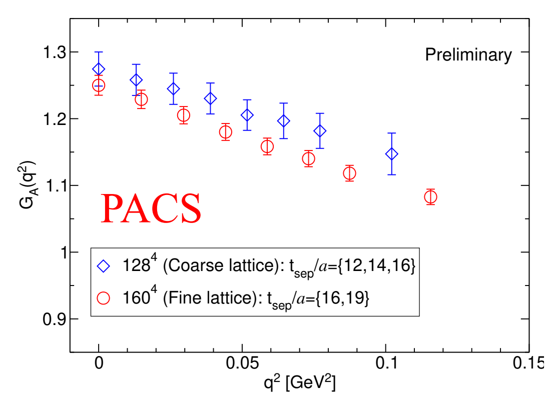
Comparison with unpublished data



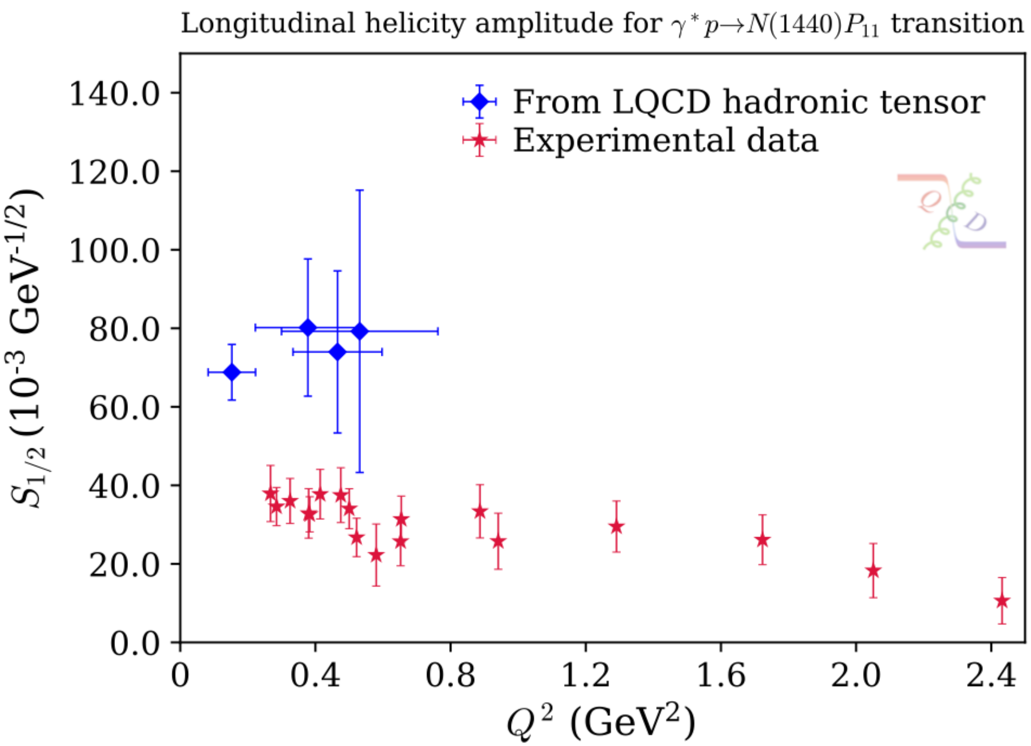








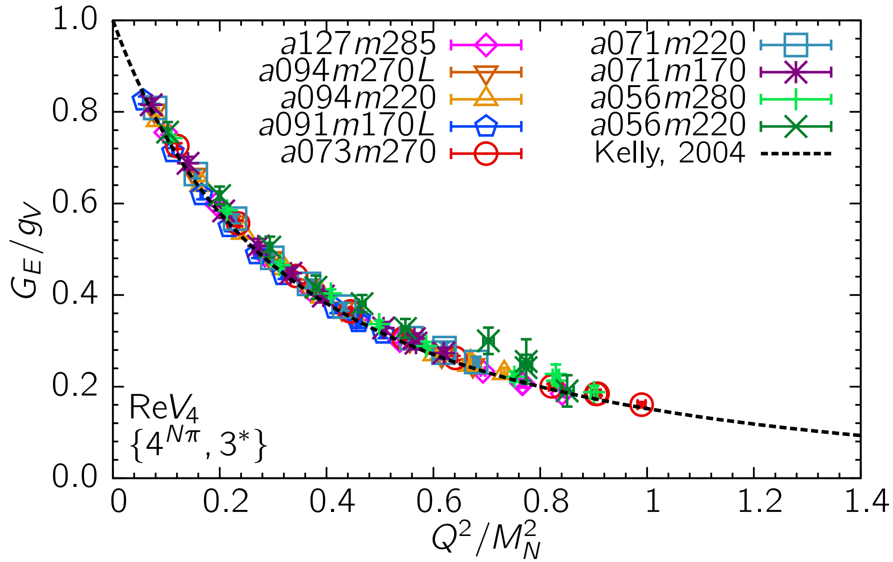
Roper transition helicity amplitude using $\langle NV_\mu V_\nu N \rangle$ hadronic tensor



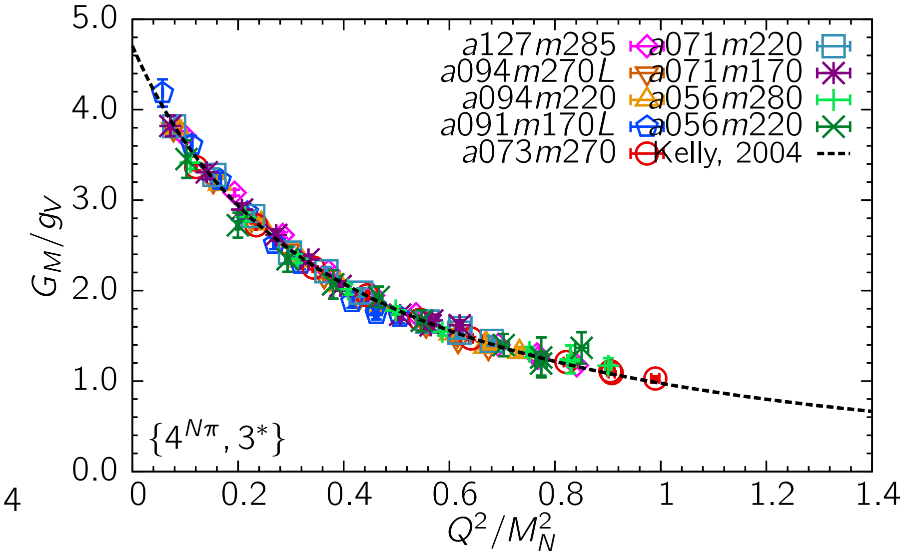
Extracted using a simple spectral decomposition of $H_{\mu\nu}$ that gives reasonable estimates

Talk by Raza Sufian for χQCD

Electric & Magnetic FF



Electric

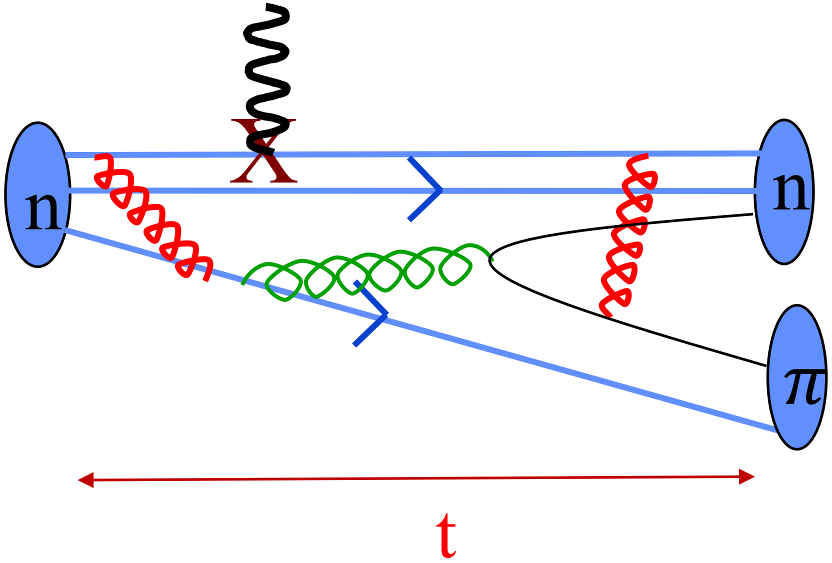


Magnetic

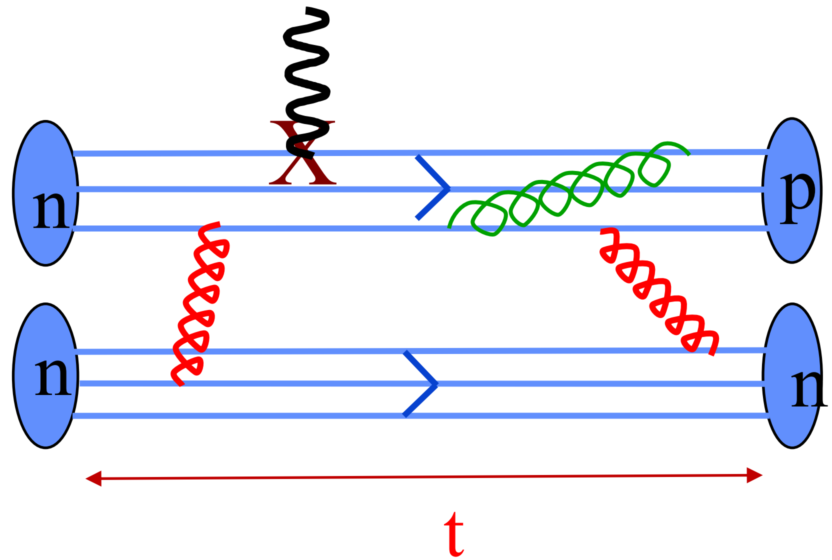
- The extraction of electric and magnetic form factors is insensitive to the details of the excited states
- Vector meson dominance → $N\pi\pi$ state should contribute (some evidence)
- The form factors do not show significant dependence on the lattice spacing or the quark mass
- Good agreement with the Kelly curve. Validates the lattice methodology
- Improve precision and get data over larger range of parameter values

Much harder:

Multi-hadron states



$$\langle\, n\, \pi^{+} \mid J^{+}_{\mu}(q) \mid n \,\rangle$$



$$\langle\, n\, p \mid J^{+}_{\mu}(q) \mid n\, n \,\rangle$$

See

- Barca et al, 2211.12278 , 2110.11908
- NPLQCD Collaboration, *Phys.Rev.Lett.* 120 (2018) 15, 152002
- Nuclear matrix elements from lattice QCD for electroweak and beyond-Standard-Model processes, 2008.11160 [hep-lat]

Looking ahead

- Challenges in lattice calculations of nucleon matrix elements:
  - Signal to noise degrades as $e^{-(M_N - 1.5 M_\pi)t}$
  - removing multi-hadrons excited states to get ground state ME
  - including multi-hadrons in initial and/or final state for transition ME
- Continue to develop a robust analysis strategy for removing dominant excited states in various nucleon matrix elements
- Improve chiral and continuum extrapolation. Simulate at more $\{a, M_\pi\}$
- Current $0.04 < Q^2 < 1$ GeV^2 Extend to larger Q^2 for DUNE
- Transition matrix elements
- Goal: Perform a comprehensive analysis of scattering data with input of lattice results for g_A, $G_E(Q^2)$, $G_M(Q^2)$, $G_A(Q^2)$, $\tilde{G}_P(Q^2)$

Improvements in algorithms and computing power are needed to reach few percent precision