Studying gauged Yukawa models and their supersymmetric limit on the lattice

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Supersymmetric gauge theories and lattice simulations

- Supersymmetric gauge theories provided important insights, lattice simulations can complement these findings, SUSY broken on the lattice.
- successful simulations: $\mathcal{N} = 1$ and 4 supersymmetric Yang-Mills theory; lower dimensional SUSY gauge theories

$\mathcal{N} = 1$ supersymmetric Yang-Mills theory:

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{\lambda} (\not{D} + m_g) \lambda$$

- λ adjoint Majorana fermion
- SUSY transformations: $\delta A_\mu = -2i\bar{\lambda}\gamma_\mu \varepsilon$, $\delta\lambda = -\sigma_{\mu\nu} F_{\mu\nu} \varepsilon$
- SUSY can be restored by fine tuning m_g: chiral limit corresponds to SUSY limit

Supersymmetric theories with scalar fields in 4 dimensions

Apart from $\mathcal{N} = 1$ SYM, all other theories require scalar fields, general theory space:

- gauged theories with fermion and scalar fields, Yukawa interactions, flat directions $V(\phi) = 0$
- $\mathcal{N} = 2$ and 4 supersymmetric Yang-Mills: adjoint fermions and adjoint scalar fields
- SQCD: fermions in adjoint and fundamental representation, complex scalar fields
- fine tuning problem with scalar fields: order 10 parameters

Lattice perturbation theory and weak coupling regime

Lattice perturbation theory with Wilson fermions
$\mathcal{N} = 2$ supersymmetric Yang-Mills

- perturbative relevant directions away from SUSY RG trajectory [I. Montvay, Nucl.Phys. B445 (1995)]
- additional phases of the theory have to be considered (one-loop effective potential)

supersymmetric QCD

- perturbative calculations of mass tuning and Yukawa couplings [M. Costa, H. Panagopoulos,Phys.Rev.D 99 (2019)],talk by Herodotos Herodotou
- simulations and perturbative calculations in weak coupling regime, [B. Wellegehausen, A. Wipf, Lattice2018], [GB, S. Piemonte, Lattice 2018]

Witten index and fermion-boson cancellation

Witten index $\tilde{Z} = \mathrm{Tr}\left[(-1)^F e^{-RH}\right]$ in SYM:

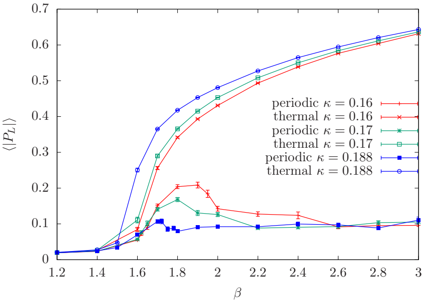
- on the lattice: periodic fermion boundary conditions (compactification radius R)
- pairing of SUSY states: Witten index stable under volume deformations
- expect no phase transition, when shrinking one of the lattice dimensions

R absence of phase transition in small limit provides signal for SUSY.

Compactified SYM on the lattice

- fermion boundary conditions: thermal $\rightarrow$ periodic
- at small m_g (large κ) no signal of deconfinement
- nearly flat effective Polyakov line potential
- similar cancellation of perturbative PL potential expected for SQCD

[O. Aharony, et al. Nucl.Phys.B 499 (1997)]



[GB,Piemonte,JHEP 1412 (2014) 133]

$N_f = 1$ supersymmetric QCD

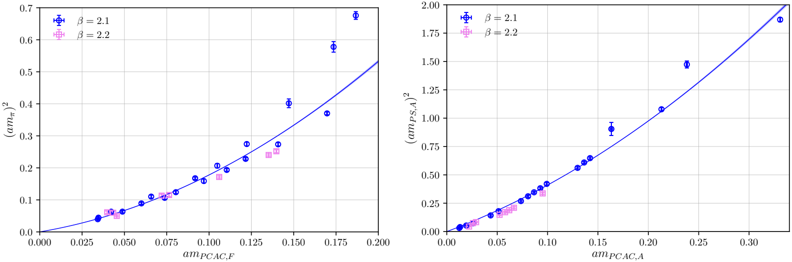
- add $N_c \oplus \bar{N}_c$ chiral matter superfield ($N_c = 2$)
- SYM + quarks ψ and squarks Φ_i with covariant derivatives, mass terms and

$$
\begin{aligned}
& i\sqrt{2}g\bar{\lambda}^a \left(\Phi_1^\dagger T^a P_+ + \Phi_2 T^a P_-\right)\psi \\
& - i\sqrt{2}g\bar{\psi}\left(P_- T^a \Phi_1 + P_+ T^a \Phi_2^\dagger\right)\lambda^a \\
& + \frac{g^2}{2}\left(\Phi_1^\dagger T^a \Phi_1 - \Phi_2^\dagger T^a \Phi_2\right)^2 .
\end{aligned}
$$

Expectation from continuum effective potential at $N_f < N_c$
"no vacuum" in chiral limit ($\Phi \to \infty$) due to flat directions

Simulations without scalar fields

SQCD without scalar fields = SU(2) gauge theory coupled to 1 adjoint Majorana + N_f fundamental Dirac



Tuning of two different fermion masses possible in lattice simulations. [GB, S. Piemonte, Phys.Rev.D 103 (2021)]

Yukawa interactions

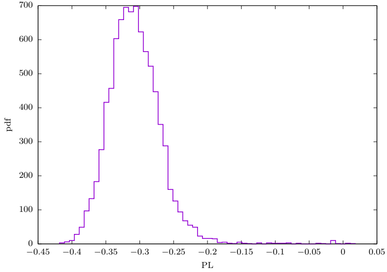
SQCD Yukawa interactions couple adjoint Majorana and fundamental Dirac fermions.
Doubling of the matrix size, Pfaffian $\mathrm{Pf}(\mathcal{M})$ instead of determinant

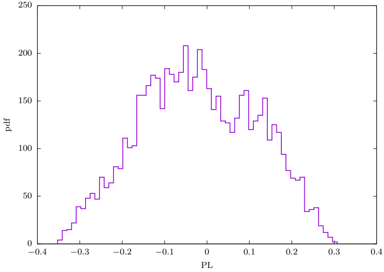
$$\frac{1}{2}(\bar{\psi}, \psi, \lambda)\mathcal{M}\begin{pmatrix}\bar{\psi}\\ \psi\\ \lambda\end{pmatrix} = \frac{1}{2}(\bar{\psi}, \psi, \lambda)\begin{pmatrix}0 & D_f & Y_1\\ -D_f^T & 0 & -Y_2^T C^T\\ -Y_1^T & CY_2 & CD_a\end{pmatrix}\begin{pmatrix}\bar{\psi}\\ \psi\\ \lambda\end{pmatrix}$$

Clover improvement added to D_a and D_f to match improved pure fermion limit, tree level Symanzik improved action.

Wilson scalar fields

- fermion-boson cancellation requires same derivative operators and masses
- larger lattice artefacts, by smaller SUSY breaking with scalar Wilson mass term
- first test: tree level Yukawa and scalar coupling

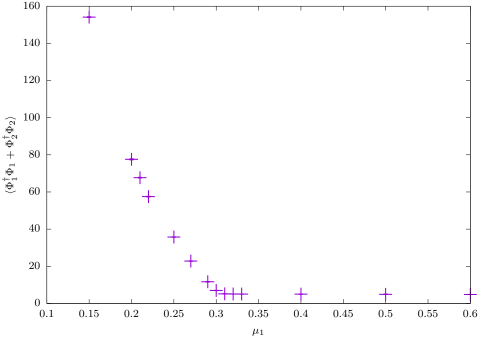




Scalar field divergence

Scalar field divergence in flat direction more relevant than expected:

- divergence already in rather heavy fermion mass regime
- can be solved by adding large scalar mass



Interesting regime not accessible at stronger couplings.

Flat directions of scalar potential

Wilson fermions get additive mass renormalization, two regimes of the simulations:

- large scalar fields, corresponds to weak coupling regime: smaller fermion mass
- smaller scalar fields, larger gauge fluctuations: larger fermion mass

Second phase with large scalar expectation values due to Wilson fermions (lattice artefact). In order to reduce the effect: stout smeared links in matter sector. Region with $(m_\pi)_F \sim 0.6$, $(m_\pi)_A \sim 0.3$ can be reached ($\beta = 1.7$).

Conclusions

- severe fine tuning problem with scalar field
- "Wilson scalars" might provide a better way to approach relevant regime
- Polyakov line effective potential and R dependence of compactified theory provide signals for SUSY tuning
- on the lattice: phase space reduced by additional lattice artefact phases
- SQCD with larger N_f or $\mathcal{N} = 2$ SYM might reduce problem
- discretizations without additive mass renormalization might solve problem