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Finite volume effects near the chiral crossover

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and
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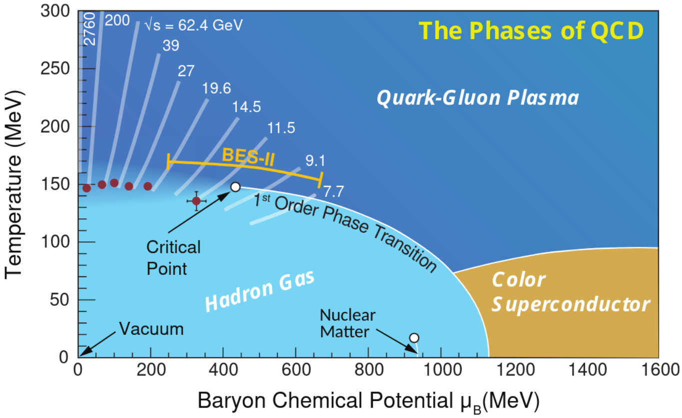
Lattice 2023



Bzdak et. al. 1906.00936

What are the reasons for the structures?

QCD Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi - \frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a}$$

- Exact global and local SU(3) gauge theory

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- Exact global and local SU(3) gauge theory

Center Symmetry $\mathcal{Z}_3$ for $m \to \infty$

- Invariance under $U_4(\vec{x}, t_0) \to zU_4(\vec{x}, t_0), z \in \{1, e^{\pm i2\pi/3}\}, U \in \mathrm{SU}(3)$
- SSB at high temperatures (deconfinement)
- Order parameter Polyakov loop: $|\langle P \rangle| \sim e^{-F/T}$

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Chiral symmetry for $m \to 0$

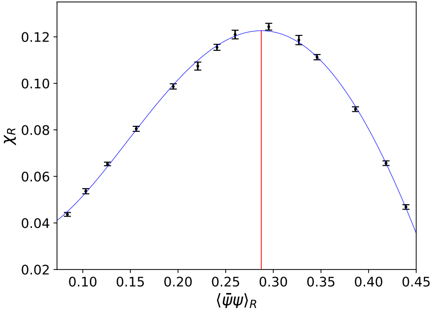
- $\mathcal{L}_F = \bar{\psi}_L i\not{D}\psi_L + \bar{\psi}_R i\not{D}\psi_R$
- SSB at moderate temperatures
- Order parameter chiral condensate $\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\rangle$

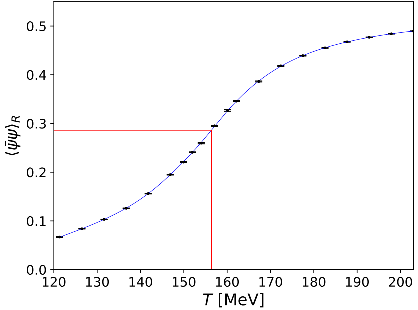
Chiral Observables

$$\langle\bar\psi\psi\rangle = \frac{T}{V}\frac{\partial\log Z}{\partial m} \qquad \langle\bar\psi\psi\rangle_R = -\left[\langle\bar\psi\psi\rangle_T - \langle\bar\psi\psi\rangle_{T=0}\right]\frac{m}{f_\pi^4}$$

$$\chi = \frac{T}{V}\frac{\partial^2\log Z}{\partial m^2} \qquad \chi_R = \left[\chi_T - \chi_{T=0}\right]\frac{m^2}{f_\pi^4}$$

How to get precisely the inflection point of $\langle\bar\psi\psi\rangle$ or the maximum of χ?





Width of the transition

$$(\Delta T)^2 = -\chi(T_c)\left[\frac{\mathrm{d}^2\chi}{\mathrm{d}T^2}(T_c)\right]^{-1}$$

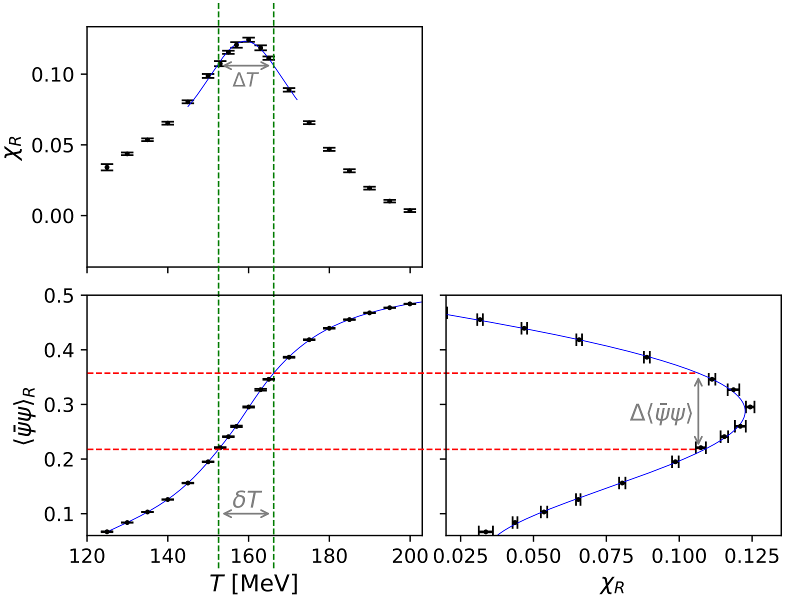
How to calculate the 2nd temperature derivative?

$$\left(\Delta\left\langle\bar{\psi}\psi\right\rangle\right)^2 = -\chi(T_c)\left(\left.\frac{\mathrm{d}^2\chi}{\mathrm{d}\left\langle\bar{\psi}\psi\right\rangle^2}\right|_{\left\langle\bar{\psi}\psi\right\rangle_c}\right)^{-1} \Rightarrow \Delta T = \Delta\left\langle\bar{\psi}\psi\right\rangle\left(\frac{\mathrm{d}\left\langle\bar{\psi}\psi\right\rangle}{\mathrm{d}T}\right)^{-1}$$

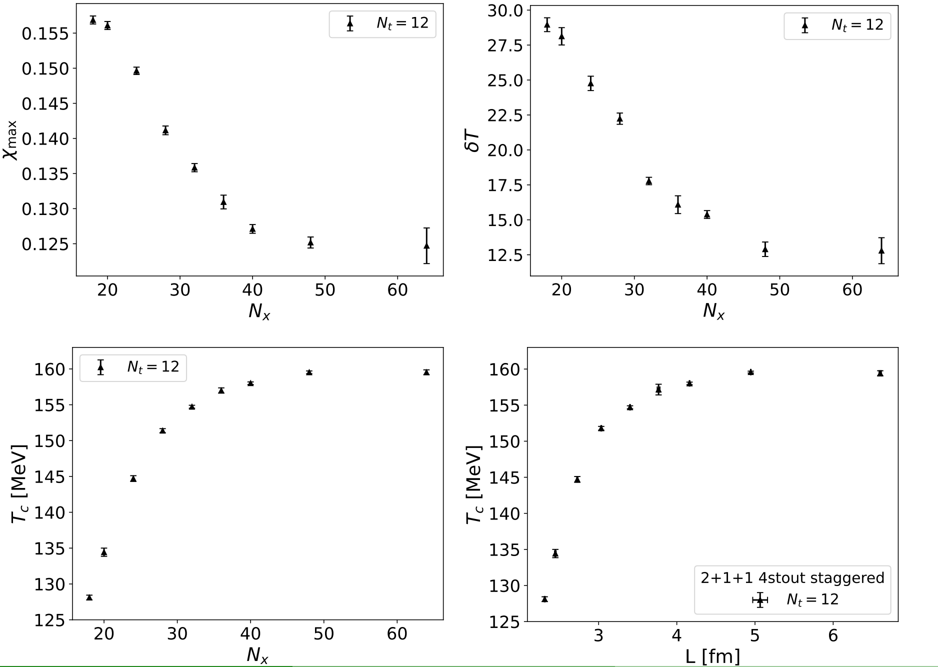
Approximate $T-$derivative by symmetric difference quotient

$$\delta T = \left\langle\bar{\psi}\psi\right\rangle^{-1}\left(\left\langle\bar{\psi}\psi\right\rangle_c + \frac{\Delta\left\langle\bar{\psi}\psi\right\rangle}{2}\right) - \left\langle\bar{\psi}\psi\right\rangle^{-1}\left(\left\langle\bar{\psi}\psi\right\rangle_c - \frac{\Delta\left\langle\bar{\psi}\psi\right\rangle}{2}\right)$$

Let's illustrate this approach



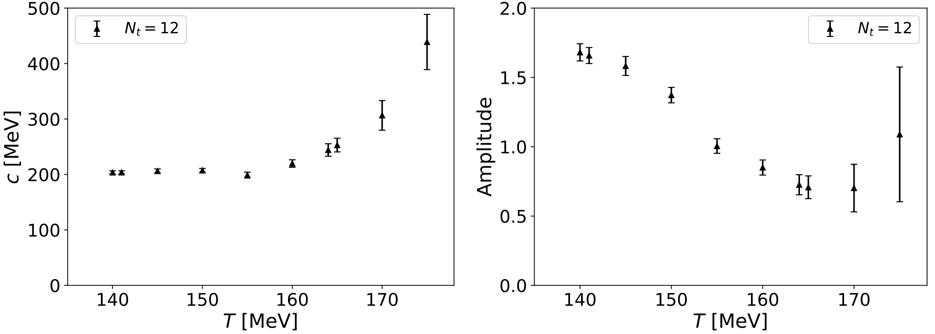
χ_R
0.10
0.05
0.00
ΔT
$\langle\bar{\psi}\psi\rangle_R$
0.5
0.4
0.3
0.2
0.1
δT
120
140
160
180
200
T [MeV]
$\Delta\langle\bar{\psi}\psi\rangle$
0.025
0.050
0.075
0.100
0.125
χ_R

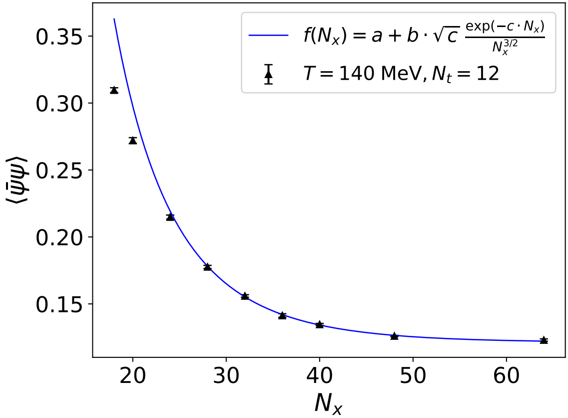

$N_t = 12$
0.155
0.150
0.145
0.140
0.135
0.130
0.125
$\chi_{\max}$
20 30 40 50 60
N_x
$N_t = 12$
30.0
27.5
25.0
22.5
20.0
17.5
15.0
12.5
δT
20 30 40 50 60
N_x
$N_t = 12$
160
155
150
145
140
135
130
125
T_c [MeV]
20 30 40 50 60
N_x
T_c [MeV]
3 4 5 6
L [fm]
2+1+1 4stout staggered
$N_t = 12$

Volume dependence of the condensate

Features

- Solve $\langle\bar{\psi}\psi\rangle$ for $T \in [140, 180]$: Exponential behaviour for full range
- $f(N_x) = a + b \cdot \exp(-c \cdot N_x)$: c has mass dimension





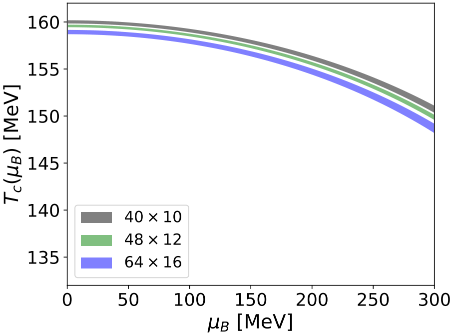
Features

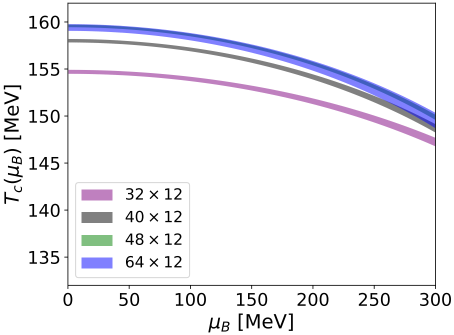
- Chiral PT prediction for $T = 0$ [Adhikari Phys.Rev.D 107] : $4.5 \cdot \frac{\sqrt{m_\pi}}{F_\pi^2} \frac{e^{-m_\pi N_x}}{(2\pi N_x)^{3/2}}$
- Choose temperature below T_c: $\underline{T = 140 \text{ MeV}}$
- Result: $m_\pi = 131 \pm 10$ MeV in the range of $N_x \in [28, 64]$

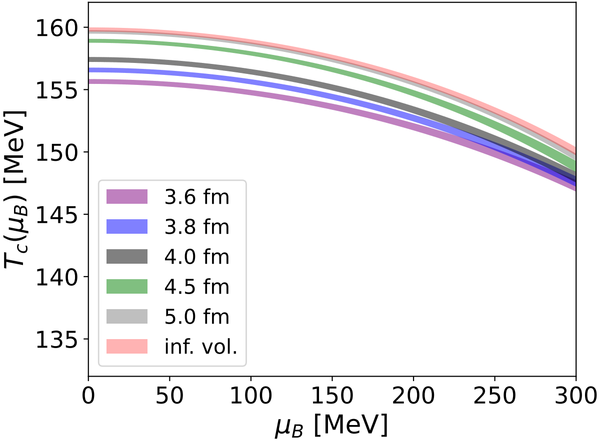
Phase diagram

Transition line in a finite box

- Circumvent sign problem by imag. $\hat{\mu}_B$ simulations
- Determine transition line by $\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2$



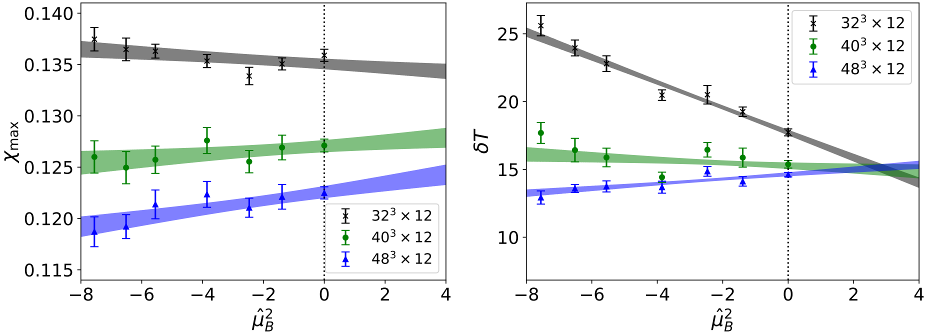




Transition line in a finite box

- Solve cubic equation for $T_c(\mu_B)$ for every μ_B
- Box-size: $L = \frac{N_x}{N_t T_c(\mu_B)}$: Iterate $T_c(N_x)$ for every μ_B to match L [fm]
- Volume effects seem to decrease for larger μ_B

Strength and width of the crossover at finite μ_B

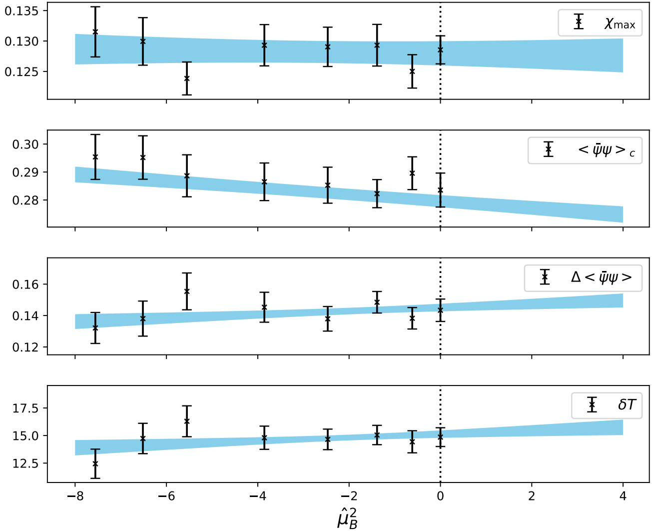


Does the crossover turn into a real transition?

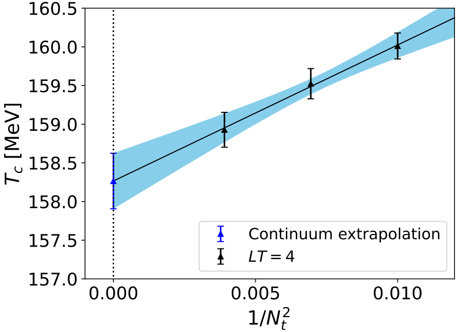
- $\hat{\mu}_B^2 \leq 0$: $\chi_{\max}$ decreases in descending order of the vol. $\Rightarrow$ weak crossover
- No sign of a stronger transition
- δT seems to be more sensitive to Roberge-Weiss transition

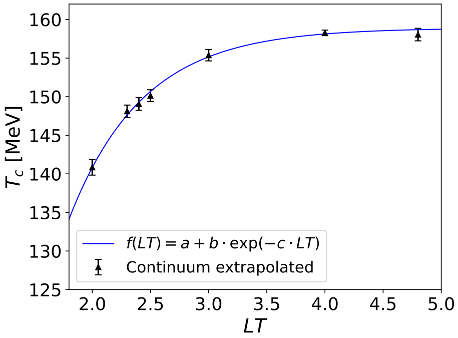
Continum extrapolated observables as functions of $\hat{\mu}_B^2$ at $LT = 4$

Borsanyi, R.K. et. al. PRL (2020)



Continuum and infinite volume extrapolated T_c





Infinite volume limit of T_c

- Inf. vol. lim. on continuum extrapolated values: $\underline{T_c = 158.90 \pm 0.63 \text{ MeV}}$

Summary

Phys. quark mass simulations at $\mu_B = 0$

- Observables show exponential behaviour at fixed N_t as functions of N_x
- Strength and width of the transition increase for $LT < 3$
- Condensate follows $\chi-$PT predictions for $T < T_c$: $m_\pi = 131 \pm 10$ MeV

Finite μ_B and continuum extrapolations

- Finite volume effects on T_c seem to get weaker for increasing μ_B
- $\hat{\mu}_B^2 < 0$: $\chi_{\max}$ decreases in descending order of the volume
- δT seems to be more sensitive to Roberge-Weiss transition than full χ
- Mild $\hat{\mu}_B^2$ of all observables in the continuum limit
- Infinite volume limit on continuum extrapolated results:

$$\underline{T_c(\mu_B = 0) = 158.90 \pm 0.63 \text{ MeV}}$$

Simulation setup

$\mu_B = 0$

- $N_t = 10$: $N_x = 20, 24, 28, 32, 40, 48$
- $N_t = 12$: $N_x = 18, 20, 24, 28, 32, 36, 40, 48, 64$
- $N_t = 16$: $N_x = 32, 40, 48, 64, 80$

imag. μ_B

- $N_t = 10$: $N_x = 40$
- $N_t = 12$: $N_x = 32, 40, 48, 64$
- $N_t = 16$: $N_x = 64$

Details of the transition width

$$(\Delta T)^2 = -\chi(T_c)\left[\frac{\mathrm{d}^2\chi}{\mathrm{d}T^2}(T_c)\right]^{-1}$$

$$(\Delta T)^2 = -\chi(T_c)\left[\left.\frac{\mathrm{d}\chi}{\mathrm{d}\left\langle\bar{\psi}\psi\right\rangle}\right|_{\left\langle\bar{\psi}\psi\right\rangle_c}\left.\frac{\mathrm{d}^2\left\langle\bar{\psi}\psi\right\rangle}{\mathrm{d}T^2}\right|_{T_c} + \left.\frac{\mathrm{d}^2\chi}{\mathrm{d}\left\langle\bar{\psi}\psi\right\rangle^2}\right|_{\left\langle\bar{\psi}\psi\right\rangle_c}\left(\left.\frac{\mathrm{d}\left\langle\bar{\psi}\psi\right\rangle}{\mathrm{d}T}\right|_{T_c}\right)^2\right]^{-1}$$

First term in the bracket is zero, since $\bar{\psi}\psi$ has inflection point at T_c.

$$\Delta T = \sqrt{-\chi(T_c)\left(\left.\frac{\mathrm{d}^2\chi}{\mathrm{d}\left\langle\bar{\psi}\psi\right\rangle^2}\right|_{\left\langle\bar{\psi}\psi\right\rangle_c}\right)^{-1}}\left(\left.\frac{\mathrm{d}\left\langle\bar{\psi}\psi\right\rangle}{\mathrm{d}T}\right|_{T_c}\right)^{-1}$$

$$:= \Delta\left\langle\bar{\psi}\psi\right\rangle\left(\left.\frac{\mathrm{d}\left\langle\bar{\psi}\psi\right\rangle}{\mathrm{d}T}\right|_{T_c}\right)^{-1}.$$