Spectroscopy of heavy-light mesons

L. Gayer, S.M. Ryan (Trinity College Dublin), D.J. Wilson (Cambridge University)

Lattice 2023, FermiLab, August 1st 2023
THE SPECTRUM OF $B$, $B_S$ & $B_C$ MESONS: MOTIVATION

- Heavy quarks provide a rich environment for spectroscopy - with many and unexpected exotic states discovered already.

- Completing the story in heavy quark spectroscopy:
  - Bottomonium: JHEP02 (2021) 213.

- Unlike charm sector only a handful of states determined experimentally in any of $B$, $B_S$, $B_C$: map the spectrum of excited and exotic states to $J \leq 4$ in a lattice calculation.

- Fruitful scattering analyses in $D\pi$, $DK$ [JHEP 10 (2016) 011, JHEP 07 (2021) 123, JHEP 02 (2021) 100]. Similar picture in $B\pi$ etc?
LATTICE DETAILS

- Symanzik-improved anisotropic gauge action with tree-level tadpole-improved coefficients and $N_f = 2 + 1$.
- Anisotropic clover fermion action with stout-smeared spatial links.

- $\xi = a_s/a_t = 3.5$; $a_s \approx 0.12$ fm, $a_t^{-1}(m_\Omega) = 5.67(4)$ GeV.

- $20^3 \times 128$ volume; $m_\pi \sim 396$ MeV. $16^3, 24^3$ volumes available for volume dependence study.

- Distillation for quark propagation.

- Operators of definite momenta constructed with up to 3 derivatives.

\[
\begin{array}{cccccc}
\Lambda & A_1 & A_2 & E & T_2 & T_1 \\
\Lambda^+ & 18 & 10 & 26 & 36 & 44 \\
\Lambda^- & 18 & 10 & 26 & 36 & 44 \\
\end{array}
\]

- Mass-dependent anisotropy tuning for heavy quarks.
**LATTICE DISPERSION RELATIONS**

- Fermion action: mass-dependent tuning of $m_q$ and $\xi$: tuned at $\eta_b$.
- Dispersion relations in heavy-light sector $(a_t E)^2 = (a_t M)^2 + \left( \frac{2\pi}{\xi L/a_s} \right)^2 n^2$.

<table>
<thead>
<tr>
<th></th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>3.365(14)</td>
</tr>
<tr>
<td>$B^*$</td>
<td>3.357(21)</td>
</tr>
<tr>
<td>$B_s$</td>
<td>3.371(11)</td>
</tr>
<tr>
<td>$B_s^*$</td>
<td>3.352(14)</td>
</tr>
<tr>
<td>$B_c$</td>
<td>3.447(5)</td>
</tr>
<tr>
<td>$B_c^*$</td>
<td>3.478(13)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.574(26)</td>
</tr>
<tr>
<td>$\eta_b$</td>
<td>3.590(15)</td>
</tr>
</tbody>
</table>
RECIPE FOR (MESON) SPECTROSCOPY

- Construct a basis of local and non-local operators $\bar{\Psi}(x) \Gamma D_i D_j \ldots \Psi(x)$ from distilled fields [PRD80 (2009) 054506].

- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 \rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

- Solve generalised eigenvalue problem $C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$

  - eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t} \left[ 1 + O(e^{-\Delta E t}) \right]$ yield principal correlators
  - eigenvectors: $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$ related to overlaps - helps identify continuum spin.
Results

$B$, $B_s$, $B_c$ mesons

Caveat Emptor

- Spectra determined from single-hadron operators.
- Relatively heavy ($\sim 400\text{MeV}$) pions
- Single lattice spacing, single volume
THE LATTICE B MESON SPECTRUM
THE LATTICE $B_S$ MESON SPECTRUM
THE LATTICE $B_c$ SPECTRUM

$\frac{M-M_{\eta b}/2}{a_t m}$

$B_c$

$A_1^-$ $A_2^-$ $E^-$ $T_2^-$ $T_1^-$ $A_1^+$ $A_2^+$ $E^+$ $T_2^+$ $T_1^+$

$J = 0$ $J = 1$ $J = 2$ $J = 3$ $J = 4$
Volume Dependence: A Study of B Mesons
THE $B$ MESON SPECTRUM
THE $B_s$ MESON SPECTRUM
THE LATTICE $B_c$ SPECTRUM
Evidence of lightest hybrid supermultiplets.

$1^-$: mixtures of spin-singlet, spin-triplet.
A number of near-threshold states in positive parity B and Bs spectrum

Elastic $B\pi, A_1^+, I = 3/2$: weakly repulsive interaction. $I = 1/2$ underway.
A study of heavy meson spectroscopy extended to heavy-light sector.

Results for the $B$, $B_s$ and $B_c$ meson spectra presented:

- Dispersion relations are consistent between heavy and heavy-light sectors, for parameters tuned once at $\eta_b$.
- Rotation breaking effects are small.
- Evidence of a hybrid supermultiplet in $B$, $B_s$, $B_c$ at energy scale approx 1.3 GeV.
  - similar characteristics in charmonium, open-charm and light mesons.

The study can be extended to larger volumes and lighter pion masses.

Paving the way for investigation of studies in $B\pi$, $BK$ and heavy tetraquarks.
A study of heavy meson spectroscopy extended to heavy-light sector.

<table>
<thead>
<tr>
<th>this work (MeV)</th>
<th>experiment - PDG (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>39(2)</td>
<td>45.21 ± 0.21</td>
</tr>
<tr>
<td>38.0(8)</td>
<td>48.6 ± 1.5</td>
</tr>
<tr>
<td>905.79(71)</td>
<td>907.75 ± 0.37 ± 0.27</td>
</tr>
<tr>
<td>602(3)</td>
<td>598(1)</td>
</tr>
</tbody>
</table>

Results for the B, Bs and Bc meson spectra presented:

- Dispersion relations are consistent between heavy and heavy-light sectors, for parameters tuned once at $\eta_b$.
- Rotation breaking effects are small.
- Evidence of a hybrid supermultiplet in $B, Bs, Bc$ at energy scale approx 1.3 GeV.  
  - similar characteristics in charmonium, open-charm and light mesons.

- The study can be extended to larger volumes and lighter pion masses.
- Paving the way for investigation of studies in $B\pi, BK$ and heavy tetraquarks.

Thanks for listening!
The Lattice Dispersion Relation

- Fermion action: mass-dependent tuning of $m_q$ and $\xi$
- $M_{\text{latt}}^{\eta_b} = M_{\text{exp}}^{\eta_b}$ and a relativistic dispersion relation recovered.

\[ (a_t E)^2 = (a_t M)^2 + \left( \frac{2\pi}{\xi L/a_s} \right)^2 n^2. \]

- $n \leq (2, 0, 0)$ in fits; no significant $(a_s p)^4$ term.
- Rest, kinetic masses consistent.
- Heavy-light dispersion relations determined using these tuned parameters.
\( a_t E_0 = 1.1126(1) \quad a_t E_1 = 1.2153(10) \quad a_t E_2 = 1.2446(11) \quad a_t E_3 = 1.2495(11) \quad a_t E_4 = 1.2814(81) \quad a_t E_5 = 1.3309(43) \)

\( a_t E_6 = 1.3429(45) \quad a_t E_7 = 1.3458(41) \quad a_t E_8 = 1.3531(47) \quad a_t E_9 = 1.3554(47) \quad a_t E_{10} = 1.3902(67) \)
EXTENDING SIMILAR WORK - OPEN CHARM FOR $J \leq 4$
IS THERE A HEAVY-LIGHT HYBRID SUPERMULTIPLLET AS IN D?

JHEP05 (2013) 021