The mixing of two-pion and vector-meson states using staggered fermions

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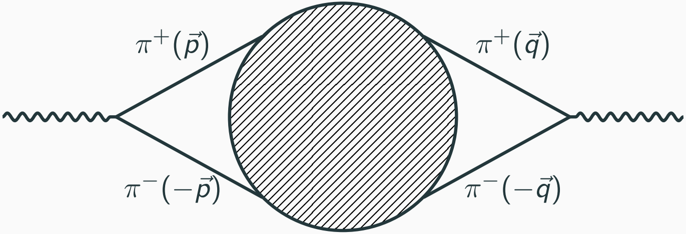
Introduction

- High precision computation of anomalous magnetic moment of the muon a_μ
- Contribution of $\pi\pi$ states (HVP)
- Investigate mixing of $\pi\pi$ and ρ (γ) states using group theory applied on the staggered symmetry group **[Lahert et al. (2021))]**



The staggered symmetry group

- The staggered is action is invariant under certain transformations (symmetry group $\mathcal{G}$)
  1. **Shift** S_μ in a given direction μ by one lattice spacing
  2. **Rotation** $R_{\mu\nu}$ in the $\mu\nu$-plane by $\frac{\pi}{2}$
  3. **Spatial inversion** (parity transformations) I_S
  4. **Charge conjugation** C_0
  5. **Taste transformation** Ξ_μ
- Restriction of $\mathcal{G}$ by restriction to a given time slice
- No temporal shifts and rotation
- Symmetry group $\mathcal{H}$
- Irreps of $\mathcal{H}$ correspond to different quantum numbers of states (momentum, spin, parity, charge conjugation quantum number, taste)
  [Kilcup/Sharpe (1987), Golterman/Smit (1984)]

Constructing the ρ

- Consider taste singlet ρ-meson at rest
- Which two-pion-states have the same quantum number and how large is their contribution?
- We choose the two-pion state

$$\pi^+\pi^- - \pi^-\pi^+$$

in order to have the correct charge conjugation quantum number $c = -1$

Constructing the ρ

- Reducible $\pi\pi$-state can be decomposed in irreps:

$$D^{\pi}(\xi = \xi_1, p = \vec{p}_1) \otimes D^{\pi}(\xi = \xi_2, p = \vec{p}_2) = n(\xi_1, \xi_2, \vec{p}_1, \vec{p}_2; \rho) D^{\rho}(\xi = 0, \vec{p} = 0) \oplus \ldots,$$

if

$$\xi_1 = \xi_2, \quad \vec{p}_1 = -\vec{p}_2, \quad \vec{p}_1 \neq \vec{0}.$$

with the multiplicity computed using the characters

$$n(\xi, \vec{p}; \rho) = \frac{1}{|\mathcal{H}|} \sum_{R, I_s, C_0, \xi_4} \chi_{\rho}^{*}(R, I_s) \chi_{2\pi}^{\xi_\mu, \vec{p}}(R, I_s, \vec{S}, \Xi_\mu)$$

Constructing the ρ

- Independently of the choice of ξ_4 and c we get the following multiplicities:

	$\|\vec{p}\|^2 = 0$	$\|\vec{p}\|^2 = 1$	$\|\vec{p}\|^2 = 2$	$\|\vec{p}\|^2 = 3$	$\|\vec{p}\|^2 = 4$
$\|\vec{\xi}\|^2 = 0$	0	1	1	1	1
$\|\vec{\xi}\|^2 = 1$	0	2	3	2	2
$\|\vec{\xi}\|^2 = 2$	0	2	3	2	2
$\|\vec{\xi}\|^2 = 3$	0	1	1	1	1

Constructing the ρ

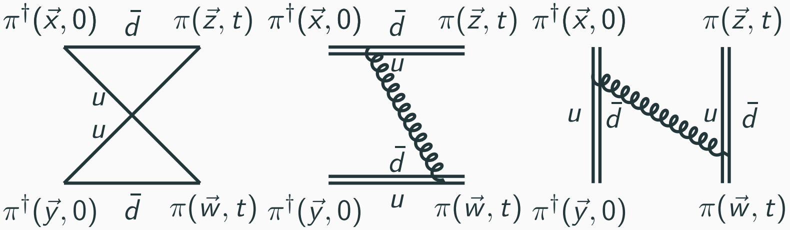
- Basic vectors $\left|\rho_\mu^{\alpha\beta}\right\rangle$ of ρ representation can be computed using Clebsch-Gordon coefficients
- α and β label the taste and momentum orbit

$$\left|\rho_\mu^{\alpha\beta}\right\rangle = \sum_{\vec{p}\in\{\vec{p}^\alpha\}} \sum_{\xi\in\{\xi^\beta\}} c^{\rho_\mu}_{\vec{p},\xi} \pi(\vec{p},\xi)\pi(-\vec{p},\xi)\left|0\right\rangle$$

- Symmetry group is finite, CG coefficients can be computed on a desktop PC in a few seconds using Wigner's theorem **[Sakata (1974)]**
- Example:

$$\left|\rho_x^{1\bar{5}}\right\rangle = \frac{1}{\sqrt{2}}\left[\pi(e_x,\bar{5}),\pi(-e_x,\bar{5})\right]\left|0\right\rangle \quad \left(\bar{5} = (1,1,1,1)^T\right)$$

Two-pion states



- The last diagram does not contribute due to antisymmetry in x and y
- Total correlation function:

$$C^{AB}_{\pi\pi}(|\vec{p}|,|\vec{q}|) = \sum K^{A,B}_{\vec{p},\vec{q}}\left(\Pi_d - 2\Pi_c\right)$$

GEVP

- Extract different energy states from the two-pion correlators

$$C(t_0 + \Delta t)v = \lambda C(t_0)v$$

- Solve eigenvalue problem of

$$M = C^{-0.5}(t_0) \cdot C(t_0 + \Delta t) \cdot C^{-0.5}(t_0)$$

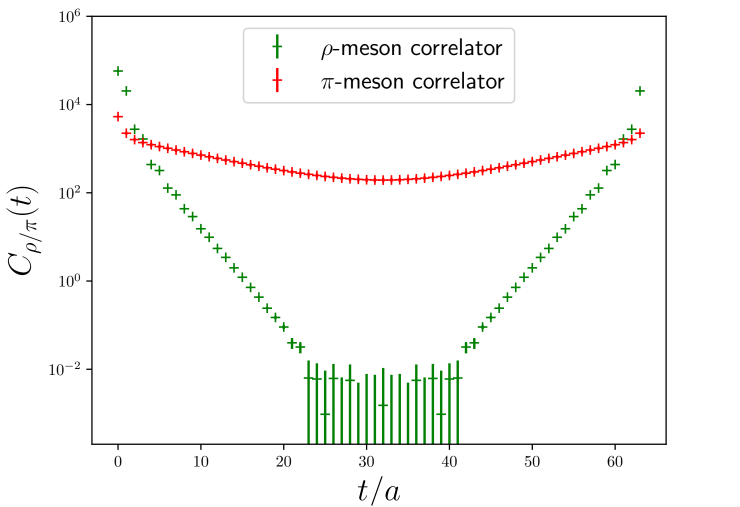
for given t_0 and Δt (e.g. 1 and 2) and compute eigenvalues for all Δt

- Provides orthogonal eigenvectors, which helps with state identification

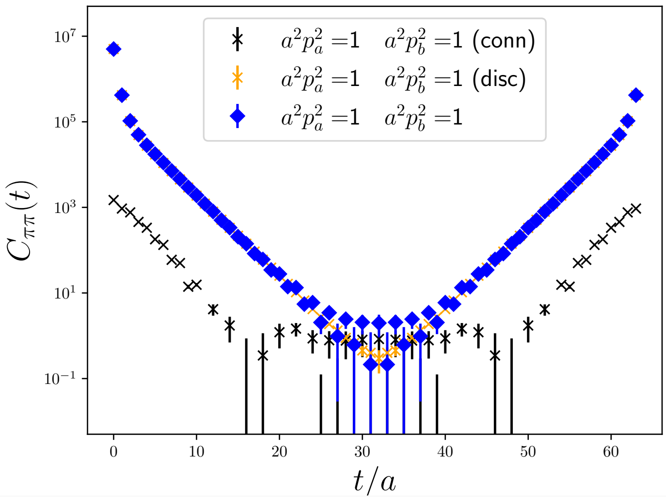
Lattice and implementation details

- Symanzik improved gauge action, `4stout` one-link staggered fermions ($N_f = 2 + 1 + 1$)
- $32^3 \times 64$ box, $\beta = 3.7000$, $m_l = 0.00205349$
- Connected diagram : sequential inversion on each time slice ($\mathcal{O}\left(\#\text{tastes} \times \#\text{momenta} \times N_t\right)$)
- Preconditioning of $\not{D}$ by precalculation of eigenvectors, optimized number of eigenvectors, time is still dominated by the inversions

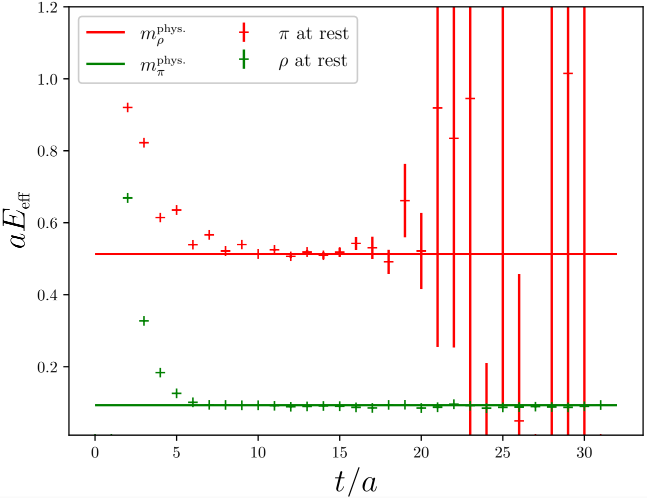
Correlation functions ρ and π meson



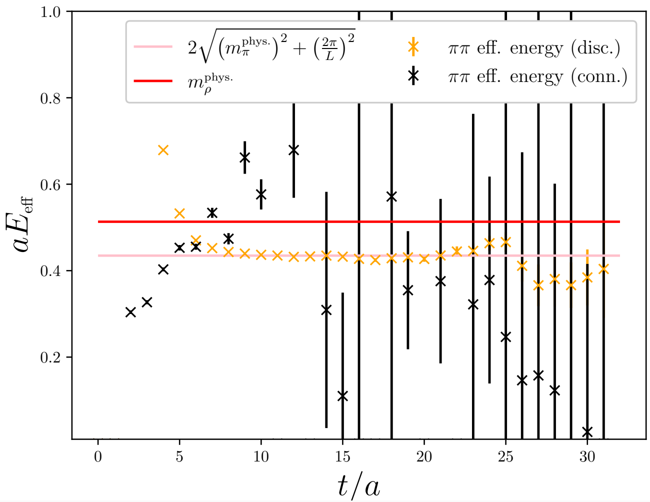
Correlation functions $\pi\pi$-states



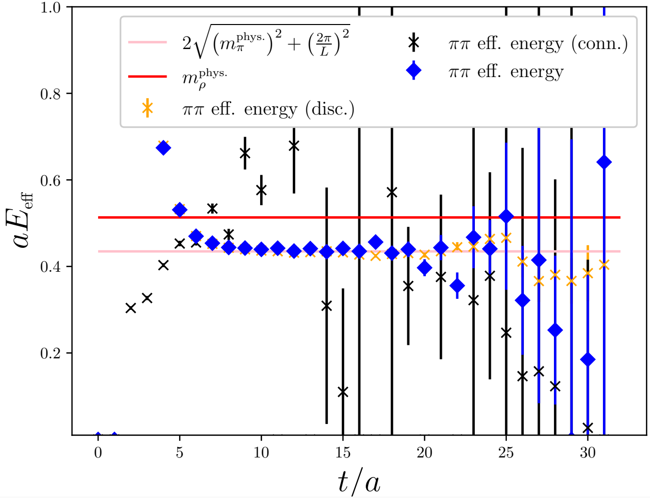
Energy extraction ρ and π meson



Energy extraction $\pi\pi$-states



Energy extraction $\pi\pi$-states



Conclusion and Outlook

- Conclusion
  - Derivation of a framework for the mixing of $\pi\pi$ and ρ
  - Implementation of $\pi\pi$-correlators at $p \neq 0$
  - First tests on small lattices
- Outlook
  - Including all tastes and higher momenta (up to $\|\vec{p}\|^2 = 4$)
  - Computation of off-diagonal elements (esp. $(\pi\pi, \rho)$)
  - Extract states with GEVP
  - Get rid of N_t-scaling and apply to larger and finer lattices
  - Reconstruct the tail of the vector-vector correlator in $g_\mu - 2$