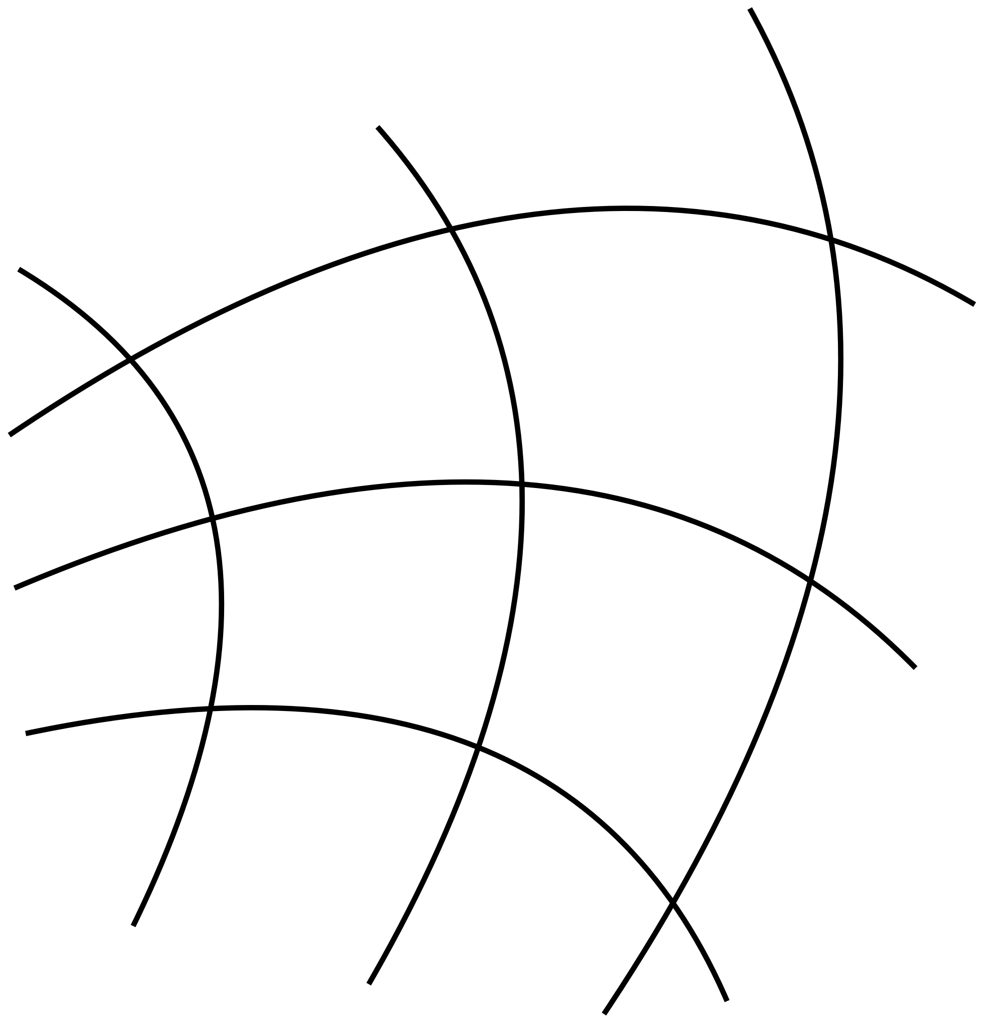
Euclidean weak-field gravity on the lattice



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Lattice 2023 Fermilab 1 Aug 2023

Euclidean path integral for GR

- GR action (no cosmological constant) with $\kappa = c^4/(16\pi G_N) \sim 10^{35}\,\mathrm{fm}^{-2}$

$$S_{\mathrm{GR}} = \int d^4x\,\kappa\sqrt{-\det \boldsymbol{g}}\,R, \qquad Z = \int d[\boldsymbol{g}]\,e^{\mathrm{i}S_{\mathrm{GR}}(\boldsymbol{g})}$$

- Wick rotate $t \to -\mathrm{i}t_E$ (or, complex metric $\sqrt{-\det \boldsymbol{g}} \to -\mathrm{i}\sqrt{\det \boldsymbol{g}_E}$)

$$S_{\mathrm{GR}} \to -\mathrm{i}\int d^4x_E\,\kappa\sqrt{\det \boldsymbol{g}_E}\,R_E \equiv \mathrm{i}S_{\mathrm{GR},E}, \qquad Z_E = \int d[\boldsymbol{g}_E]\,e^{-S_{\mathrm{GR},E}(\boldsymbol{g}_E)}$$

- drop subscript E (work exclusively with Euclidean signature)

$$S_{\mathrm{GR}} = -\int d^4x\,\kappa\sqrt{\det \boldsymbol{g}}\,R, \qquad Z = \int d[\boldsymbol{g}]\,e^{-S_{\mathrm{GR}}(\boldsymbol{g})}$$

- for $S_{\mathrm{GR}} \geq 0$, generate realistic snapshots of spacetime with probability $p(\boldsymbol{g}) = e^{-S_{\mathrm{GR}}(\boldsymbol{g})}$

… weak-field limit

$$S_{\mathrm{GR}} = -\int d^4x\, \kappa \sqrt{\det \boldsymbol{g}}\, R$$

- small, dynamic $\boldsymbol{h}(x)$ on background flat spacetime $\boldsymbol{\eta} = \mathrm{diag}(1,1,1,1)$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)\,, \qquad |\,h_{\mu\nu}\,| \ll 1$$

- to leading order in $\boldsymbol{h}$,

$$\sqrt{\det \boldsymbol{g}} = 1 + \frac{\mathrm{tr}\,\boldsymbol{h}}{2} + \mathcal{O}(h^2)$$

$$R = \partial_{\mu\nu} h_{\mu\nu} - \partial^2 \mathrm{tr}\,\boldsymbol{h} + \mathcal{O}(h^2)$$

- at $\mathcal{O}(h)$, $\sqrt{\det \boldsymbol{g}}\, R$ a total derivative, so leading contribution to $\mathscr{L}_{\mathrm{GR}}$ is $\mathcal{O}(h^2)$

't Hooft and Veltman, 1974

$$\mathscr{L}^{(2)}_{\mathrm{GR}} = \frac{\kappa}{2}\left(\frac{1}{2}(\partial_\rho h_{\mu\nu})^2 - \partial_\rho h_{\mu\nu}\partial_\nu h_{\rho\mu} - \frac{1}{2}(\partial_\mu \mathrm{tr}\,\boldsymbol{h})^2 + \partial_\nu h_{\mu\nu}\partial_\mu \mathrm{tr}\,\boldsymbol{h}\right)$$

… history of path integral quantized GR

- Misner proposed, with Minkowski metric, as approach to quantum gravity Misner, 1957

- Hawking et al. revived with Euclidean metric
  - positive action conjecture York, 1972; Gibbons, Hawking, and Perry, 1978
  - used to calculate black hole entropy/area law, $\mathcal{S} = A/4$ Hartle and Hawking, 1976
  - positive action conjecture proven Gibbons and Pope, 1979; Schoen and Yau, 1979

- issues
  - nonrenormalizable ’t Hooft and Veltman, 1974; Goroff and Sagnotti, 1985
  - complex metric potentially problematic Witten, 2021

- weak-field limit with static background ’t Hooft and Veltman, 1974

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \qquad |\,h_{\mu\nu}\,| \ll 1$$

  - flat background avoids potential complex metric issues (just Wick rotate)
  - effective theory, hence nonrenormalizability Donoghue, 1995; Burgess, 2004

Outline

- path integral quantized, Euclidean, weak-field GR
  - discretize
  - gauge fix
  - positive action conjecture
- preliminary results
  - discretization and finite volume effects
  - temperature dependence
- summary

… discretize

$$S_{\mathrm{GR}} = \int d^4x\, \mathscr{L}^{(2)}(x) + \mathcal{O}(h^3)$$

- finite volume and n^th order forward finite difference approximation, write $\delta^{(fn)}$

$$\int d^4x = a^4 \sum_x + \mathcal{O}(\mathrm{FV}) \qquad \text{and} \qquad \partial_\nu h_{\alpha\beta} = \delta_\nu^{(fn)} h_{\alpha\beta} + \mathcal{O}(a^n)$$

$$\mathscr{L}^{(2,fn)}_{\mathrm{GR}} = \frac{\kappa}{2}\left(\frac{1}{2}(\delta_\rho^{(fn)} h_{\mu\nu})^2 - \delta_\rho^{(fn)} h_{\mu\nu} \delta_\nu^{(fn)} h_{\rho\mu} - \frac{1}{2}(\delta_\mu^{(fn)} \mathrm{tr}\, \boldsymbol{h})^2 + \delta_\nu^{(fn)} h_{\mu\nu} \delta_\mu^{(fn)} \mathrm{tr}\, \boldsymbol{h}\right)$$

- discrete, Euclidean weak-field GR action is

$$S_{\mathrm{GR}} = -a^4 \sum_x \mathscr{L}^{(2,fn)}(x) + \mathcal{O}(h^3) + \mathcal{O}(a^n) + \mathcal{O}(\mathrm{FV})$$

weak-field
discretisation
finite volume

- lattice spacing enters simulation via input value for $a^2\kappa$
  - low energy effective theory at scale $\mu = a^{-1}$
  - must have $\mu \ll m_{\mathrm{Pl}}$, or equivalently, $a \gg \ell_P \sim 10^{-20}\,\mathrm{fm}$

… gauge fix

$$S_{\rm GR} = \int d^4x\, \mathscr{L}^{(2,fn)}(x) + \mathcal{O}(h^3, a^n, \mathrm{FV})$$

- GR has 2 physical degrees of freedom, $\boldsymbol{h}$ has 16 components
  - $\boldsymbol{h}$ is symmetric (16 - 6 = 10)
  - gauge fixing (10 - 4 = 6)
  - 4 constraints from Bianchi identity $\nabla^\mu\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = 0$

- gauge fix to harmonic gauge

$$\partial_\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h = 0$$

  - choose to constrain diagonals
  - dynamic spacetime parametrized by $h_{\alpha\beta}\,,\ \alpha < \beta$

$$\begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ \cdot & h_{11} & h_{12} & h_{13} \\ \cdot & \cdot & h_{22} & h_{23} \\ \cdot & \cdot & \cdot & h_{33} \end{pmatrix}$$

… constrain spacetime on the boundary

$$S_{\mathrm{GR}} = -\kappa \int_V \sqrt{\det \boldsymbol{g}}\, R$$

- if S_{GR} positive definite
  - generate realistic snapshots of spacetime with $\mathrm{prob}(\boldsymbol{h}) = e^{-S_{\mathrm{GR}}(\boldsymbol{h})}$
  - also required for $\delta S_{\mathrm{GR}} = 0 \Rightarrow 2R_{\mu\nu} - Rg_{\mu\nu} = 16\pi T_{\mu\nu}$
- it’s not, a problem known since the 1970s

… constrain spacetime on the boundary

$$S_{\mathrm{GR}} = -\kappa \int_V \sqrt{\det \boldsymbol{g}}\, R \quad .$$

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- it’s not, a problem known since the 1970s
- solution is **positive action conjecture**

All 4D Riemannian asymptotically Euclidean manifolds have $S_{\mathrm{GR}} \geq 0$, with $S_{\mathrm{GR}} = 0$ iff flat.

$$\hat{\boldsymbol{g}} = \mathbf{1} \qquad \hat{n}\cdot\partial\hat{\boldsymbol{g}}\Big|_{\partial V} = 0$$

$$\boxed{\hat{\boldsymbol{g}} = \boldsymbol{g}\Big|_{\partial V}}$$

$$S_{\mathrm{GR}} = -\kappa\int_V \sqrt{\det \boldsymbol{g}}\, R \;+\; 2\kappa\int_{\partial V}\sqrt{\det\hat{\boldsymbol{g}}}\left(\hat{g}^{\mu}_{\nu}\partial_\mu n^\nu + \frac{1}{2}\hat{g}^{\nu\rho}n^\mu\partial_\mu g_{\nu\rho}\right)$$

- if S_{GR} positive definite
  - generate realistic snapshots of spacetime with $\mathrm{prob}(\boldsymbol{h}) = e^{-S_{\mathrm{GR}}(\boldsymbol{h})}$
  - also required for $\delta S_{\mathrm{GR}} = 0 \Rightarrow 2R_{\mu\nu} - Rg_{\mu\nu} = 16\pi T_{\mu\nu}$

- it’s not, a problem known since the 1970s
- solution is **positive action conjecture**

All 4D Riemannian asymptotically Euclidean manifolds have $S_{\mathrm{GR}} \geq 0$, with $S_{\mathrm{GR}} = 0$ iff flat.

- implemented analytically via surface term (e.g. Gibbons-Hawking-York)
  - constrains asymptotic behaviour of metric
- **instead** of adding term to S_{GR}, **impose asymptotic behaviour explicitly** on metric

Proof uses $R(x) \leq 0$ for all asymptotic Euclidean 4D Riemannian manifolds.

Schoen and Yau, 1979

Require $\boldsymbol{g}\big|_{\partial V} = \mathbf{1}$ and $\hat{n} \cdot \partial\boldsymbol{g}\big|_{\partial V} = 0$, and **should** observe $R(x) \leq 0$ for all x

observe violations: perhaps because…

- conjecture applies asymptotically, $\boldsymbol{g}\big|_{\partial V} = \mathbf{1} + \mathcal{O}(1/\sqrt{V})$
- but I impose in finite volume

Since prob = $e^{-S_{\mathrm{GR}}}$, this is problematic. Therefore, constrain $\boldsymbol{h}$ such that

$$\boldsymbol{g}\big|_{\partial V} = \mathbf{1} \text{ and } \hat{n} \cdot \partial\boldsymbol{g}\big|_{\partial V} = 0\ ,$$

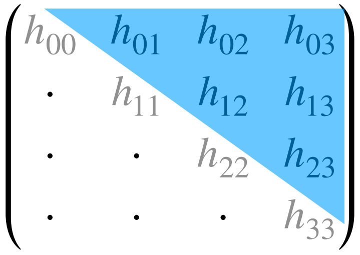
and also require $R(x) \leq 0$ for all x.

… sketch of GR update code

Markov chain updates (initially flat) with probability $p(\boldsymbol{h}) = e^{-S_{\mathrm{GR}}(\boldsymbol{h})}$

```
for t=0,…,Nt-1:
    for x=0,…,Nx-1:
        for y=0,…,Ny-1:
            for z=0,…,Nz-1:
```

jiggle spacetime



$$\begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ \cdot & h_{11} & h_{12} & h_{13} \\ \cdot & \cdot & h_{22} & h_{23} \\ \cdot & \cdot & \cdot & h_{33} \end{pmatrix}$$

```
                for $\alpha$=0,1,2:
                    for $\beta$=$\alpha$+1,…,3:

                        $\widetilde{h}_{\alpha\beta}$(t,x,y,z) = “jiggled $h_{\alpha\beta}$(t,x,y,z)”

                gauge fix
                constrain nearby $\boldsymbol{h}$s so $\boldsymbol{g}|_{\partial V} = \mathbf{1}$ and $\hat{n} \cdot \partial \boldsymbol{g}|_{\partial V} = 0$
```

enforce $R(x) \leq 0$;
1-10% acceptance ⟵

```
                if S_GR($\widetilde{\boldsymbol{h}}$) $\geq$ 0:
```

Markov chain update

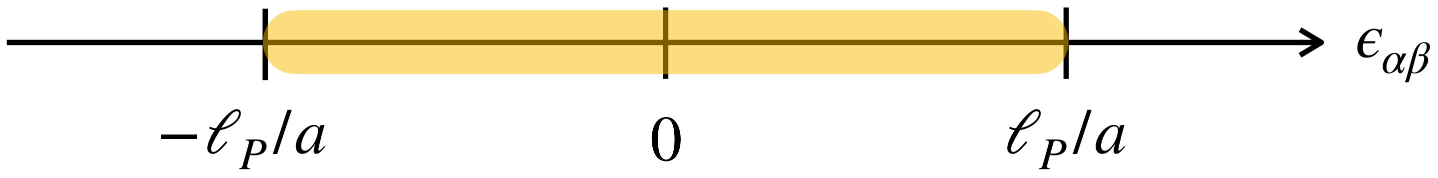
```
                    dS_GR = S_GR($\widetilde{\boldsymbol{h}}$) - S_GR($\boldsymbol{h}$)
                    if exp(-dS_GR) > random(0,1):
                        $\boldsymbol{h} = \widetilde{\boldsymbol{h}}$
```

… how much to jiggle $\boldsymbol{h}$?

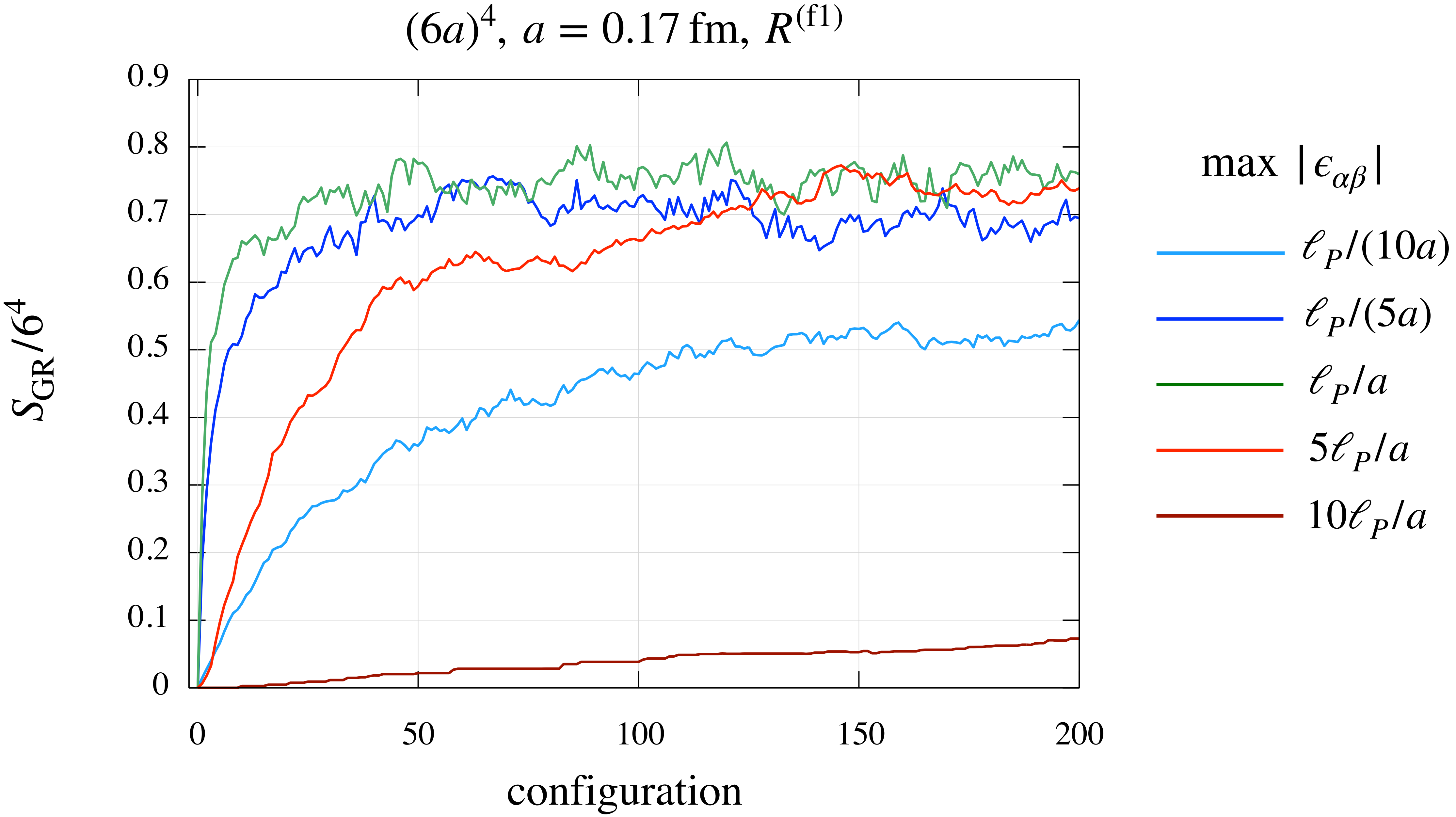
- for QCD, $U_\mu \in \mathrm{SU}(3)$, and can unambiguously define nearby

$$U_\mu \to \widetilde{U}_\mu = e^{i\varepsilon^a t^a} U_\mu \quad \text{for } |\varepsilon^a| \ll 1$$

- what is nearby for: $h_{\alpha\beta} \to \widetilde{h}_{\alpha\beta} = h_{\alpha\beta} + \epsilon_{\alpha\beta}$?
    - Planck length ℓ_P gives order one S_{GR}, $|\epsilon_{\alpha\beta}| \sim \ell_P/a$
    - for lattice spacing $a \sim 0.1 \text{ fm}$, $|\epsilon_{\alpha\beta}| \sim \dfrac{\ell_P}{a} \sim 10^{-19}$ (single precision ok)
    - choose $\epsilon_{\alpha\beta}$ randomly from

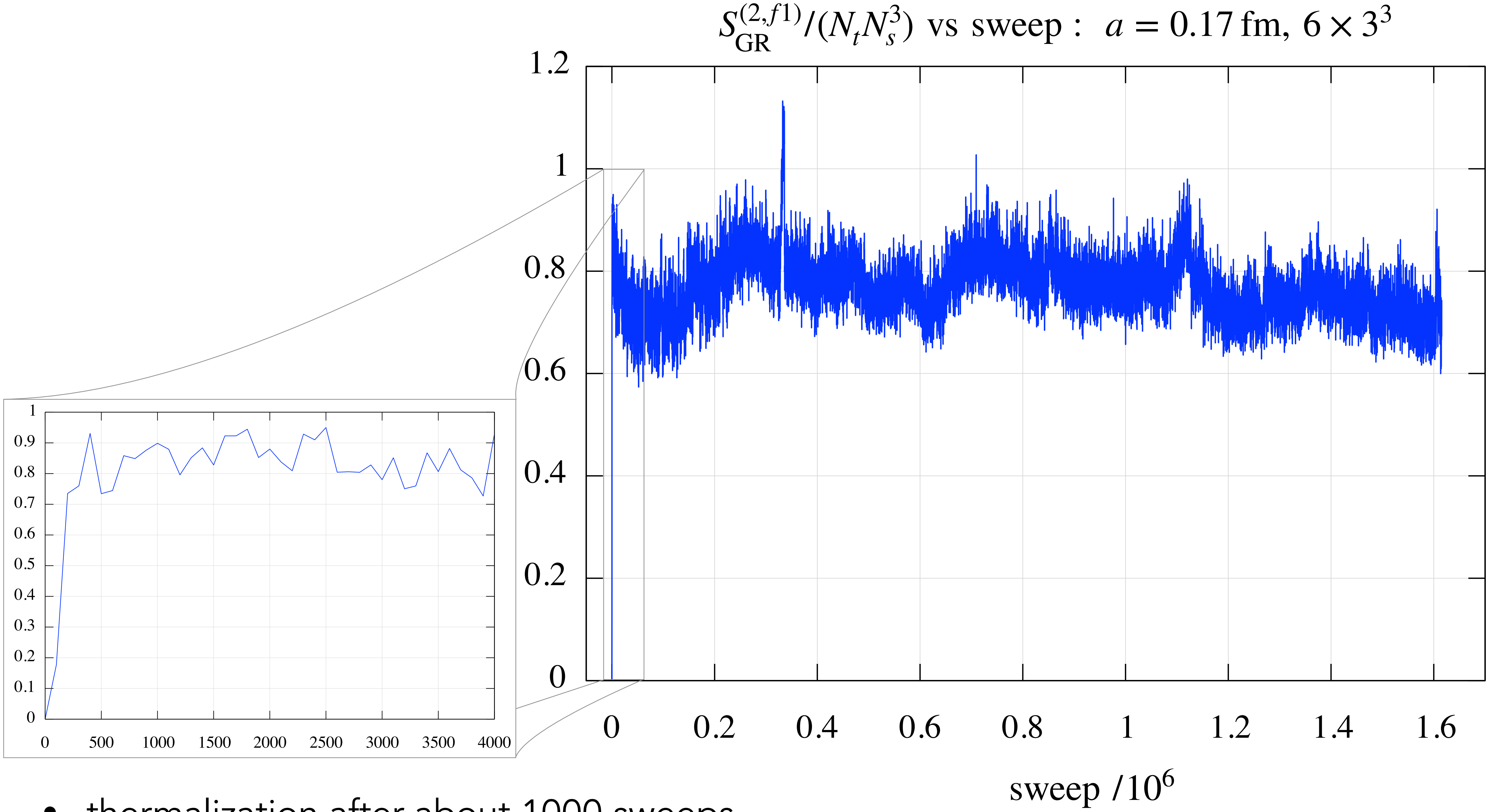


… how much to jiggle $\boldsymbol{h}$?

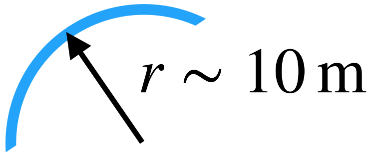


- Planck length jiggles give most efficient approach to thermalization
- larger jiggles, acceptance too unlikely
- smaller jiggles, too many updates needed to thermalize

… curved spacetime configurations

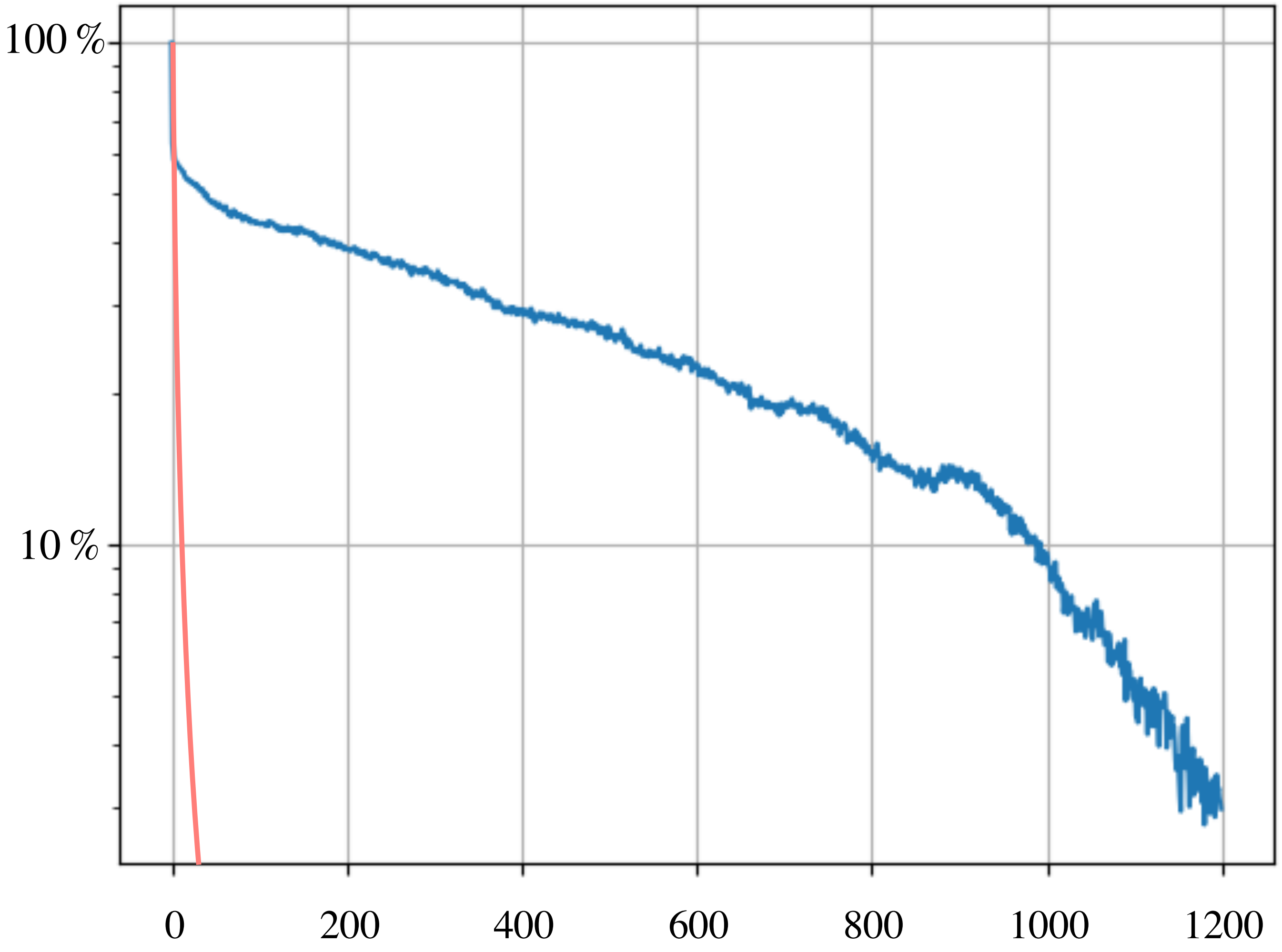


- thermalization after about 1000 sweeps
- for this $\mu = a^{-1}$, spacetime has $\mathrm{avg}(R) \sim -10^{-2}\,\mathrm{m}^{-2}$
- curvature from nonzero temperature
- spacetimes satisfy GR sanity checks, symmetries of $\Gamma^{\mu}_{\nu\rho}$ and $R^{\mu}_{\nu\rho\sigma}$

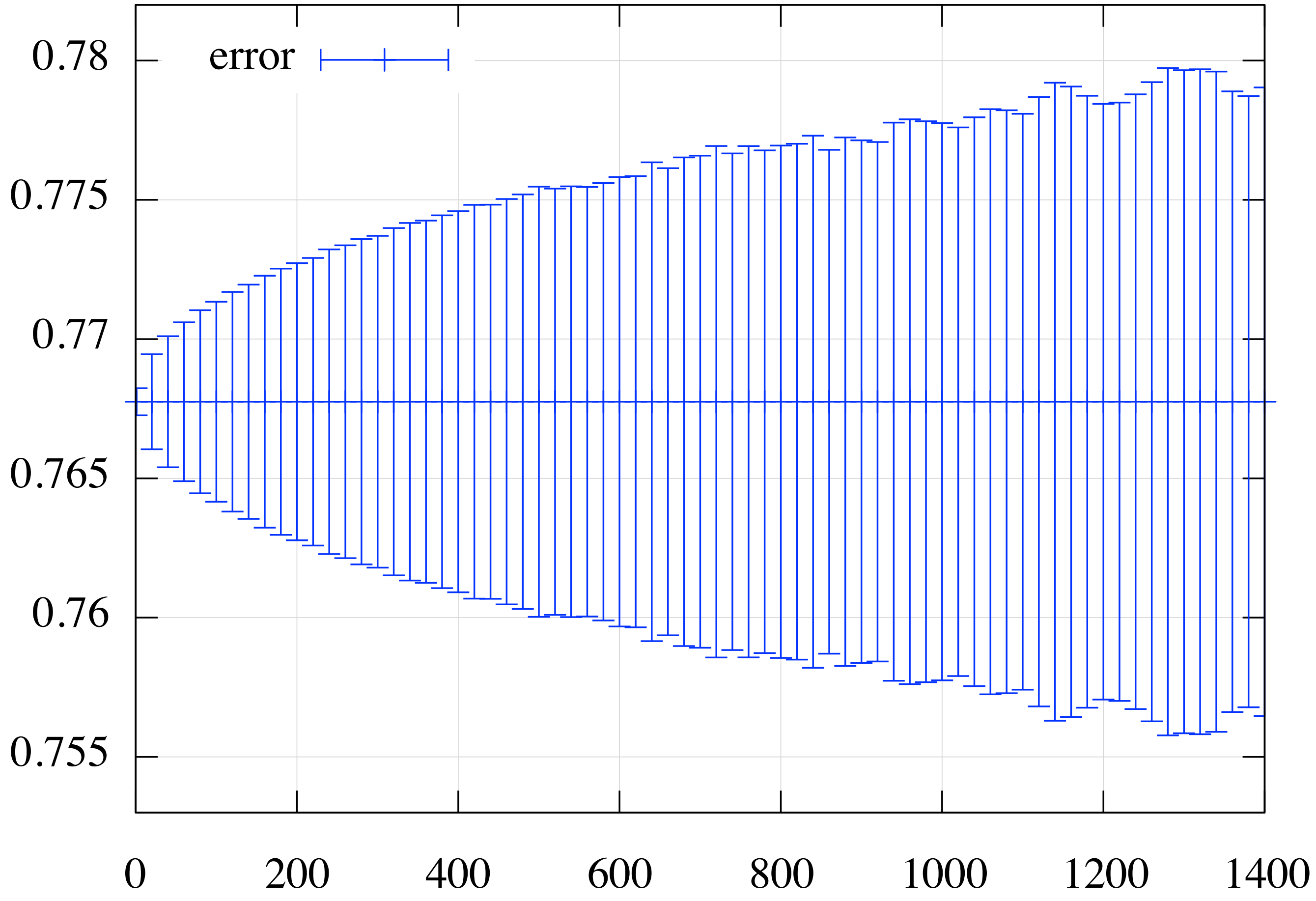


… autocorrelations

$\mathscr{A}(\text{lag})$ vs lag : $f1,\ a = 0.17\,\text{fm},\ 6\times 3^3$



error vs bin size : $f1,\ a = 0.17\,\text{fm},\ 6\times 3^3$



$$\mathscr{A}(\text{lag}) = \frac{\sum_{i=1}^{N}(O_i - \text{mean}_i(O))(O_{i+\text{lag}} - \text{mean}_i(O))}{\text{var}(O)\sqrt{N-1}}$$

- 1.6×10^6 sweeps → save every 100 → bin 1000 →16 configurations
- about 1000x worse than quenched QCD (factor of 10–100 from requiring positive action)
- $f2$ requires about 10x sweeps to thermalise but about half the bin size

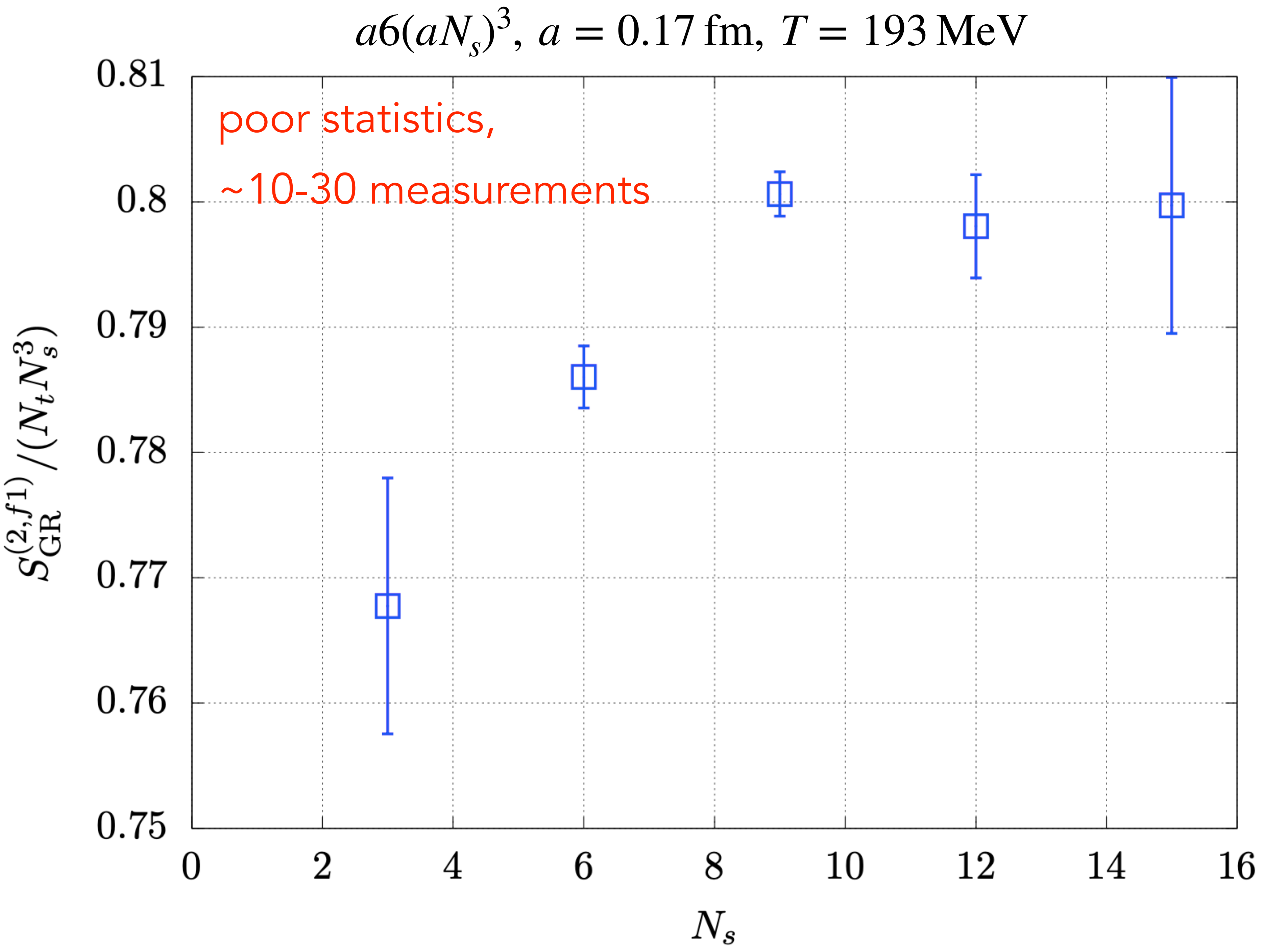
… GR discretization effects

- nonrenormalizability complicates continuum extrapolation
  - difficult to run observables to common scale $\mu = a^{-1}$
  - difficult to disentangle running from discretization effects

- estimate size at fixed lattice spacing using multiple discretizations
  - $R^{(f1)}(a, \mu) = R(0, \mu) + \mathcal{O}(a)$
  - $R^{(f2)}(a, \mu) = R(0, \mu) + \mathcal{O}(a^2)$ …
  - observe 14% difference between $R^{(f1)}$ and $R^{(f2)}$ for $a = 0.17\,\mathrm{fm}$, 6^4
  - if $\mathcal{O}(a^2) \ll \mathcal{O}(a)$, this is an estimate of $\mathcal{O}(a)$ effects

- how do $\mathcal{O}(a^n)$ errors scale in vacuum GR?
  - no GR-related IR scale
  - possible lattice scales are $(aN_s)^{-1}$ and $(aN_t)^{-1}$, so maybe $\mathcal{O}(N_s^{-1}, N_t^{-1})$?

… GR finite volume effects

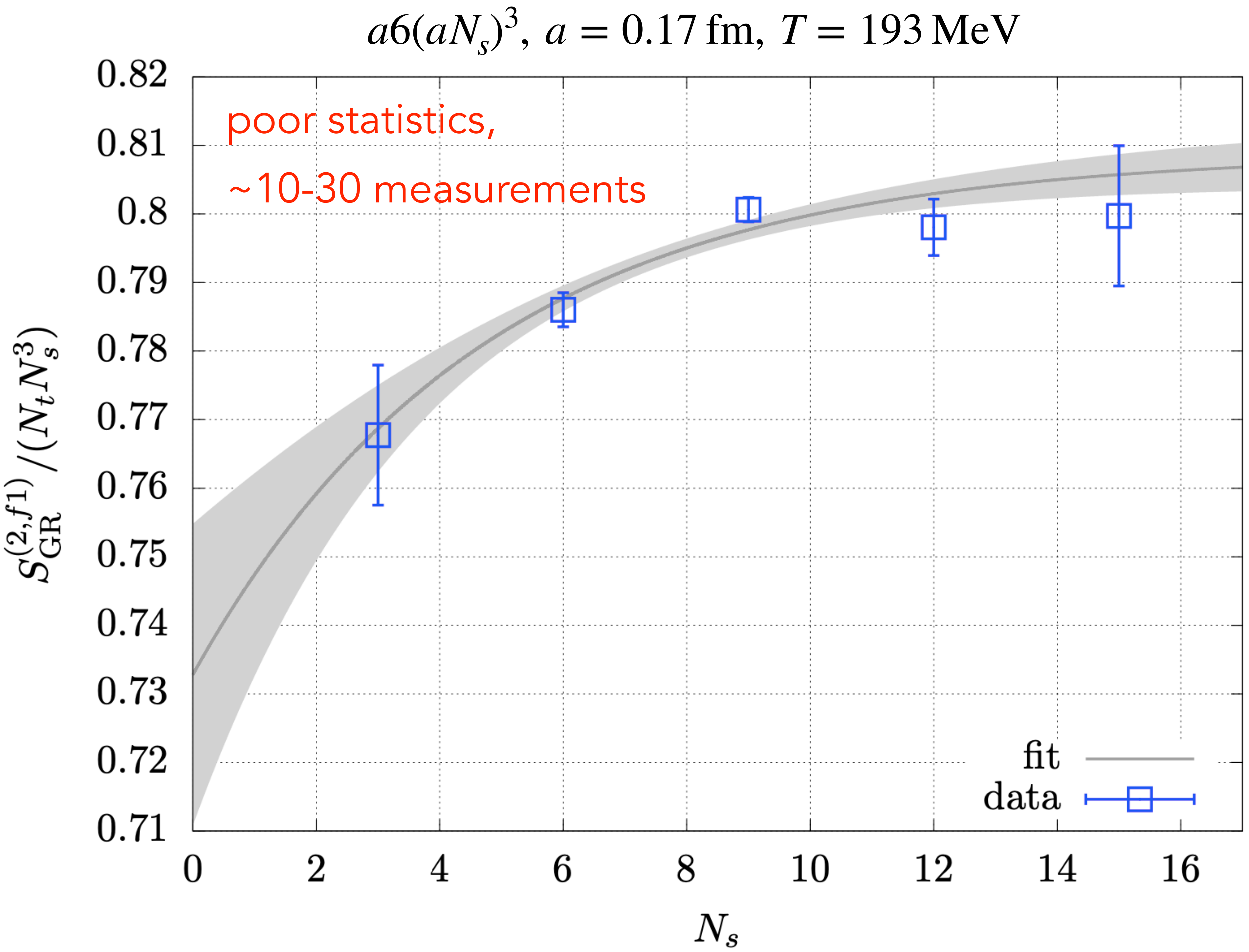


poor statistics,

~10-30 measurements

- might be able to extrapolate to infinite volume
- limited statistics

… scale of GR finite volume effects



$a6(aN_s)^3,\ a = 0.17\,\mathrm{fm},\ T = 193\,\mathrm{MeV}$

poor statistics,

~10-30 measurements

$\mathrm{fit}(N_s) = a - be^{-cN_s}$

Bayesian fit:

$\chi^2/\mathrm{dof} = 7.0/5$

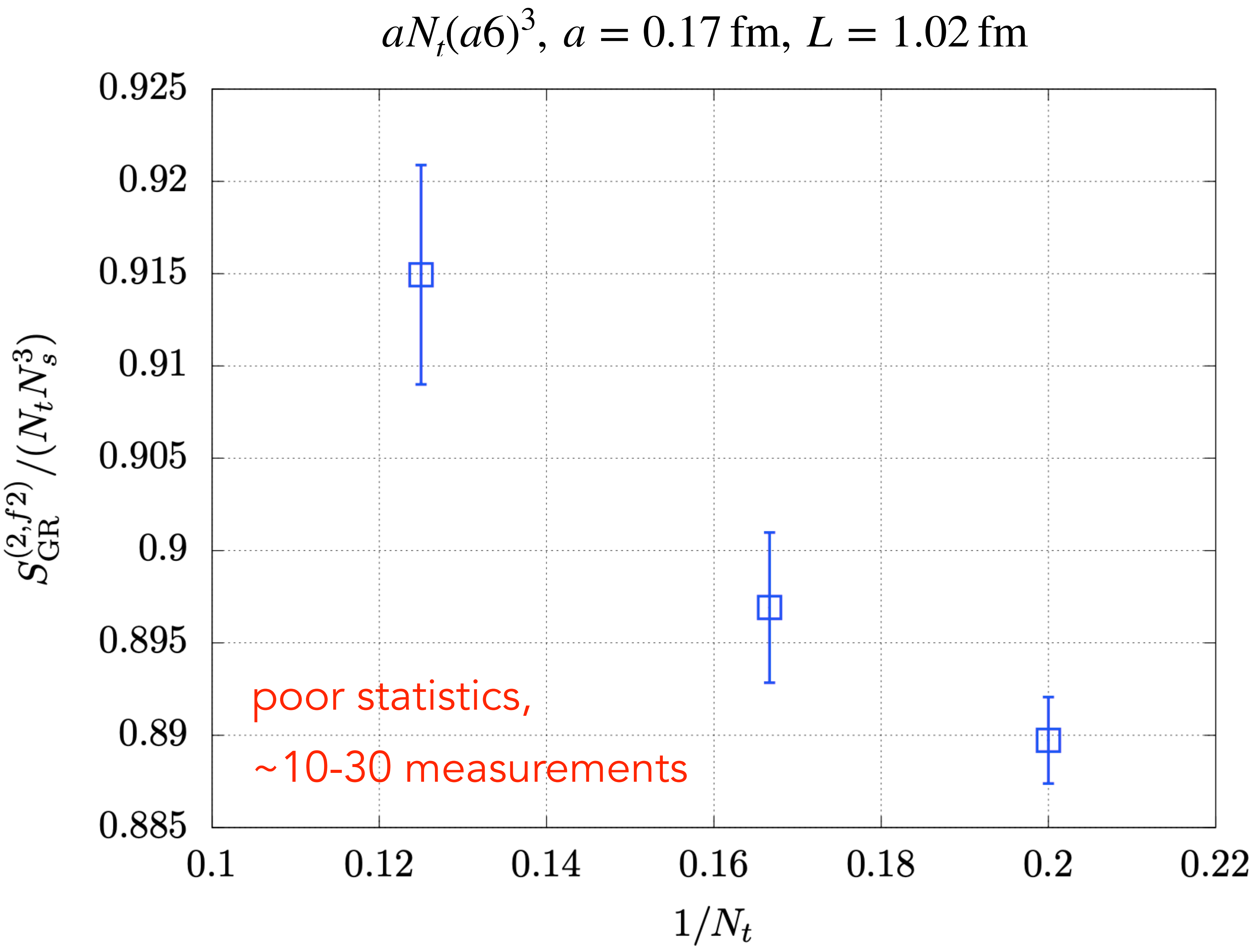
$a = 0.8089(52) \quad [1.0(5)]$

$b = 0.076(20) \quad [0.5(5)]$

$c = 0.213(67) \quad [0.17(8)]$

- rate of decay $\exp(-\Lambda L)$ implies scale, $\Lambda = 247(78)\,\mathrm{MeV} \approx T$
- infinite volume radius of curvature, $r = 8.86(3)\,\mathrm{m}$

… GR temperature dependence



- relativistic thermodynamics (for static ideal fluid), $T\sqrt{g_{00}} = \text{constant}$ Tolman, 1930
- quantum corrections to Newtonian gravity decrease with T Brandt, Frenkel, McKeon, Sakoda, 2023
- suggests $\partial_T R < 0$

Summary and outlook

- preliminary lattice implementation of weak-field, Euclidean GR
  - low energy effective theory of quantum gravity
  - complementary to analytic, perturbative efforts (both weak-field)

- needs further study/understanding
  - autocorrelations/statistics
  - discretisation effects

- working on coupling to QCD vacuum
  - if gravity quantum, coupling too small for machine precision, PI factorizes
  - if gravity classical
    - back reaction of quantum theory on a classical background (believe novel)
    - no-go theorem enhanced cross-talk

      see, e.g., Oppenheim, 2021

Thank you.

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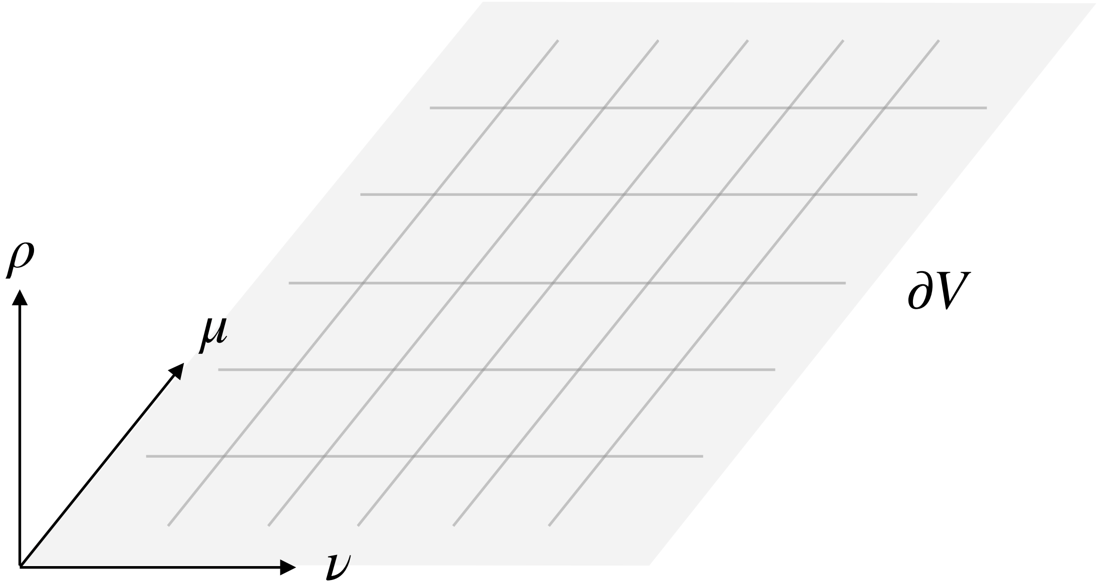
… constrain spacetime on the boundary

- constrain induced metric on boundary

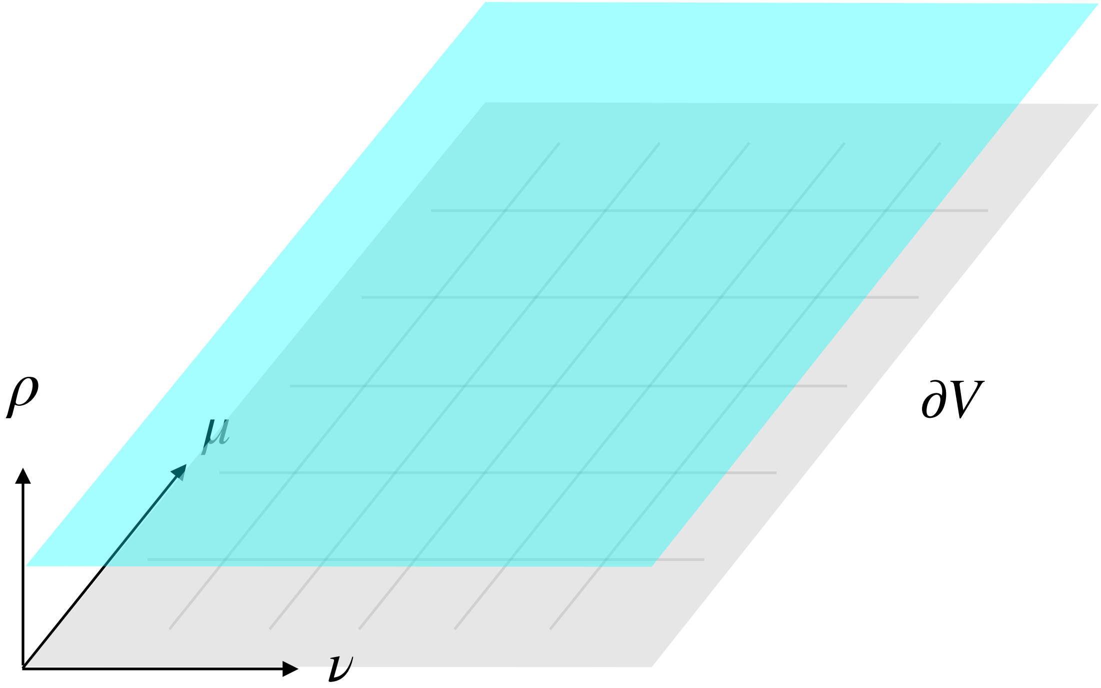
$$\boldsymbol{g}\Big|_{\partial V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \text{for } x_\rho \in \partial V, \;\; h_{\mu\nu}(x) = 0 \text{ for } \mu, \nu \neq \rho$$



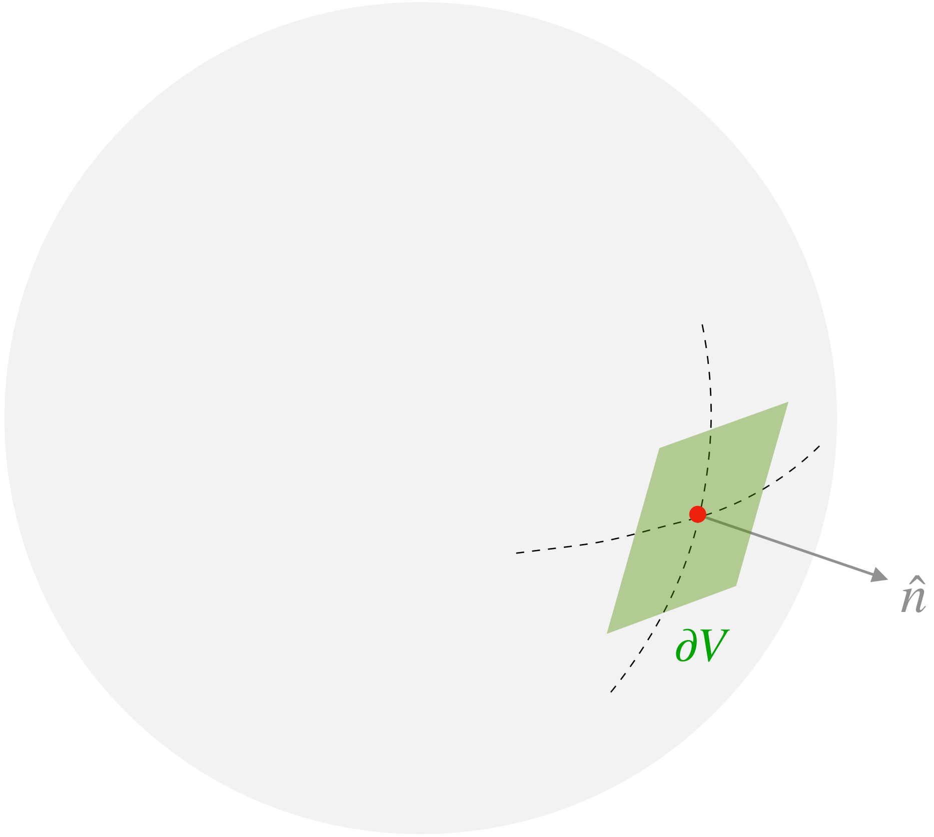
- constrain induced metric to **asymptote** to Euclidean metric on ∂V, $\hat{\rho} \cdot \partial \boldsymbol{g}\Big|_{\partial V} = 0$
  - for 1st order forward finite difference,

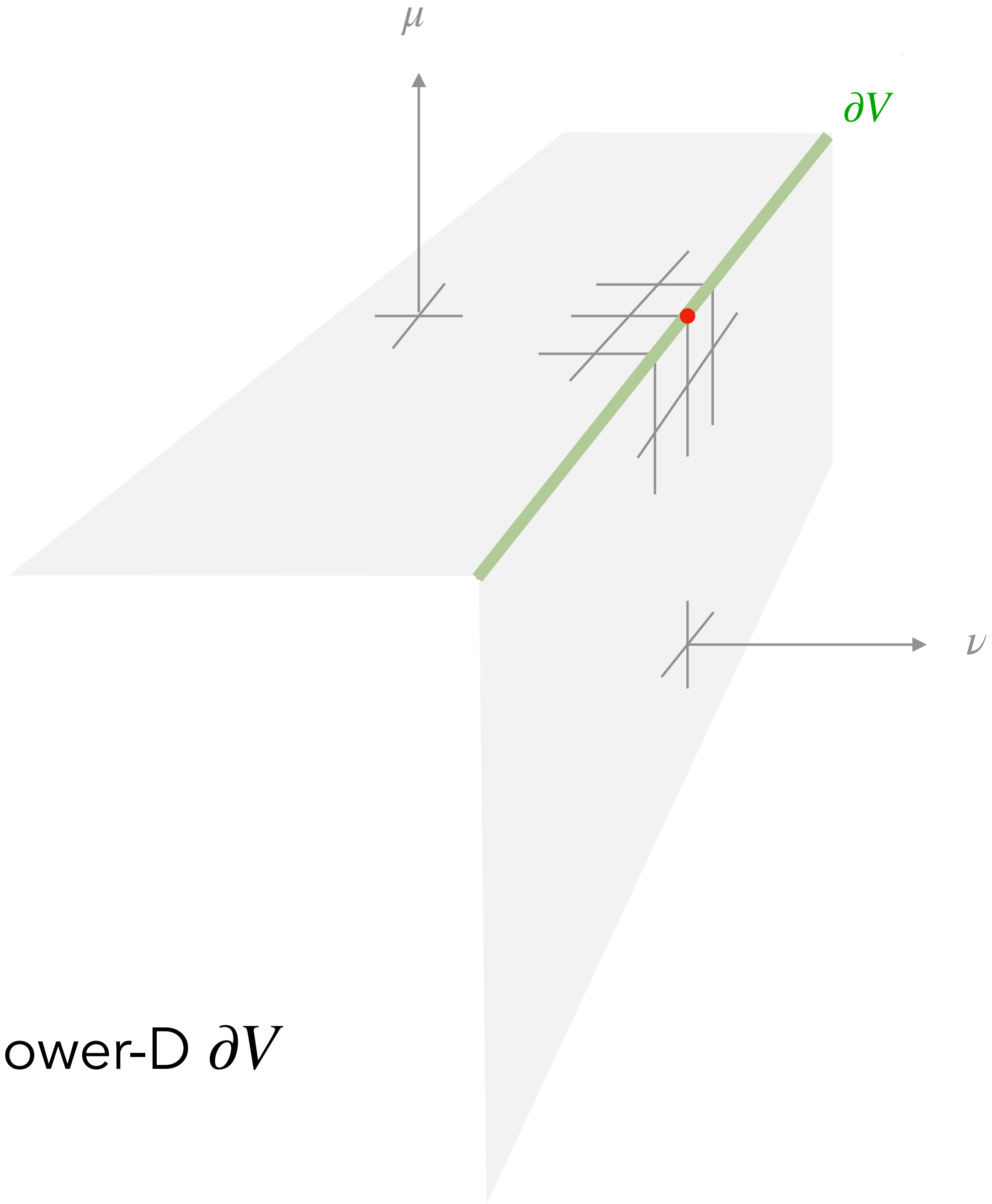
$$h_{\mu\nu}(x + a\hat{\rho}\,) = h_{\mu\nu}(x) = 0 \text{ for } \mu, \nu \neq \rho$$



… constrain spacetime on the boundary

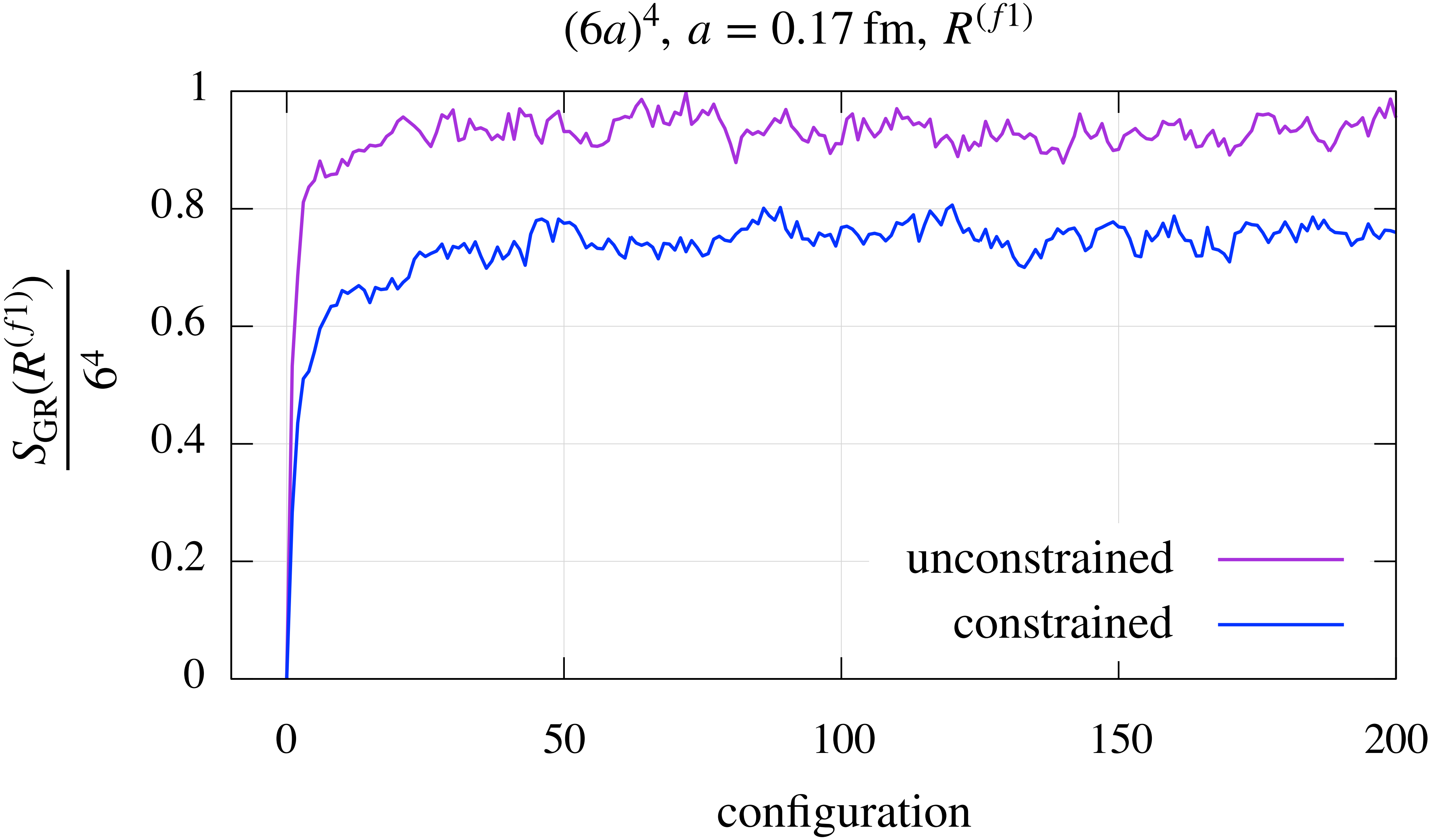
- with spherical symmetry ∂V always 3D





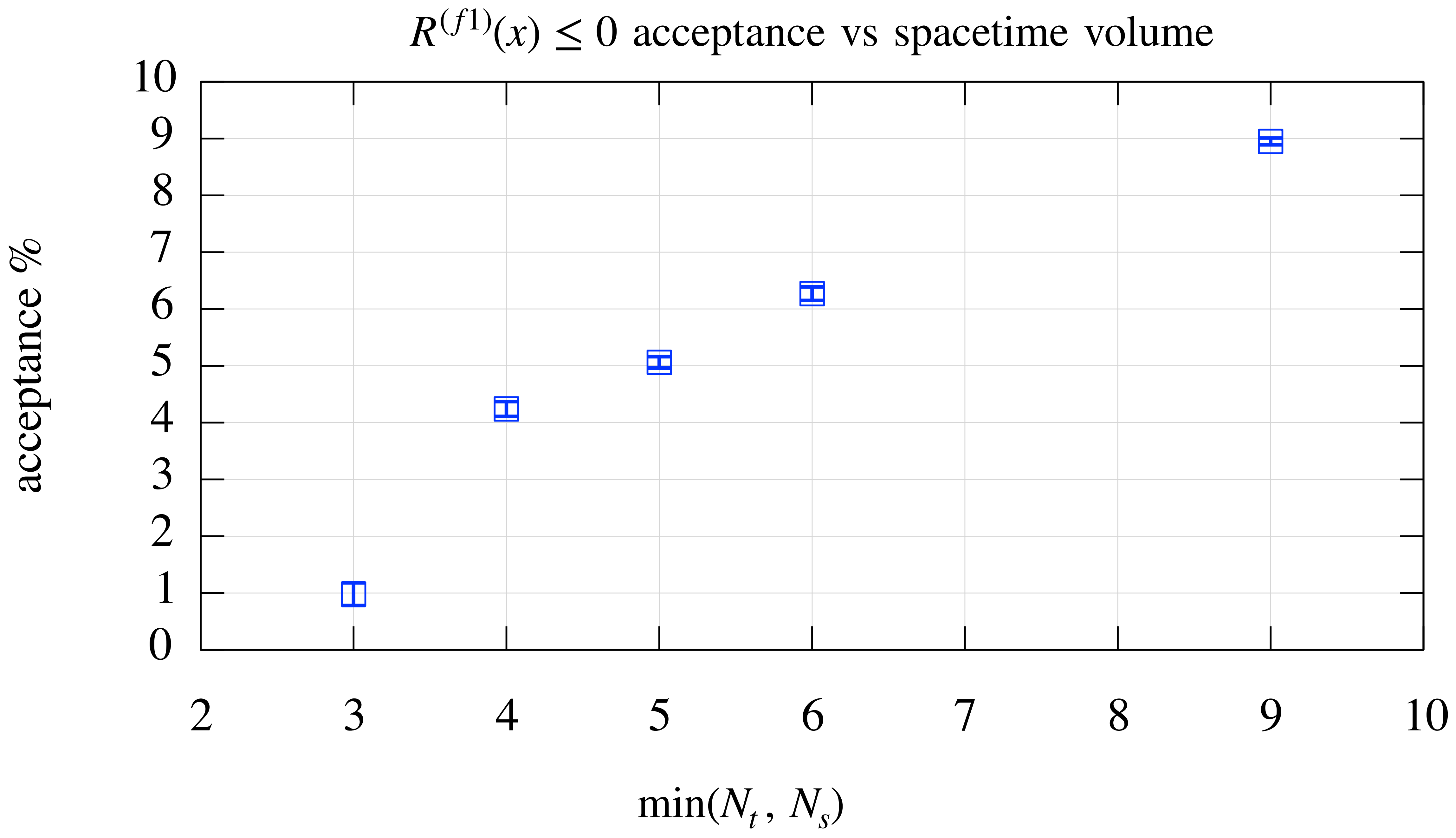
- edges and corners of hypercube have D < 3
- on edges and in corners, constraint imposed on lower-D ∂V

… constrain spacetime on the boundary



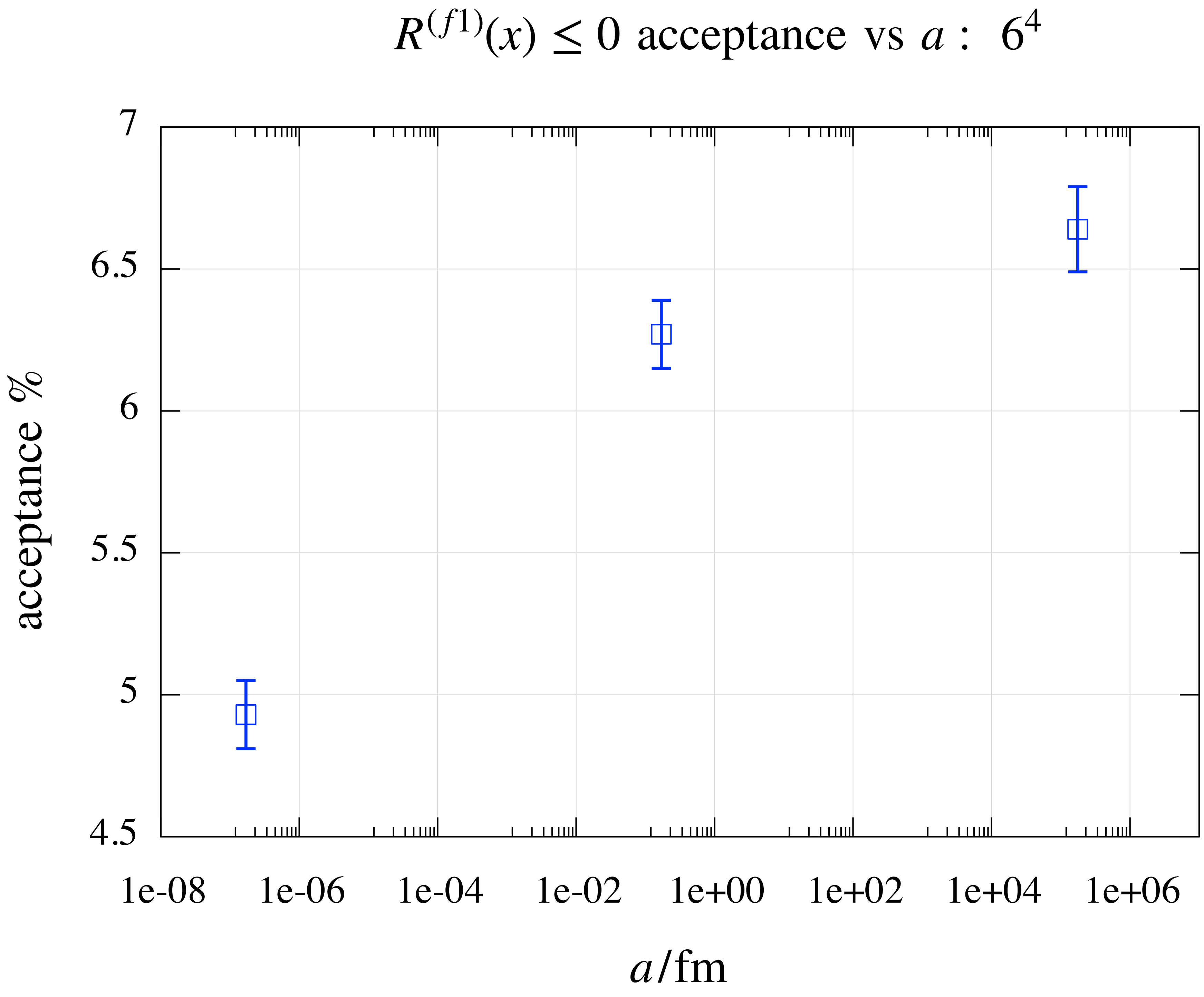
- positive action conjecture "*if spacetime asymptotes to flat, then* $S_{\rm GR} \geq 0$"
  - demanding $S_{\rm GR} \geq 0$ doesn't ensure asymptotic flatness
  - without asymptotic flatness, $\delta S_{\rm GR} = 0 \not\Rightarrow$ Einstein field equations
- increased curvature in bulk without boundary constraint

… positive action constraint



- local violations of positive action conjecture are (at least in part) due to finite volume

… positive action constraint



- Minor impact on autocorrelation from lattice spacing

Add QCD and look for interplay

- path integral quantised GR
  - fine as low energy EFT for quantum gravity
  - historically useful beyond weak-field with nontrivial background metric, e.g. black hole area-entropy law

- my motivation was preparation for next step

- to avoid pathologies in coupling classical GR to quantum theory of QCD

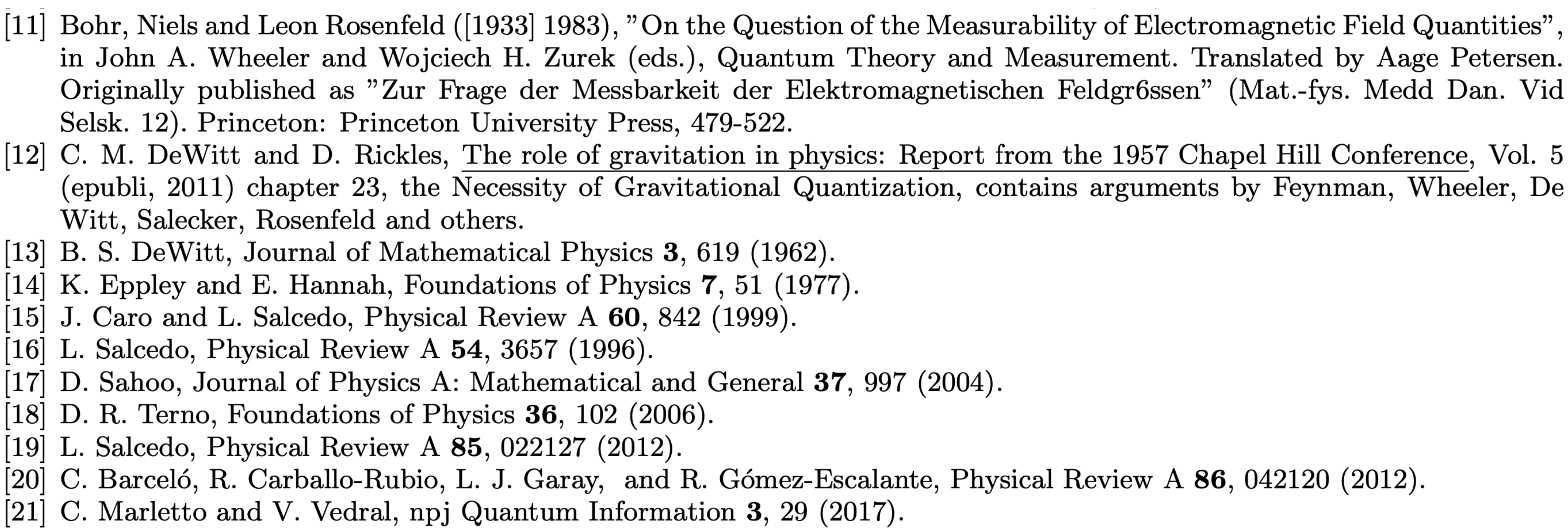
e.g., Oppenheim (2021)

$$\underset{\swarrow\ \textit{classical}}{2R_{\mu\nu} - Rg_{\mu\nu}} = \underset{\searrow\ \textit{quantum}}{16\pi T_{\mu\nu}}$$

  - couple GR to quantum fluctuations of QCD, not to expectation values
  - ⇒ GR must live in the path integral and respond to QCD fluctuations

No-go theorem references

taken from Oppenheim (2021)



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… interaction of GR and QCD

- expected QCD and GR interaction via cross-term

$$\sqrt{\det \boldsymbol{g}}\,\mathscr{L} = \mathscr{L}_{\mathrm{GR}} + \mathscr{L}_{\mathrm{QCD}} + \frac{\mathrm{tr}\,\boldsymbol{h}}{2}\mathscr{L}_{\mathrm{QCD}} + \ldots$$

- cross-term negligible: $\mathrm{tr}\,\boldsymbol{h}\mathscr{L}_{\mathrm{QCD}} \sim \mathscr{O}(10^{-17})$, $\mathscr{L}_{\mathrm{GR}} \sim \mathscr{O}(1)$, and $\mathscr{L}_{\mathrm{QCD}} \sim \mathscr{O}(10)$

- however, to couple classical and quantum theories, leading order terms are linked by contributions to $S_{\mathrm{QCD}} + S_{\mathrm{GR}}$

$$\text{prob}(\boldsymbol{U}, \boldsymbol{h}) \; = \exp\left[-\left(S_{\mathrm{QCD}}(\boldsymbol{U}) + S_{\mathrm{GR}}(\boldsymbol{h})\right)\right]$$

- as path integral samples paths in $(\boldsymbol{U}, \boldsymbol{h})$ space, QCD and GR collaborate in their contributions to action

… phenomenological applications

- extent of impact of QCD vacuum on curvature, with T as proxy for age of universe

- temperature study of GR + QCD simulation
  - what happens to spacetime across QCD confining phase transition

- Einstein field equations

$$2R_{\mu\nu} - Rg_{\mu\nu} = 16\pi T_{\mu\nu}$$

  - LHS without QCD should give $T^{(V)}_{\mu\nu}$ for spacetime without matter

$$T^{(V)}_{\mu\nu} = -\frac{\Lambda}{16\pi}g_{\mu\nu}$$

  - add QCD and compare LHS to $T_{\mu\nu}$ from LQCD

- effect of GR on QCD entanglement