Fine grinding localized updates via gauge equivariant flows in the 2D Schwinger model

BERGISCHE UNIVERSITÄT WUPPERTAL

31. July 2023

Lattice 2023

Fermilab - Virtual Contribution - 16:40

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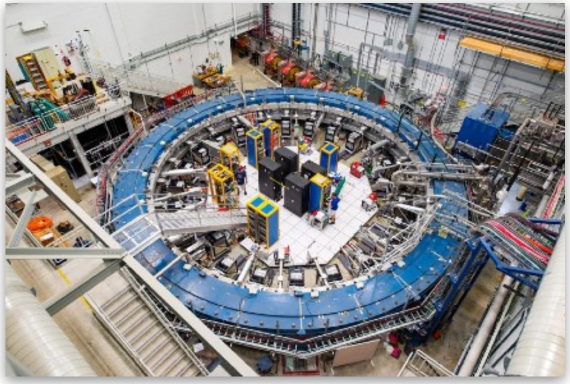
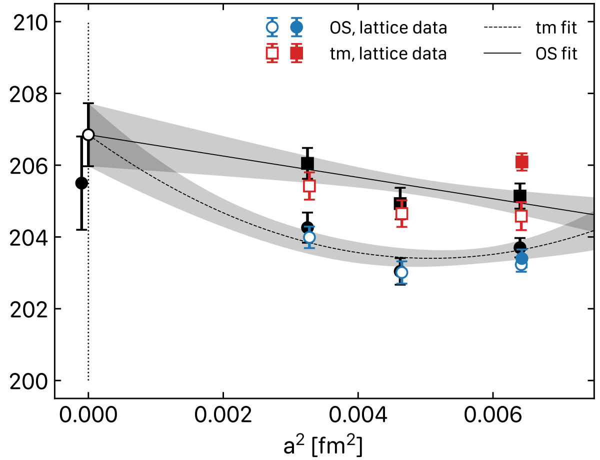
Motivation



Critical slowing down

- towards fine lattice spacing, autocorrelation increases proportional to the inverse of the lattice spacing



- In gauge theories, this is even more severe due to topological freezing
  - Currently not possible to simulate a < 0.04 fm with periodic BC

How to fix ?
Concept of MCMC:

1. Propose U' according to $T_0(U \to U')$
2. Accept-reject $P_{acc}(U \to U') = \min\left[1, \frac{\tilde{\rho}(U)\rho(U')}{\rho(U)\tilde{\rho}(U')}\right]$

Works if:

- (1.) proposal can decorrelate/ has high topological tunnelling rate
- (2.) Acceptance rate is high

Acceptance rate

Acceptance step:

$$P_{acc}(U \to U') = \min\left[1, \frac{\tilde{\rho}(U)\rho(U')}{\rho(U)\tilde{\rho}(U')}\right]$$

Distributions $(\tilde{\rho}(U)\rho(U'))/(\rho(U)\tilde{\rho}(U'))$ log-normal distributed

❖ for the acceptance rate follows

$$P_{acc} = \mathrm{erfc}\{\sqrt{\sigma^2(\Delta S)/8}\}$$

with

$$\Delta S = \ln\{\rho(U')\} - \ln\{\rho(U)\} + \ln\{\tilde{\rho}(U)\} - \ln\{\tilde{\rho}(U')\}$$

Creutz, Phys. Rev. D38 (1988) 1228–1238

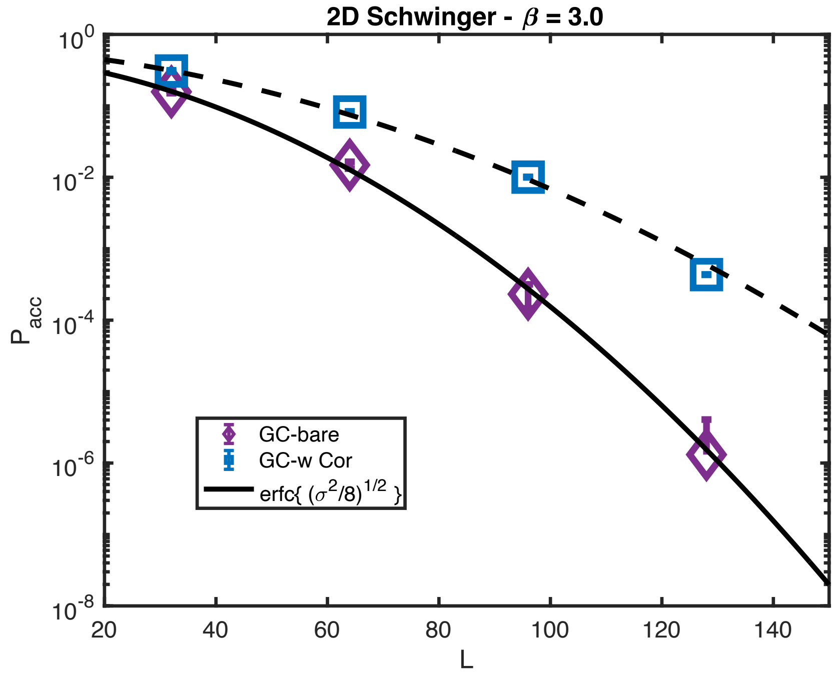
How to control the variance $\sigma^2(\Delta S)$

❖ Using correlations between proposal and target distribution: $\max|\mathrm{cov}(\rho, \tilde{\rho})|$

❖ Reduce degrees of freedom within the proposal
- e.g. Domain decomposition

Boltzmann weight

$$\rho(U) = \det D^2(U)\, e^{-\beta S_g(U)}$$



1. Propose via $\propto e^{-\beta S_g(U)}$
2. Correct via $\propto \det D^2(U)$

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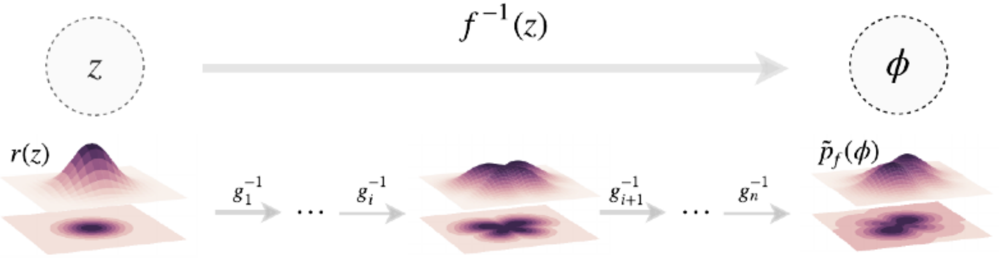
Proposal

Propose U' according to

$$T_0(U \to U')$$

Allow topological tunneling

- ❖ One choice: trivialising flows
  - start with uniform distribution $r(U_0)$
  - Flow back to target distribution using coupling layers $f^{-1}(U_0) \to U$



Albergo et al., Phys.Rev.D 100 (2019) 3, 034515

Flow distribution is given by the Jacobian over the coupling layers

$$\tilde{\rho}(U) = r(f(U)) \cdot \left|\det \frac{\partial f(U)}{\partial U}\right|$$

Idea: train coupling layers: $\min\left(\sigma^2(\Delta S)\right)$

❖ Train Networks: $\tilde{\rho}(U) \approx \rho(U)$

Kanwar et al., Phys.Rev.Lett.125 (2020) 12, 121601

Albergo et al., Phys.Rev.D 100 (2019) 3, 034515

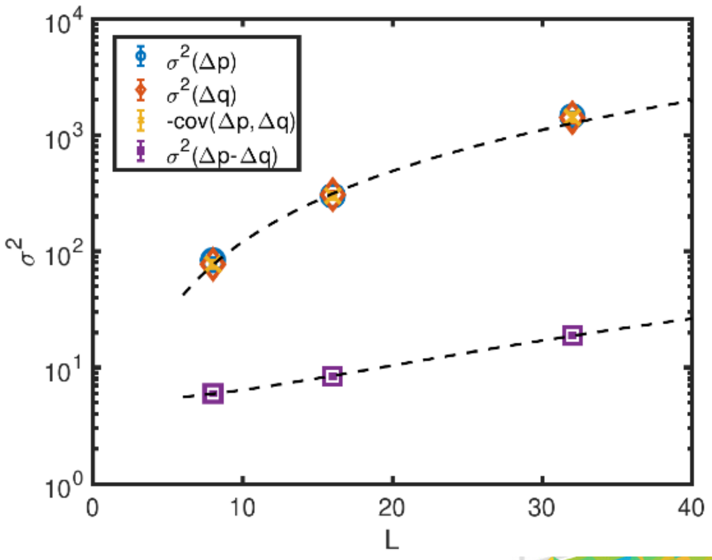
Boyda et al., Phys.Rev.D 103 (2021) 7, 074504

Albergo et al., arXiv:2101.08176

Volume fluctuations

❖ Localized models:

$$\sigma^2(S) = \langle S^2 \rangle - \langle S \rangle^2 \quad = V(a_0 + a_1 e^{-d} + a_2 e^{-\sqrt{2}d} + \ldots)$$



❖ Variance scales with the volume, acceptance rate is rapidly 0

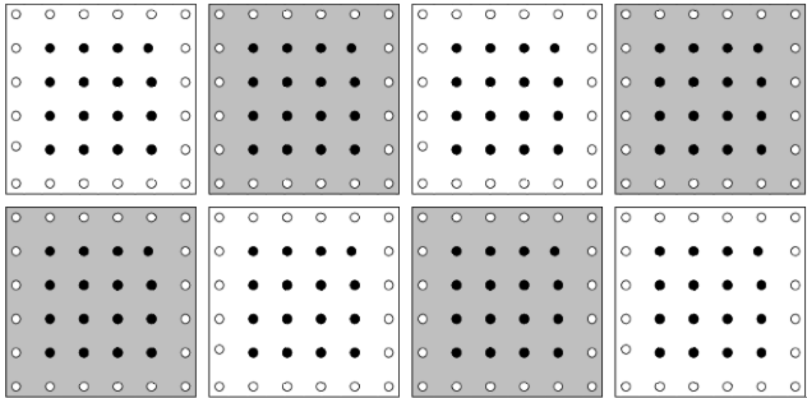
Localization: Domain Decomposition

Idea: Decomposition of lattice into domain

Separate action into:

$$S_{global} = \sum_{blk} S_{local} + I(S_{global}, S_{local})$$

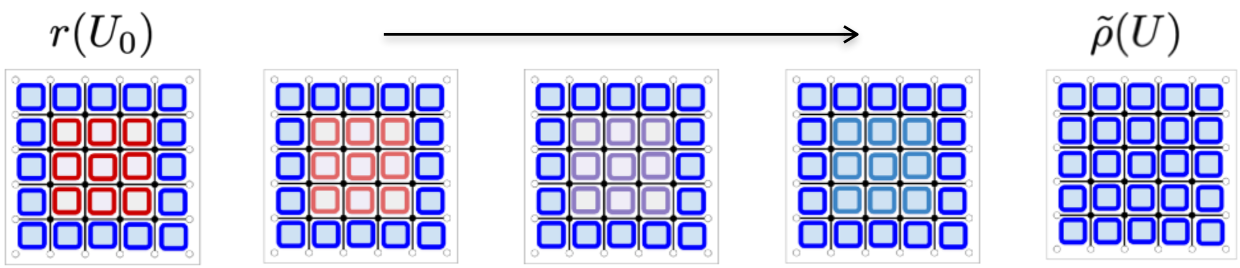
Decomposition straightforward for ultra local lattice actions

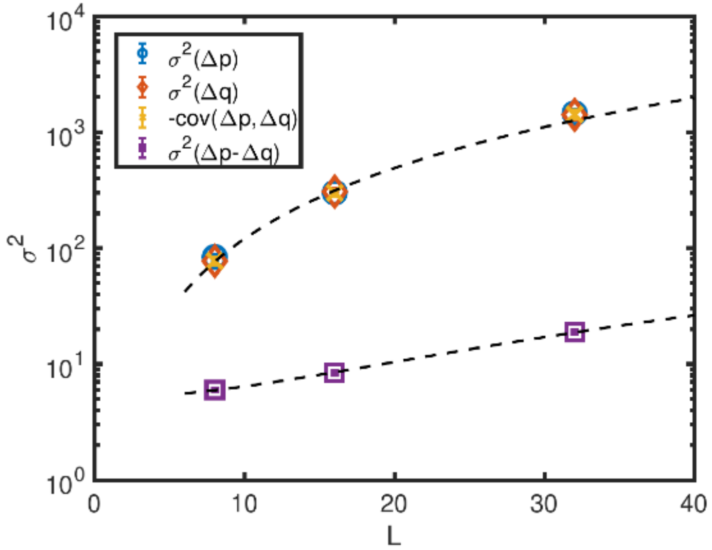


Taken from: M. Luscher, CPC 165 (2005) 199-220

Domain Decomposition of normalizing flow

- update only links/variables inside blocks by creating maps of active links within each block



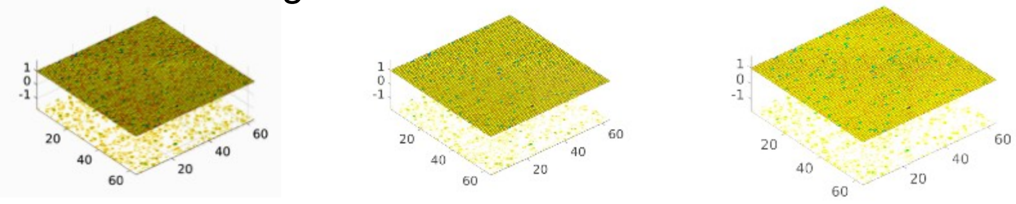


However: becomes difficult for larger volumes (needed in 4D-SU(3))

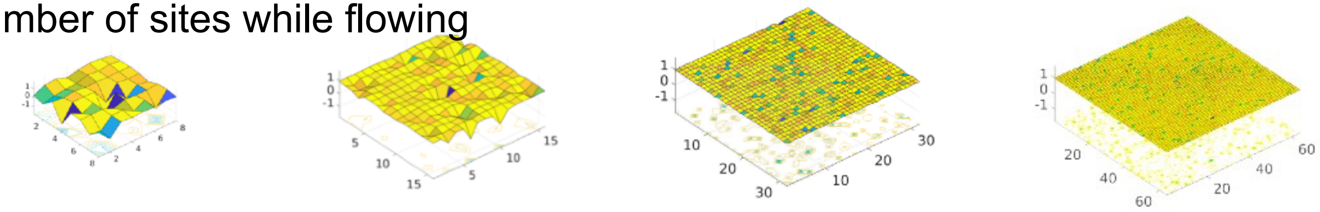
Fine graining flows

Flow maps:

Standard Map: Keep L/a fixed
❖ Physical lattice size is decreasing



Natural Projection: Keep physical box size fixed
❖ extend number of sites while flowing

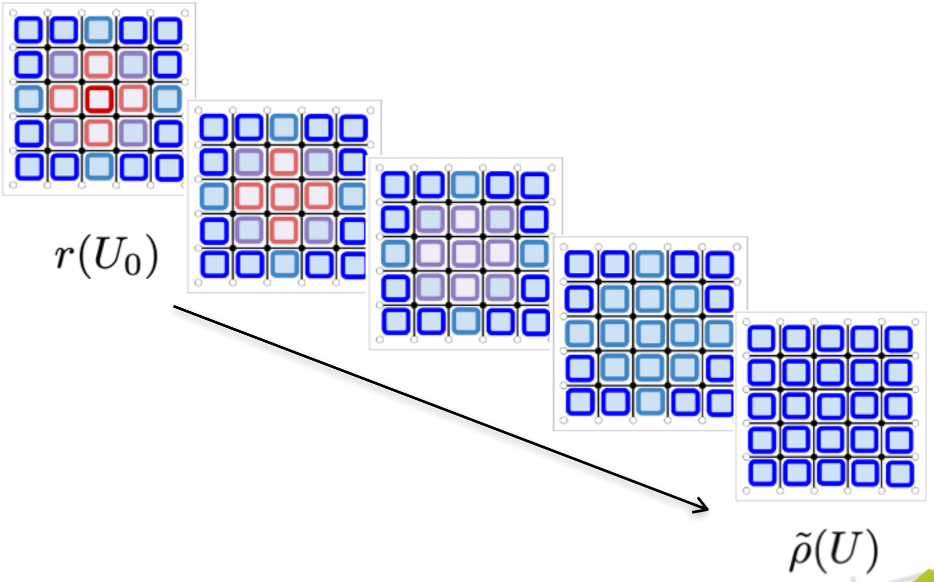


Idea: Effective coarse to fine graining

Introduce a smoothing mapping within a larger lattice
❖ place local defect and smooth out

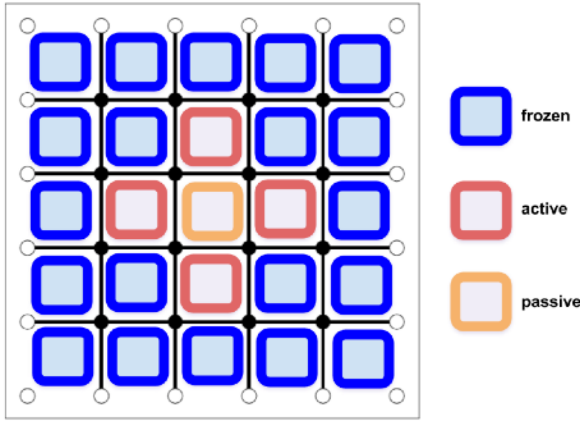
- like multi-tempering approaches
  - successfully applied in 4D-SU(N)

C. Bonati et al., PRD 99, 054503 (2019)

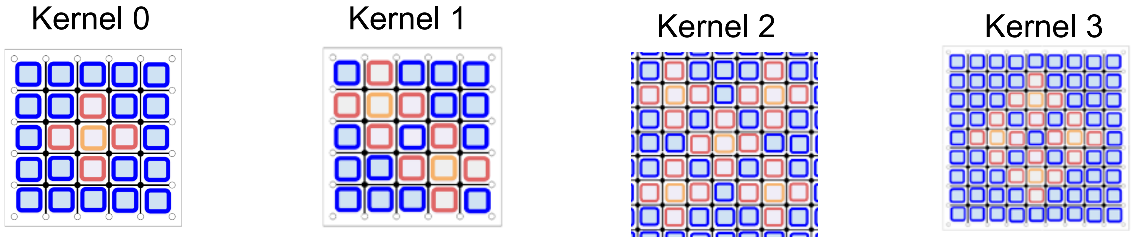


Here: use local flow transformations
❖ needs adjustments/modifications
  ❖ Maps: localization of updates
  ❖ Training conditions / loss function
❖ Train for topological tunneling

Training techniques



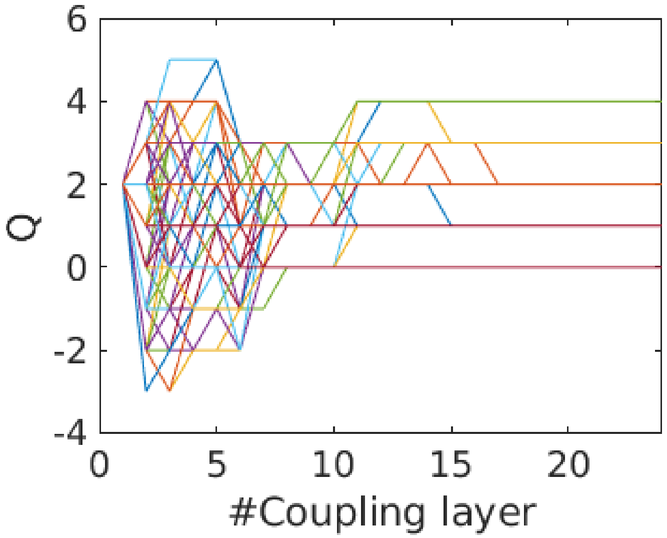
Localized update:
Center symmetric update
❖ only randomise all 4 links of center plaquette
❖ in 2D use a max. compact map
✦ active to passive ratio = 4:1



In line with Kanwar et al., Phys.Rev.Lett.125 (2020) 12, 121601

Modification of the loss-function:
Training for topological transitions
❖ by modification of the loss-function

$$L = \ln(\tilde{\rho}(U')/\rho(U')) \cdot |Q(U') - Q(U)|$$



Train transitions using only four uniformed links
❖ Correlations need to be smeared out
❖ otherwise fancy plaquette update

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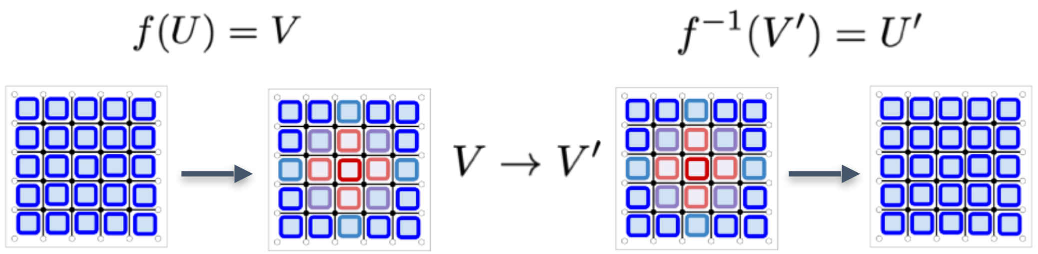
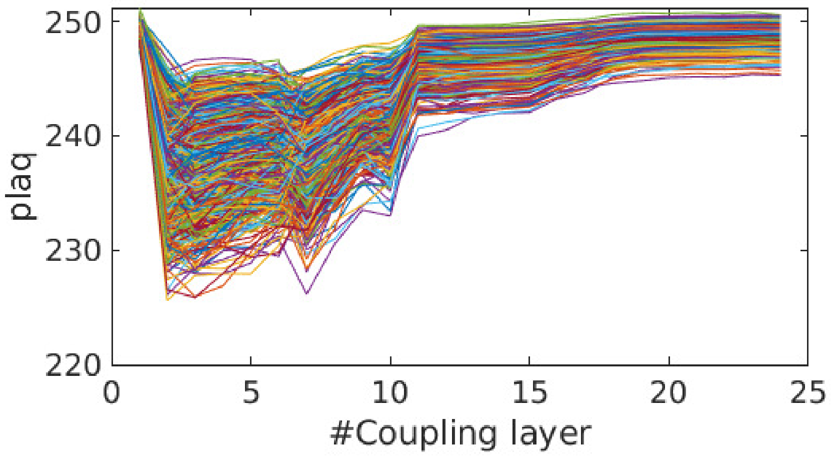
Grinding the fine graining



Graining needs change of update procedure:
Requires back transformation before the update

- Currently: for training fix backward transformation and update occasionally
- Allows for new loss-function
  - Works in combination with topo. loss and fixed backward transformation

$$L = \ln\{\rho(U')\} - \ln\{\rho(U)\} + \ln\{\tilde{\rho}_{fixed}(U)\} - \ln\{\tilde{\rho}_{train}(U')\}$$



Grinding training:
Current training setup: relative long training chain

- Pre-train on L=8 with pBC
- Retrain on L=16 with fixed $\tilde{\rho}$
- Build up chain via intermediate loss
- Fine tune with updated $\tilde{\rho}$

Use HMC generated configs

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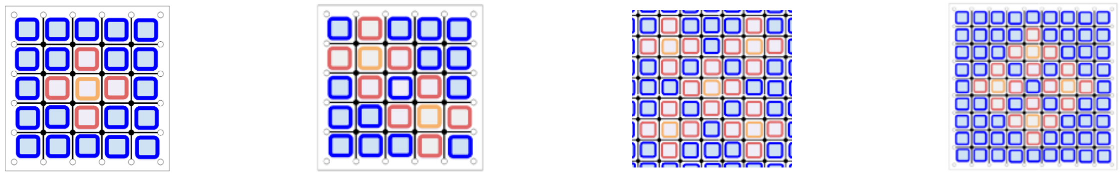
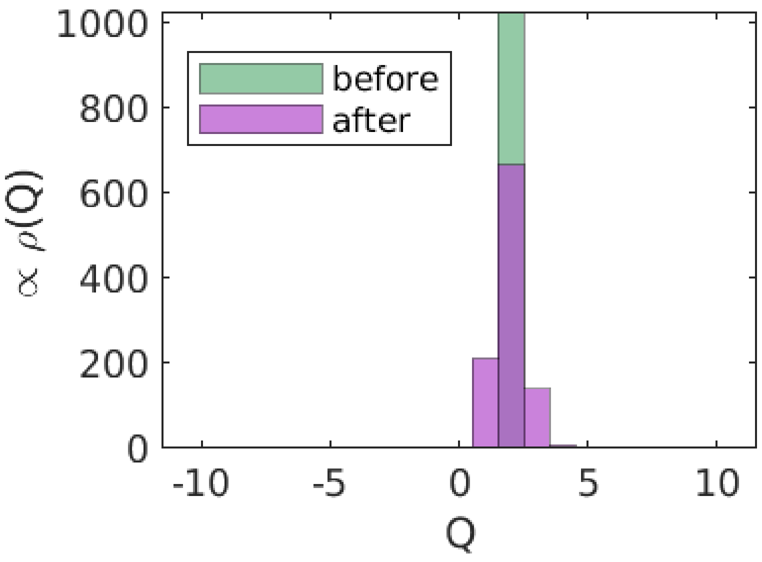
Fine Graining Flows

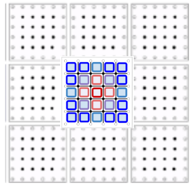
Test fine graining updates

- Updates at $\beta = 11.25$
- Kernel 1-3 used
  Use 3x[1, 3, 2] + 2 with shifts

Kernel 0 Kernel 1 Kernel 2 Kernel 3

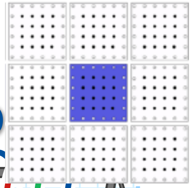




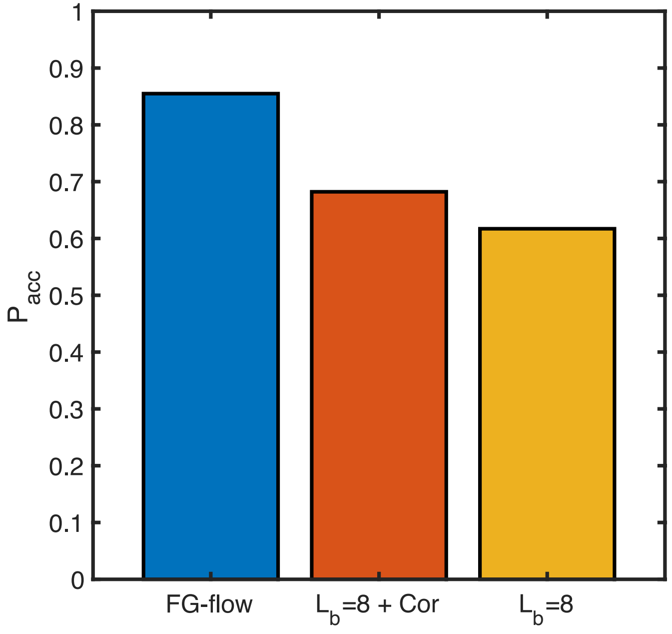


Local updates also increase block determinants acceptance rate

$$P^{(b)}_{acc}(U \to U') = \min\left[1, \frac{\det(D^b(U'))^2}{\det(D^b(U))^2}\right]$$



- Using an Fine Grain - flow update within a L=16 box
  - Acceptance rates of Pacc ~ 0.85 can be reached



- Flow updates within L=8 box is reaching Pacc ~ 0.6 %

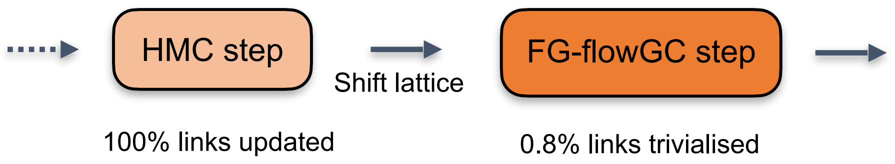
Results:
Tunneling rate

Tunneling rate:

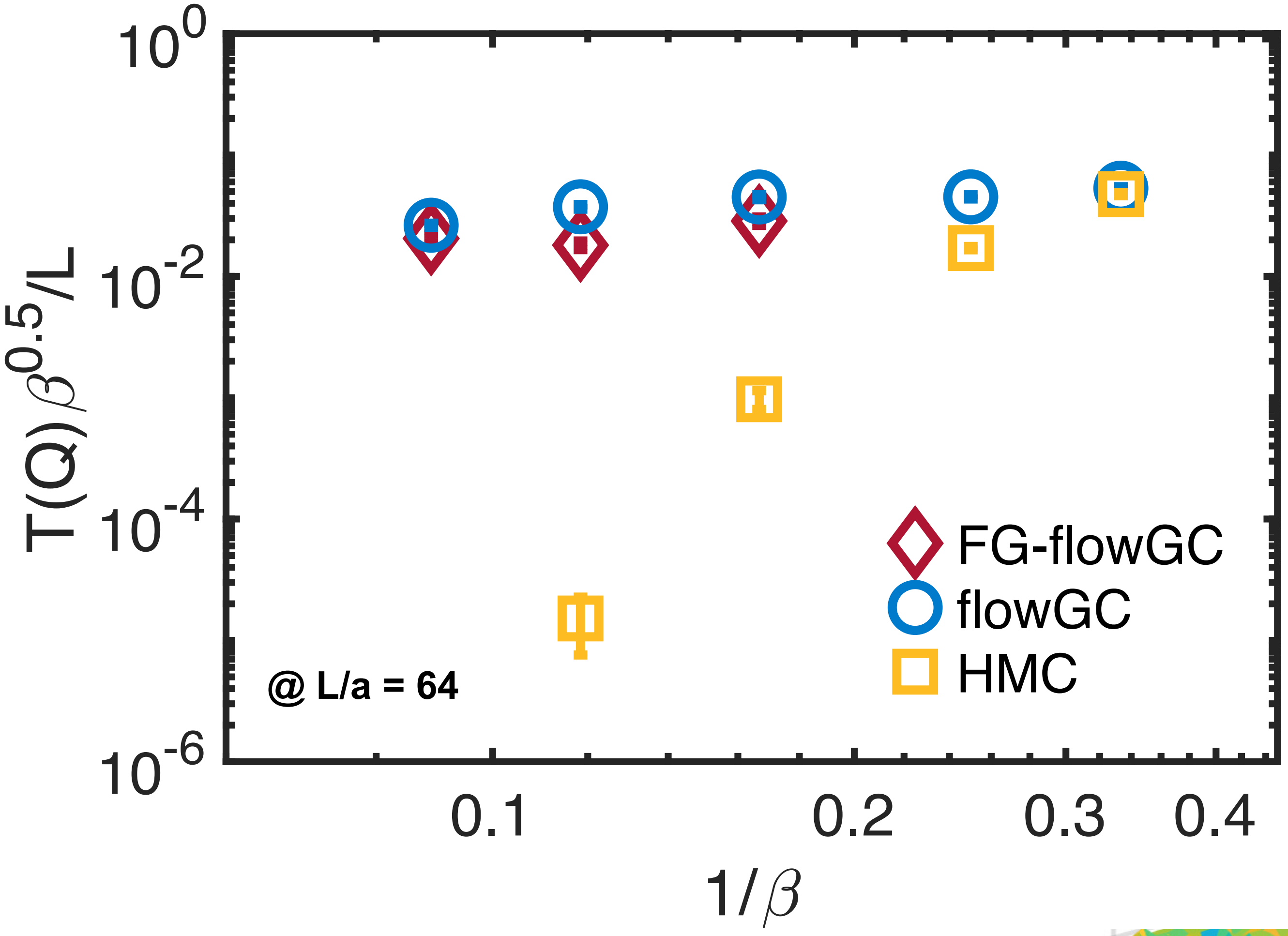
$$T(Q) = \langle |Q_i - Q_{i+1}| \rangle$$

Flow enables simulations beyond beta > 6.0

FG-flowGC:



flowGC:



Global Corrections with Fermions

Recursive Domain Decomposition

Action with fermions:

$$\rho(U) = Z^{-1} \left(\prod_j^{N_f} \det D_j(U) \right) e^{-\beta_g(U)}$$

with $\det D(U)$ is a *localised* action

- distance interaction decays with

$$cov(x,y) \propto \exp\{-m_{PS}|x-y|\}$$

Idea: using exact decomposition of fermion action:

$$\det D = \det S_{red} \cdot \det S_{pink} \cdot \det D_{blue}$$

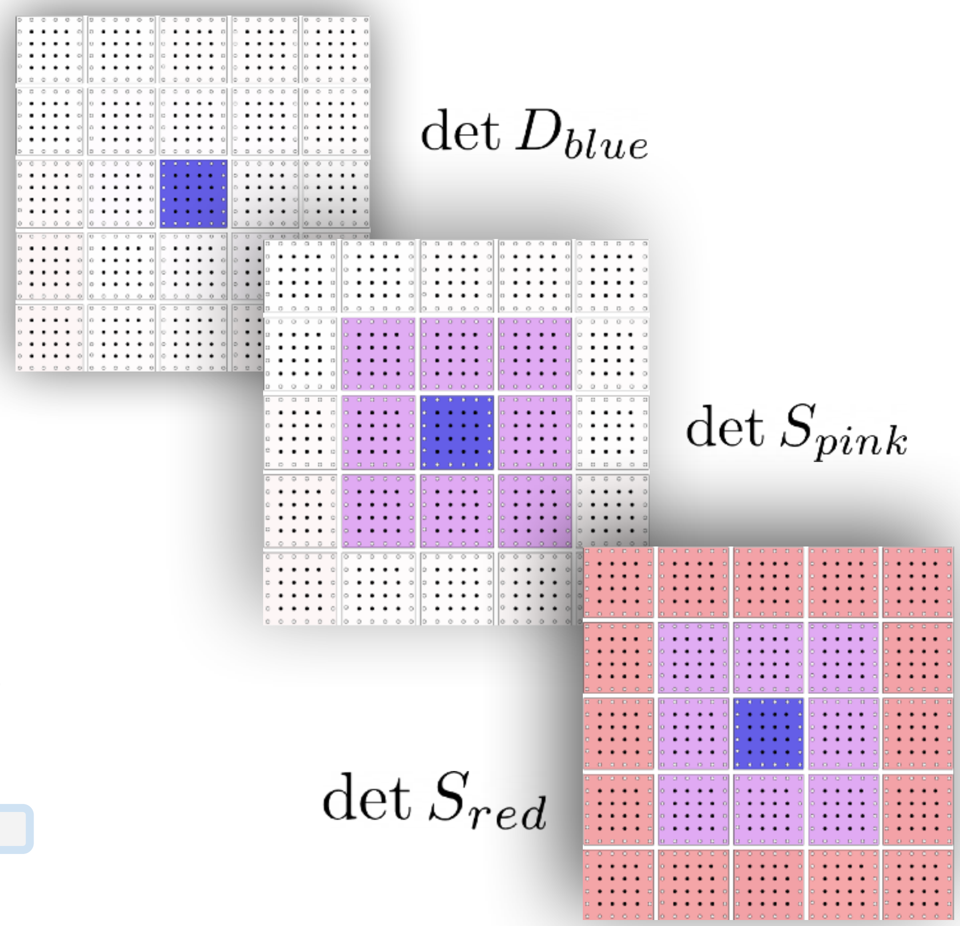
effective long range decomposition of the fermion determinant

M. Luscher, CPC 165 (2005) 199-220

J.F. et al., CPC 184 (2013) 1522-1534

M. Cè et al., Phys.Rev.D 93 (2016) 9, 094507

M. Cè et al., Phys.Rev.D 95 (2017) 3, 034503



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Correction steps With Fermions

Fermion Corrections via hierarchical Filter and flow-based pure gauge updates

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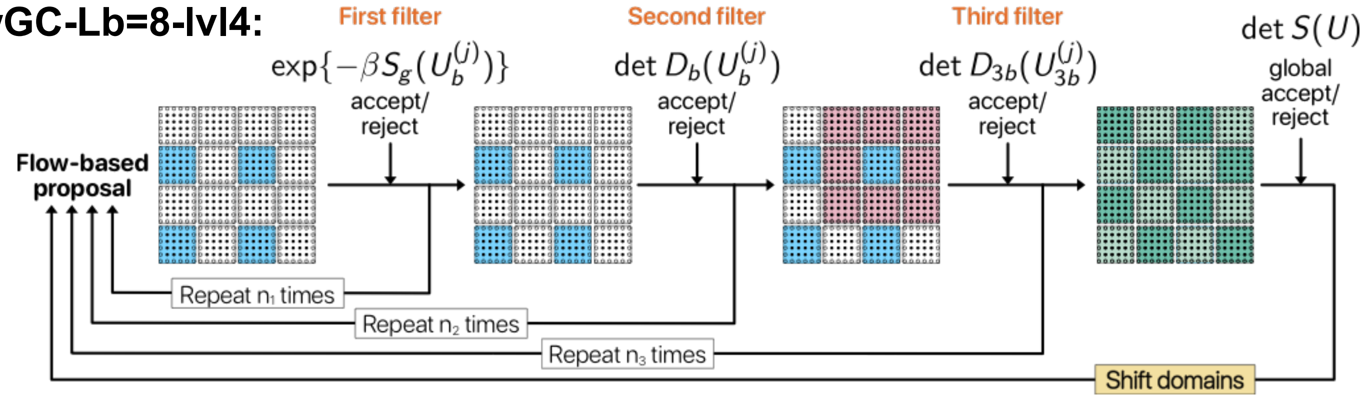
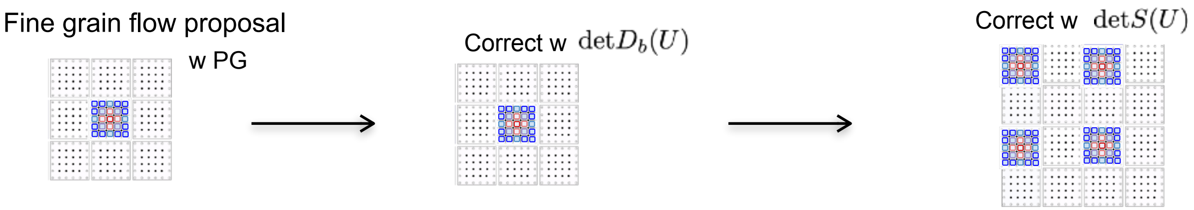
flowGC-Lb=8-lvl4:

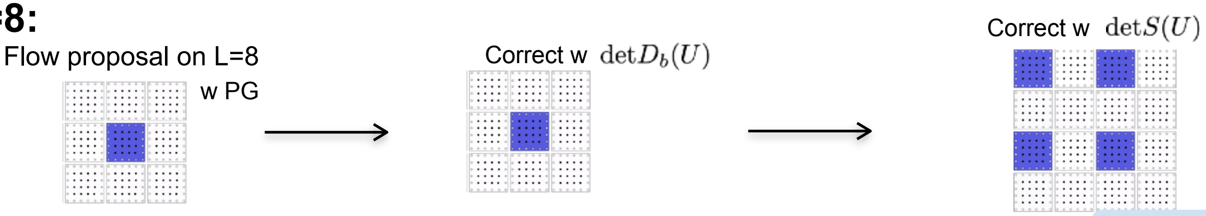


GC-bare:

FG-flowGC: New

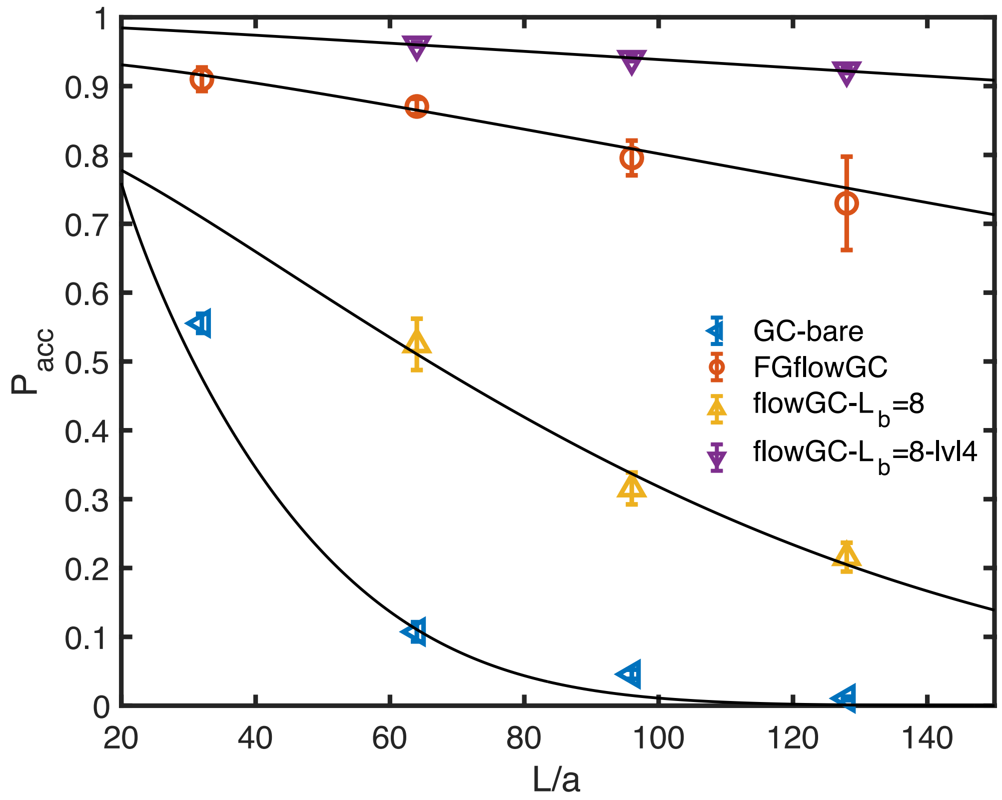


flowGC-Lb=8:



J.F., arXiv:2201.02216

2D - Schwinger Model
Global acceptance rate



$\beta = 6.0$ **@ z = 0.2**

N. Christian et al., Nucl.Phys.B 736 (2006)

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Conclusion

How to design proposals with topological tunnelling ?

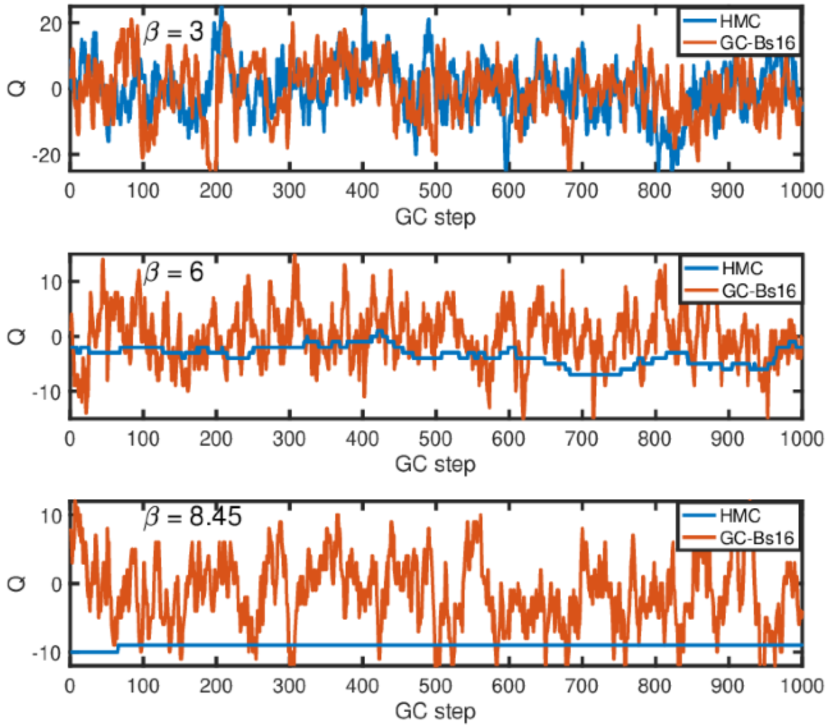
Fine graining updates in 2D-U(1)

❖ Use flow maps to smooth out defect
  - currently grinding task to train
  - Minimal change of degrees of freedom

Leads to

❖ higher acceptance rate within the fermion correction step

❖ Leads to topological decoupling



Next step:

❖ improve training procedure, less grinding …

❖ apply procedure to 4D-SU(3)

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Thank you

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