Neutron Electric Dipole Moment from Isovector Quark Chromo-Electric Dipole Moment

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Abstract

We present results from our lattice QCD study of the contribution of the isovector quark chromo-electric dipole moment (qcEDM) operator to the nucleon electric dipole moments (nEDM). The calculation was carried out on four 2+1+1-flavor of highly improved staggered quark (HISQ) ensembles using Wilson-clover quarks to construct correlation functions. We use the non-singlet axial Ward identity including corrections up to O(a) to show how to control the power-divergent mixing of the isovector qcEDM operator with the lower dimensional pseudoscalar operator. Results for the nEDM are presented after conversion to the MSbar scheme at the leading-log order.

(arXiv:2304.09929 [hep-lat])

Outline

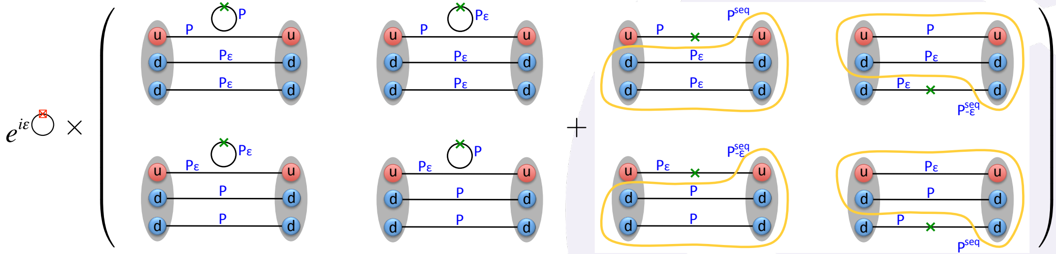
Introduction
Quark Chromo-Electric Dipole Moment

- Dimension-5 operator arising from Dimension-6 operator beyond electroweak symmetry breaking.
- $SU(3)$ color analog of quark electric dipole moment: $\bar{\psi}\Sigma \cdot \widetilde{G}\tau\psi$.
- Breaks P, CP and chiral symmetry.
- Is fermion-bilinear, can be incorporated by Schwinger-source trick:

$$\mathcal{P} = \left[\not{D} + m - \frac{r}{2}D^2 + c_{\rm SW}\Sigma \cdot G\right]^{-1} \to \left[\not{D} + m - \frac{r}{2}D^2 + \Sigma \cdot \left(c_{\rm SW}G + i\,\epsilon\tau\widetilde{G}\right)\right]^{-1}$$

- $a^{-1}\epsilon$ needs to be small to avoid multiple insertions.

Introduction
3-point function



- Disconnected loops from the fermion determinant vanish for isovector qcEDM.
- Isoscalar part of electromagnetic current gives nonvanishing disconnected loops.
- Ignore all disconnected contributions.

Introduction
HISQ Ensembles from MILC

- Mixed action calculation: Clover on HISQ
- Tadpole-improved tree-level c_{SW}
- $M_\pi L \gtrsim 4$
- $M_\pi^{\mathrm{sea}} \approx M_\pi^{valence}$

ID	a (fm)	M_π^{sea} (MeV)	M_π^{val} (MeV)	$L^3 \times T$	N_{conf}	ϵ	ϵ_5
$a12m310$	$0.1207(11)$	$305.3(4)$	$310.2(2.8)$	$24^3 \times 64$	1013	0.008	0.0024
$a12m220L$	$0.1189(09)$	$217.0(2)$	$227.6(1.7)$	$40^3 \times 64$	475	0.001	0.0003
$a09m310$	$0.0888(08)$	$312.7(6)$	$313.0(2.8)$	$32^3 \times 96$	447	0.008	0.0024
$a06m310$	$0.0582(04)$	$319.3(5)$	$319.3(0.5)$	$48^3 \times 144$	72	0.009	0.0012

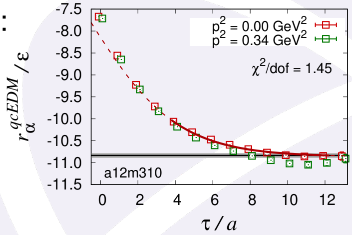
Introduction
Nucleon wave function

The wave-function of the nucleon in standard basis:

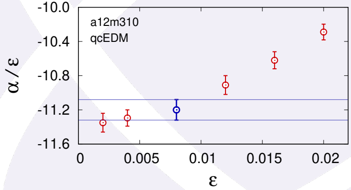
$$N_\alpha = e^{-i\alpha_N}\epsilon^{abc}\left[\psi_d^{aT}(\gamma_0\gamma_2)\gamma_5\frac{1\pm\gamma_4}{2}\psi_u^b\right]\psi_d^c$$

$$\alpha_N = \lim_{\tau\to\pm\infty}\frac{\Im\,\mathrm{Tr}\,\gamma_5(1\pm\gamma_4)\langle N_0(0)\bar{N}_0(\tau)\rangle}{\Re\,\mathrm{Tr}(1\pm\gamma_4)\langle N_0(0)\bar{N}_0(\tau)\rangle}$$

$$\approx -\frac{r\epsilon}{8ma}\frac{a^2\langle\Omega|\bar{\psi}\Sigma\cdot G\psi|\Omega\rangle}{\langle\Omega|\bar{\psi}\psi|\Omega\rangle}$$



α_N linear in ϵ and momentum-independent.



$$\langle N_\alpha(p')|J)\mu^{\mathrm{EM}}|N_\alpha(p)\rangle$$

$$= \bar{u}(p')\left[\gamma_\mu F_1 + \Sigma_{\mu\nu}\frac{q^\nu}{2M_N}(F_2 - iF_3\gamma_5)\right]u(p)$$

Mixing
Power divergences

- $C \equiv \bar{\psi}\Sigma \cdot \tilde{G}\tau_3\psi$ has power divergent mixing with $P \equiv \bar{\psi}\gamma_5\tau_3\psi$.
- Allowed even with good chiral symmetry.
- Does not mix with $G\tilde{G}$ due to isospin invariance even when chiral symmetry is broken.

Isovector CPV mass term P can be rotated away by nonsinglet axial rotation! No effect?

AWI for Wilson-like fermions

$$Z_A(m)\partial_\mu A_3^\mu + iac_A\partial^2 P_3 + 2imP_3 = iaK\tilde{C}_3 + O(a^2)$$

where

- $\tilde{C}_3 \equiv C - a^{-2}AP_3^2$ is free of power divergence.
- K comes from c_{SW} mistuning.

Apart from field redefinitions,

$$\frac{2am}{K}\frac{P_3}{a} \sim \frac{2am}{2am+K}aC_3 + O(a^2)$$

and is power-divergence free

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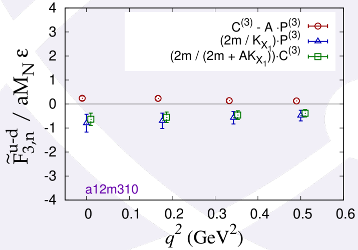
Mixing

P_3 and C_3 both measure $\tilde{C}_3$

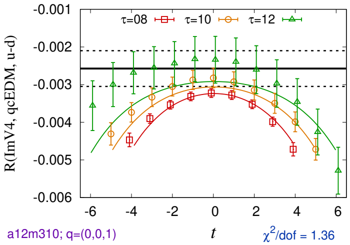
Ensemble	$\tilde{F}_3^{\gamma_5} / \tilde{F}_3^{\mathrm{qcEDM}}$					$\frac{K_{X1}}{2am + AK_{X1}}$
	$Q^2 = 1$	$Q^2 = 2$	$Q^2 = 3$	$Q^2 = 4$	$Q^2 = 5$	
a12m310	0.879(17)	0.863(14)	0.867(18)	0.844(23)	0.864(13)	0.694(48)
a12m220L	0.81(10)	0.769(77)	0.869(75)	0.98(18)	0.94(11)	0.7807(70)
a09m310	1.063(35)	1.042(40)	1.078(45)	1.006(58)	1.039(44)	0.740(61)
a06m310						0.859(64)

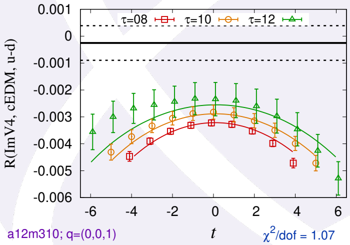
- $\tilde{F}_3 = F_3 + O(Q^2)$ defined because of better signal.
- C_3 and P_3 determinations within 10–20%
- Difference $O(a^2)/am$ enhanced by small am!



Excited state contributions
$N\pi$ intermediate state

- With CP violation, interpolating operator couples to $N\pi$ intermediate state
- Difficult to see in 2-pt function: volume suppression.
- Vector current can couple strongly to two pions: *may* be enhanced in 3-pt function.
- χ^2 surface has flat direction: results uncertain.





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Renormalization
Mixing with quark EDM

- All power-law mixing subtracted.
- Mixing with only dimension-5 operators.
- Only dimension-5 operator qEDM: $\bar{\psi}\Sigma \cdot \tilde{F}\tau_3\psi$; mixing $O(\alpha_{\rm EM}) \sim 1\%$.

But, $\int d^4x \tilde{C}_3 J_\mu^{\rm EM} A^\mu$ has mixing with qEDM at $O(\alpha_s)$!

$$F_3(\vec{O}_{\overline{\rm MS}}) = U \begin{pmatrix} \left(\frac{\alpha_s(\mu)}{\alpha_s(a^{-1})}\right)^{-\gamma_{11}/\beta_0} & 0 \\ 0 & \left(\frac{\alpha_s(\mu)}{\alpha_s(a^{-1})}\right)^{-\gamma_{22}/\beta_0} \end{pmatrix} U^{-1}\ F_3(\vec{O}_L(a))$$

$$U = \begin{pmatrix} 1 & -\frac{\gamma_{12}}{\gamma_{11}-\gamma_{22}} \\ 0 & 1 \end{pmatrix} \qquad \vec{O} = \begin{pmatrix} \rm qcEDM \\ \rm qEDM \end{pmatrix}$$

Extrapolation
Chiral behavior

- qcEDM operator transforms to qcMDM under chiral rotation.
- CP-violation can be removed by chiral rotation!
- Physical CP-violation arises from mismatch between chiral condensate direction and operator.
- On chiral rotation to fix the chiral condensate, CP-violating term becomes:

$$-\frac{md_3}{\sqrt{m^2+\bar{r}^2 d_3^2}}(C_3-\bar{r}P_3) \qquad 2\bar{r}\equiv\frac{\langle\Omega|\bar{\psi}\Sigma\cdot G\psi|\Omega\rangle}{\langle\Omega|\bar{\psi}\psi|\Omega\rangle}$$

- CP-violaion absent if qcEDM only chiral-breaking in the theory.
- But for $d_3 \ll m$, no mass suppression.

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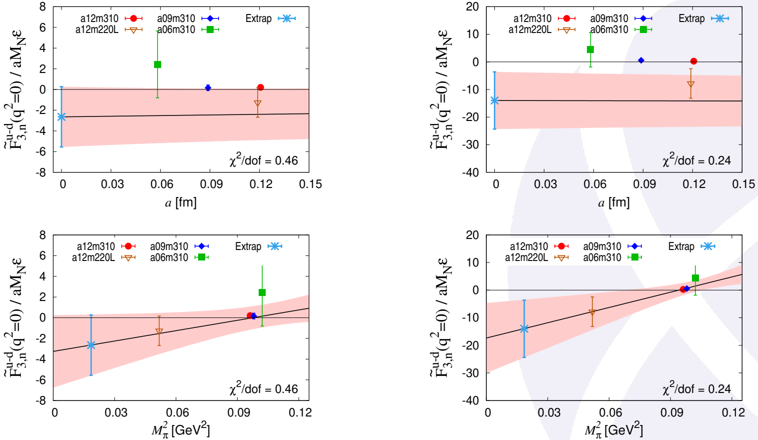
Extrapolation
Continuum-Chiral Fits



Standard

$N\pi$-fit

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Conclusions
and Future Directions

- Power-divergence in isovector chromo-EDM present even with good chiral symmetry.
- The power-divergent mixing is with P_3 which implements chiral rotation, but no CP-violation in the continuum.
- Any lattice artifact in it is enhanced by $1/ma$. Important to demonstrate control.
- Perturbative $O(a)$-improved Wilson fermions still have large uncertainty, though chiral rotation agrees with χPT at 10%.
- Control over Excited State Contamination needs to be demonstrated.
- Disconnected diagrams and possible chiral+isospin breaking mixing with $\Theta G\tilde{G}$ need to be considered.