$B_{(s)}$-mixing parameters from all-domain-wall-fermion simulations

Justus Tobias Tsang
joined work between the **RBC/UKQCD** and **JLQCD** Collaborations
in particular F. Erben, T. Kaneko, R. Mukherjee

Lattice2023, Fermilab, USA

2 August 2023

Outline

1. Motivation
2. Simulation Set-up
3. Analysis
4. Conclusion

Flavour Physics: The Cabibbo-Kobayashi-Maskawa matrix

experiment ≈ CKM-factors × non-perturbative inputs × known terms

CKM Matrix

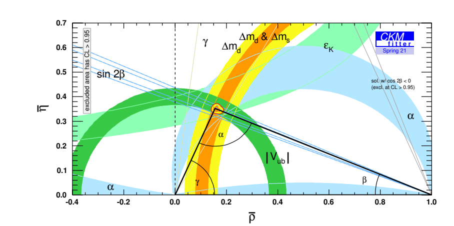
- parameterises transitions
  up-type ↔ down-type

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- **fundamental SM parameters**
- **unitary matrix in the SM:**

$$VV^\dagger = \mathbb{1}_3$$

Unitarity Triangle



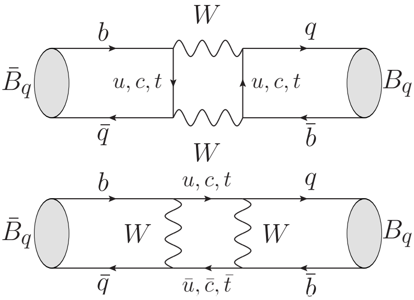
yellow & orange bands = exp. measurements of $\Delta m_{d(s)}$ + SM predictions for mixing parameters

⇒ **Tests CKM unitarity!**

Non-unitarity of CKM ⇔ New Physics Beyond the SM

Neutral $B_{(s)}$ meson mixing - background

Neutral mesons oscillate:



where $q = d, s$

mass eigenstate $\neq$ flavour eigenstate

$$|B_{L,H}\rangle = p\,|B_q^0\rangle \pm q\,|\bar{B}_q^0\rangle$$

$\Rightarrow$ **splittings** in mass eigenstates:

- mass splitting $\Delta m_q \equiv m_H - m_L$
- width splitting $\Delta\Gamma_q \equiv \Gamma_L - \Gamma_H$

Time dependence:

$$|B_q^0(t)\rangle = g_+(t)\,|B_q^0\rangle + \frac{q}{p} g_-(t)\,|\bar{B}_q^0\rangle$$

$$|\bar{B}_q^0(t)\rangle = \frac{p}{q} g_-(t)\,|B_q^0\rangle + g_+(t)\,|\bar{B}_q^0\rangle$$

Occurs at loop level in SM $\Rightarrow$ **sensitive probe of new physics**!

Neutral $B_{(s)}$ meson mixing - theory

Short distance dominated $\Rightarrow$ described by $\mathcal{H}^{\Delta b=2}$ eff. weak Hamiltonian. OPE factorises this into

- **Perturbative model-dependent Wilson coefficients** $C_i(\mu)$
- **Non-perturbative model-independent matrix elements**

$$\left\langle B^0_{(s)} \right| \mathcal{H}^{\Delta b=2} \left| \bar{B}^0_{(s)} \right\rangle = \sum_i C_i(\mu) \left\langle B^0_{(s)} \right| \mathcal{O}_i^{\Delta b=2}(\mu) \left| \bar{B}^0_{(s)} \right\rangle$$

- 5 independent (parity even) operators $\mathcal{O}_i$, only $\mathcal{O}_1$ relevant for Δm:

$$\mathcal{O}_1 = \left(\bar{b}_a \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_a\right) \left(\bar{b}_b \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_b\right) = \mathcal{O}_{VV+AA}$$

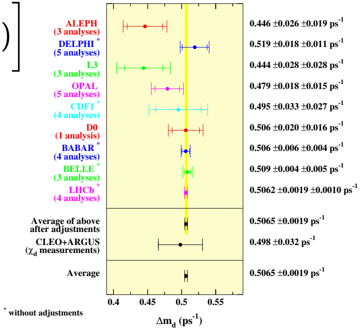
- Define bag parameters: $\hat{B}^{(i)}_{B_q} = \left\langle \bar{B}^0_q \right| \mathcal{O}_i \left| B^0_q \right\rangle / \left\langle \bar{B}^0_q \right| \mathcal{O}_i \left| B^0_q \right\rangle_{VSA}$

$$\Delta m_q = \left|V^*_{tb} V_{tq}\right|^2 \times f^2_{B_q} \hat{B}^{(1)}_{B_q} \times m_{B_q} \mathcal{K}$$

$\Rightarrow$ **Non-perturbative matrix elements calculable on the lattice**

Neutral $B_{(s)}$ meson mixing - experiment

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}t\right) \pm \cos(\Delta m_q t)\right]$$



[HFLAV 2206.07501]

Δm experimentally known sub percent!

B_d^0: Many results

B_s^0: "Only" CDF, CMS and LHCb

$$\Delta m_d = 0.5065(19)\mathrm{ps}^{-1}$$

$$\Delta m_s = 17.765(06)\mathrm{ps}^{-1}$$

$SU(3)$ breaking ratios give access to $|V_{td}/V_{ts}|$

$$\xi^2 \equiv \frac{f_{B_s}^2 \hat{B}_{B_s}^{(1)}}{f_{B_d}^2 \hat{B}_{B_d}^{(1)}} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Delta m_s}{\Delta m_d}\frac{m_B}{m_{B_s}}$$

How to simulate the b-quark?

Effective action for b

- Can tune to m_b
- Inherent systematic errors which are hard to estimate/reduce

Relativistic action for b

- Theoretically cleaner and systematically improvable
- Need to control extrapolation in heavy quark mass

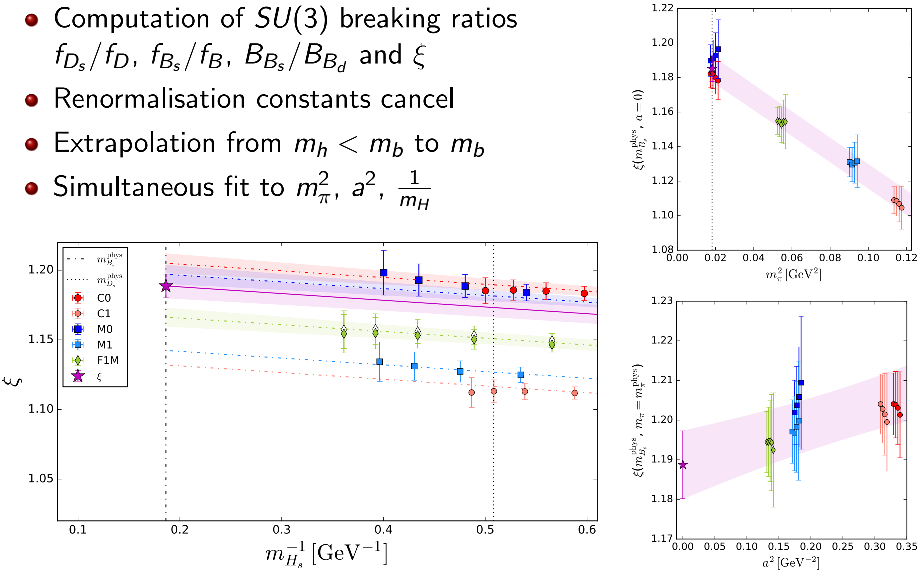
Different properties:

- computational cost
- chirality
- tuning errors
- systematic errors
- cut-off effects
- renormalisation

- Results in literature mostly use effective action for b
- We uses **chirally symmetric Domain Wall Fermions for all quarks**
  $\Rightarrow$ Automatically $O(a)$-improved
  $\Rightarrow$ Continuum-like renormalisation pattern and fully non-perturbative

Recall: RBC/UKQCD'18: strategy [1812.08791]

- Computation of $SU(3)$ breaking ratios f_{D_s}/f_D, f_{B_s}/f_B, B_{B_s}/B_{B_d} and ξ
- Renormalisation constants cancel
- Extrapolation from $m_h < m_b$ to m_b
- Simultaneous fit to m_π^2, a^2, $\frac{1}{m_H}$



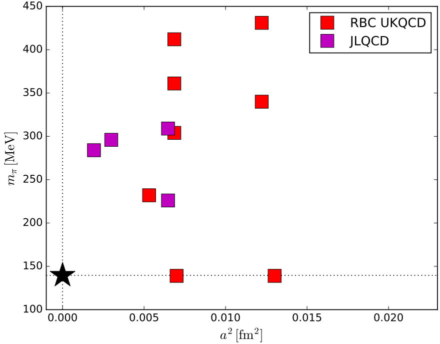
RBC/UKQCD/JLQCD: ongoing work

RBC/UKQCD'18 dominated by

- chiral-continuum fit

$\Rightarrow$ heavy quark extrapolation

$\Rightarrow$ estimates of higher order $1/m_H$ terms



- Supplement existing dataset with finer JLQCD ensembles
  $\Rightarrow$ **reduce extrapolation** in m_H significantly
- all domain wall fermion set-up
  $\Rightarrow$ **physical block-diagonal renormalisation pattern**
- all 5 operators $\hat{B}^{(i)}_{B_d}$ and $\hat{B}^{(i)}_{B_s}$
- (3+3) lattice spacings, 2 ensembles with physical pion mass
  $\Rightarrow$ good control over all required limits
- new correlator fitting strategy

Computational set-up

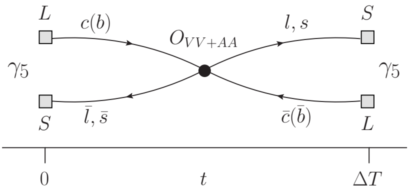
UKQCD/RBC $N_f = 2+1$

- 8 ens, 3 a^{-1} : $[1.7, 2.7]\,\mathrm{GeV}$, 2 m_π^{phys}, Iwasaki gauge, DWF sea, $M_5 = 1.8$.
- val l and s same as sea, s tuned
- val h: 3 stout with $\rho = 0.1$, $M_5 = 1.0$.

$\Rightarrow$ **Mixed action**

JLQCD $N_f = 2+1$

- 7 ens, 3 a^{-1} : $[2.5, 4.5]\,\mathrm{GeV}$, $m_\pi^{\mathrm{min}} \sim 230\,\mathrm{MeV}$, Symanzik gauge, DWF sea, $N_\rho = 3$ stout, $\rho = 0.1$, $M_5 = 1.0$
- l, s, h all use same action
- 2 m_s bracket m_s^{phys}.
- 1 pair of ens only differ in V



- Z_2-sources on every 2nd (4th) time plane
  - $\Rightarrow$ many (16-64) translations
  - $\Rightarrow$ many (5-17) ΔT per conf.
- 4-6 heavy quark masses with $am_h < 0.7$

rich but challenging dataset.

Correlation function fit strategy

Goals:

- Correlated fits including contribution from first excited state
- Inclusion of multiple source-sink separations to constrain excited state
- Two independent correlator analyses (JTT and Felix Erben) to mitigate subjective choices

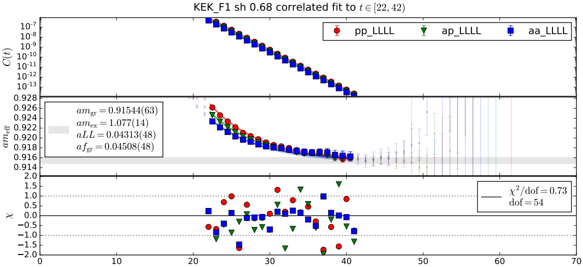
Strategy (JTT):

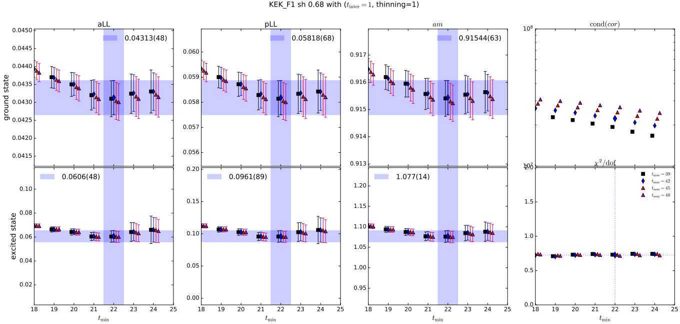
1. Stability study of 2-point functions to determine t_{min}^{2pt}, t_{max}^{2pt}
2. Simultaneously re-fit $C_2(t)$ and $C_3(t, \Delta T, \mathrm{op})$ keeping the two-point $[t_{\mathrm{min}}^{2pt}, t_{\mathrm{max}}^{2pt}]$, selecting δ such that $t_{\mathrm{min}}^{3pt} = t_{\mathrm{min}}^{3pt} + \delta$ and select ΔT_{min} and ΔT_{max}.
3. Check stability for
   - $\delta \pm 1$,
   - range of choices of ΔT
   - consistency of shared parameters between different fits

Correlation functions and their fits: Two points

Example fit:

- heavy-strange
- $a^{-1} \sim 4.5\,\mathrm{GeV}$, $m_\pi \sim 280\,\mathrm{MeV}$
- $M_{hs} \approx 4.1\,\mathrm{GeV} \sim 75\% M_{B_s}^{\mathrm{phys}}$



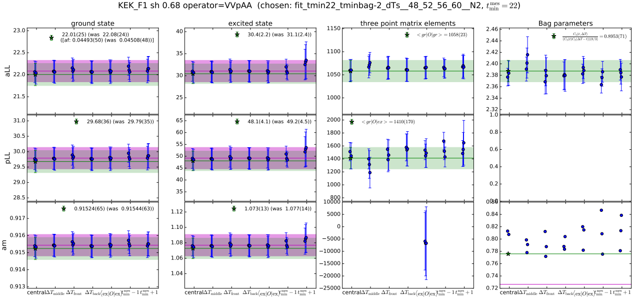


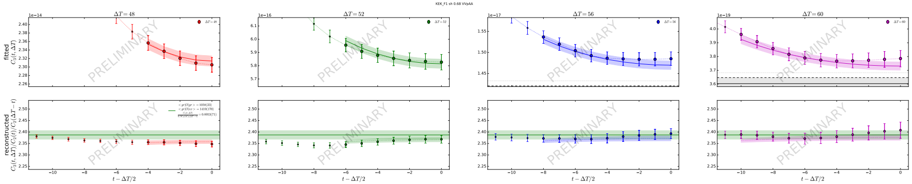
- Perform stability analysis for all parameters.
- Starting point for combined C_2 and C_3 fit

Correlation functions and their fits: Three points

- same spectrum between 2pt, 3pt
- $\langle gr | O | gr \rangle$, $\langle gr | O | ex \rangle$ $(\langle ex | O | ex \rangle)$
- joined fit in $(t, \Delta T)$ to $C_2(t)$ and $C_3(t, \Delta T)$
- stability analysis

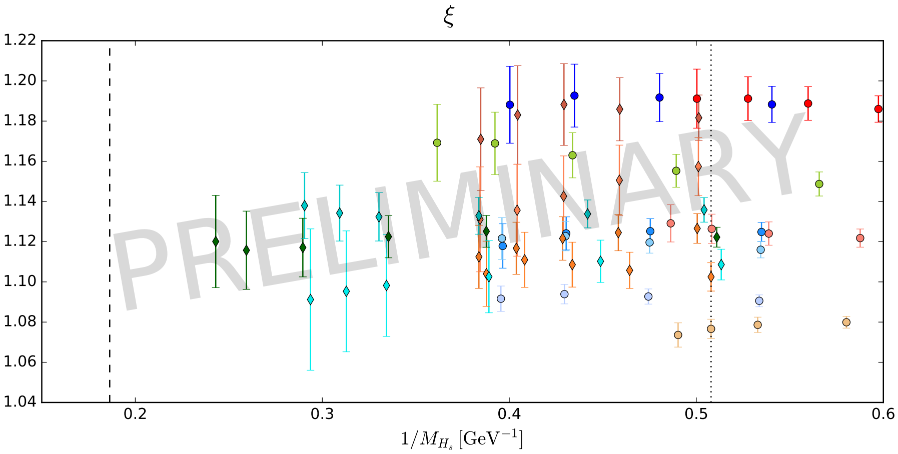
example: $\mathcal{O}_{VV+AA}$





$\Delta T/a = 48$ $\Delta T/a = 52$ $\Delta T/a = 56$ $\Delta T/a = 60$

RBC/UKQCD/JLQCD: preliminary data – ratios



- Mild behaviour with $1/M_{H_s}$
- Data a lot closer to physical *b*-quark mass
- Precision on individual datapoints $\lesssim O(1\%)$.
- Starting to do global fits

4q-NPR for mixing operators (Rajnandini Mukherjee)

- Analogously to $K - \bar{K}$ [1708.035 52] the 5 operators $\mathcal{O}_1 = \mathcal{O}_{VV+AA}$, $\mathcal{O}_2 = \mathcal{O}_{VV-AA}$, $\mathcal{O}_3 = \mathcal{O}_{SS-PP}$, $\mathcal{O}_4 = \mathcal{O}_{SS+PP}$, $\mathcal{O}_5 = \mathcal{O}_{TT}$ mix.
- DWF $\Rightarrow$ block diagonal renorm. matrix (up to $O(am_{\mathrm{res}})$)

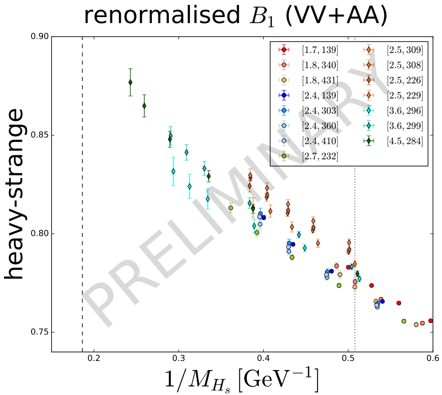
$$Z_{ij}^{RI}(\mu, a) = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix}$$

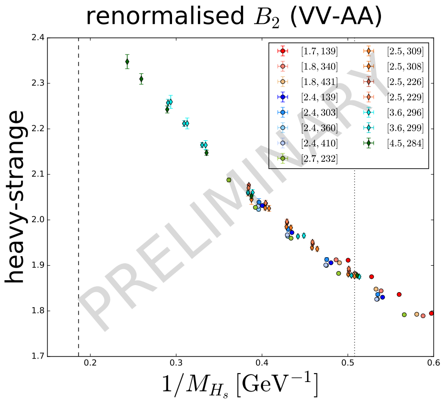
- $B_{(s)} - \bar{B}_{(s)}$: Mixed action for RBC/UKQCD $\Rightarrow$ Mixed action NPR.
- Generalise idea from 1701.02644 for fully non-perturbative mixed action renormalisation to four quark operators

$$\frac{\mathcal{P}[\Lambda_A](ll)\mathcal{P}[\Lambda_A](hh)}{(\mathcal{P}[\Lambda_A](lh))^2} = \frac{(Z_A^{\mathrm{lh}})^2}{Z_A^{ll} Z_A^{hh}}$$

fully NPR complete for all 15 ensembles $\times$ 5 operators and combinations of single and mixed actions. ✓

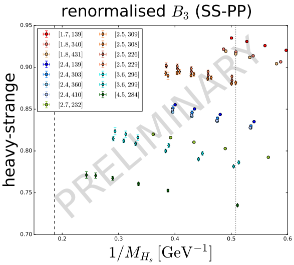
RBC/UKQCD/JLQCD: beyond ratios I

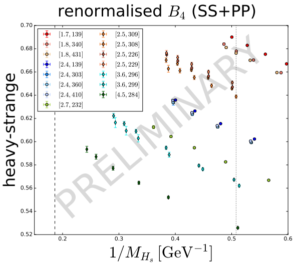


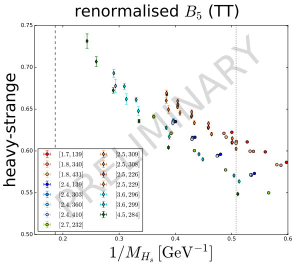


- a^{-1}, M_π, m_s, V effects not parameterised yet. . . .
- . . . but seem to be mild
- Benign $1/M_{Hs}$ behaviour

RBC/UKQCD/JLQCD: beyond ratios II





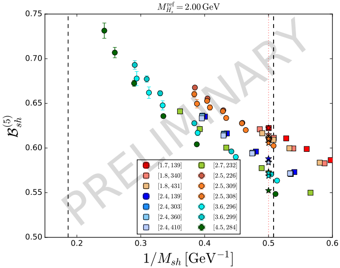


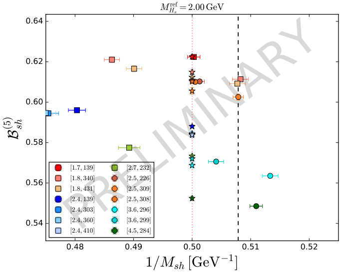
- O1,O2: Mild discretisation effects
- O3,O4: Large discretisation effects
- O5: Mass dependent discretisation effects

TASK:

1. Parameterise dependence on $(a, am_h; m_l, m_s, m_h)$
2. Evaluate stability of this description

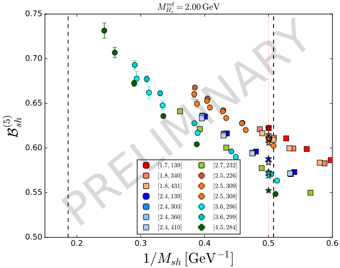
interpolate data to $M_{H_s}^{\text{ref}}$, then assess m_l, m_s, a dependence (expl: $\mathcal{B}_{sh}^{(5)}$)

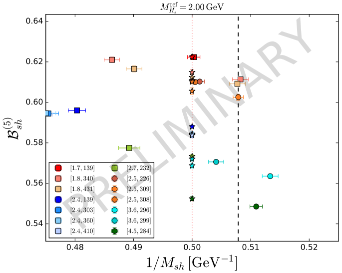




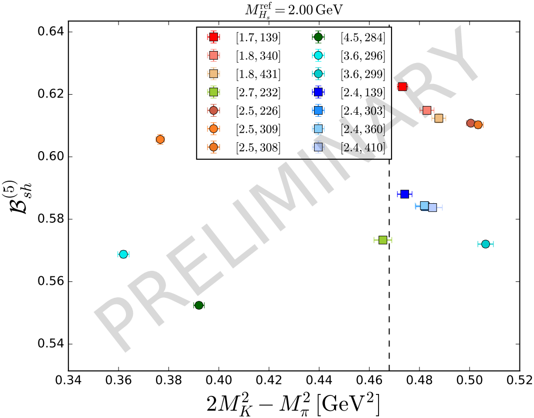
Strategy I: Parameterising m_l, m_s, a – at fixed m_h

interpolate data to $M_{H_s}^{\rm ref}$, then assess m_l, m_s, a dependence (expl: $\mathcal{B}_{sh}^{(5)}$)





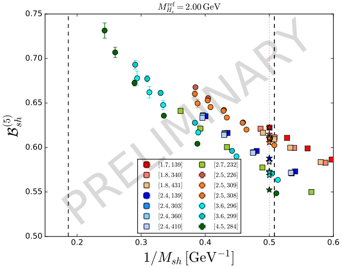
- $m_s \propto 2M_K^2 - M_\pi^2$ ✓

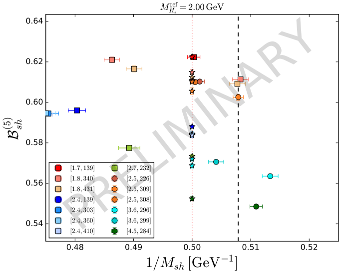


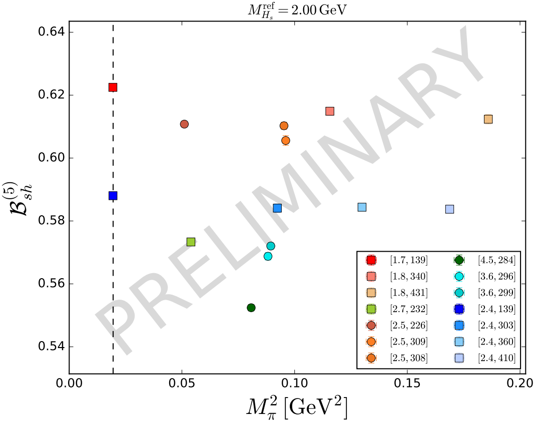
Strategy I: Parameterising m_l, m_s, a – at fixed m_h

interpolate data to $M_{H_s}^{\rm ref}$, then assess m_l, m_s, a dependence (expl: $\mathcal{B}_{sh}^{(5)}$)

- $m_s \propto 2M_K^2 - M_\pi^2$ ✓
- $m_l \propto M_\pi^2$ ✓

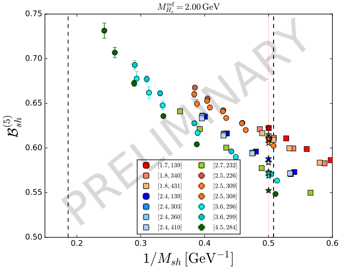


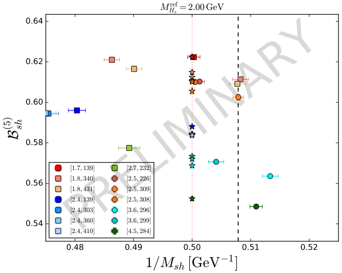




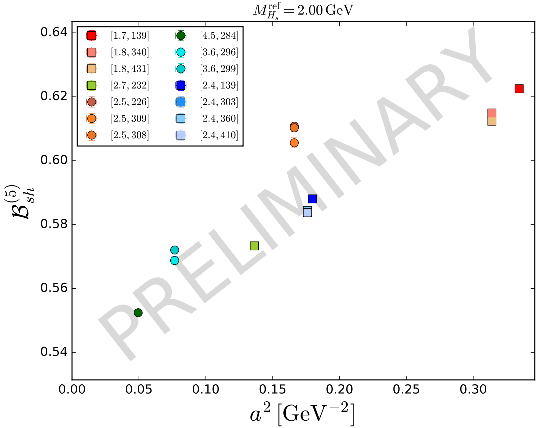
Strategy I: Parameterising m_l, m_s, a – at fixed m_h

interpolate data to $M_{H_s}^{\mathrm{ref}}$, then assess m_l, m_s, a dependence (expl: $\mathcal{B}_{sh}^{(5)}$)





- $m_s \propto 2M_K^2 - M_\pi^2$ ✓
- $m_l \propto M_\pi^2$ ✓
- a^{2n}: 2 CLs + universality constr.(✓)



Strategy II: Global extrapolation to physical values

Expected mass behaviours:

m_l: $g_\chi(M_\pi) \equiv C_\chi(M_\pi^2 - M_\pi^2|_{\rm phys})$ + test chiral logs (✓)

m_s: $g_K(M_\pi, M_K) \equiv C_s[(2M_K^2 - M_\pi^2)|_{\rm sim} - (2M_K^2 - M_\pi^2)|_{\rm phys}]$ (✓)

m_b: Set m_b via M_{B_s}. Try expansion in powers of $1/M_{H_s}$ (+ heavy logs?).

$$g_h(M_{H_s}) \equiv \mathcal{O}_{\rm static} + C_h^{(1)}(\Lambda/M_{H_s}) + C_h^{(2)}(\Lambda/M_{H_s})^2 \text{ (✓)}$$

Currently exploring different fitansaetze: starting with

$$\mathcal{O}(M_\pi, M_K, m_h; a, am_q, am_{\rm res}) = g_\chi(M_\pi) + g_K(M_\pi, M_K) + g_h(M_{H_s}) + f_{CL}^{JLQCD}(a^2)\delta_{i,JLQCD} + f_{CL}^{UKRBC}(a^2)\delta_{i,UKRBC}$$

where $X \in \{JLQCD, UKRBC\}$

f_{CL}^X to incorporate leading order cut-off effects:

- $C_{CL}^{(1)}(a\Lambda_{\rm QCD})^2$, $C_{am_q^2}(am_q)^2$, $C_{am_{\rm res}} am_{\rm res}$
- mass-dependent $C_{CL}^{(1)} \equiv C_{CL}^{(1)}(M_{H_s})$ - exploit insights from Strategy I

Summary and Outlook

Huge data-set

- 15 ensembles with varying $a, L, m_l (\rightarrow M_\pi), m_s$.
- wide range of heavy-quark masses (4-6) per ensemble $M_{H_s}^{\max} \sim 0.75 M_{B_s}^{\text{phys}}$
- 3+3 lattice spacings (2 different discretisations)
- 2 M_π^{phys} ensembles.
- 2 pairs of ensembles which only differ in m_s.
- 1 pair which differs in V.

Fitting strategy & extrapolations

1. Two independent correlation function analyses ✓
2. RI/sMOM renormalisation analysis ✓(preliminary)
3. Fit to parameterise (ongoing)
   - $M_\pi \rightarrow M_\pi^{\text{phys}}$, $m_s \rightarrow m_s^{\text{phys}}$ ✓
   - $M_{H_s} \rightarrow M_{B_s}$ (✓)
   - $a \rightarrow 0$ [+ universality constraint!]

   for $\hat{B}_{B_q}^{(i)}$, f_{B_q}, $f_{B_q}\sqrt{\hat{B}_{B_q}^{(i)}}$, ξ, ratios.
4. Assess robustness of fit and all systematic uncertainties

GOAL: <u>Controlled prediction of all observables and their correlations</u>