Radiative corrections to neutron decay: the $\gamma - W$ box diagram using Lattice QCD

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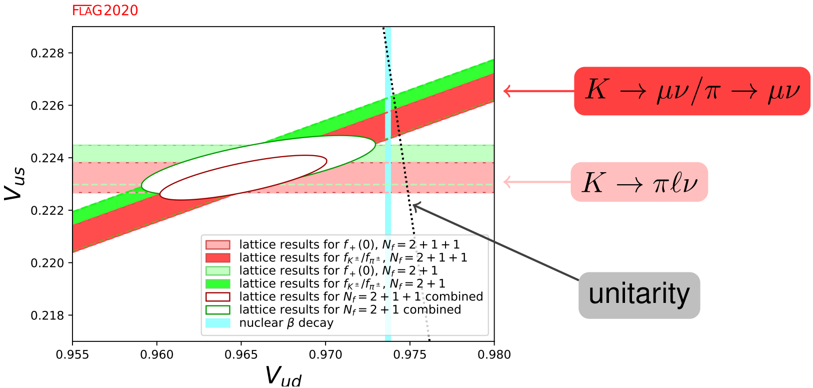
Status

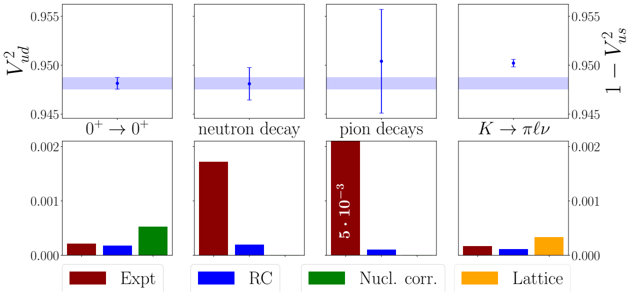


Two anomalies (deviations from black line showing $\Delta_{\mathrm{CKM}} = 0$):

1. kaon decays and LQCD show $\sim 2\sigma$ tension with unitarity
2. including superallowed β decays brings the discrepancy to $\sim 4\sigma$

Is this a hint of BSM physics?

$|V_{ud}|$ and $|V_{us}|$ are the targets of our calculation



- $|V_{ud}|$ extracted from β decays of pions, neutrons, and nuclei
  Current: $|V_{ud}|^2 = 0.94809(27)$ dominated by $0^+ \to 0^+$ β decays
- $|V_{us}|^2 = 0.05040(36)$ from kaon decays ($K \to \pi e \bar{\nu}_e$, $K \to \mu \bar{\nu}_\mu$)
- $|V_{ub}|^2 \approx (2 \pm 0.4) \times 10^{-5}$ is tiny, no impact on the unitarity test
- Uncertainty in Δ_{CKM} has comparable contributions from $|V_{ud}|$ and $|V_{us}|$

Status of $|V_{ud}|^2$, and improvements in experiments

- $|V_{ud}|$ extracted from neutron β decay
  Current: $|V_{ud}|^2 = 0.9487(22)_{\mathrm{RC}}(37)_{\tau_{\mathrm{n}}}(68)_{\lambda}$ from UCNτ@LANL
  Current: $|V_{ud}|^2 = 0.9467(22)_{\mathrm{RC}}(86)_{\tau_{\mathrm{n}}}(68)_{\lambda}$ from PDG avg.

- $|V_{us}|^2 = 0.04976(25)$ from kaon semileptonic decays ($K \to \pi e \bar{\nu}_e$)

- Anticipate error in $\lambda = g_A/g_V$ will decrease from $\sim 0.05\%$ to $\sim 0.01\%$:
  uncertainty in Δ_{CKM} goes from (68) to $(14)_{\lambda}$

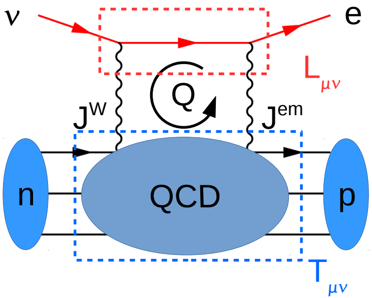
- Anticipate error in τ_n will decrease from ~ 0.34 sec to ~ 0.15:
  uncertainty in Δ_{CKM} goes from (37) to $(16)_{\tau_n}$

Impact of Improvements in $\lambda \equiv g_A/g_V$, τ_n and RC on Δ^{CKM}

When	$\|V_{ud}\|^2$	Δ^{CKM}
Current	$0.9487(22)_{\mathrm{RC}}(37)_{\tau_\mathrm{n}}(68)_\lambda$	2σ
Future λ Meas.	$0.9487(22)_{\mathrm{RC}}(37)_{\tau_\mathrm{n}}(14)_\lambda$	$\sim 3\sigma$
Future τ_n Meas.	$0.9487(22)_{\mathrm{RC}}(16)_{\tau_\mathrm{n}}(14)_\lambda$	$\lesssim 5\sigma$
Updates on RC	$0.9487(7)_{\mathrm{RC}}(16)_{\tau_\mathrm{n}}(14)_\lambda$	$\sim 5\sigma$

Table: To find $\Delta^{CKM} = |V_{ud}|^2 + |V_{us}|^2 - 1$, $|V_{us}|^2 = 0.04976(25)$ was used. Future λ measurement was assumed to be 0.01% precision, and future τ_n measurement uncertainty is assumed to be .15s. Our updates on RC is the best possible uncertainty without improvement in OPE.

The calculation: $\gamma - W$ Box Diagram



Uncertainty in radiative correction in $|V_{ud}|$ dominated by $\gamma - W$ box diagram!

- $\Delta_{\rm np}$ given by the product of leptonic, $L^{\mu\nu}$, and hadronic, $T_{\mu\nu}$, parts

$$\Delta_{\rm np} = \int_0^{+\infty} dQ^2 \int_{-Q}^{Q} dQ_0 \frac{1}{Q^4} \frac{1}{Q^2 + m_W^2} L^{\mu\nu}(Q, Q_0) T_{\mu\nu}^{VA}(Q, Q_0)$$

- Lattice QCD needed for $T^{\mu\nu}$ in $0.1 < Q^2 < 2\mathrm{GeV}^2$

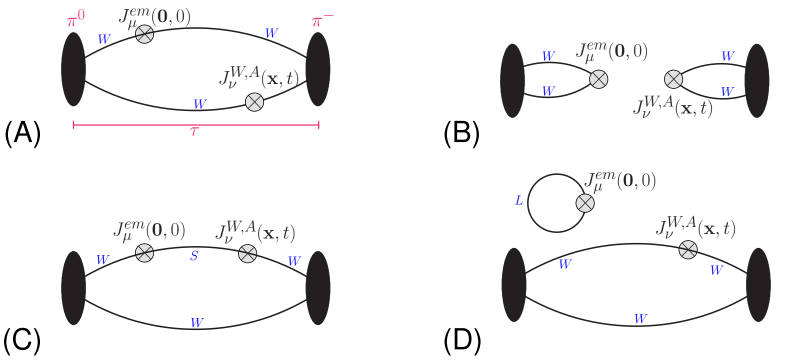
$$T_{\mu\nu}^{VA} = \frac{1}{2} \int d^4x\, e^{iQ\cdot x} \langle N_f(p)| T\left[J_\mu^{em}(0,0) J_\nu^{W,A}(\vec{x},t)\right] |N_i(p)\rangle$$

$\gamma - W$ Box Diagram – Contd.

- The uncertainty in the integral over Q^2 is dominated by $T^{\mu\nu}$, especially in the $0 < Q^2 \lesssim 2$ GeV^2 regime where QCD gives large corrections
- The best estimates in the literature combine robust theoretical information on the behavior of the integrand at $Q^2 \sim 0$, where it is determined by the nucleon elastic form factors, and at large $Q^2 \gtrsim 2\,\mathrm{GeV}^2$, where operator product expansion and perturbation theory are reliable
- Lattice QCD calculations aim to reduce the uncertainty in the non-perturbative region, $0.1 < Q^2 \lesssim 2$ GeV^2. This is currently being $approximated$ using models

We are determining the RC to pion/kaon/neutron decay

RC to pion decay:



- No signal degradation with source-sink separation $\longrightarrow$ No excited state error
- Much simpler contractions to construct correlation functions
- charged kaon decay can be computed using the same contractions

Steps in the calculation

- Fix lattice to Coulomb gauge
- Generate wall source propagators at source and sink times ($\boldsymbol{p} = 0$) (labeled W)
- Diagram (A): Spin-color contraction of Ws from source and sink with current insertion on 2 different quark lines
- Diagram (C): Compute additional propagator S from $J_\mu^{\rm em}$ point. Perform contractions of W and S propagators at site $(\boldsymbol{x}, t)$ with insertion of $J_\nu^{W,A}$
- Diagram (D): Calculate stochastic estimate of disconnected quark loop L with the insertion of the electromagnetic current. Calculate the correlation of loop L with the 3-point function with insertion of $J_\nu^{W,A}$ at point $(\boldsymbol{x}, t)$.

- $\mathcal{M}_H(Q^2)$ is calculated from $H^{VA}_{\mu\nu} = \langle n(p)^{s'}|T\left[J^{em}_\mu(0,0)J^{W,A}_\nu(\vec{x},t)\right]|p(p)^s\rangle$
- The relevant hadronic tensor is:

$$T^{VA}_{\mu\nu} = \frac{1}{2}\int d^4x\, e^{iQ\cdot x} P\langle n(\boldsymbol{p}=0,s)|T\left[J^{em}_\mu(0,0)J^{W,A}_\nu(\vec{x},t)\right]|p(\boldsymbol{p}=0,s')\rangle$$

  We use the spin projector $P = (1+\gamma_4)/2$ for neutron decay
- The spin-independent part of $T^{VA}_{\mu\nu}$ has only one term

$$T^{VA}_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta T_3 + \dots$$

- Knowing T_3 as a function of Q^2, the γW-box correction is given by

$$\Box^{VA}_{\gamma W} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}\mathcal{M}_H(Q^2)$$

8 clover-on-HISQ Ensembles used for pion and kaon decay

- Interpolated to the physical pion mass using results at $M_\pi \approx 310, 220, 130$ MeV
- Performed continuum extrapolation from $a \approx 0.15, 0.12, 0.09, 0.06$ fm

EnsID	a [fm]	M_π^{val} [MeV]	$L^3 \times T$	$m_\pi L$	τ/a	R^2 [fm^2]	N_{conf}
a06m310	0.0582(04)	319.6(2.2)	$48^3 \times 144$	4.52	62	5.42	168
a09m130	0.0871(06)	138.1(1.0)	$64^3 \times 96$	3.90	40	6.07	45
a09m220	0.0872(07)	225.9(1.8)	$48^3 \times 96$	4.79	40	6.07	93
a09m310	0.0888(08)	313.0(2.8)	$32^3 \times 96$	4.51	40	6.31	156
a12m220	0.1184(09)	227.9(1.9)	$32^3 \times 64$	4.38	30	5.61	150
a12m220L	0.1189(09)	227.6(1.7)	$40^3 \times 64$	5.49	30	5.65	150
a12m310	0.1207(11)	310.2(2.8)	$24^3 \times 64$	4.55	30	5.83	179
a15m310	0.1510(20)	320.6(4.3)	$16^3 \times 48$	3.93	24	9.12	80

Table: Values of a, $M_\pi^{\rm sea}$ from [Bazavov et al., 2013] and $M_\pi^{\rm val}$ from [Gupta et al., 2018]

$\mathcal{M}_H(Q^2)$ as a function of integration radius R

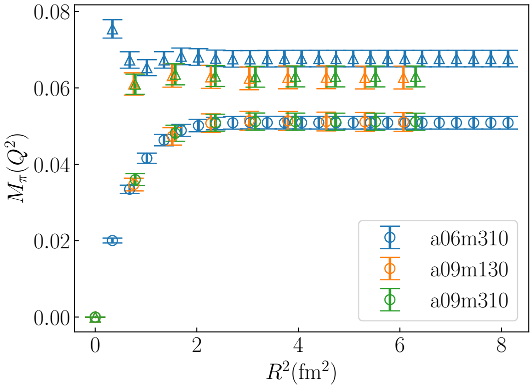
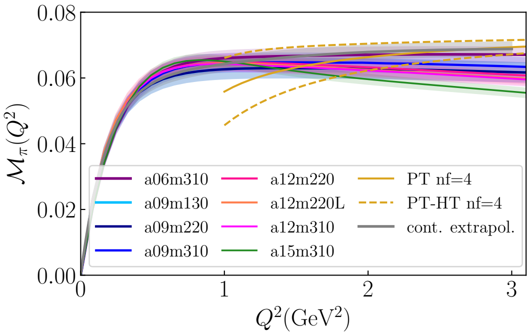


Figure: triangle marker $Q^2 = .317\mathrm{GeV}^2$ and circle marker $Q^2 = 3.0\mathrm{GeV}^2$

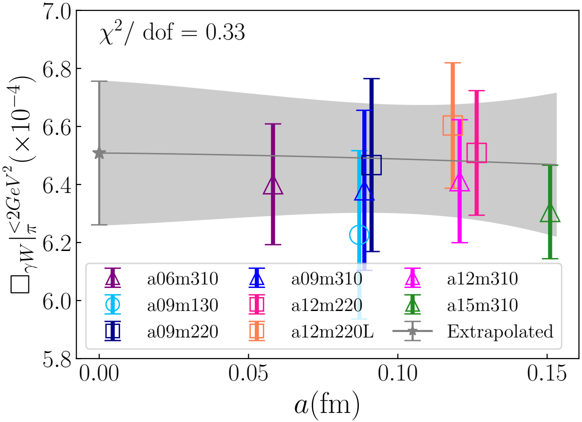
- $\mathcal{M}_H\left(Q^2\right) = -\frac{1}{6}\frac{1}{F_+^H}\frac{\sqrt{Q^2}}{m_H}\int d^4x\omega(t,\vec{x})\epsilon_{\mu\nu\alpha 0}x_\alpha\mathcal{H}_{\mu\nu}^{VA}(t,\vec{x})$
- $\int d^4x$ is done within a finite radius R on lattice
- Result saturates at some radius R

Q^2-dependence of $\mathcal{M}_H(Q^2)$ for the pion



- For $Q^2 \gtrsim 2\mathrm{GeV}^2$, results from coarse (large a) ensembles were farther away from the PT result
- Taking the continuum limit ($a \to 0$) with $\alpha_s(a^{-1})$, the PT result and continuum extrapolation overlap already for $2\mathrm{GeV}^2 < Q^2 < 3\mathrm{GeV}^2$
- The surplus in the low-Q^2 regime compensates for deficiency in high-Q^2 for coarse ensembles
  $\to$ small a-dependence of the integral

Integrated box contribution below $Q^2 \leq 2\mathrm{GeV}^2$



Leading a- and m_π-dependence (in χPT) is expected to be

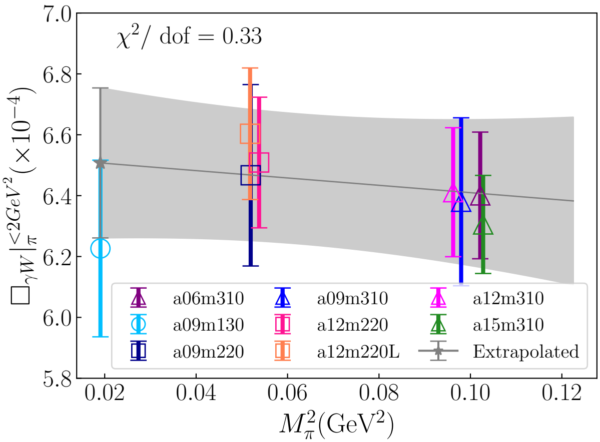
$$\square_{\gamma W}^{VA}|_{\pi}^{Q^2\leq 2\mathrm{GeV}^2}(m_\pi, a) = c_0 + c_1 m_\pi^2 + c_2 a\alpha_s(a^{-1}) \quad (1)$$

After extrapolation to physical point

$$\square_{\gamma W}^{VA}|_{\pi}^{Q^2\leq 2\mathrm{GeV}^2} = 6.51(25)\times 10^{-4} \quad (2)$$

Mild dependence on a and M_π

Integrated box contribution for pion



Our lattice result at physical point

$$\Box_{\gamma W}^{VA}|_{\pi}^{Q^2 \leq 2\mathrm{GeV}^2} = 0.651(25) \times 10^{-3}$$

is combined with pQCD result

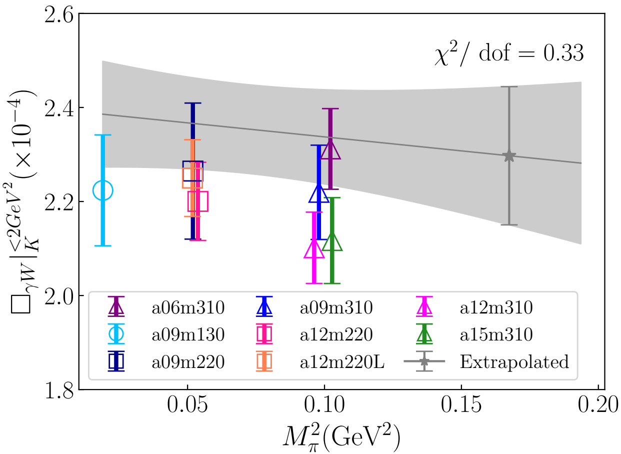
$$\Box_{\gamma W}^{VA}|^{Q^2 > 2\mathrm{GeV}^2} = 2.159(6)(7) \times 10^{-3}$$

$$\Rightarrow \quad \Box_{\gamma W}^{VA}|_{\pi} = 2.810(26) \times 10^{-3}$$

$$\text{cf.)} \;\; \Box_{\gamma W}^{VA}|_{\pi} = 2.830(11)(26) \times 10^{-3}$$

[Xu Feng, et al., PRL (2020)]

Integrated box contribution for kaon



PT for $Q^2 > 2\mathrm{GeV}^2$ gives

$$\square_{\gamma W}^{VA}|^{Q^2>2\mathrm{GeV}^2} = 2.159(6)(7)\times 10^{-3}$$

Lattice result at physical point for $Q^2 \leq 2\mathrm{GeV}^2$

$$\square_{\gamma W}^{VA}|_K^{Q^2\leq 2\mathrm{GeV}^2} = 0.230(15) \times 10^{-3}$$

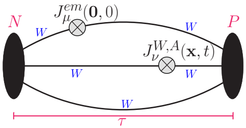
Combining with PT for $Q^2 > 2\mathrm{GeV}^2$

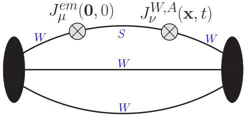
$$\Rightarrow \quad \square_{\gamma W}^{VA}|_K = 2.389(17) \times 10^{-3}$$

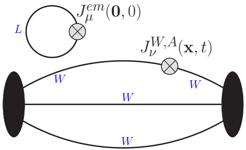
$$\text{cf.)} \ \square_{\gamma W}^{VA}|_K = 2.437(44) \times 10^{-3}$$

[Xu Feng, et al., PRD (2021)]

RC to neutron decay:

(A)


(C)


(D)


- No diagram B
- S2N falls exponentially for nucleon correlators
- Excited state effects can be significant
- Spin projector $P = (1+\gamma_4)/2$ cancels spin dependent terms in $T_{\mu\nu}^{VA}$
- Requires much larger statistics

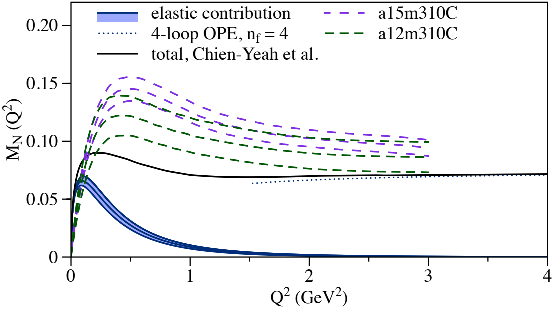
Ensembles being simulated for neutron decay

- Carried out neutron γW box measurement on 3 different HISQ-Clover ensembles
- Different lattice spacing can be used for continuum extrapolation ($a \to 0$)
- Results from various values of m_π are interpolated to the physical pion mass

EnsID	a [fm]	M_π^{val} [MeV]	$L^3 \times T$	$m_\pi L$	τ/a	R^2 [fm^2]	N_{conf}
a12m220	0.1184(09)	227.9(1.9)	$32^3 \times 64$	4.38	10	5.61	365
a12m310	0.1207(11)	310.2(2.8)	$24^3 \times 64$	4.55	10	5.83	898
a15m310	0.1510(20)	320.6(4.3)	$16^3 \times 48$	3.93	8	9.12	1168

Table: The three HISQ-Clover lattice ensembles used in this work. The values of a, M_π^{sea} and M_π^{val} are reproduced from Ref. [Gupta et al., 2018].

Comparison of $\mathcal{M}_H(Q^2)$ for the neutron



(Preliminary)

- In $Q^2 \gtrsim 2\mathrm{GeV}^2$ regime, results from coarse (large a) ensembles tend towards perturbation theory results
- Tendency to reproduce the quasielastic peak near $\sim .1\mathrm{GeV}^2$.

Summary

- Computed electroweak γW-box corrections to the semileptonic pion decay and kaon decay on 8 ensembles. Results published (accepted in PRD)
  - $\Box^{VA}_{\gamma W}|_\pi = 2.810(26) \times 10^{-3}$
  - $\Box^{VA}_{\gamma W}|_K = 2.389(17) \times 10^{-3}$

- Preliminary analysis of radiative correction to neutron decay shows signal
- Increasing statistics and the number of ensembles

- Goal: Provide precision results for RC to pion, kaon, neutron decays to improve the extraction of $|V_{ud}|^2$ and $|V_{us}|^2$

Acknowledgement

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- The calculations used the CHROMA software suite
- We thank DOE for allocations at NERSC and OLCF
- We thank USQCD for allocations at JLAB
- We thank Institutional Computing at Los Alamos National Laboratory for allocations.
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Appendix

Two ways to probe BSM physics:

Two ways to probe Beyond the Standard Model (BSM) physics:

1. Directly produce new particles in high energy experiments
2. look for tiny deviations from the SM predictions =
   Confront precision measurements with accurate predictions of the SM

Improve the theoretical input to test the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix

- The SM has only one-type of charged-current interactions

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W^+_\mu \, \overline{U}_i \, \gamma^\mu (1-\gamma_5) \, (V_{\rm CKM})_{ij} \, D_j \; + \; \text{h.c.} \, , \qquad \begin{aligned} \overline{U} &= (\bar{u}, \bar{c}, \bar{t}) \\ D^T &= (d, s, b) \end{aligned}$$

- Unitarity of $V_{\rm CKM}$ would imply no BSM physics

$$\Delta_{\rm CKM} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

$|V_{ud}|^2$ from neutron decay: $\delta|V_{ud}|^2 \approx (8 \to 3) \times 10^{-4}$

- $|V_{ud}|^2$ from neutron decay is given by the master formula

$$|V_{ud}|^2 = \left(\frac{G_\mu^2 m_e^5}{2\pi^3} f\right)^{-1} \frac{1}{\tau_n(1+3g_A^2)(1+\mathrm{RC})} = \frac{5099.3(4)\mathrm{s}}{\tau_n(1+3g_A^2)(1+\mathrm{RC})}$$

τ_n: free neutron lifetime
g_A: axial coupling obtained from the neutron β decay (asymmetry parameter A)
G_μ: Fermi constant extracted from muon decays,
$f = 1.6887(1)$ is a phase space factor

- Experimental Proposal: $\delta\tau_n \approx$ 0.1 sec, $\delta A/A \approx 0.1$ % .
- Theory Proposal: reduce uncertainty in RC to the 10% level, ie, by a factor of two

This will test the SM up to scales of 15 TeV, which are inaccessible at the LHC

- $\mathcal{M}_H(Q^2)$ can be calculated from $H_{\mu\nu}^{VA} = \langle\pi^0(p)|T\left[J_\mu^{em}(0,0)J_\nu^{W,A}(\vec{x},t)\right]|\pi^-(p)\rangle$
- The relevant hadronic tensor is:

$$T_{\mu\nu}^{VA} = \frac{1}{2}\int d^4x\, e^{iQ\cdot x}\langle\pi^0(p)|T\left[J_\mu^{em}(0,0)J_\nu^{W,A}(\vec{x},t)\right]|\pi^-(p)\rangle$$

- The spin-independent part of $T_{\mu\nu}^{VA}$ has only one term

$$T_{\mu\nu}^{VA} = i\epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta T_3 + \dots$$

- Knowing T_3 as a function of Q^2, the γW-box correction is given by

$$\square_{\gamma W}^{VA} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}\mathcal{M}_H(Q^2)$$

Radiative correction to $\beta-$decay

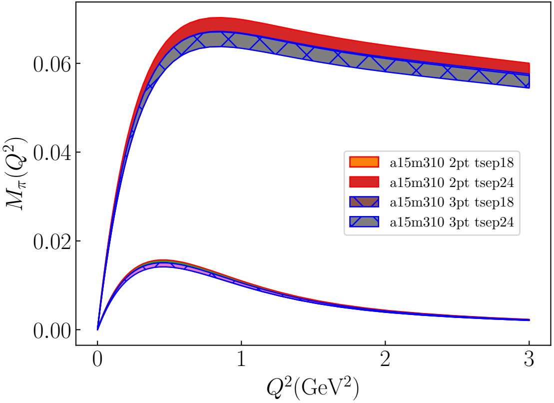
- The theoretical error in $|V_{ud}|$, is dominated by the uncertainty in the RC, which is expressed as the sum of three terms

$$\mathrm{RC} = \frac{\alpha_{\mathrm{em}}}{2\pi}\left\{\bar{g}(E_m) + 4\ln\frac{m_Z}{m_p} + \Delta_{\mathrm{np}}\right\}$$

- The first two terms dominate the RC but have very small uncertainties:
  - $\bar{g}(E_m)$, where E_m is the electron endpoint energy, arises from the emission of soft photons, integrated over the allowed phase space
  - $\ln(m_Z/m_p)$ encodes perturbative short-distance γ-Z_{boson} loop effects
  - Together they give 0.036, or about 95% of RC
- $\alpha_{\mathrm{em}}\Delta_{\mathrm{np}}/(2\pi) \sim 0.002$: This non-perturbative long distance effect is comparatively small, but its estimated uncertainty, $\sim 20\%$, dominates the theory error budget.

Lattice QCD is the only known *controlled* method to determine Δ_{np} at the 10% level and to reach $\delta|V_{ud}|^2 \leq 3 \times 10^{-4}$

$\mathcal{M}_H(Q^2)$ using 2-, 3-, 4-pt correlation functions



- $\mathcal{M}_H\left(Q^2\right) =$
  $-\frac{1}{6}\frac{1}{F_+^H}\frac{\sqrt{Q^2}}{m_H}\int d^4x\omega(t,\vec{x})\epsilon_{\mu\nu\alpha 0}x_\alpha \mathcal{H}_{\mu\nu}^{VA}(t,\vec{x})$
- $\mathcal{H}_{\mu\nu} = \frac{2m_\pi C_{4pt}}{C_{2pt}}$
- Combine with prefactor $F_+^\pi = \frac{C_{3pt}}{C_{2pt}}$
- ratio $\mathcal{H}_{\mu\nu}/F_+^\pi = \frac{2m_\pi C_{4pt}}{C_{3pt}}$ has better S2N due to cancellation of correlated fluctuations.

Determination of $|V_{ud}|$

- $\Gamma_{\pi\ell 3} = \frac{G_\mu^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3}(1+\delta) I_\pi$
- χPT result: $\delta = 0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}}$
- $\delta = \frac{\alpha_e}{2\pi}\left[\bar{g} + 3\ln\frac{m_Z}{m_p} + \ln\frac{m_Z}{m_W} + \tilde{a}_g\right] + \delta_{HO}^{QED} + 2\Box_{\gamma W}^{VA}$
- Earlier result on lattice: $\delta = 0.0332(1)_{\gamma W}(3)_{HO}$ [Xu Feng, et al., PRL (2020)]
- Our preliminary result: $\delta = 0.0332(1)_{\gamma W}(3)_{HO}$

Bazavov, A. et al. (2013).
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