Nucleon Electromagnetic Form Factors at Large Momentum from Lattice QCD

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(Lattice Hadron Physics collaboration)

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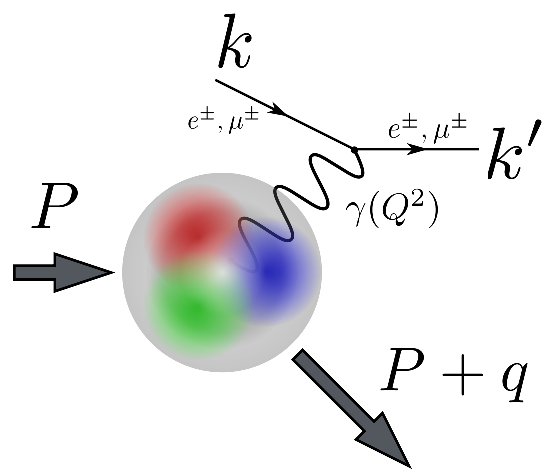
Outline

- Nucleon vector form factors at large momentum
- Challenges for large-Q^2 hadron structure on lattice
- Preliminary results and comparison to experiment & phenomenology
- Examining systematic effects
- Summary and Outlook

Nucleon Elastic E&M Form Factors



$$Q^2 = -(k-k')^2 > 0$$

Elastic e^-p amplitude

(Dirac) (Pauli)

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]U_P$$

Sachs Electric $$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$$

Magnetic $$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Elastic e^-p cross-section

- $G_{E,M}$ from ϵ-dep. at fixed $\tau(Q^2)$ ("***Rosenbluth separation***")
- dominated by G_M at large Q^2
- 2γ corrections at $Q^2 \gtrsim 1$ GeV^2

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\rm Mott}}{1+\tau}\left[G_E^2 + \frac{\tau}{\epsilon}G_M^2\right]$$

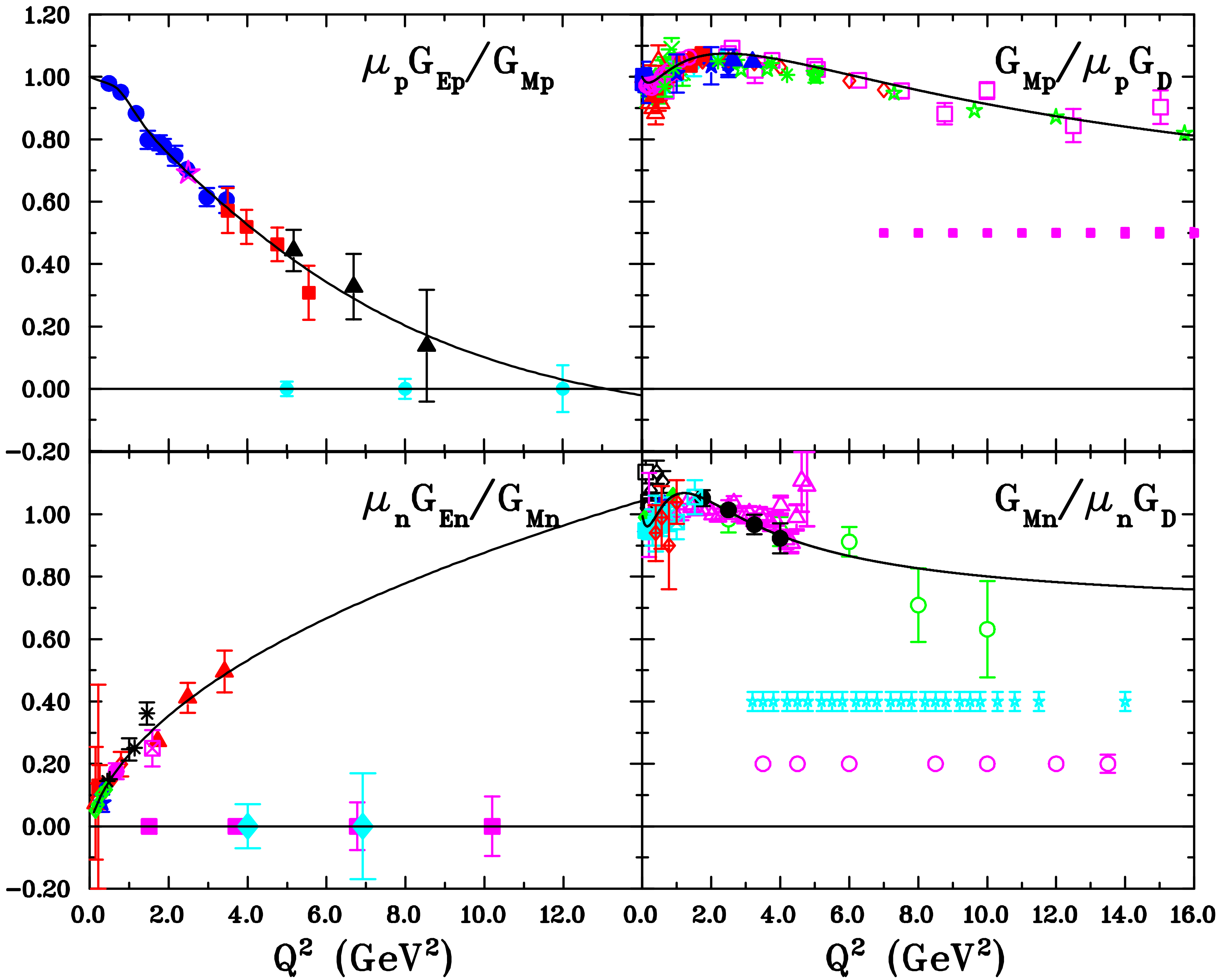
$$\tau = \frac{Q^2}{4M_N^2} \qquad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

Polarization transfer: polarized e^-beam
+ detect polarization of recoil nucleon
(alt.: transverse asymmetry on pol. target)

- *G_E/G_M ratio* (only small radiative corrections)

$$P_t/P_l \propto G_E/G_M$$

Recent/Ongoing Experiments

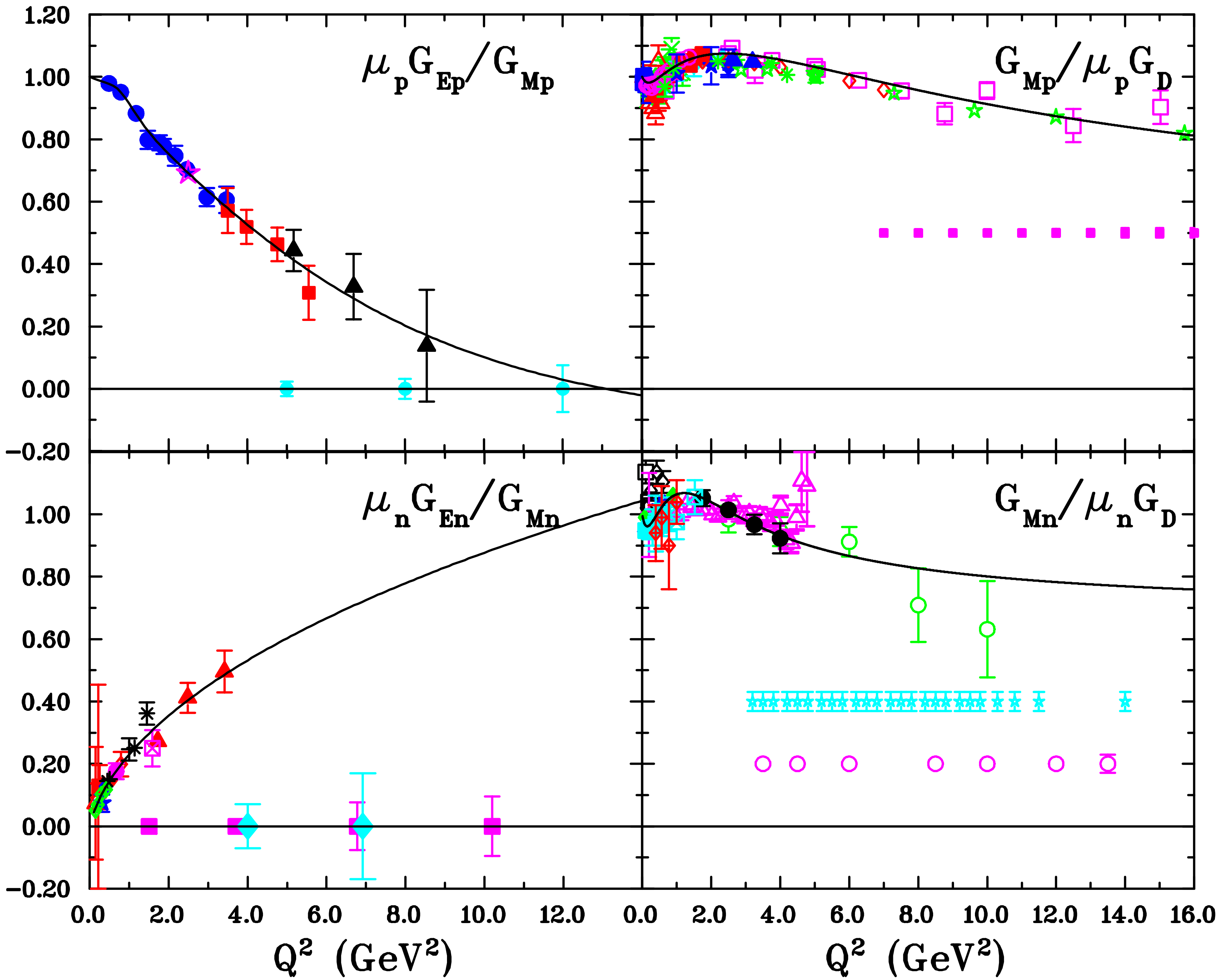


Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

Experiments at JLab@12GeV

- Hall A (HRS, SBS):
  - G_{Mp} @ $Q^2 \lesssim 17.5$ GeV^2
  - G_{Ep}/G_{Mp} @ $Q^2 \lesssim 15$ GeV^2 ;
  - G_{Mn} @ $Q^2 \lesssim 18$ GeV^2
  - G_{En}/G_{Mn} @ $Q^2 \lesssim 10.2$ GeV^2 ;
- Hall B (CLAS12):
  - G_{Mn} @ $Q^2 \lesssim 14$ GeV^2
- Hall C :
  - G_{En}/G_{Mn} @ $Q^2 \lesssim 6.9$ GeV^2

Recent/Ongoing Experiments



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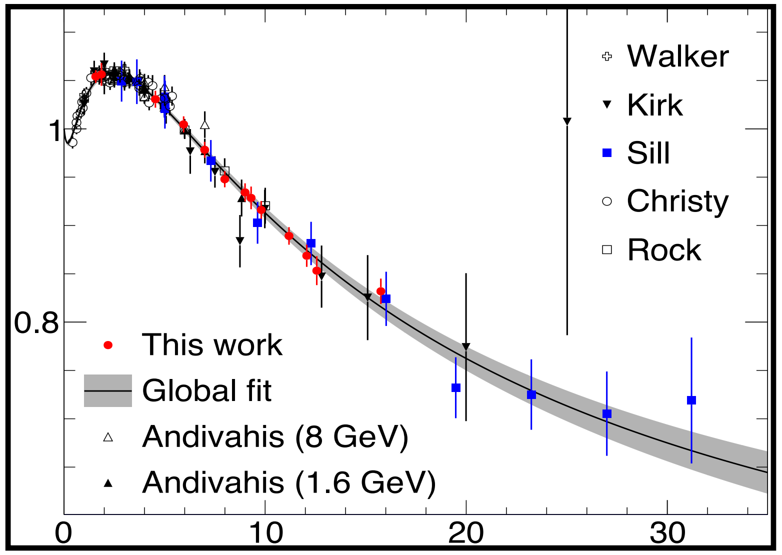
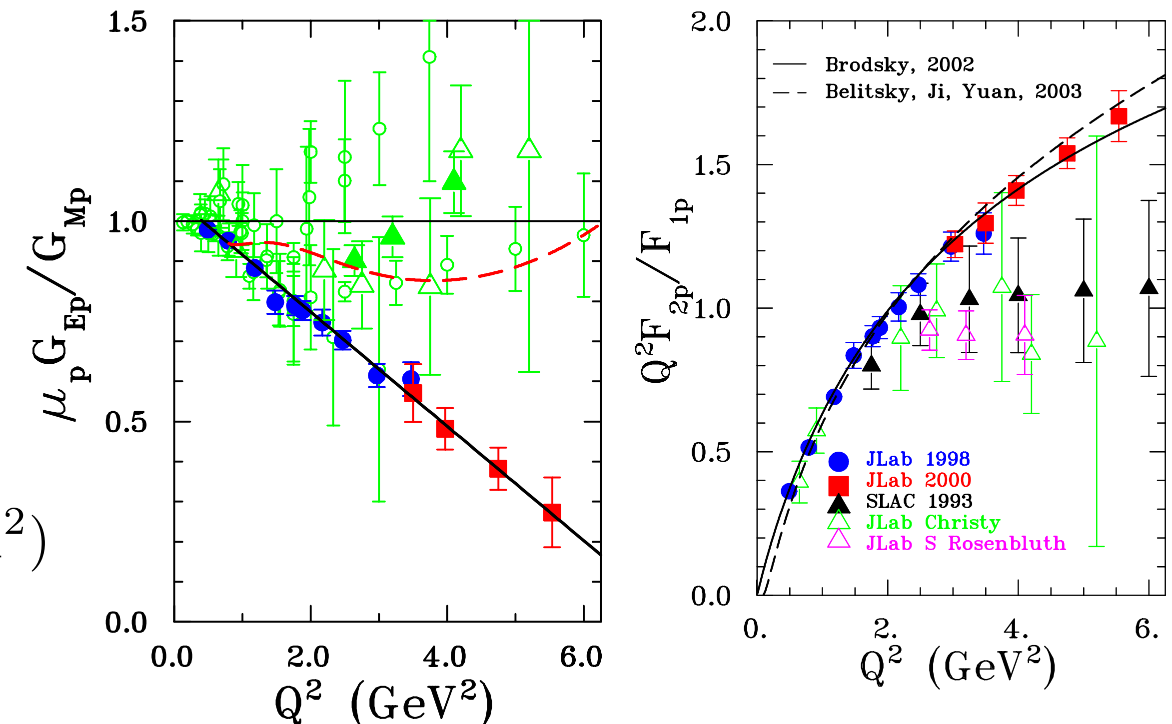


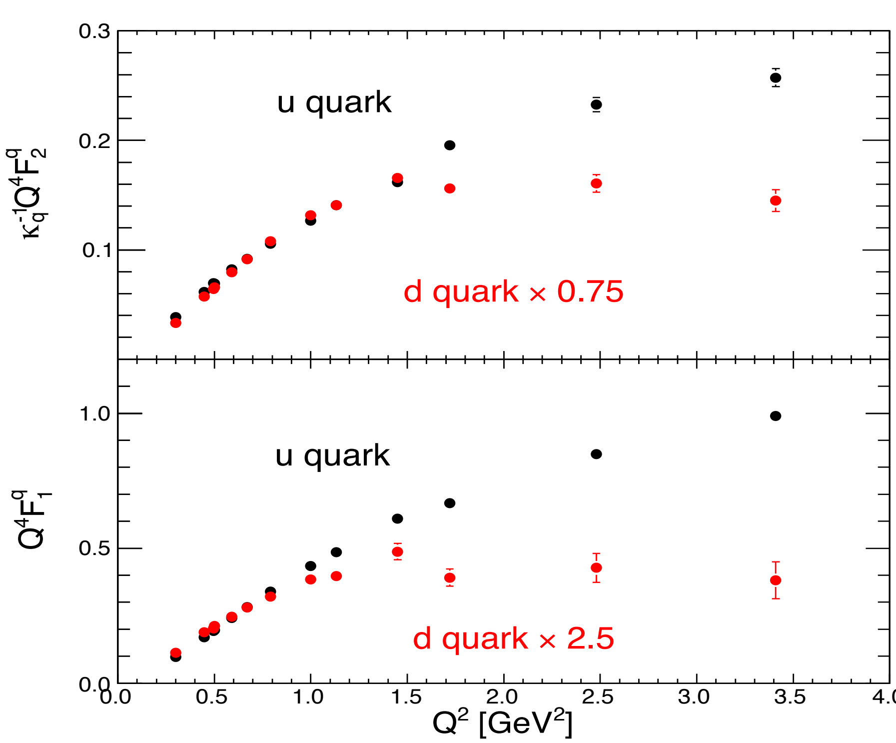
New G_{Mp} data from Hall A
[Christy et al, PRL'22]

Nucleon Form Factors: Open Questions

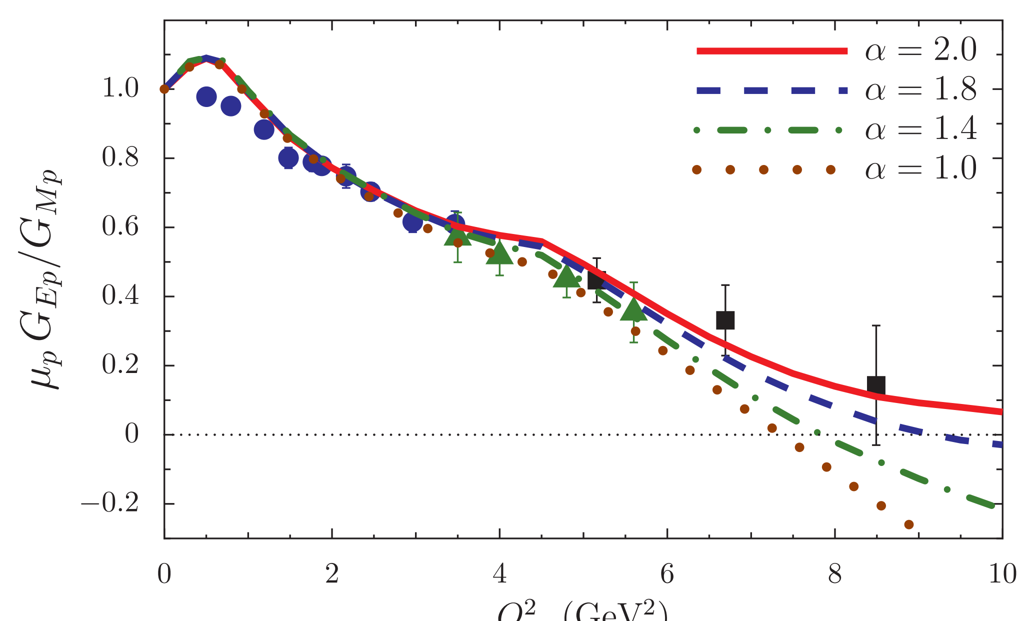
- *Are model descriptions of the nucleon viable ?*
  Nucleon models disagree beyond explored range
- *Role of diquark correlations in elastic scattering ?*
  Neutron & proton G_E/G_M at/above Q² = 8 GeV²
- *Scale of transition to perturbative QCD ?*
  (F_2/F_1) scaling at large Q^2 : $Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$
- *What are contributions from u and d flavors?*
  Proton <u>and</u> neutron data needed in wide Q^2 range



[Research Mgmt. Plan for SBS(JLab Hall A)]



[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]



Dyson-Schwinger Eqns : quarks & 0⁺, 1⁺ diquarks
(α ≈ rate of transition const.quarks → pQCD with Q^2)
[Cloet, Roberts, Prog.Part.Nucl.Phys 77:1 (2014)]

Challenges at Large Q^2

- Discretization effects:
  O(a) Correction to current operator

$$(V_\mu)_I = [\bar{q}\gamma_\mu q] + c_V\, a\, \underbrace{\partial_\nu[\bar{q} i \sigma_{\mu\nu} q]}_{\propto Q}$$

- Stochastic noise grows faster with T [Lepage'89]:

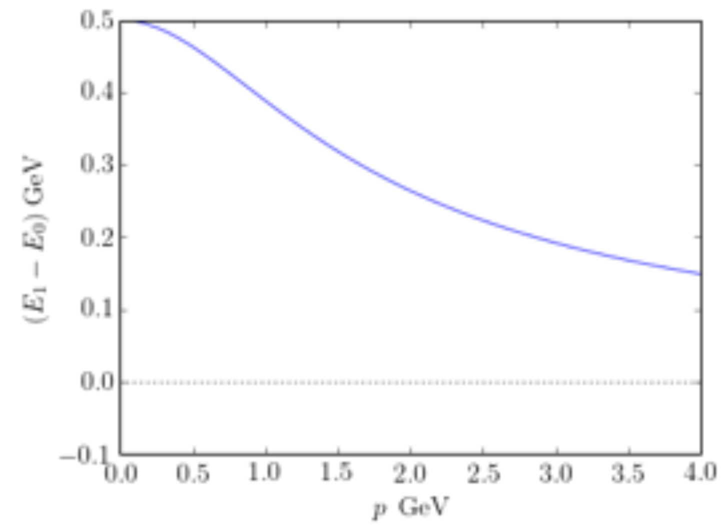
$$\begin{array}{lll} \text{Signal} & \langle N(T)\bar{N}(0)\rangle & \sim e^{-E_N T} \\ \text{Noise} & \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 & \sim e^{-3m_\pi T} \\ \text{Signal/Noise} & & \sim e^{-(E_N - \frac{3}{2}m_\pi)T} \end{array}$$

SNR reduction
at 1 fm/c ***~ O(10^{-4})***
(phys.quarks, $Q^2 \approx 12$ GeV^2)

- Excited states: boosting "shrinks" the energy gap

$$E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$$

  - N(~1500): pN→1.5 GeV ⇒ ΔE = 500 → 300 MeV

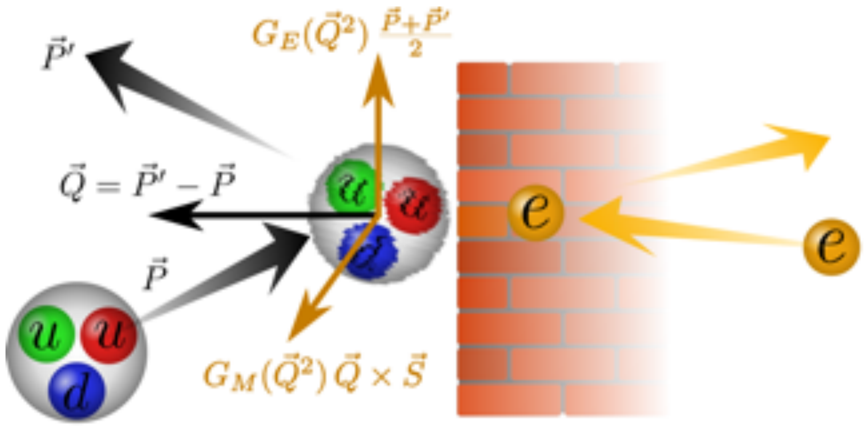


- Quark-disconnected contributions:
  negligible (≲1%) at $Q^2 \leq 1$ GeV^2, unknown at large Q^2

- Large p_N: no reliable EFT/ChPT for m_π-, lattice size-extrapolation

Large statistics required to suppress MC noise in lattice correlators

Accessing Large Q^2 : Breit Frame on a Lattice



"Brick-Wall" frame

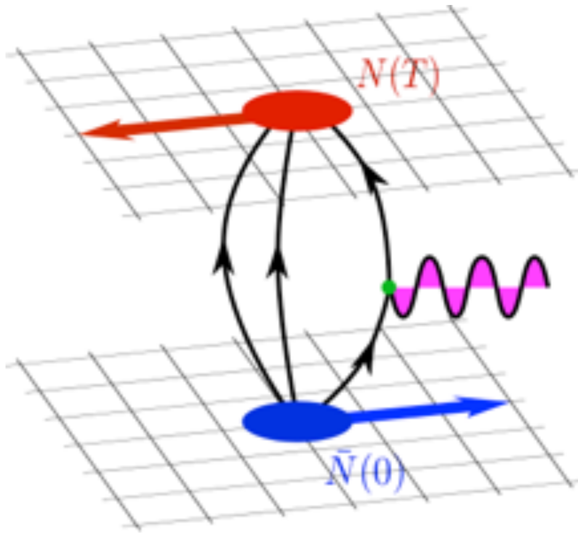
Minimize $E_{in,out}$ for target Q^2 :

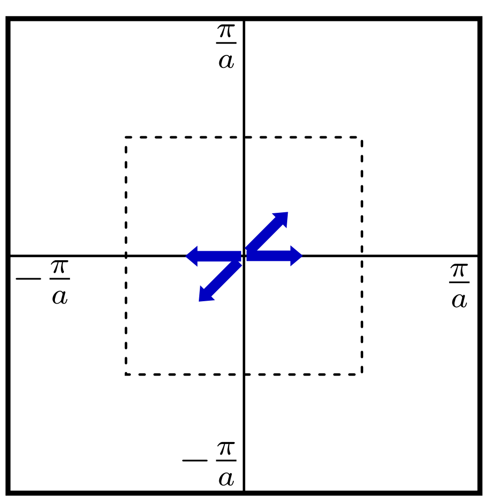
$$Q^2 = (\vec{p}_{in} - \vec{p}_{out})^2 - (E_{in} - E_{out})^2$$

Back-to-back $Q^2 = 4\vec{p}^2$

For $(Q^2)_{max}$ = *10 GeV²* $(E_N \approx 1.9\ GeV)$

$$|\vec{p}| = \frac{1}{2}\sqrt{Q^2_{\max}} \approx 1.6\ \text{GeV}$$





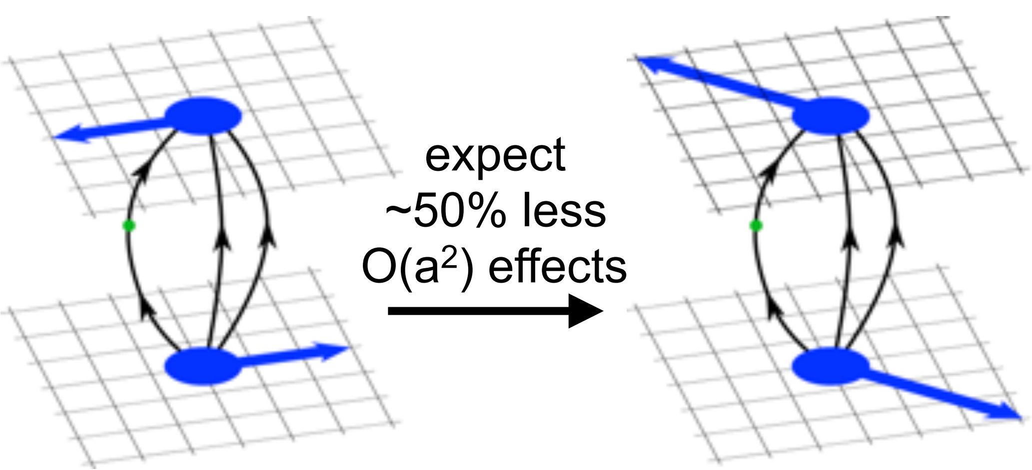
lattice kinematics for $Q^2 \approx$ 10 GeV²

Nucleon momentum ~ Brillouin zone

$$\langle N\bar{N}\rangle^{-1}(p) \stackrel{?}{=} -i\not{p}^{\text{lat}} + m_N$$

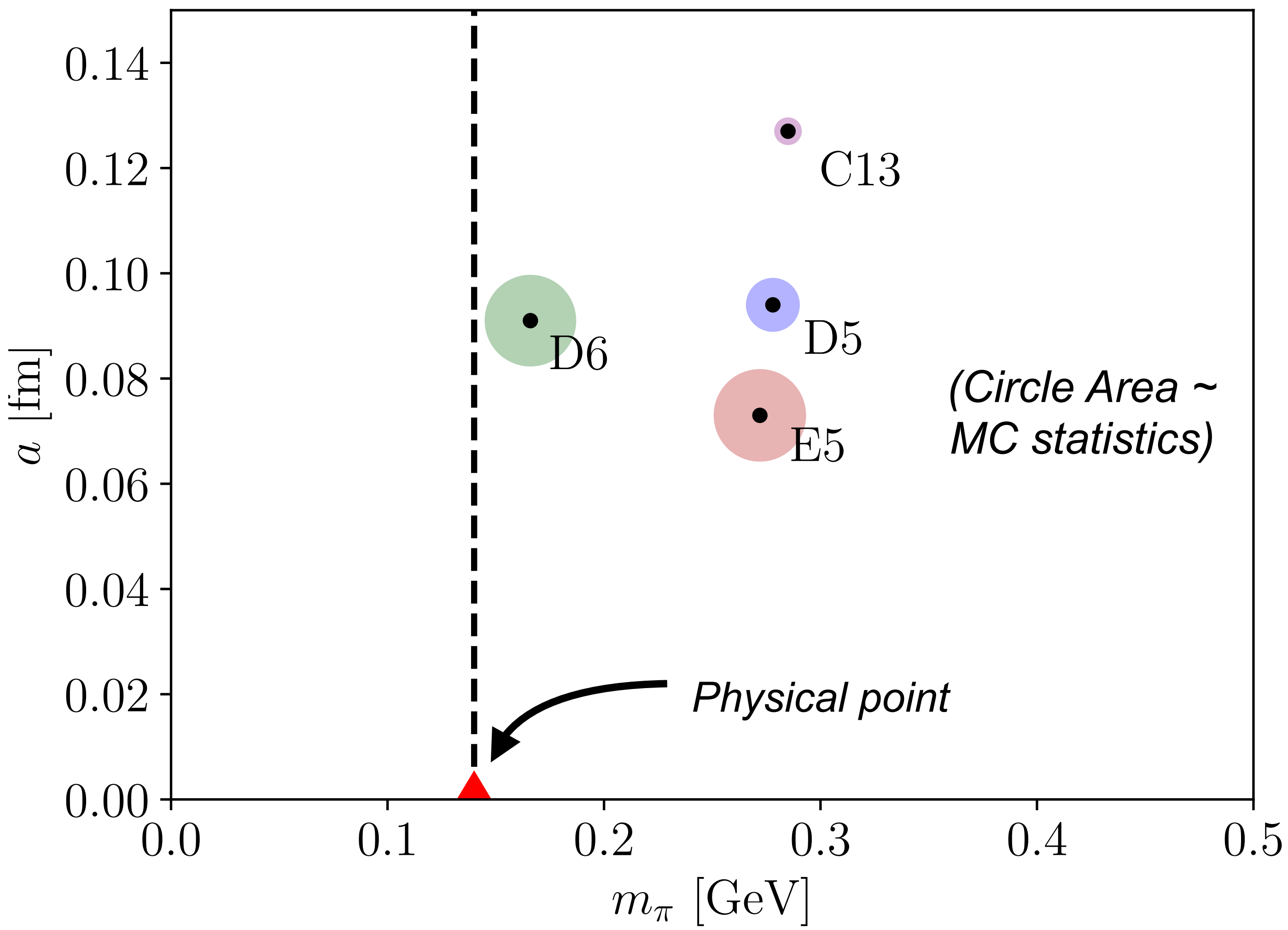
$$p_\mu^{\text{lat}} = k_\mu + O(k^3)$$

⇒ **expect** O(a^2) corrections from lattice nucleon spinor



Present QCD Calculation Parameters

- $N_F = 2+1$ clover-improved Wilson fermion ensembles (JLab / W&M / LANL / MIT)
- Lattice spacing $a \approx 0.073 \div 0.091$ fm
- Light quark masses approaching physical : $m\pi = 170 \div 280$ MeV
- Large physical volume $L \gtrsim 3.7\,(m\pi)^{-1}$
- Source-sink separation t_{sep}= 0.51 ÷ 1.09 fm
- Momentum smearing, AMA sampling
- Estimate disconnected contributions



2022/23:

- MC Statistics ~250k on D6 (48^3 x 96), E5 (48^3 x 128)
- Disconnected contractions on D6 (1000+ configs)

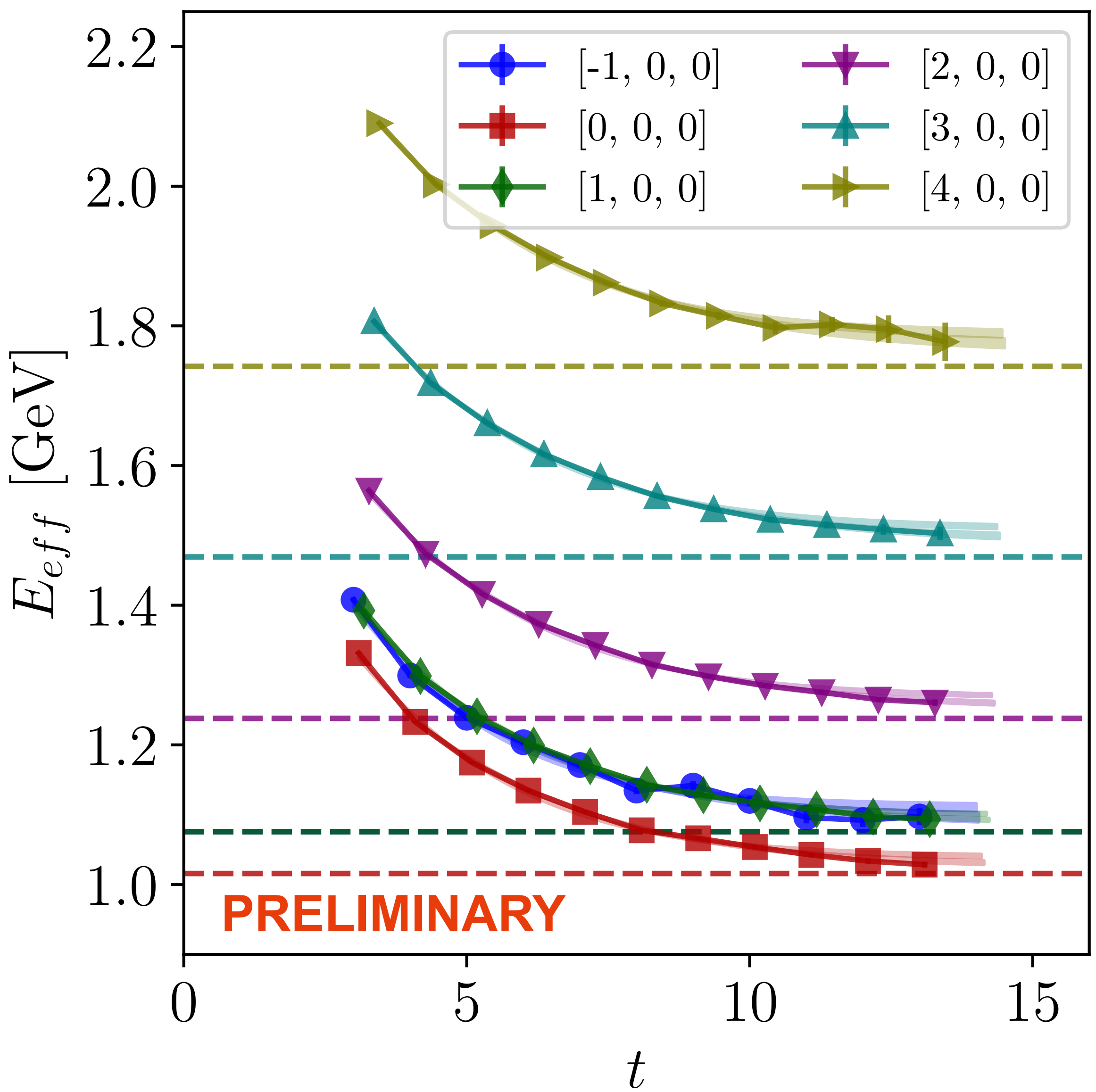
Made possible by new nVidia A100 clusters

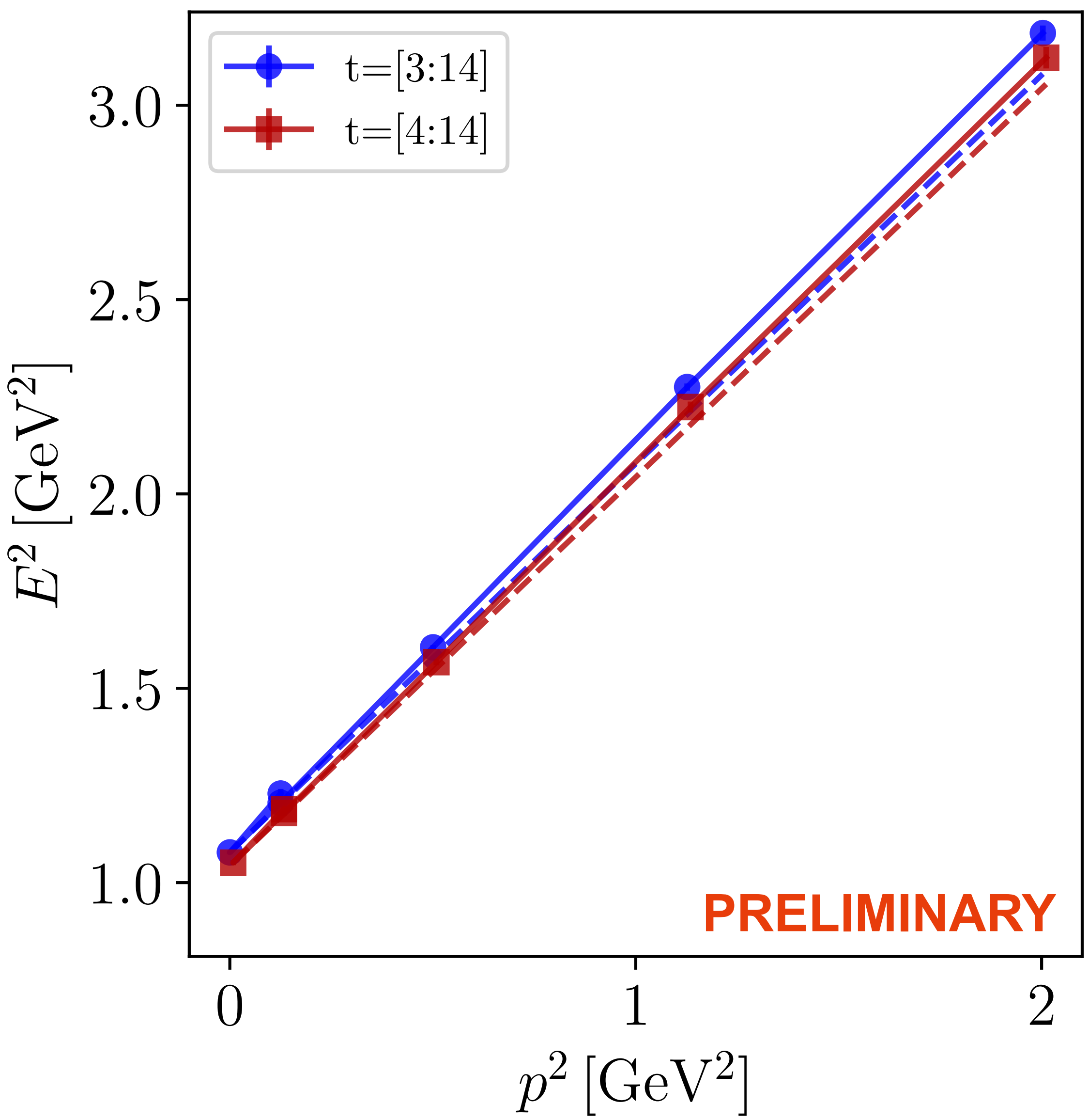
- Perlmutter [NERSC]
- Juwels-Booster [Fz. Juelich]

Many thanks to the QUDA team!
[K. Clark, R.Babich, R.Brower, M.Wagner, E.Weinberg, and many others]

Lattice Nucleon Energy & Dispersion Relation (E5)

- E5 : mπ = 272 MeV , spacing a = 0.073 fm , 266k MC samples





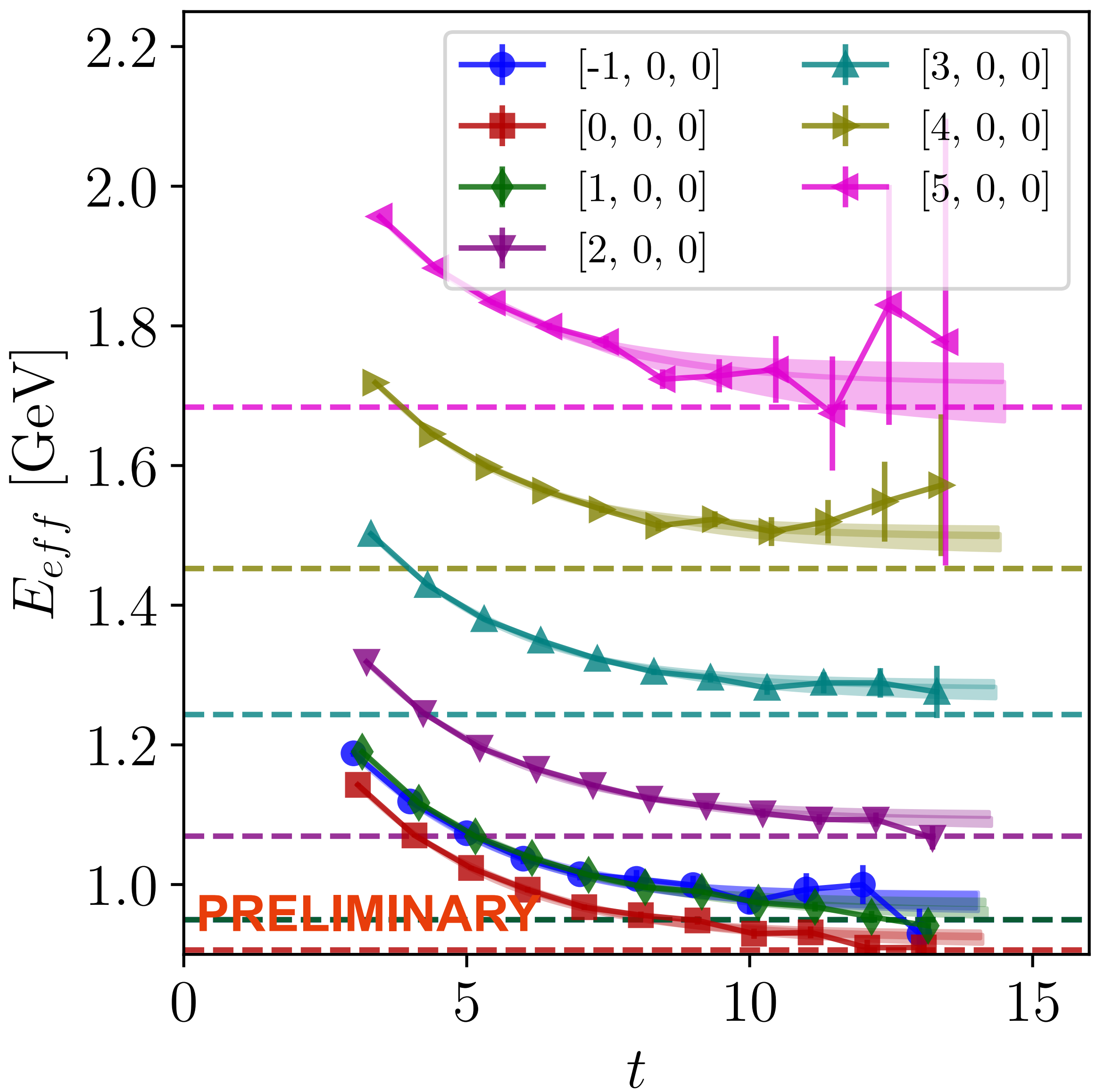
- Effective energy and 2-state fits

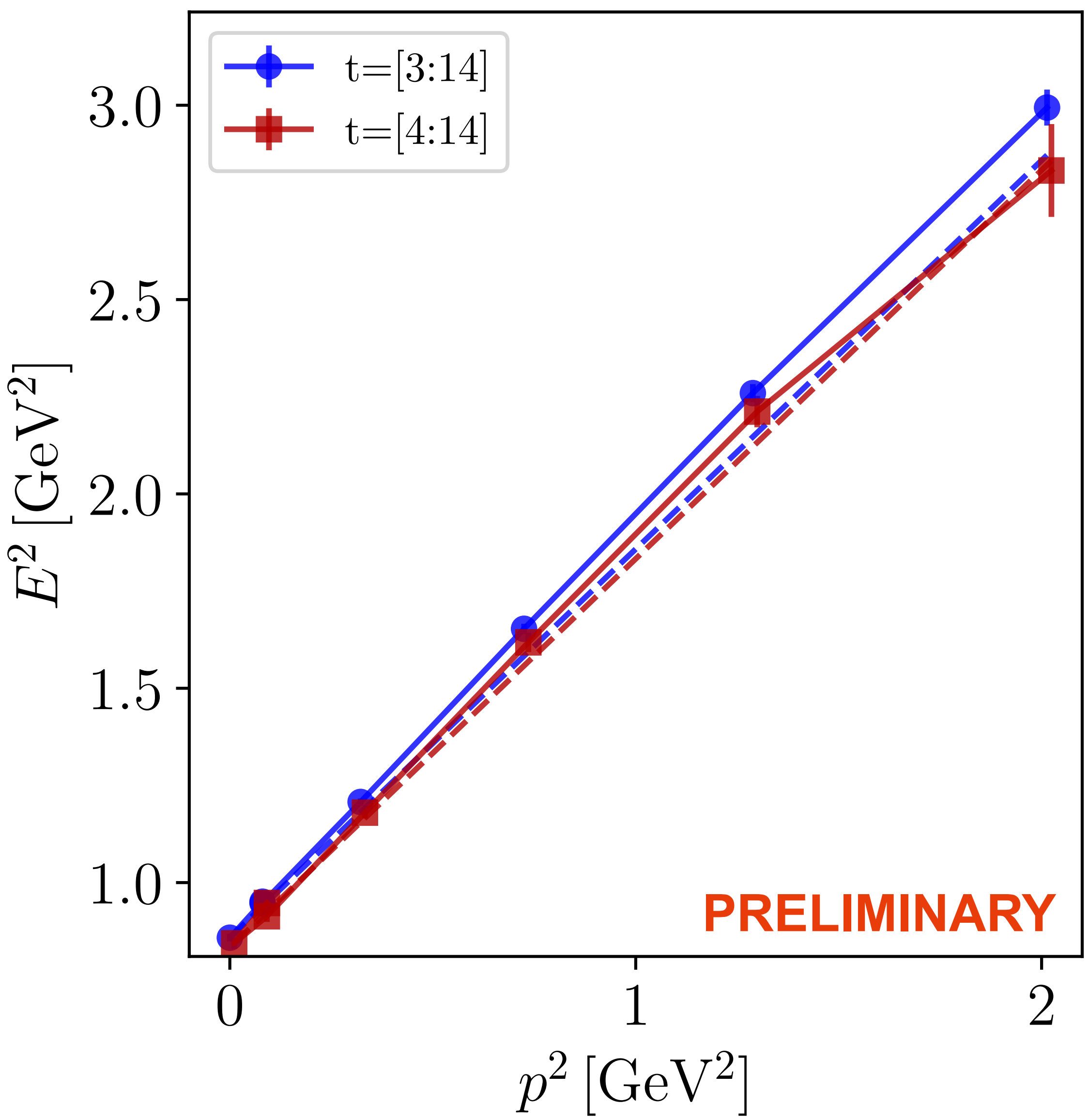
$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

- Dispersion relation
  Dashed lines: cont. $E^2(p) = E^2(0) + p^2$

Lattice Nucleon Energy & Dispersion Relation (D6)

- D6 : $m\pi = 166$ MeV , spacing $a = 0.091$ fm , 261k MC samples





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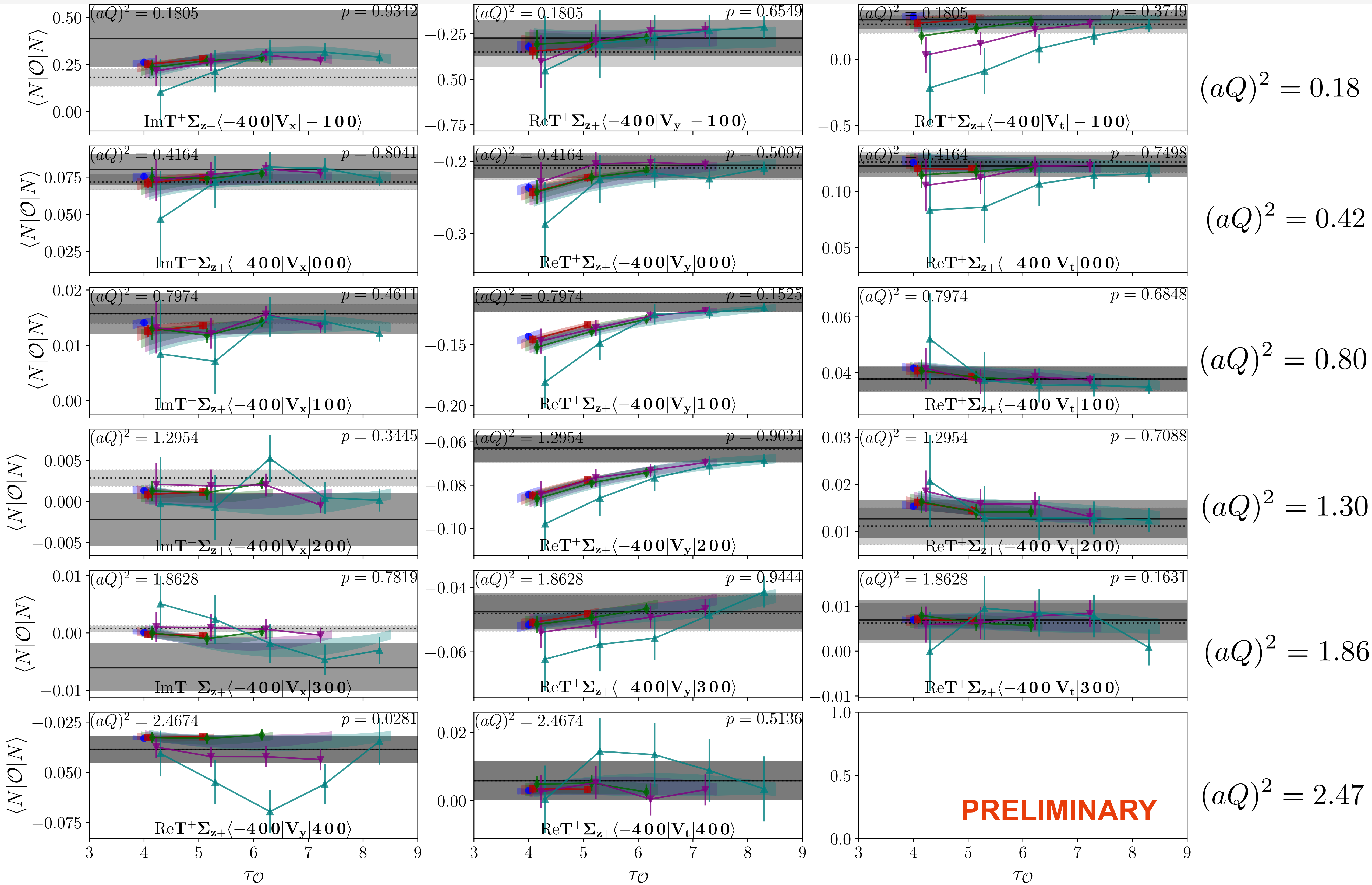
- Dispersion relation
  Dashed lines: cont. $E^2(p) = E^2(0) + p^2$

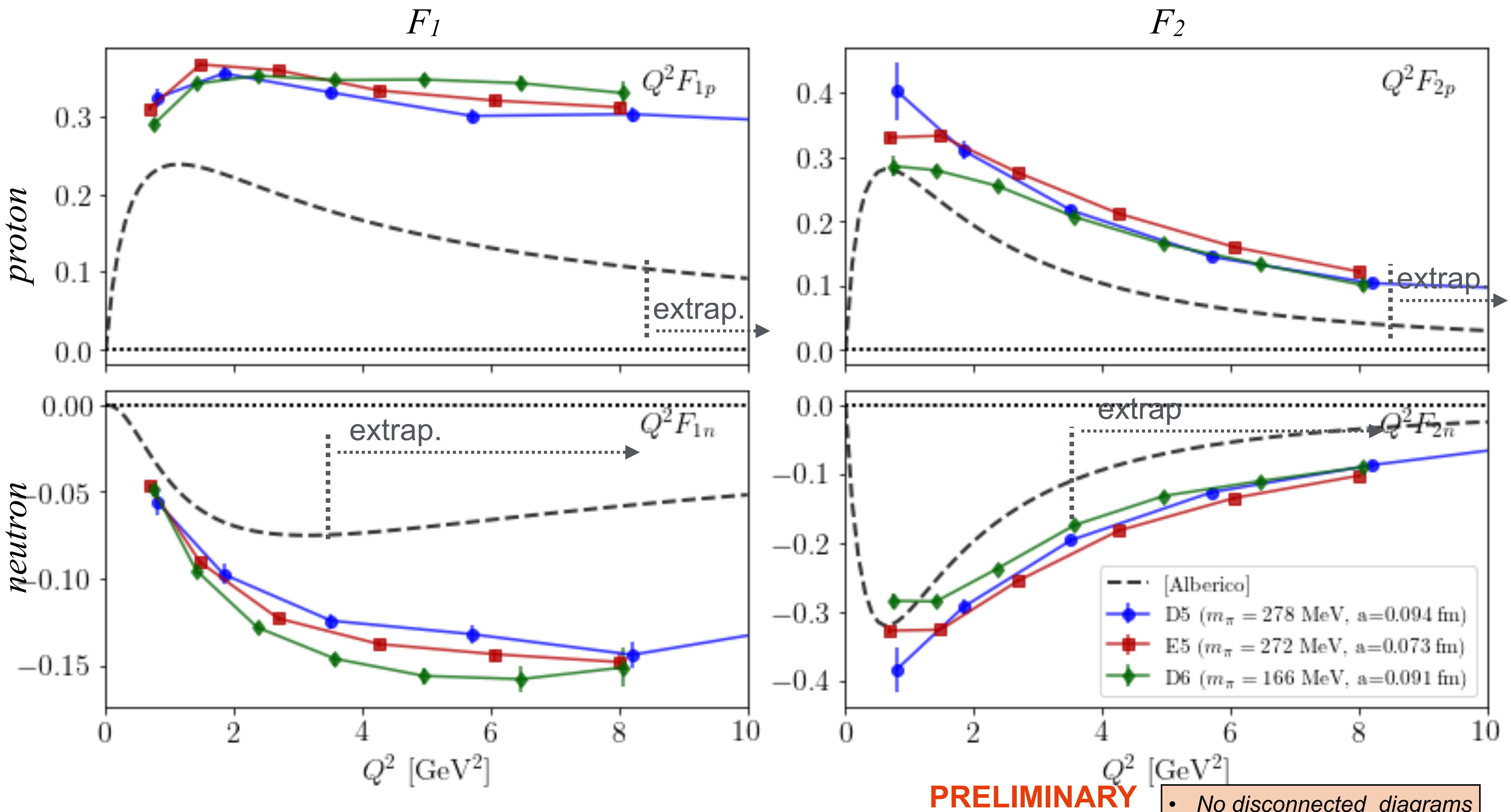
Nucleon Matrix Element & Form Factor Fits (D5)



- 2-state fit t_{sep}=0.73 ÷1.09 fm (8a ÷12a); energies fixed from 2-state fits to ⟨NN̄⟩

Nucleon Form Factors

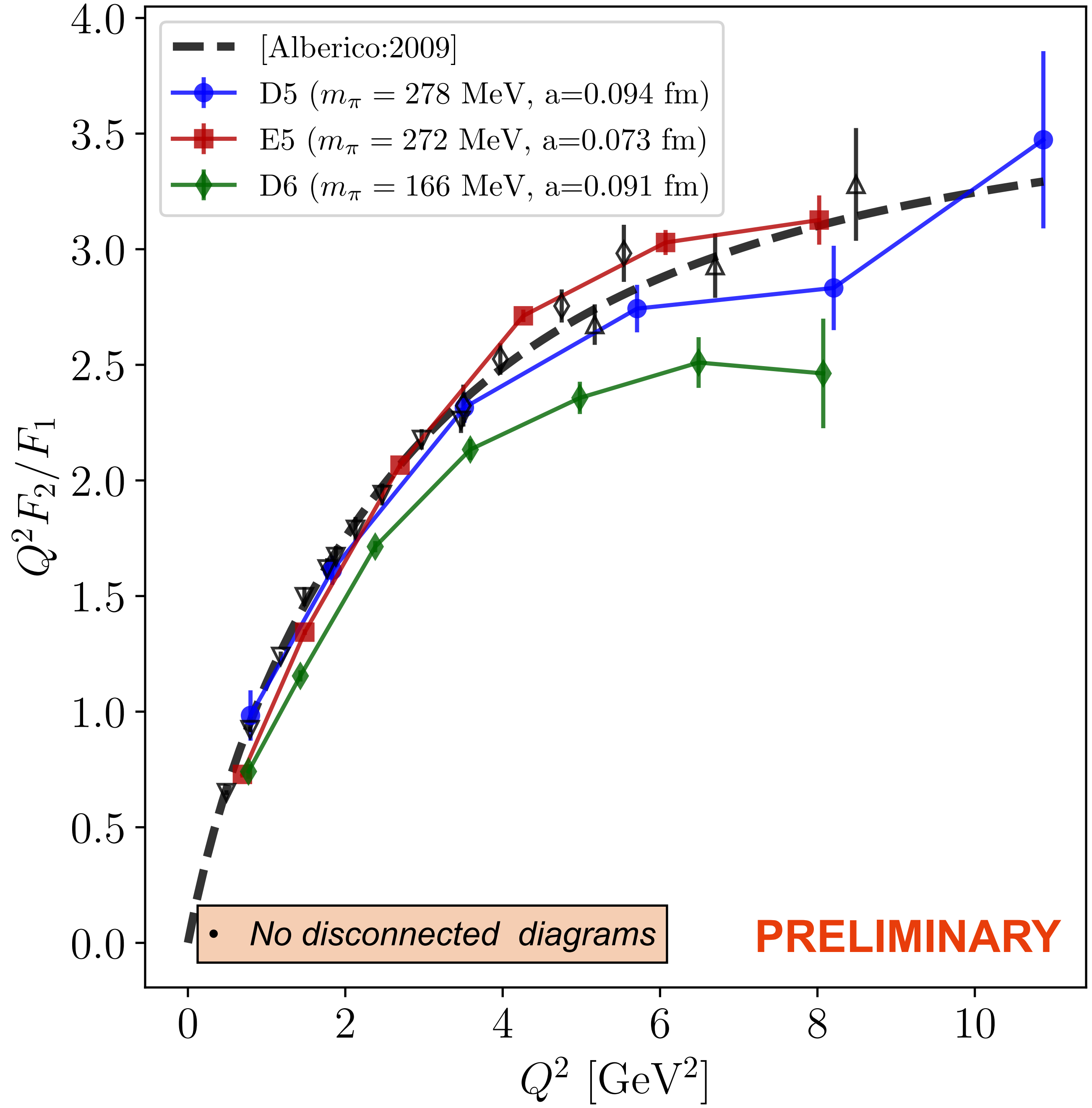
- 2-state fits to extract the ground state
- discrepancy x(2..2.5) for Q2>2GeV2: exc.states? discretization? quark mass
- *Phenomenology curves* : [Alberico et al, PRC79:065204 (2008)]

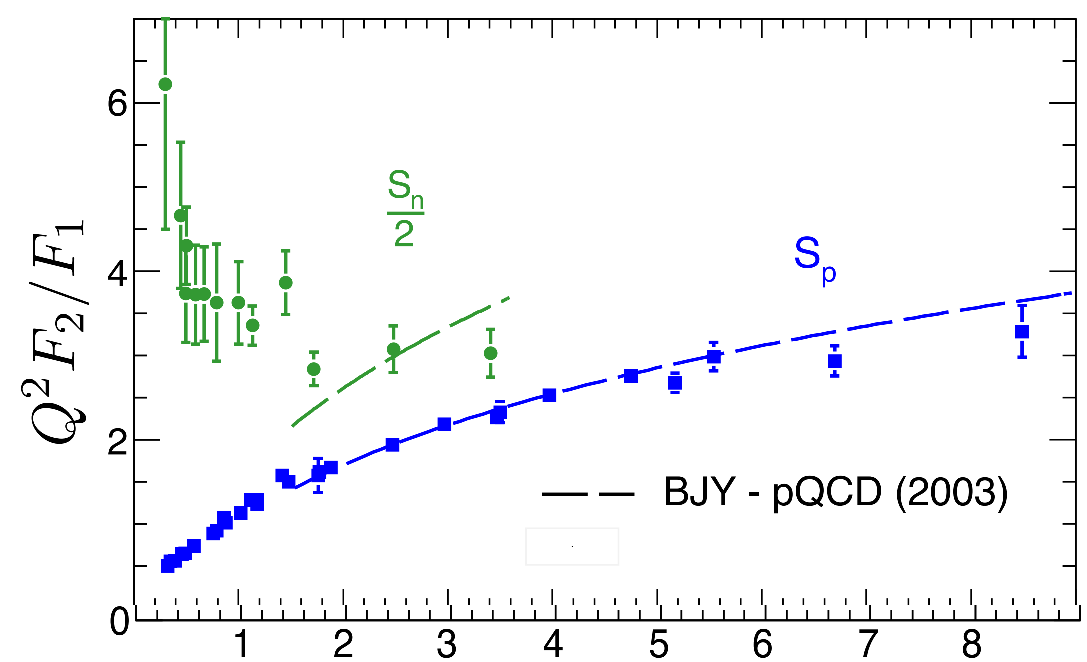


- *No disconnected diagrams*

Proton F_2/F_1 Ratio

- Lattice data: 2-state fits
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Comparison to experimental data (black points)

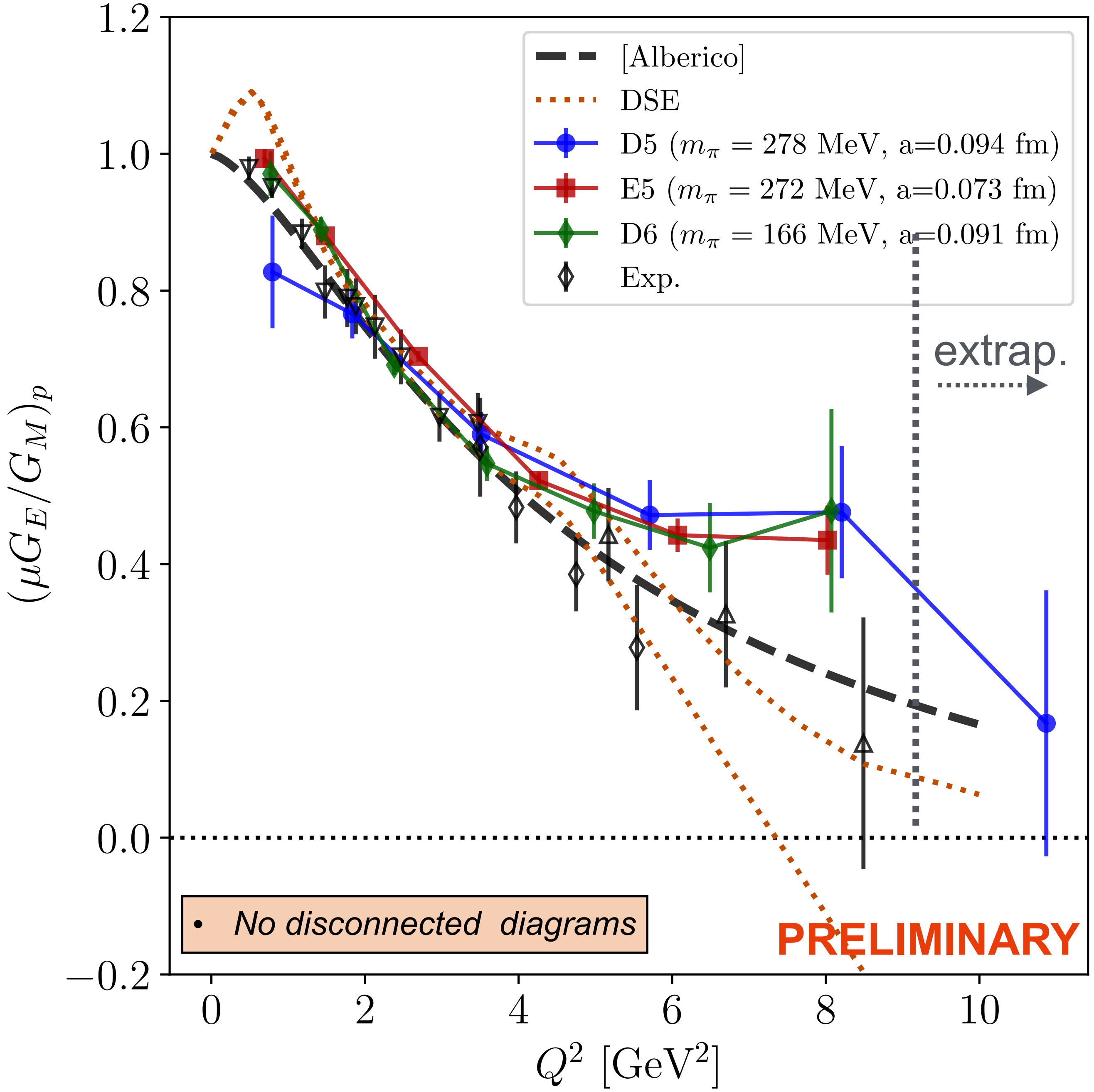


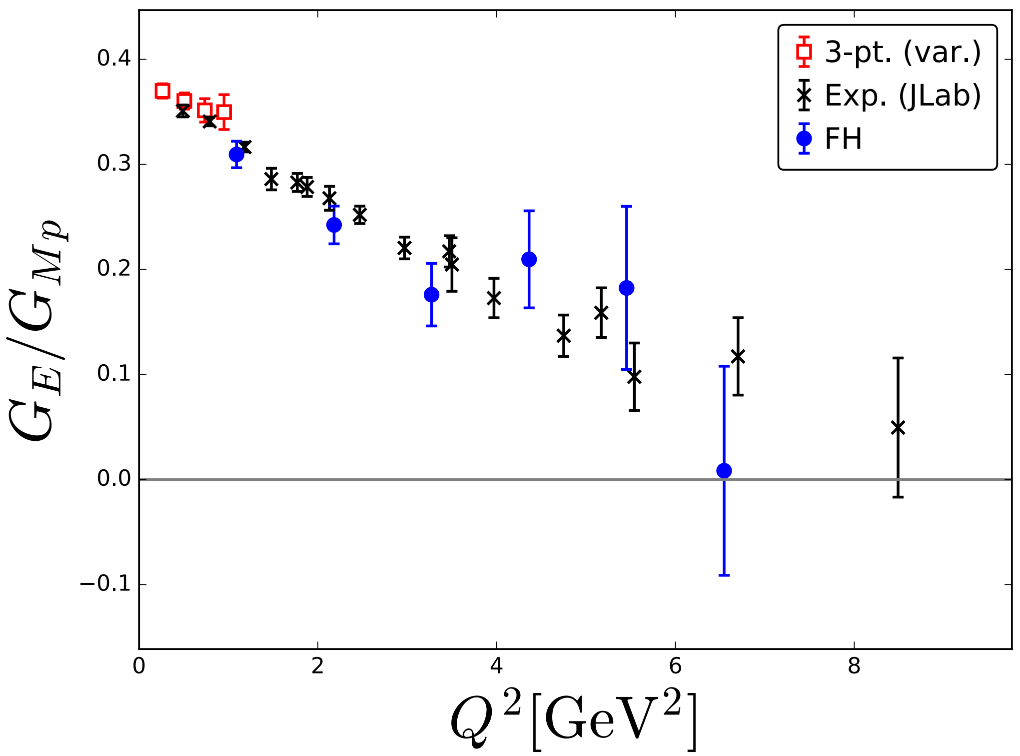


[G.D.Cates, et al, PRL106:252003 (2011)]

Proton G_E/G_M Ratio

- Lattice data: 2-state fits
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Comparison to experimental data (black points)

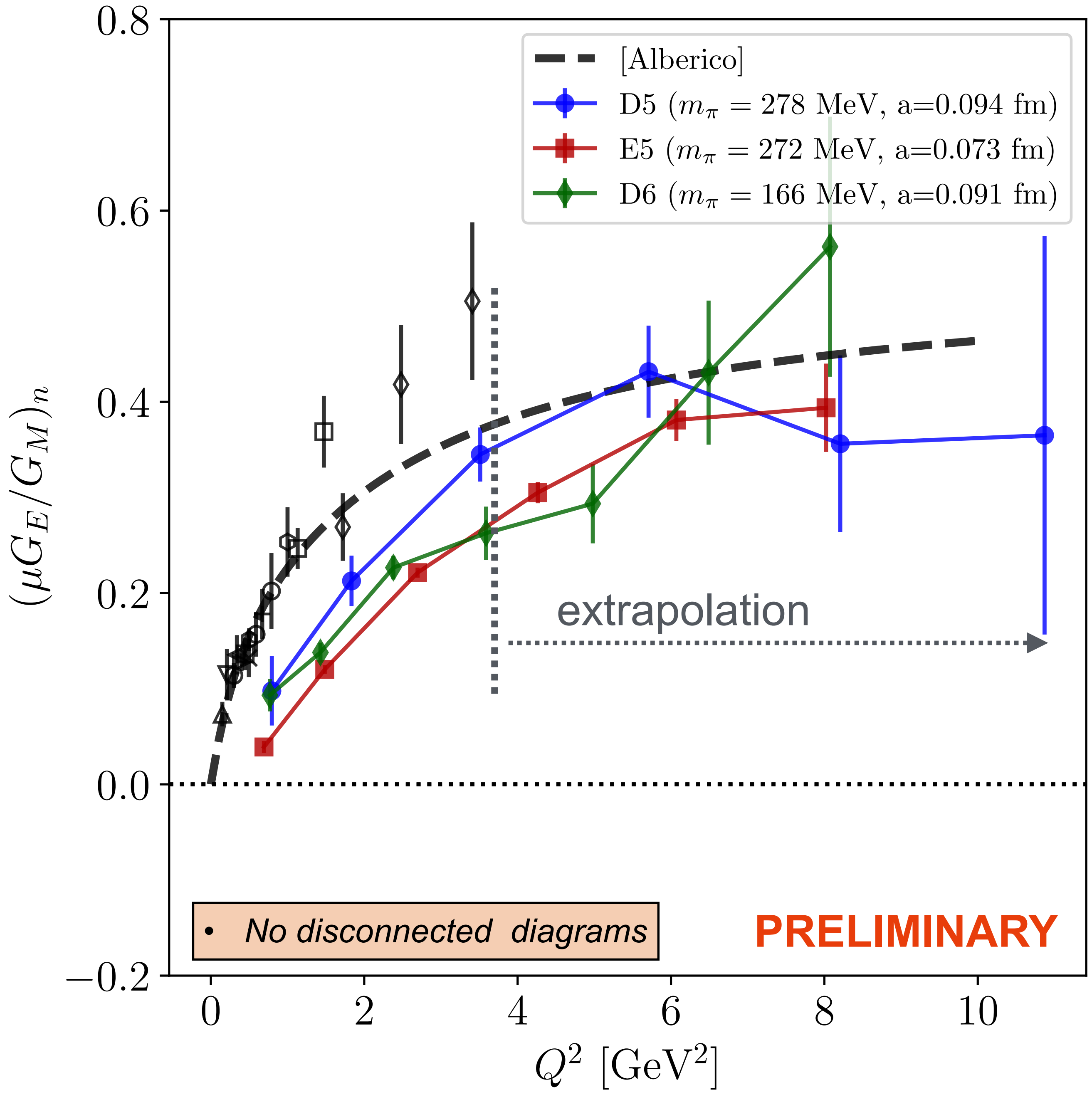




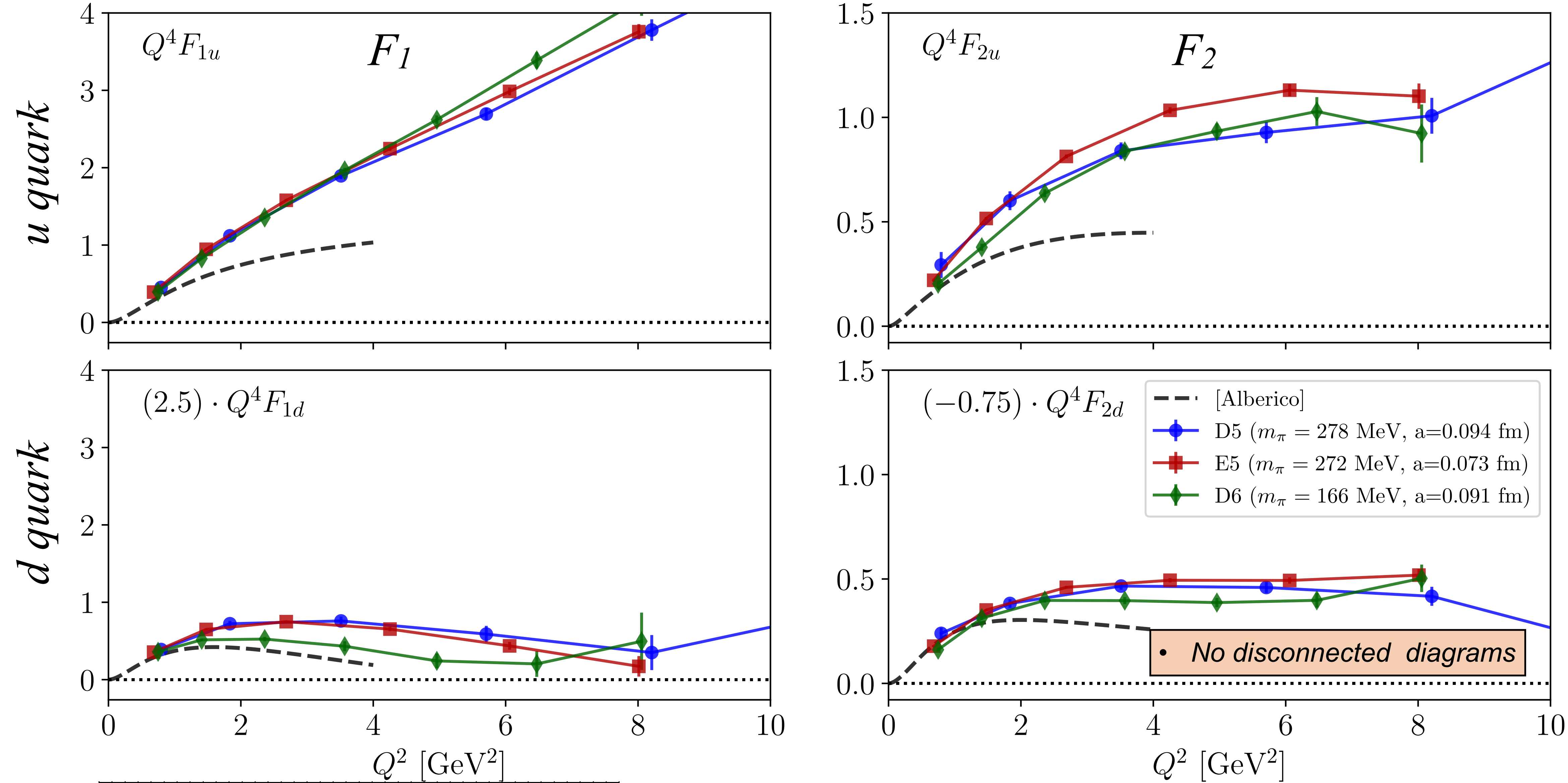
Earler calculation: (a=0.074 fm, m_π=470MeV)
Feynman-Hellman method
[Chambers et al (CSSM), PRD96: 114509]

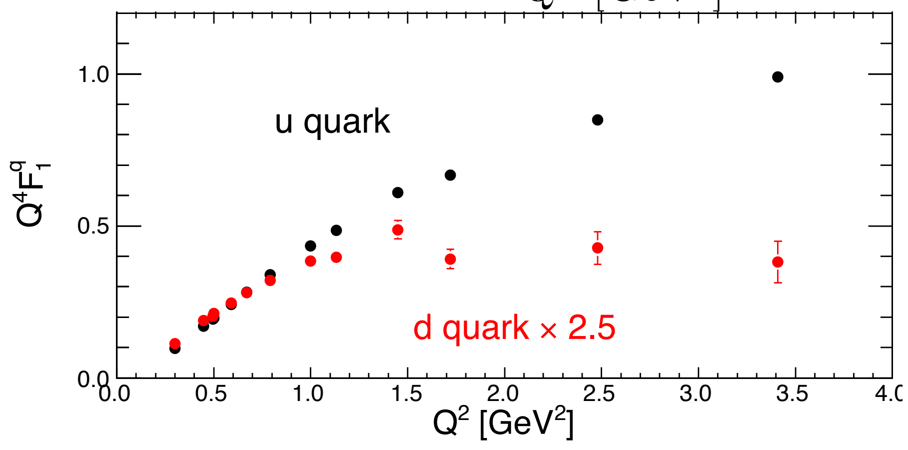
Neutron G_{En}/G_{Mn} Ratio

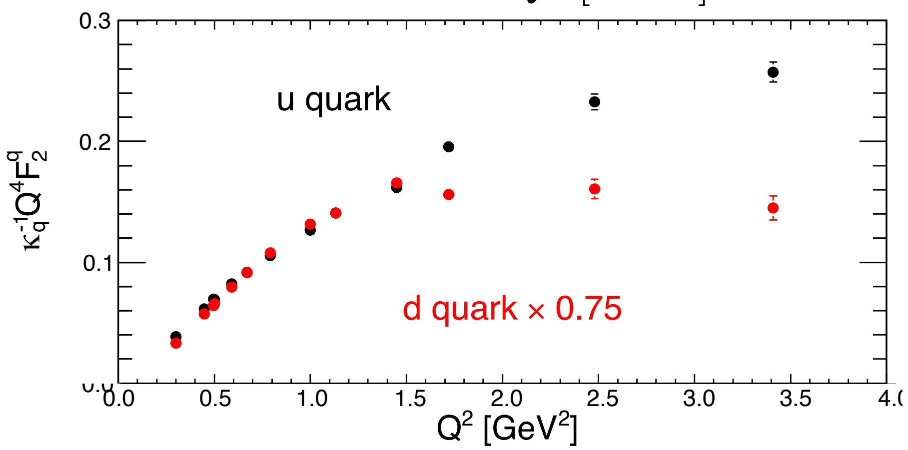
- Lattice data: 2-state fits
- Phenomenology curves : [Alberico et al, PRC79:065204 (2008)]
- Comparison to experimental data (black points)



Light-Flavor Decomposition (Proton)







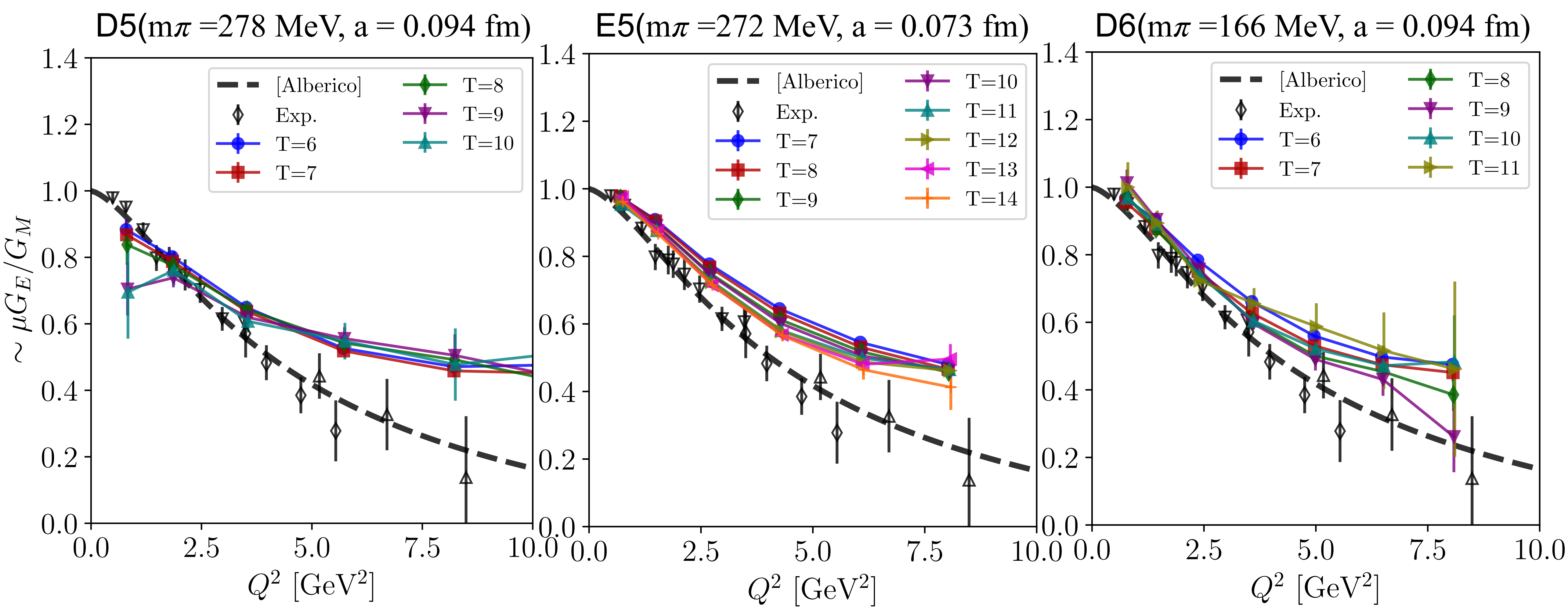
- Similar qual.features of flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

Examine Exc.States : Proton G_E/G_M

- Robust estimator from nucleon-current correlators: avoid lattice correlators fits to ~Σ exp(–Et)

$$\mathrm{Re}\,\langle p'\hat{x}|J_t|p\hat{x}\rangle \;\propto\; \cosh\frac{\lambda'+\lambda}{2} G_E$$
$$\mathrm{Re}\,\langle p'\hat{x}|J_y|p\hat{x}\rangle \;\propto\; \sinh\frac{\lambda'-\lambda}{2} G_M$$

where $\begin{pmatrix} p^{(\prime)} & = m_N \sinh\lambda^{(\prime)} \\ E^{(\prime)} & = m_N \cosh\lambda^{(\prime)} \end{pmatrix}$



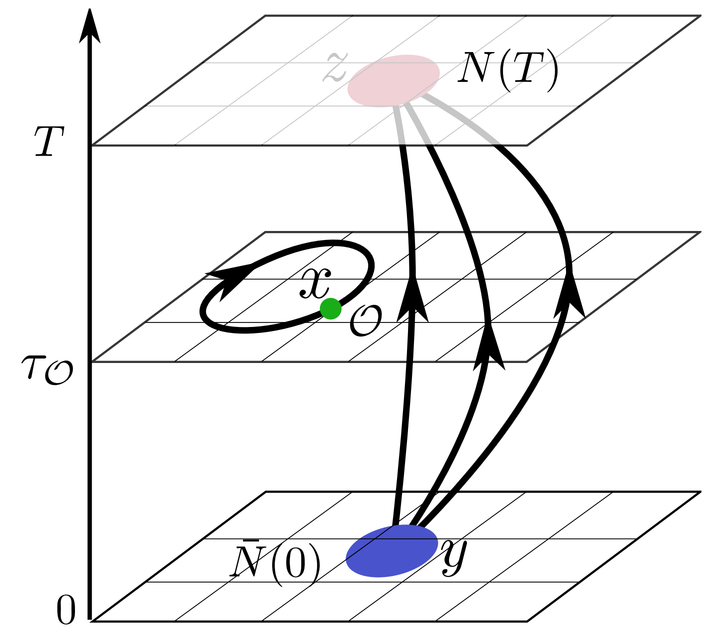
$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle} \overset{T\to\infty}{=} G_E/G_M$$

Disconnected Quark Loops

- Stochastic evaluation: $\begin{cases} \xi(x) = \text{ random } Z_2\text{-vector} \\ E\left[\xi^\dagger(x)\xi(y)\right] = \delta_{x,y} \end{cases}$

$$\sum_x e^{iqx} \not{D}^{-1}(x,x) \approx \frac{1}{N_{MC}} \sum_i^{N_{MC}} \xi^\dagger_{(i)} \big(e^{iqx} \not{D}^{-1} \xi_{(i)}\big)$$

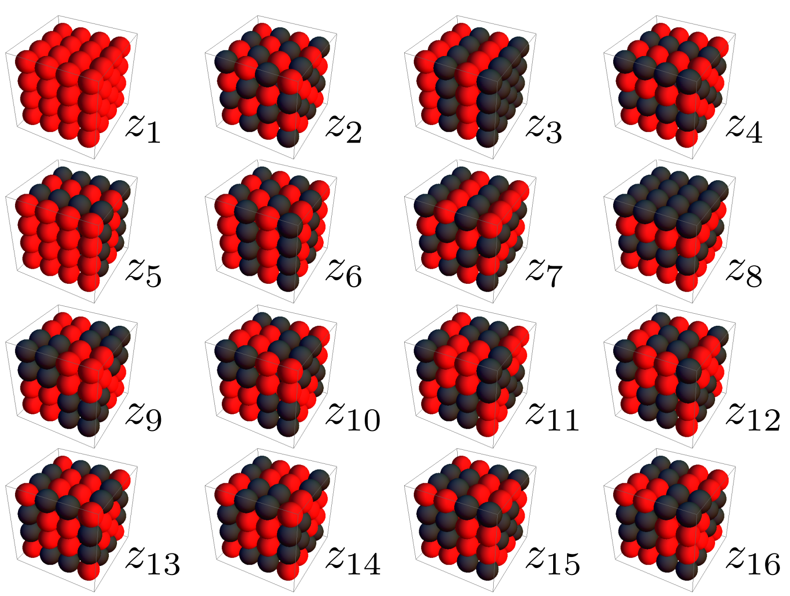
$$\mathrm{Var}\Big(\sum_x \not{D}^{-1}(x,x)\Big) \sim \frac{1}{N_{MC}} \qquad \text{(contributions from } \not{D}^{-1}(x \neq y))$$



- Exploit $\not{D}^{-1}(x,y)$ falloff to reduce $\sum_{x\neq y} |\not{D}^{-1}(x,y)|^2$:

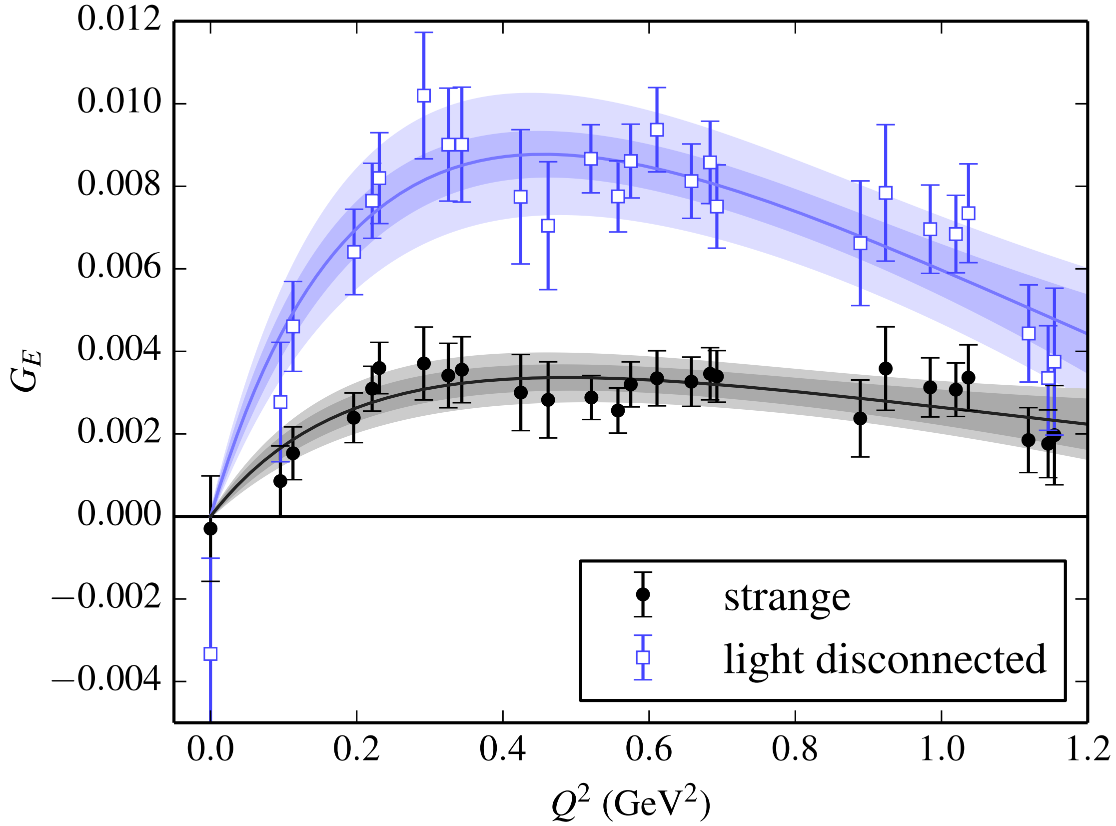
Hierarchical probing method [K.Orginos, A.Stathopoulos, '13] :
In sum over $N=2^{nd+1}$ 3D(4D) Hadamard vectors,
near-(x,y) terms cancel:

$$\frac{1}{N} \sum_i z_i(x) z_i(y)^\dagger = \begin{cases} 0, & 1 \leq |x-y| \leq 2^k, \\ 1, & x = y \text{ or } 2^k < |x-y| \end{cases}$$



- Further decrease variance by deflating low-lying, long-range modes [A.Gambhir's PhD thesis]

Prior work: Disc.Light & Strange Quark F.F's

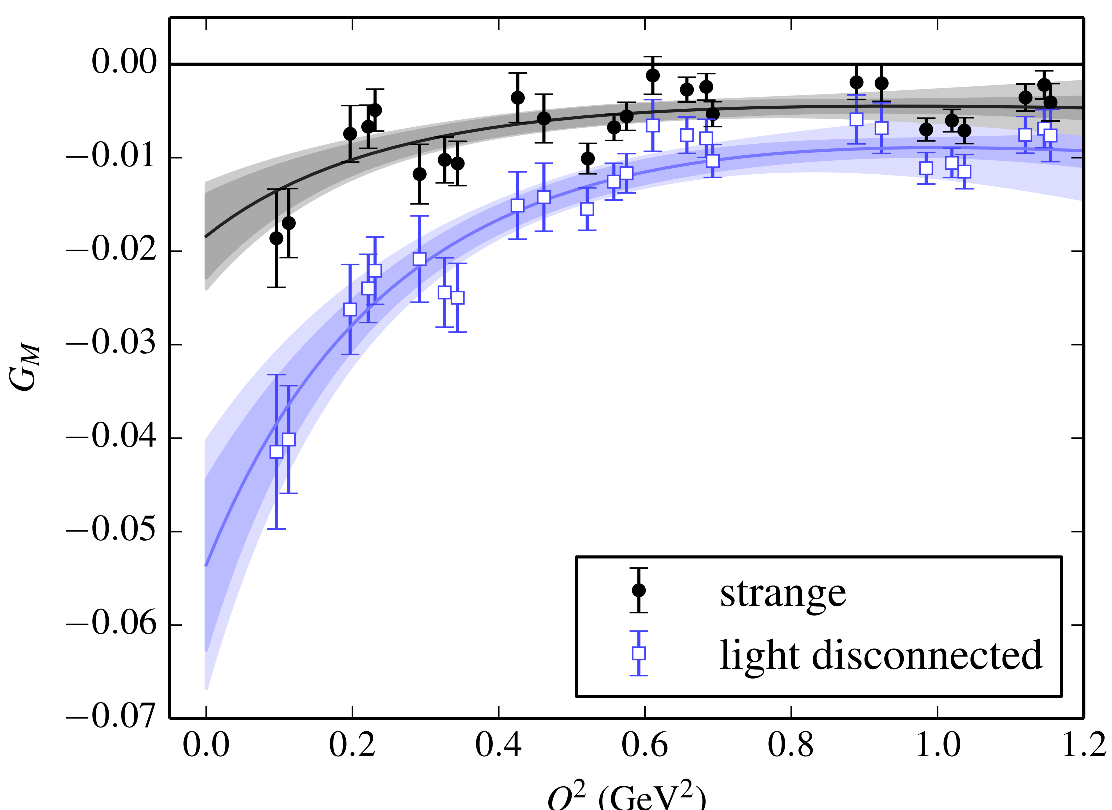


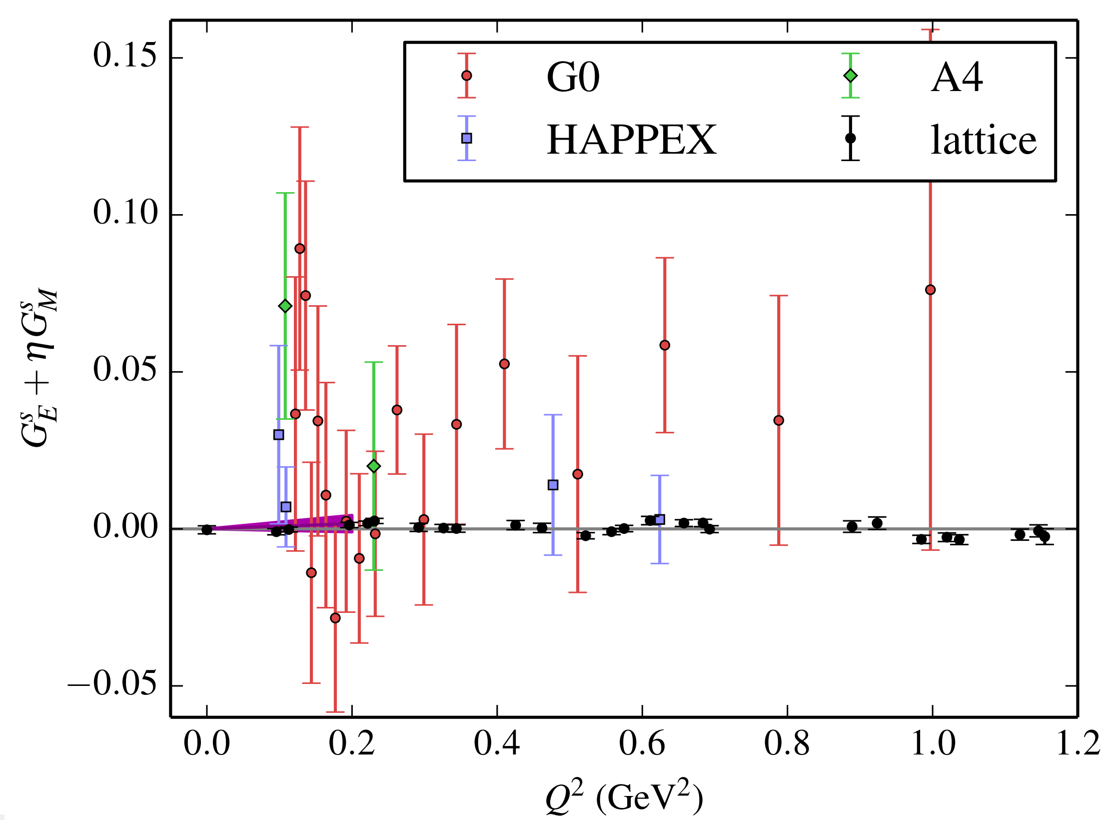
N_f=2+1 dynamical fermions, $m_\pi \approx$ 320 MeV
(C13 ensemble)

$$|(G_E^{u/d})_{\rm disc}| \lesssim 0.010 \text{ of } |(G_E^{u-d})_{\rm conn}|$$
$$|(G_E^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

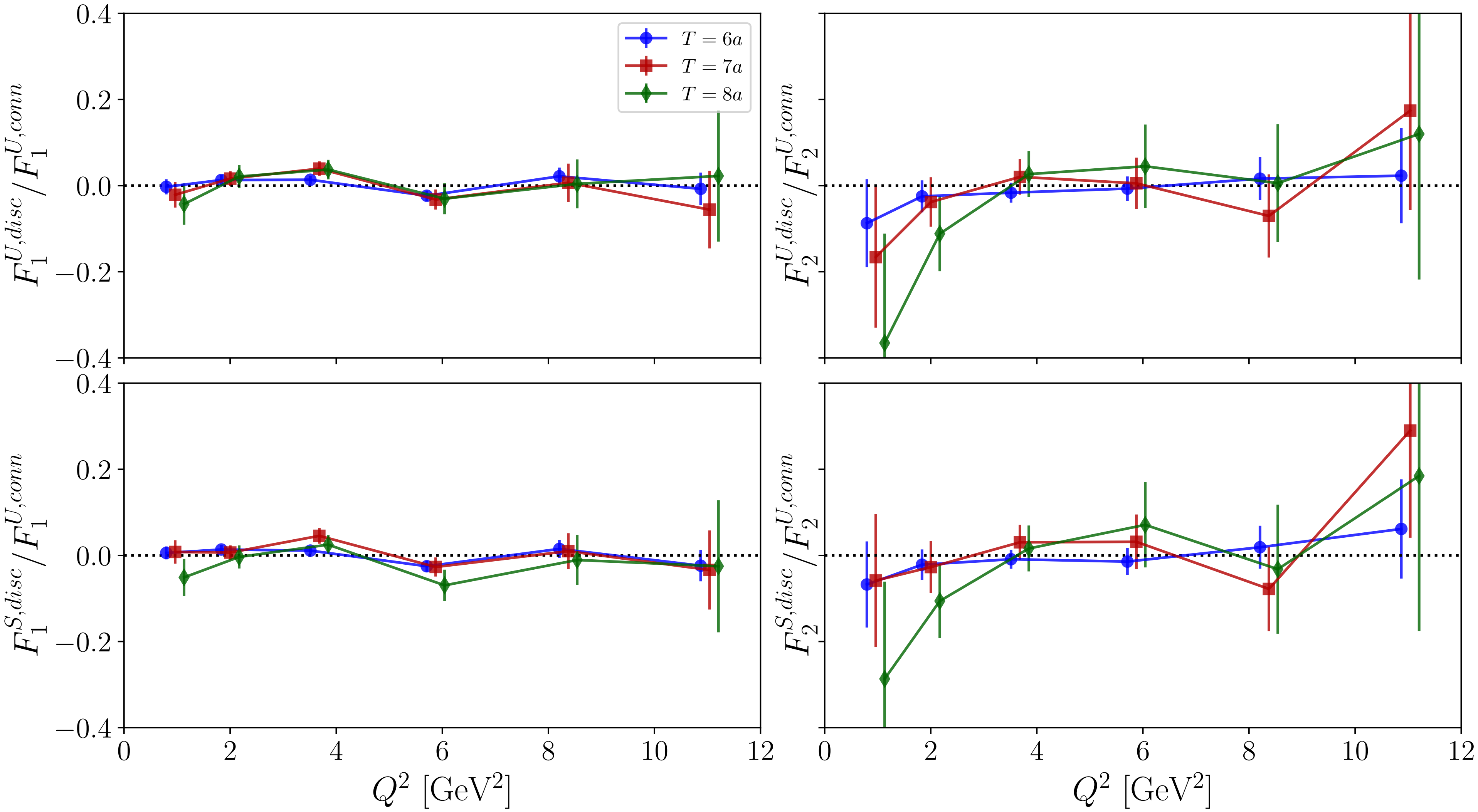
$$|(G_M^{u/d})_{\rm disc}| \lesssim 0.015 \text{ of } |(G_M^{u-d})_{\rm conn}|$$
$$|(G_M^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_M^{u-d})_{\rm conn}|$$

[J. Green, S. Meinel, S.S. et al;
PRD92:031501 (2015)]





Disconnected Light, Strange vs. Connected

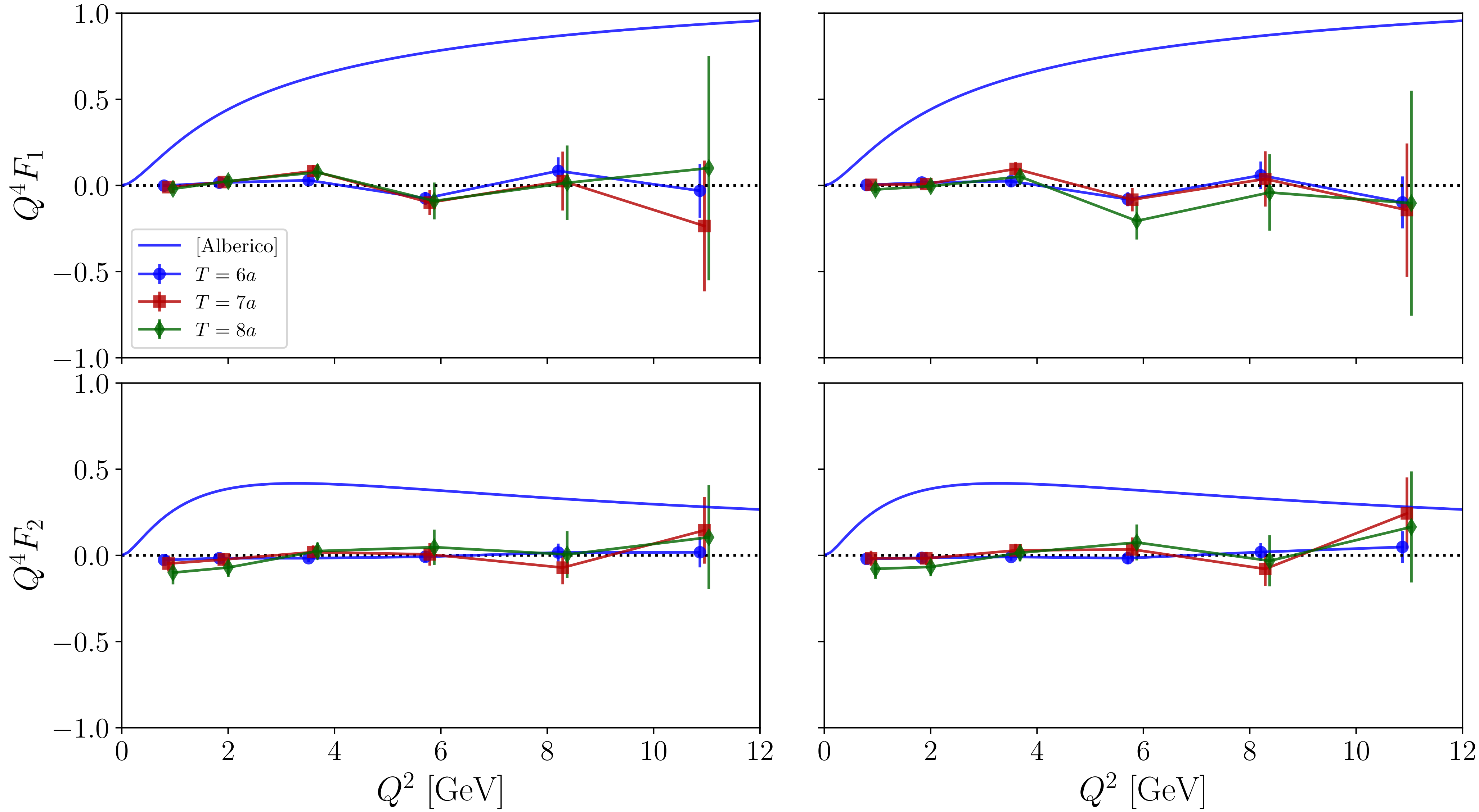


- D5 ensemble(mπ=280 MeV, a=0.094 fm), 1346 configs,
- 512 HP vectors ; UD: also deflation with 500 DdagD evecs [Stathopoulos et al (2013) ; Gambhir et al 2017]
- s-, disconnected *u,d*- contributions are small also at high Q^2 up to ≲ 10 GeV^2

$$|F_1^s| \lesssim |(F_1^{u/d})_{\rm disc}| \lesssim \ 10\% \text{ of } |F_2^{u,d}|$$

$$|F_2^s| \lesssim |(F_2^{u/d})_{\rm disc}| \lesssim \ 20\% \text{ of } |F_2^{u,d}|$$

Disconnected Light, Strange vs. Connected



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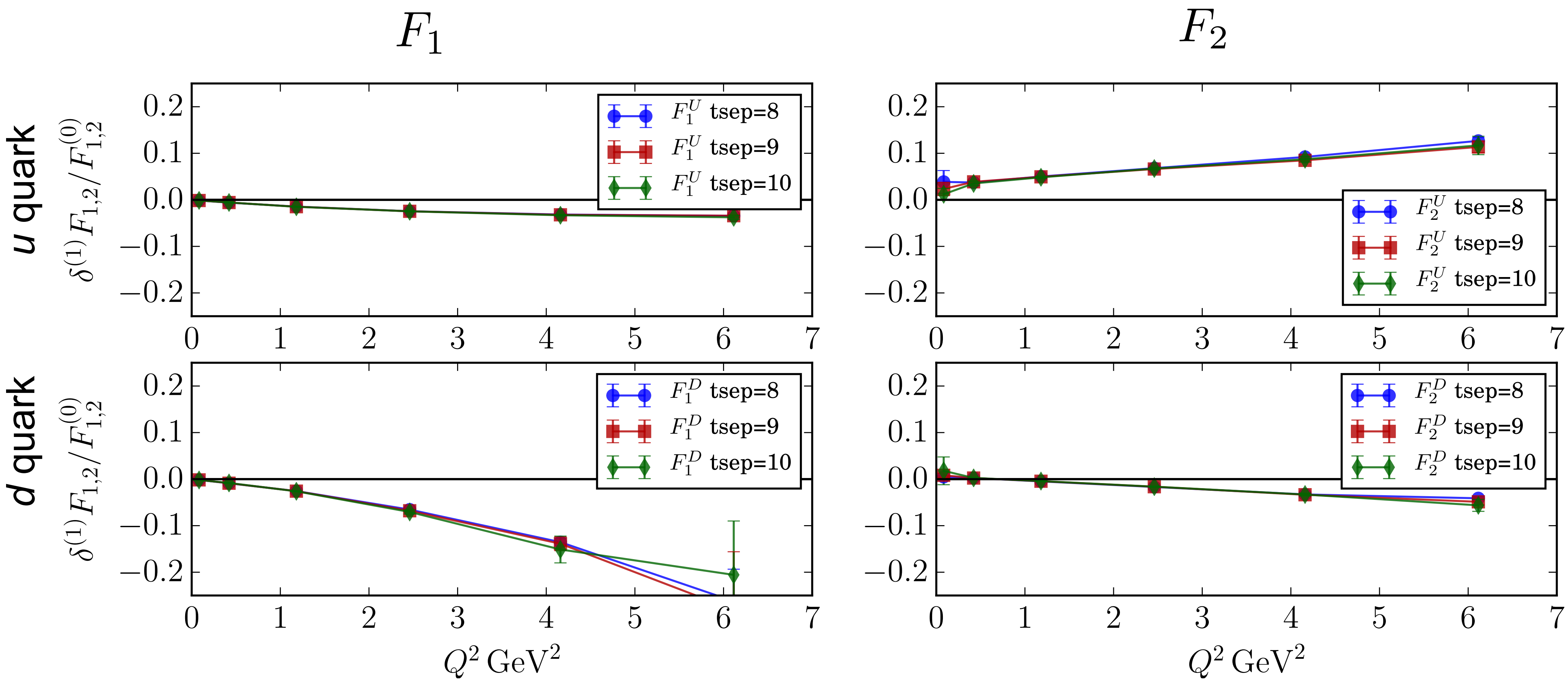
$$|F_1^s| \lesssim |(F_1^{u/d})_{\rm disc}| \lesssim \ 10\% \text{ of } |F_2^{u,d}|$$
$$|F_2^s| \lesssim |(F_2^{u/d})_{\rm disc}| \lesssim \ 20\% \text{ of } |F_2^{u,d}|$$

O(a) Vector Current Correction

- *No disconnected diagrams*

Improved vector current $(V_\mu)_I = \bar{q}\gamma_\mu q + c_V\, a\partial_\nu \bar{q} i\sigma_{\mu\nu} q$

O(a¹) correction : form factors of $a\,\langle N|\partial_\nu(\bar{q}i\sigma^{\mu\nu}q)|N\rangle$



Relative magnitude of $O(a^1)$ effects : $\{O(a^1)\} / \{O(a^0)\}$ form factors
(assuming c_V=0.05)

- improvement coefficient c_V : must be computed on lattice from WI
- perturbation theory: cV ≈ –0.01$C_F(g_0)^2$

Summary

- Preliminary results for high MC-statistics high-momentum form factors
  up to $Q^2 \lesssim 12$ GeV^2, two lattice spacings $a \gtrsim 0.07$ fm, two pion masses $m\pi \gtrsim 170$ MeV
  (No quark-disconnected contributions yet)

- Form factor results overshoot experimental data x(2 ... 2.5) ;
  G_E/G_M ratios in qualitative agreement
  ~~Discretization?~~
  Excited states?
  Non-physical quark masses?
  Quark-Disconnected contributions?

- Comparison to experiment important to validate lattice methods
  for computing relativistic nucleon matrix elements
  Impact on lattice methodology for TMDs, PDFs, DAs calculation

BACKUP