Twist-3 axial GPDs of the proton from lattice QCD

Martha Constantinou

Temple University

The 40th International Symposium on Lattice Field Theory

August 3, 2023

Collaborators

- S. Bhattacharya, Temple University/RIKEN BNL
- K. Cichy, Adam Mickiewicz University
- J. Dodson, Temple University
- A. Metz, Temple University
- Josh Miller, Temple University
- A. Scapellato, Temple University
- F. Steffens, University of Bonn

arXiv:2306.05533v1 [hep-lat] 8 Jun 2023

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]

Collaborators

- S. Bhattacharya, Temple University/RIKEN BNL
- K. Cichy, Adam Mickiewicz University
- J. Dodson, Temple University
- A. Metz, Temple University
- Josh Miller, Temple University
- A. Scapellato, Temple University
- F. Steffens, University of Bonn

arXiv:2306.05533v1 [hep-lat] 8 Jun 2023

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]

Outline

★ Motivation - twist-3 classification

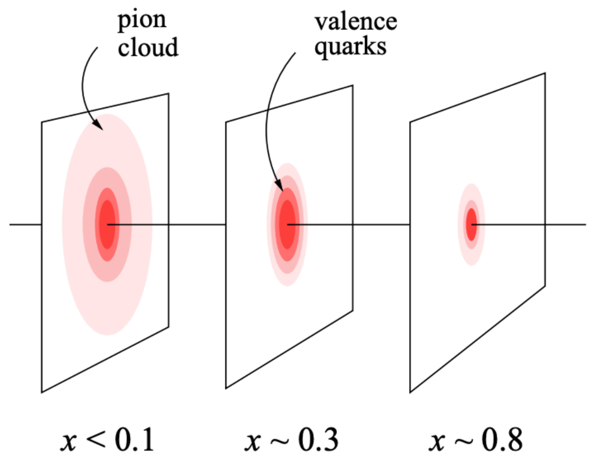
★ Methodology and computational setup

★ Lattice results

★ Light-cone GPDs

★ Consistency checks

Motivation for GPDs studies



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

$1_{\text{mom}} + 2_{\text{coord}}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal mom. transfer

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x - \xi + i\epsilon} dx$

  (SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs

- GPDs phenomenology more complicated than PDFs (multi-dimensionality)

- and more challenges …

Essential to complement the knowledge on GPD from lattice QCD

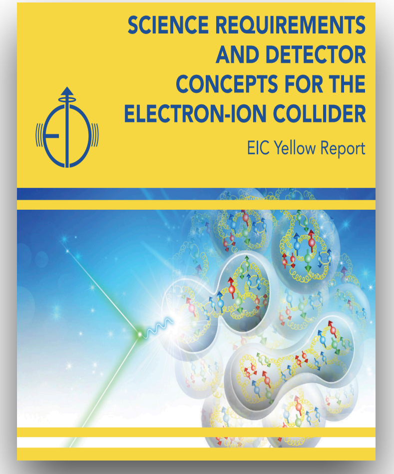
Hadron structure at core of nuclear physics

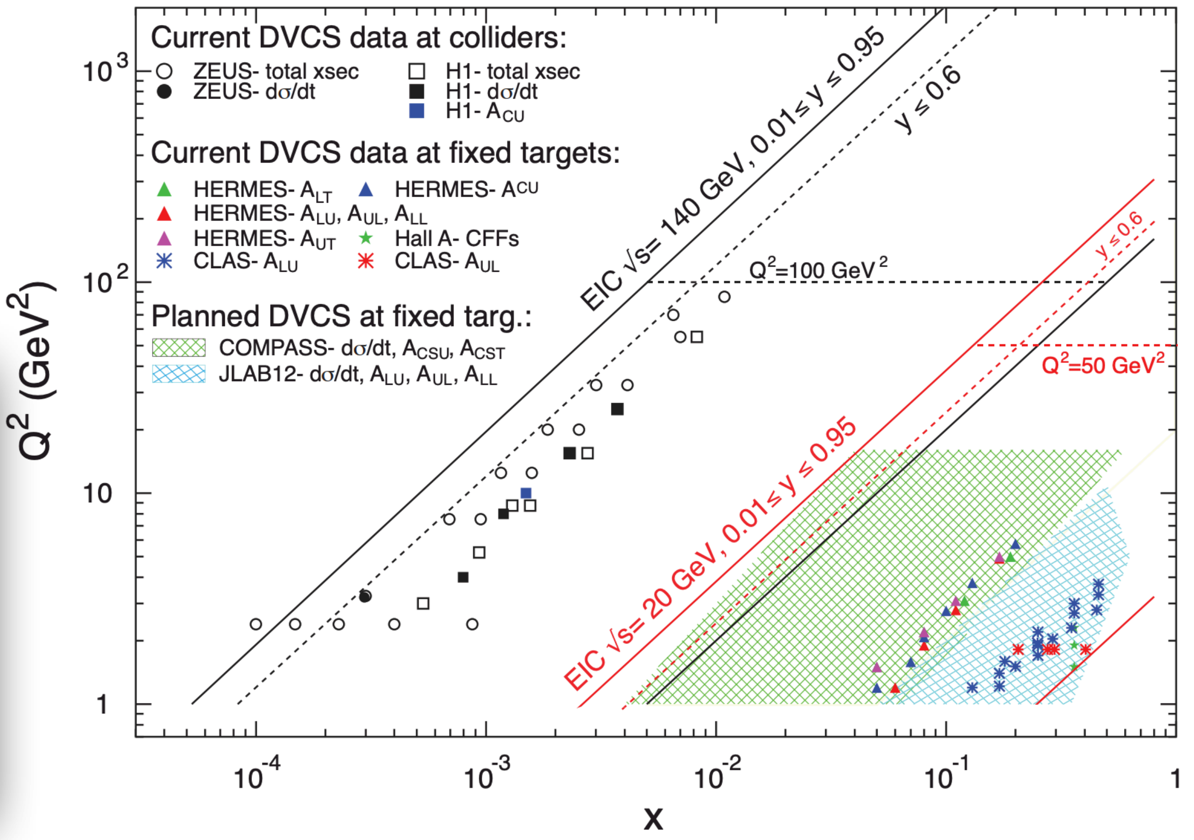
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





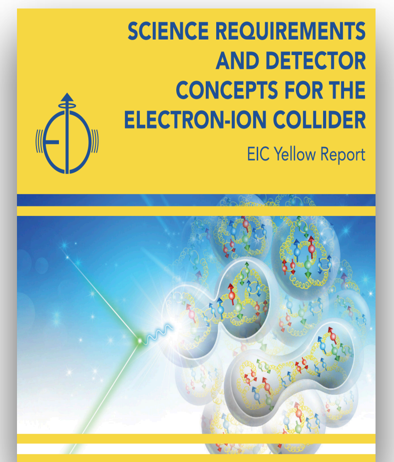
Hadron structure at core of nuclear physics

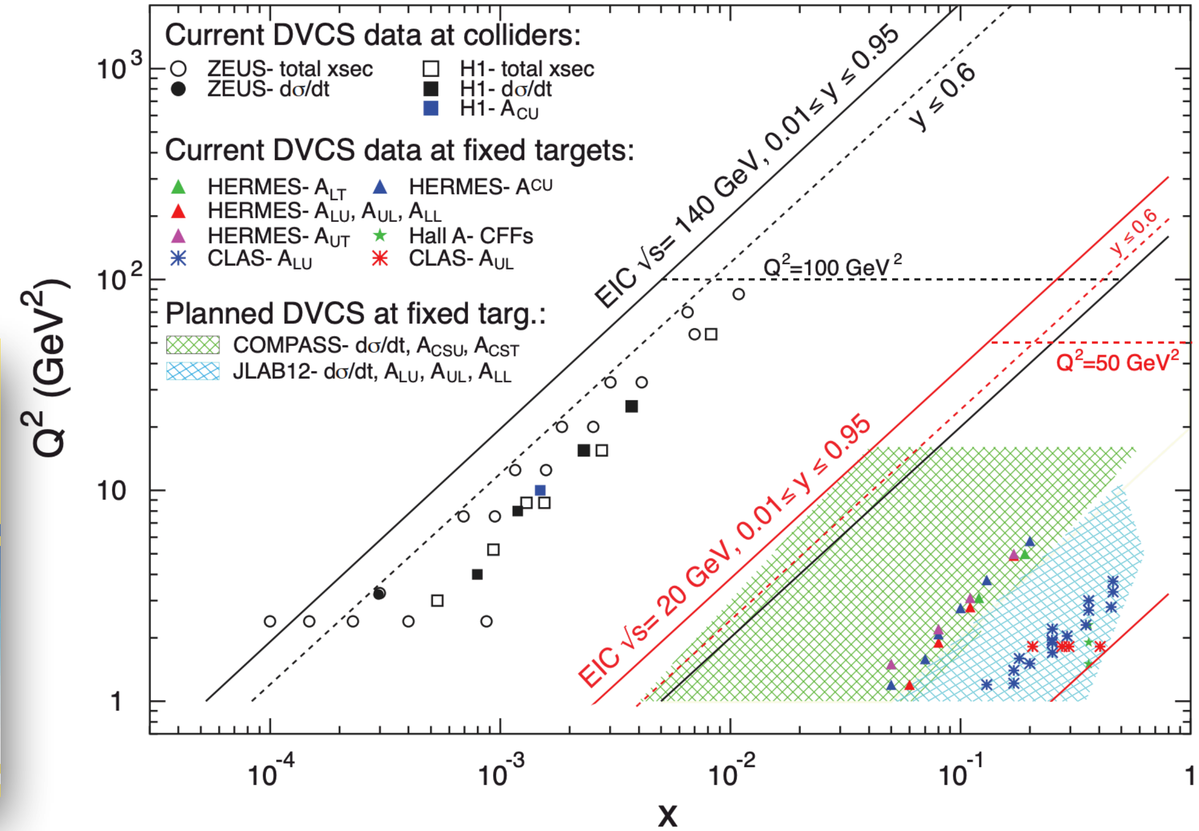
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





U.S. DEPARTMENT OF ENERGY | Office of Science

QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number: DE-SC0023646

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

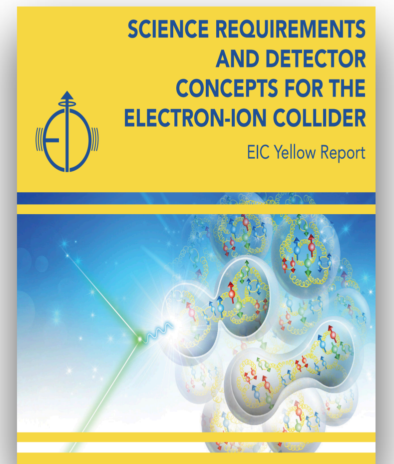
Hadron structure at core of nuclear physics

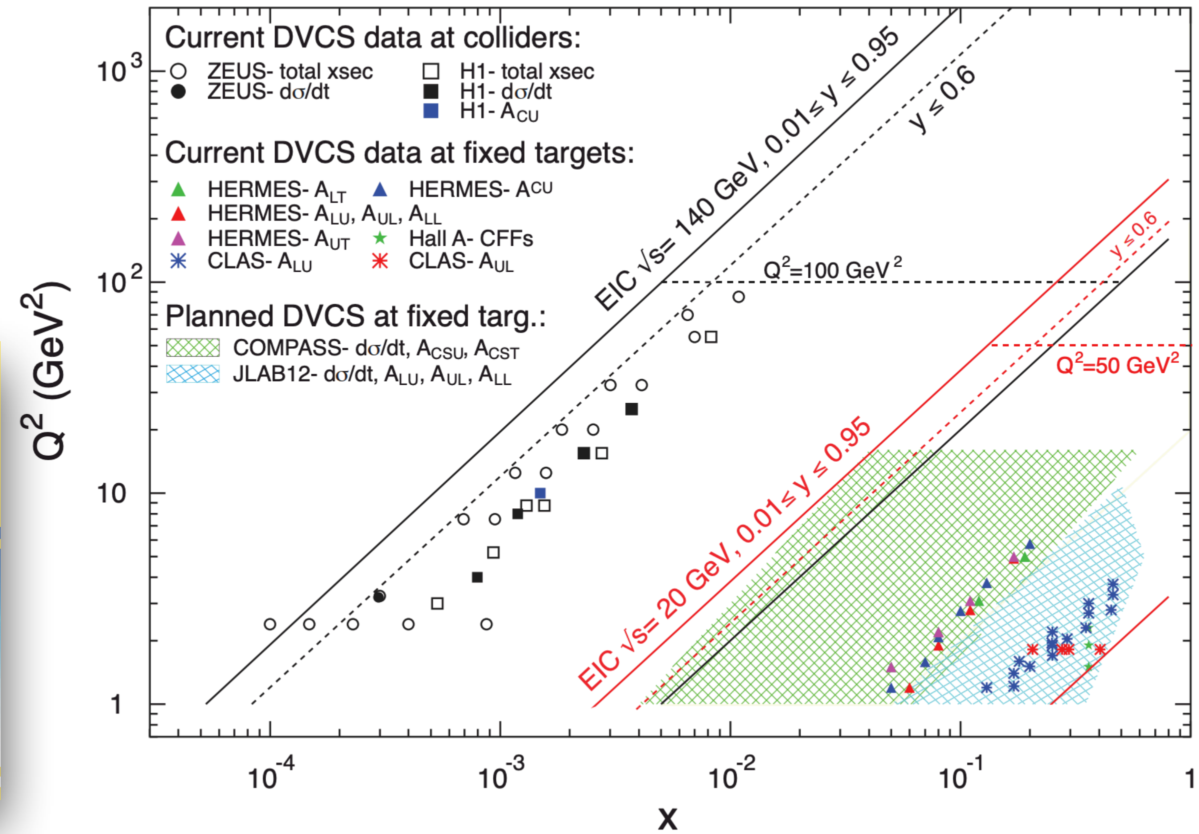
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





U.S. DEPARTMENT OF ENERGY | Office of Science

PHENOMENOLOGY · THEORY · LATTICE QCD

QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number:
DE-SC0023646

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

Advances of lattice QCD are timely

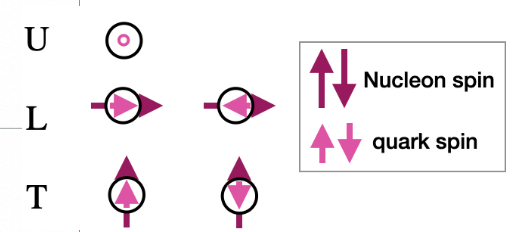
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) **Twist-3** $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

★ Twist-2: probabilistic densities - a wealth of information exists

★ Twist-3: poorly known, but very important:
- as sizeable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

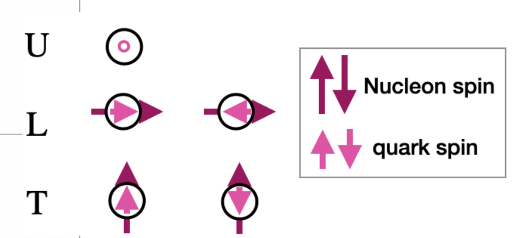
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) **Twist-3** $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

★ Twist-2: probabilistic densities - a wealth of information exists

★ Twist-3: poorly known, but very important:
- as sizeable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

While twist-3 $f_i^{(1)}$ share some similarities with twist-2 $f_i^{(0)}$ in their extraction, there are several challenges both experimentally and theoretically

GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of PDFs/GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

Access of PDFs/GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

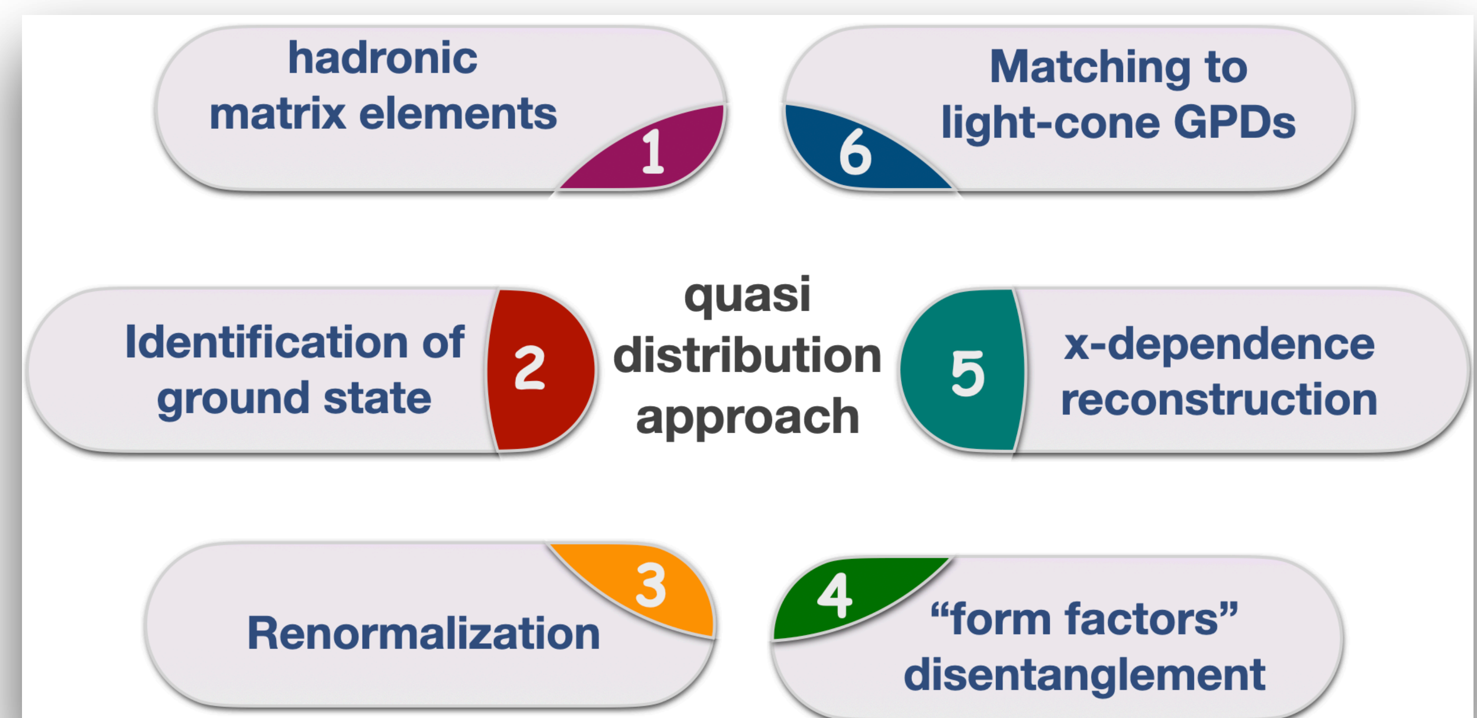
$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

Accessing -t dependence:
Computationally intensive

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$



Access of PDFs/GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

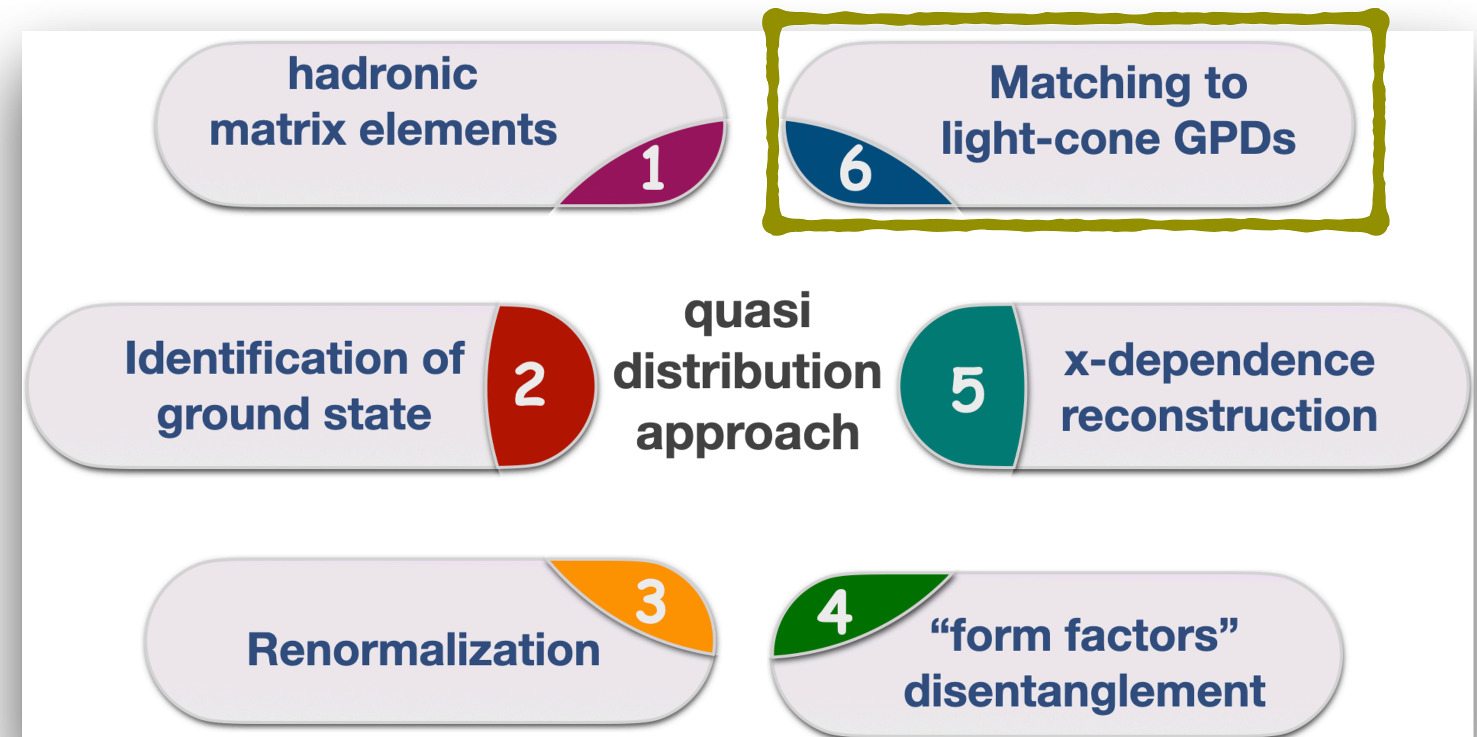
$$\tilde{q}^{\mathrm{GPD}}_{\Gamma}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Accessing -t dependence:
Computationally intensive

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

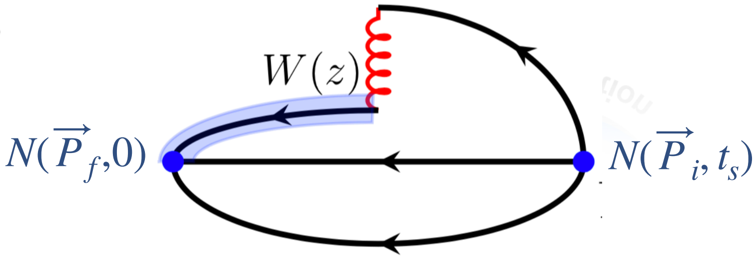


Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4



★ Calculation of connected diagram

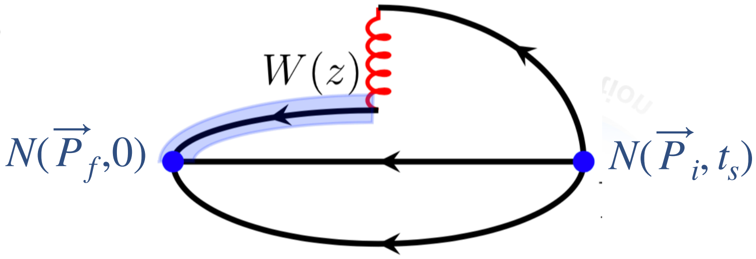
P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
± 0.83	$(0,0,0)$	0	2	194	8	3104
± 1.25	$(0,0,0)$	0	2	731	16	23392
± 1.67	$(0,0,0)$	0	2	1644	64	210432
± 0.83	$(\pm 2,0,0)$	0.69	8	67	8	4288
± 1.25	$(\pm 2,0,0)$	0.69	8	249	8	15936
± 1.67	$(\pm 2,0,0)$	0.69	8	294	32	75264
± 1.25	$(\pm 2,\pm 2,0)$	1.38	16	224	8	28672
± 1.25	$(\pm 4,0,0)$	2.76	8	329	32	84224

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4



★ Calculation of connected diagram

P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
±0.83	(0, 0, 0)	0	2	194	8	3104
±1.25	(0, 0, 0)	0	2	731	16	23392
±1.67	(0, 0, 0)	0	2	1644	64	210432
±0.83	(±2, 0, 0)	0.69	8	67	8	4288
±1.25	(±2, 0, 0)	0.69	8	249	8	15936
±1.67	(±2, 0, 0)	0.69	8	294	32	75264
±1.25	(±2, ±2, 0)	1.38	16	224	8	28672
±1.25	(±4, 0, 0)	2.76	8	329	32	84224

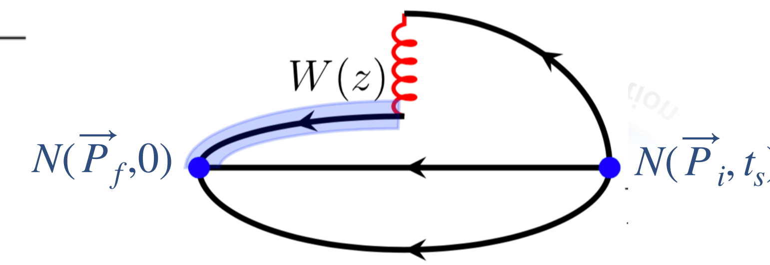
Symmetric frame computationally expensive

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

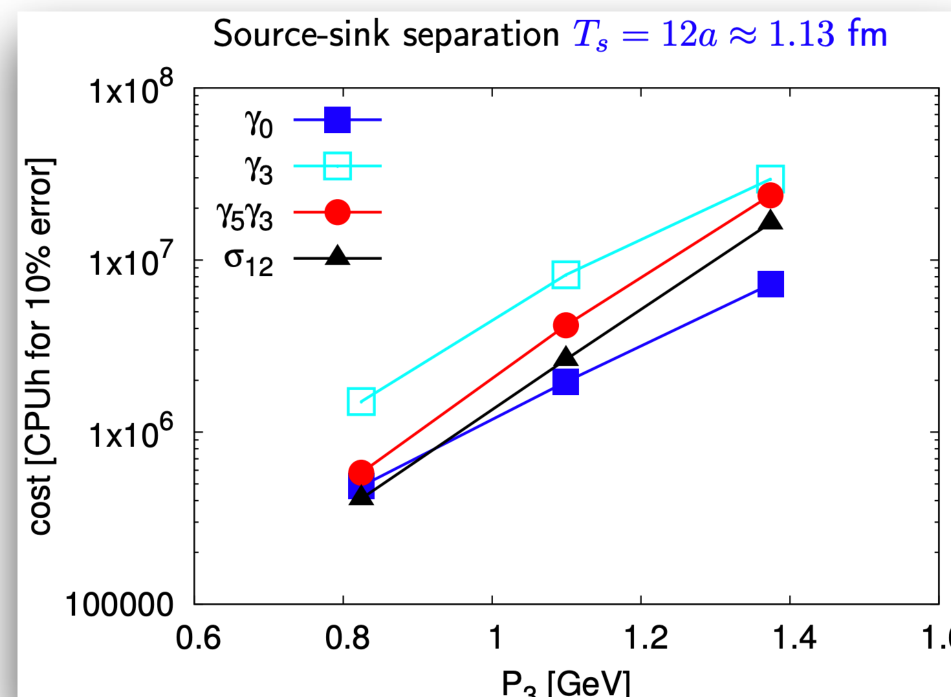
[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4



★ Calculation of connected diagram

P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
±0.83	(0,0,0)	0	2	194	8	3104
±1.25	(0,0,0)	0	2	731	16	23392
±1.67	(0,0,0)	0	2	1644	64	210432
±0.83	(±2,0,0)	0.69	8	67	8	4288
±1.25	(±2,0,0)	0.69	8	249	8	15936
±1.67	(±2,0,0)	0.69	8	294	32	75264
±1.25	(±2,±2,0)	1.38	16	224	8	28672
±1.25	(±4,0,0)	2.76	8	329	32	84224



Symmetric frame computationally expensive

Suppressing gauge noise and reliably extracting the ground state comes at a significant computational cost

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned}
F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[& P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3) \\
&+ \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3) \\
&+ \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)
\end{aligned}$$

Theoretical setup

★ **Correlation functions in coordinate space**

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ **Parametrization of coordinate-space correlation functions**

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar\psi(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec z_\perp=\vec 0_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned} F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar u(p_f,\lambda')\Big[& P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde H}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde E}(x,\xi,t;P^3) \\ & + \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde E+\widetilde G_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde H+\widetilde G_2}(x,\xi,t;P^3) \\ & + \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde G_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde G_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda) \end{aligned}$$

★ Twist-3 contributions to helicity GPDs: $\Gamma = \gamma^j\gamma_5,\ j=1,2$

Theoretical setup

★ Correlation functions in coordinate space

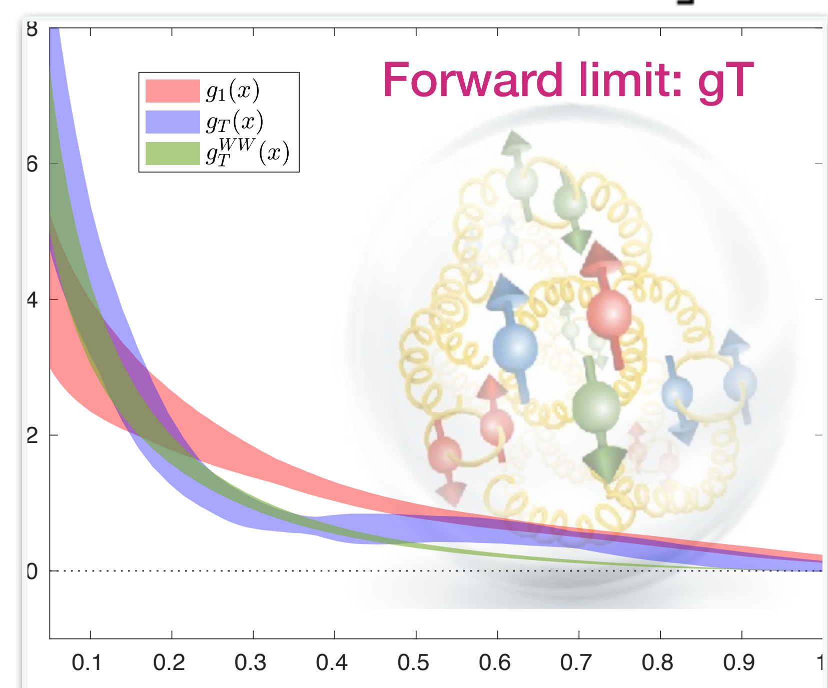
$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar\psi(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned} F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar u(p_f,\lambda')\Big[& P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde H}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde E}(x,\xi,t;P^3) \\ & + \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde E+\widetilde G_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde H+\widetilde G_2}(x,\xi,t;P^3) \\ & + \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde G_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde G_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda) \end{aligned}$$

★ Twist-3 contributions to helicity GPDs: $\Gamma = \gamma^j\gamma_5,\ j=1,2$



[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

Decomposition

★ Requirement: four independent matrix elements

$P_3\,[\mathrm{GeV}]$	$\vec{q}\,[\frac{2\pi}{L}]$	$-t\,[\mathrm{GeV}^2]$
± 0.83	$(0,0,0)$	0
± 1.25	$(0,0,0)$	0
± 1.67	$(0,0,0)$	0
± 0.83	$(\pm 2,0,0)$	0.69
± 1.25	$(\pm 2,0,0)$	0.69
± 1.67	$(\pm 2,0,0)$	0.69
± 1.25	$(\pm 2,\pm 2,0)$	1.38
± 1.25	$(\pm 4,0,0)$	2.76

★ Average kinematically equivalent matrix elements

$$\Pi^1(\Gamma_0) = C\left(-F_{\widetilde{H}+\widetilde{G}_2}\frac{P_3\Delta_y}{4m^2} - F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_y(E+m)}{2m^2}\right),$$

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2P_3}\right)$$

$$\Pi^1(\Gamma_2) = i\,C\left(-F_{\widetilde{H}+\widetilde{G}_2}\frac{\Delta_x\Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x\Delta_y(E+m)}{8m^3} - F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x\Delta_y(E+m)}{4m^2P_3}\right),$$

$$\Pi^1(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_x(E+m)}{2m^2P_3}\right),$$

$$\Pi^2(\Gamma_0) = C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{P_3\Delta_x}{4m^2} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x(E+m)}{2m^2}\right),$$

$$\Pi^2(\Gamma_1) = i\,C\left(-F_{\widetilde{H}+\widetilde{G}_2}\frac{\Delta_x\Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x\Delta_y(E+m)}{8m^3} - F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x\Delta_y(E+m)}{4m^2P_3}\right),$$

$$\Pi^2(\Gamma_2) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_x^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_y^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x^2(E+m)}{4m^2P_3}\right)$$

$$\Pi^2(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_y(E+m)}{2m^2P_3}\right),$$

Consistency Checks

★ Sum Rules (generalization of Burkhardt-Cottingham)

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

★ Sum Rules (generalization of Efremov-Leader-Teryaev)

[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

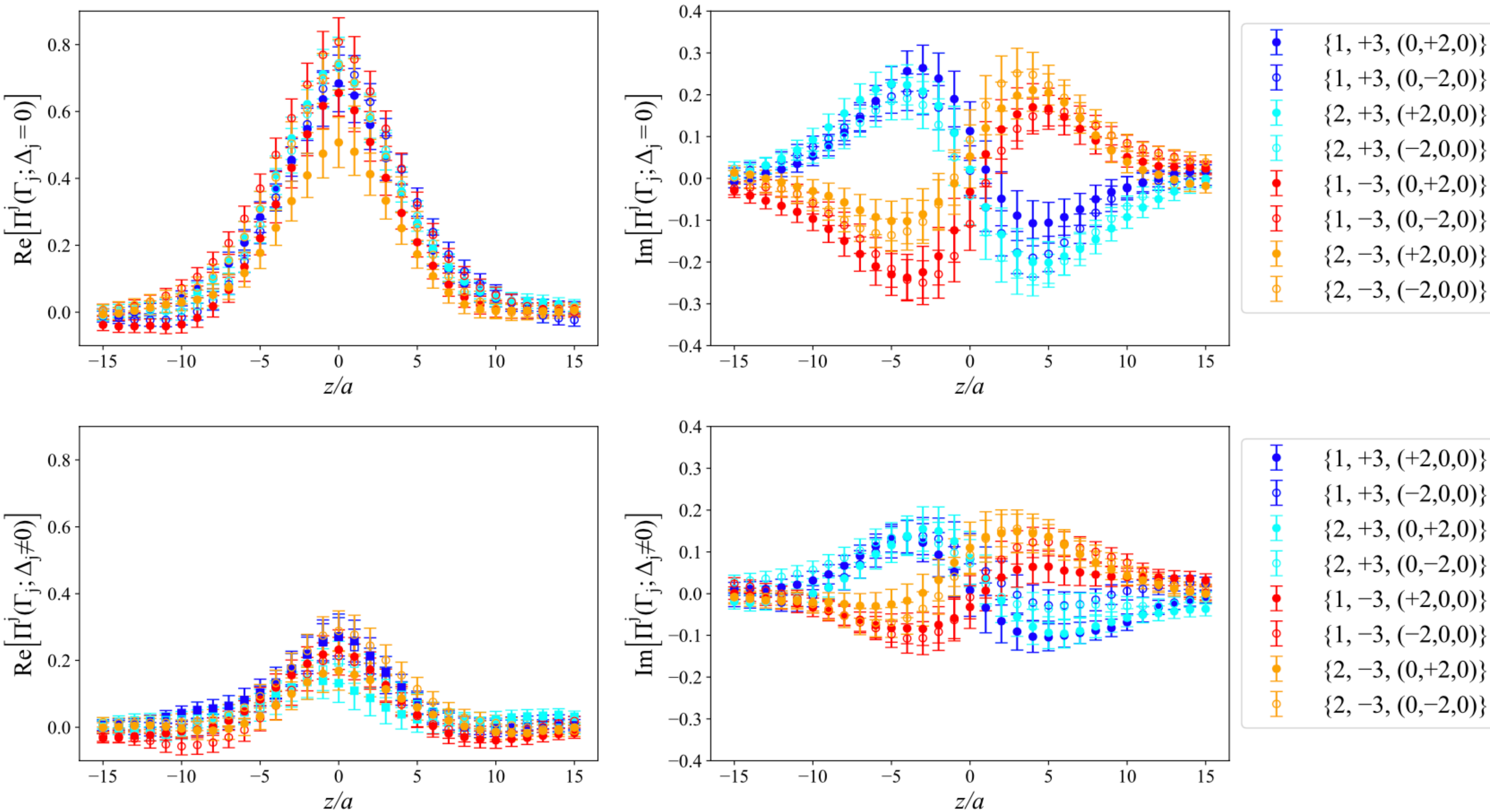
$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = \frac{\xi}{4} G_E \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4} G_E(t)$$

G_E : electric FF

Lattice Results - Matrix Elements

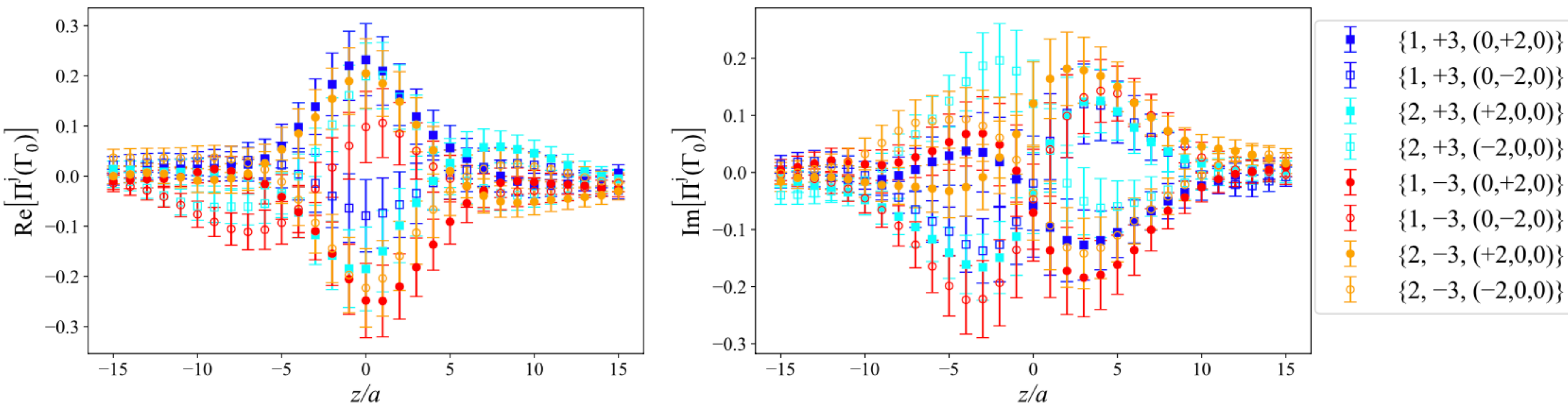
★ Bare matrix elements

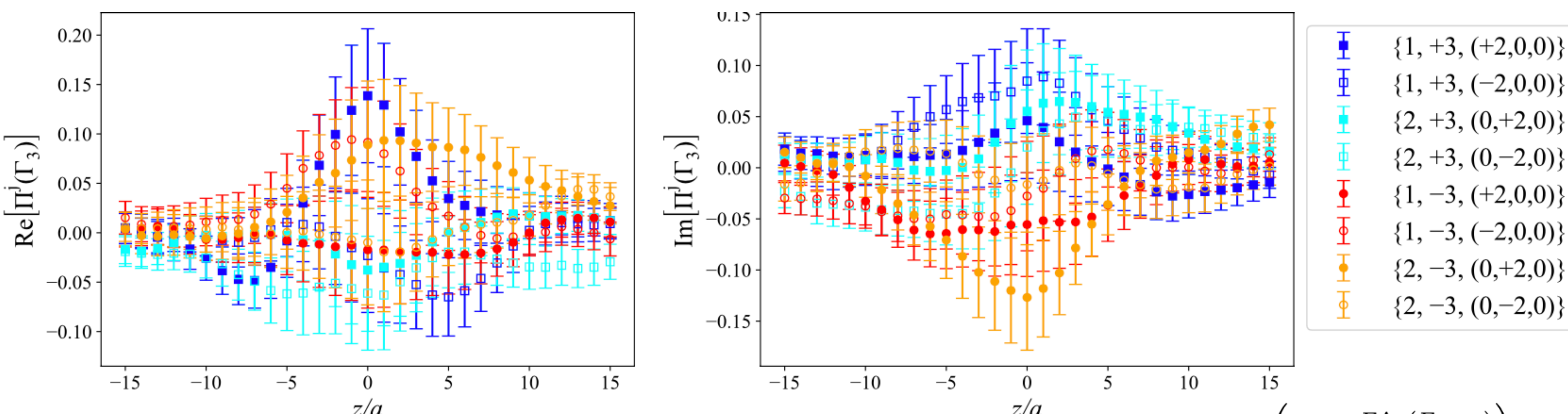
$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\,\frac{\left(4m(E+m)+\Delta_y^2\right)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\,\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\,\frac{\text{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2 P_3}\right)$$



Lattice Results - Matrix Elements

★ Bare matrix elements

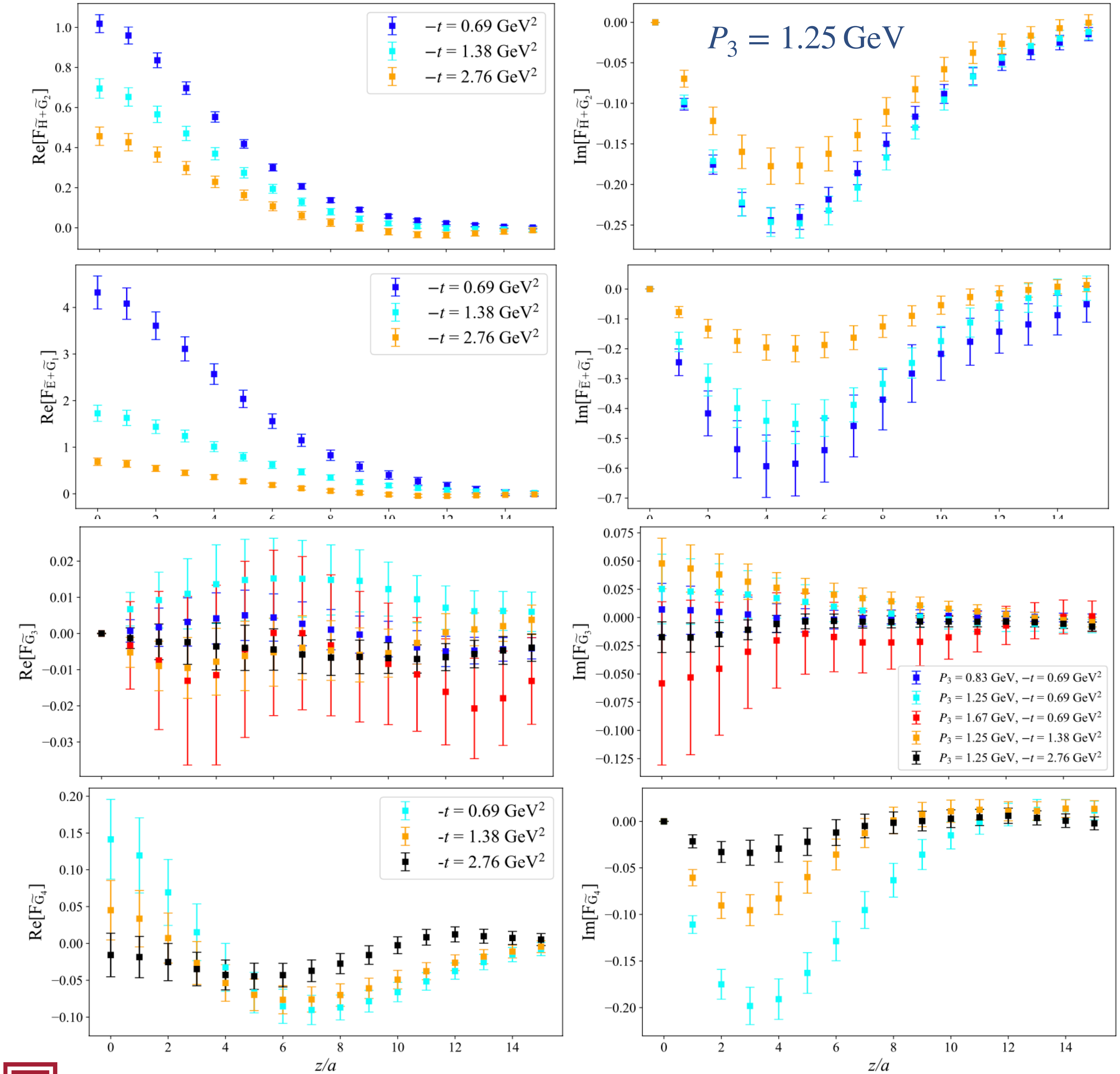




$$\Pi^1(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_x(E+m)}{2m^2P_3}\right)$$

★ Suppressed signal compared to γ_0, γ_3 operators

Lattice Results - quasi-GPDs



$$\int dx\, x\, \widetilde{G}_3 = \frac{\xi}{4} G_E(t)$$

Indeed, numerically found to be zero within uncertainties at ξ=0

Reconstruction of x-dependence & matching

★ quasi-GPDs transformed to momentum space using Backus Gilbert
[G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)]

★ Matching formalism to 1 loop accuracy level

$$F_X^{\rm MMS}(x,t,P_3,\mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C_{\gamma_j\gamma_5}^{\rm MMS,\overline{MS}}\left(\frac{x}{y},\frac{\mu}{yP_3}\right) G_X^{\overline{\rm MS}}(y,t,\mu) \;+\; \mathcal{O}\left(\frac{m^2}{P_3^2},\frac{t}{P_3^2},\frac{\Lambda_{\rm QCD}^2}{x^2P_3^2}\right)$$

★ Operator dependent kernel

PHYSICAL REVIEW D **102,** 034005 (2020)

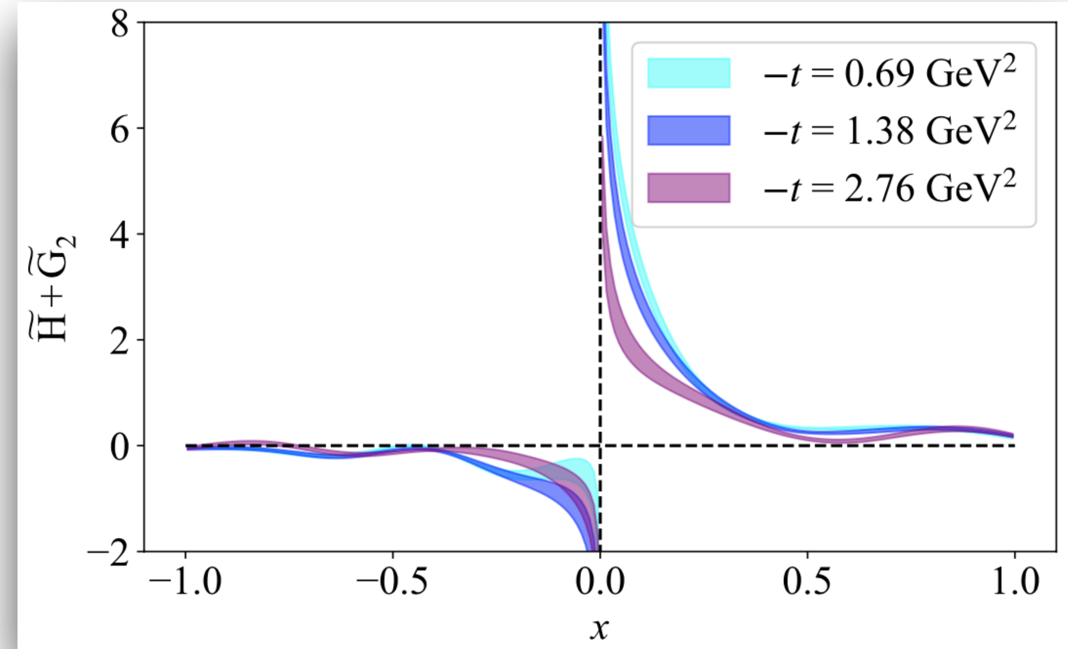
One-loop matching for the twist-3 parton distribution $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

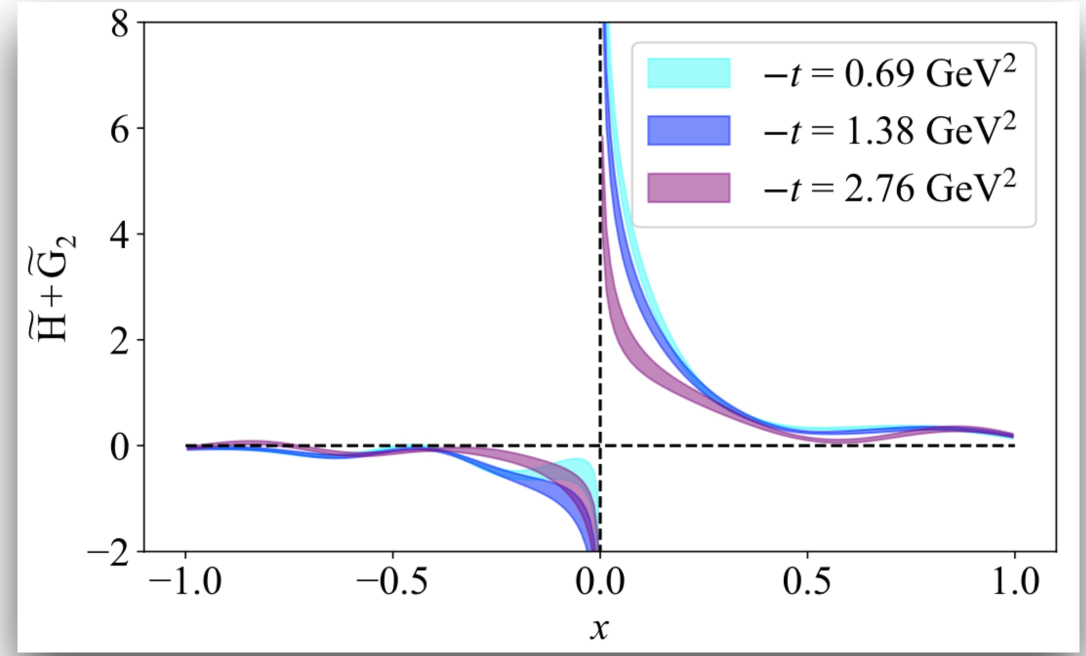
$$C^{(1)}_{\rm MMS}\left(\xi,\frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0 \\ \delta(\xi) \\ 0 \end{cases} + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{\xi}{1-\xi}+\frac{3}{2\xi}\right]_+ & \xi>1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2}+\frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi}-\frac{\xi}{1-\xi}+\frac{3}{2(1-\xi)}\right]_+ & \xi<0\,, \end{cases}$$

★ Matching does not consider mixing with q-g-q correlators
[V. Braun et al., JHEP 05 (2021) 086]

Lattice Results - light-cone GPDs

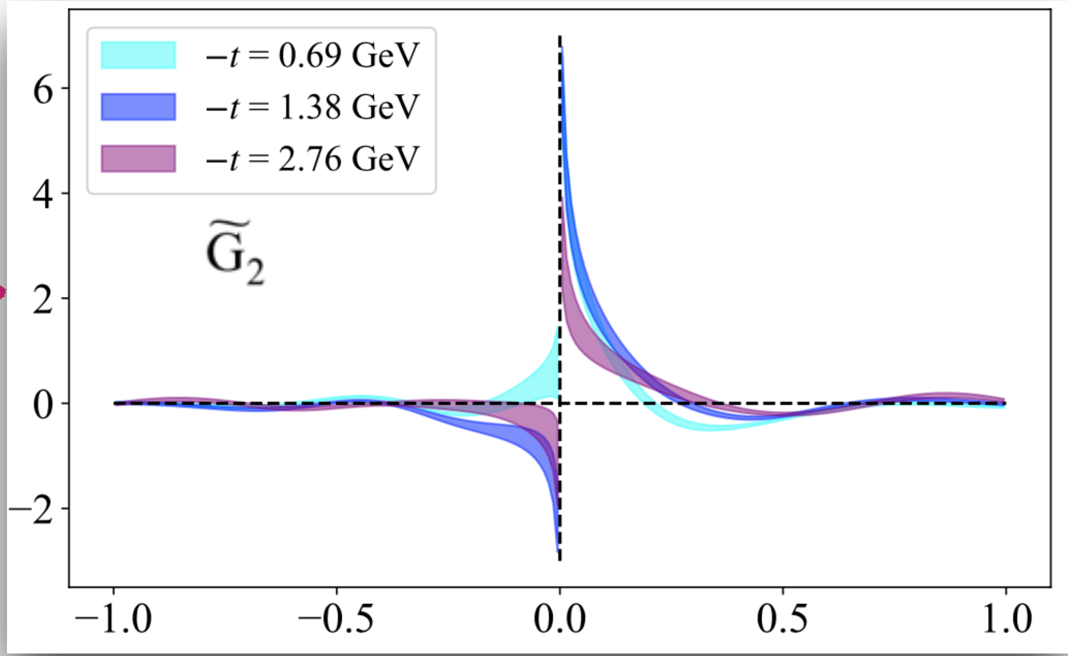


Lattice Results - light-cone GPDs

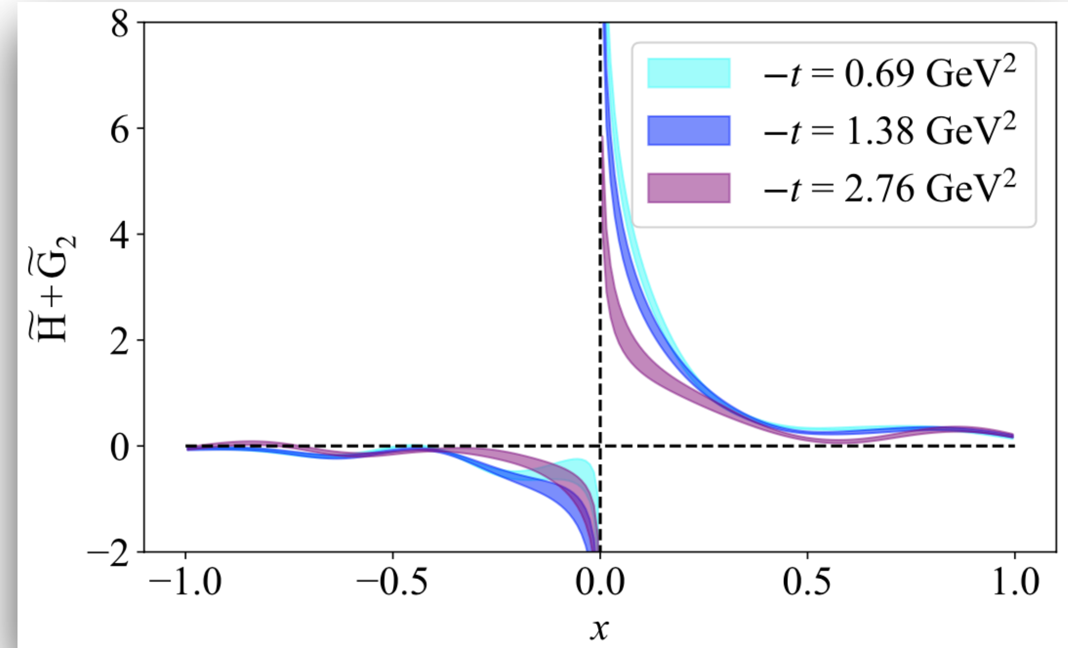


Isolating $\widetilde{G}_2$

using $\widetilde{H}$

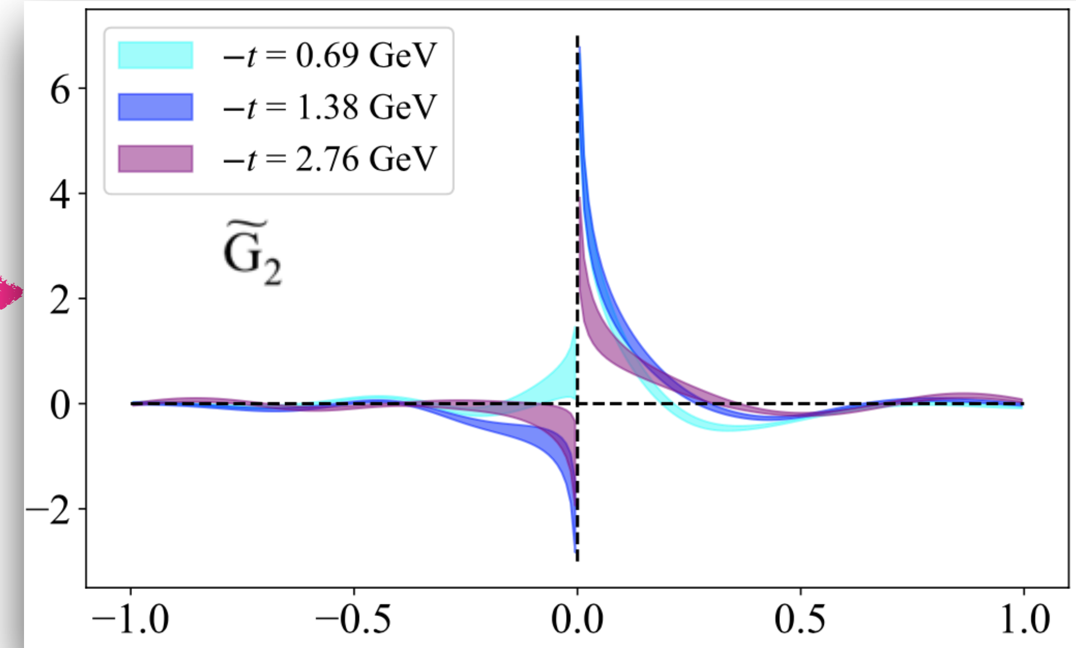


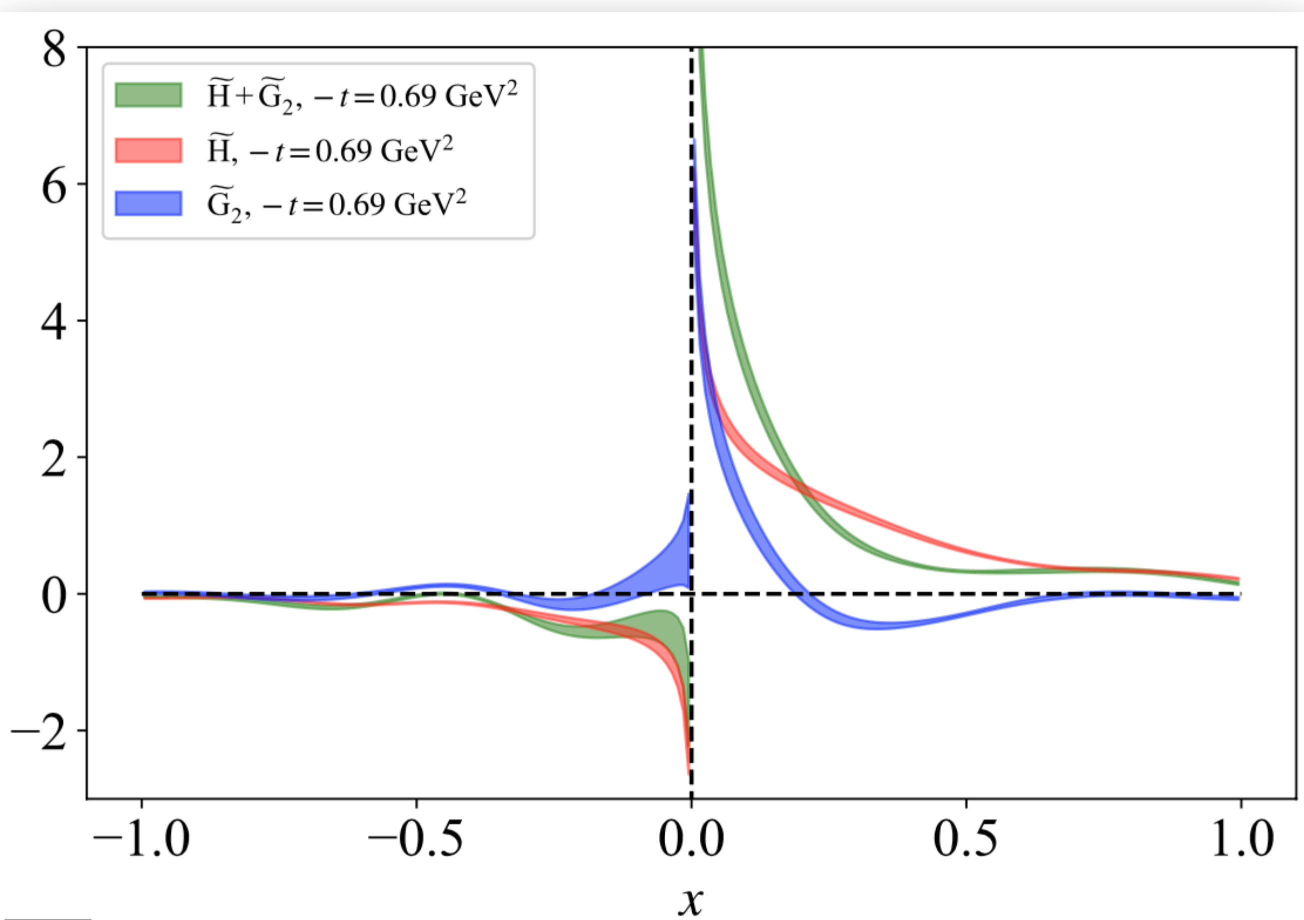
Lattice Results - light-cone GPDs



Isolating $\widetilde{G}_2$

using $\widetilde{H}$





Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

- ★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness
- ★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

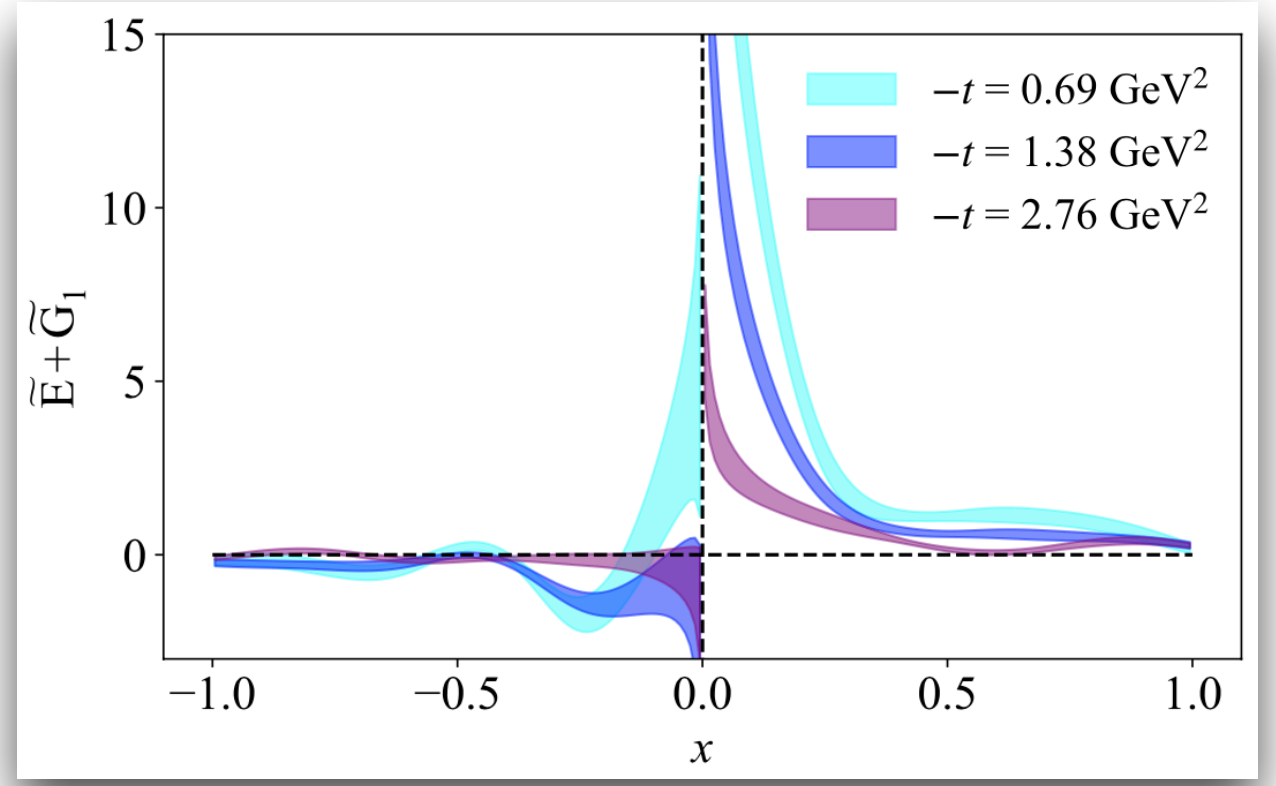
$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x, \xi, t; P^3)$$

Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :



★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

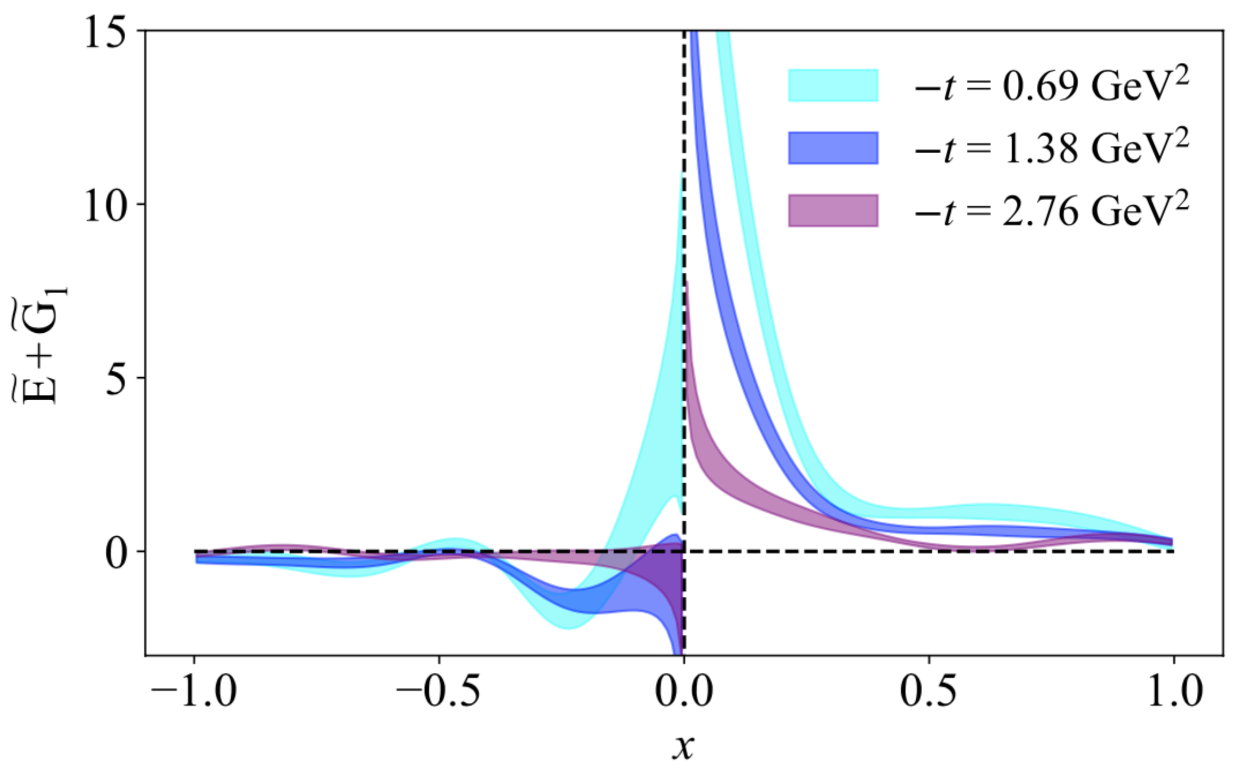
$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

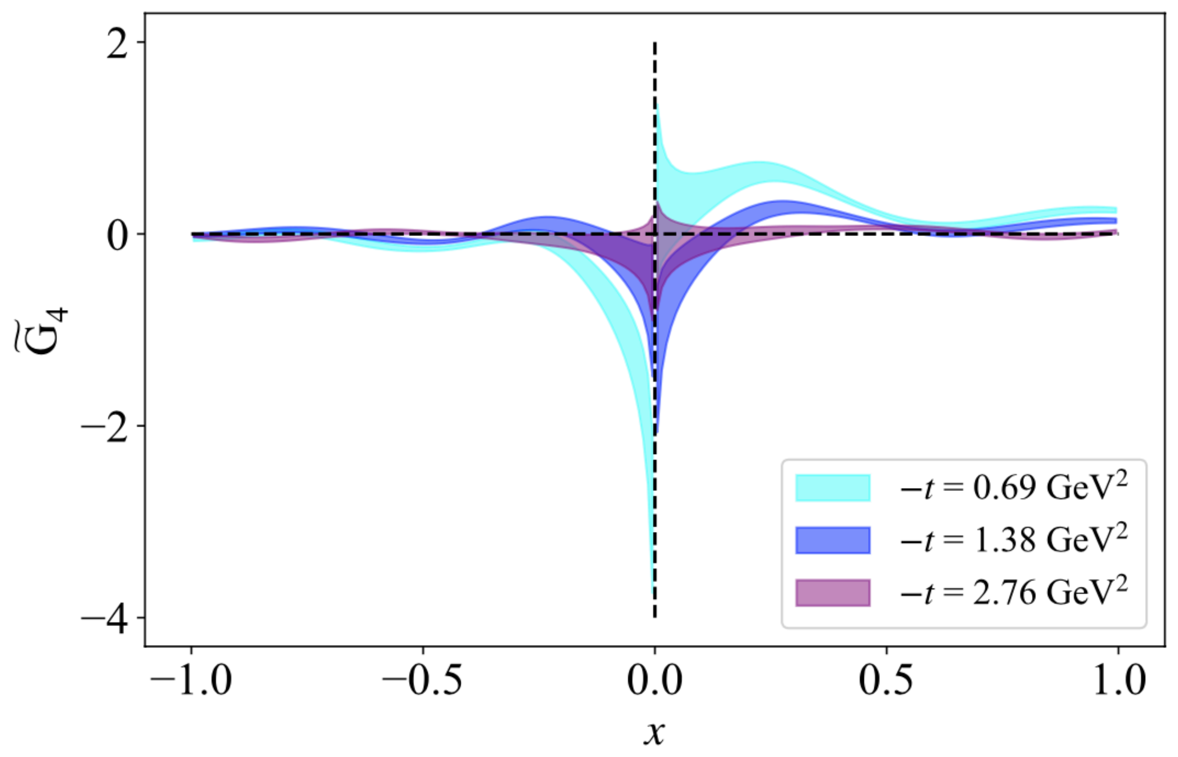
Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :





★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

Consistency checks

★ Norms satisfied

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

Consistency checks

★ Norms satisfied

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

Consistency checks show encouraging results

Consistency checks

★ Norms satisfied

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

Consistency checks show encouraging results

★ Alternative kinematic setup can be utilized

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

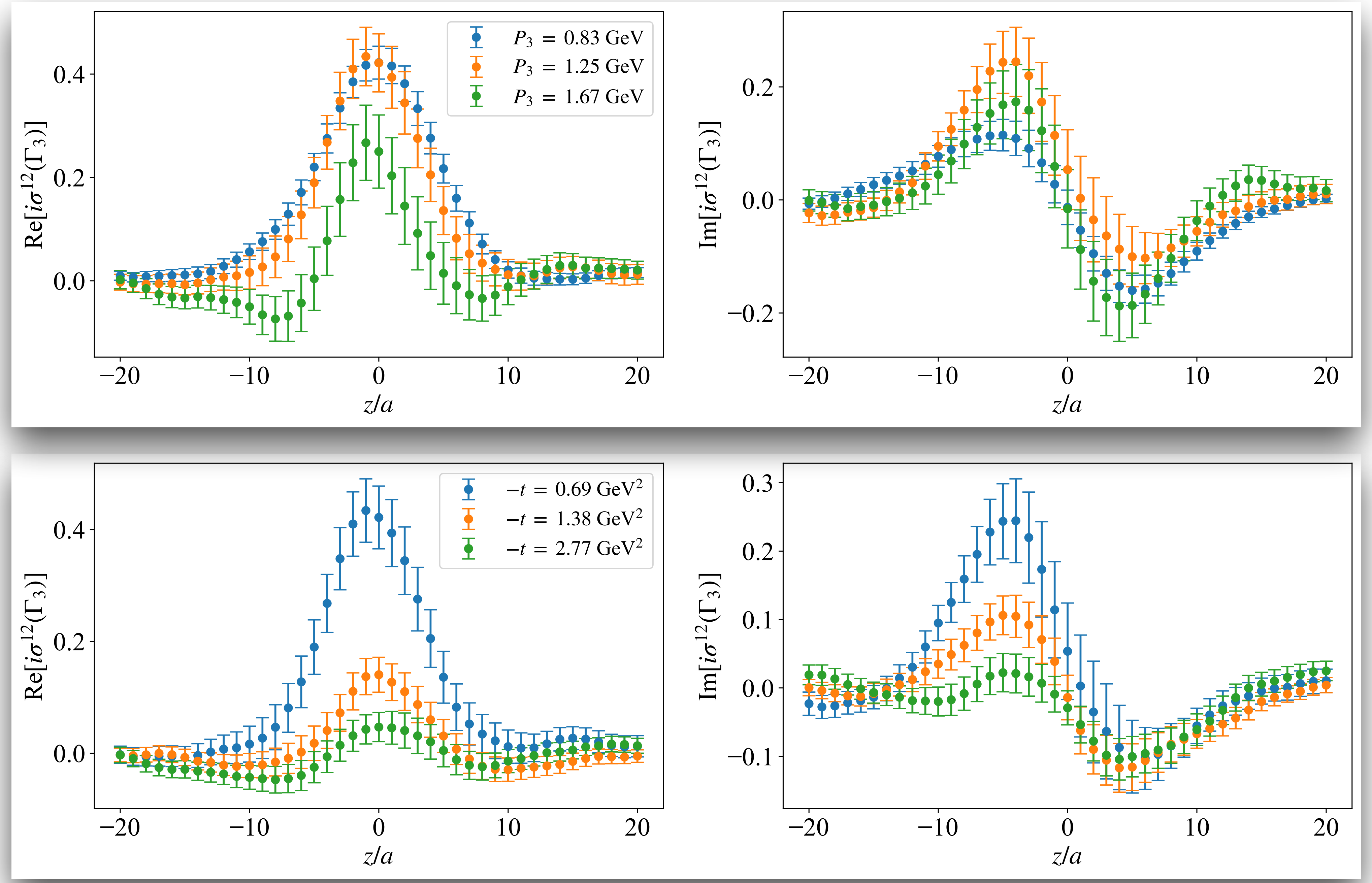
$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\Big(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\Big) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\Big(z_3 P_0 P_3^2 - z_3 P_0^3\Big) A_1$$

Extension to twist-3 tensor GPDs

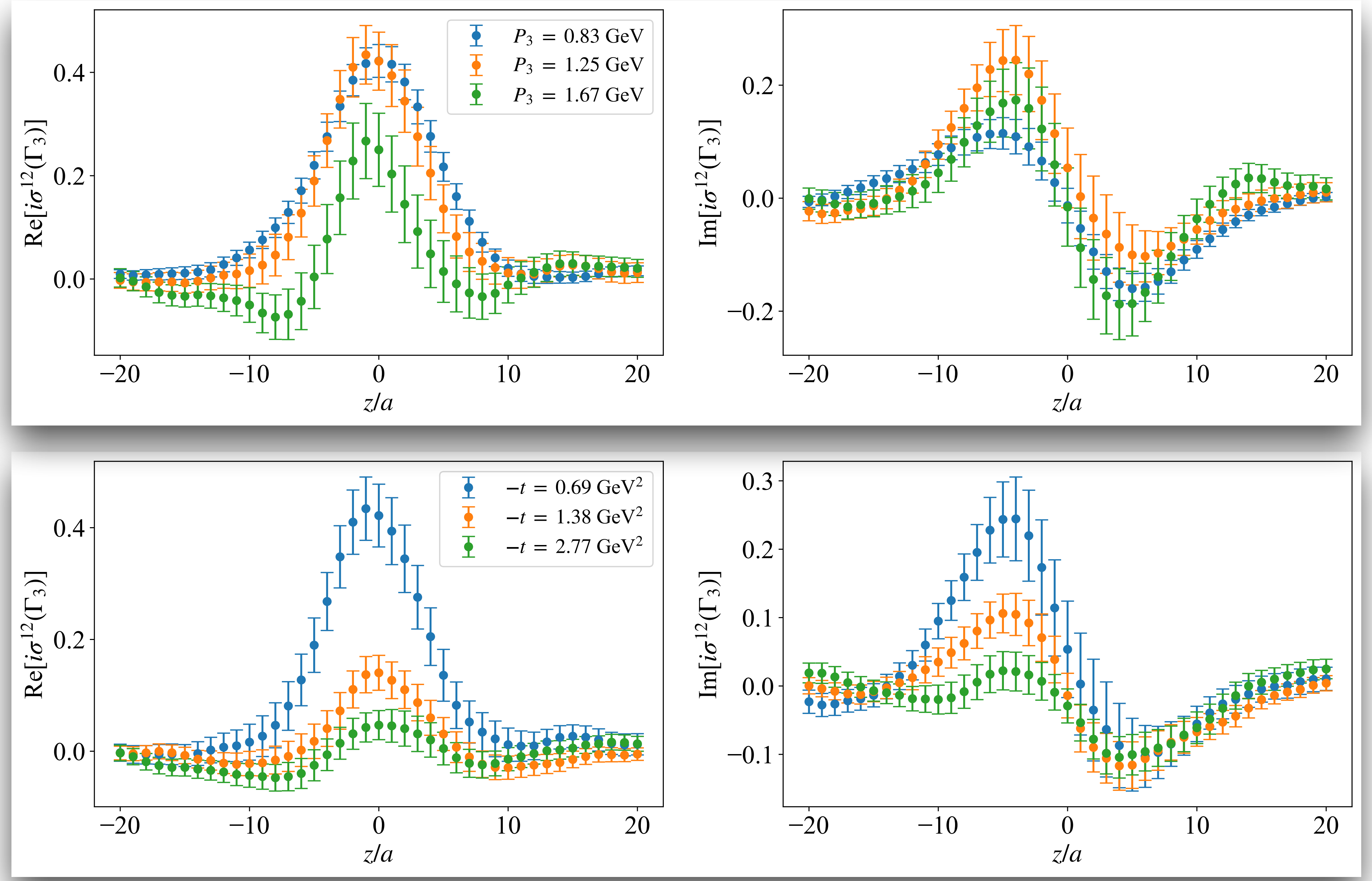
Extension to twist-3 tensor GPDs



Extension to twist-3 tensor GPDs

★ **Parametrization** [Meissner et al., *JHEP* 08 (2009) 056]

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p')\left(\gamma^+\gamma_5\,\widetilde{H}_2' + \frac{P^+\gamma_5}{M}\,\widetilde{E}_2'\right)u(p)$$



Summary

★ LaMET formalism is applicable beyond leading twist

★ We address computationally expensive calculations
GPDs with signal comparable to PDFs

★ Several improvements needed
- mixing with quark-gluon-quark correlator

★ Synergy with phenomenology is an exciting prospect!

QUARK-GLUON TOMOGRAPHY COLLABORATION
Award Number:
DE-SC0023646

Thank you

DOE Early Career Award (NP)
Grant No. DE-SC0020405

Backup slides

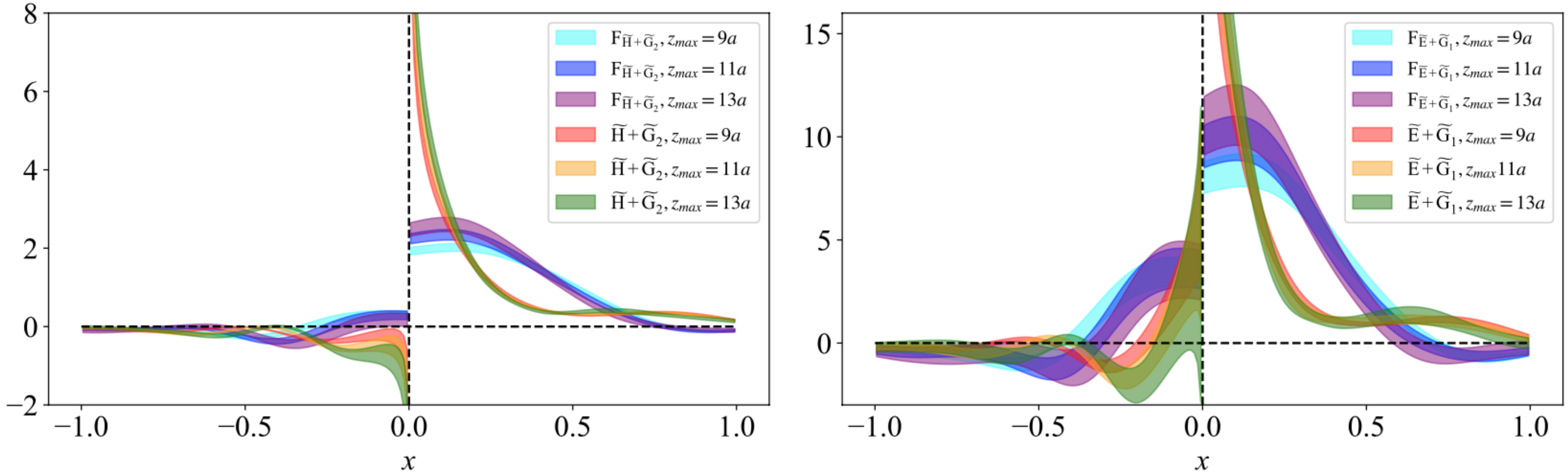


FIG. 10. $z_{\max}$ dependence of $F_{\widetilde{H}+\widetilde{G}_2}$ and $\widetilde{H}+\widetilde{G}_2$ (left), as well as $F_{\widetilde{E}+\widetilde{G}_1}$ and $\widetilde{E}+\widetilde{G}_1$ (right) at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

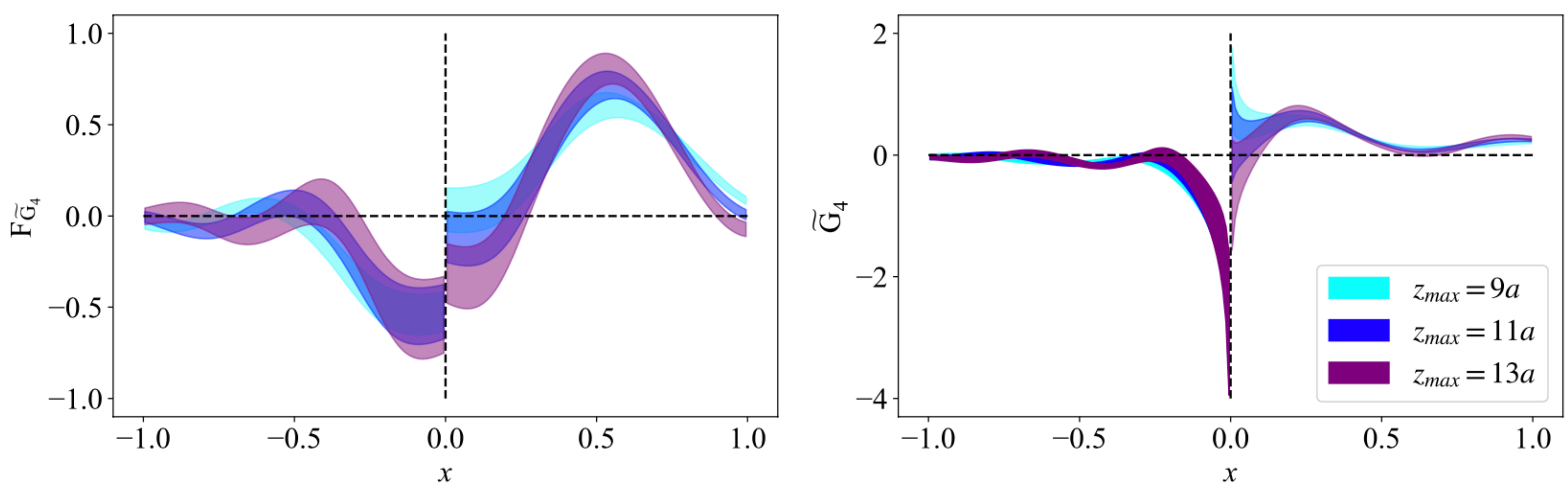


FIG. 11. $z_{\max}$ dependence of $F_{\widetilde{G}_4}$ and $\widetilde{G}_4$ at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in $\overline{\text{MS}}$ scheme at a scale of 2 GeV.