A qubit regularization of asymptotic freedom without fine tuning

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work with **S. Maiti & D. Banerjee**, Saha Institute and **S. Chandrasekharan,** Duke University [arXiv:2307.06117]

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Outline

- **Asymptotically free QFTs & Wilson's RG**
- **Qubit regularization in the Euclidean spacetime** [Singh, Chandrasekharan, Phys. Rev. D 100, 054505 (2019)]
- **Asymptotic freedom via decoupled fix point** [Bhattacharya et al. PRL 126 (2021), 172001]
- **Rich universal behaviour recovered from UV to IR**
- **Elegant solution for the XY model without fine tuning**

[Maiti, Banerjee, Chandrasekharan, MKM, 2307.06117]

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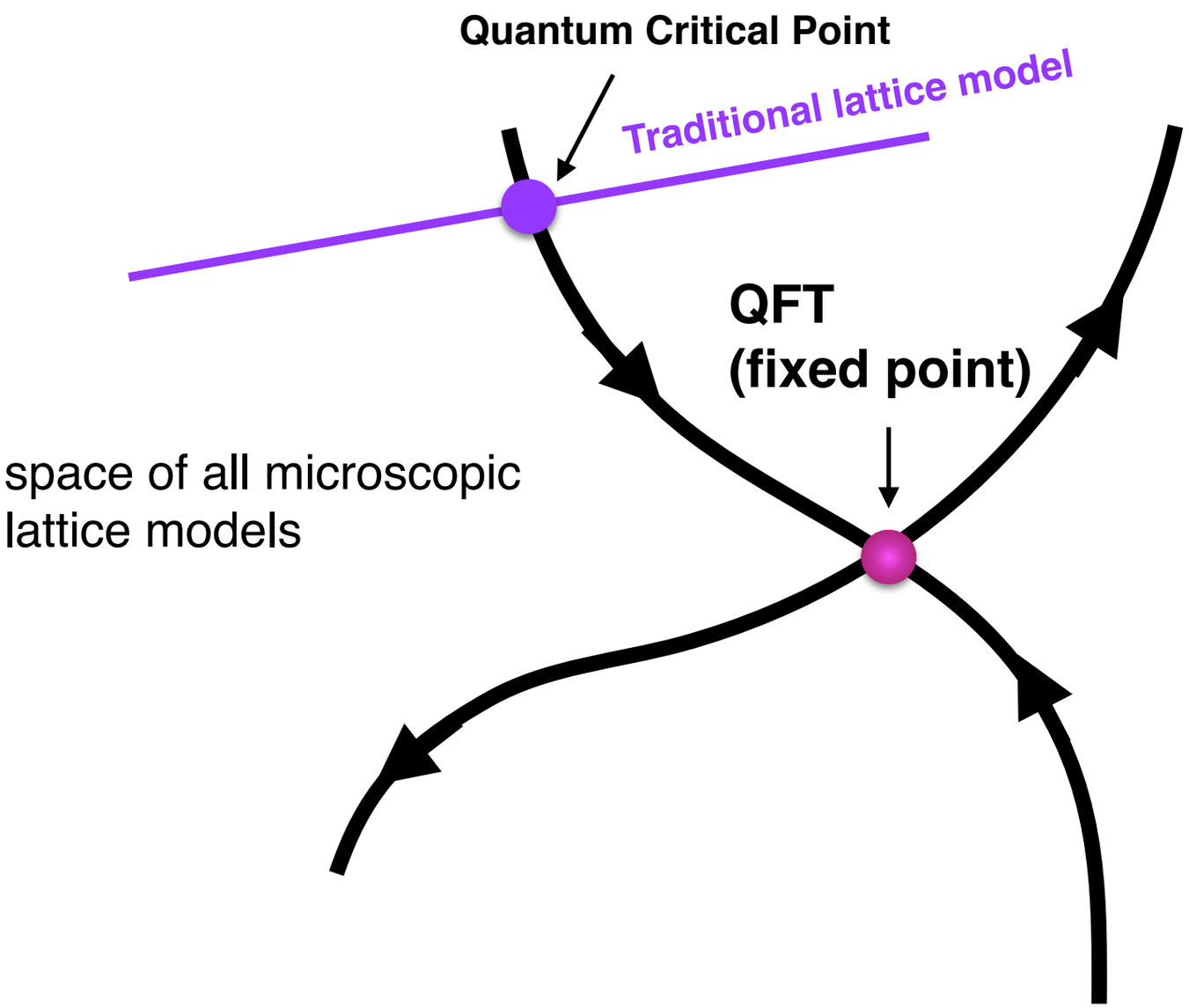
[Maiti, Banerjee, Chandrasekharan, MKM, 2307.06117]

Sandip Maiti

Debasish Banerjee

Shailesh Chandrasekharan

Magic of Wilson's Renormalization Group



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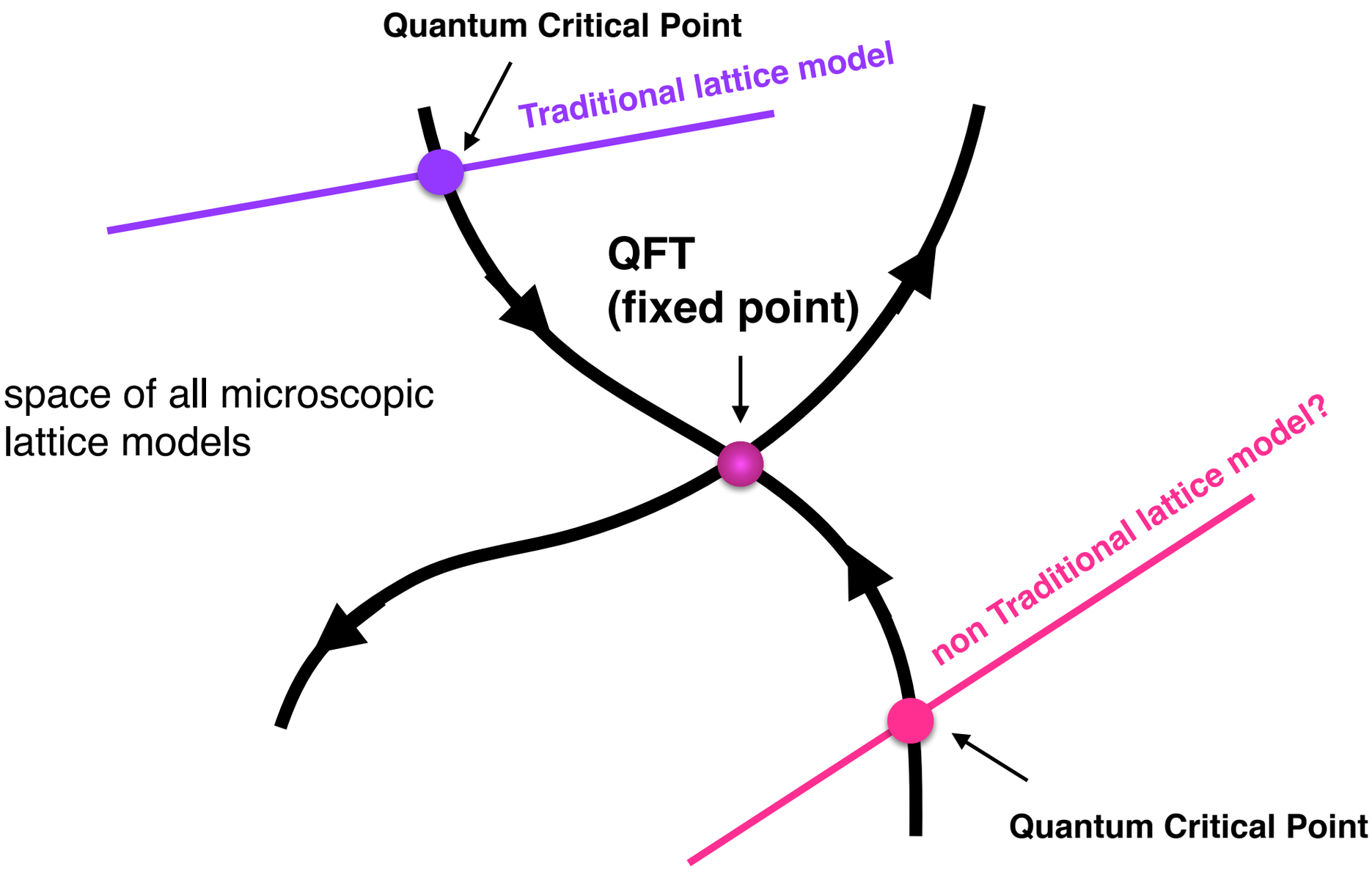

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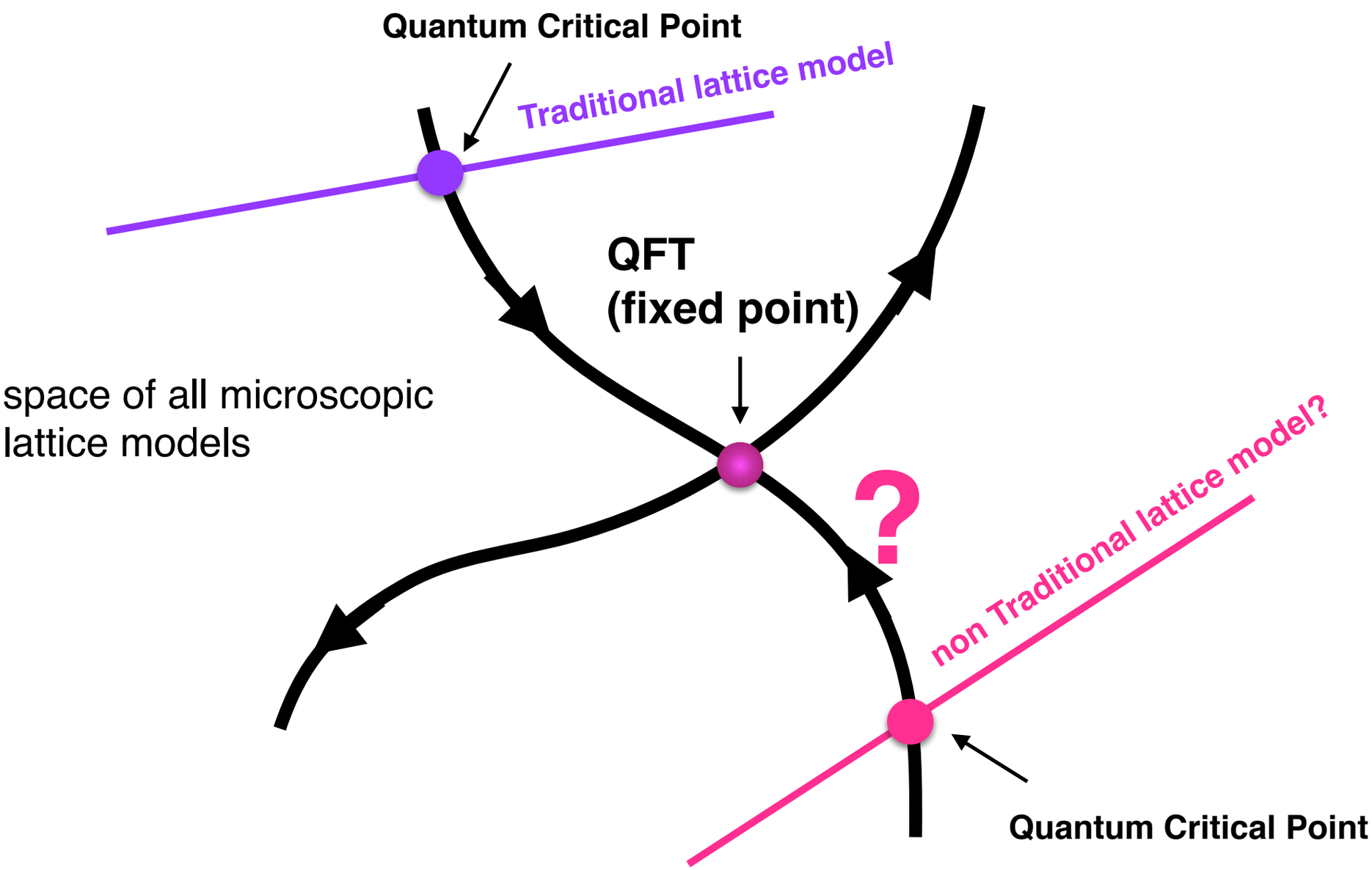


- **e.g. qubit regularized models** [see also talk by H. Liu@LATT23 on Monday 16.20]

Advantages of Qubit Regularization

[Wiese, Nucl. Phys. B Proc. Suppl. 73, 146 (1999)]

[Brower, Chandrasekharan, Riederer, Wiese, Nucl. Phys. B 693, 149 (2004)]

[Banerjee, Bögli, Dalmonte, Rico, Stebler, Wiese, Zoller, PRL110, 125303 (2013)]

[Singh, Chandrasekharan, Phys. Rev. D 100, 054505 (2019)]

[Bhattacharya et al. PRL 126 (2021), 172001]

- **Lattice formulation of continuum QFTs with a finite local Hilbert space**

- **In finite lattice volume: Hilbert space finite → suitable for quantum simulations**

- **Continuum QFT emerges as usual:**

  - **continuum limit:** $a \to 0$

  - **thermodynamic limit:** $L \to \infty$

Wilson's Renormalization Group

Regularized models << physical UV length scales (Gaussian Theory!) << physical IR length scale (mass gap)

Universal continuum physics

Non-universal, lattice physics

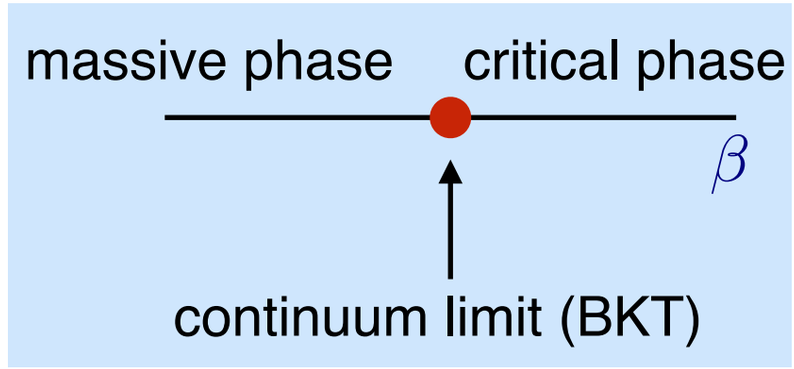
Unlike the traditional (lattice regularization) approach there are no guarantees for a connection between the qubit regularized theory to the Gaussian theory!

Demonstrated in this work for the XY model (qubit regularization in Euclidean spacetime)

XY Model: different regularizations

- **Lattice regularization: traditional (bosonic) XY model:**

$$Z = \prod_i \int_0^{2\pi} d\theta_i \; exp\Big(\beta \sum_{\langle i,j \rangle} cos(\theta_i - \theta_j)\Big)$$



XY Model: different regularizations

[Banerjee, Chandrasekharan Phys. Rev. D 81, 125007 (2010)]

[Singh, Chandrasekharan, Phys. Rev. D 100, 054505 (2019)]

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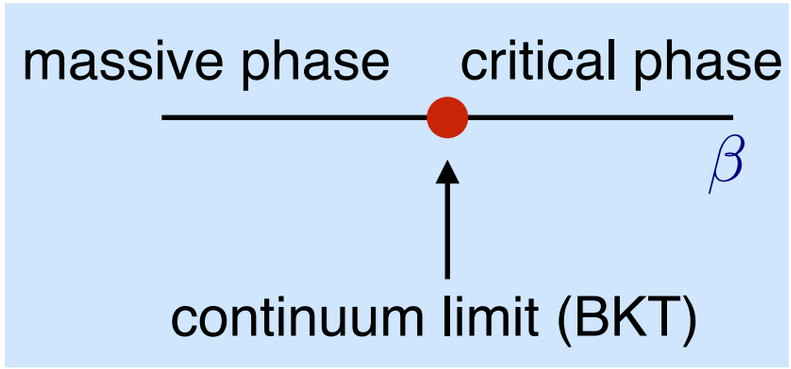

- **Qubit regularization (fermionic) XY model:**

$$Z = \int [d\bar{\psi} d\psi] \; [d\bar{\chi} d\chi] \; e^{\lambda \sum_i \; \bar{\psi}_i \psi_i \bar{\chi}_i \chi_i} \times e^{\sum_{\langle ij \rangle} \left(\bar{\psi}_i \psi_i \bar{\psi}_j \psi_j + \bar{\chi}_i \chi_i \bar{\chi}_j \chi_j\right)}$$

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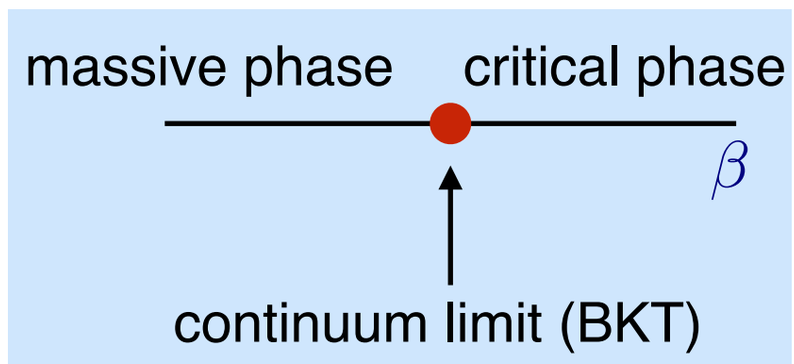

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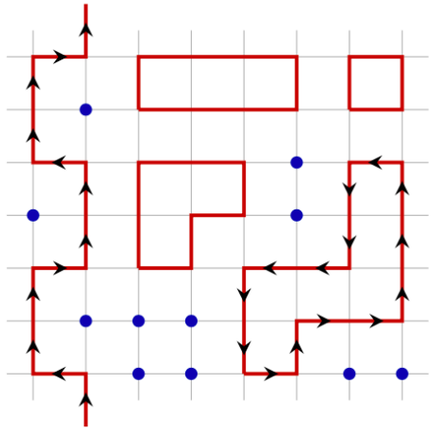
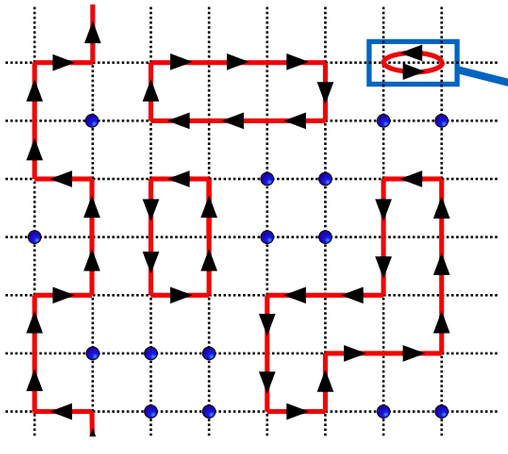
λ - weight of an instanton (inter-layer dimer)

Loop configurations in the O(3) model



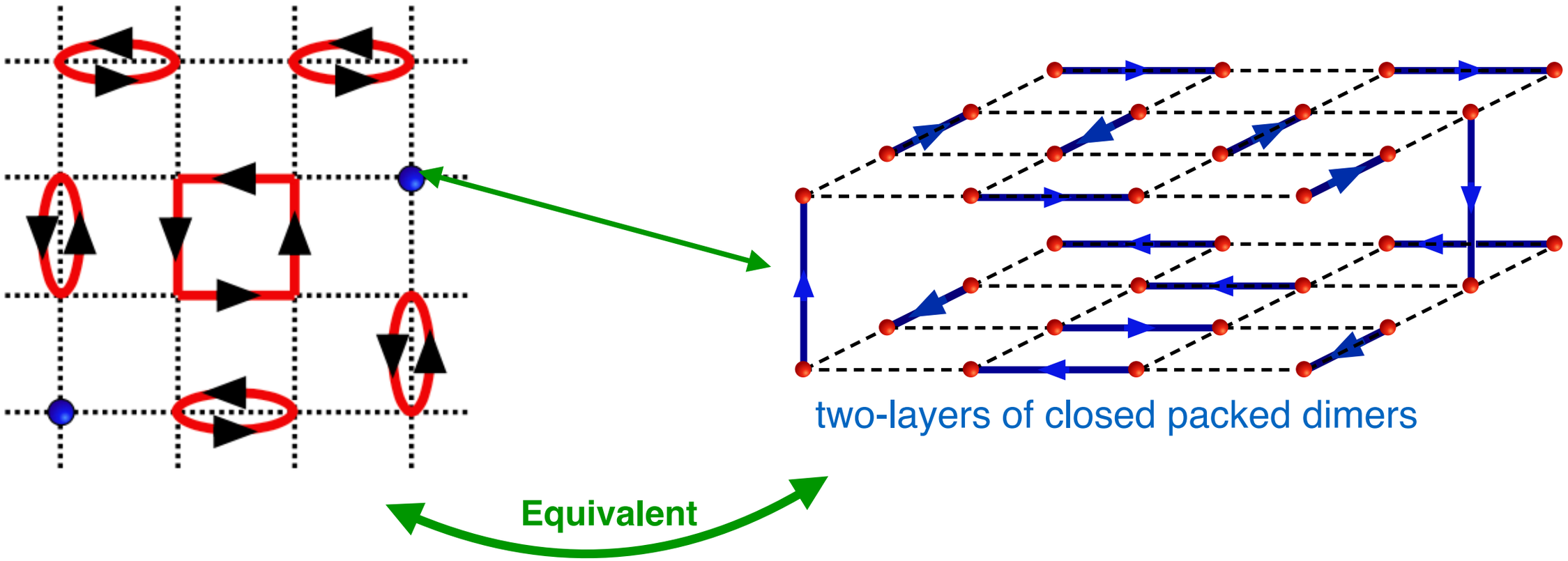
Loop configurations in the O(2) model



Closed loops on a single bond are now allowed

Qubit regularization of asymptotic freedom: XY model

[Maiti, Banerjee, Chandrasekharan, MKM, 2307.06117]



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- **Simulated using a variant of worm algorithm**
  [Adams, Chandrasekharan, Nucl. Phys. B 662, 220 (2003)]

- **The two layers decouple in the limit:** $L \to \infty$, $\lambda \to 0$

- **Asymptotic freedom of BKT emerges in the limit:** $\lambda \to 0, L \to \infty$ **without fine-tuning!**

Asymptotically free theories and SSF

[Caracciolo, et.al., PRL 75, 1891 (1995)]

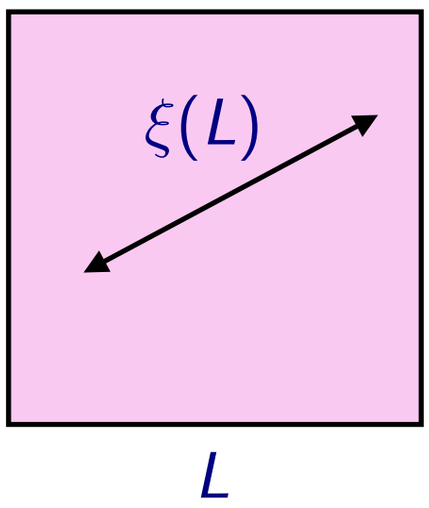
[Hasenbusch, J. Phys A 38, (2005) 5869]

[Maiti, Banerjee, Chandrasekharan, MKM, 2307.06117]

- **Asymptotically free theories:**
  - **have no free parameters**
  - **develop a non-perturbative mass-gap**
  - **contain rich physics all the way from the UV to the IR**

- **Physics encoded in Step-Scaling Function SSF** [Luscher, Weisz, Wolff, 1991]

$$\xi(2L)/\xi(L) \quad \text{vs.} \quad \xi(L)/L$$



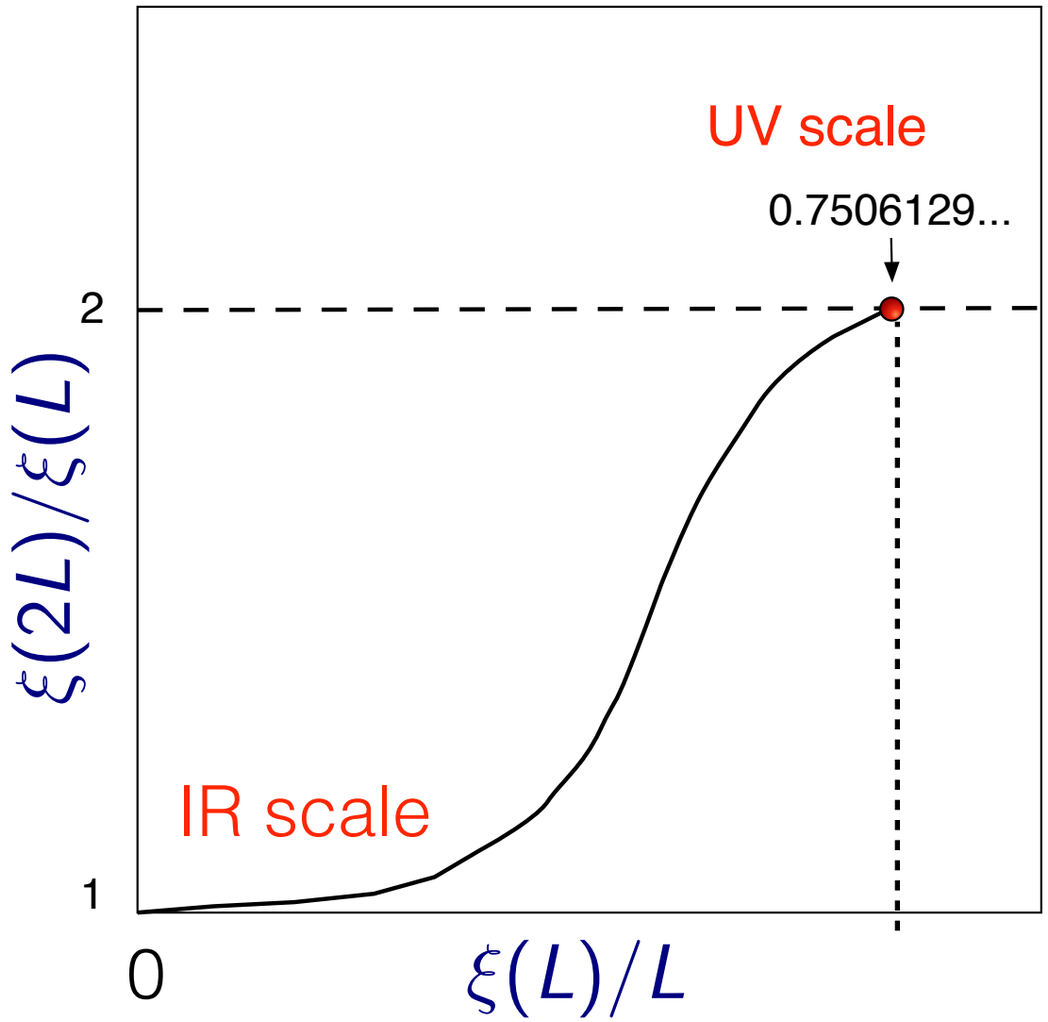
$$\xi(L) = \frac{\sqrt{\frac{G(0)}{G(2\pi/L)} - 1}}{2\sin(\pi/L)}$$

$$G(p) = \sum_{j\equiv(x,t)} e^{ipx}\langle O^{+}(t,x)O^{-}(0,0)\rangle$$

- **Lattice reg. (bosonic) XY:**
  - $O_j^{+} = e^{i\theta_j}, O_j^{-} = e^{-i\theta_j}$

- **Qubit reg. (fermionic) XY:**
  - $O_j^{+} = O_j^{-} = \bar{\psi}_j\psi_j$

SSF in the XY Model

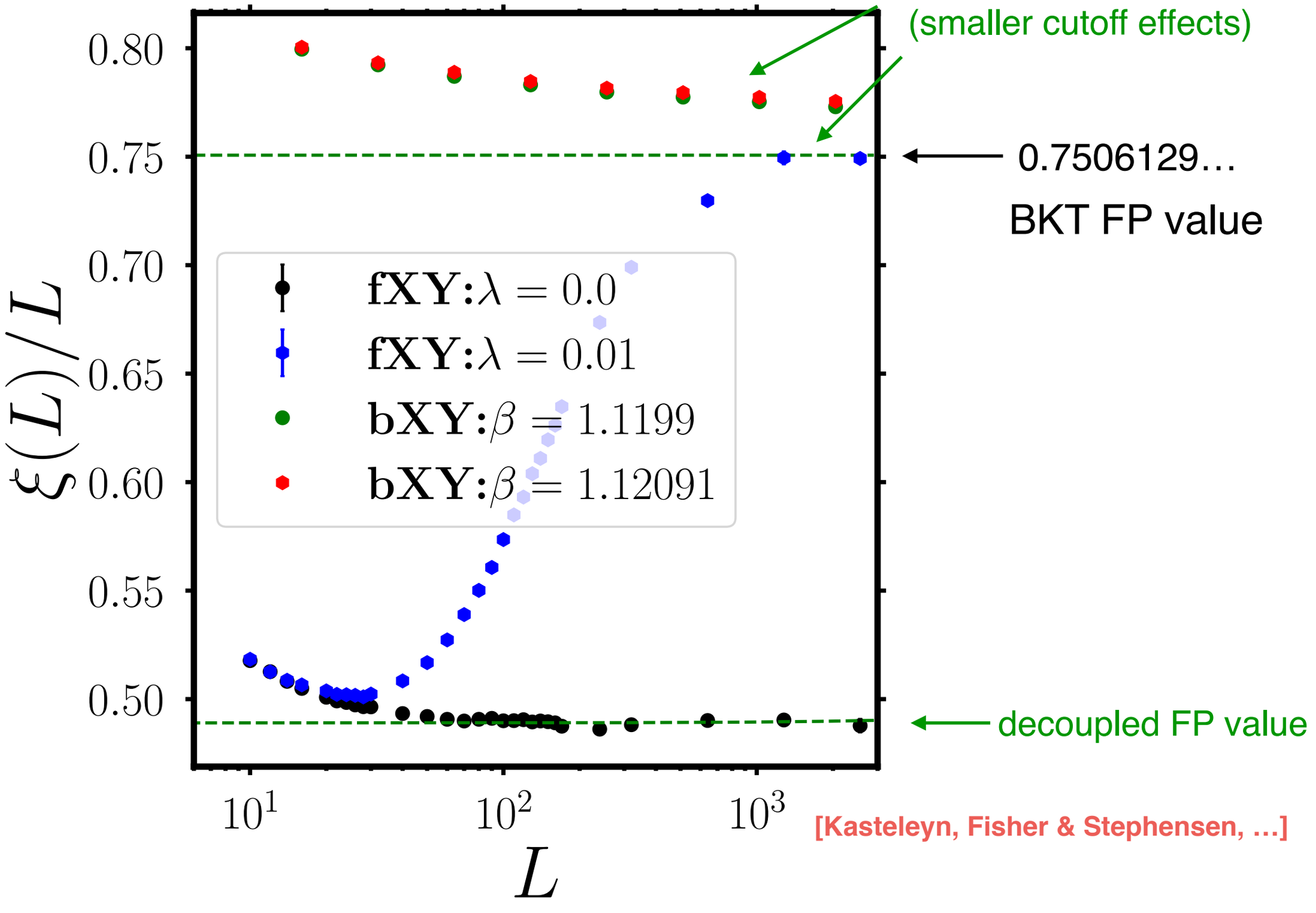


- **We computed the universal curve in two different lattice models: BKT transition without fine tunning**
- **Universal parameters reproduced: β_c, Υ,... [Hasenbusch, J. Phys A 38, (2005) 5869]**
- **No complete universal curve in the existing literature for Eucliden box ($L_\tau \to \infty$: [Balog et al., Nucl.Phys. B675 (2003), Balog et al., Nucl.Phys.B Proc.Suppl. 129 (2004)])**

Magic of Wilson's RG at work

[Maiti, Banerjee, Chandrasekharan, MKM, arXiv:2307.06117]

Qubit model is much better than the traditional model! (smaller cutoff effects)



[Kasteleyn, Fisher & Stephensen, …]

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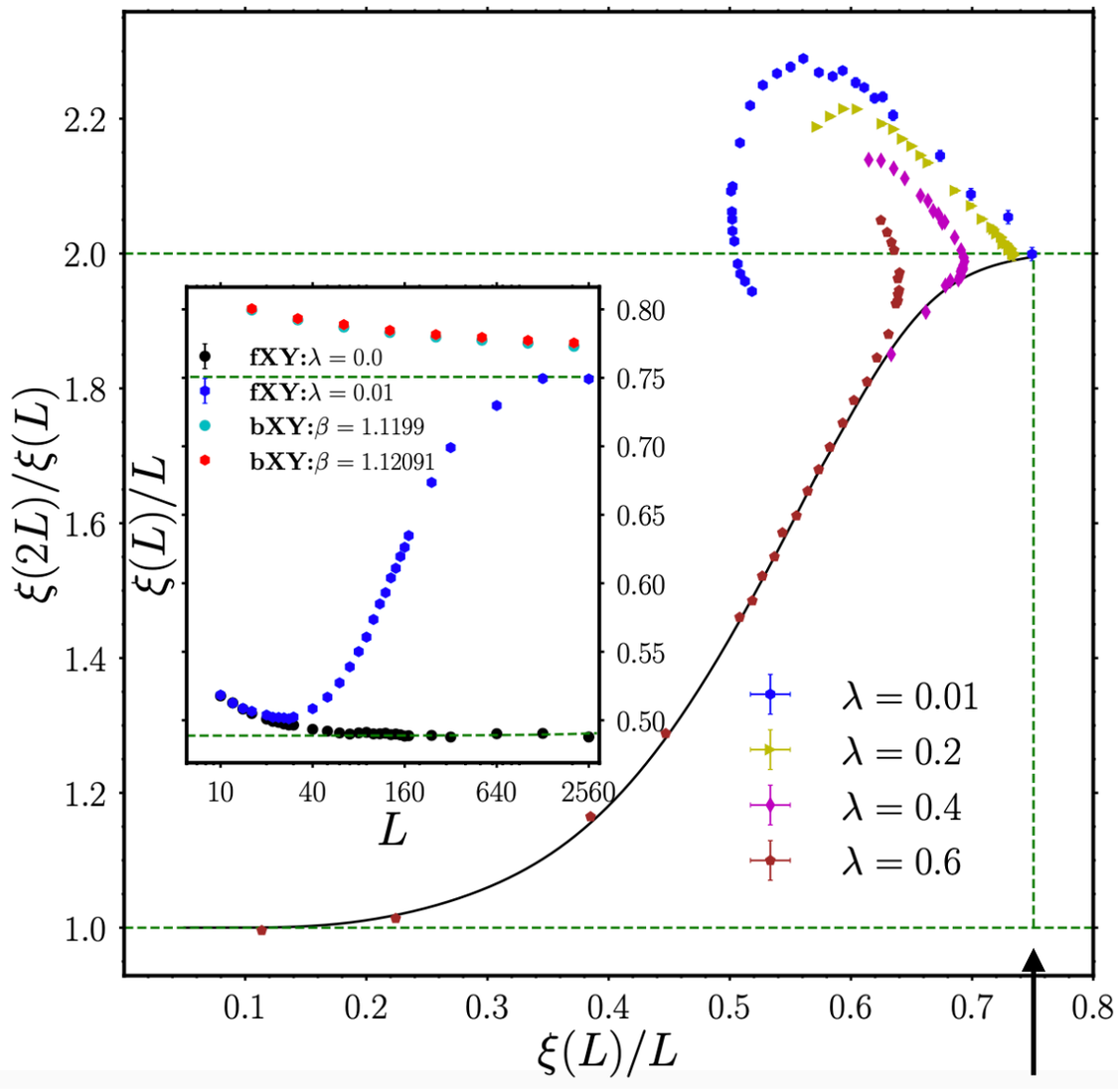


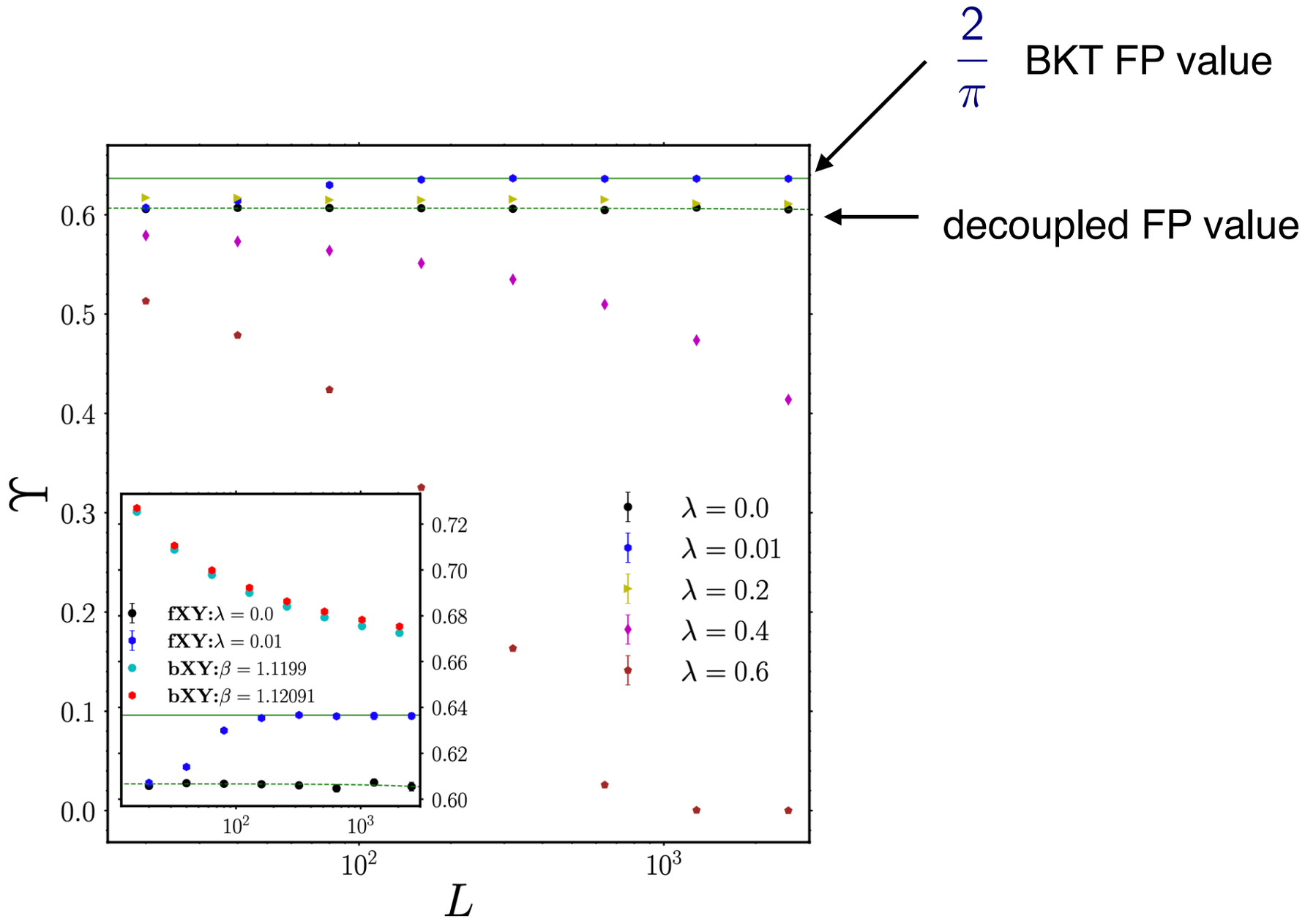
0.7506129…

BKT FP value

- **Obtained universal curve from UV to IR when the two layers decouple**
- **Fermionic XY: the UV physics emerges when $\lambda \to 0$ instead of $\lambda = 0$ (then take $L \to \infty$ limit)**

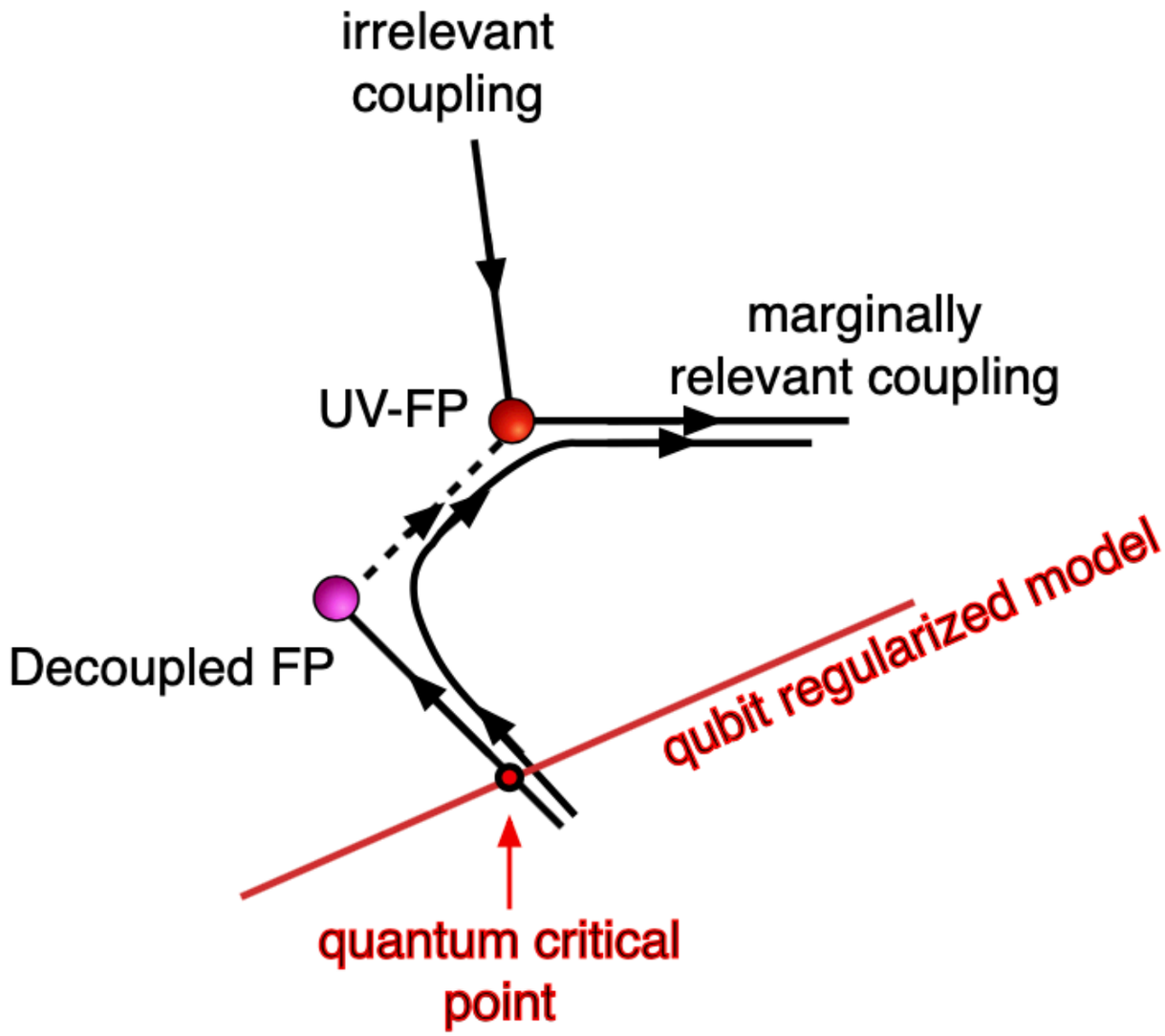
Helicity modulus: $\Upsilon = \langle Q_\omega^2 \rangle$

[Maiti, Banerjee, Chandrasekharan, MKM, arXiv:2307.06117]



- Q_ω - spatial winding number of bosonic worldlines
- Difference in Υ between $\lambda \to 0$ and $\lambda = 0$
- In the massive phase Υ must vanish

Wilson's RG working in an unexpected way



- At the decoupled FP: two differently qubit regularized models describe the physics of a critical system containing two decoupled theories
- When a small non-zero coupling is introduced between the theories, the long distance physics flows towards the desired universal physics of the UV-FP theory
- Our non-traditional qubit regularized model gives the cont. physics of the XY model

Summary & Outlook

- **Many ways to regularize QFTs: qubit-regluarization approach**
- **Recover asymptotically-free massive QFTs using qubit-degrees of freedom**
- **Capture UV and IR physics without fine-tuning: universal step-scaling function of the traditional XY model**
- **Smaller cut-off effects in universal quantities**
- **Similar mechanism discovered in O(3)** [Bhattacharya et al. PRL 126 (2021), 172001]
- **Can Gaussian fixed point be recovered for qubit regularization in higher dimensions?**
- **Can this approach be extended to non-abelian gauge theories?** [Liu, Chandrasekharan, Symmetry 14 (2022) 2, 2112.02090, H. Liu@LATT23, Mon. 16.20]