Towards hadronic D decays at the SU(3) flavour symmetric point

Maxwell T. Hansen

August 3rd, 2023

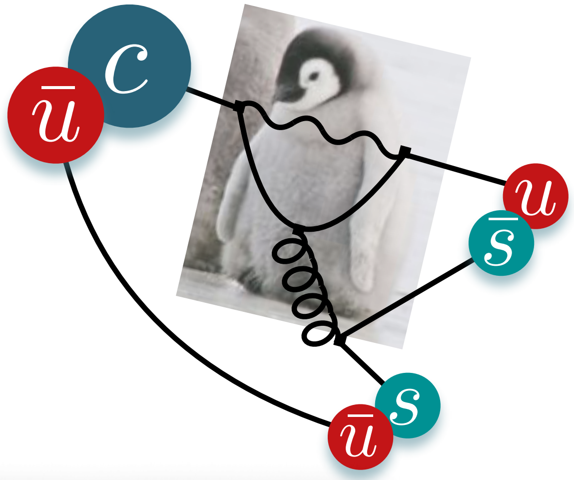
- ongoing work with ***F Joswig***, F Erben, M Di Carlo, N Lachini, S Paul, A Portelli

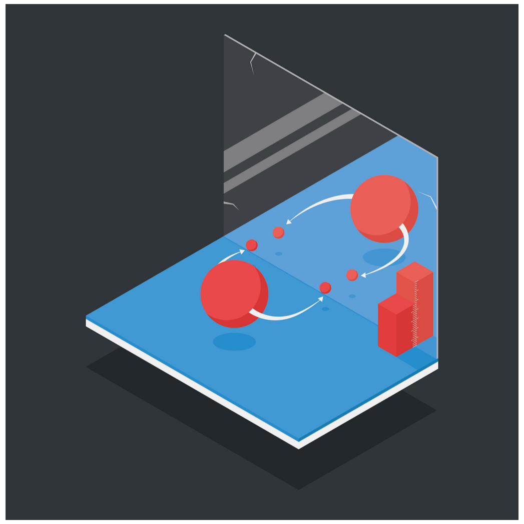
THE UNIVERSITY *of* EDINBURGH

Motivation

- SM is well known to have CPV, $\mathrm{Im}[V_{\mathrm{CKM}}] \neq 0$ …but not enough for ***baryogenesis***!
- 2019: LHCb observed CP violation in *hadronic charm decays* $D \to \pi\pi, K\overline{K}$

$$\Delta A_{\mathrm{CP}} = A_{\mathrm{CP}}(K^-K^+) - A_{\mathrm{CP}}(\pi^-\pi^+) \underset{\text{LHCb}}{=} (-15.4 \pm 2.9) \times 10^{-4}$$





• LHCb (PRL, 2019) •

Is this new physics?

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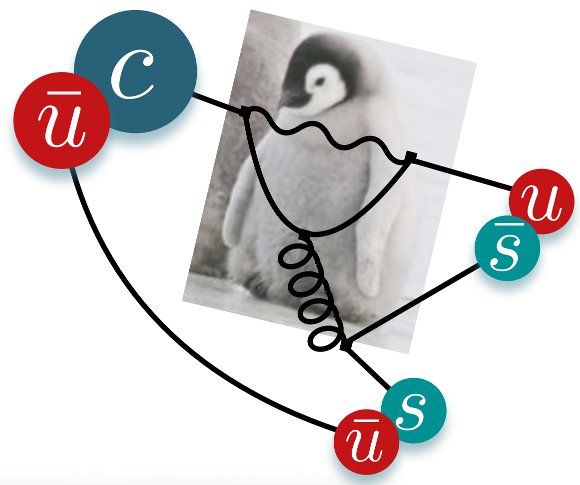

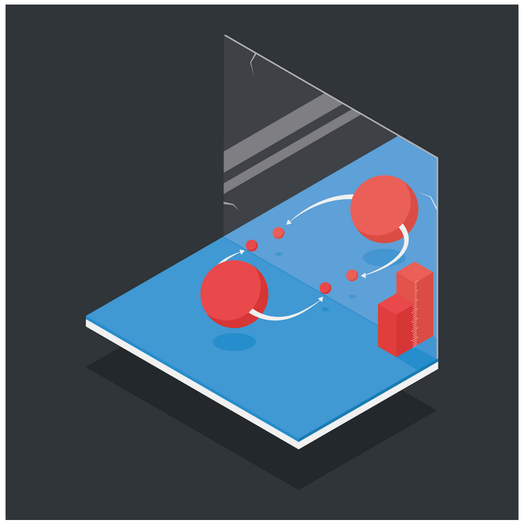

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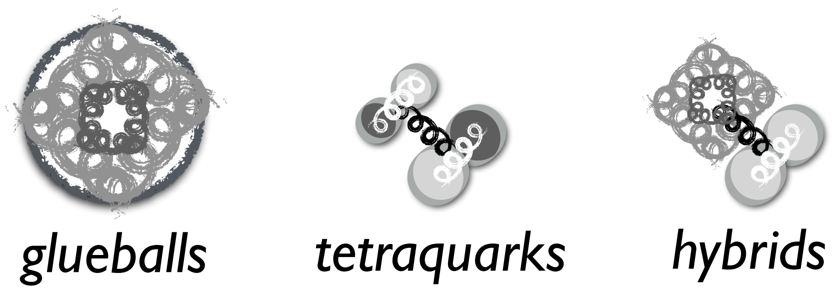
Is this new physics?

- Lattice QCD can provide the Standard Model prediction (correctly treating all complicated QCD dynamics)



- Here we present progress on the first model calculation

$D \to K\pi$ at the $SU(3)_F$ point

Hadronic D decays: Lattice Calculation

- Calculation comes with many challenges

$$A(D \to h_1 h_2) = \mathcal{C}^{\mathsf{LL}}_{n,L,h_1 h_2} \left[\lim_{a \to 0} Z^{\overline{\mathrm{MS}}} \langle n, L | \mathcal{H}_W | D, L \rangle \right]$$

- Non-perturbative renormalization of four-quark operators
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Gold standard here is the RBC/UKQCD calculation of $K \to \pi\pi$

This work is far from that level of calculation: building strategies/understanding feasibility

• R. Abbott et al., RBC/UKQCD, Phys.Rev.D 102 (2020) 5, 054509 •

Computational set-up: *gauge field ensembles*

- Lattices generated by the OPEN LATtice initiative

  • A Francis, Friday 9:00am, Curia II •

- Three flavors of stabilised Wilson fermions with $m_\pi = m_K$

Label	$T \times L^3/a^4$	β	κ	a (fm)	m_π (MeV)	$m_\pi L$
a12m400	96×24^3	3.685	0.1394305	0.12	410	5.988(28)
a094m400	96×32^3	3.8	0.1389630	0.094	410	6.201(19)
a064m400	96×48^3	4.0	0.1382720	0.064	410	6.383(14)

- Results for the finer two ensembles = new relative to last year's presentation

  • F Joswig, Lattice2022 •

- Similar physical volumes across different lattice spacings (~7% variation)

Computational set-up: *software*

- Distillation framework is fully open source and based on
  - **Grid:** data parallel C++ library (*github.com/paboyle/Grid*)
  - **Hadrons:** Grid-based workflow management system (*github.com/aportelli/Hadrons*)

- Initially developed for domain wall fermions
  → flexibility of **Grid & Hadrons** allows us to use it for Wilson fermions

- Improved solvers for Wilson-clover type fermions are needed

- Data analysis based on
  - **pyerrors:** python framework for error computation using the Γ method (*github.com/fjosw/pyerrors*)

    - U Wolff, Comput.Phys.Commun. 156 (2004) 143-153,. 176 (2007)

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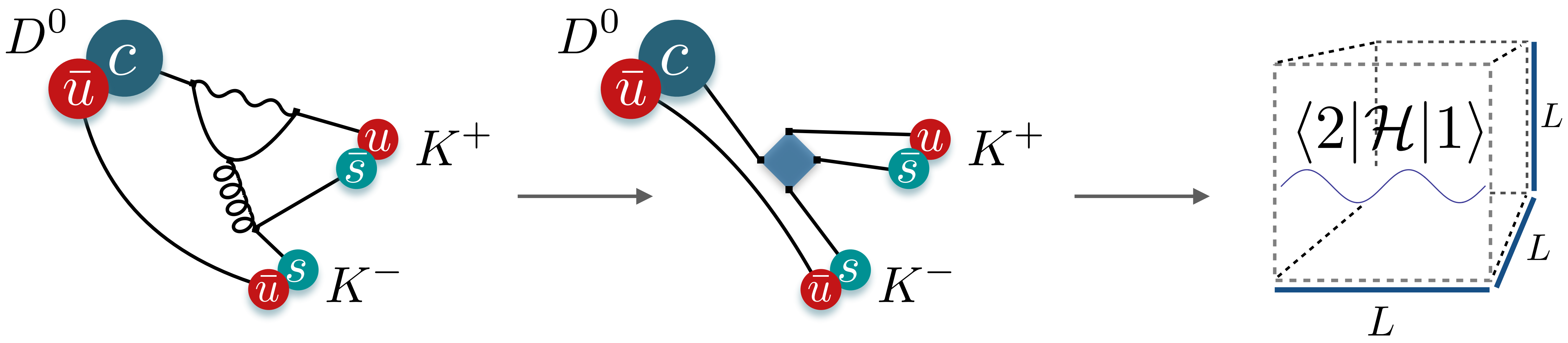
Hadronic D decays

- Integrating out electroweak physics → basis of four-quark operators



- Four-quark operators can be challenging with Wilson quarks
  - Power-divergent mixing $\mathcal{O}_{\text{dim }6} + \frac{1}{a^n}\mathcal{O}_{\text{dim }(6-n)}$
  - Operator mixing $\mathcal{O}_{\text{dim }6} + c\,\mathcal{O}_{\text{other dim }6}$
  - Lack of O(a) improvement $\mathcal{O}_{\text{dim }6} + a\mathcal{O}_{\text{dim }7}$

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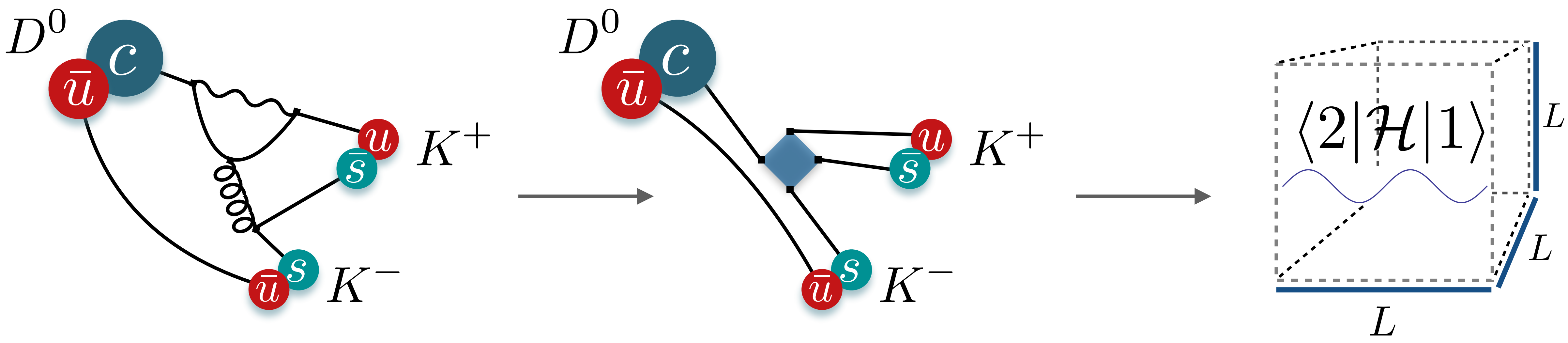

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- First two issues: not present for $\overline{D}^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^-\pi^+$ decays

- Third issue: addressed by multiple lattice spacings (and lower precision goal)

Lack of mixing

- Four distinct flavours → no power-divergent mixing

$$Q_1^{\bar{d}s} = (\bar{d}u)_{V-A}(\bar{c}s)_{V-A}, \qquad Q_2^{\bar{d}s} = (\bar{d}_a u_b)_{V-A}(\bar{c}_b s_a)_{V-A}\,,$$

- Discrete symmetries of $SU(4)_F$ theory highly constraining (even for Wilson quarks)

$$\mathcal{P} = \text{parity}\,, \qquad \mathcal{C} = \text{charge conjugation}\,, \qquad \mathcal{S} = (\psi_2 \leftrightarrow \psi_4)\,,$$

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- Use basis with definite $\psi_2 \leftrightarrow \psi_4$ exchange symmetry

$$Q^{[\mathcal{S}=\pm]}_{[\mathcal{P}=-]} = O^{\pm}_{VA} + O^{\pm}_{AV} \qquad O^{\pm}_{\Gamma_a\Gamma_b} = \frac{1}{2}\left[(\overline{\psi}_1\Gamma_a\psi_2)(\overline{\psi}_3\Gamma_b\psi_4) \pm (\overline{\psi}_1\Gamma_a\psi_4)(\overline{\psi}_3\Gamma_b\psi_2)\right]$$

- Parity-negative part of $(V-A)^2$ operator does not mix under renormalization

$$\begin{pmatrix} 1\pm \\ 2\pm \\ 3\pm \\ 4\pm \\ 5\pm \end{pmatrix}^{\text{RI-scheme}} = \begin{pmatrix} \square & & & & \\ & \square & \square & & \\ & \square & \square & & \\ & & & \square & \square \\ & & & \square & \square \end{pmatrix} \begin{pmatrix} 1\pm \\ 2\pm \\ 3\pm \\ 4\pm \\ 5\pm \end{pmatrix}^{\text{Wilson lattice}} .$$

• Donini et al. [hep-lat/9902030] •

Non-perturbative renormalization

- Regularization independent (RI) momentum-subtraction schemes
  - Scale comes from lattice momenta (MOM vs SMOM)
  - Perturbative conversion to $\overline{\text{MS}}$
- Strategy
  - • Martinelli et al., Nucl.Phys.B 445 (1995) •
  - Calculate amputated vertex function on Landau-gauge-fixed background
  - Demand "projected vertex = tree vertex" at a given scale

$$\lim_{m_R \to 0} Z_q^{-1} Z_{\mathcal{O}} \mathcal{V}_{\mathcal{O}}(p^2)\Big|_{p^2=\mu^2} = 1$$

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$$Z_{\mathcal{O}}^{\overline{\text{MS}}\leftarrow\text{latt}}(\mu_{\overline{\text{MS}}}, a) = Z_{\mathcal{O}}^{\overline{\text{MS}}\leftarrow\text{RI}}(\mu_{\overline{\text{MS}}}, \mu_{\text{RI}}) \cdot Z_{\mathcal{O}}^{\text{RI}\leftarrow\text{latt}}(\mu_{\text{RI}}, a) + O(a^2\mu_{\text{RI}}^2, \alpha(\mu)^n)$$

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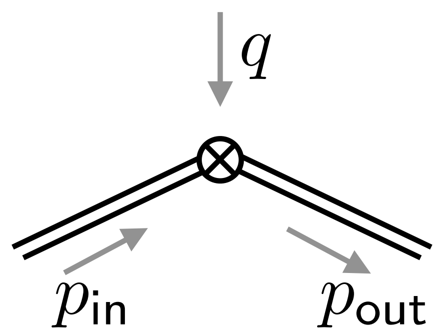
- Use twisted boundaries to define different momentum trajectories

twist-1	$p_{\text{in}} = (p,p,p,-p)$,	$p_{\text{out}} = (p,p,p,p)$,	$q = (0,0,0,2p)$
twist-2	$p_{\text{in}} = (p,p,0,0)$,	$p_{\text{out}} = (p,0,p,0)$,	$q = (0,-p,p,0)$
twist-4	$p_{\text{in}} = (0,0,0,2p)$,	$p_{\text{out}} = (p,p,p,p)$,	$q = (p,p,p,-p)$



Example: $Q^{[\mathcal{S}=-]}_{[\mathcal{P}=-]} = O^-_{VA} + O^-_{AV}$ **SMOM**

- First focus on large momenta

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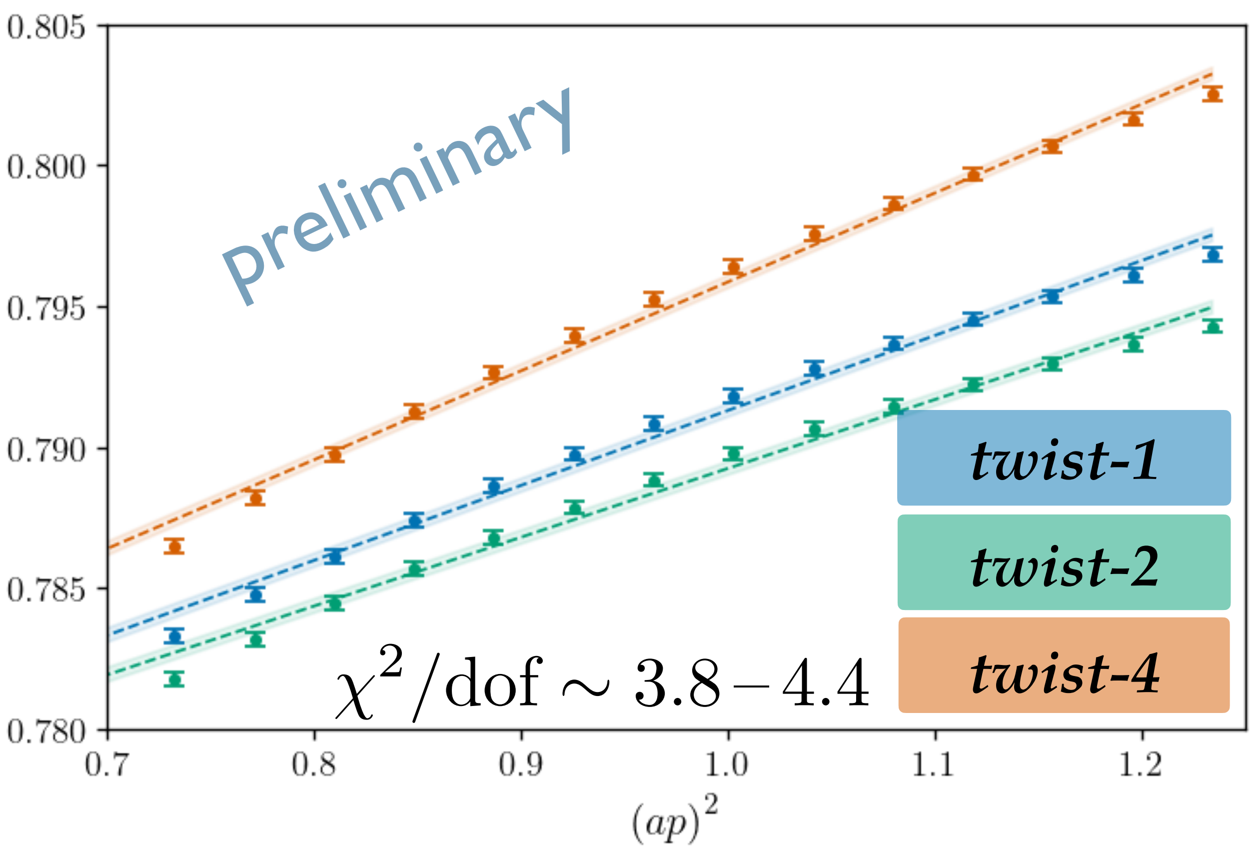

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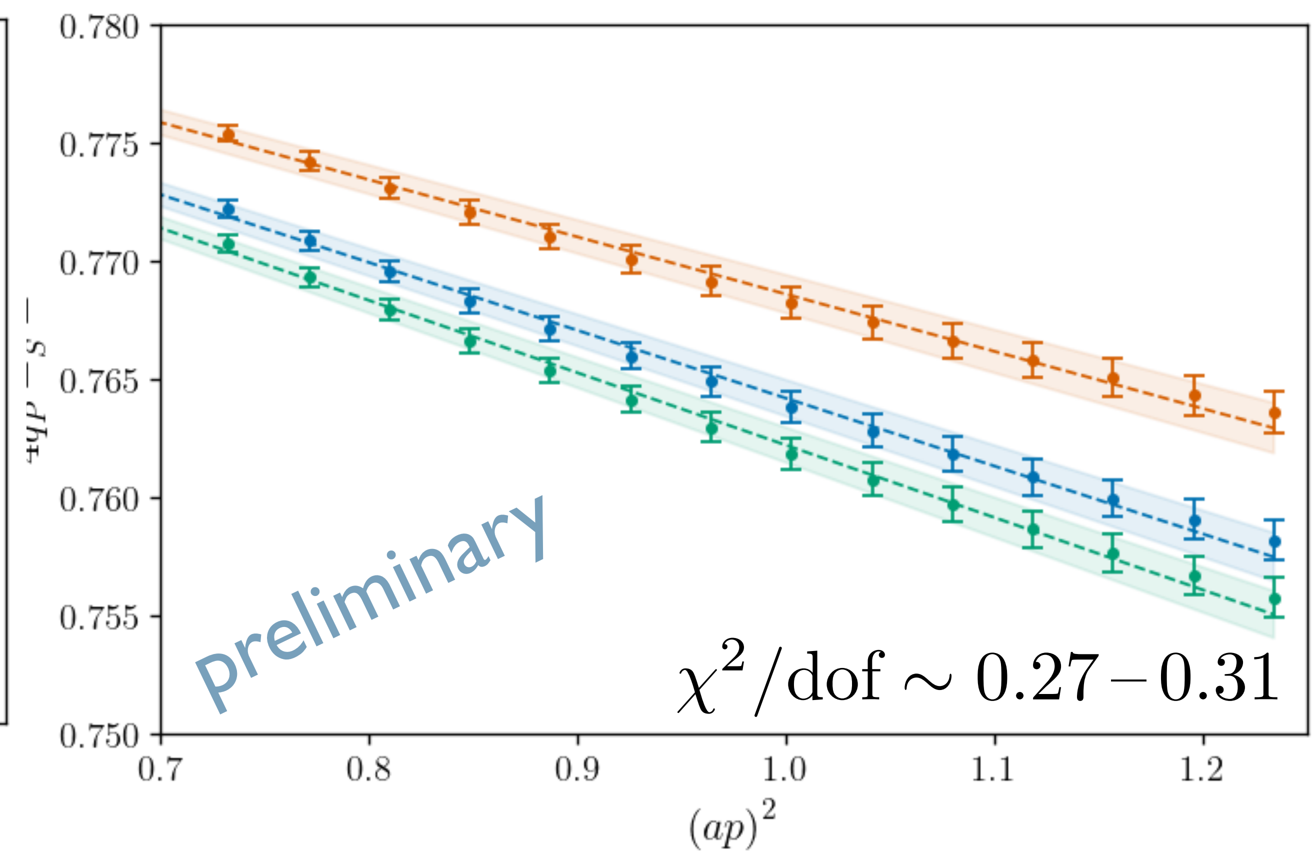

- Perturbative conversion at one loop only, seems to reduce curvature

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- Full momentum range

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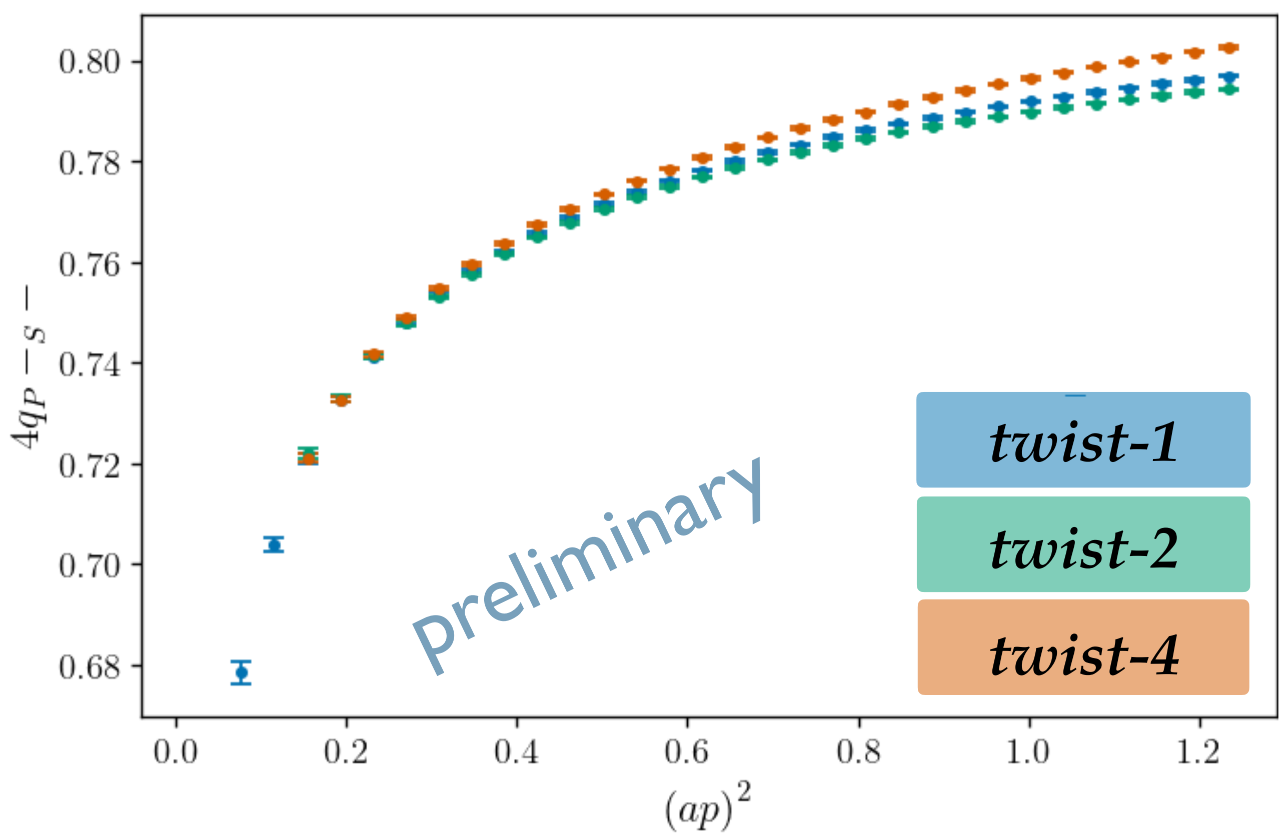

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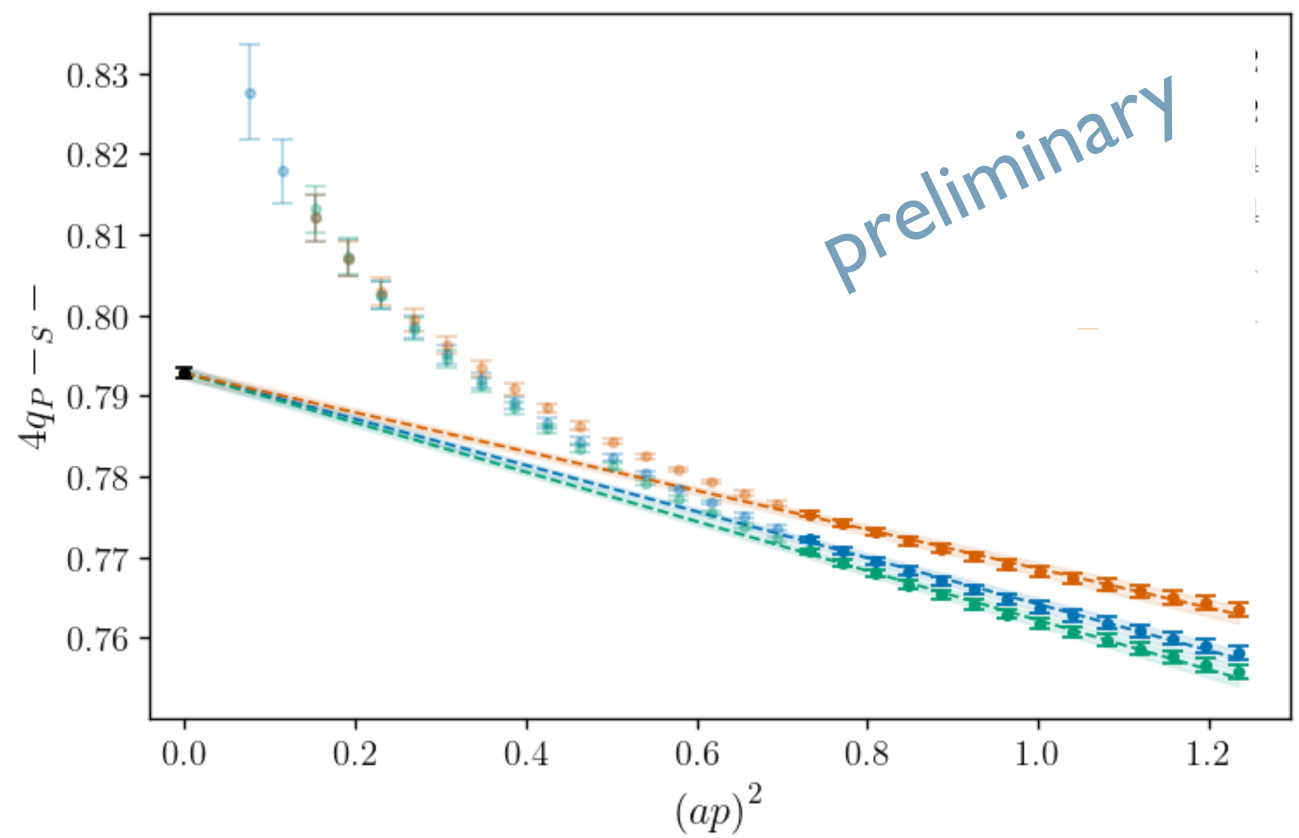

- Fit away a^2p^2 in the large momentum regime
- Twist trajectories fit independently, consistency is encouraging
- Renormalization looks likely to be sub-dominant uncertainty here

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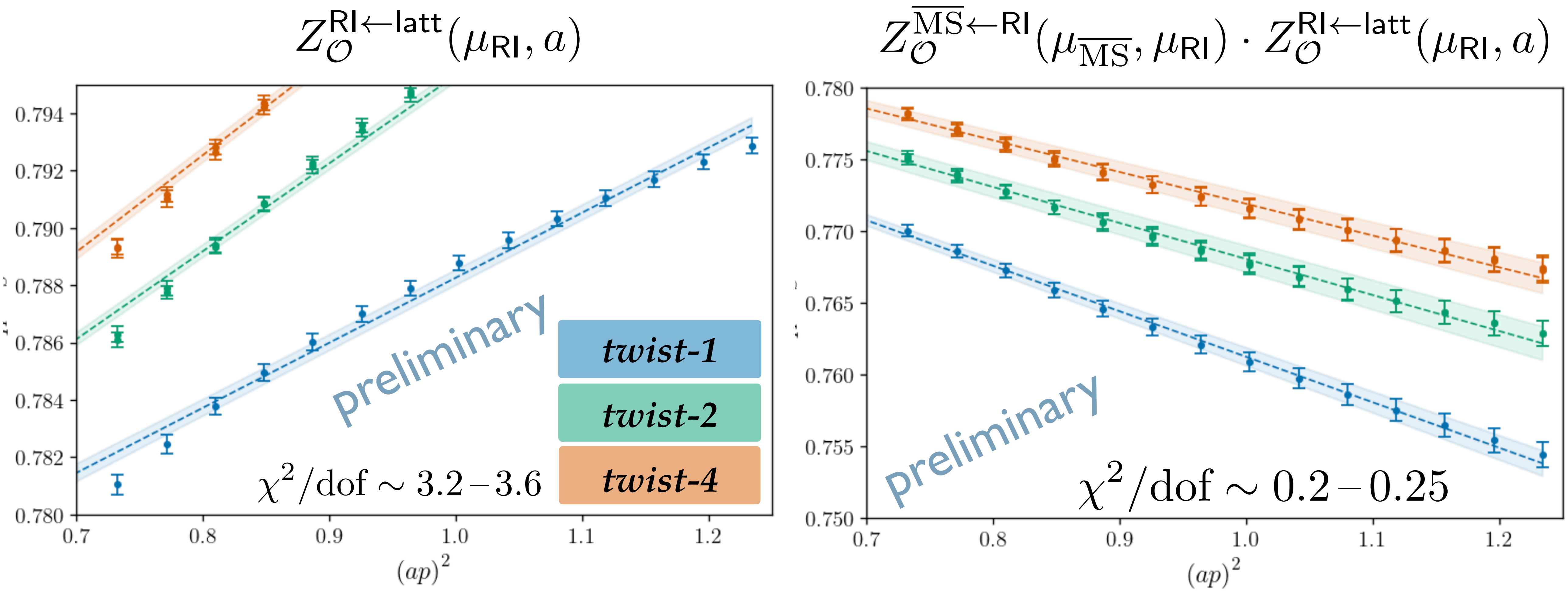

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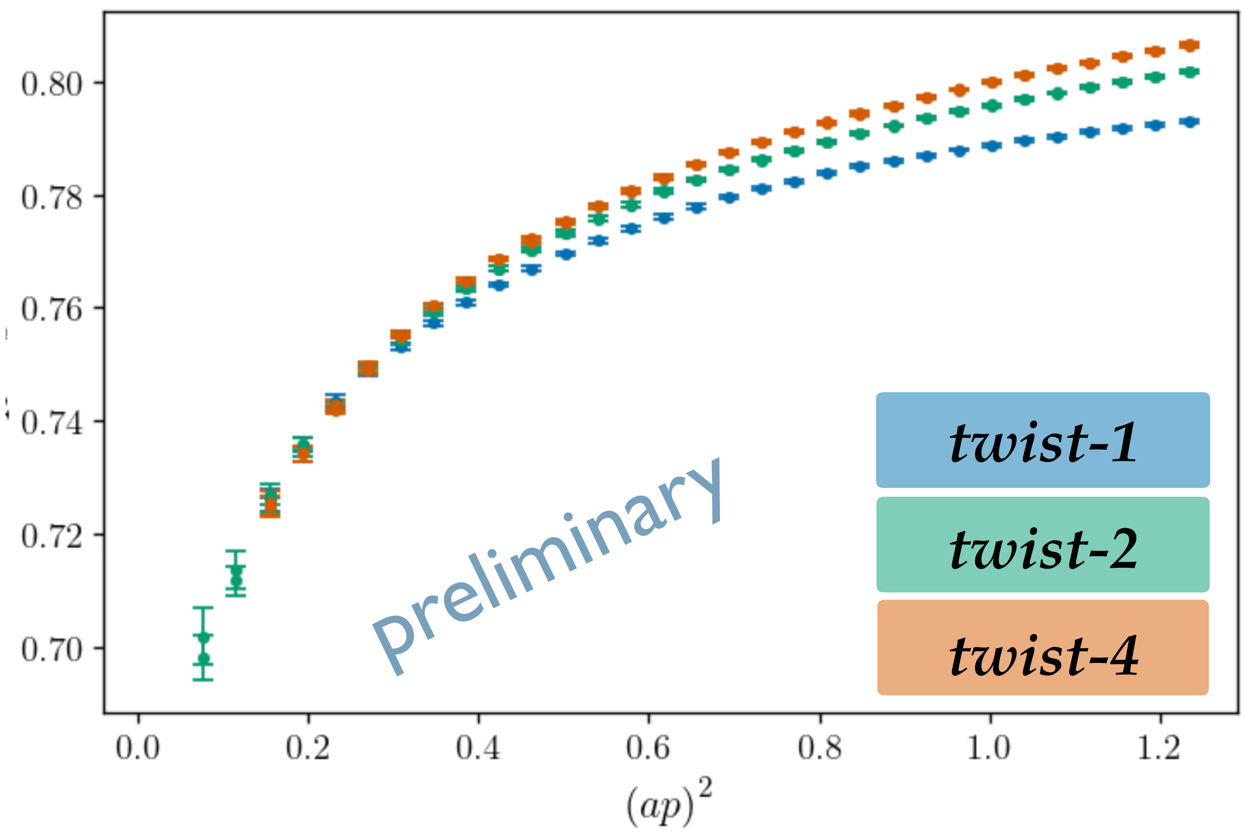

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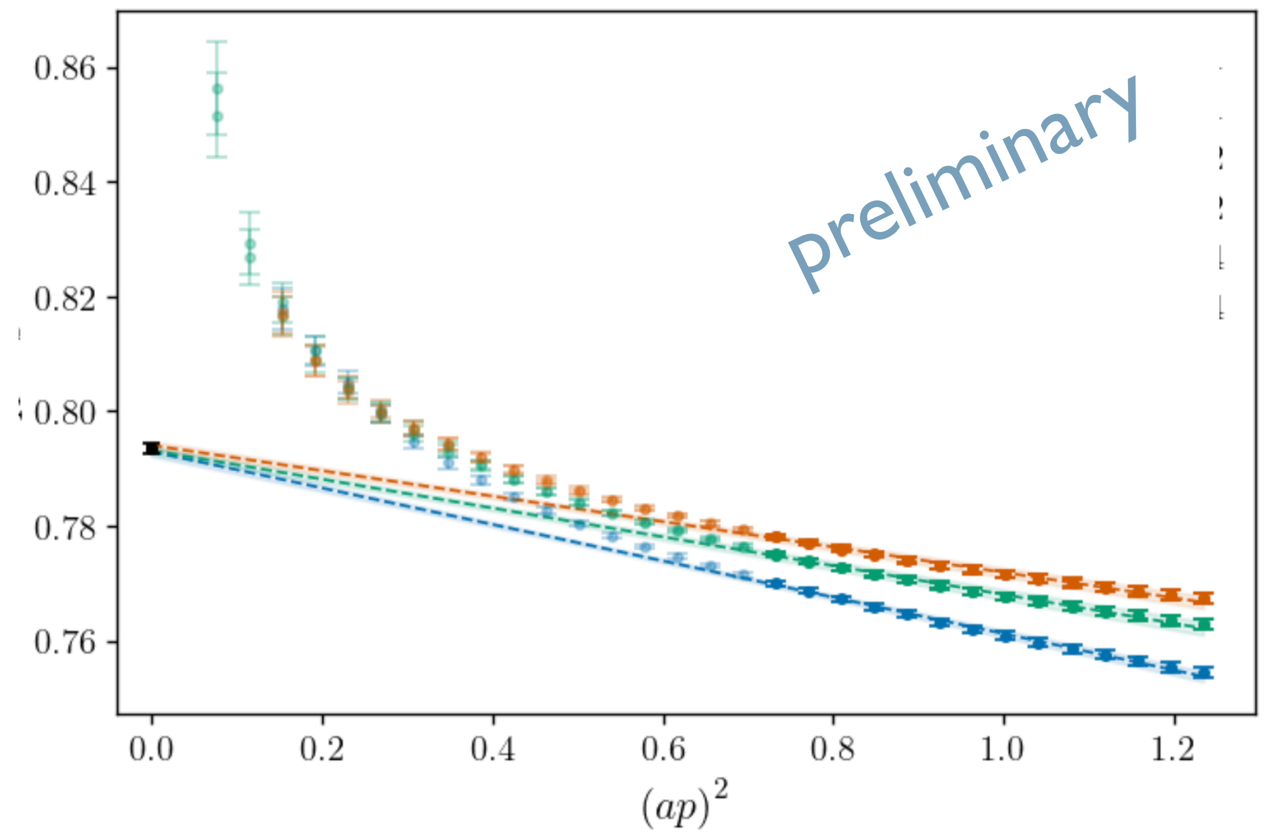

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- *Clear consistency between MOM and SMOM!*

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Operator construction

- Need a broad basis of operators to reliably create excited states
- Feasible thanks to distillation
  - Quark-field smearing (projection into low-modes of the covariant laplacian)
  - We use exact distillation with $N_{\text{vec}} = 60$ eigenvectors
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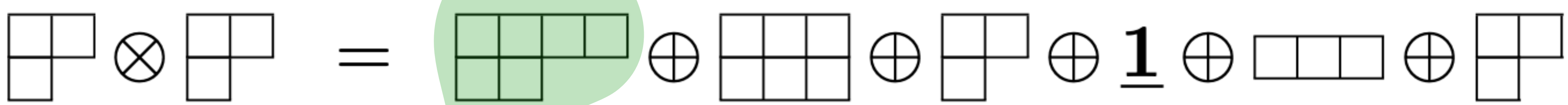


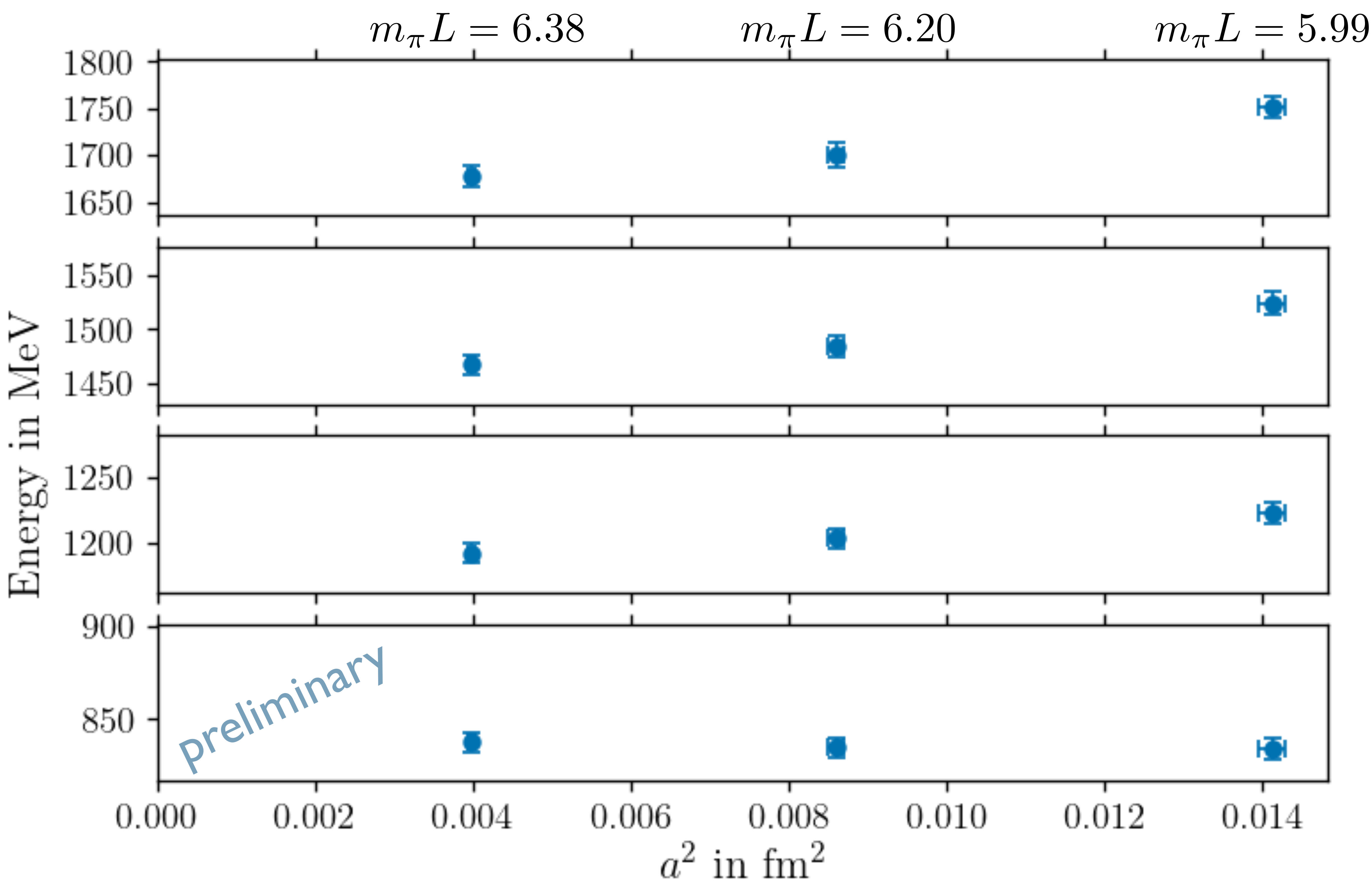
$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \overline{\mathbf{10}} \oplus \mathbf{8} \oplus \mathbf{1} \oplus \mathbf{10} \oplus \mathbf{8}.$$

Contains $I = 3/2\ K\pi$

- Will lead to the Cabibbo-enhanced and doubly Cabibbo-suppressed *D* decay amplitudes

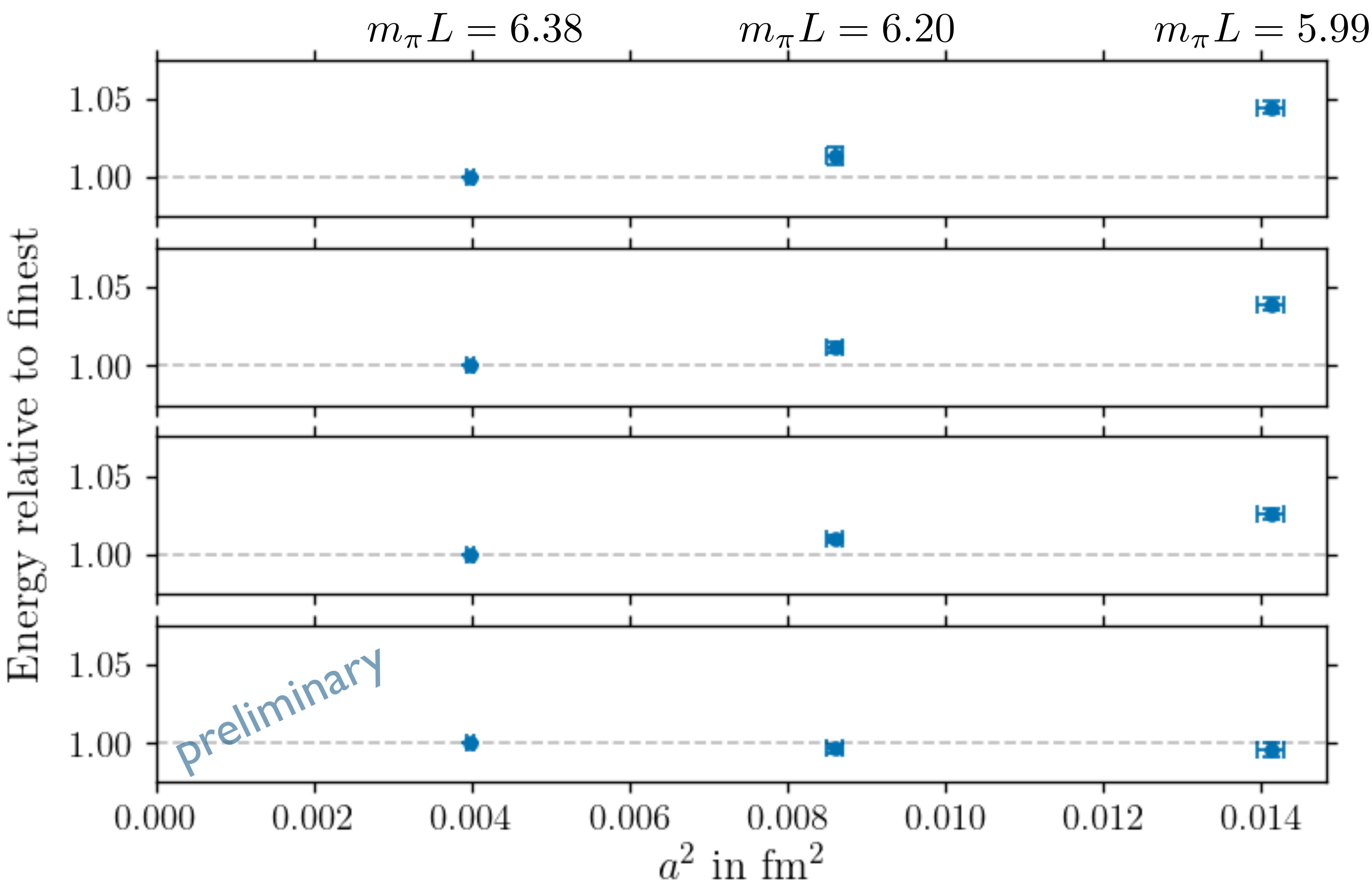
Extracted energy spectrum

- Variation of energies = mixture of volume and cutoff effects



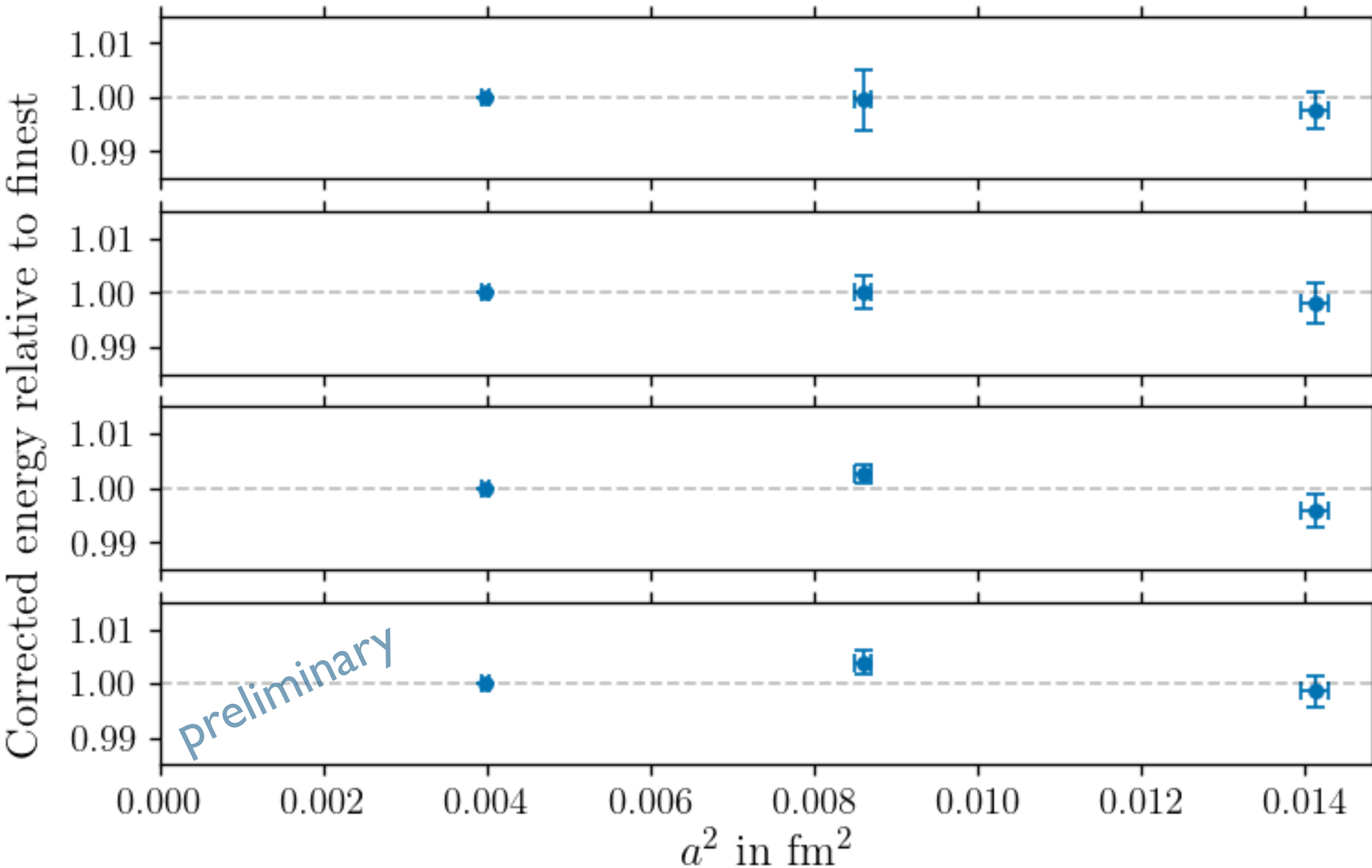
Extracted energy spectrum

- Up to ~5% effect… commiserate with ~5% volume mistuning

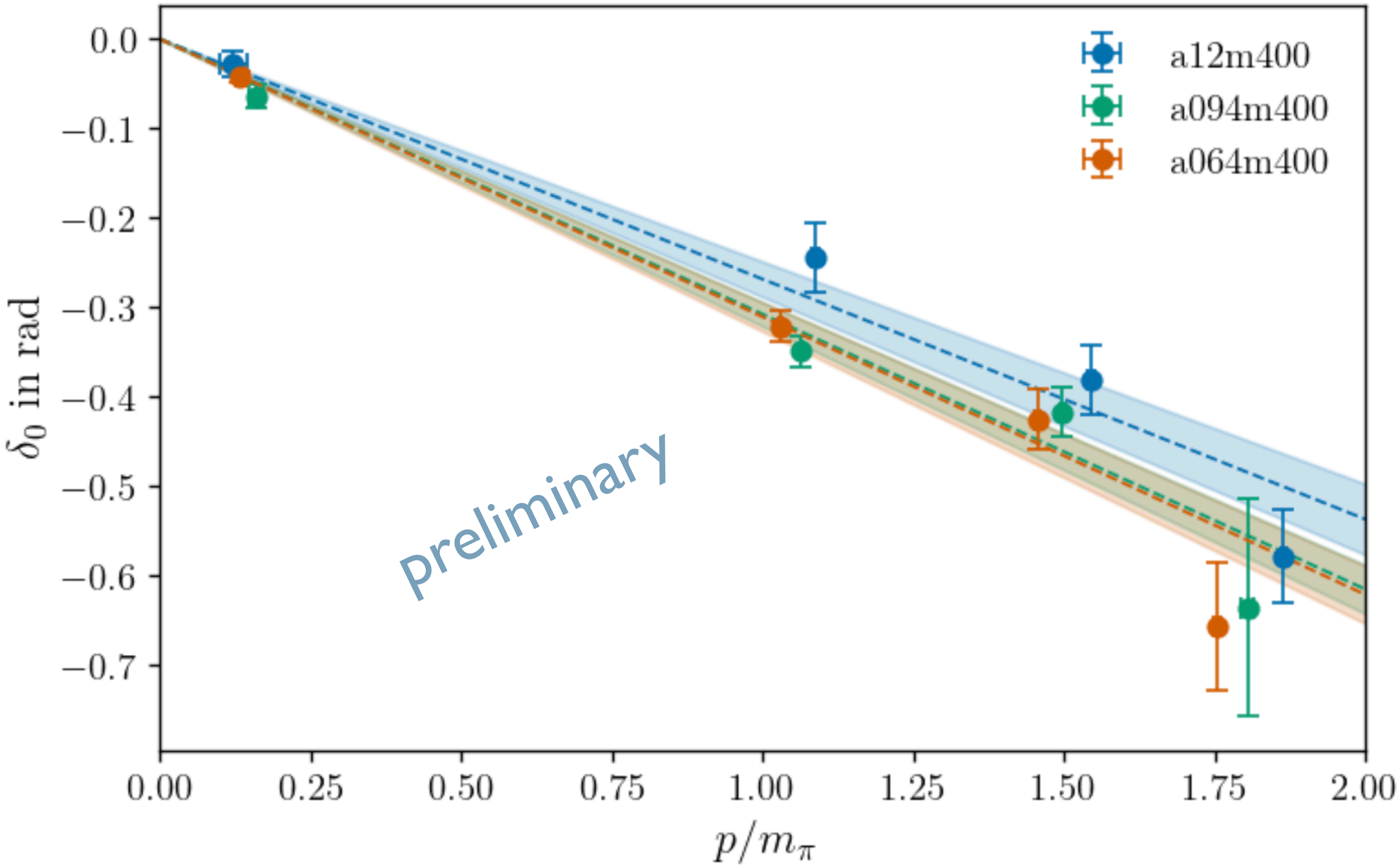


Extracted energy spectrum

- Rescaling by non-interacting ratio → cutoff effects unresolvable



Phase shift tells consistent story



- Next steps
  - Complete and analyze moving frame data
  - More careful phase-shift analysis

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Lellouch-Lüscher formalism

- At the $SU(3)_F$ point, four-particle threshold is just open for $E_{K\pi} = M_D$

- Standard Lellouch-Lüscher formula can be applied

$$\left|\mathcal{C}^{\mathsf{LL}}\right|^2 = 8\pi \left(q\frac{\partial \phi}{\partial q} + k\frac{\partial \delta_0}{\partial k} \right)_{k=k_n} \frac{E_n^2 m_D}{k_n^3}$$

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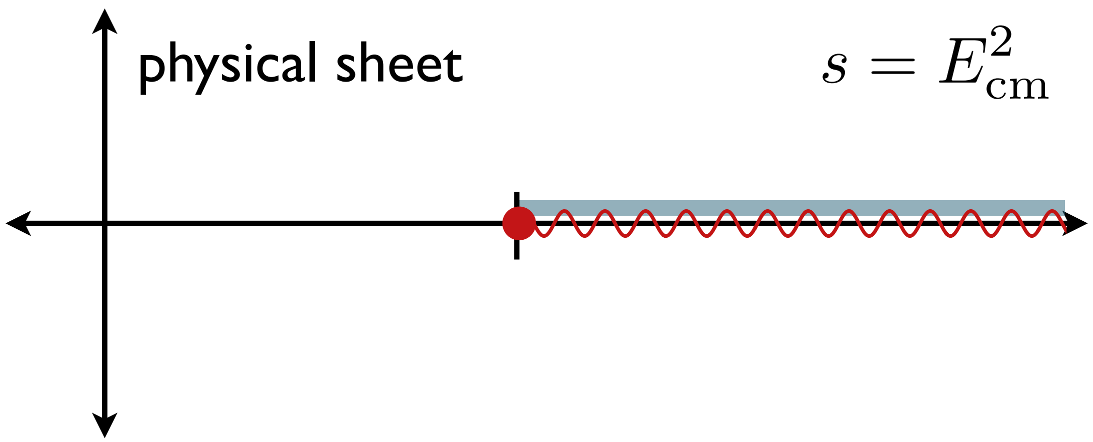
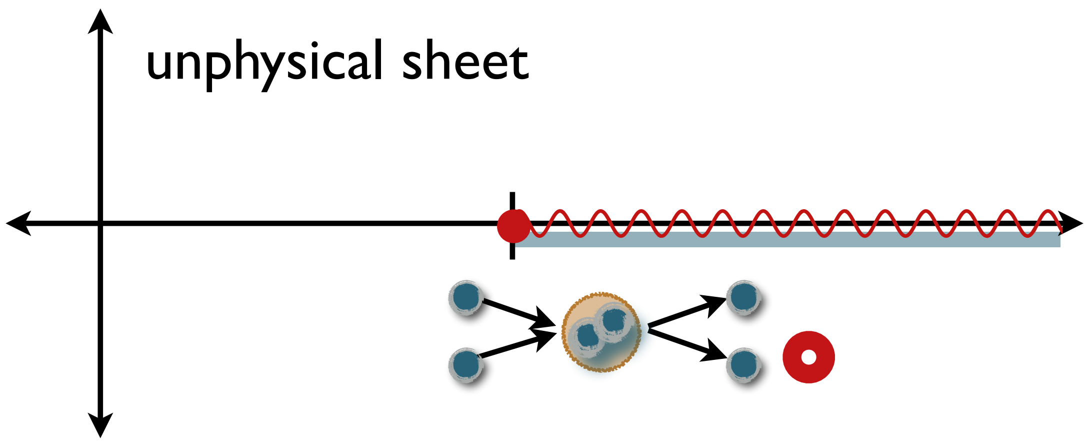
- Applies for all energies: $f(E) = \langle E, \pi K, \mathrm{out} | \mathcal{H}_W(0) | D \rangle$
- Extrapolate (interpolate?) to $E = M_D$ for physical amplitude
- Take advantage of K-matrix based ideas to motivate fit forms

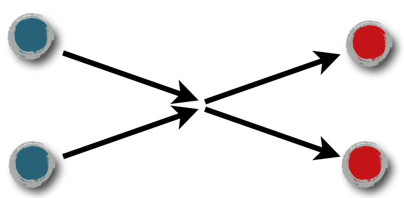
$$\mathcal{A} = \frac{1}{1 - \mathcal{K}_2 \rho} \mathcal{H}$$





Towards lighter masses

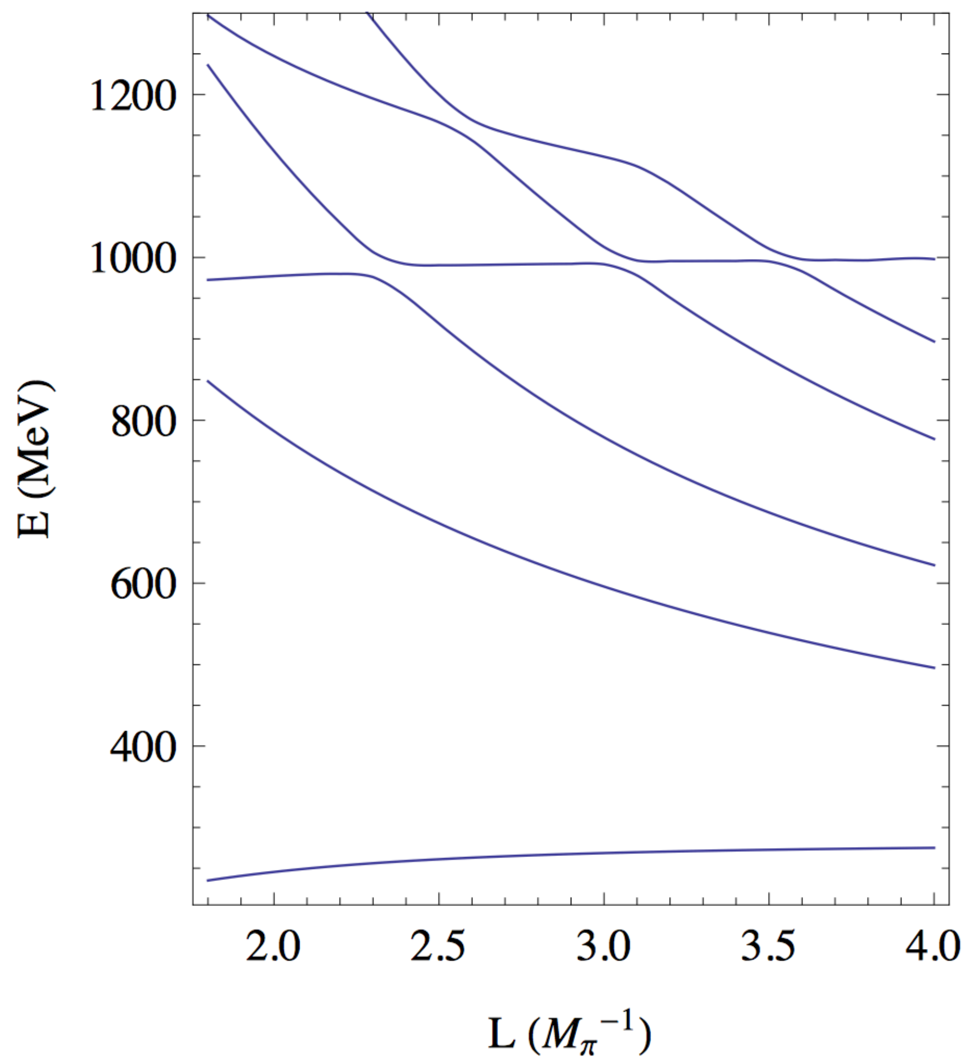
infinite-volume
scattering amplitudes



$\pi\pi \to K\overline{K}$

know field theoretic relation,
exact up to $e^{-M_\pi L}$

finite-volume energies



• MTH, Sharpe, Phys.Rev. D86 (2012) 016007 •

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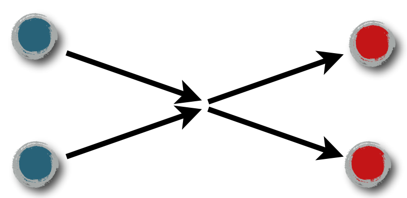

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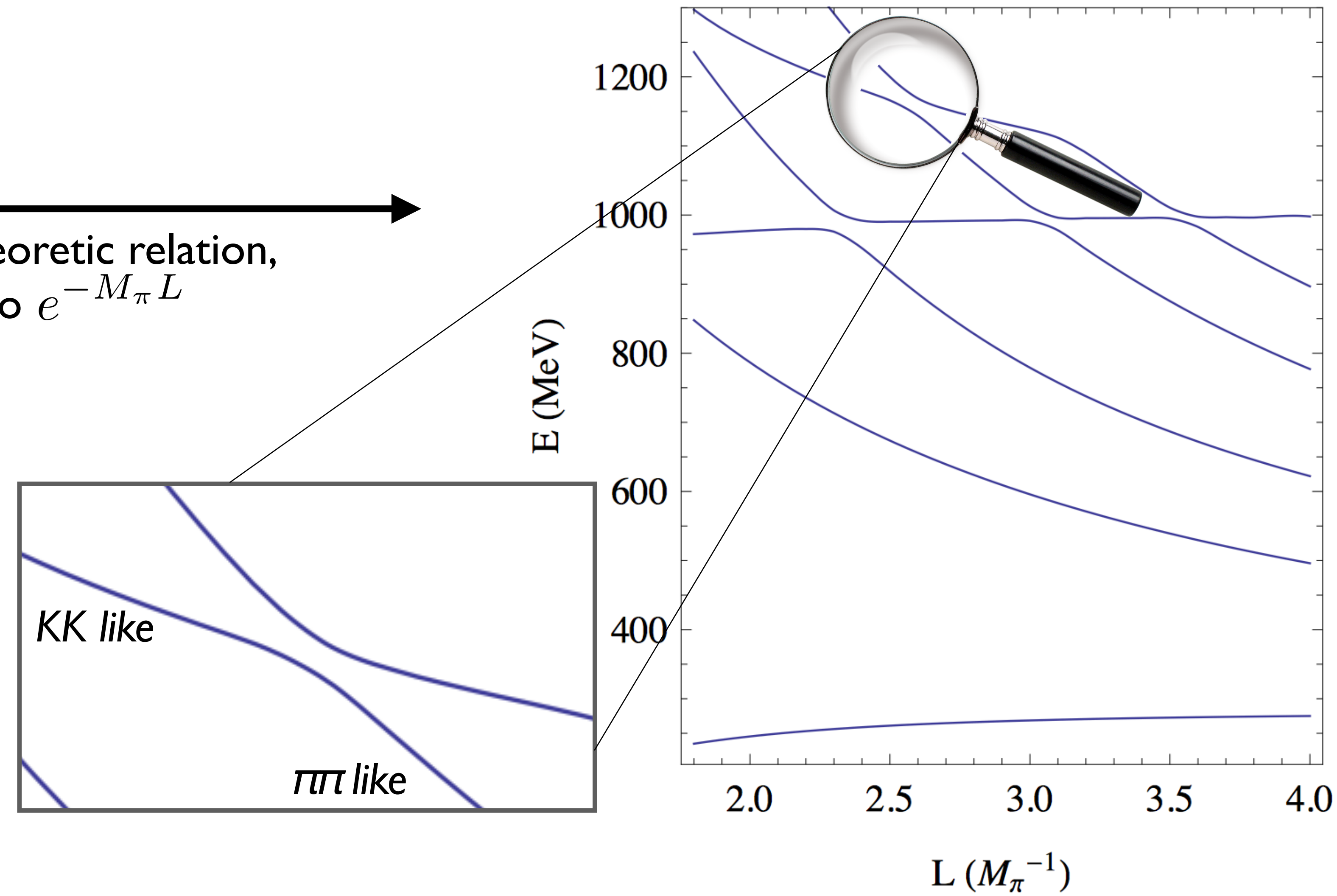

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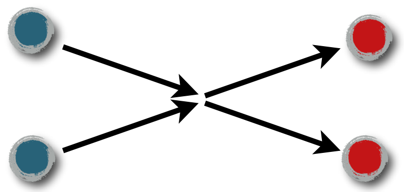

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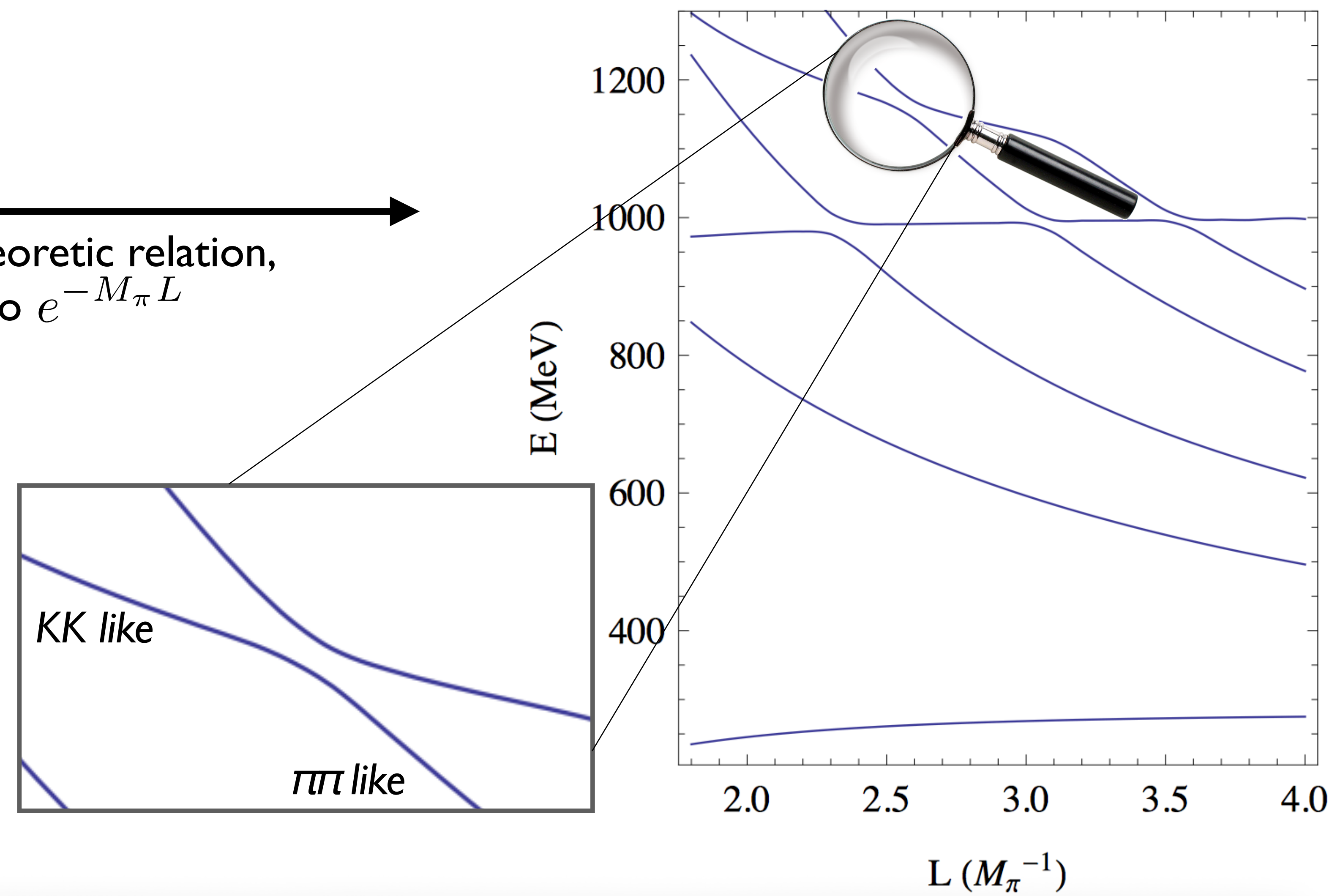
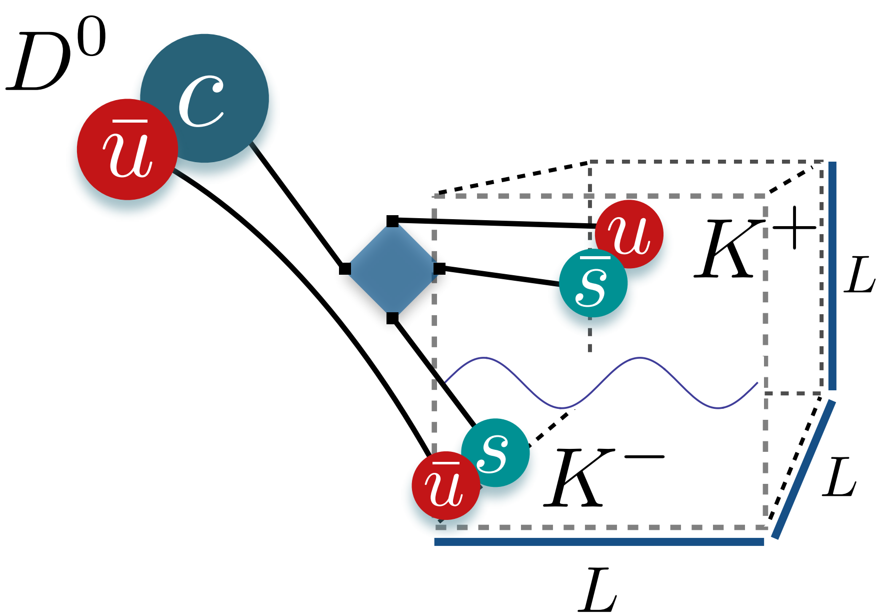
finite-volume energies





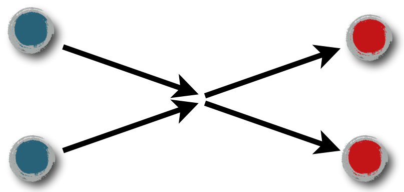
$$|n, L\rangle = c_\pi^{(n)}(L)\,|\pi\pi, \mathrm{out}\rangle + c_K^{(n)}(L)\,|K\overline{K}, \mathrm{out}\rangle$$

path to physical amplitude

• MTH, Sharpe, Phys.Rev. D86 (2012) 016007 •

Towards lighter masses

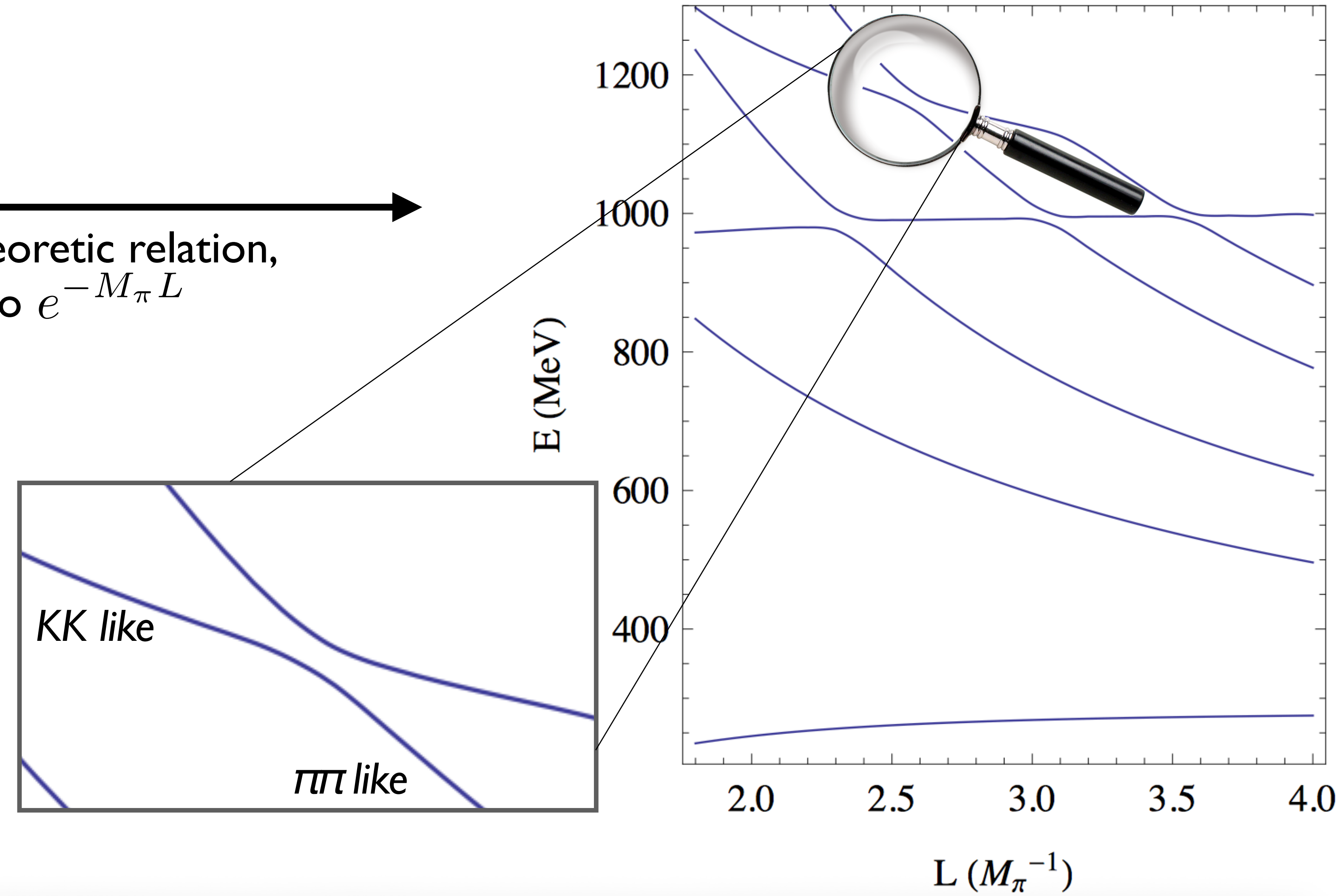
infinite-volume
scattering amplitudes

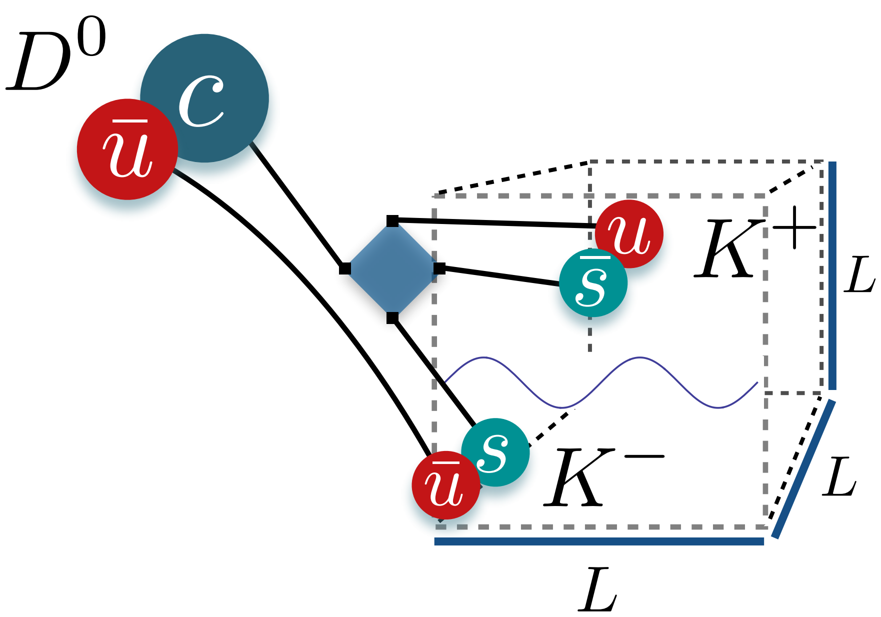


$\pi\pi \to K\overline{K}$

know field theoretic relation,
exact up to $e^{-M_\pi L}$

finite-volume energies





$$|n, L\rangle = c_\pi^{(n)}(L)\,|\pi\pi, \mathrm{out}\rangle + c_K^{(n)}(L)\,|K\overline{K}, \mathrm{out}\rangle$$

path to physical amplitude

biggest challenge = treating $\pi\pi\pi\pi$ etc, channels

$$|n, L\rangle = c_\pi^{(n)}(L)\,|\pi\pi, \mathrm{out}\rangle + c_K^{(n)}(L)\,|K\overline{K}, \mathrm{out}\rangle + c_{4\pi}^{(n)}|\pi\pi\pi\pi, \mathrm{out}\rangle + \cdots$$

• MTH, Sharpe, Phys.Rev. D86 (2012) 016007 •

Hadronic D decays: Lattice Calculation

- Calculation comes with many challenges

$$A(D \to h_1 h_2) = \mathcal{C}^{\mathsf{LL}}_{n,L,h_1 h_2}\left[\lim_{a\to 0} Z^{\overline{\mathrm{MS}}}\langle n, L|\mathcal{H}_W|D, L\rangle\right]$$

- Non-perturbative renormalization of four-quark operators ●●●●○
- Reliable creation of excited multi-hadron final states ●●●○○
- Removal of discretization effects (enhanced by the charm mass) ●●●○○
- Formalism to relate finite-volume matrix elements to the amplitudes ●●●●○
- Extraction of the matrix element from three-point functions ●●○○○

Thanks for listening!

Funded by UKRI Future Leaders Fellowship

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