Simulating $\mathbb{Z}_2$ lattice gauge theory on a quantum computer

Charles et al. - 2305.02361 [hep-lat]

Hank Lamm

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This work is an outcome of the QCIPU program @ Fermilab

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NASA
3-week summer school for 17 students + year-long internship for 4-5 students with goal to develop diverse quantum workforce with skills needed to succeed in academia and industry

**Young field provides opportunity to build inclusive community**

- Students paid competitive hourly wage
  - *Essential* to enable participation by students from all socioeconomic backgrounds.
- Topical lectures by experts in the field
  - Quantum physics & math, quantum algorithms, error mitigation & correction, quantum hardware. Self-contained and accessible to all preparation levels.
- Pair programming on quantum simulators & real devices
  - Computational exercises in Python + Qiskit on classical and quantum algorithms. Final project simulating 1+1d lattice gauge theory on real devices.
- Panels and informal discussions on career opportunities
  - Panelists from both academia and industry. Information about applying to and paying for graduate school especially important for first-generation college students.
- Year-long interns perform publishable research

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**PHYSICAL REVIEW D 106, 114501 (2022)**

*Primitive quantum gates for an SU(2) discrete subgroup: Binary tetrahedral*

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*Simulating Z_2 lattice gauge theory on a quantum computer*

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Hank Lamm | Simulating $\mathbb{Z}_2$ on Quantum Computer
Quantum Computing for Particle Physics, it’s a need

• The world is quantum, and we are lucky anything is amenable to classical computers
  – Large-scale quantum computers can tackle computations in HEP otherwise inaccessible
  – This opens up new frontiers & extends the reach of LHC, LIGO, EIC & DUNE

• Ab initio cross sections for colliders and neutrino experiments
• Cosmic inflation and the evolution of matter asymmetry in the early universe
• Explorations of BSM, supersymmetry, and quantum gravity
• Hadronization and Hydrodynamics in Heavy-Ion collisions

While broad, these topics often are formulated as lattice field theories

Quantum Simulation for High-Energy Physics
Bauer, Davoudi et al. - PRX Quantum 4 (2023) 2, 027001
Wonderful survey of physics questions, methods, and outstanding problems in field
2 is the smallest and only even prime number

1+1d $\mathbb{Z}_2$ lattice gauge theory on **quantum computer**

Premature optimization is the root of all evil
The ladder of discrete gauge theories in HEP calculations

Coherence Time Increasing

Gluon Field Digitization for Quantum Computers
NuQS collaboration - Phys.Rev.D 100 (2019) 11, 114501
Demonstrated that S(1080) approximates certain 3+1d SU(3) observables

Digitising SU(2) gauge fields and the freezing transition
Understanding the scaling of freezing transitions with approximations

\[ \beta_{f,U(1)} = \frac{\log(1 + \sqrt{2})}{1 - \cos\left(\frac{2\pi}{N}\right)} \approx \kappa_2 N^2, \text{ which extends to } \beta_{f,\text{SU}(N_c)} \approx \kappa N^{N_c^2 - 1}/2 \]

But whereas \( Z_N \) can be taken to \( \infty \), limited number for \( \text{SU}(N_c) \)

\[ \beta \propto \frac{1}{\log(a)} \implies a_f \propto e^{-\beta_f} \]
A tale of two Hamiltonians

Kogut-Susskind Hamiltonian [with $O(a^2)$ errors]

$$H = \sum_{n=1}^{N_s-1} \left[ \frac{1}{2} \sigma_{n,n+1}^x + \frac{\eta}{2} \left( \vec{\psi}_n \sigma_{n,n+1}^z \psi_{n+1} + h.c. \right) \right] + m_0 \sum_{n=1}^{N_s} (-1)^n \vec{\psi}_n \psi_n,$$

Qubit Hamiltonian via Jordan-Wigner

$$H = \frac{1}{2} \sum_{n=0}^{N_s-1} \sigma_{n,n+1}^z - \frac{m_0}{2} \sum_{n=0}^{N_s-1} (-1)^n Z_n + \frac{\eta}{4} \sum_{n=0}^{N_s-2} (X_n X_{n+1} + Y_n Y_{n+1}) \sigma_{n,n+1}^z.$$

Hamiltonian Formulation of Wilson’s Lattice Gauge Theories
Kogut & Susskind *Phys.Rev.D* 11 (1975) 395-408
Formulated $O(a^2)$ lattice Hamiltonian for LGT with staggered matter

**Always remember:** lattice Hamiltonian is a choice

Improved Hamiltonians for Quantum Simulation of Gauge Theories
Carena, Lamm, Li, Liu *PRL* 129 (2022) 5
Developed quantum circuits for $O(a^3)$ pure-gauge Hamiltonian

Quantum Simulation of Lattice QCD with Improved Hamiltonians
Ciavarella 2307.05593 [hep-lat]
Formulated Hamiltonian with reduced truncation errors

Improved Fermion Hamiltonians for quantum simulations
Gustafson & Van de Water - in prep (Talk @ 4:20 PM on Thurs.)
Formulating Hamiltonians for ASQTAD fermions
Hamiltonian Gates for Trotterization with restricted connectivity

Restricting to longest linear graph
for heavy-polygon with $p$ sides: $\frac{N_p - 2}{N_p - 1} \leq 86\%$ BAD!

$\text{ibm_nairobi} \rightarrow 43\%$ WORSE!
Performing scale setting with 2-pt Minkowski correlator

- Want to measure a correlator after preparing in a superposition of vacuum and “particle” state

\[ C(t) = \langle \phi(N_s) \mid U^\dagger(t) \, O \, U(t) \mid \phi(N_s) \rangle = \cos(Mt) + \ldots, \]

- Trotterization introduces discretization errors into correlator, and thus scale setting \( M \)

\[ \mathcal{C}(t/\varepsilon) = \langle \phi(N_s) \mid U^\dagger(t/\varepsilon)^N t \, O \, U(t/\varepsilon)^N t \mid \phi(N_s) \rangle \]

“Particle” state and operator insertion are given by “meson” excitation operator

\[ O = X_n \sigma^{z}_{n,n+1} X_{n+1} \]

\[ \mathcal{U}_{sp} = \]

- \( |p_0\rangle \)
- \( |\sigma_{0,1}\rangle \)
- \( |e_1\rangle \)
- \( |\sigma_{1,2}\rangle \)
- \( |p_2\rangle \)
- \( |\sigma_{2,3}\rangle \)
- \( |e_3\rangle \)
First error mitigation: Pauli Twirling

Converts **coherent errors** to **stochastic ones**

Example: CNOT gate

Simulating one-dimensional quantum chromodynamics on a quantum computer:
Real-time evolutions of tetra- and pentaquarks
Y. Y. Atas et al., 2207.03473 [quant-ph]
Early example of use in 1+1d SU(3) lattice simulation
Second error mitigation: Readout error mitigation

Error arising from readout typically \textbf{\textasciitilde20\%} albeit dependent on correlator value

Genuine 12-qubit entanglement on a superconducting quantum processor
Good example of inversion matrix method for readout error mitigation
Third error mitigation: Rescaling

Self-mitigating Trotter circuits for SU(2) lattice gauge theory on a quantum computer
First demonstration of rescaling in a lattice simulation
Fourth error mitigation: Dynamical Decoupling

Perform gates operators on spectators to **prevent noise**

For our lattice simulations on IBM devices, we found **XY4** the best balance

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Dynamical decoupling for superconducting qubits: a performance survey
N. Ezzell et al. 2207.03670 [quant-ph]
Nice state-of-the-art review
Putting it all together

Error mitigation allows up to $6x$ longer evolution
Multiple volumes, multiple masses

\[
\begin{array}{c}
G(\kappa) \\
\quad t/\epsilon
\end{array}
\]

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\begin{array}{c}
G(\kappa) \\
\quad t/\epsilon
\end{array}
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\begin{array}{c}
G(\kappa) \\
\quad t/\epsilon
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\begin{array}{c}
G(\kappa) \\
\quad t/\epsilon
\end{array}
\]

\[
\begin{array}{c}
\Delta a M \\
\quad a M_0
\end{array}
\]

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<th>(m_0)</th>
<th>(\epsilon)</th>
<th>(a t M)</th>
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Endgame

- The road to quantum practicality in HEP will be long and winding
- Error mitigation provides small, but non-trivial extensions in evolution time
- Scale setting directly on the quantum computer is possible
- Current QCIPU project is computing hadronic tensor in this theory