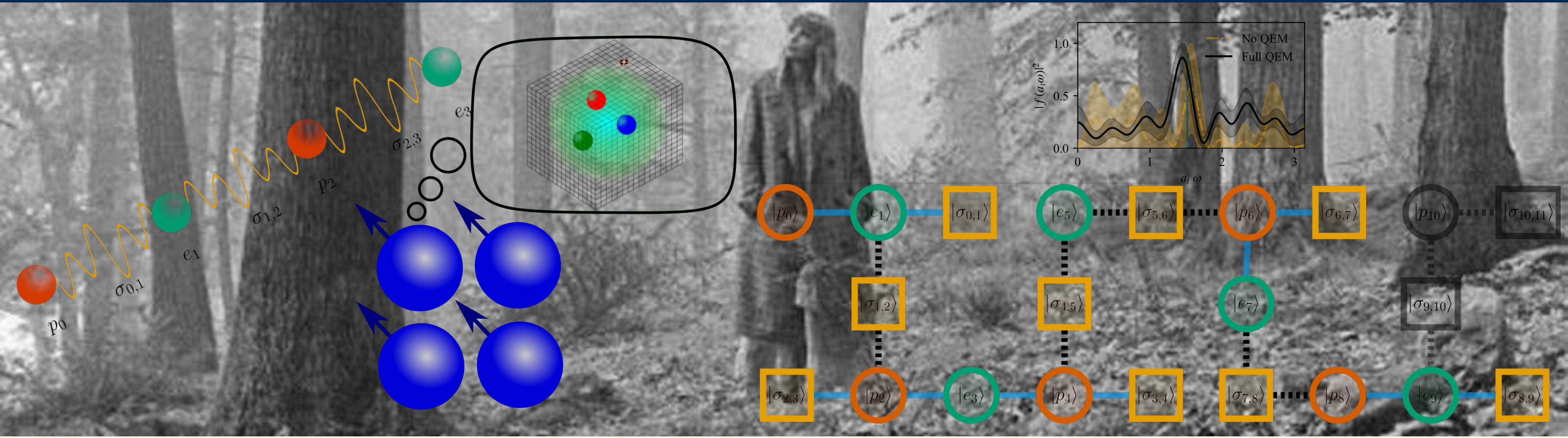


Simulating Z_2 lattice gauge theory on a quantum computer

Charles *et al.* - *2305.02361 [hep-lat]*

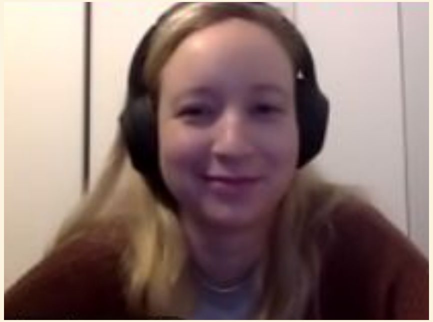
Hank Lamm

August 3, 2023

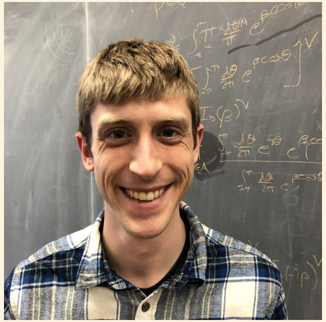
This work is an outcome of the QCIPU program @ Fermilab

Ruth Van de Water
Fermilab

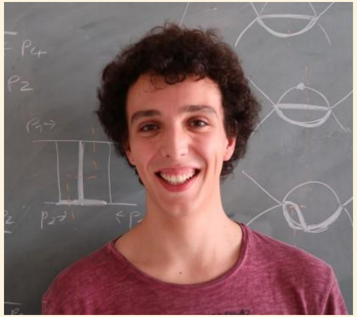
Clement Charles
→ Grad Student @ Maryland



Elizabeth Hardt
Argonne



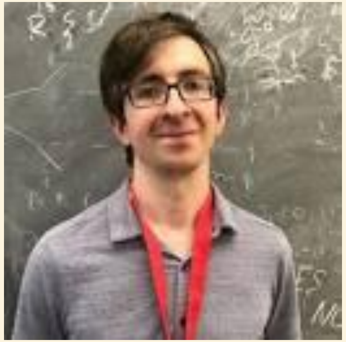
Michael Wagman
Fermilab



Florian Herren
Fermilab

Sara Starecheski
Undergrad @ UIUC

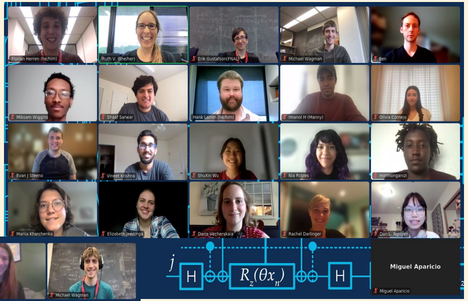
Norman Hogan
Grad Student @ NCSU



Erik Gustafson
NASA

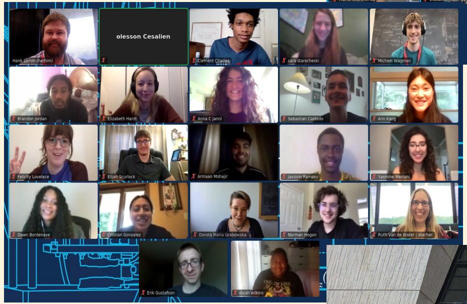
Quantum Computing Internship For Physics Undergraduates 2020 - Present

3-week summer school for 17 students + year-long internship for 4-5 students with goal to develop diverse quantum workforce with skills needed to succeed in academia and industry



Young field provides opportunity to build inclusive community

- Students paid competitive hourly wage
  - ***Essential*** to enable participation by students from all socioeconomic backgrounds.
- Topical lectures by experts in the field
  - Quantum physics & math, quantum algorithms, error mitigation & correction, quantum hardware. Self-contained and ***accessible to all preparation levels***.
- Pair programming on quantum simulators & real devices
  - Computational exercises in Python + Qiskit on classical and quantum algorithms. Final project simulating 1+1d lattice gauge theory on real devices.
- Panels and informal discussions on career opportunities
  - Panelists from ***both academia and industry***. Information about applying to and paying for graduate school especially important for first-generation college students.
- Year-long interns perform publishable research



PHYSICAL REVIEW D **106,** 114501 (2022)

Primitive quantum gates for an $SU(2)$ discrete subgroup: Binary tetrahedral

Erik J. Gustafson[1,*] Henry Lamm[1,†] Felicity Lovelace,[2,‡] and Damian Musk[3,§]
[1]*Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA*
[2]*Department of Physics, University of Illinois at Chicago, Chicago, Illinois 60607, USA*
[3]*Stanford University Online High School, Redwood City, California 94063, USA*

(Received 6 September 2022; accepted 9 November 2022; published 8 December 2022)

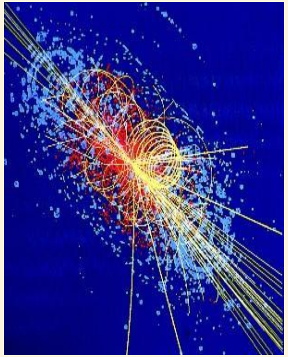
FERMILAB-PUB-23-171-SQMS-T

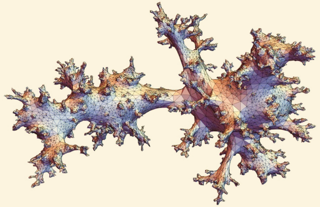
Simulating $\mathbb{Z}_2$ lattice gauge theory on a quantum computer

Clement Charles,[1,2] Erik J. Gustafson,[3,4,5] Elizabeth Hardt,[6,7] Florian Herren,[3] Norman Hogan,[8] Henry Lamm,[3] Sara Starecheski,[9,10] Ruth S. Van de Water,[3] and Michael L. Wagman[3]
[1]*Department of Physics, The University of the West Indies, St. Augustine Campus, Trinidad & Tobago*
[2]*Physics Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*
[3]*Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
[4]*Quantum Artificial Intelligence Laboratory (QuAIL), NASA Ames Research Center, Moffett Field, CA, 94035, USA*
[5]*USRA Research Institute for Advanced Computer Science (RIACS), Mountain View, CA, 94043, USA*
[6]*Department of Physics, University of Illinois at Chicago, Chicago, Illinois 60607, USA*
[7]*Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[8]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA*
[9]*Department of Physics, Sarah Lawrence College, Bronxville, NY 10708, USA*
[10]*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA*
(Dated: May 5, 2023)

Quantum Computing for Particle Physics, it's a need

- **The world is quantum, and we are lucky anything is amenable to classical computers**
  - Large-scale quantum computers can tackle computations in HEP otherwise inaccessible
  - This opens up new frontiers & extends the reach of LHC, LIGO, EIC & DUNE

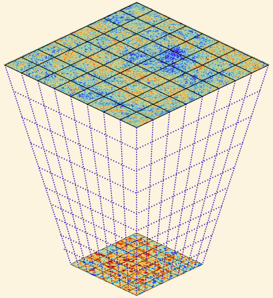


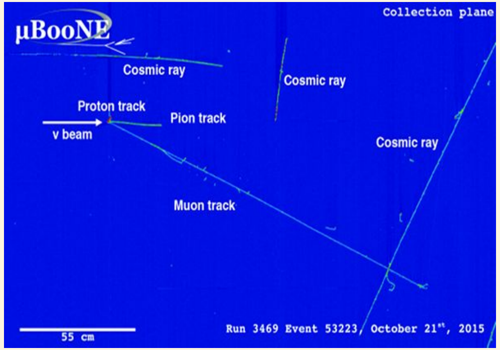


- ***Ab initio* cross sections for colliders and neutrino experiments**
- **Cosmic inflation and the evolution of matter asymmetry in the early universe**
- **Explorations of BSM, supersymmetry, and quantum gravity**
- **Hadronization and Hydrodynamics in Heavy-Ion collisions**

While broad, these topics often are formulated as **lattice field theories**

Quantum Simulation for High-Energy Physics
Bauer, Davoudi *et al.* - *PRX Quantum 4 (2023) 2, 027001*
Wonderful survey of physics questions, methods, and outstanding problems in field





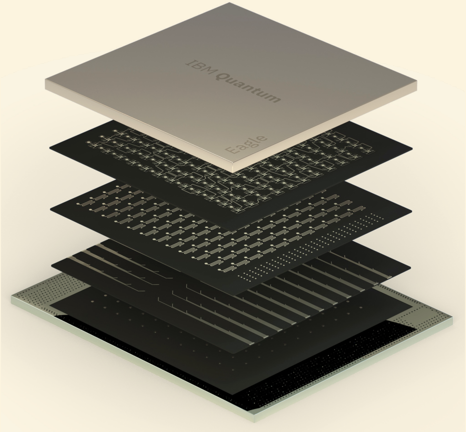
Premature optimization is the root of all evil

2 is the smallest and only even prime number

Premature optimization is the root of all evil

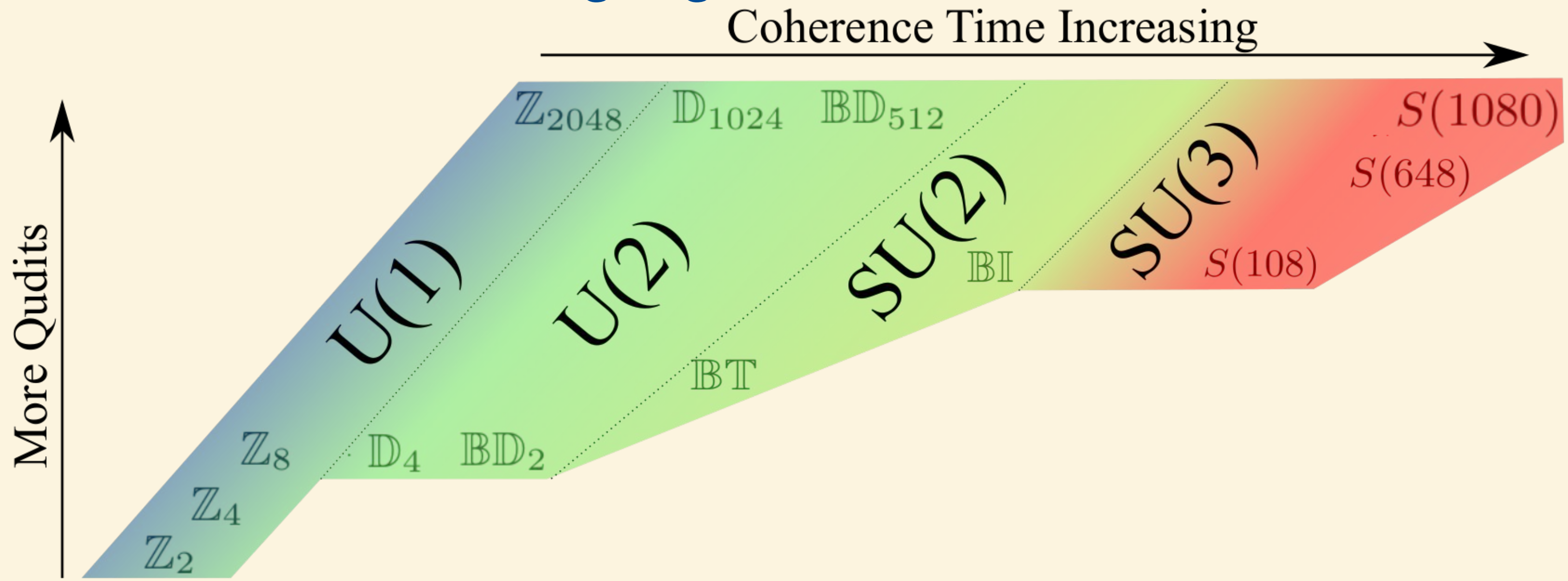
2 is the smallest and only even prime number

1+1d Z_2 lattice gauge theory on **quantum computer**



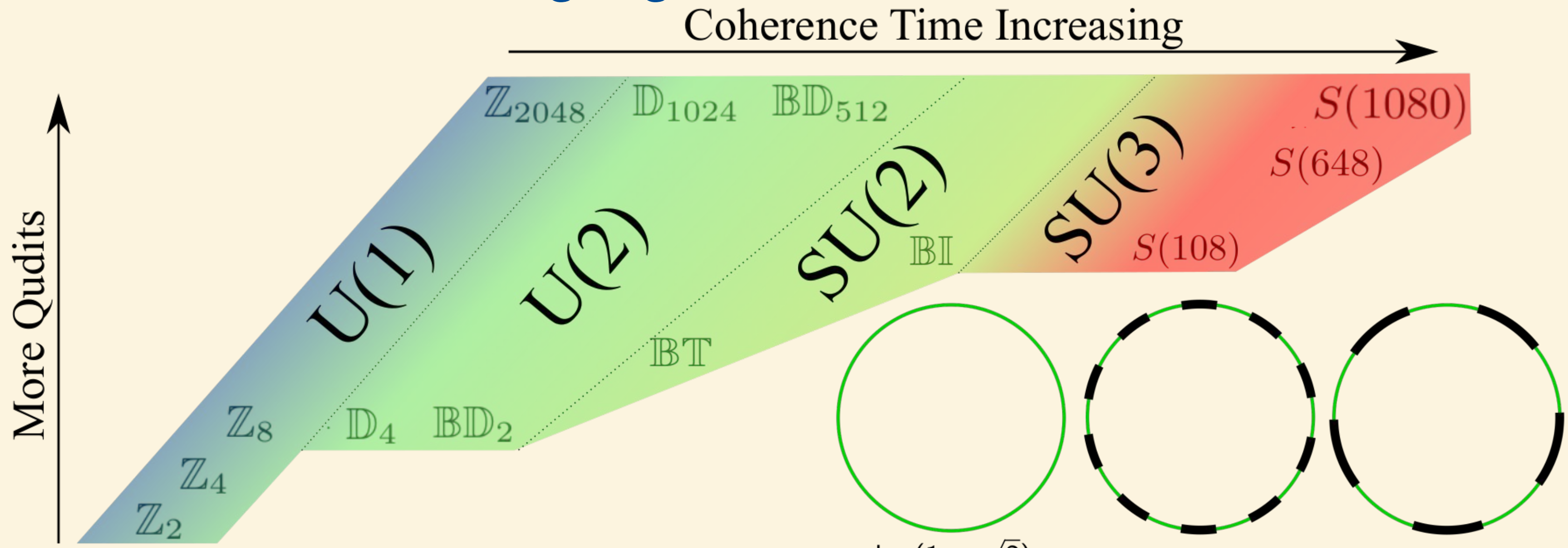
Experiments with a Gauge-Invariant Ising System
Michael Creutz, Laurence Jacobs, and Claudio Rebbi *Phys. Rev. Lett. 42, 1390 (1979)*
Early simulation of 3+1d 8^4 Euclidean Z_2 pure gauge theory

The ladder of discrete gauge theories in HEP calculations



Gluon Field Digitization for Quantum Computers
NuQS collaboration - *Phys.Rev.D 100 (2019) 11, 114501*
Demonstrated that S(1080) approximates certain 3+1d SU(3) observables

The ladder of discrete gauge theories in HEP calculations



$$\beta_{f,U(1)} = \frac{\log(1+\sqrt{2})}{1-\cos\left(\frac{2\pi}{N}\right)} \approx \kappa_2 N^2,\ \text{which extends to}\ \beta_{f,SU(N_c)} \approx \kappa N^{\frac{N_c^2-1}{2}}$$

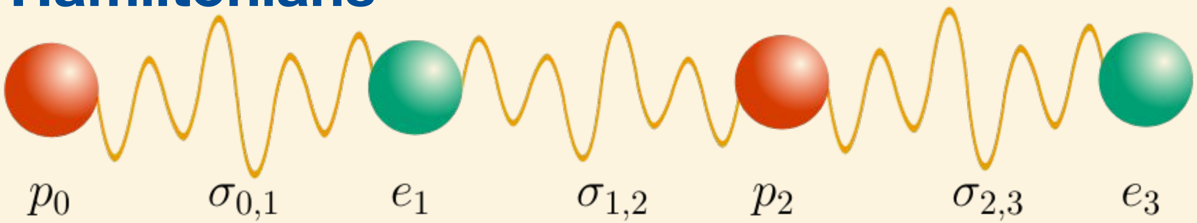
But whereas $\mathbb{Z}_N$ can be **taken to** ∞, **limited** number for $SU(N_c)$

$$\beta \propto \frac{1}{\log(a)} \implies a_f \propto e^{-\beta_f}$$

Gluon Field Digitization for Quantum Computers
NuQS collaboration - *Phys.Rev.D 100 (2019) 11, 114501*
Demonstrated that S(1080) approximates certain 3+1d SU(3) observables

Digitising SU(2) gauge fields and the freezing transition
Hartung *et al.* - *Eur.Phys.J.C 82 (2022) 3, 237*
Understanding the scaling of freezing transitions with approximations

A tale of two Hamiltonians



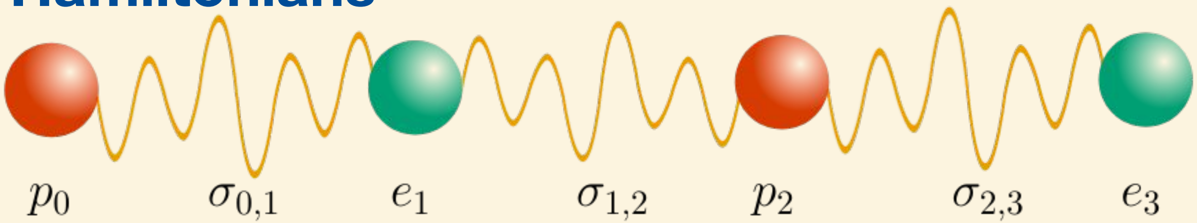
A tale of two Hamiltonians

Kogut-Susskind Hamiltonian [with O(a^2) errors]

$$H = \sum_{n=1}^{N_s-1}\left[\frac{1}{2}\sigma^x_{n,n+1} + \frac{\eta}{2}(\bar{\psi}_n\sigma^z_{n,n+1}\psi_{n+1} + h.c.)\right] + m_0\sum_{n=1}^{N_s}(-1)^n\bar{\psi}_n\psi_n,$$

Hamiltonian Formulation of Wilson's Lattice Gauge Theories
Kogut & Susskind *Phys.Rev.D 11 (1975) 395-408*
Formulated O(a^2) lattice Hamiltonian for LGT with staggered matter

A tale of two Hamiltonians



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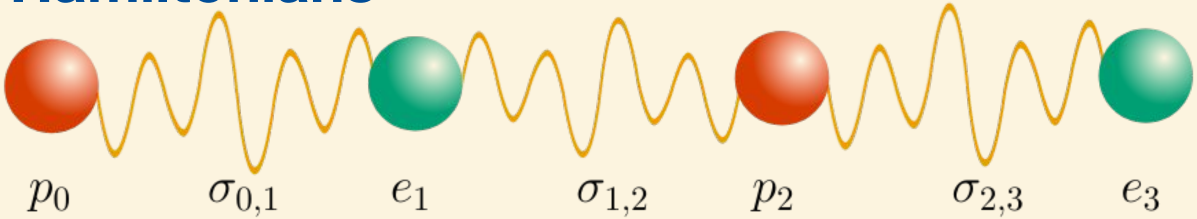
Qubit Hamiltonian via Jordan-Wigner

$$H = \frac{1}{2}\sum_{n=0}^{N_s-1}\sigma^x_{n,n+1} - \frac{m_0}{2}\sum_{n=0}^{N_s-1}(-1)^n Z_n$$

$$+ \frac{\eta}{4}\sum_{n=0}^{N_s-2}(X_nX_{n+1} + Y_nY_{n+1})\sigma^z_{n,n+1}.$$

A tale of two Hamiltonians



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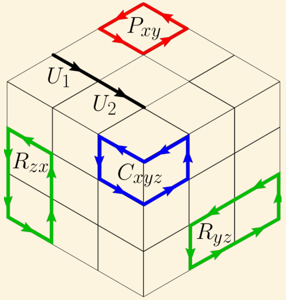
Hamiltonian Formulation of Wilson's Lattice Gauge Theories
Kogut & Susskind *Phys.Rev.D 11 (1975) 395-408*
Formulated O(a^2) lattice Hamiltonian for LGT with staggered matter

Alway remember: lattice Hamiltonian is **a choice**

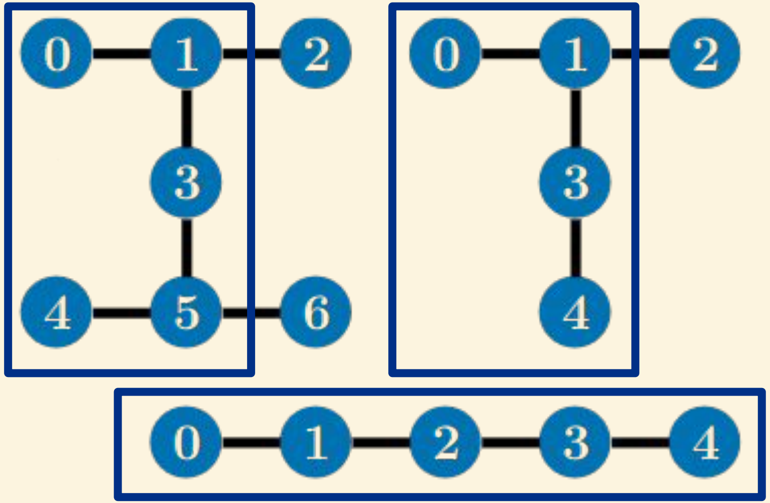
Improved Hamiltonians for Quantum Simulation of Gauge Theories
Carena, Lamm, Li, Liu *PRL 129 (2022) 5*
Developed quantum circuits for O(a^4) pure-gauge Hamiltonian

Quantum Simulation of Lattice QCD with Improved Hamiltonians
Ciavarella *2307.05593 [hep-lat]*
Formulated Hamiltonian with reduced truncation errors



Improved Fermion Hamiltonians for quantum simulations
Gustafson & Van de Water - in prep (Talk @ 4:20 PM on Thurs.)
Formulating Hamiltonians for ASQTAD fermions

Hamiltonian Gates for Trotterization with restricted connectivity

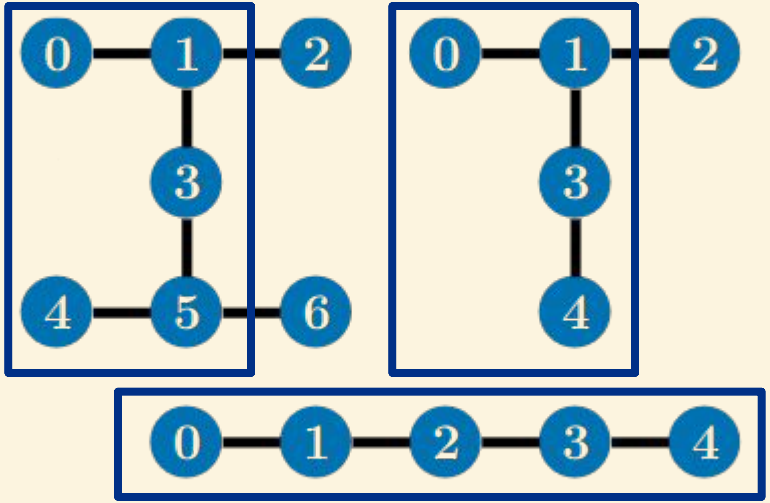


Restricting to longest linear graph

for heavy-polygon with p sides: $\frac{N_p - 2}{N_p - 1} \leq 86\%$ BAD!

$\texttt{ibm\_nairobi} \rightarrow 43\%$ WORSE!

Hamiltonian Gates for Trotterization with restricted connectivity



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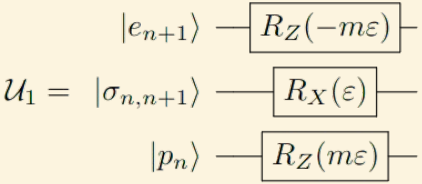

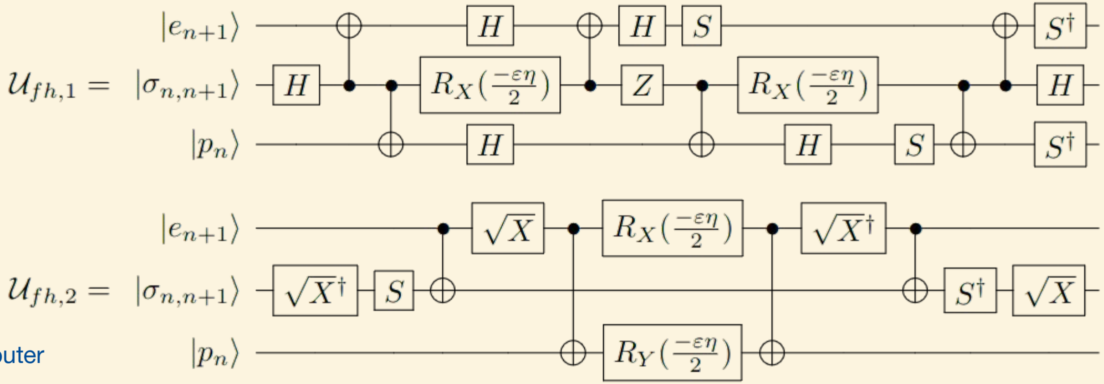

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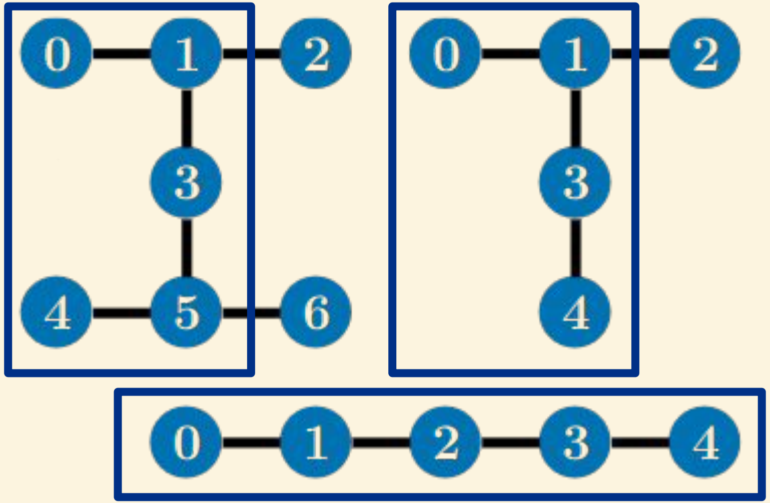

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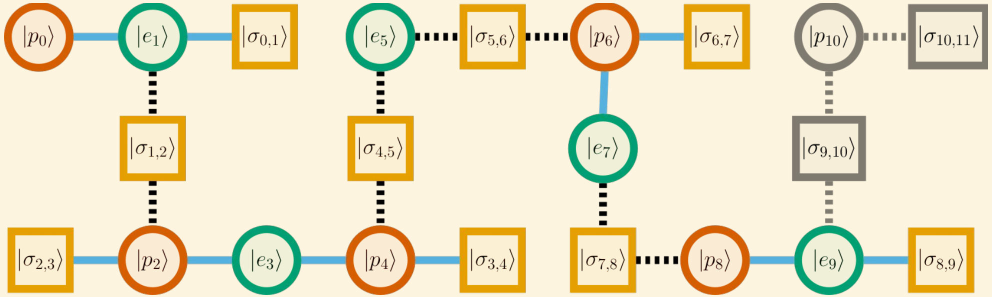

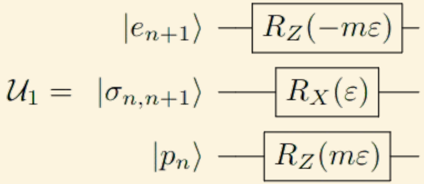

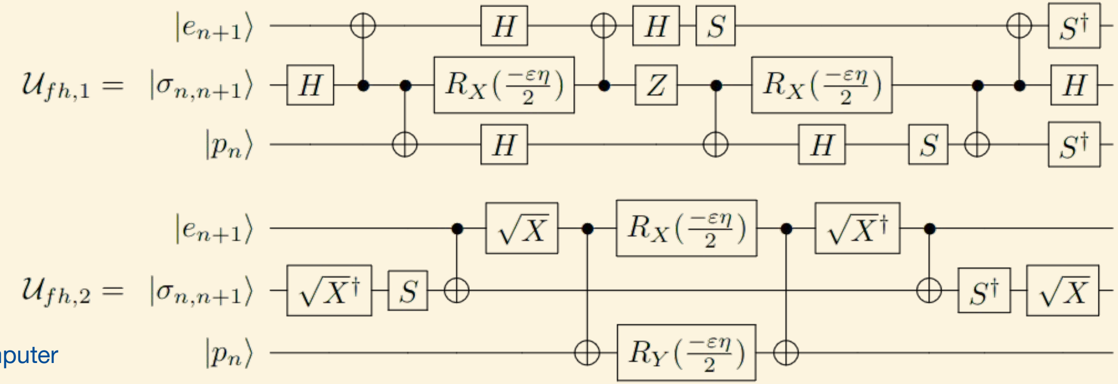

Performing scale setting with 2-pt Minkowski correlator

- Want to measure a correlator after preparing in a superposition of vacuum and "particle" state

$$
\begin{aligned}
C(t) &= \langle \phi(N_s) \big| \mathcal{U}^\dagger(t)\; O\; \mathcal{U}(t) \big| \phi(N_s) \rangle \\
&= \cos(Mt) + \ldots,
\end{aligned}
$$

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$$C(t) = \langle\phi(N_s)|\,\mathcal{U}^\dagger(t)\; O\; \mathcal{U}(t)\,|\phi(N_s)\rangle$$
$$= \cos(Mt) + \ldots,$$

- Trotteriztion introduces discretization errors into correlator, and thus scale setting M

$$\mathfrak{C}(t/\varepsilon) = \langle\phi(N_s)|\,\mathcal{U}^\dagger(t/\varepsilon)^{N_t}\; O\; \mathcal{U}(t/\varepsilon)^{N_t}\,|\phi(N_s)\rangle$$

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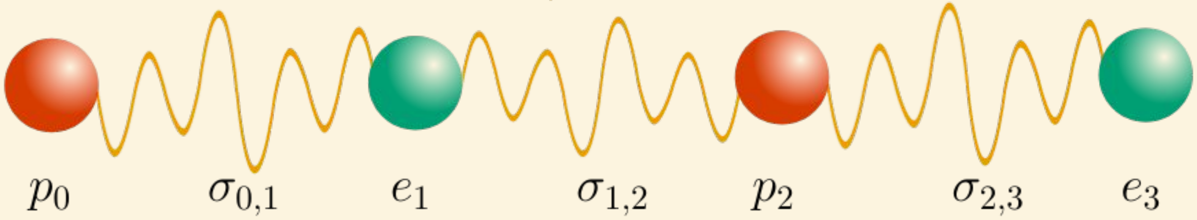
"Particle" state and operator insertion are given by **"meson" excitation** operator

$$O = X_n \sigma^z_{n,n+1} X_{n+1}$$



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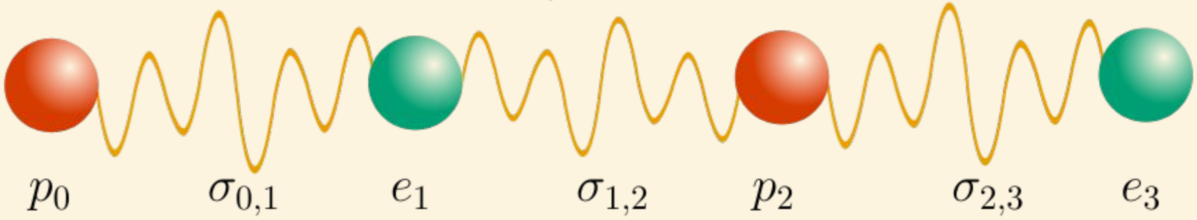
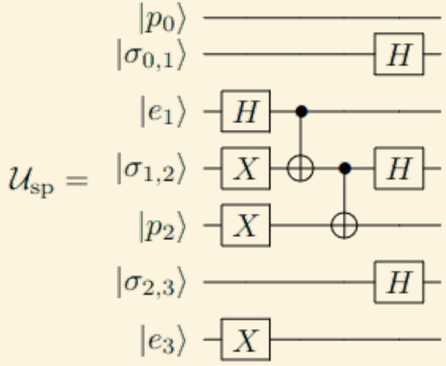
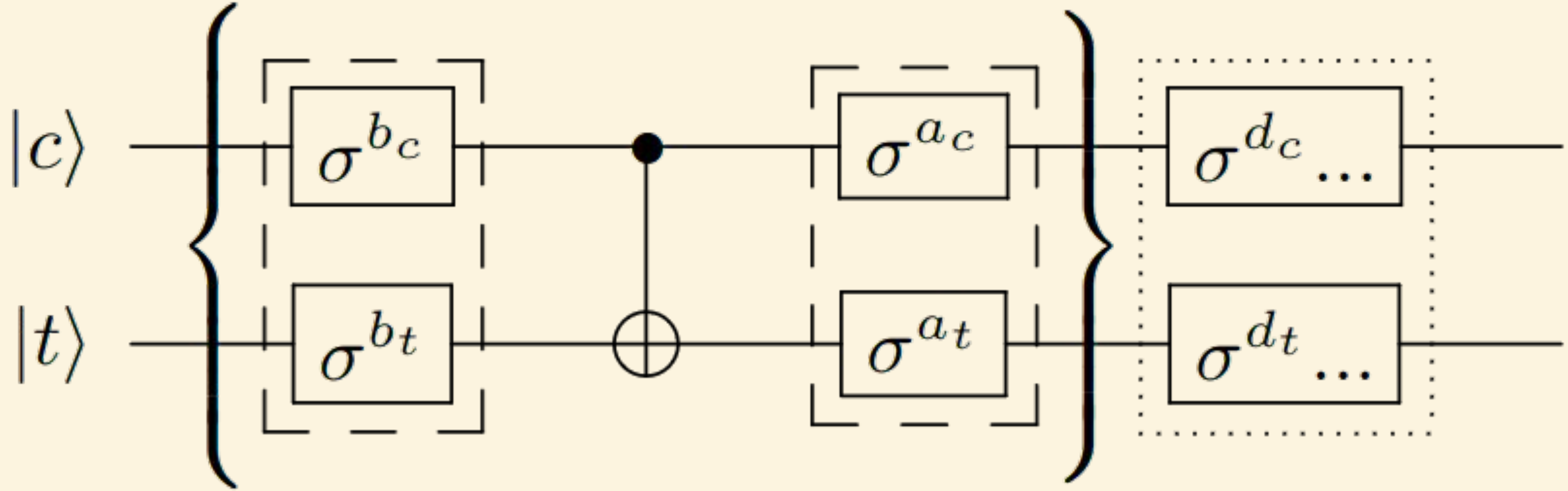




First error mitigation: Pauli Twirling

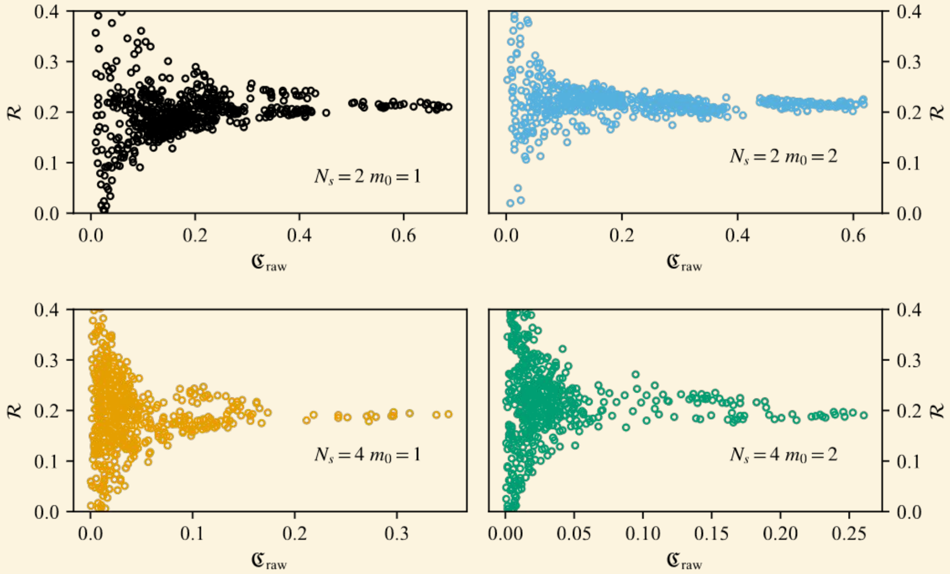
Converts **coherent errors** to **stochastic ones**

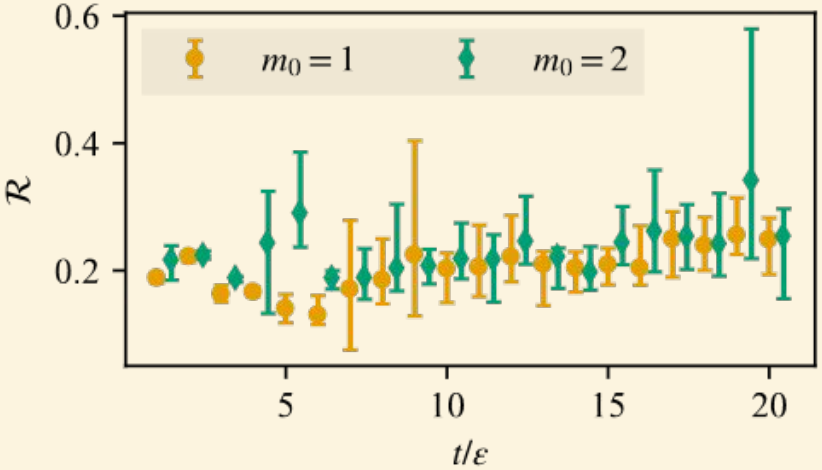
Example: CNOT gate



Simulating one-dimensional quantum chromodynamics on a quantum computer: Real-time evolutions of tetra- and pentaquarks
Y. Y. Atas *et al.*, 2207.03473 [quant-ph]
Early example of use in 1+1d SU(3) lattice simulation

Second error mitigation: Readout error mitigation

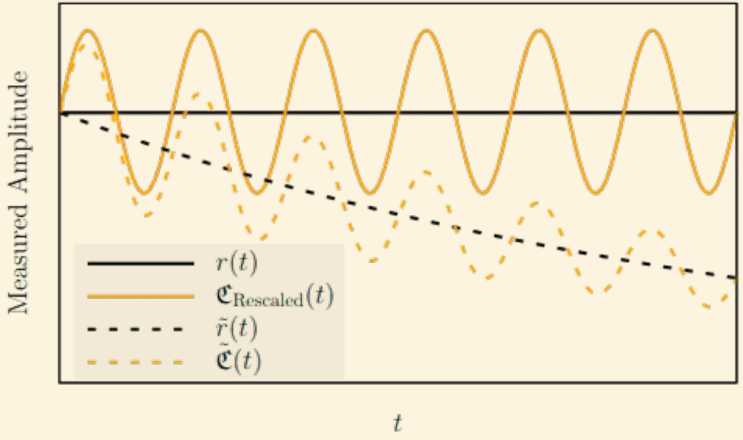




Error arising from readout typically **~20%** albeit dependent on correlator value

Genuine 12-qubit entanglement on a superconducting quantum processor
M. Gong et al. Phys. Rev. Lett. 122, 110501 (2019)
Good example of inversion matrix method for readout error mitigation

Third error mitigation: Rescaling



Self-mitigating Trotter circuits for SU(2) lattice gauge theory on a quantum computer
S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, Phys. Rev. D 106
First demonstration of rescaling in a lattice simulation

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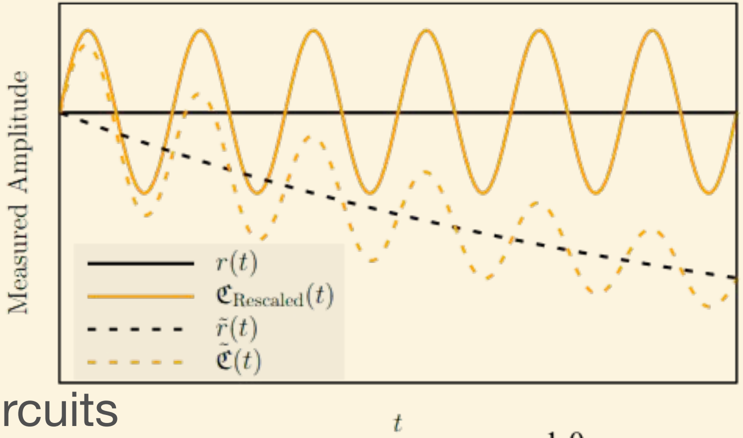
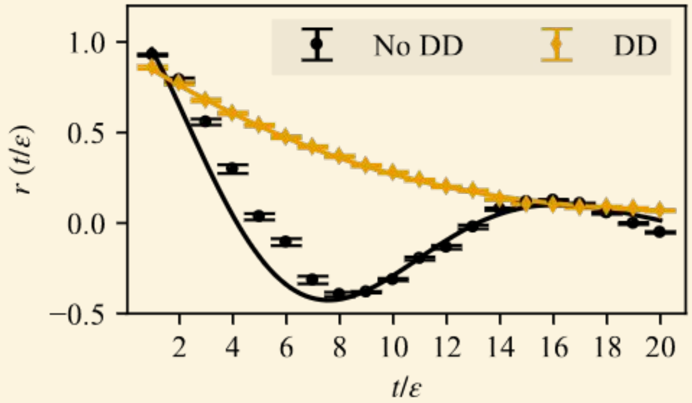
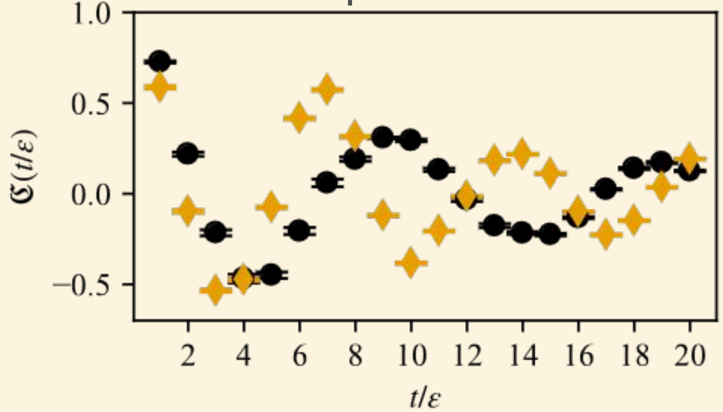


Rescaling circuits



2-pt correlator



Self-mitigating Trotter circuits for SU(2) lattice gauge theory on a quantum computer
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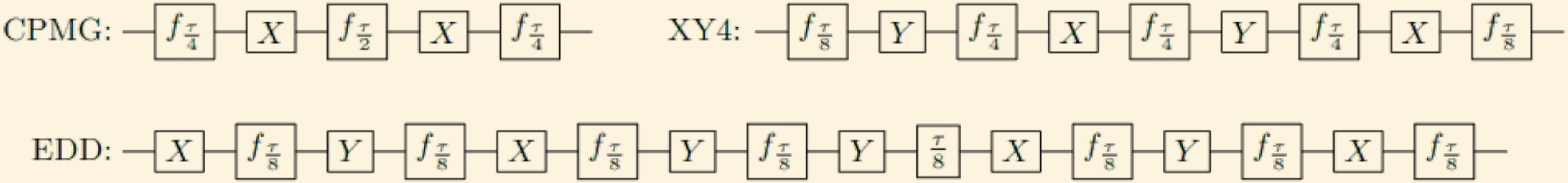
First demonstration of rescaling in a lattice simulation

Fourth error mitigation: Dynamical Decoupling

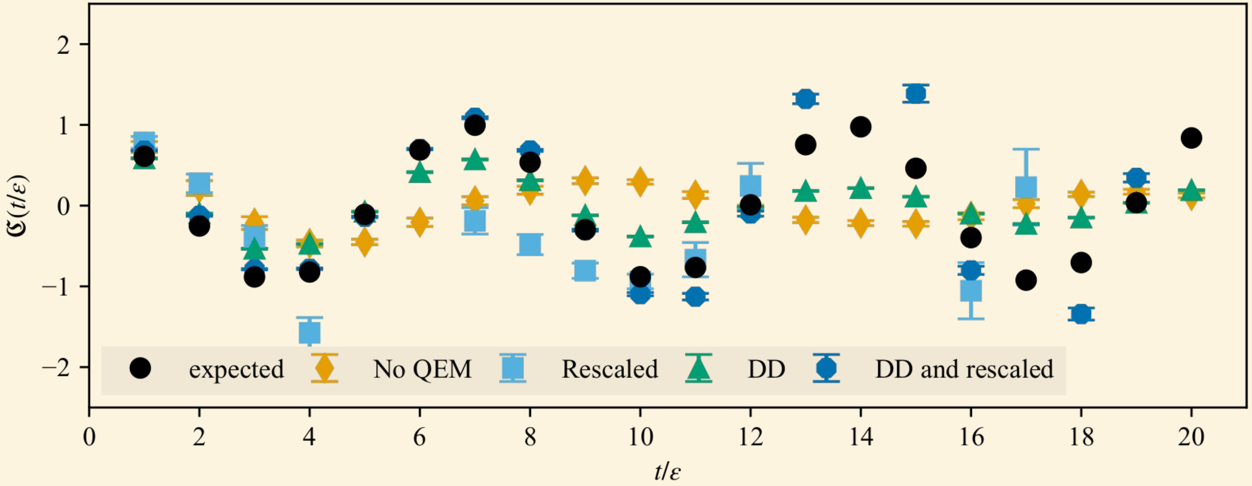
Perform gates operators on spectators to **prevent noise**



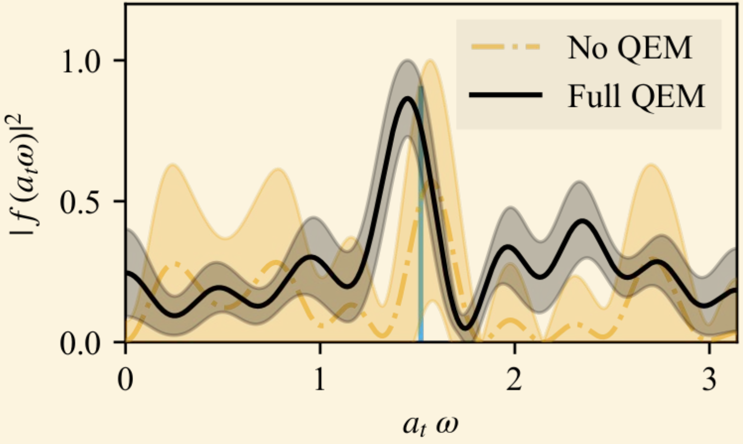
For our lattice simulations on IBM devices, we found **XY4** the best balance

Dynamical decoupling for superconducting qubits: a performance survey
N. Ezzell et al. *2207.03670 [quant-ph]*
Nice state-of-the-art review

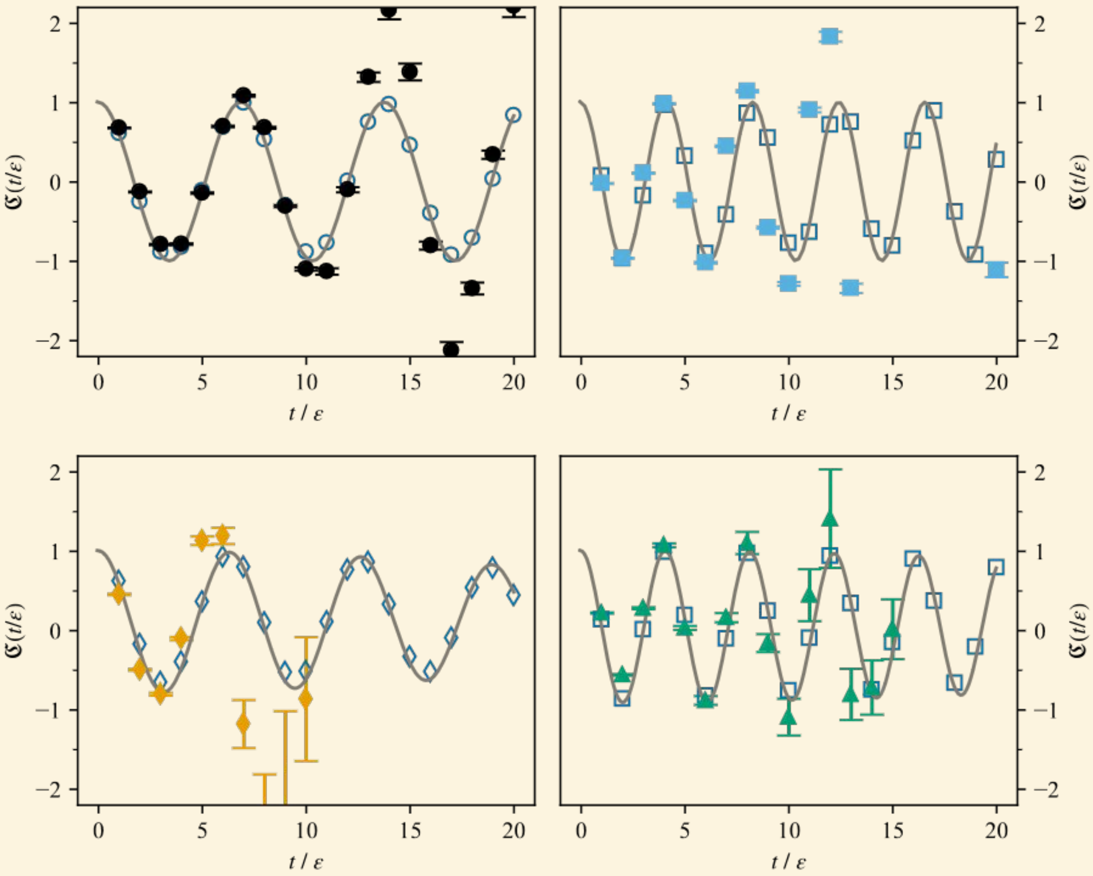
Putting it all together



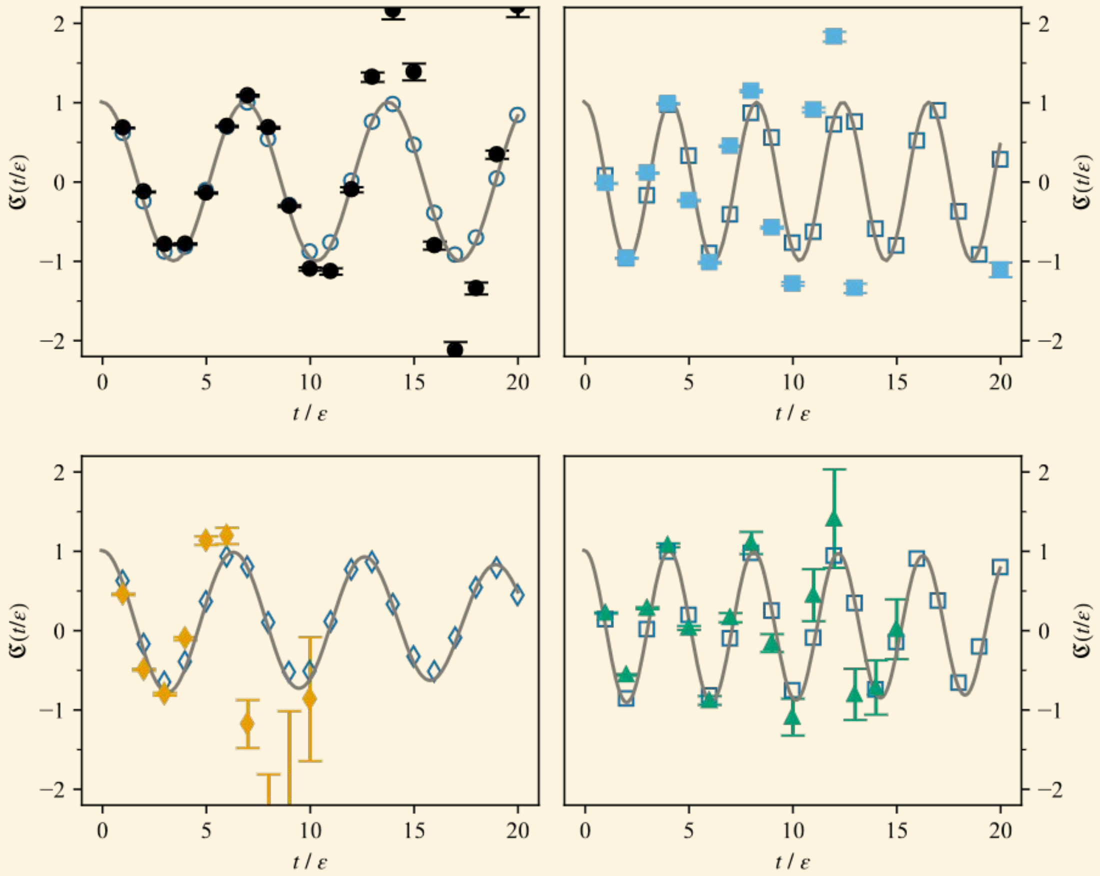
Error mitigation allows up to **6x** longer evolution

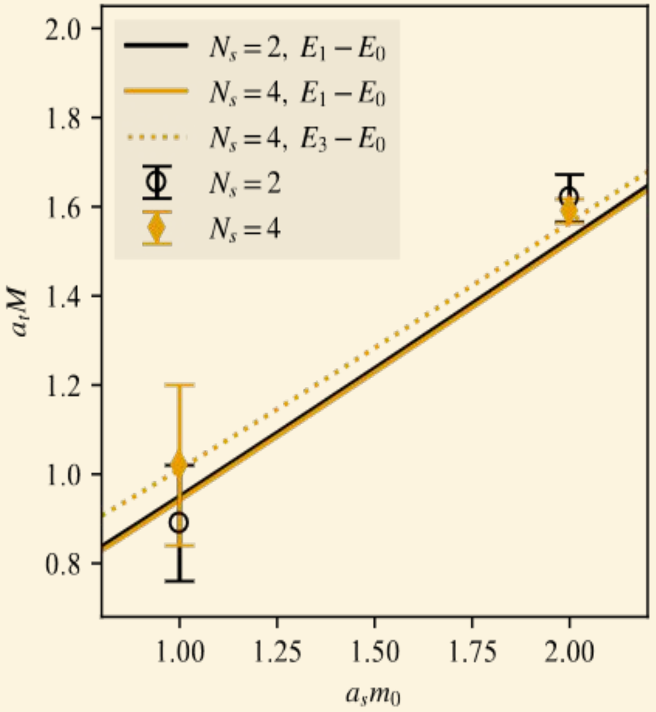


Multiple volumes, multiple masses



Multiple volumes, multiple masses

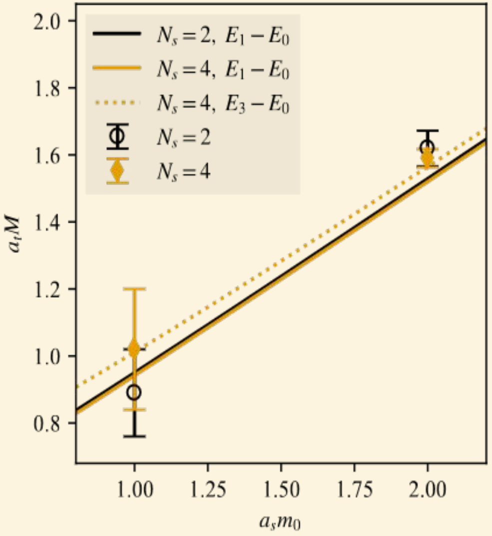




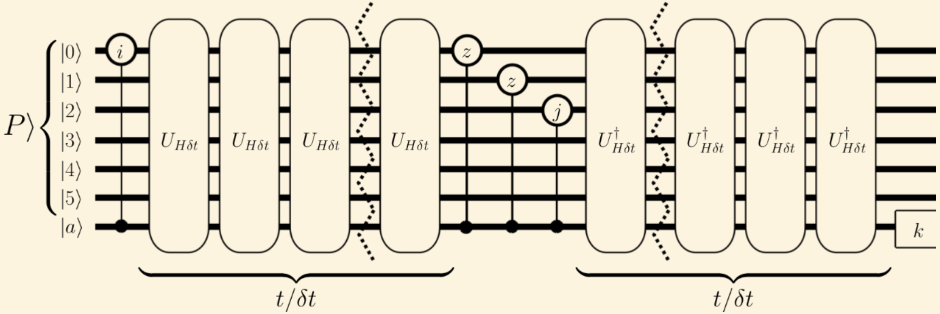
N_s	m_0	ε	$a_t M$	$(a_t M)_{\text{exact}}$
2	1.0	0.3	0.89(13)	0.9473
4	1.0	0.3	1.02(18)	0.9386
2	2.0	0.3	1.619(53)	1.5204
4	2.0	0.3	1.591(27)	1.5168

Endgame

- The road to quantum practicality in HEP will be **long** and **winding**
- Error mitigation provides small, but non-trivial **extensions in evolution time**
- **Scale setting directly** on the quantum computer is possible
- Current QCIPU project is computing **hadronic tensor** in this theory



$$\langle P|\chi^\dagger(tn^\mu)\chi(0)|P\rangle = \sum_{i,j,k=\{x,y\}} \frac{c_{ij}}{4}\langle P, a|U_{i,j,k}|P, a\rangle$$



Parton Physics on Quantum Computers
Lamm, Lawrence, Yamauchi - *Phys.Rev.Res* 2 (2020) 1, 013272
Formulation of Practical HEP Quantum Advantage Problem