Flavor diagonal nucleon charges from clover fermions on MILC HISQ ensembles

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Physics from flavor diagonal nucleon charges

- $\boldsymbol{g_A^q = \Delta q}$: Quark contributions to the nucleon spin

$$\frac{1}{2} = \sum_{u,d,s,\cdots} \left(\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\Delta q} + L_q\right) + J_g$$

X. Ji (1997),
L_q: orbital angular momentum of the quark
J_g: total angular momentum of the gluons

- $\boldsymbol{g_T^q}$: Quark EDM contributions to the neutron EDM d_n

nEDM collab. (2020)

$$|d_n| = |d_u^\gamma \boldsymbol{g_T^u} + d_d^\gamma \boldsymbol{g_T^d} + d_s^\gamma \boldsymbol{g_T^s} + \cdots| \le 1.8 \times 10^{-26}\ e\ \mathrm{cm}$$

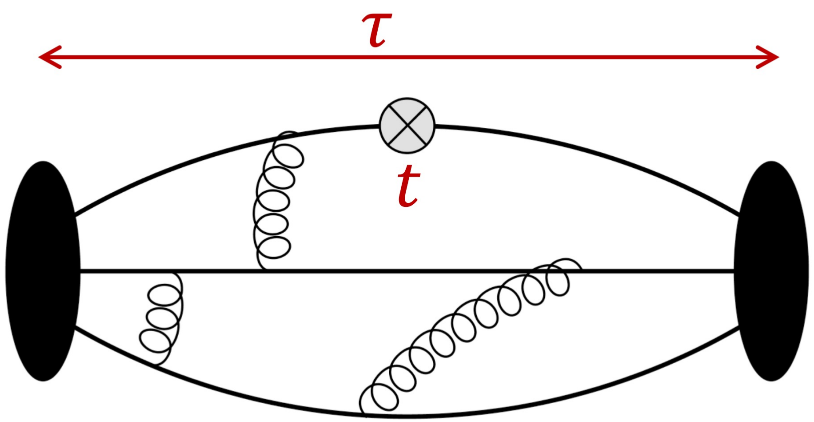
- $\boldsymbol{g_S^q = \frac{\partial M_N}{\partial m_q}}$: Slope of the nucleon mass with respect to the quark mass

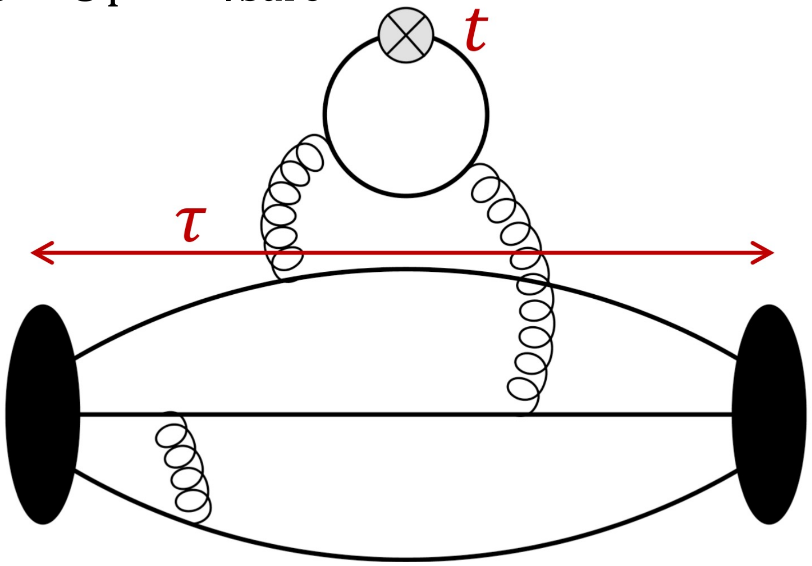
$\sigma_{\pi N} = m_l \boldsymbol{g_S^{u+d}}$: Quark contributions to the nucleon mass
$\sigma_s = m_s \boldsymbol{g_S^s}$

Connected and disconnected diagrams

- Calculation of flavor diagonal charges are complicated due to the additional contribution of the disconnected diagrams.

$$g_\Gamma^f|_{\text{bare}} = g_\Gamma^{f,\text{conn}}|_{\text{bare}} + g_\Gamma^{f,\text{disc}}|_{\text{bare}}$$





Calculated with covariant Gaussian source smearing, multiple source-sink separation $0.9 \lesssim \tau \lesssim 1.4$, accelerated with coherent sequential inversions and the truncated solver method with bias correction. [PNDME, PRD98, 034503 (2018)]

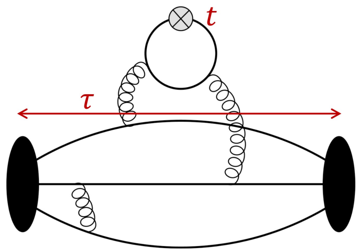
All-to-all quark propagator estimated by stochastic method using Z_4 random sources, accelerated with the truncated solver method with bias correction and hoping parameter expansion. [PNDME. PRD92,094511 (2015)]

Disconnected on 2+1+1-flavor HISQ Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	$N_{\rm conf}^{\rm conn}$	$N_{\rm conf}^{\rm disc}$ light/strange
a15m310	~0.15	320	3.93	1917	1917 / 1917
a12m310	~0.12	310	4.55	1013	1013 / 1013
a12m220	~0.12	228	4.38	744	958 / 870
a09m310	~0.09	313	4.51	2263	1017 / 1024
a09m220	~0.09	226	4.79	964	712 / 847
a09m130	~0.09	138	3.90	1290	1270 / 994
a06m310	~0.06	320	4.52	500	808 / 976
a06m220	~0.06	235	4.41	649	1001 / 1002

PNDME, PRD98, 034503 (2018)
: Statistics for connected diagrams

Analyzed for the disconnected diagrams



- Ensembles generated by MILC Collaboration
- 8 ensembles including one physical $M_\pi^{\rm phys}$ ensemble
- HYP smeared $N_f = 2 + 1 + 1$ MILC HISQ lattices,
- Clover fermion with a tree-level tadpole improved c_{SW}

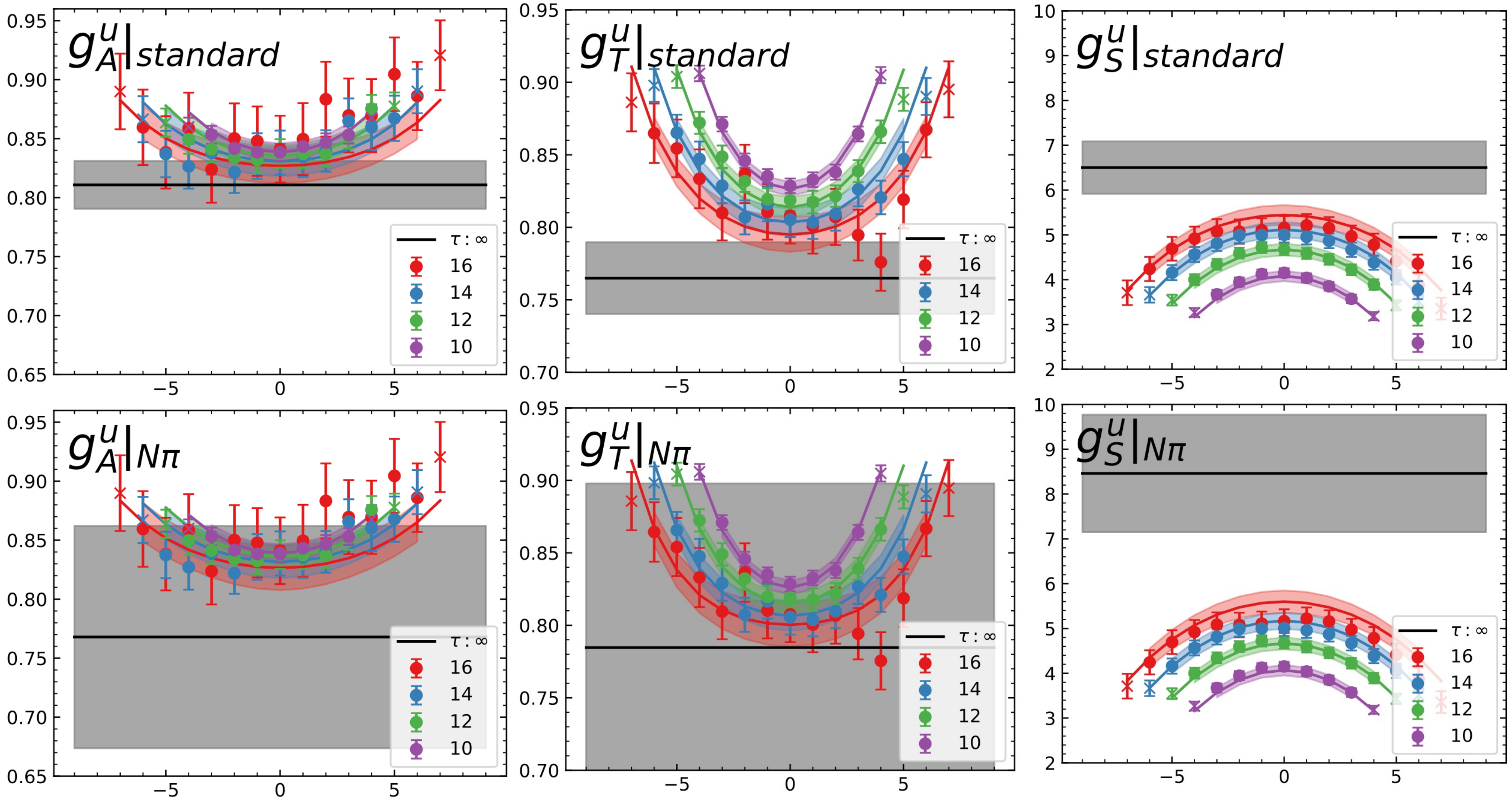
Removing excited state contaminations (ESC)

- Simultaneous fits to 2- and 3-point (connected + disconnected) functions using empirical Bayesian prior on the excited mass spectrum M_i and A_i

$$C^{2\text{pt}}(\tau) = \sum_{i=0} |\mathcal{A}_i|^2 e^{-M_i \tau}, \quad C_\Gamma^{3\text{pt}}(\tau;t) = \sum_{i,j=0} \mathcal{A}_i \mathcal{A}_j^* \langle i|O_\Gamma|j\rangle e^{-M_i t - M_j (t-\tau)},$$

- Repeat the analysis to quantify the model variation of the results by choosing different sets of (τ, t_{skip}) and number of states in the excited state fits (2 or 3^*-state fits)
  - t_{skip}: number of data points next to the source and the sink for each τ, skipped in the excited state fits
  - τ : source-sink separation
- The Final results are taken from the average over the model values, weighting each by its Akaike information criteria weights. [SP, *PoS* LATTICE2022 118]

ESC from $\boldsymbol{N\pi}$ and $\boldsymbol{N\pi\pi}$



[SP, *PoS* LATTICE2022 118]

- We carry out two types of analyses:
  1. The "**standard**" fit to $C_{2\text{pt}}(\tau)$ uses wide priors for all the excited-state amplitudes, A_i, and masses, M_i, to stabilize the fits.
  2. The "$\boldsymbol{N\pi}$" fit in which a narrow prior is used for M_1 with the central value given by the non-interacting energy of the lowest allowed $N\pi$ or $N\pi\pi$ state on the lattice

- For g_Γ^S, the leading multi-hadron ES is expected to be ΣK → "standard" analysis

Operator mixing calculation in RI-sMOM

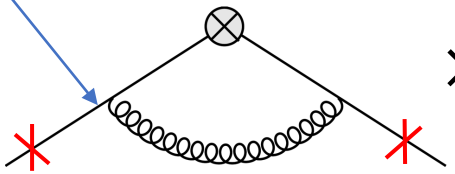
- We explicitly evaluated the 3×3 flavor (u, d, s) mixing matrices in **RI-sMOM**

$$g_\Gamma^f = \sum_{f'} Z_\Gamma^{ff'} g_\Gamma^{f'}|_{\mathrm{bare}}$$

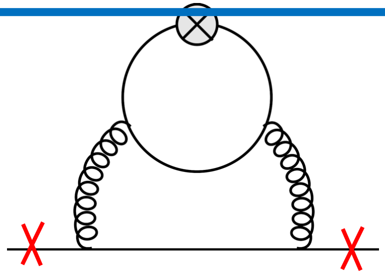
Landau gauge fixed quark propagators using momentum source with $p \propto (1,1,1,1)$

Projected amputated Green's function
$\mathrm{Tr}[(..)\mathbb{P}] \equiv \Lambda_\Gamma^{PA}$

$$(Z_\Gamma^{-1})^{ff'} = \sum_{f'} \frac{1}{Z_\psi^f} \mathrm{Tr}[($$



$$\times \delta^{ff'} -$$

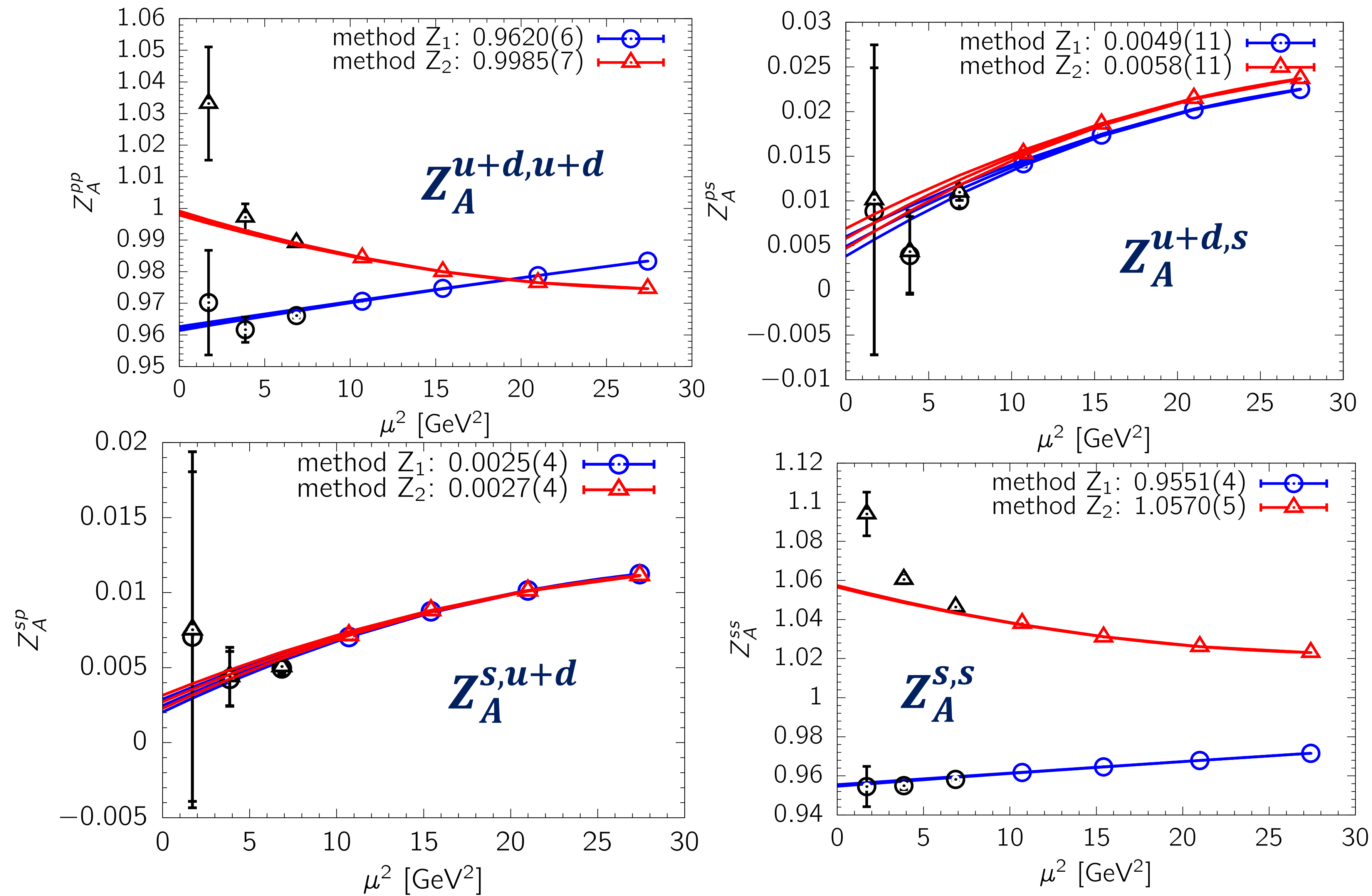


$$)\mathbb{P}(p', p)]$$

- **Z_1 method**: $Z_\psi(p) \equiv \frac{i}{12p^2} \mathrm{Tr}[S^{-1}(p) p \cdot \gamma]$
- **Z_2 method**: $Z_\psi^{\mathrm{VWI}}(p) \equiv \Lambda_V^{PA}(p)/g_V$

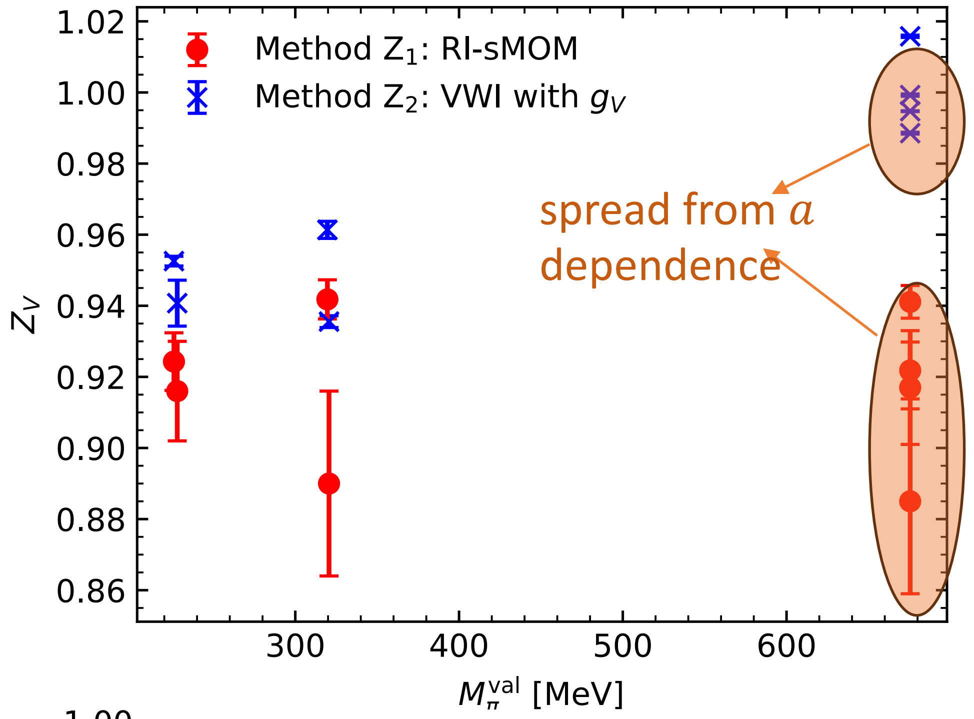
Using Vector Ward Identity (VWI),
$g_V Z_V = 1$
And g_V from separate nucleon matrix element calculation

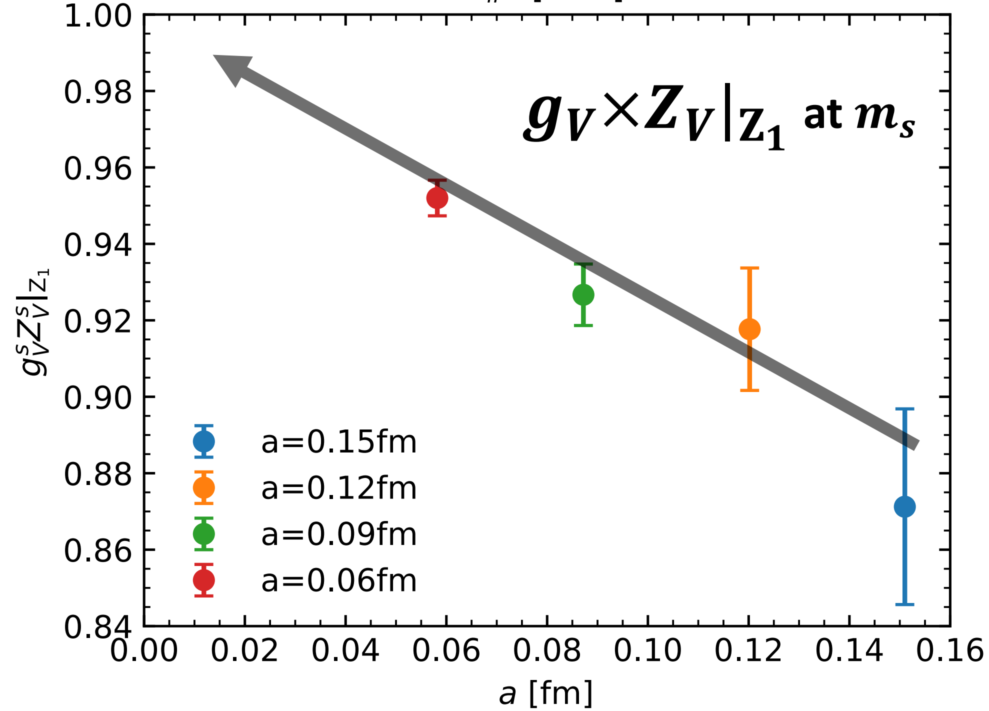
Ex) $Z_A^{\overline{\mathrm{MS}},2\mathrm{GeV}}(\mu) = \mathrm{Z} + c_1\mu^2 + c_2\mu^4$ extrapolation



- Removing $O(\mu^2 a^2)$ artifact after the perturbative matching and RG running to $\overline{\mathrm{MS}}$, 2GeV
- Diagonals of mixing matrix $(Z_A^{u+d,u+d}, Z_A^{s,s})$ show different μ-dependence (lattice artifact)
- Light and strange flavor mixing is a sub-percent contribution

$\rightarrow Z_\Gamma|_{Z_1} - Z_\Gamma|_{Z_2}$ becomes smaller as $a \rightarrow 0$





Z_V from methods $\mathbf{Z_1}$, $\mathbf{Z_2}$

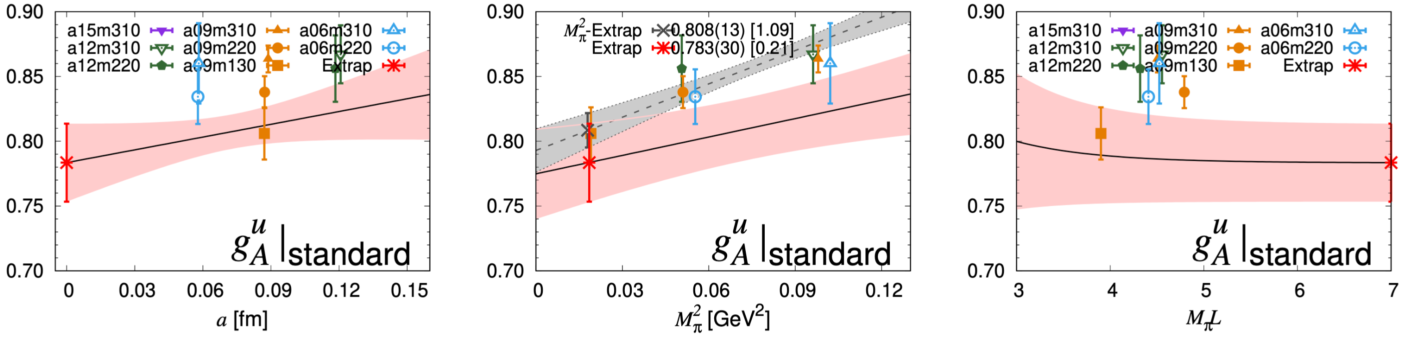
- $Z_V|_{Z_1}$ and $Z_V|_{Z_2}(= 1/g_V)$ have different M_π^{val} and a dependence
- $g_V \times Z_V|_{Z_1}$ deviates from 1 (Vector Ward-Identity) at large quark mass

- To study the systematic effect in two different methods, $\{\mathbf{Z_1}, \mathbf{Z_2}\}$ we do chiral-continuum extrapolate $g_\Gamma|_{Z_1}$ and $g_\Gamma|_{Z_2}$, separately, and compare the results.

Chiral-Continuum (Finite Volume) Extrapolation

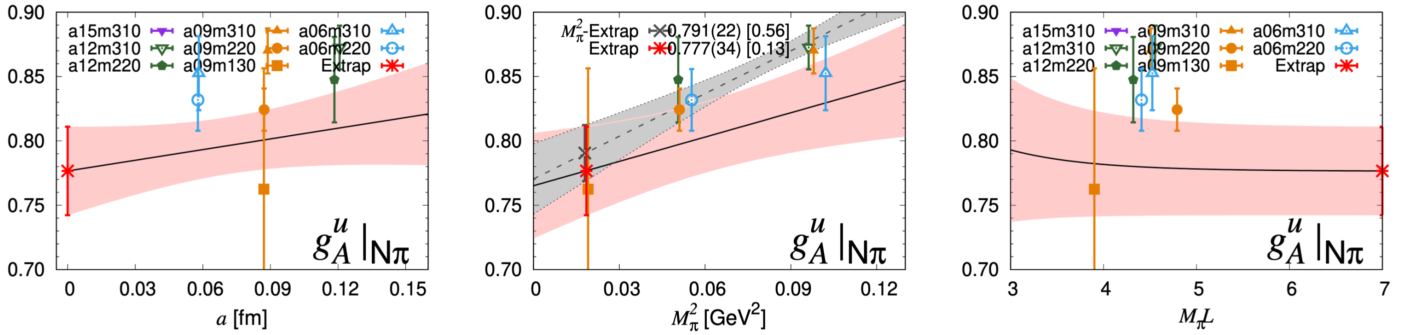
- We compare following CC(FV) extrapolation results:
  - ESC: “standard” vs “$N\pi$” analysis (except for g_Γ^S)
  - Renormalization: Method Z_1 vs Z_2
  - Extrapolation: CC vs CCFV

g_A^u extrapolation: ESC “standard” vs ”$N\pi$”



(f) $g_A^u|_{Z_2}$, $\{4\}$

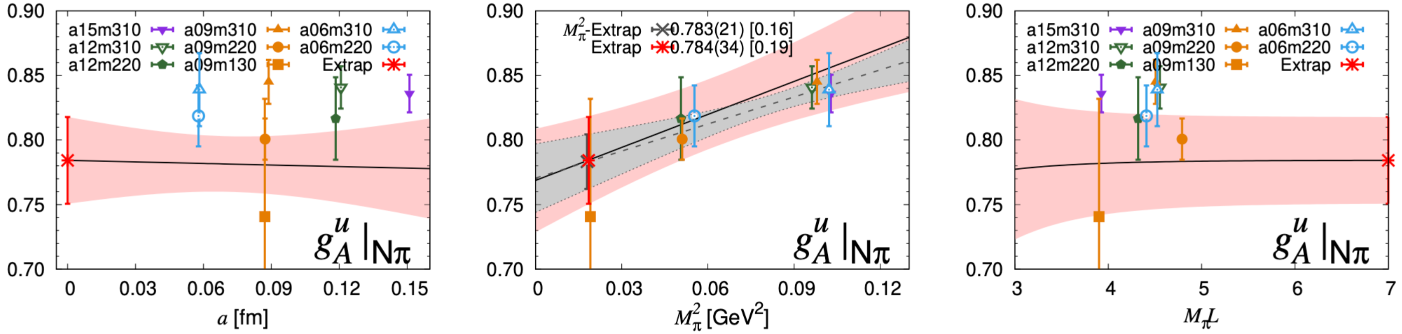
0.783(30)



(h) $g_A^u|_{Z_2}$, $\{4^{N\pi}\}$

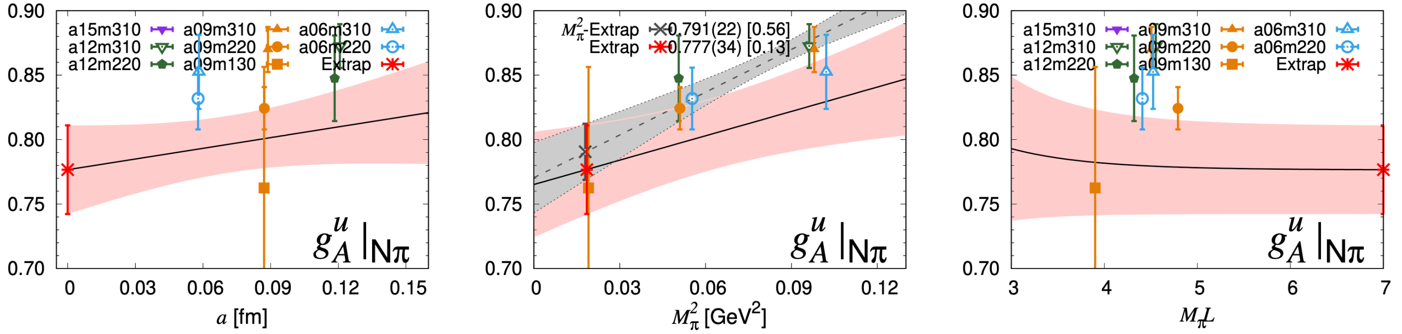
0.777(34)

g_A^u extrapolation: Renorm. "$\mathbf{Z_1}$" vs "Z_2"



(g) $g_A^u|_{Z_1}$, $\{4^{N\pi}\}$

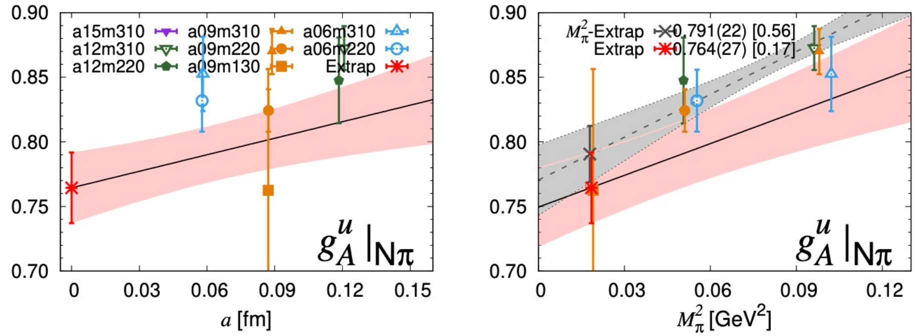
0.784(34)



(h) $g_A^u|_{Z_2}$, $\{4^{N\pi}\}$

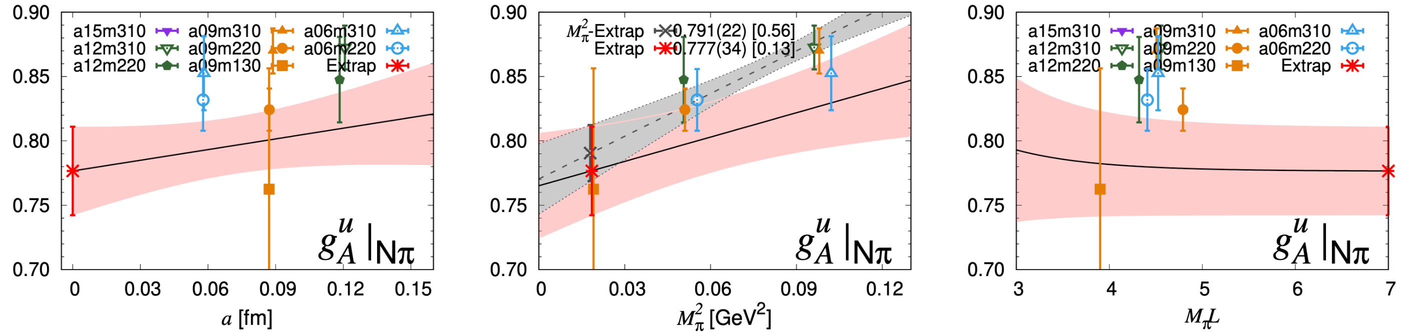
0.777(34)

g_A^u extrapolation: CC vs CCFV



(d) $g_A^u|_{Z_2}$, $\{4^{N\pi}\}$, $d_3 = 0$

0.764(27)



(h) $g_A^u|_{Z_2}$, $\{4^{N\pi}\}$

0.777(34)

Results past and present for g_Γ^q

	2023 (PRELIMINARY)			PNDME 2018	PRD 98, 091501 (2018) PRD 98, 094501 (2018)
q	g_A^q	g_T^q	g_S^q	g_A^q	g_T^q
u	0.780(34)(7)(8)(3)	0.784(28)(11)(0)(18)	8.8(13)(2)	0.777(25)(30)	0.784(28)(10)
d	-0.415(37)(2)(32)(37)	-0.202(12)(2)(16)(4)	8.7(9)(1)	−0.438(18)(30)	−0.204(11)(10)
s	-0.052(11)(2)(1)	-0.0016(12)(0)(1)	0.45(11)(3)	−0.053(8)	−0.00319(72)

(Error notation $g_A^u = 0.780(34)_{\mathbf{STAT}}(7)_{\mathbf{NPR}}(8)_{\mathbf{ESC}}(3)_{\mathbf{FV}}$)

- NPR and FV errors are all smaller or comparable to the statistical error
- ESC error is larger in $g_{A,T}^d$ than $g_{A,T}^u$
- FV effect for g_S^q is under investigation

- $g_S^{u,d}$: with "$N\pi$" analysis is motivated by the ChPT analysis of nucleon sigma term Gupta et al., PRL 127, 242002 (2021)

	2021 (st.)	2021 ($N\pi$)	2023 (st.)	2023 ($N\pi$)
$\sigma_{\pi N}$	41.9(4.9)	59.6(7.4)	44(5)(0)	60(6)(1)
σ_s			42(10)(3)	68(12)(4)

- m_{ud} and m_s taken from from FLAG 21

Summary

- We analyzed flavor diagonal nucleon charges using clover fermion on 8 MILC HISQ lattices
- Excited state fits: "standard" and "$N\pi$" analysis
  - $g_{A,T}^{u,d}$: not sensitive, $g_S^{u,d}$ ($\sigma^{\pi N}$): sensitive to the $N\pi/N\pi\pi$ state mass prior
- Renormalization:
  - Z_1 and Z_2 methods give consistent result
  - no significant flavor mixing, especially at smaller a
- Chiral-continuum extrapolation
  - Finite volume correction is small for $M_\pi L > 4$
  - Leading chiral logarithm $M_\pi^2 \log M_\pi^2$: cannot resolve
- In progress
  - Comparison with clover-on-clover calculation
  - More statistics for the physical pion mass MILC HISQ ensemble $a09m130$

Acknowledgements

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