Meron-Cluster Algorithms for Quantum Link Models

Joao C. Pinto Barros
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Together with: T. Budde and M. K. Marinkovic
Outline

1. What model and Why

2. Meron Cluster Algorithms

3. Satisfying Gauss’ Law
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1. **What** model and **Why**

2. Meron Cluster Algorithms

3. Satisfying Gauss’ Law
What is it about?

- An efficient algorithm to simulate fermions coupled to (Abelian) gauge fields;
- Operates on the Hamiltonian formalism,
- Main challenges: satisfying Gauss’ law and deal with the fermion sign problem.

E. Huffman’s talk: (Thursday, Algorithms, 17:20)

- Gauss’ law satisfied with post-selection of configurations on the target sector;
- Sign problem solved using Meron-cluster algorithm approach.
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This talk: (Soon on arXiv)
• Gauss’ law satisfied by building configurations constructively;
• Sign problem solved as a consequence of satisfying Gauss’ law (strictly 1 + 1-d).
What models are considered?

Abelian gauge theories in $1 + 1$ dimensions.

For $U(1)$ quantum link models with spin $S$:

\[
H = -t \sum_n c_n^+ S_n^+ c_{n+1} + \text{h.c.} + m \sum_n (-1)^n c_n^+ c_n + g \sum_n (S_n^z)^2 + U \sum_n (c_n^+ c_n - 1/2) (c_{n+1}^+ c_{n+1} - 1/2)
\]

- We can simulate any spin $S$ can be made arbitrary;
- In the worst case, it scales like $\mathcal{O}(S^2)$;
- Very efficient for $m = 0$ but can work at arbitrary $m$. Restricted to $t = 2U$. 
Why it is important

- Provide reliable benchmarks for Quantum Simulators;

- Provide a step toward the construction of efficient algorithms for more complicated theories (e.g. QCD);

- Cross-validation and better performance over other methods (e.g. tensor networks for large spin $S$);

- Construction of the algorithm by re-writing the partition provides a new way of describing the model;
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Mechanics of the Meron-Cluster Algorithm

Construction of the path integral

\[ \langle O \rangle = \lim_{\beta \to \infty} \frac{\text{Tr}(O e^{-\beta H})}{\text{Tr}(e^{-\beta H})} \]
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Configuration space:

S. Chandrasekharan, U.-J. Wiese - PRL, 1999
E. Huffman, D. Banerjee - arXiv:2305.08917
How to Include Gauge Fields?

Simplest case: spin 1/2 Quantum Link Model

\[ H = -t \sum_n c_n^\dagger \sigma_n^z c_{n+1} + \text{h.c.} + m \sum_n (-1)^n c_n^\dagger c_n + U \sum_n (c_n^\dagger c_n - 1/2) (c_{n+1}^\dagger c_{n+1} - 1/2) \]

All we need to take care of is to satisfy Gauss’ law

\[ G_n = E_{n+1} - E_n - \rho_n, \quad G_n \ket{\psi} = 0. \quad \rho_n = c_n^\dagger (n) c(n) + \frac{1 - (-1)^n}{2} \]
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Positive and negative charges always alternate at every time slice;
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Clusters can be classified according to their contribution to the total number of charges in the system:

- **Neutral** clusters: bring no net charge into the system;
- **Positive**: bring net positive charge into the system;
- **Negative**: bring net negative charge into the system;
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Neutral clusters are non-winding;
Charged clusters wind;
Positive and negative charged clusters alternate.
Satisfying Gauss’ in the Spin 1/2 Quantum Link Model
Conditional Flipping: The Meron-Automaton Solution

Topological structure of the clusters is encoded in a tree. Charge clusters are at the top and are associated with neighboring clusters.
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Neutral cluster combinatorics: exhaust all valid possibilities as we follow the tree.

Charged cluster combinatorics: sample words from a regular language (recognized by a finite-state automaton).
Symmetry Breaking on the Spin 1/2 Quantum Link Model

Small mass limit: Pair creation is common, **Symmetric phase**

Large mass limit: Pair creation is rare, **CP-broken phase**

\[ \chi_\epsilon = \left\langle \left( \frac{1}{L} \sum_n \sigma_n^z \right)^2 \right\rangle \]

Non-universal critical mass:

\[ m_c \sim 0.24 \]

2d Ising critical exponents give curve collapse:

\[ \nu = 1, \beta = 1/8 \]
Conclusions

The Algorithm

Construction of a cluster algorithm that can update efficiently fermionic and gauge degrees of freedom satisfying Gauss' law.

- The algorithm is generalizable to arbitrary spin $S$;
- Study the approach to infinite spin - Schwinger model.
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- The algorithm is generalizable to arbitrary spin $S$;

- Study the approach to infinite spin - Schwinger model.

- Can a similar algorithm be constructed in higher dimensions?

- Can we simulate other types of constrained systems (e.g. canonical ensemble, which has fixed particle number)?
Conclusions
Overview

**Classical computers** face fundamental challenges (e.g. real-time evolution).

**Quantum simulators** hold the promise of addressing these issues.

BUT

**Improvement is not impossible.**

In fact, it is crucial to **complement**, **guide**, and **validate** quantum simulators.