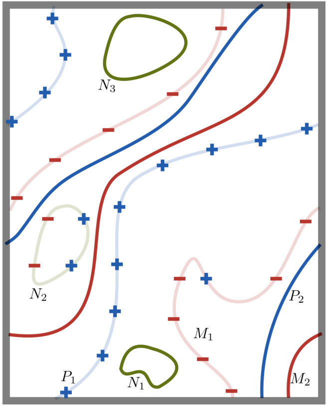
ETH zürich

ETH, Institute for Theoretical Physics
High Performance Computational Physics group

Meron-Cluster Algorithms for Quantum Link Models

Joao C. Pinto Barros
Lattice 2023, Fermilab
Together with: **T. Budde** and **M. K. Marinkovic**



Outline

1. **What** model and **Why**

2. Meron Cluster Algorithms

3. Satisfying Gauss' Law

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What is it about?

- An efficient algorithm to simulate fermions coupled to (Abelian) gauge fields;
- Operates on the Hamiltonian formalism,
- Main challenges: satisfying Gauss' law and deal with the fermion sign problem.

E. Huffman's talk: (Thursday, Algorithms, 17:20)

- Gauss' law satisfied with post-selection of configurations on the target sector;
- Sign problem solved using Meron-cluster algorithm approach.

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This talk: (Soon on arXiv)

- Gauss' law satisfied by building configurations constructively;
- Sign problem solved as a consequence of satisfying Gauss' law (strictly $1+1$-d).

Thea Budde 

Marina Krstic Marinkovic 

What models are considered?

Abelian gauge theories in $1+1$ dimensions.

For $U(1)$ quantum link models with spin S:

$$H = -t\sum_n c_n^\dagger S_n^+ c_{n+1} + \text{h.c.} + m\sum_n (-1)^n c_n^\dagger c_n + g\sum_n (S_n^z)^2 + U\sum_n \left(c_n^\dagger c_n - 1/2\right)\left(c_{n+1}^\dagger c_{n+1} - 1/2\right)$$

- We can simulate any spin S can be made arbitrary;
- In the worst case, it scales like $\mathcal{O}\left(S^2\right)$;
- Very efficient for $m=0$ but can work at arbitrary m. Restricted to $t=2U$.

Why it is important

- Provide reliable benchmarks for Quantum Simulators;
- Provide a step toward the construction of efficient algorithms for more complicated theories (e.g. QCD);
- Cross-validation and better performance over other methods (e.g. tensor networks for large spin S);
- Construction of the algorithm by re-writing the partition provides a new way of describing the model;

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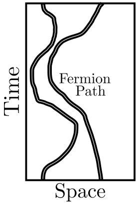
Mechanics of the Meron-Cluster Algorithm

Construction of the path integral $\langle \mathcal{O} \rangle = \lim_{\beta\to\infty} \frac{\mathrm{Tr}\left(\mathcal{O} e^{-\beta H}\right)}{\mathrm{Tr}\left(e^{-\beta H}\right)}$



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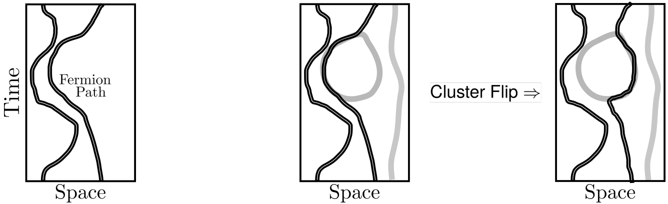

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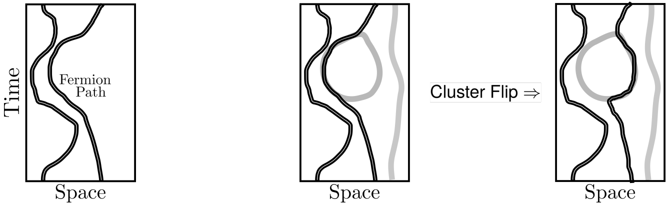
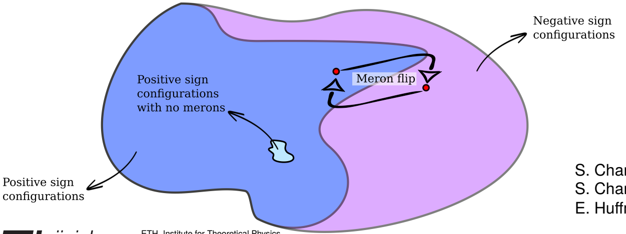


Configuration space:



S. Chandrasekharan, U.-J. Wiese - PRL, 1999
S. Chandrasekharan, J. Cox, J. C. Osborn, U.-J. Wiese - NPB, 2003
E. Huffman, D. Banerjee - arXiv:2305.08917

How to Include Gauge Fields?

Simplest case: spin 1/2 Quantum Link Model

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All we need to take care of is to satisfy Gauss' law

$$G_n = E_{n+1} - E_n - \rho_n, \quad G_n \left|\psi\right\rangle = 0. \quad \rho_n = c^\dagger(n)\, c(n) + \tfrac{1-(-1)^n}{2}$$

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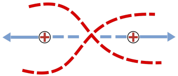
Positive and negative charges always alternate at every time slice;



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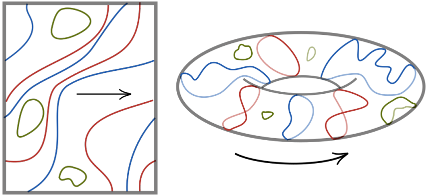
Satisfying Gauss' Law in the Spin 1/2 Quantum Link Model

- Clusters can be classified according to their contribution to the total number of charges in the system:
  - **Neutral** clusters: bring no net charge into the system;
  - **Positive**: bring net positive charge into the system;
  - **Negative**: bring net negative charge into the system;



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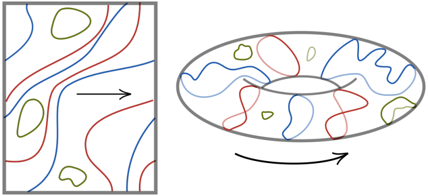
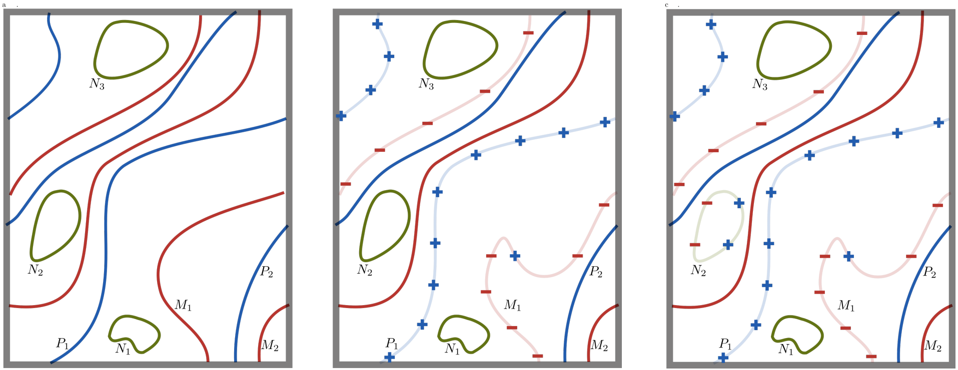


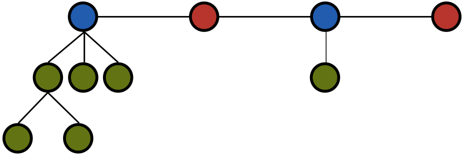
- Neutral clusters are non-winding;
- Charged clusters wind;
- Positive and negative charged clusters alternate.

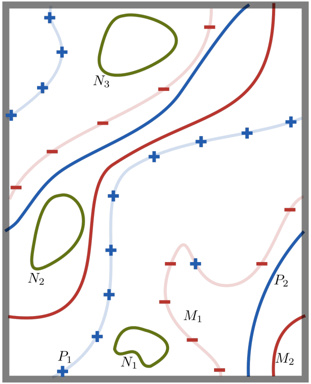
Satisfying Gauss' in the Spin 1/2 Quantum Link Model



Conditional Flipping: The Meron-Automaton Solution

Topological structure of the clusters is encoded in a tree.
Charge clusters are at the top and are associated with neighboring clusters.





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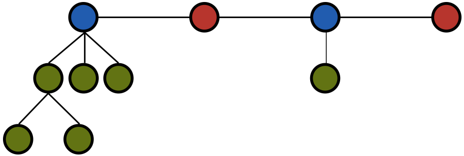

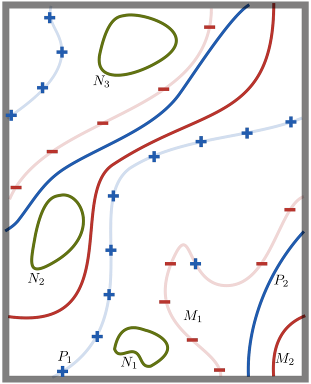

Neutral cluster combinatorics: exhaust all valid possibilities as we follow the tree.

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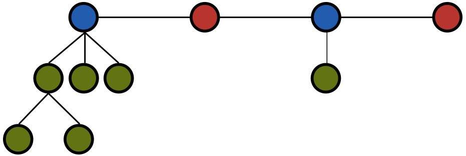

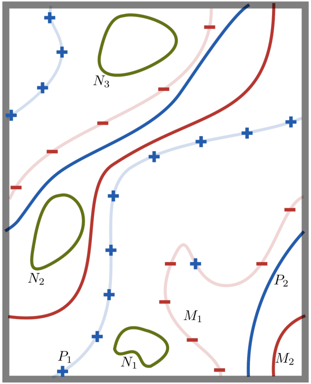

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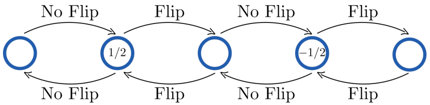
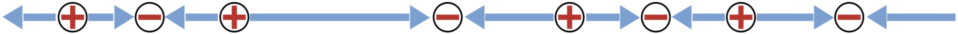


Charged cluster combinatorics: sample words from a regular language (recognized by a finite-state automaton).

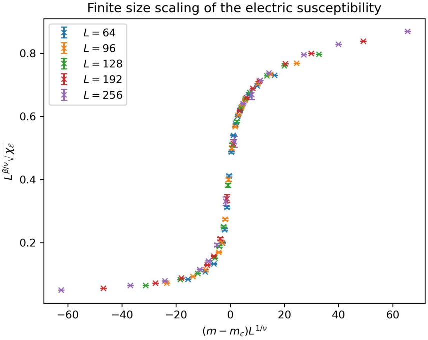
CP Symmetry Breaking on the Spin 1/2 Quantum Link Model

Small mass limit: Pair creation is common, **Symmetric phase**



Large mass limit: Pair creation is rare, CP**-broken phase**

$$\chi_\epsilon = \left\langle \left(\frac{1}{L} \sum_n \sigma_n^z \right)^2 \right\rangle$$



Non-universal critical mass:
$m_c \sim 0.24$

2d Ising critical exponents give curve collapse:
$\nu = 1,\ \beta = 1/8$

Conclusions
The Algorithm

Construction of a cluster algorithm that can update efficiently fermionic and gauge degrees of freedom satisfying Gauss' law.

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- Study the approach to infinite spin - Schwinger model.

- Can a similar algorithm be constructed in higher dimensions?
- Can we simulate other types of constrained systems (e.g. canonical ensemble, which has fixed particle number)?

Conclusions
Overview

Classical computers face fundamental challenges (e.g. real-time evolution).

Quantum simulators hold the promise of addressing these issues.

BUT

Improvement is not impossible.

In fact, it is crucial to **complement**, **guide**, and **validate** quantum simulators.