

**Form factors for semileptonic B-decays
with HISQ light quarks and
clover b-quarks in Fermilab interpretation**

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Fermilab Lattice and MILC collaborations

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Semileptonic B-decays

♠ $B \rightarrow \pi l^+ l^-, B \rightarrow K l^+ l^-$

- Flavor-changing neutral current (FCNC) interactions
- Leading standard model contributions are suppressed
 \Rightarrow New-physics effects may be larger.

- $\frac{d\Gamma}{dq^2} = (\text{known factors}) \times |V_{tb} V_{tf}^*|^2 \times \{|f_+(q^2)|^2, |f_0(q^2)|^2, |f_T(q^2)|^2\}$

♠ $B \rightarrow \pi l \nu, B_s \rightarrow K l \nu$

- Precise determination of $|V_{ub}|$

- $\frac{d\Gamma}{dq^2} = (\text{known factors}) \times |V_{ub}|^2 \times \{|f_+(q^2)|^2, |f_0(q^2)|^2\}$

Form factors

♠ For $B \rightarrow L$ decays ($B \in \{B, B_s\}, L \in \{\pi, K\}$),

$$f_{\parallel}(E_L) = \frac{\langle L | \mathcal{V}^0 | B \rangle}{\sqrt{2M_B}}, \quad (1)$$

$$f_{\perp}(E_L) = \frac{\langle L | \mathcal{V}^i | B \rangle}{\sqrt{2M_B}} \frac{1}{k^i}, \quad (2)$$

$$f_{\mathcal{T}}(E_L) = \frac{M_B + M_L}{\sqrt{2M_B}} \frac{\langle L | \mathcal{T}^{0i} | B \rangle}{\sqrt{2M_B}} \frac{1}{k^i}. \quad (3)$$

where E_L is the L -meson recoil energy, $M_{B(L)}$ are the $B(L)$ -meson mass, and k^i is the L -meson momentum in the B -meson rest frame.

♠ f_+ and f_0 are linear combinations of f_{\parallel} and f_{\perp} .

FNAL-HISQ campaign

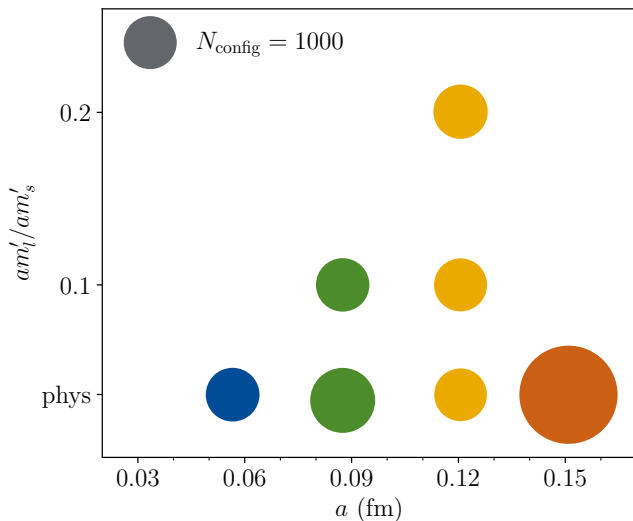
♠ $N_f = 2 + 1 + 1$ MILC HISQ gauge ensemble

: one-loop improved Lüscher-Weisz gluons
+ HISQ sea quarks

♠ Valence quarks: **HISQ light and strange**

+ **clover bottom in Fermilab interpretation**

Lattice setup



Data analysis overview

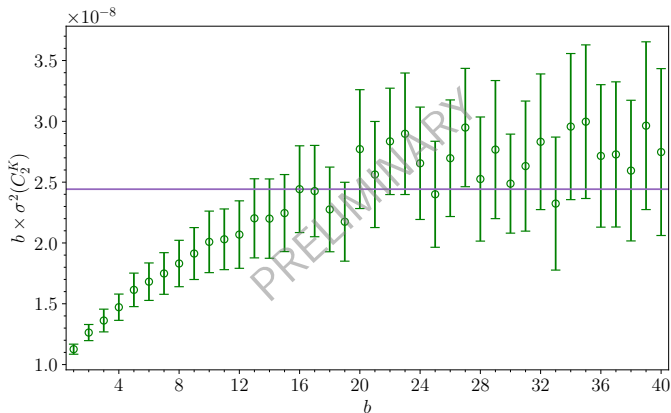
1. Binning: reduce the autocorrelation
2. Effective mass/amplitude: priors for 2pt function fitting
3. Fit 2pt function: ground state energies are used in computing ratio, excited states as priors
4. Compute the ratio

$$R(t, T) = \frac{C_3^{B \rightarrow L}(t, T)}{\sqrt{C_2^L(t) C_2^B(T-t)}} \sqrt{\frac{2E_L^{(0)}}{e^{-E_L^{(0)}t} e^{-E_B^{(0)}(T-t)}}$$

5. **Fit the ratio**: extract form factors
6. Chiral-continuum fit
7. Z-expansion

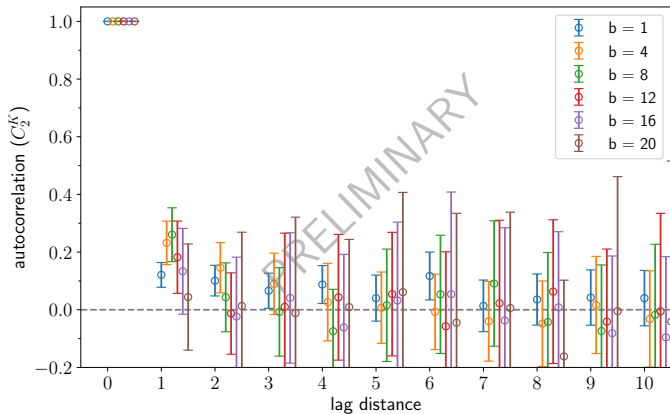
Binning

- ♠ Binning (blocking) with **bin size b** : $(\tilde{C}[b])_i = \frac{1}{b} \sum_{j=1}^b C_{(i-1)*b+j}$
- ♠ $\sigma^2(\tilde{C}[b]) = \frac{1}{b}\sigma^2(C) +$ (contribution from **autocovariances**)



Autocorrelation

♠ Binning reduces the autocorrelation



Effective mass and amplitude

♠ Effective mass and amplitude

$$aM_{\text{eff}}(t) = \cosh^{-1} \left(\frac{C_2(t+1) + C_2(t-1)}{2C_2(t)} \right) \quad (4)$$

$$A_{\text{eff}}(t) = \frac{C_2(t)}{e^{-M_{\text{eff}}t} + e^{-M_{\text{eff}}(Nt-t)}} \quad (5)$$

♠ To suppress oscillating state contributions,

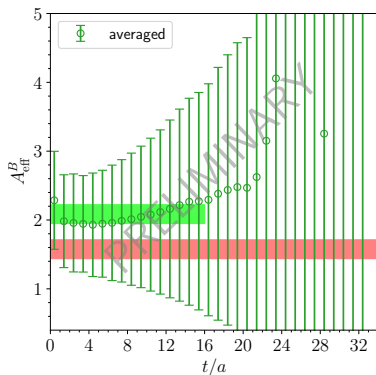
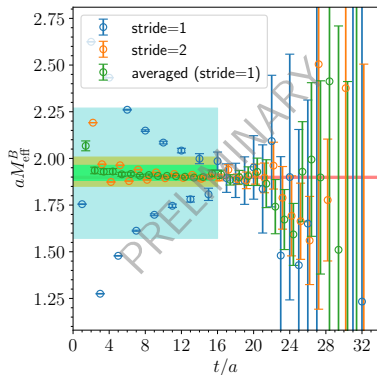
1. Stride-two method

$$aM_{\text{eff}}(t) = \frac{1}{2} \cosh^{-1} \left(\frac{C_2(t+2) + C_2(t-2)}{2C_2(t)} \right) \quad (6)$$

2. Averaging

$$\bar{C}_2(t) \simeq \frac{e^{-M_{\text{eff}}t}}{4} \left[\frac{C_2(t)}{e^{-M_{\text{eff}}t}} + \frac{2C_2(t+1)}{e^{-M_{\text{eff}}(t+1)}} + \frac{C_2(t+2)}{e^{-M_{\text{eff}}(t+2)}} \right] \quad (7)$$

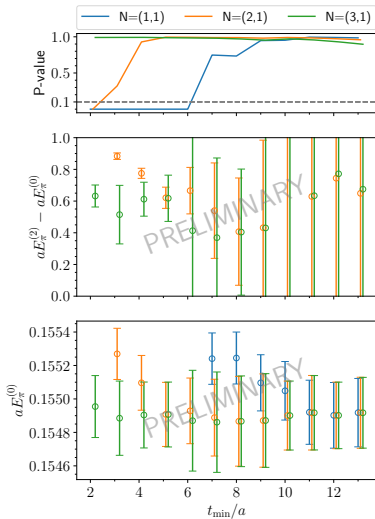
Effective mass and amplitude



- Used as priors with some extended widths in the 2pt function fitting.
- Red bands are fit posteriors for ground state.

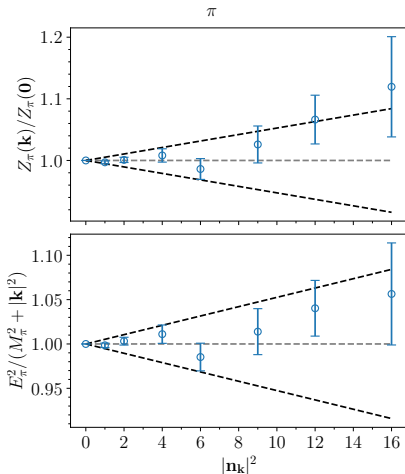
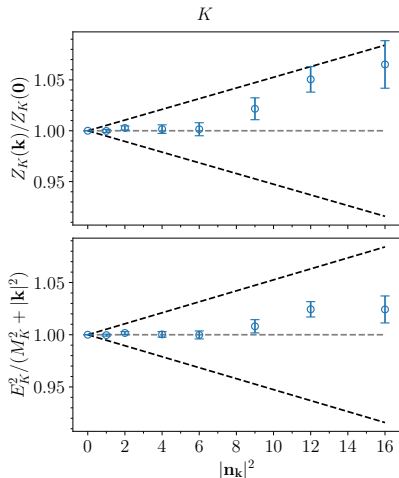
2pt function fitting

- $N = (N_{no}, N_o)$
: number of states
- (deaugmented) P-value:
computed by removing the
augmented term in χ_{aug}^2
- ♠ Find t_{min} giving consistent
energies
 - $N = (1, 1): t_{min}/a = 11$
 - $N = (2, 1): t_{min}/a = 5$
 - $N = (3, 1): t_{min}/a = 2$



Dispersion relation

♠ Replace non-zero momentum energies to dispersion relation.



Averaging

♠ Averaging suppresses oscillating state contributions

$$\bar{C}_2(t) \equiv \frac{e^{-E^{(0)}t}}{4} \left[\frac{C_2(t)}{e^{-E^{(0)}t}} + \frac{2C_2(t+1)}{e^{-E^{(0)}(t+1)}} + \frac{C_2(t+2)}{e^{-E^{(0)}(t+2)}} \right], \quad (8)$$

$$\begin{aligned} \bar{C}_{3,\mu}^{B \rightarrow L}(t, T) \equiv & \frac{e^{-E_L^{(0)}t} e^{-m_B^{(0)}(T-t)}}{8} \left[\frac{C_{3,\mu}^{B \rightarrow L}(t, T)}{e^{-E_L^{(0)}t} e^{-m_B^{(0)}(T-t)}} + \frac{C_{3,\mu}^{B \rightarrow L}(t, T+1)}{e^{-E_L^{(0)}t} e^{-m_B^{(0)}(T+1-t)}} \right. \\ & + \frac{2C_{3,\mu}^{B \rightarrow L}(t+1, T)}{e^{-E_L^{(0)}(t+1)} e^{-m_B^{(0)}(T-t-1)}} + \frac{2C_{3,\mu}^{B \rightarrow L}(t+1, T+1)}{e^{-E_L^{(0)}(t+1)} e^{-m_B^{(0)}(T-t)}} \\ & \left. + \frac{C_{3,\mu}^{B \rightarrow L}(t+2, T)}{e^{-E_L^{(0)}(t+2)} e^{-m_B^{(0)}(T-t-2)}} + \frac{C_{3,\mu}^{B \rightarrow L}(t+2, T+1)}{e^{-E_L^{(0)}(t+2)} e^{-m_B^{(0)}(T-t-1)}} \right]. \quad (9) \end{aligned}$$

Ratio

♠ With averaged correlators,

$$\begin{aligned} & \frac{\overline{C}_3^{B \rightarrow L}(t, T)}{\sqrt{\overline{C}_2^L(t) \overline{C}_2^B(T-t)}} \\ &= \frac{\sum_{m,n} (-1)^{m(t+1)} (-1)^{n(T-t-1)} \overline{A}_{mn} e^{-E_L^{(m)} t} e^{-E_B^{(n)} (T-t)}}{\sqrt{\left(\sum_m (-1)^{m(t+1)} \overline{Z}_m^L e^{-E_L^{(m)} t} \right) \left(\sum_n (-1)^{n(T-t-1)} \overline{Z}_n^B e^{-E_B^{(n)} (T-t)} \right)}} \end{aligned} \quad (10)$$

Ratio

$$\bar{R}(t, T) = \frac{\bar{C}_3^{B \rightarrow L}(t, T)}{\sqrt{\bar{C}_2^L(t) \bar{C}_2^B(T-t)}} \sqrt{\frac{2E_L^{(0)}}{e^{-E_L^{(0)}t} e^{-E_B^{(0)}(T-t)}}} \quad (11)$$

$$\simeq \frac{\bar{A}_{00} \sqrt{2E_L^{(0)}}}{\sqrt{\bar{Z}_0^L \bar{Z}_0^B}} \left[1 + \sum_m (-1)^{m(t+1)} \left(\frac{\bar{A}_{m0}}{\bar{A}_{00}} - \frac{1}{2} \frac{\bar{Z}_m^L}{\bar{Z}_0^L} \right) e^{-\delta E_L^{(m)}t} \right] \quad (12)$$

$$+ \sum_p (-1)^{p(T-t-1)} \left(\frac{\bar{A}_{0p}}{\bar{A}_{00}} - \frac{1}{2} \frac{\bar{Z}_p^B}{\bar{Z}_0^B} \right) e^{-\delta E_B^{(p)}(T-t)} \quad (13)$$

$$+ \sum_{m,p=1} (-1)^{m(t+1)} (-1)^{p(T-t-1)} \frac{\bar{A}_{mp}}{\bar{A}_{00}} e^{-\delta E_L^{(m)}t} e^{-\delta E_B^{(p)}(T-t)} \quad (14)$$

$$+ (\text{other cross terms}) \quad (15)$$

- We expect (12), (13) \gg (14), (15).

Fit model

♠ Splitting non-oscillating and oscillating states in Eq. (12), Eq. (13), we define our fit model:

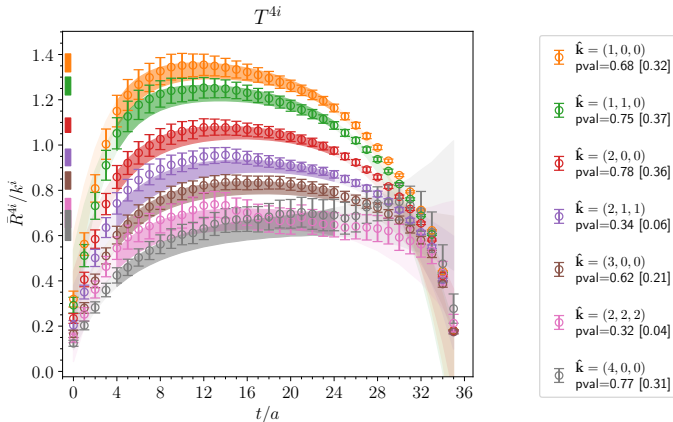
$$\begin{aligned} \bar{R}(t, T) \sim F_0 & \left[1 + \sum_{\mathbf{m}} F_{(\mathbf{m})}^L e^{-\delta E_L^{(\mathbf{m})} t} + \sum_{\mathbf{n}} (-1)^{t+1} F_{(\mathbf{n})}^L e^{-\delta E_L^{(\mathbf{n})} t} \right. \\ & \left. + \sum_{\mathbf{p}} F_{(\mathbf{p})}^B e^{-\delta E_B^{(\mathbf{p})} (T-t)} \sum_{\mathbf{q}} (-1)^{T-t-1} F_{(\mathbf{q})}^B e^{-\delta E_B^{(\mathbf{q})} (T-t)} \right] \end{aligned} \quad (16)$$

♠ $N = (\mathbf{m}, \mathbf{n})(\mathbf{p}, \mathbf{q})$

- $B \rightarrow K: (2, 0)(2, 1)$
- $B \rightarrow \pi: (2, 0)(2, 1)$
- $B_s \rightarrow K: (2, 1)(2, 1)$

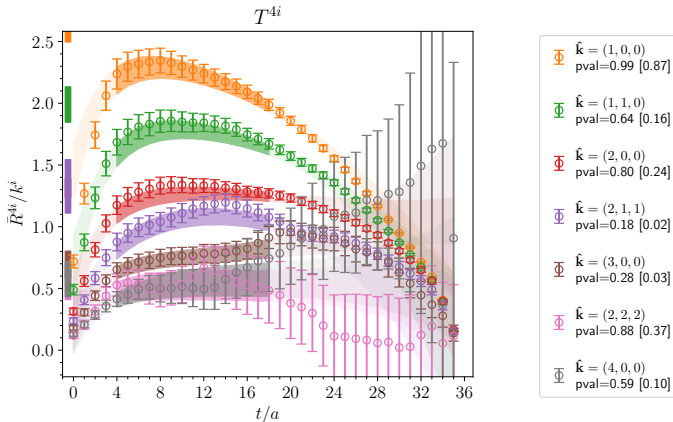
Ratio - preliminary results

- $a \simeq 0.06$ fm, $m'_l/m'_s = \text{physical}$
- $B \rightarrow K$



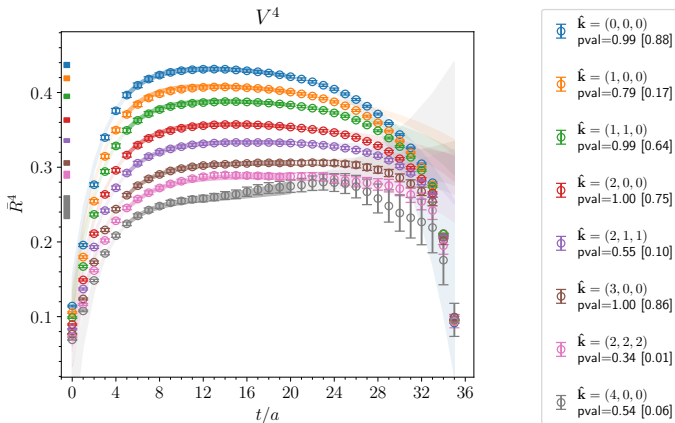
Ratio - preliminary results

- $a \simeq 0.06$ fm, $m'_l/m'_s = \text{physical}$
- $B \rightarrow \pi$



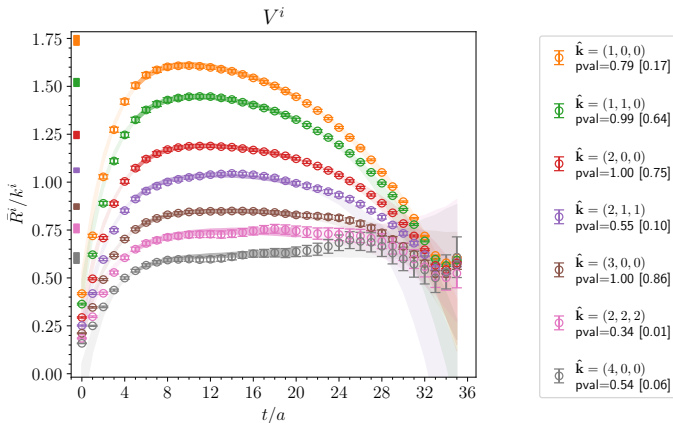
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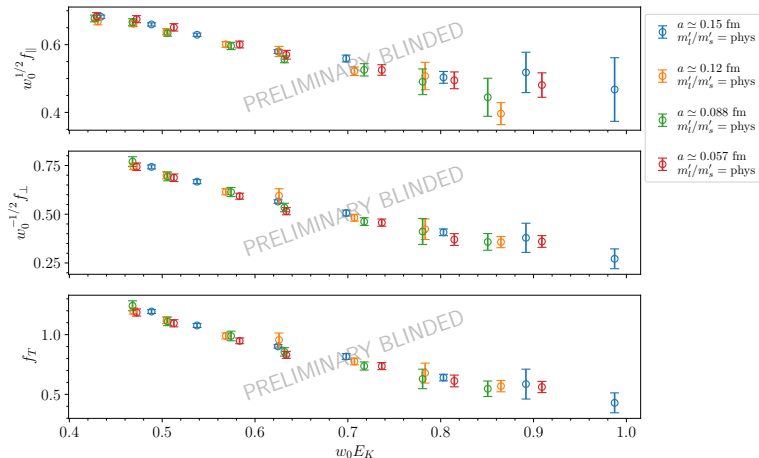
Ratio - preliminary results

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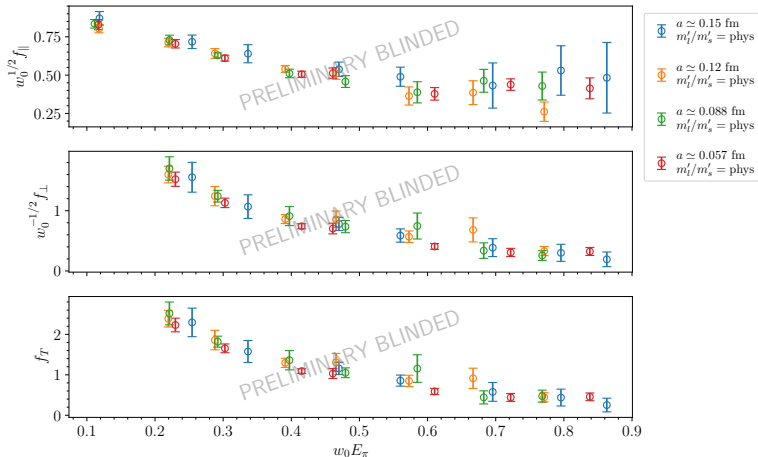
Form factors - preliminary results

- $B \rightarrow K$



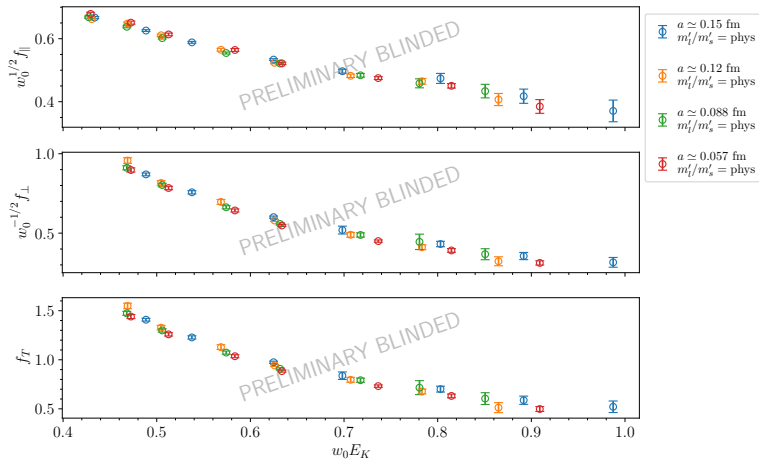
Form factors - preliminary results

- $B \rightarrow \pi$



Form factors - preliminary results

- $B_s \rightarrow K$



Plan

- Chiral-continuum fit
- Z-expansion
- Decay rates
- $|V_{ub}|/|V_{cb}|$

Thank you for your attention. :)