Form factors for semileptonic B-decays with HISQ light quarks and clover b-quarks in Fermilab interpretation

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Fermilab Lattice and MILC collaborations

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Semileptonic B-decays

♠ $B \to \pi\ell^+\ell^-, B \to K\ell^+\ell^-$

- Flavor-changing neutral current (FCNC) interations
- Leading standard model contributions are suppressed

  $\Rightarrow$ New-physics effects may be larger.
- $\frac{d\Gamma}{dq^2} = \text{(known factors)} \times |V_{tb}V_{tf}^*|^2 \times \{|\boldsymbol{f_+(q^2)}|^2, |\boldsymbol{f_0(q^2)}|^2, |\boldsymbol{f_T(q^2)}|^2\}$

♠ $B \to \pi\ell\nu, B_s \to K\ell\nu$

- Precise determination of $|V_{ub}|$
- $\frac{d\Gamma}{dq^2} = \text{(known factors)} \times |V_{ub}|^2 \times \{|\boldsymbol{f_+(q^2)}|^2, |\boldsymbol{f_0(q^2)}|^2\}$

Form factors

- For $B \to L$ decays $(B \in \{B, B_s\}, L \in \{\pi, K\})$,

$$f_{\parallel}(E_L) = \frac{\langle L|\mathcal{V}^0|B\rangle}{\sqrt{2M_B}}\,, \tag{1}$$

$$f_{\perp}(E_L) = \frac{\langle L|\mathcal{V}^i|B\rangle}{\sqrt{2M_B}}\frac{1}{k^i}\,, \tag{2}$$

$$f_T(E_L) = \frac{M_B + M_L}{\sqrt{2M_B}}\frac{\langle L|\mathcal{T}^{0i}|B\rangle}{\sqrt{2M_B}}\frac{1}{k^i}\,. \tag{3}$$

where E_L is the L-meson recoil energy, $M_{B(L)}$ are the $B(L)$-meson mass, and k^i is the L-meson momentum in the B-meson rest frame.

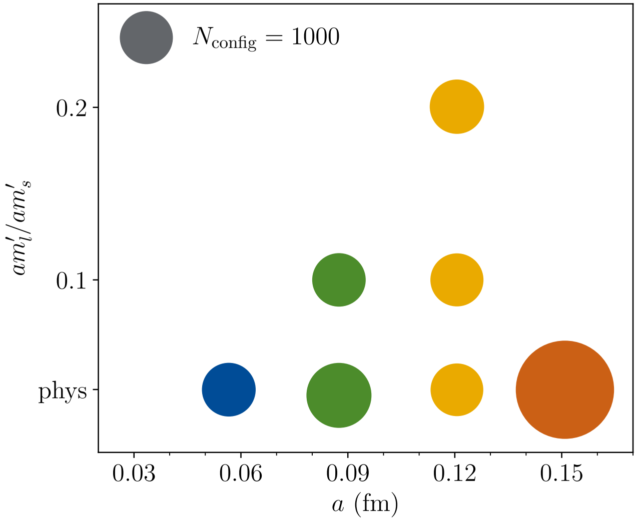
- f_+ and f_0 are linear combinations of $f_{\parallel}$ and $f_{\perp}$.

FNAL-HISQ campaign

- **$N_f = 2+1+1$ MILC HISQ gauge ensemble**
  : one-loop improved Lüscher-Weisz gluons
  + HISQ sea quarks

- Valence quarks: **HISQ light and strange**
  + **clover bottom in Fermilab interpretation**

Lattice setup



Data analysis overview

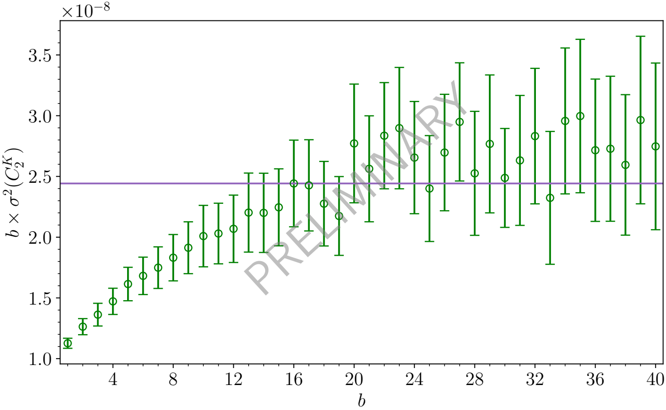
1. Binning: reduce the autocorrelation
2. Effective mass/amplitude: priors for 2pt function fitting
3. Fit 2pt function: ground state energies are used in computing ratio, excited states as priors
4. Compute the ratio

$$R(t,T) = \frac{C_3^{B\to L}(t,T)}{\sqrt{C_2^L(t) C_2^B(T-t)}} \sqrt{\frac{2E_L^{(0)}}{e^{-E_L^{(0)} t} e^{-E_B^{(0)}(T-t)}}}$$

5. **Fit the ratio**: extract form factors
6. Chiral-continuum fit
7. Z-expansion

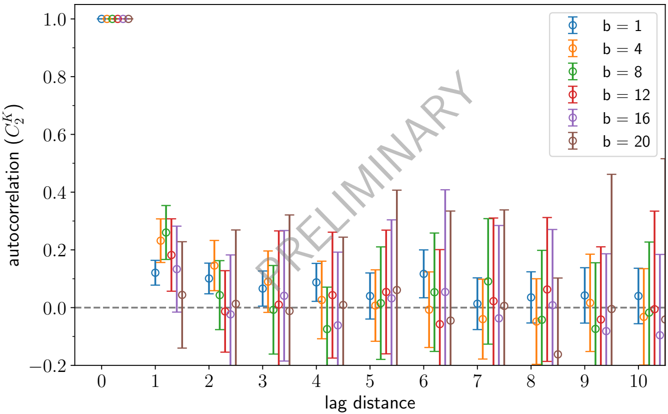
Binning

- Binning (blocking) with **bin size b**: $(\widetilde{C}[b])_i = \frac{1}{b}\sum_{j=1}^{b} C_{(i-1)*b+j}$
- $\sigma^2(\widetilde{C}[b]) = \frac{1}{b}\sigma^2(C) +$ (contribution from **autocovariances**)



Autocorrelation

- Binning reduces the autocorrelation



Effective mass and amplitude

- Effective mass and amplitude

$$aM_{\text{eff}}(t) = \cosh^{-1}\left(\frac{C_2(t+1)+C_2(t-1)}{2C_2(t)}\right) \tag{4}$$

$$A_{\text{eff}}(t) = \frac{C_2(t)}{e^{-M_{\text{eff}}t} + e^{-M_{\text{eff}}(N_t-t)}} \tag{5}$$

- To suppress oscillating state contributions,

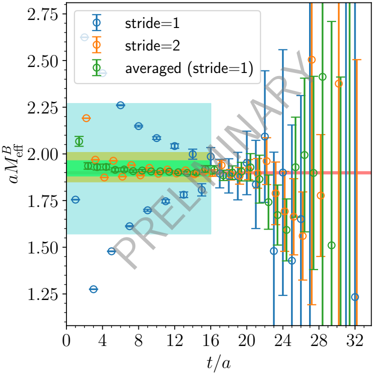
1. Stride-two method

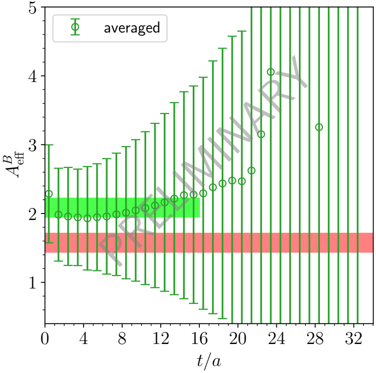
$$aM_{\text{eff}}(t) = \frac{1}{2}\cosh^{-1}\left(\frac{C_2(t+2)+C_2(t-2)}{2C_2(t)}\right) \tag{6}$$

2. Averaging

$$\overline{C}_2(t) \simeq \frac{e^{-M_{\text{eff}}t}}{4}\left[\frac{C_2(t)}{e^{-M_{\text{eff}}t}} + \frac{2C_2(t+1)}{e^{-M_{\text{eff}}(t+1)}} + \frac{C_2(t+2)}{e^{-M_{\text{eff}}(t+2)}}\right] \tag{7}$$

Effective mass and amplitude





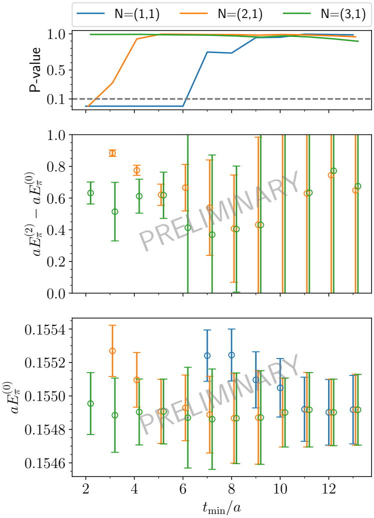
- Used as priors with some extended widths in the 2pt function fitting.
- Red bands are fit posteriors for ground state.

2pt function fitting

- $N = (N_{no}, N_o)$
  : number of states
- (deaugmented) P-value: computed by removing the augmented term in χ^2_{aug}

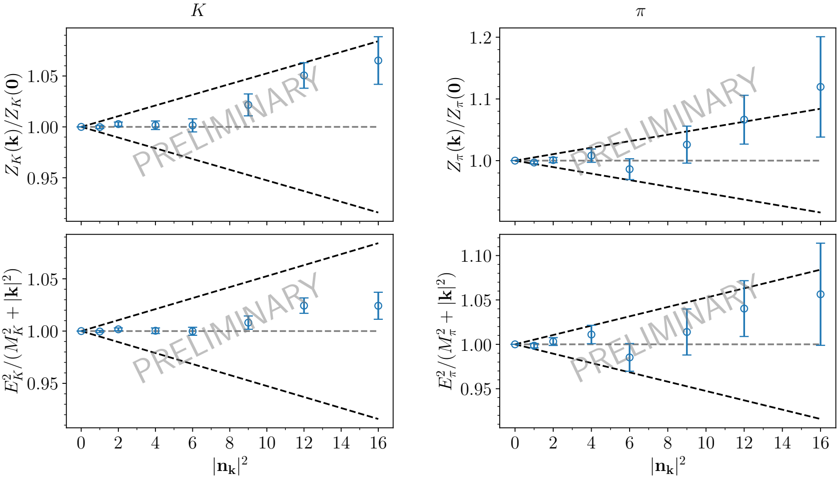
♠ Find t_{min} giving consistent energies

- $N = (1,1)$: $t_{\text{min}}/a = 11$
- $N = (2,1)$: $t_{\text{min}}/a = 5$
- $N = (3,1)$: $t_{\text{min}}/a = 2$



Dispersion relation

- Replace non-zero momentum energies to dispersion relation.



Averaging

- Averaging suppresses oscillating state contributions

$$\overline{C}_2(t) \equiv \frac{e^{-E^{(0)}t}}{4}\left[\frac{C_2(t)}{e^{-E^{(0)}t}} + \frac{2C_2(t+1)}{e^{-E^{(0)}(t+1)}} + \frac{C_2(t+2)}{e^{-E^{(0)}(t+2)}}\right], \tag{8}$$

$$\begin{aligned}\overline{C}^{B\to L}_{3,\mu}(t,T) \equiv \frac{e^{-E_L^{(0)}t}e^{-m_B^{(0)}(T-t)}}{8}\Bigg[&\frac{C^{B\to L}_{3,\mu}(t,T)}{e^{-E_L^{(0)}t}e^{-m_B^{(0)}(T-t)}} + \frac{C^{B\to L}_{3,\mu}(t,T+1)}{e^{-E_L^{(0)}t}e^{-m_B^{(0)}(T+1-t)}}\\ &+ \frac{2C^{B\to L}_{3,\mu}(t+1,T)}{e^{-E_L^{(0)}(t+1)}e^{-m_B^{(0)}(T-t-1)}} + \frac{2C^{B\to L}_{3,\mu}(t+1,T+1)}{e^{-E_L^{(0)}(t+1)}e^{-m_B^{(0)}(T-t)}}\\ &+ \frac{C^{B\to L}_{3,\mu}(t+2,T)}{e^{-E_L^{(0)}(t+2)}e^{-m_B^{(0)}(T-t-2)}} + \frac{C^{B\to L}_{3,\mu}(t+2,T+1)}{e^{-E_L^{(0)}(t+2)}e^{-m_B^{(0)}(T-t-1)}}\Bigg].\end{aligned} \tag{9}$$

Ratio

- With averaged correlators,

$$\frac{\overline{C}_3^{B\to L}(t,T)}{\sqrt{\overline{C}_2^L(t)\overline{C}_2^B(T-t)}} = \frac{\sum_{m,n}(-1)^{m(t+1)}(-1)^{n(T-t-1)}\overline{A}_{mn}\, e^{-E_L^{(m)}t}e^{-E_B^{(n)}(T-t)}}{\sqrt{\left(\sum_m(-1)^{m(t+1)}\overline{Z}_m^L e^{-E_L^{(m)}t}\right)\left(\sum_n(-1)^{n(T-t-1)}\overline{Z}_n^B e^{-E_B^{(n)}(T-t)}\right)}} \tag{10}$$

Ratio

$$\overline{R}(t,T) = \frac{\overline{C}_3^{B\to L}(t,T)}{\sqrt{\overline{C}_2^L(t)\overline{C}_2^B(T-t)}}\sqrt{\frac{2E_L^{(0)}}{e^{-E_L^{(0)}t}e^{-E_B^{(0)}(T-t)}}} \tag{11}$$

$$\simeq \frac{\overline{A}_{00}\sqrt{2E_L^{(0)}}}{\sqrt{\overline{Z}_0^L\overline{Z}_0^B}}\Bigg[1 + \sum_m (-1)^{m(t+1)}\left(\frac{\overline{A}_{m0}}{\overline{A}_{00}} - \frac{1}{2}\frac{\overline{Z}_m^L}{\overline{Z}_0^L}\right)e^{-\delta E_L^{(m)}t} \tag{12}$$

$$+ \sum_p (-1)^{p(T-t-1)}\left(\frac{\overline{A}_{0p}}{\overline{A}_{00}} - \frac{1}{2}\frac{\overline{Z}_p^B}{\overline{Z}_0^B}\right)e^{-\delta E_B^{(p)}(T-t)} \tag{13}$$

$$+ \sum_{m,p=1}(-1)^{m(t+1)}(-1)^{p(T-t-1)}\frac{\overline{A}_{mp}}{\overline{A}_{00}}e^{-\delta E_L^{(m)}t}e^{-\delta E_B^{(p)}(T-t)} \tag{14}$$

$$+ \text{(other cross terms)} \tag{15}$$

- We expect (12), (13) $\gg$ (14), (15).

Fit model

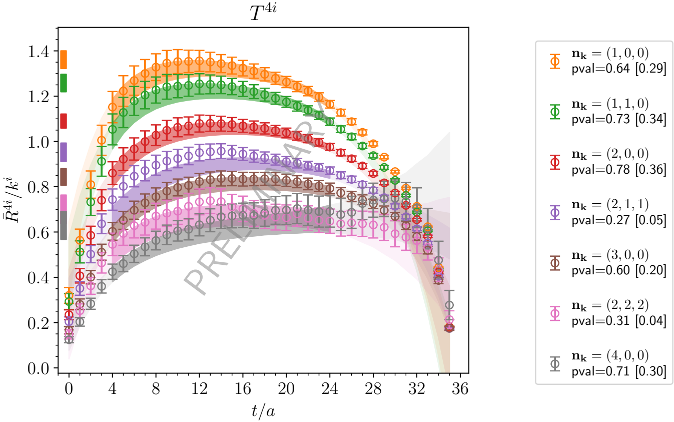
- Splitting non-oscillating and oscillating states in Eq. (12), Eq. (13), we define our fit model:

$$\overline{R}(t,T) \sim F_0\Big[1 + \sum_{m} F^L_{(m)} e^{-\delta E_L^{(m)} t} + \sum_{n} (-1)^{t+1} F^L_{(n)} e^{-\delta E_L^{(n)} t} + \sum_{p} F^B_{(p)} e^{-\delta E_B^{(p)} (T-t)} \sum_{q} (-1)^{T-t-1} F^B_{(q)} e^{-\delta E_B^{(q)} (T-t)}\Big] \quad (16)$$

- $N = (m, n)(p, q)$
  - $B \to K$: $(2,0)(2,1)$
  - $B \to \pi$: $(2,0)(2,1)$
  - $B_s \to K$: $(2,1)(2,1)$

Ratio - preliminary results

- $a \simeq 0.06$ fm, $m_l'/m_s' =$ physical
- $B \to K$



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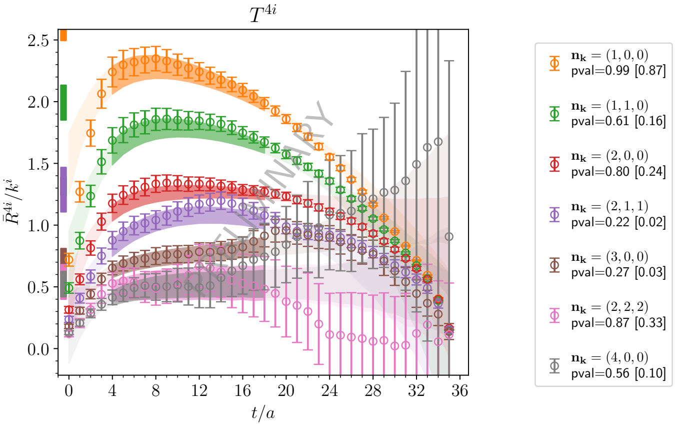

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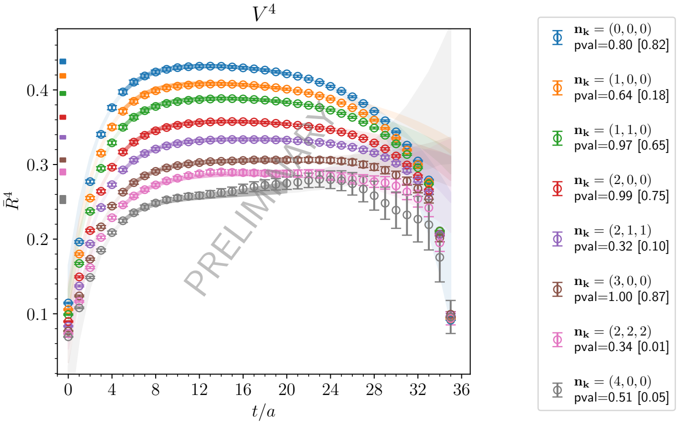

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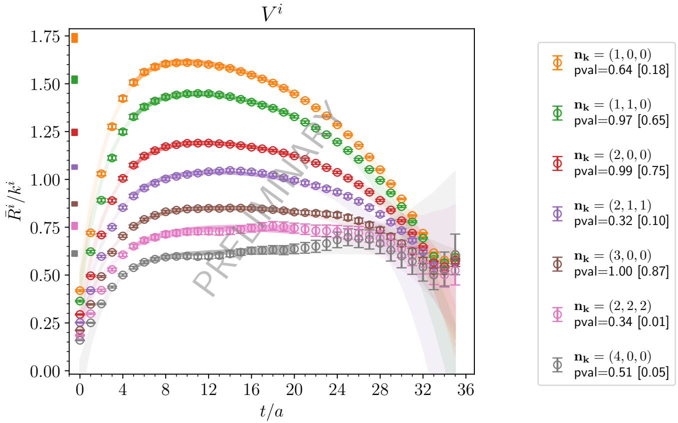
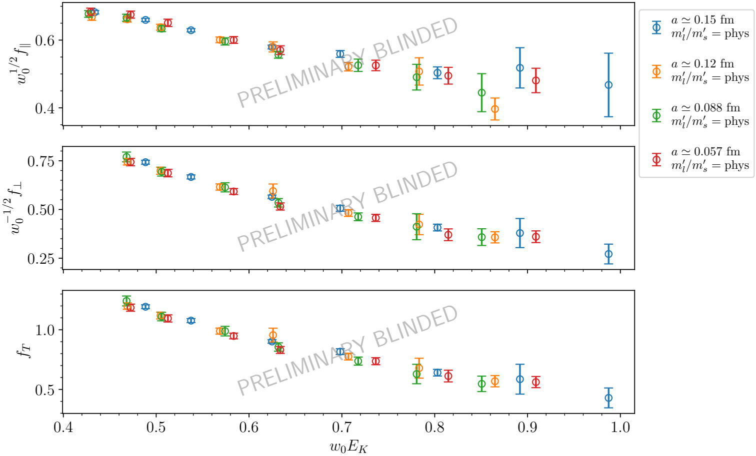


Form factors - preliminary results

- $B \to K$



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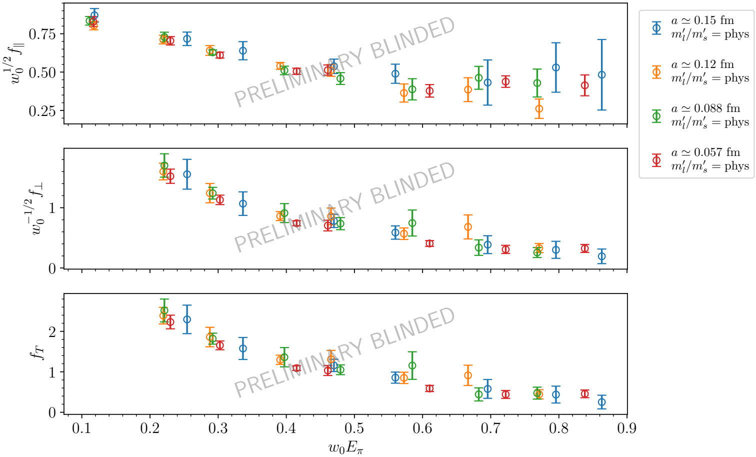
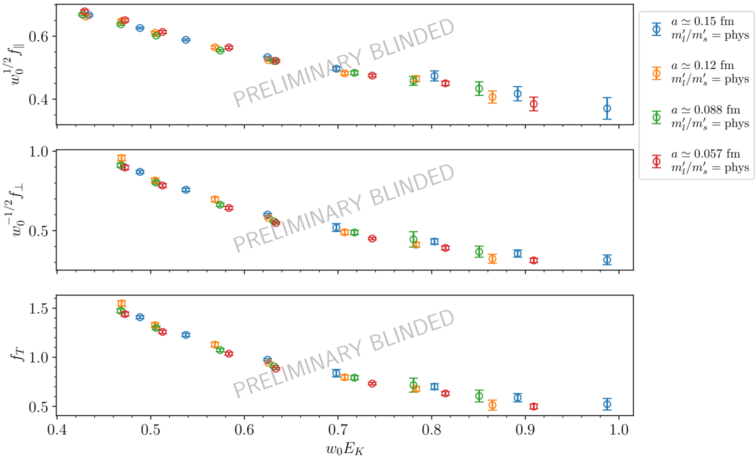


Form factors - preliminary results

- $B_s \to K$



Plan

- Chiral-continuum fit
- Z-expansion
- Decay rates
- $|V_{ub}|/|V_{cb}|$

Thank you for your attention. :)