Extracting Instantons from the Lattice

+ standing topological waves

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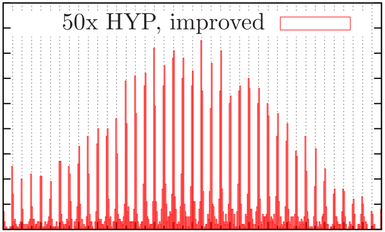
Setup: Ensemble & topological charge

Ensemble:

1. full QCD: twisted mass $n_f = 2 + 1 + 1$ ensemble
2. $L = 32, \ T = 64$
3. approximately 5000 configurations

pure gluonic definition of the topological charge density $q(x)$:

1. $q(x) = tr[F_{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x)]$
2. "Highly-improved lattice field-strength tensor" [hep-lat/0203008v1]
3. $\rightarrow \mathcal{O}(a^4)$-improved topological charge
4. Use 4D HYP-smearing to address different scales
5. works very well: no multiplicative renormalization needed



B55.32 the Q histogram aligns well with integer numbers (grid)

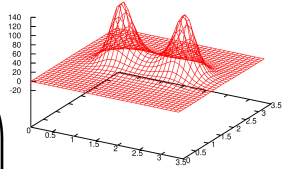
Instantons & Merons

Merons

- Localised pure gauge field excitation $\Rightarrow$ named pseudoparticle
- Classical solution of the Yang-Mills equations of motions
  $\Longleftrightarrow$ local minima of the action
  $\Longrightarrow$ high weight in PI
- Path integral saddlepoint approximation around minima leads to:

$$A_\mu^{Meron}(x) = \frac{x_\mu}{x^2+\rho^2}\Sigma_{\mu\nu}\,, \quad \Sigma = \frac{1}{2}\begin{pmatrix} 0 & +\sigma_3 & -\sigma_2 & -\sigma_1 \\ -\sigma_3 & 0 & +\sigma_1 & -\sigma_2 \\ +\sigma_2 & -\sigma_1 & 0 & -\sigma_3 \\ +\sigma_1 & +\sigma_2 & +\sigma_3 & 0 \end{pmatrix}$$

  with size parameter $\rho = 0$
- Topological charge $q = 1/2$
- Why merons? classical solution, long range tails $\rightarrow$ area law
- Instanton = 2 Merons

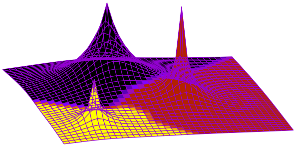


Action of Dimeron in an SU(2) toy model

Identification of topological sectors

Basic algorithm:

1. compute gradient field for $\gamma(x) = q(x)$ or $\gamma(x) = q(x)z(x)$ or $\gamma(x) = \mathrm{sign} q(x)z(x)$
2. identify extrema $>$ cut-off $=$ seed sectors
3. grow by an additional layer around each maximum as long the points lay down-hill
4. and do not belong to another sector
5. alternate between sector
6. stop: no new points



Sectors associated with 3 peaks

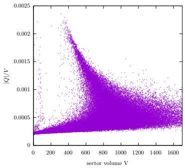
Overlapping sectors

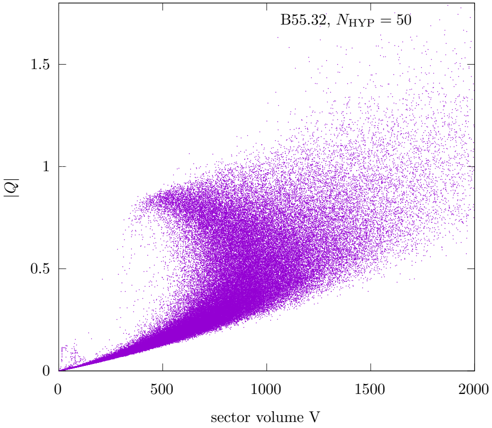
1. shared content
2. equally redistributed shared content

Sectors of the Shark fin

Ensemble:

- topological excitations have higher density
- and self-duality
- ...
- will be used as prototype to identify pseudoparticles





Filtering for the supposed Instantons

Dimensions

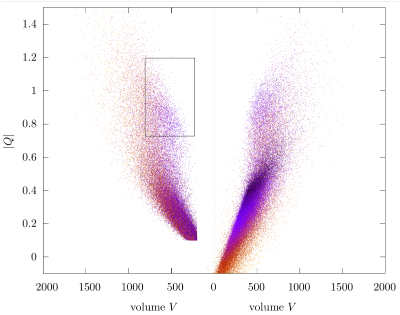
1. Basic quantities: Q, Z (self-duality), S & size
2. shared content per sector & basic quantity
3. redistributed content

Remove correlations

- $|Q|$ - all other quantities
- same with V
- high correlation between Q and size
- and between the other "technical dimensions"

Select data from Fin region

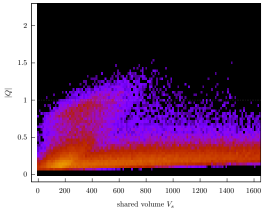
- square in $(V, |Q|)$
- $0.7 < q < 1.2$
- compute 2σ environment $\forall$ quantities but $V, |Q|$



supposed instanton region (LHS), tilting the sector cloud in $(V, |Q|, Z_{\mathrm{shared}})$ removes some of the $|Q|$ — V-correlation $\rightarrow$ higher density in that region (RHS)

Discard all sectors $\notin$ 2σ: non-linear deviations of the data cloud from supposed instanton sectors cannot be noise $\rightarrow$ remove physically different entities we are not interested in

Different approaches & the resulting clusters

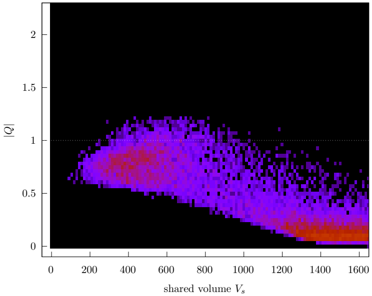


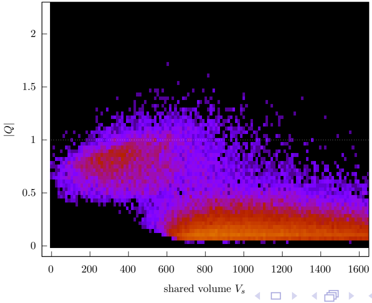
Instantons don't like to share...

Variants

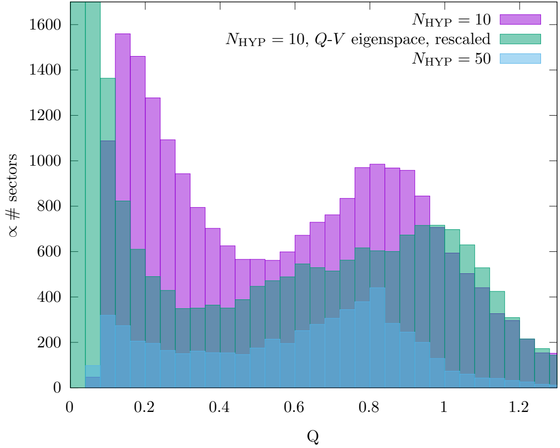
1. TR: $\gamma \equiv q(x) \cdot z(x)$
2. TL: $\gamma \equiv \mathrm{sign}(\mathrm{q}(\mathrm{x}))q(x) \cdot z(x)$
   with redistribution
3. BL: $\gamma \equiv \mathrm{sign}(\mathrm{q}(\mathrm{x}))q(x) \cdot z(x)$

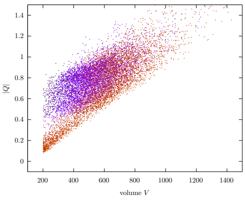
The shared volume might not be the physically most interesting dimension, but the instanton cluster separates quite well from the noise





Topological charge distribution of the sectors



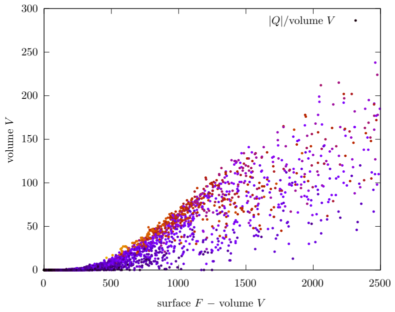


filtered fin, heat= Z_{shared} /w $|Q|$ — V-correlation

- The $Q \in [0.5, 1]$ is rather stable when varying the parameters (green,blue)
- Rotate in Q-V-plane to account for noise?
- Q needs to be rescaled - st. the Peak remains at $Q \approx 1$
- $\rightarrow$ possible Meron peak starts to appear (green)h

- Normalisation: $\#\text{sectors} \propto V^{-1}$ $\rightarrow$ divide by VS
- Peak close to $Q = 1$ - but there is some loss
- starts at $Q \approx 0.5$ $\rightarrow$ Meron+Instanton-Cluster?

End of Part I:



sectors are almost surface-like &
top excitations a little less...

Add-on Part: Oscillating AC & standing topological waves

Uncover AC osc.:

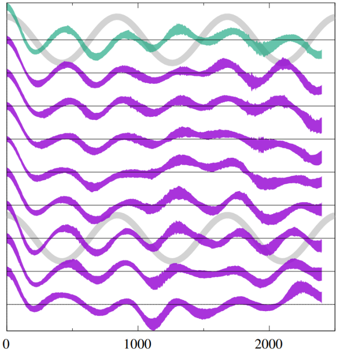
1. remove τ average and IR filter

$$c_\lambda(t,\tau) \equiv \sum_{\tau'=1}^{N}[c(t,\tau') - \langle c\rangle_\tau(t)]\frac{1}{\sqrt{2\pi\lambda^2}}e^{-\frac{(\tau-\tau')^2}{2\lambda^2}}$$

2. normalise each time slice, respectively

$$\bar{c}(t,\tau) \equiv \frac{c_\lambda(t,\tau) - \langle c_\lambda\rangle_\tau(t)}{\sigma_\tau[c](t)}$$

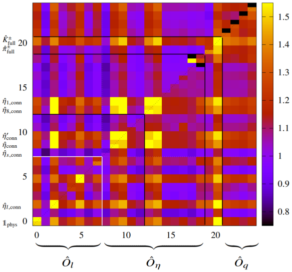
- std error computation doesn't apply → effective Nr measurements via entropy [PoS(LATTICE2022)045]



osc. in t-slices over config nr τ $< q(\tau_0)(\tau_0+\tau) >_{\tau_0}$

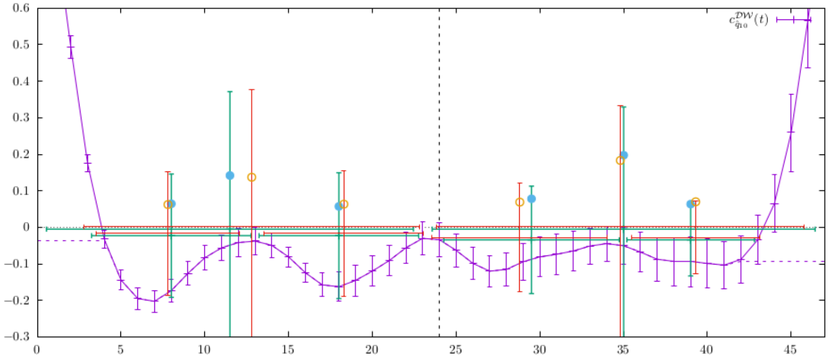
An effective number of independent DOF

$$N_{\text{eff}}[c] \equiv \frac{\mathcal{H}[p, Corr]}{\mathcal{H}[p, \mathbb{1}]} = \frac{\log\left(\sqrt{2\pi \det^2(Corr)}\right) + N_{\text{conf}}/2}{\log\left(\sqrt{(2\pi\sigma_f^2)^{N_{\text{conf}}}}\right) + N_{\text{conf}}/2}$$



Intensity of the AC-oscillation large for pseudoscalars singlets
[ML: Non-renormalizability of the HMC algorithm, arXiv:1103.1810v2]

The standing topological wave:

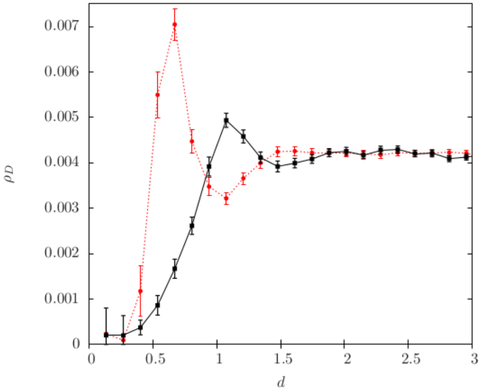


The crosses are estimations of the valley's surface (vertical bar) + the width of the valley (horizontal bar).

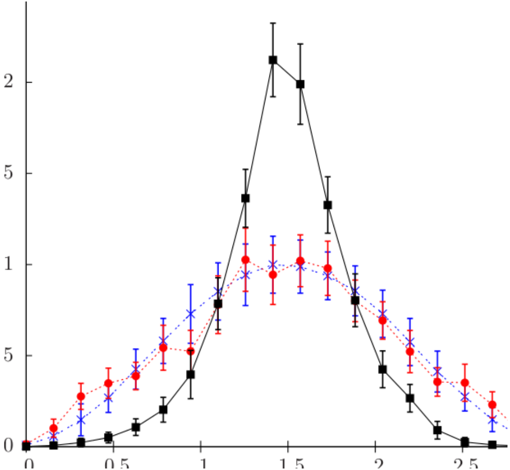
Dots on bars = error ⇒ evidence for wave

RHS: correlator that crossed the boundary of the lattice box.

Oscillations from SU(2) toy model (Dimerons)



Probability to find pseudoparticle wit the same charge (black) or opposite charge (red)



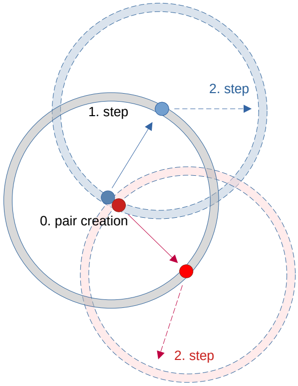
Colour orientation between neighbouring merons is non-random

HMC, a 5D-theory

The Hamiltonian used for Molecular Dynamics part of the HMC introduces canonical momenta P for the fields U and essentially defines a 5d-theory.
The momenta are defined by the choice of the kinetic term in H_{MD} and the Hamiltonian EOM.

$$H_{\mathrm{MD}} \equiv \frac{1}{2}\sum_{x,\mu} P^2_{x,\mu} + S_{\mathrm{G}}[U] + S_{\mathrm{PF}}[U,\phi,\phi^\dagger]$$

- AC oscillations originate from 5d-theory.
- theory includes QCD $\rightarrow$ potentially complex
- similar modes seen with Fourier Acceleration **[Sheta:2021hsd]**:
  - bco weak coupling limit & landau gauge fixing?
  - fields decouple & perform simple harmonic motions
  - showed oscillations for vec potentials A_μ
- not weakly coupled, here.



measure for pair annihilation after 2 updates
<< other configuration space

Randomization of momenta $P_{x,\mu} \propto e^{-P^2/2}$ at the beginning of each trajectory shall ensure ergodicity $\rightarrow$ identify bypass?

1. 2nd law of thermodynamics introduces arrow of time, nevertheless.
2. fixed trajectory length: system left in equivalent dynamical states \w lots of potential energy V & grad $V \neq 0$ @end of trajectory ...